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OF THE

AMERICAN SOCIETY

OF

CIVIL ENGINEERS

(INSTITUTED 1852)

VOLUME 90

JUNE, 1927

Edited by the Secretary, under the direction of the Committee on Meetings and Publications.

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NEW YORK
PUBLISHED BY THE SOCIETY

1927

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Paper No. 1598

MOMENTS IN RESTRAINED AND CONTINUOUS BEAMS BY THE METHOD OF CONJUGATE POINTS*

By L. H. Nishkian† and D. B. Steinman,‡ Members Am. Soc. C. E.

With Discussion by Messrs. F. E. Richart, Charles S. Whitney, Edward Godfrey, Felix H. Spitzer, C. A. P. Turner, Hardy Cross, George Higgins, E. G. Walker, Donald Derickson, W. M. Wilson, S. M. Cotten, A. T. Granger, John J. Gould, Richard G. Doerfling, R. McC. Beanfield, Walter Ruppel, Jules D. Gedo, and L. H. Nishkian and D. B. Steinman.

Synopsis

The object of this paper is to present a rapid and direct graphic solution of the relation expressed by the Theorem of Three Moments, and to indicate the application of this solution to a number of problems of interest to structural engineers.

The method is first developed for the simplest problem of continuous beams, assuming constant moment of inertia, symmetrical loads, and freely supported ends. It is then shown how the method can be extended to problems of restrained ends, unsymmetrical loading, and varying moment of inertia, so as to yield a simple graphic solution for practically any problem involving restrained or continuous beams.

It is shown how the graphic solution can be applied to the determination of bending moments, shears, and reactions for any given loading, and to the construction of influence lines for moving loads.

The application of the method is further extended to the graphic solution of rigid portals and other indeterminate frames; to problems involving the

* Published in October, 1925, *Proceedings*.

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settlement of supports; and to short-cut graphic determinations of secondary stresses in bridge trusses.

A number of concepts are introduced which can be applied to advantage in the solution of many other problems, and diagrams and tables are furnished to facilitate practical application.

Practical usefulness and clarity of presentation have been the governing considerations. The essentials of argument, method, and concept are presented in the body of the paper, and the necessary mathematical demonstrations are given in condensed form in an Appendix.

Introduction

The proper treatment of restrained and continuous beams is of constantly increasing importance in structural design. Before the general use of reinforced concrete, the application of the Theorem of Three Moments was required only in special structures. To-day, concrete buildings and bridges cannot be economically designed without a careful analysis of the moments in the continuous beams and slabs. The present methods of such analysis, however, are so involved and tedious that most building ordinances have established arbitrary coefficients for moments in a continuous beam. Needless waste of material naturally results.

The writers are hopeful that an easy method for obtaining moments in continuous beams will gradually eliminate the use of arbitrary coefficients, thus giving the engineer an opportunity to exercise his ingenuity and judgment toward obtaining maximum efficiency and economy. The use of prescribed coefficients makes the work mechanical and unscientific. Arbitrary coefficients for continuous beams as given in most building ordinances result in moments which may be 50% too small or 60% too large.

The availability of simple, graphic solutions should also promote the increased use of certain efficient forms of reinforced concrete construction which come under the classification of Indeterminate Frames. These structural forms have recently come into extensive use abroad in response to economic conditions which demanded maximum efficiency; but the development of the same class of structures in the United States appears to have been retarded by a prevailing impression that the required analysis is involved and tedious. The deterring influence of such an impression is generally under-estimated.

The methods presented in this paper should be helpful to the designer of steel structures as well as to the specialist in reinforced concrete. Of particular interest to the bridge engineer should be the proposal of methods for the expeditious determination of secondary stresses. The tediousness of the standard analytical methods has unquestionably been an obstacle to the more frequent investigation of these stresses in bridges. This paper offers some simple graphic methods that will quickly give the secondary stresses in any members of a truss, with sufficient accuracy for practical requirements. The availability of such methods removes the prevailing excuse for ignoring these important elements of structural stress, and should be conducive to improvements in specifications and in design.

Definitions and Symbols

The following notation and nomenclature are used throughout this paper, and are illustrated in Fig. 1.

R_1, R_2, R_3, ..., = supports in a continuous beam. The verticals through these points of support are called Reaction Lines, or R-lines.

l_1, l_2, l_3, ..., = span to right of R_1, R_2, R_3, ..., respectively. The vertical center lines of these respective spans are called C-lines. The one-third lines, or verticals through the one-third points of the respective spans, are called U and V-lines (U-line at left and V-line at right, respectively, in each span). By interchanging the one-third segments between the U and V-lines adjacent to each intermediate support, a new vertical is obtained near the R-line; this new vertical is called the Transposition Line, or T-line. (For unequal moments of inertia, see Fig. 15; for varying moments of inertia, see Fig. 22.)

A_1, A_2, A_3, ..., = simple-beam moment areas on the span, l_1, l_2, l_3, ..., respectively; that is A_1 is the area of the moment graph for the applied loading on the span, l_1, if l_1 were a simple beam, etc. Verticals through the centers of gravity of these respective moment areas are called G-lines. For symmetrical loading in any span, the G-line coincides with the C-line, as indicated in Fig. 1.

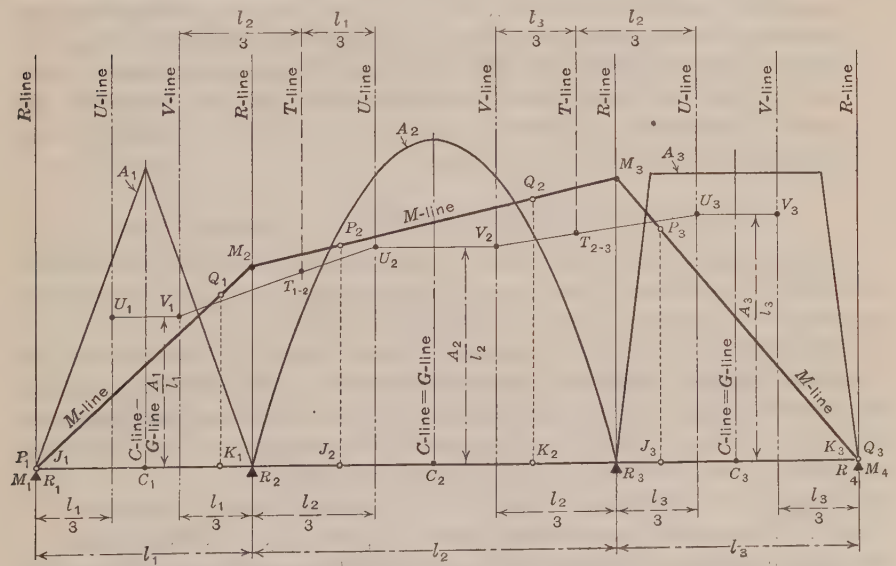


Fig. 1.—Notation Reference Diagram for the Method of Conjugate Points.

M_1, M_2, M_3, ..., = moments at supports, R_1, R_2, R_3, ..., respectively. If these moments are laid off (with reversed sign) on the respective R-lines and connected by a straight line in each span, the resulting line is the moment datum line and is called the Closing Line, or M-line. The ordinates of the simple-beam moment graphs (A_1, A_2, A_3, ...), measured from this datum line, are the resultant bending moments sought.

U_1, V_1; U_2, V_2; U_3, V_3; ...; = points on the one-third verticals, or U and V-lines in the respective spans. The heights of these points, for symmetrical loading, equal $\frac{A_1}{l_1}$, $\frac{A_2}{l_2}$, $\frac{A_3}{l_3}$, ..., respectively. (For unsymmetrical loading, see Fig. 14; for varying moments of inertia, see Fig. 22.)

$T_{1\text{-}2}$, $T_{2\text{-}3}$, ..., = points on the T-lines where these are intersected by the straight lines joining the adjacent U-V points (the lines, V_1-U_2, V_2-U_3, ..., respectively).

P_1, Q_1; P_2, Q_2; P_3, Q_3; ...; = points, two in each span, on the moment datum line or closing line of the continuous beam. These are called the Conjugate Points. The P-points are on the left and the Q-points are on the right in the respective spans. Consecutive P-points are mutually conjugate, each being obtainable from the preceding one by a simple graphic construction; similarly, consecutive Q-points are mutually conjugate. Commencing at the left end of the continuous beam, the P-points are successively located; commencing at the right end, the Q-points are successively located. Joining the P and Q-points in each span by a straight line extending to common intersections over the supports, the desired closing line (M-line) is obtained.

J_1, K_1; J_2, K_2; J_3, K_3; ...; = the projections on the base line, of the respective conjugate points, P_1, Q_1; P_2, Q_2; P_3, Q_3; .... The locations of these J and K-points are found to be independent of the loading, and therefore they are called the Fixed Points of the respective spans. There are two in each span, the J-point at the left and the K-point at the right. Under certain conditions in unloaded spans, the conjugate points, P, Q, coincide with the fixed points, J, K.

1.—The Theorem of Three Moments in Simplified Form

The Theorem of Three Moments, in its usual form, expresses the relation between the moments over any three consecutive supports of a continuous beam in terms of the lengths of span and the applied loading.

The starting point for the proposed graphic solution of continuous and restrained beams is a modified form of the Theorem of Three Moments, wherein the moment relation is expressed in terms of the simple-beam moment areas. The corresponding fundamental equation is simpler in form than the usual expressions, and more convenient for graphic solution.

Consider two consecutive spans of a continuous beam, as represented in Fig. 2. The simple-beam moment graphs, A_1 and A_2, are drawn above the base line in the respective spans. The moments at the supports, M_1, M_2, and M_3, due to continuity, are laid off on the respective reaction lines, and are then connected by straight lines in each span. These straight lines form a supplementary moment graph which must be added, algebraically, to the simple-beam moment graphs to give the resultant moments in the continuous spans. The total moment diagram in each span then consists of the sum of the M-trapezoid (which may be divided into two triangles as indicated) and the simple-beam moment graph, A_1 or A_2.

For the present, it is assumed that the loading is symmetrical in each span, and that the moment of inertia, I, of the beam is constant. (The modifications for unsymmetrical loads and varying I will be introduced subsequently.)

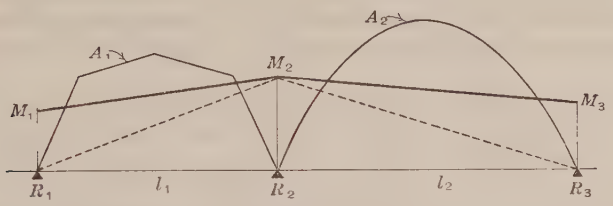


FIG. 2.—MOMENT AREAS IN TWO SPANS OF A CONTINUOUS BEAM.

A simple way of writing the Equation of Three Moments is to apply one of the established principles of the well-known Method of Moment Areas, namely: The angular deflections at the ends of any span are equal to the respective end reactions produced by loading that span with its moment area, divided by $E\,I$. At the common support (R_2, Fig. 2), the two spans must have equal but opposite angular deflections, because the beam is continuous. Consequently, the total moment areas in the two spans (considered as imaginary loadings) must produce equal but opposite reactions (or a zero combined reaction) at the intermediate support.

On applying this condition to the moment areas represented in Fig. 2, observing that the centers of gravity of the M-triangles are at the respective one-third points of the spans and that the simple-beam moment areas are centered on their spans, the following expression is obtained:

$$\frac{1}{E\,I}\left[\frac{1}{3}\left(\frac{1}{2}\,M_1\,l_1\right)+\frac{2}{3}\left(\frac{1}{2}\,M_2\,l_1\right)+\frac{1}{2}\,A_1\right]=-\frac{1}{E\,I}\left[\frac{1}{3}\left(\frac{1}{2}\,M_3\,l_2\right)+\frac{2}{3}\left(\frac{1}{2}\,M_2\,l_2\right)+\frac{1}{2}\,A_2\right]$$

This relation is re-written as follows:

$$(M_1+2\,M_2)\,l_1+(M_3+2\,M_2)\,l_2=-3\,(A_1+A_2)\ldots\ldots\ldots\ldots(1)$$

Equation (1) is a general expression for the Theorem of Three Moments for all types of loads such that the simple-beam moment areas center on their respective spans, for beams of constant moment of inertia.

2.—THE CONCEPT OF CONJUGATE POINTS

After the simple-beam moment graphs are drawn, it is required to locate the correct closing line, or M-line. The latter will serve as a datum line from which the resultant moment ordinates may be measured.

Evidently, the correct closing line must satisfy Equation (1). There are, however, an infinite number of lines that will satisfy this equation. Even if a point, P_1, is known as a point through which the correct closing line will pass, an infinite number of lines satisfying the equation can be drawn through P_1. Notwithstanding this difficulty, the following analysis leads to a solution of the problem.

In Fig. 3, let the two M-lines indicated represent any two lines passing through any given point, P_1, and satisfying Equation (1). Let P_2 be the second point of intersection of these two lines. From the geometry of Fig. 3 and the condition imposed by Equation (1), it is easy to establish the necessary relations between the co-ordinates, x_1, y_1, and x_2, y_2, of the two points. These relations are found to be expressed by Equations (2) and (3):

$$\left(3 - \frac{l_1}{x_1}\right) l_1 = \left(\frac{l_2}{x_2} - 3\right) l_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$\frac{l_1^2 y_1}{x_1} + \frac{l_2^2 y_2}{x_2} = -3 (A_1 + A_2) \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

(For the derivation of these equations, see the Appendix.)

Equation (2) fixes the abscissa relationship of the two points, P_1 and P_2. It should be noted that the abscissa relationship is independent of the applied loading and of the ordinates of the points. Equation (3) determines the ordinate relationship and expresses it in terms of the moment areas, A_1 and A_2.

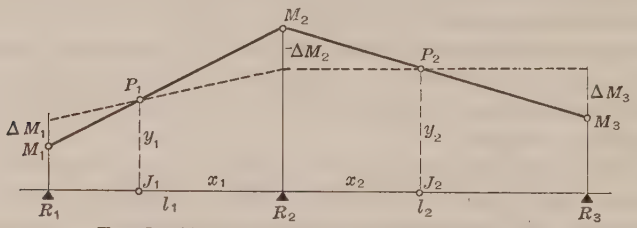


FIG. 3.—THE CONCEPT OF CONJUGATE POINTS.

If one of the points, P_1, is known, Equations (2) and (3) suffice to determine the location of the other point of intersection, P_2. As the two lines assumed were any two lines passing through P_1 and satisfying Equation (1), it follows that all lines passing through P_1 and satisfying Equation (1) will also pass through the point, P_2. Therefore, if the correct closing line passes through P_1, it will also pass through P_2. Accordingly, Equations (2) and (3) provide a means of determining successive points (in consecutive spans) through which the correct closing line will pass, after one such point is known.

Inspection of these governing equations (Equations (2) and (3)), shows that they are symmetrical and that the two points, P_1 and P_2, are interchangeable in each equation; hence the relations between the two points are reversible or mutual. As P_1 and P_2 are coupled by a definite, reciprocal relationship, they are appropriately named conjugate points.

3.—GRAPHIC CONSTRUCTION OF CONJUGATE POINTS

In order to arrive at a graphic solution of the problem, it is necessary to translate the governing equations (Equations (2) and (3)) into an equivalent geometrical construction.

The writers have devised a number of graphic constructions for the location of the conjugate points, and present the following as probably the simplest and most flexible solution.

To represent the influence of the applied loading, two points, U and V, are first marked in each span (see Fig. 1). These points are located on the one-third lines, at a height equal to $\frac{A}{l}$, that is,

$$U_1 = V_1 = \frac{A_1}{l_1}; \; U_2 = V_2 = \frac{A_2}{l_2}; \text{ etc} \ldots\ldots\ldots\ldots\ldots (4)$$

By interchanging the one-third span segments between the U and V-lines adjacent to each intermediate support, a new vertical is obtained near the reaction line; this new vertical is called the transposition line, or T-line (Fig. 1). (If two consecutive spans are equal, their T-line obviously coincides with the R-line.)

Where the T-line is intersected by the straight line joining the adjacent U-V-points, a new point is obtained which is called the T-point. Thus (Fig. 1), the line, V_1-U_2, locates the point, T_{1-2}, the line, V_2-U_3, locates the point, T_{2-3}, etc. The T-point is a significant point and represents, graphically, the combined influence of the loadings applied on the two adjoining spans. From the geometry of the construction, as indicated in Fig. 4, it is easily shown that the height of the T-point represents the important function:

$$T_{1-2} = \frac{A_1 + A_2}{l_1 + l_2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

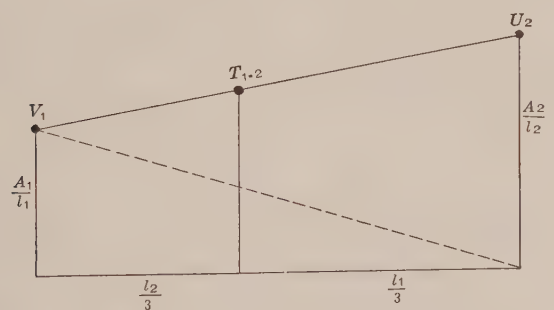


FIG. 4.—LOCATION OF THE T-POINT.

After the T-points are thus located, the construction proceeds, as follows: Let P_1 (Fig. 5) be a known point; it is required to find its conjugate point, P_2. Through P_1 and T_{1-2} draw a "pennant diagram", $P_1\ H_1\ B\ H_2$ (Fig. 5), as follows: From P_1 draw a line in any direction, intersecting the $\frac{1}{3}$-line (V-line) at some point, H_1, and the reaction-line (R-line) at some point, B; from H_1 draw a line through T_{1-2}, intersecting the $\frac{1}{3}$-line (U-line) of the next span in some point, H_2; on the line joining B with H_2, the required point, P_2, will lie. It should be noted that the pennant diagram has its apices on the reaction line and the one-third lines, and its base passes through the T-point.

The foregoing is a general graphic construction representing the relations expressed by Equations (2) and (3). (For proof, see the Appendix.)

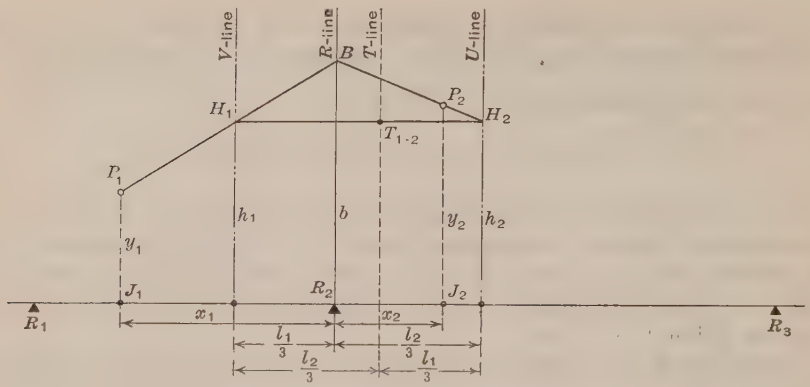


FIG. 5.—GENERAL CONSTRUCTION OF CONJUGATE POINTS.

From the proposition that this construction satisfies Equations (2) and (3), the following useful corollaries are at once deduced.

Corollary 1.—As the initial line, $P_1 H_1$ (Fig. 5), of the pennant diagram, may be drawn in any direction, a second pennant diagram starting from P_1 will intersect the first pennant diagram in the required point, P_2. (See Fig. 6.)

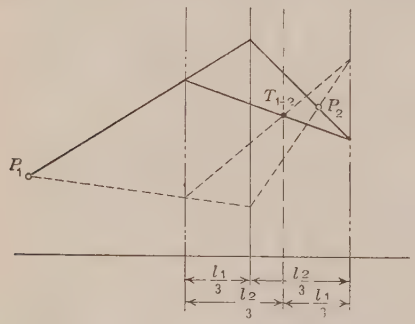


FIG. 6.

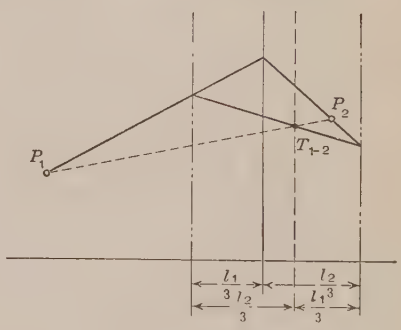


FIG. 7.

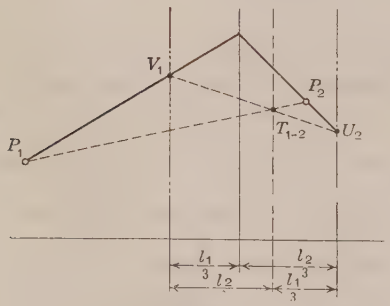


FIG. 8.

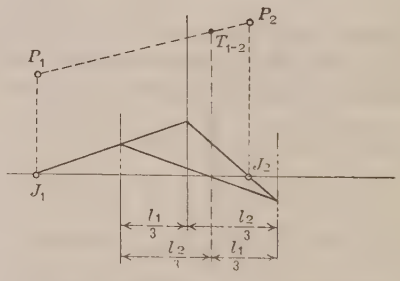


FIG. 9.

FOUR VARIATIONS OF THE GENERAL CONSTRUCTION FOR CONJUGATE POINTS.

Corollary 2.—If the initial line from P_1 is taken through T_{1-2}, the pennant diagram (Fig. 5) reduces to a straight line. Hence, P_1-T_{1-2}-P_2 is a straight line. (See Fig. 7.)

Corollary 3.—To save lines, the initial line from P_1 may be drawn through V_1, utilizing V_1-U_2 as the base of the pennant diagram. (See Fig. 8.)

Corollary 4.—As the pennant-diagram construction (Fig. 4) satisfies Equation (2) (which defines the abscissa relationship), it can be used for locating the fixed points at the feet of the ordinates of the conjugate points. (See Fig. 9.)

These four corollaries yield four respective constructions for locating conjugate points. These four variations are shown in Figs. 6, 7, 8, and 9, respectively.

The construction indicated in Fig. 8 is apparently the simplest, requiring the fewest new lines. It will generally be used as the preferred method throughout the remainder of this paper.

The construction indicated in Fig. 9 offers advantages when more than one loading condition has to be considered, as in the construction of influence lines.

Some alternative constructions, which may be of interest to the reader, are indicated in Fig. 10. Instead of the $\frac{1}{3}$-lines and their transposition point, T_{1-2}, the $\frac{1}{2}$-lines (or C-lines) are used with a corresponding transposition point, S_{1-2}. The constructions of Figs. 6, 7, 8, and 9, however, appear to be simpler and of greater general usefulness.

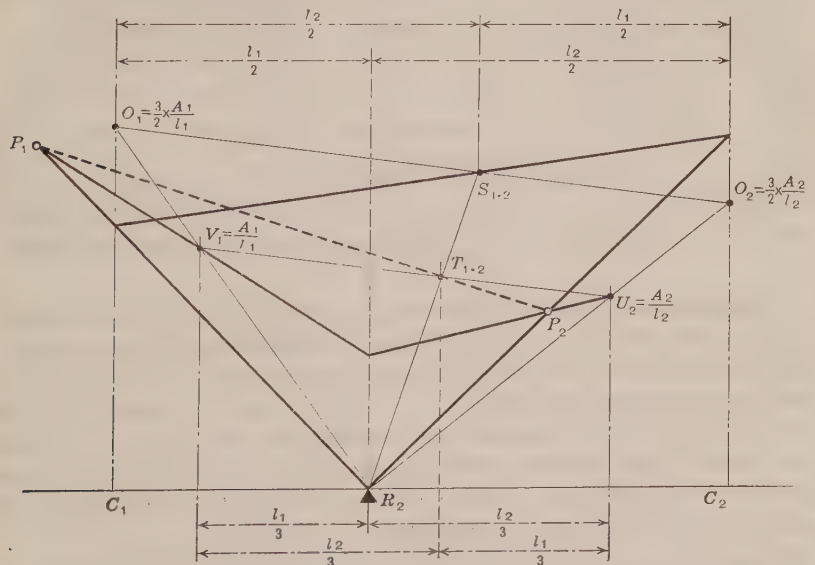


FIG. 10.—SOME ALTERNATIVE CONSTRUCTIONS FOR CONJUGATE POINTS.

4.—APPLICATION OF THE METHOD OF CONJUGATE POINTS

The foregoing may now be applied to a continuous beam of any number of spans and with any loading such that the simple-beam moment area on

each span centers on that span. The case with freely supported ends will be taken first. The procedure will be, as follows (see Fig. 1):

1.—Represent the continuous beam to scale and draw on each span the moment graph on the assumption that the individual spans are simple beams. It is now necessary to locate the true datum line (or M-line) for the continuous beam.

2.—For freely supported ends the M-line must pass through the extreme left and right supports. Commencing with the left support as the first known point, P_1, through which the M-line must pass, the conjugate point, P_2, in the second span can be found by the method of Fig. 8 (or one of its variations). In the same manner, a point, P_3, in the third span can be found conjugate to the point, P_2, in the second span. This will be repeated until the end span is reached.

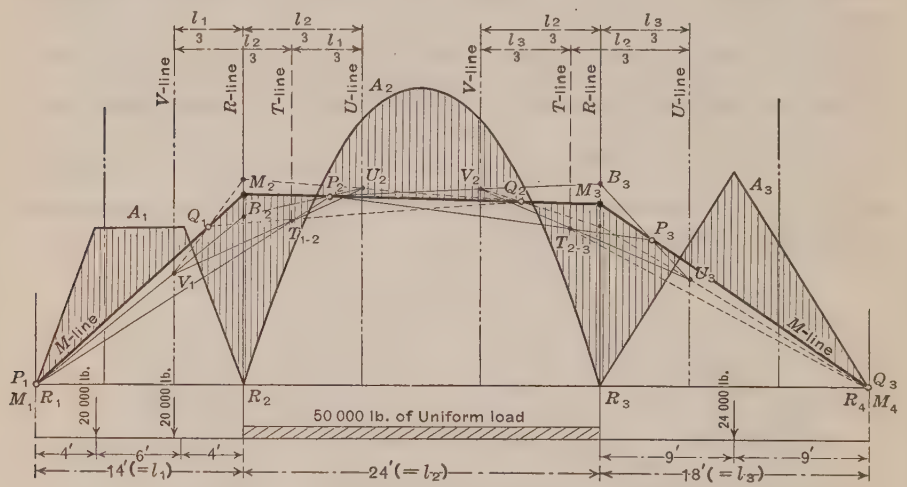


FIG. 11.—THE METHOD OF CONJUGATE POINTS APPLIED TO A CONTINUOUS BEAM WITH ENDS FREELY SUPPORTED.

3.—Beginning at the extreme right support and working to the left, another set of conjugate points, ..., Q_3, Q_2, Q_1, can be located in similar manner.

4.—There are now two points, P and Q, in each span, through which the M-line, or datum line, must pass. If each pair of points, P_1 and Q_1, P_2 and Q_2, P_3 and Q_3, ..., are connected by a straight line, and this line is extended in each span to the reaction verticals, the broken line thus formed is the required M-line, or true datum line. Common intercepts on the intermediate support verticals will check the correctness of the work.

The application to an actual problem is given in Fig. 11, which is constructed in accordance with the foregoing principles, the detailed procedure being, as follows:

(*a*) Draw the ordinary moment graphs, A_1, A_2, A_3, in the respective spans, assuming the latter to be simple beams.

(*b*) On the $\frac{1}{3}$-lines of each span locate the U-V points, such that:

$$U_1 = V_1 = \frac{A_1}{l_1};\ U_2 = V_2 = \frac{A_2}{l_2};\ U_3 = V_3 = \frac{A_3}{l_3}$$

(*c*) Locate the T-line near each intermediate reaction vertical by transposing the adjoining one-third span segments.

(*d*) Connect V_1-U_2 and V_2-U_3. Where these connecting lines cut the T-lines, mark the respective T-points, T_{1-2} and T_{2-3}.

(*e*) Draw P_1-V_1 intersecting the R_2-vertical at B_2. Draw B_2-U_2. A line from P_1 through T_{1-2} intersects B_2-U_2 in the conjugate point, P_2.

(*f*) Draw P_2-V_2 intersecting the R_3-vertical at B_3. Draw B_3-U_3. A line from P_2 through T_{2-3} intersects B_3-U_3 in the conjugate point, P_3.

(*g*) The conjugate points, Q_2 and Q_1, may be located in similar manner by starting at the other end, Q_3, and working toward the left.

(*h*) Connect P_1-Q_1, P_2-Q_2, and P_3-Q_3. If these lines have common intercepts on the R and R-verticals, the correctness of the work is checked.

(*i*) The broken line last drawn is the required closing line, or M-line. Vertical intercepts between this datum line and the previously drawn simple-beam moment graphs represent the bending moments at the respective points of the beam. These intercepts are indicated by the areas shaded in Fig. 11. Ordinates above the closing line represent positive bending moments, and ordinates below the closing line represent negative bending moments.

It should be noted that the closing line represents the moments of continuity with reversed sign. This reversal of the M-line about the base axis is for convenience, permitting direct graphic subtraction of the moments to replace the less convenient addition that would otherwise be required.

5.—Application to Beams with Restrained Ends

If the end of a beam is restrained, or "fixed", instead of freely supported, or "free", the construction illustrated in Fig. 11 requires modification because the M-line will no longer pass through the end of the beam.

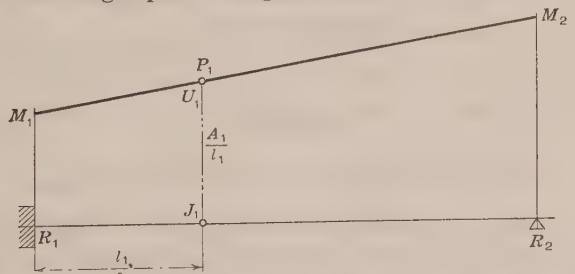


Fig. 12.—Construction for Beam with Restrained End.

A restrained end is equivalent to continuity with an imaginary adjoining span of zero length, $l_0 = 0$, and zero end moment, $M_0 = 0$. For such condition, the Theorem of Three Moments, Equation (1), reduces to:

$$(M_2 + 2\,M_1)\,l_1 = -3\,A_1 \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

The datum line (M-line) must satisfy this equation.

It is easily shown, by the geometry of Fig. 12, that any line satisfying Equation (6) will pass through the point, U_1, on the one-third vertical near the fixed end. Consequently, the true M-line must pass through U_1, and this point, therefore, may be used as the initial conjugate point, P_1, for the construction of the datum line.

Hence for any continuous beam with fixed ends the same procedure may be followed as that given for free ends, except that the initial point must be taken at U_1 instead of R_1. Similarly, if the extreme right end of the beam is fixed, the last V-point must be used instead of the last R-point as an initial conjugate point.

Fig. 13 shows the solution of the problem illustrated in Fig. 11 on the assumption that the ends are fixed. The procedure is identical with that followed in the construction of Fig. 11, except that the initial conjugate points, P_1 and Q_3, are taken at the extreme U-V points (U_1 and V_3, respectively), instead of at the extreme R-points (R_1 and R_4, respectively). The resultant bending moments are represented by the vertical intercepts of the shaded areas in Fig. 13.

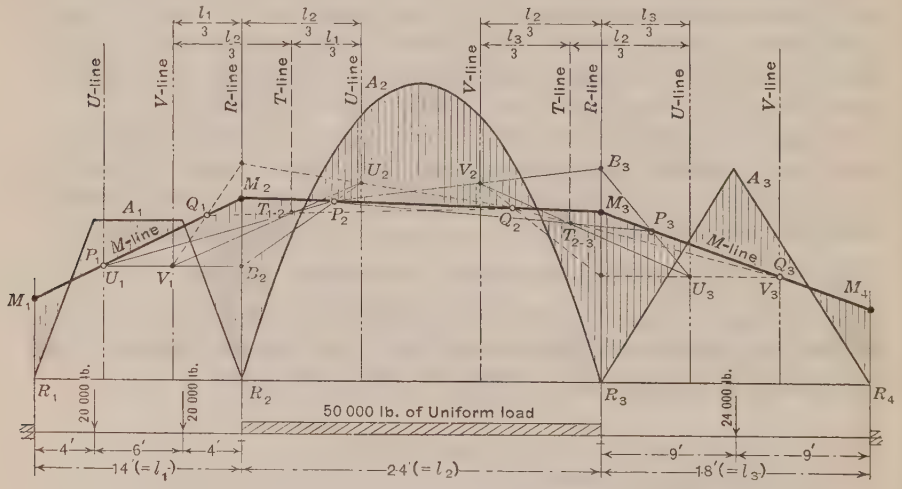


FIG. 13.—METHOD OF CONJUGATE POINTS APPLIED TO A CONTINUOUS BEAM WITH ENDS RESTRAINED.

6.—MODIFICATION FOR UNSYMMETRICAL LOADING

In the foregoing, it has been assumed that the simple-beam moment area in each span centered on that span. The modification necessary for unsymmetrical loads will now be given.

If the simple-beam moment area in any span does not center on the span, as will generally be true for unsymmetrical loading, the G-line, or centroid of the moment area, will be at some distance, $g\,l$, instead of $\frac{1}{2}\,l$, from the end of the span. (See Fig. 14.)

A reference to the method of establishing the simplified Equation of Three Moments (as given in Section 1) will show that the only modification necessary for unsymmetrical loading is to change the coefficient of A from $\frac{1}{2}$ to g in the equation preceding Equation (1). The three-moment relation then becomes:

$$(M_1 + 2\,M_2)\,l_1 + (M_3 + 2\,M_2)\,l_2 = -6\,(g_1\,A_1 + g_2\,A_2) \ldots\ldots\ldots (7)$$

Equation (7) is the simplified Theorem of Three Moments generalized to include the case of unsymmetrical loading.

Only slight modification of the graphic solution is necessary to satisfy this modification of the fundamental equation. The only change necessitated in the construction is a revised height of the U-V points, and is shown in Fig. 14. Mark a point, O, at the height, $\frac{A}{l}$, on the center line of the span; two lines drawn through O from the ends of the span will intercept the G-line at the required heights of U and V. The remainder of the graphic solution is exactly as given heretofore.

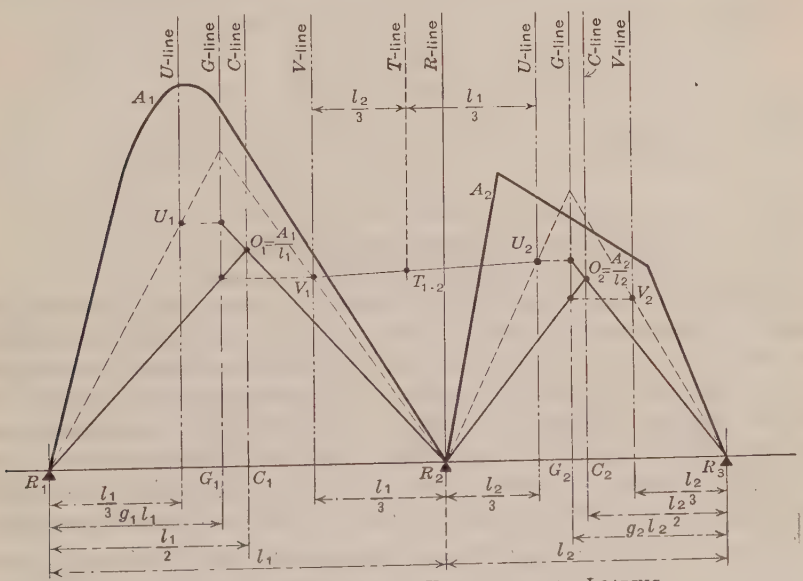


FIG. 14.—CONSTRUCTION FOR UNSYMMETRICAL LOADING.

The logic of this construction is almost obvious from a comparison of Equation (7) with Equation (1); but a detailed proof is given in the Appendix.

As a check on the correctness of the construction, it may be noted (Fig. 14) that the two rays, R-U and R-V, drawn from the ends of the span, must meet on the G-line.

7.—MODIFICATION FOR UNEQUAL MOMENTS OF INERTIA

Thus far it has been assumed that the continuous beam has the same moment of inertia in all the spans. If successive spans have differing values of I, a slight modification becomes necessary in the graphic construction.

For I constant in each span, but not the same value in successive spans, the simplified Equation of Three Moments, Equation (1), takes the form:

$$(M_1 + 2\,M_2)\,\frac{l_1}{I_1} + (M_3 + 2\,M_2)\,\frac{l_2}{I_2} = -\,3\left(\frac{A_1}{I_1} + \frac{A_2}{I_2}\right) \quad \ldots\ldots\ldots (8)$$

in which, I_1 and I_2 are the moments of inertia in the respective spans. It may be noted that Equation (8) differs from Equation (1) only in the retention of the appropriate value of I as a denominator in each term; if $I_1 = I_2$, Equation (8) obviously reduces to the simpler form of Equation (1).

This amplification of the fundamental equation is satisfied by a slight modification in the graphic solution. The only change necessary in the construction is an altered location of the T-line; the location of the "modified T-line" is shown in Fig. 15.

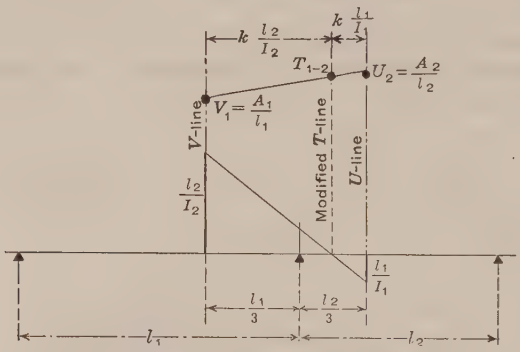


Fig. 15.—Construction for Unequal Moments of Inertia.

Heretofore, assuming unchanging moment of inertia, the T-line has been located by transposing the adjoining one-third span segments; this operation was equivalent to dividing the distance between the adjacent U and V-lines into two segments inversely proportional to the respective span lengths. For unequal moments of inertia, it is now necessary, to divide the same distance into two segments inversely proportional to the span lengths divided by their respective moments of inertia. This is done, as indicated in Fig. 15, by measuring off (in opposite directions from any base line) lengths proportional to $\frac{l_2}{I_2}$ and $\frac{l_1}{I_1}$ on the V and U-lines, respectively; the intersection of the connecting line with the base line locates the required modified T-line.

Except for this simple modification in the location of the T-lines, the graphic construction heretofore presented remains unchanged for unequal moments of inertia. (For proof, see the Appendix.)

It is readily seen that this construction for locating the T-line (Fig. 15) is more general and reduces to the previous construction (Fig. 4) as a special case when $I_1 = I_2$.

8.—The Method of "Transformed Spans"

As an alternative to the modified T-line method of allowing for unequal values of I, represented in Fig. 15, a procedure which will be referred to as the

Method of Transformed Spans, may sometimes be preferred. This procedure consists simply in altering the length scale of some of the spans in the drawing.

The idea is to change the delineated lengths of the spans in such manner as to produce a virtual condition equivalent to equal moments of inertia. Taking the value of I of the principal loaded span as a standard, any span having a lower value of I is lengthened, and any span having a higher value of I is shortened (in Fig. 15), the lengthening or shortening being in the ratio of the standard value of I to the I of the span being altered. When this change is made in the delineated span lengths, the graphic solution can proceed the same as for equal moments of inertia.

The correctness of this method is founded on the following principle: If the length, l, and moment of inertia, I, of any span are altered in the same ratio so as to keep the rigidity of the span $\left(\text{represented by the quotient, } r = \frac{I}{l}\right)$, unchanged, the stress conditions in the remaining spans of a structure will be unaffected.

For examples of the application of the method of transformed spans, see Figs. 17 and 18.

9.—Application to Indeterminate Frames

The foregoing methods established for continuous beams can be applied directly to indeterminate frames or bents by regarding the rigidly connected members as spans of a continuous beam.

Fig. 16 shows the construction by the Method of Conjugate Points applied to a rectangular portal frame hinged at the bases of the columns. Fig. 17 shows the construction for a similar frame with columns fixed at their bases. In this problem the closing line passes through the one-third points, instead of the ends, of the outer spans.

In order to illustrate the two alternate methods of allowing for the condition of unequal moments of inertia, the device of the modified T-line is used in the construction of Fig. 16, and the device of transformed spans in that of Fig. 17. If the frames were trapezoidal (sloping columns) instead of rectangular, the constructions given in Figs. 16 and 17 would remain unchanged.

In Figs. 16 and 17, the frames and the applied loading are assumed to be symmetrical; consequently, the graphic construction needs to be carried out for only one side of the structure, as indicated. If the structure or loading is unsymmetrical, a small secondary correction must be introduced on account of the slight lateral displacement of the middle span; this displacement is equivalent to a yielding of the respective supports of the outer spans. (See Section 17.)

The method of conjugate points can also be applied advantageously to more complex indeterminate frames in which more than two members meet at a joint. Fig. 18 is an illustration; it represents a three-span continuous beam with rigidly connected intermediate columns.

The construction shown in Fig. 18 introduces a device which will be referred to as the Composition of Rigidities. The flexural rigidity of a member is pro-

portional to $\frac{E\,I}{l}$; if E is uniform, the rigidity may be represented by the quotient, $r = \frac{I}{l}$, or by $r = \frac{4}{3}\,\frac{I}{l}$ when the outer end of the member is restrained. If two or more members are framed into a single joint, they may be replaced, in effect, by a single member having a rigidity equal to the sum of the respective individual rigidities.

The foregoing principle is utilized in the construction of Fig. 18. The

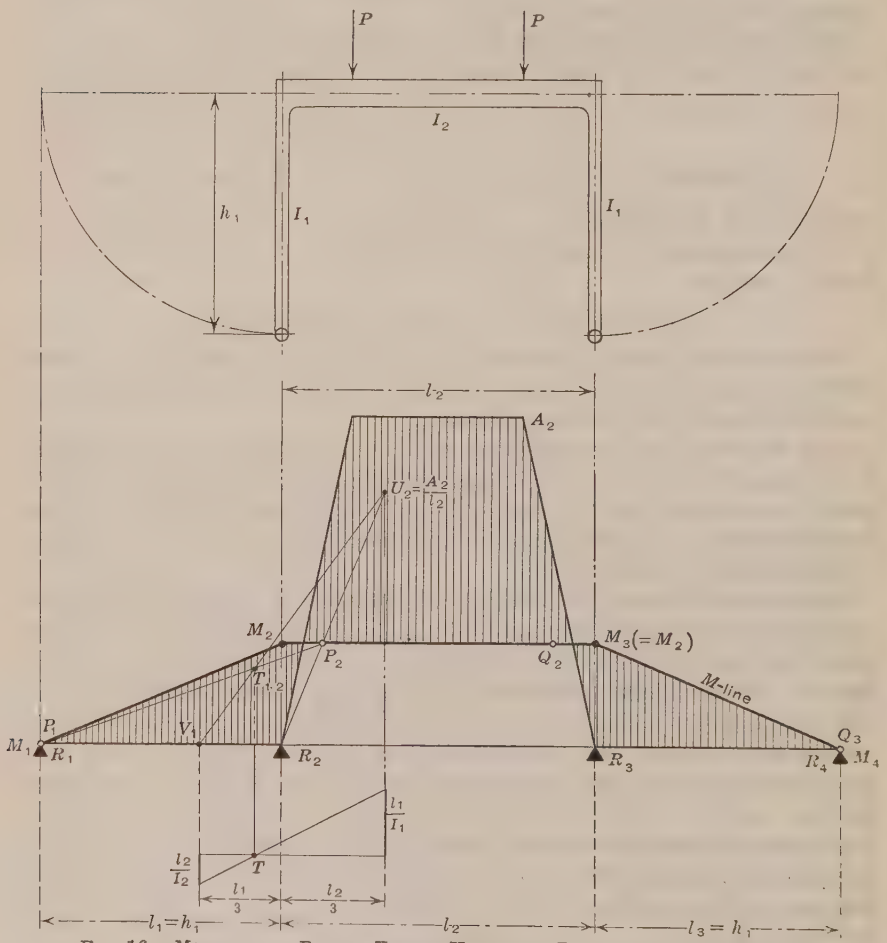


FIG. 16.—MOMENTS IN PORTAL FRAME, HINGED AT BASE, BY THE METHOD OF "MODIFIED T-LINES".

two members, l and h, at each end of the main span, l_2, are replaced by an equivalent single member, l_1. The rigidity of this equivalent single member must be:

$$r_1 = r_l + r_h$$

The nominal length of this member must then be:

$$l_1 = \frac{r_2}{r_1} \cdot l_2$$

in order that its nominal moment of inertia may be equal to the moment of inertia of the main span. (This is an application of the device of transformed spans.) With the side-span lengths, l_1, thus determined, the construction in Fig. 18 is identical with the construction heretofore presented for continuous beams and simple frames. (Compare Fig. 18 with Figs. 16 and 17.)

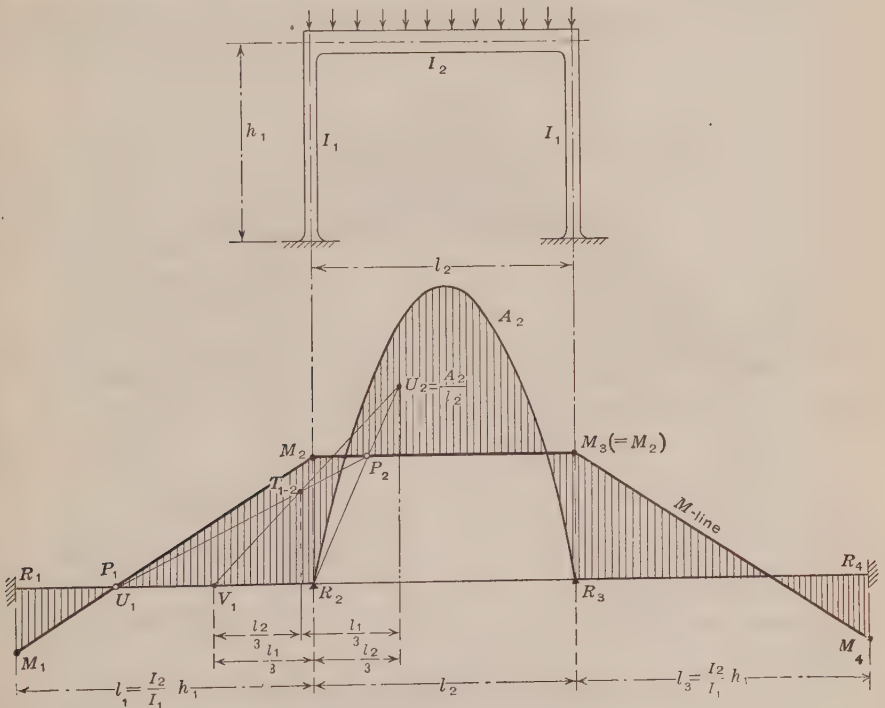


FIG. 17.—MOMENTS IN PORTAL FRAME, FIXED AT BASE, BY THE METHOD OF "TRANSFORMED SPANS".

On completion of the construction, the end moments found for the substituted equivalent members must be resolved or subdivided to give the bending moments in the respective component members. The total end moment, M_2 (Fig. 18), must be divided between the original component members in direct proportion to their respective rigidities. This final resolution of bending moments is indicated in Fig. 18; for the member with free end, the M-line goes to the free end; for the member with fixed end, the M-line goes through the outer one-third point.

This construction by the Composition of Rigidities has been explained at length because it has many useful applications; its utilization in the graphic determination of secondary stresses will be given in a subsequent section (Section 18).

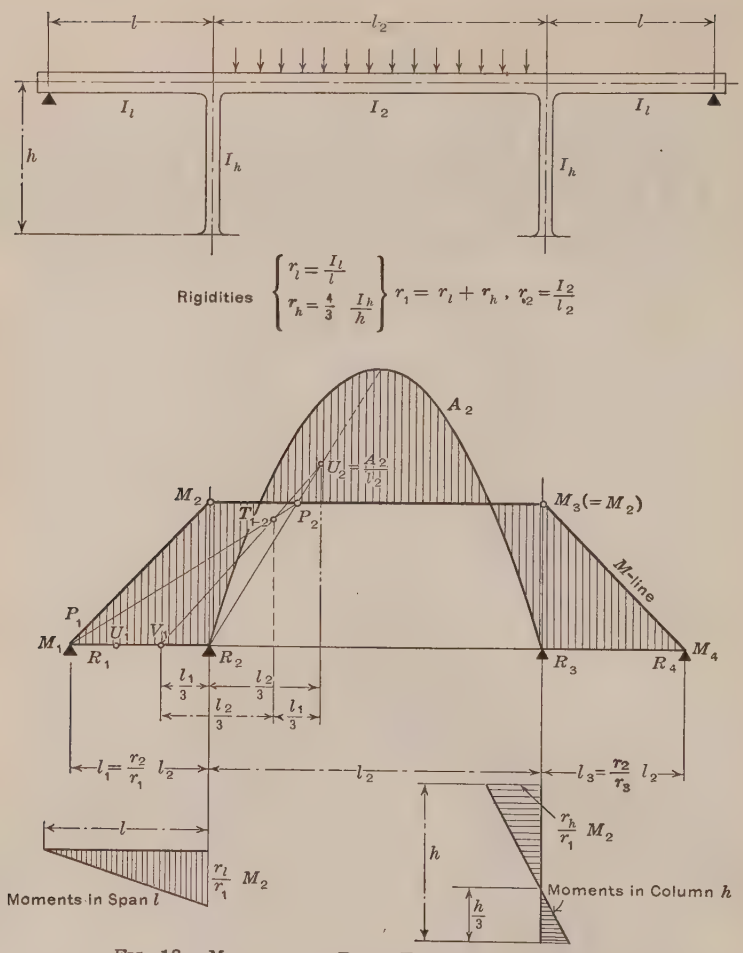


FIG. 18.—MOMENTS IN RIGID FRAME BY METHODS OF "SUMMATION OF RIGIDITIES".

10.—SIMPLIFICATION FOR TWO-SPAN BEAMS

For the common example of a two-span continuous beam, the foregoing constructions by the Method of Conjugate Points yield to a material simplification.

If the ends of a two-span continuous beam are free, the end moments have the values, $M_1 = 0$ and $M_3 = 0$. On substituting these values in Equation (1), that equation reduces to:

$$M_2 = -\frac{3}{2}\frac{A_1 + A_2}{l_1 + l_2} \qquad (9)$$

Comparing this with the value of $T_{1\text{-}2}$ given by Equation (5), it is observed that the height of the peak of the closing line (the point, M_2) is $\frac{3}{2}$ the height

of the T-point. The problem is thereby solved without requiring the location of the conjugate points.

For this case, it is convenient to mark the U-V points at $\frac{3}{2}$ their usual height, $\frac{3}{2}\frac{A}{l}$, instead of $\frac{A}{l}$, so that the resulting T-point will be $\frac{3}{2}$ its customary height; a horizontal through T_{1-2} will then locate the required point, M_2.

If the ends of a two-span continuous beam are "fixed", it is easily shown (from Equations (1) and (6)) that the moment, M_2, at the intermediate support has the value:

$$M_2 = -\frac{A_1 + A_2}{l_1 + l_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

This equation does not contain the factor, $\frac{3}{2}$, found in Equation (9). Hence, in this case, no modification is required in the height of the T-point; and a horizontal through T_{1-2} locates the required point, M_2.

Instead of using the $\frac{1}{3}$-lines (U-V lines) and their transposition point, T_{1-2}, the $\frac{1}{2}$-lines (C-lines) and their transposition point, S_{1-2}, may be used. If the heights, $\frac{A}{l}$, are laid off on the $\frac{1}{2}$-lines, the resulting S-point will have a height:

$$S_{1-2} = \frac{A_1 + A_2}{l_1 + l_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

which is identical with the value for T_{1-2} (Equation (5)). This substitution may be preferred in the application to two-span beams. (See Fig. 19, Cases 6-9 and Fig. 20, Cases 14-17.)

In these simplified constructions for two-span beams, the corrections for unsymmetrical loads and for unequal moments of inertia are made in the same manner as heretofore established for the general case, namely, as indicated in Figs. 14 and 15.

The construction is still further simplified when the number of spans is reduced to one. For a single-span beam with one end restrained, Equation (9) reduces to $M = -\frac{3}{2}\frac{A_1}{l_1}$. (See Fig. 19, Case 1.) For a single-span beam with both ends restrained, Equation (10) reduces to $M = -\frac{A_1}{l_1}$. (See Fig. 19, Case 3.) These two results also follow directly from the previously established general relation for restrained ends indicated in Fig. 12.

11.—Application to Special Cases Commonly Occurring in Practice

Applications of the foregoing constructions to the special cases that occur most frequently in practice are indicated in a series of diagrams on Figs.

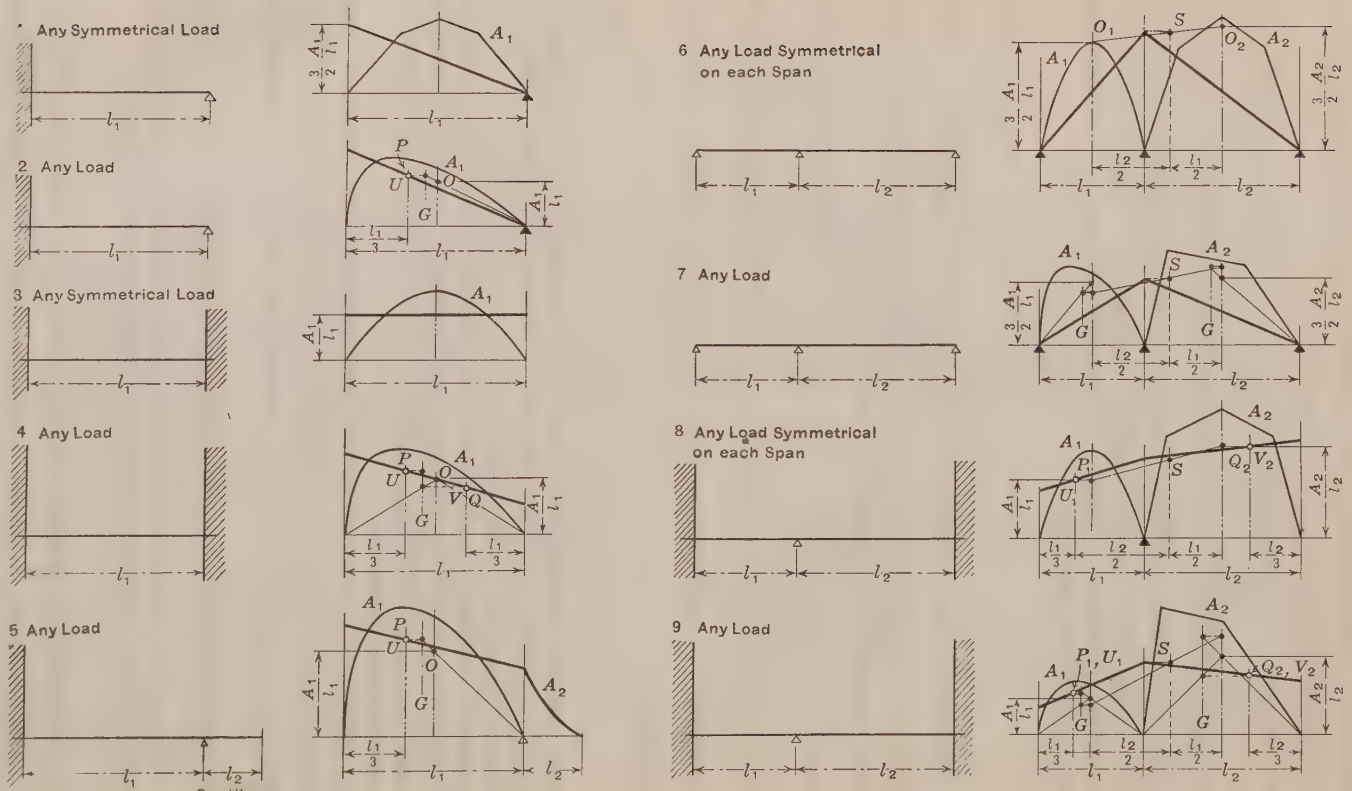


FIG. 19.—APPLICATION OF THE METHOD OF CONJUGATE POINTS TO SPECIAL CASES COMMONLY OCCURRING IN PRACTICE.

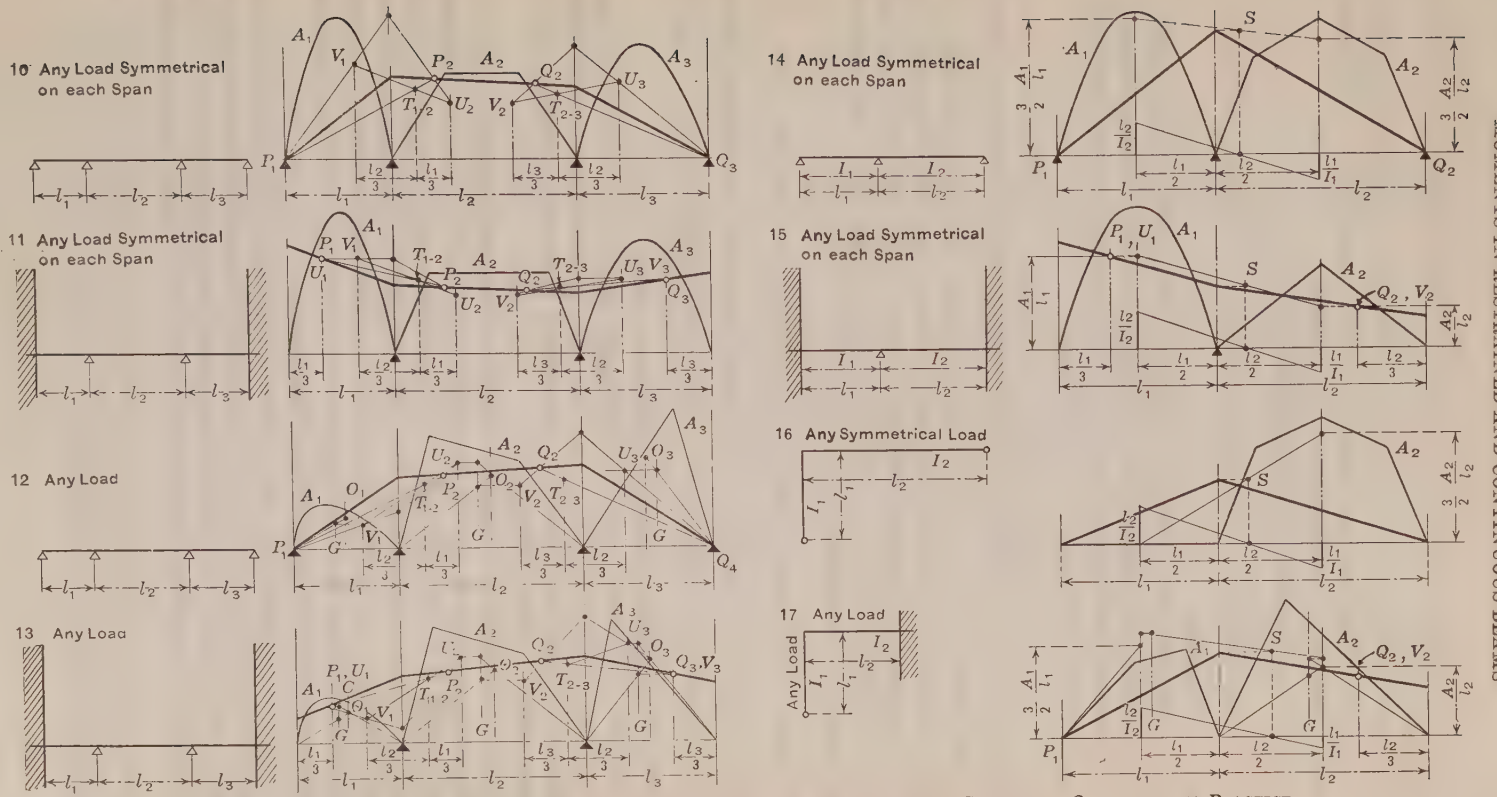

10 Any Load Symmetrical on each Span
11 Any Load Symmetrical on each Span
12 Any Load
13 Any Load
14 Any Load Symmetrical on each Span
15 Any Load Symmetrical on each Span
16 Any Symmetrical Load
17 Any Load
Any Load

FIG. 20.—APPLICATION OF METHOD OF CONJUGATE POINTS TO SPECIAL CASES COMMONLY OCCURRING IN PRACTICE.

19 and 20. Seventeen cases are included, covering practically all conditions up to three spans.

The constructions given on Figs. 19 and 20 follow the methods established in the preceding Sections, and require no further explanation. The general order of procedure for each solution is as follows:

1.—Draw the simple-beam moment graphs.

2.—On each center line, measure the height, $\frac{A}{l}$ (except where otherwise shown).

3.—From this, locate the U-V points as indicated (where necessary).

4.—Locate the T-points (or S-point) as indicated (where necessary).

5.—Locate the conjugate points, P, Q, as indicated (where necessary).

6.—Draw the closing line as indicated.

12.—Application to Continuous Beams with Varying Moment of Inertia

In the foregoing, it has been assumed that the moment of inertia, I, is constant for each span, although successive spans might have different values of I. The Method of Conjugate Points can also be applied to the more general case of continuous beams with moment of inertia varying within the individual spans, as will now be shown.

If the value of I is not constant for the length of a span, the factor, $\frac{1}{I}$, cannot be taken out from the moment-area expressions for the span, as was done in Equations (1), (7), and (8). Instead, the variable factor, $\frac{1}{I}$, must be applied to the individual ordinates of all moment areas.

When the varying values of I are known throughout the length of the span, it is a simple matter to divide the ordinates of any moment area by the corresponding values of the variable, I; the reduced moment graph thus obtained is called a **Modified Moment Area.**

Referring to Fig. 2, it is observed that there are three moment areas to be considered in each span, namely (considering the span, l_1), the left M-triangle (M_1-triangle), the right M-triangle (M_2-triangle), and the simple-beam moment area (A_1). Each of these areas must be "modified" by dividing its ordinates by the variable, I, before the analysis can be applied as heretofore. The areas and centroids of these modified moment areas must be used in writing a new expression for the Theorem of Three Moments.

As the end moments, M_1 and M_2, are not known in advance, M-triangles of unit height, $M_1 = 1$, $M_2 = 1$, are considered instead of the actual M-triangles. Similarly, for convenience of notation, the height of the A_1-area may be imagined to be reduced proportionally to a mean ordinate of unity before applying the factor, $\frac{1}{I}$. (Such scalar reductions of height do not affect the horizontal locations of the centroids.)

On dividing the ordinates of these three moment areas by the variable, I, three modified moment areas are obtained for the span. By summations, their respective mean ordinates and centroids are determined. The notation used is indicated in Fig. 21.

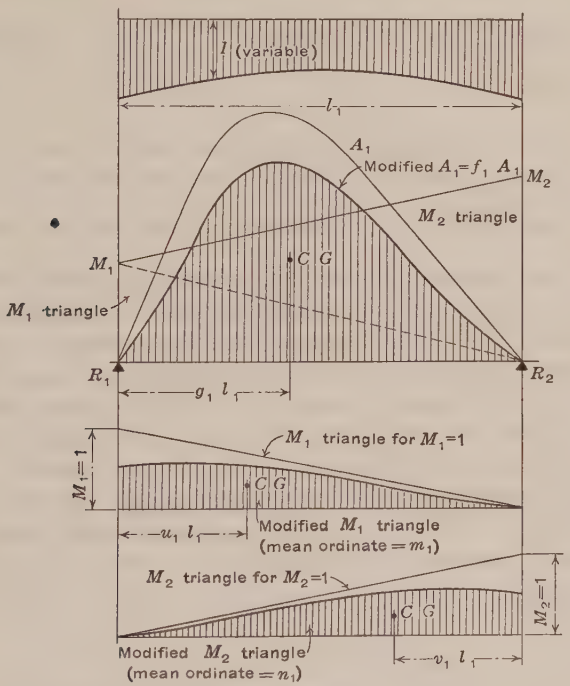


FIG. 21.—NOTATION DIAGRAM FOR VARYING I.

Let,

m_1 = mean ordinate of modified M_1-triangle, for $M_1 = 1$; = area of modified M_1 triangle, divided by $M_1 l_1$.

n_1 = mean ordinate of modified M_2-triangle, for $M_2 = 1$; = area of modified M_2-triangle, divided by $M_2 l_1$.

f_1 = mean ordinate of modified A_1-area, for $\frac{A_1}{l_1} = 1$; = modified A_1-area divided by A_1.

and, let,

u_1 = abscissa ratio of the centroid of the modified M_1-triangle. ($u_1 l_1$ = abscissa from R_1.)

v_1 = abscissa ratio of the centroid of the modified M_2-triangle. ($v_1 l_1$ = abscissa from R_2.)

g_1 = abscissa ratio of the centroid of the modified A_1-area. ($g_1 l_1$ = abscissa from R_1.)

For the adjoining span, l_2, the corresponding quantities are m_2, n_2, f_2, and u_2, v_2, g_2. The abscissas represented by u and v are to be measured from the

respective ends of the span, $u\,l$ from the left end and $v\,l$ from the right end; the abscissas represented by g are to be measured from the left end of each span. (If the span is symmetrical in variation of section, $m = n$ and $u = v$. If the applied loading is also symmetrical, $g = \frac{1}{2}$. If I is constant throughout the span, then $m = n = \frac{1}{2} \times \frac{1}{I}$, $f = 1 \times \frac{1}{I}$, $u = v = \frac{1}{3}$.)

As an alternative to the Method of Modified Moment Areas for obtaining the constants, m, n, f, and u, v, g, the same results may be obtained by the Method of Constant, $\frac{\Delta x}{I}$, that is, by dividing the beam into the elements, Δx, proportional to the respective values of I. This is easily done by any one of several graphic methods that have been published in connection with the analysis of arch ribs.* This dispenses with the necessity of dividing all moment ordinates by the variable, I. The ordinates of the unmodified moment areas, measured at the centers of the Δx-divisions, are then used instead of the uniformly spaced ordinates of the modified moment areas. The resulting values of m, n, f, and u, v, g, should be the same by either method.

Following the same reasoning as for Equations (1), (7), and (9), a more general expression of the Theorem of Three Moments may now be written to include the case of varying I. The fundamental equation takes the form:

$$(C_1 M_1 + D_1 M_2)\, l_1 + (C_2 M_2 + D_2 M_3)\, l_2 = -(E_1 A_1 + E_2 A_2) \ldots . (12)$$

in which,

$$C_1 = m_1 \times u_1 \,(= n_1 v_1) \qquad C_2 = m_2\,(1 - u_2)$$
$$D_1 = n_1\,(1 - v_1) \qquad D_2 = n_2 \times v_2 \,(= m_2 u_2)$$
$$E_1 = f_1 \times g_1 \qquad E_2 = f_2\,(1 - g_2)$$

In application to any actual problem, with the I-variation and loading known, the numerical values of m, n, f, and u, v, g, are first computed for each span by a simple tabulation and summation of the known variables; and the products, C, D, E, are then inserted in Equation (12) to obtain the governing equation for each pair of spans.

Equations (1), (7), and (8) are only special cases of the more general Equation (12), as will be found on substituting the appropriate special values of the coefficients, C, D, and E.

The Method of Conjugate Points can be applied to the solution of Equation (12) in exactly the same manner as heretofore given, if the U, V, and T-points are located with due regard to the changed coefficients in the equation. It is easily shown that these new coefficients are satisfied if the U, V, and T-points are located as follows (see Fig. 22):

1.—Locate all U and V-lines at the centroids of the modified M-triangles: that is, locate each U-line at the distance, $u \times l$, from the left end of the span

* "Plain and Reinforced Concrete Arches", Melan-Steinman, pp. 45–46.

and each V-line at the distance, $v \times l$, from the right end of the span. (Note that $v_1 = \frac{C_1}{C_1 + D_1}$, $u_2 = \frac{D_2}{C_2 + D_2}$.)

2.—Locate the T-line by dividing the distance between the adjacent U-V lines into two segments inversely proportional to $n_1\ l_1$ and $m_2\ l_2$; or, equivalently, inversely proportional to $(C_1 + D_1)\ l_1$ and $(C_2 + D_2)\ l_2$.

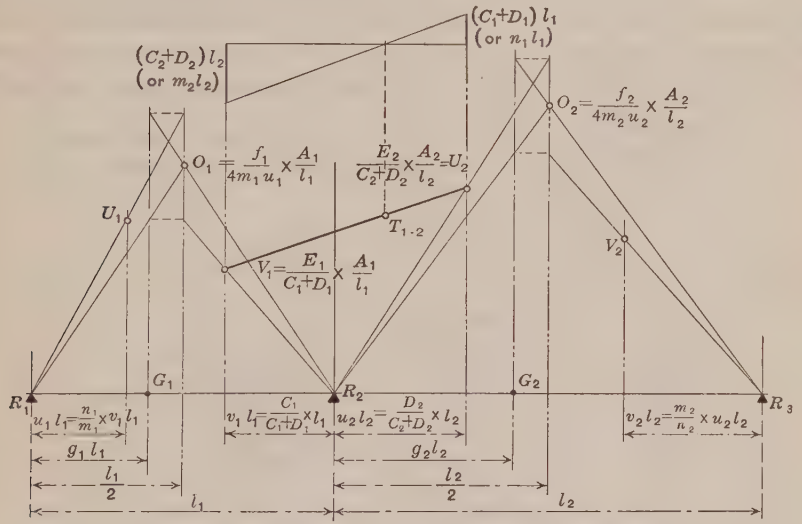


FIG. 22.—CONSTRUCTION FOR VARYING I, UNSYMMETRICAL SPANS AND LOADING.

3.—Locate the U-V points at the heights:

$$V_1 = \frac{E_1}{C_1 + D_1} \times \frac{A_1}{l_1} = \frac{f_1\ g_1}{n_1} \times \frac{A_1}{l_1}$$

$$U_2 = \frac{E_2}{C_2 + D_2} \times \frac{A_2}{l_2} = \frac{f_2\ (1 - g_2)}{m_2} \times \frac{A_2}{l_2}$$

The remainder of the procedure (for locating the closing line) is identical with that heretofore presented. (For proof, see the Appendix.)

If the beam and the loading in any span are symmetrical about the center line, the U and V-points in the span will be symmetrically located at equal heights. If unsymmetrical conditions obtain, Equation (12) is re-written for each successive pair of spans, and the new values of the numerical coefficients, C, D, and E, are used in locating the new group of V, U, and T-points; or the relations indicated in Fig. 22 may be used to locate the U and V-points.

If the special coefficients of Equations (1), (7), and (8) are substituted for C, D, and E in the foregoing rules, the latter will be found to reduce to the simpler rules previously established for the respective special cases.

A reduction of all the coefficients in Equation (12), by dividing through by any numerical factor, will not affect the construction, as the foregoing rules involve only ratios, and not absolute values, of the coefficients.

13.—Method for Varying I Applied to a Numerical Example

The foregoing method for varying I will now be illustrated by application to an actual numerical problem.

The profile of a two-span concrete beam is indicated in Table 1, which also gives the necessary tabulations and summations. The beam is assumed to be divided into sections 1 ft. long, and all variables refer to the mid-points of these sections. In the tabulated values, x denotes the abscissas, y the straight line ordinates of M-triangles of unit height, and y_a the ordinates of the simple-beam moment areas for $A_1 = 1$ and $A_2 = 1$. The respective ordinates divided by I give the "modified" ordinates. The applied loading is shown in Fig. 23.

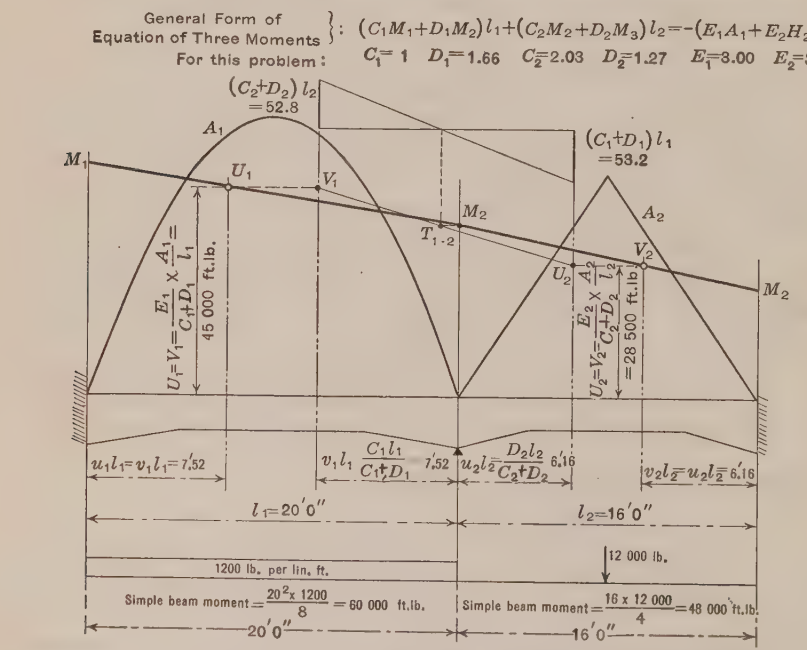


Fig. 23.—Method for Varying I, Applied to a Numerical Problem. (See Table 1.)

From Table 1:

$C_1 = 0.0289 \times 0.376 = 0.0109$ $\qquad$ $C_2 = 0.0359\ (1 - 0.385) = 0.0221$

$D_1 = 0.0289 \times (1 - 0.376) = 0.0181$ $\qquad$ $D_2 = 0.0359 \times 0.385 = 0.0138$

$E_1 = 0.0654 \times 0.500 = 0.327$ $\qquad$ $E_2 = 0.0853 \times 0.500 = 0.0427$

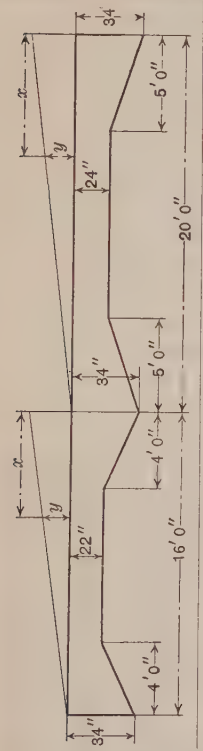
With these coefficients determined, Equation (9) may now be written for this problem as follows:

$$(0.0109\,M_1 + 0.0181\,M_2)\,l_1 + (0.0221\,M_2 + 0.0138\,M_3)\,l_2 = -\,(0.0327\,A_1 + 0.0427\,A_2)$$

or,

$$(M_1 + 1.66\,M_2)\,l_1 + (2.03\,M_2 + 1.27\,M_3)\,l_2 = -\,(3.00\,A_1 + 3.92\,A_2)$$

TABLE 1.—Calculation of Constants for Numerical Example of Beam with Varying I.



Depth, d.	$I = \frac{d^3}{1\,000}$.	x.	y.	$\frac{y}{I}$.	$\frac{y-x}{I}$.	y_a.	$\frac{y_a}{I}$.	$\frac{y_a - x}{I}$.
33	35.94	0.5	0.975	0.0271	0.014	0.007	0.0002	0.0001
31	29.79	1.5	0.925	0.0310	0.047	0.021	0.0007	0.0010
29	24.39	2.5	0.875	0.0359	0.090	0.033	0.0014	0.0034
27	19.68	3.5	0.825	0.0419	0.147	0.043	0.0022	0.0077
25	15.62	4.5	0.775	0.0496	0.223	0.052	0.0033	0.0151
24	13.82	5.5	0.725	0.0524	0.288	0.060	0.0043	0.0238
24	13.82	6.5	0.675	0.0488	0.317	0.066	0.0048	0.0309
24	13.82	7.5	0.625	0.0452	0.339	0.070	0.0051	0.0381
24	13.82	8.5	0.575	0.0416	0.354	0.073	0.0053	0.0451
24	13.82	9.5	0.525	0.0380	0.361	0.075	0.0054	0.0514
24	13.82	10.5	0.475	0.0344	0.361	0.075	0.0054	0.0568
24	13.82	11.5	0.425	0.0307	0.354	0.073	0.0053	0.0610
24	13.82	12.5	0.375	0.0271	0.339	0.070	0.0051	0.0636
24	13.82	13.5	0.325	0.0235	0.317	0.066	0.0048	0.0643
24	13.82	14.5	0.275	0.0199	0.288	0.060	0.0043	0.0627
25	15.62	15.5	0.225	0.0144	0.228	0.052	0.0033	0.0519
27	19.68	16.5	0.175	0.0089	0.147	0.043	0.0022	0.0363
29	24.39	17.5	0.125	0.0051	0.090	0.033	0.0014	0.0235
31	29.79	18.5	0.075	0.0025	0.047	0.021	0.0007	0.0129
33	35.94	19.5	0.025	0.0007	0.014	0.007	0.0002	0.0040
....	.....	.....	10.000	0.5787	4.360	1.000	0.0654	0.6536
32.5	34.33	0.5	0.969	0.0282	0.014	0.008	0.0002	0.0001
29.5	25.67	1.5	0.906	0.0353	0.053	0.023	0.0009	0.0014
26.5	18.61	2.5	0.844	0.0453	0.118	0.039	0.0021	0.0052
23.5	12.98	3.5	0.781	0.0602	0.211	0.055	0.0042	0.0147
22	10.65	4.5	0.719	0.0675	0.304	0.070	0.0066	0.0297
22	10.65	5.5	0.656	0.0616	0.339	0.086	0.0081	0.0448
22	10.65	6.5	0.594	0.0558	0.362	0.102	0.0096	0.0620
22	10.65	7.5	0.531	0.0499	0.374	0.117	0.0110	0.0825
22	10.65	8.5	0.469	0.0440	0.374	0.117	0.0110	0.0935
22	10.65	9.5	0.406	0.0382	0.362	0.102	0.0096	0.0906
22	10.65	10.5	0.344	0.0323	0.339	0.086	0.0081	0.0847
22	10.65	11.5	0.281	0.0264	0.304	0.070	0.0066	0.0759
23.5	12.98	12.5	0.219	0.0169	0.211	0.055	0.0042	0.0527
26.5	18.61	13.5	0.156	0.0084	0.113	0.039	0.0021	0.0283
29.5	25.67	14.5	0.094	0.0036	0.053	0.023	0.0009	0.0132
32.5	34.33	15.5	0.031	0.0009	0.014	0.008	0.0002	0.0035
....	.....	.....	8.000	0.5745	3.540	1.000	0.0854	0.6823

NOTE.—

$$m_1 = n_1 = \frac{0.5787}{20} = 0.0289;\ u_1 = v_1 = \frac{4.360}{20 \times 0.5787} = 0.376;\ f_1 = 0.0654;\ g_1 = \frac{0.6586}{20 \times 0.0654} = 0.500$$

$$m_2 = n_2 = \frac{0.5745}{16} = 0.0359;\ u_2 = v_2 = \frac{3.540}{16 \times 0.5745} = 0.385;\ f_2 = 0.0854;\ g_2 = \frac{0.6823}{16 \times 0.0854} = 0.500$$

Using either the coefficients of this last equation or the original factors, m, n, f, u, v, g, the principal points, U, V, T, are located as indicated in Fig. 23. The closing line is then easily obtained, as shown in Fig. 23, by applying the simplified construction of Section 10. The closing line passes through the outer U and V-points because the beam is assumed to be restrained at the ends.

14.—Use of Tabulated Constants for Cases of Varying I

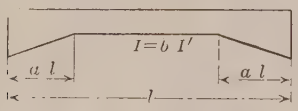
For practical use, tables may be constructed evaluating the constants, m, n, f, and u, v, g, for most common conditions, thus expediting the solution of problems of this type.

Table 2 is presented for this purpose, and gives the values of $m = n$ and $u = v$ for symmetrical beams with straight and parabolic haunches, respectively. Values for intermediate ratios not tabulated may be obtained by interpolation.

TABLE 2.—Constants for Beams of Varying I.

(Upper figures are values of $u = v$.
Lower figures, divided by I, give values of $m = n$.)

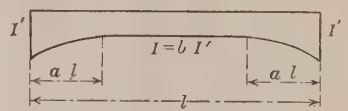
Straight Haunches.



Values of a.

	0.25	0.30	0.35	0.40	0.50
$b = 0.05$	0.425	0.434	0.441	0.444	0.434
	0.313	0.276	0.238	0.201	0.126
$b = 0.06$	0.422	0.431	0.437	0.440	0.430
	0.318	0.282	0.245	0.209	0.136
$b = 0.08$	0.416	0.425	0.430	0.431	0.422
	0.327	0.292	0.258	0.223	0.154
$b = 0.10$	0.412	0.419	0.424	0.425	0.416
	0.335	0.302	0.269	0.236	0.170
$b = 0.12$	0.407	0.415	0.419	0.420	0.410
	0.342	0.310	0.279	0.248	0.184
$b = 0.15$	0.402	0.408	0.412	0.412	0.403
	0.352	0.322	0.292	0.263	0.203
$b = 0.20$	0.395	0.400	0.402	0.402	0.394
	0.366	0.339	0.312	0.286	0.232
$b = 0.30$	0.382	0.385	0.387	0.387	0.380
	0.390	0.367	0.346	0.324	0.279
$b = 0.60$	0.356	0.360	0.358	0.357	0.359
	0.444	0.439	0.422	0.411	0.392
$b = 1.00$	0.333	0.333	0.333	0.333	0.333
	0.500	0.500	0.500	0.500	0.500

Parabolic Haunches.



Values of a.

	0.25	0.30	0.35	0.40	0.50
$b = 0.05$	0.406	0.415	0.423	0.428	0.433
	0.359	0.331	0.302	0.274	0.218
$b = 0.06$	0.403	0.412	0.419	0.425	0.429
	0.364	0.336	0.309	0.282	0.227
$b = 0.08$	0.398	0.407	0.413	0.418	0.422
	0.372	0.346	0.320	0.295	0.243
$b = 0.10$	0.395	0.403	0.408	0.413	0.416
	0.378	0.354	0.330	0.306	0.257
$b = 0.12$	0.392	0.398	0.404	0.408	0.411
	0.385	0.362	0.338	0.316	0.270
$b = 0.15$	0.387	0.393	0.399	0.402	0.405
	0.393	0.371	0.350	0.328	0.286
$b = 0.20$	0.381	0.386	0.390	0.393	0.395
	0.404	0.385	0.366	0.392	0.308
$b = 0.30$	0.371	0.375	0.378	0.380	0.382
	0.422	0.407	0.392	0.376	0.345
$b = 0.60$	0.351	0.353	0.354	0.355	0.355
	0.462	0.454	0.447	0.439	0.424
$b = 1.00$	0.333	0.333	0.333	0.333	0.333
	0.500	0.500	0.500	0.500	0.500

To illustrate the use of Table 2, it may be applied directly to the problem of the two-span beam, solved in Section 13. For the first span (having the ratio, $a = 0.25$, $b = 0.352$), Table 2 gives $u_1 = v_1 = 0.377$ (compared with 0.376 by the previous calculation) and $m_1 = n_1 = \dfrac{0.399}{13.82} = 0.0289$ (identical with the value previously calculated). For the second span (having the ratios, $a = 0.25$, $b = 0.27$), Table 2 gives $u_2 = v_2 = 0.386$ (compared with 0.385 by the previous calculation) and $m_2 = n_2 = \dfrac{0.383}{10.65} = 0.0359$ (identical with the value previously calculated).

Similar tables can easily be constructed to give the values of f and g for the common conditions of loading.

With such tables, the computations represented by Table 1 in the foregoing numerical example are dispensed with. The constants, m, n, f, and u, v, g,

can be taken from the tables and used directly in the graphic solution shown in Figs. 22 and 23.

15.—Determination of Shears and Reactions

After the closing line, or M-line, is located for any problem by the foregoing methods, the shears and reactions are easily determined either graphically or analytically.

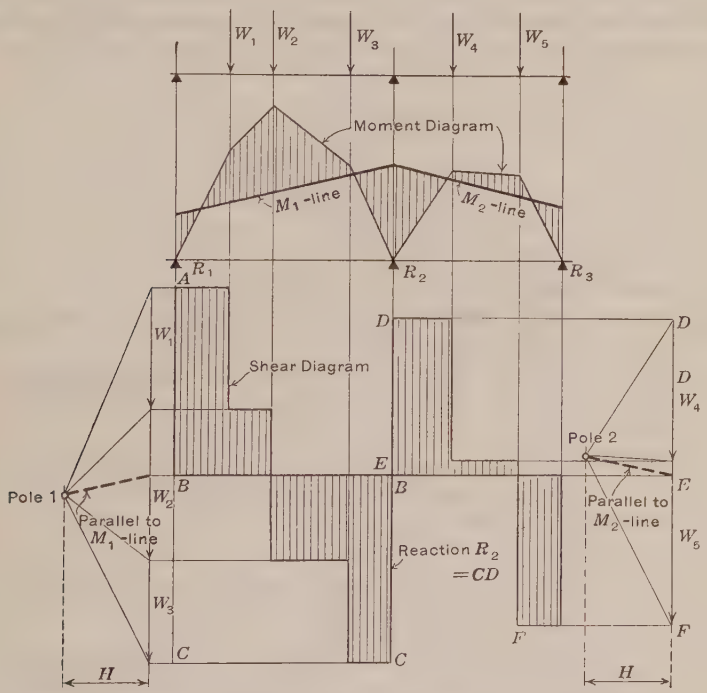


Fig. 24.—Determination of Shears and Reactions.

The graphic procedure is indicated in Fig. 24. In the pole diagram corresponding to the simple-beam moment graph of any span, a ray drawn parallel to the closing line divides the load line into two segments representing the respective end shears. The sum of the two end shears at any intermediate support gives the corresponding intermediate reaction. Horizontals from the division points of the load line intersect the load verticals in points of the desired shear diagram.

If the moment graph is curved, the outer rays of the pole diagram are drawn parallel to the terminal tangents in order to locate the pole.

It should be noted that the pole diagrams for successive spans of a continuous beam must have the same pole distance, H, because the respective moment graphs are drawn to the same vertical scale.

The foregoing construction is equivalent to the customary analytical procedure represented by the relation:

$$\text{Shear between } R_1 \text{ and } R_2 = \text{Shear for Simple Beam} + \frac{M_1 - M_2}{l_2}.$$

16.—Application to Construction of Influence Lines

If the foregoing methods are applied to a unit concentration assumed to be acting at successive points on a beam, the successive values of any moment, shear, or reaction can be plotted to produce the corresponding influence line. Each value of the moment, shear, or reaction is represented by a scaled intercept in the diagram constructed for the respective position of the moving load.

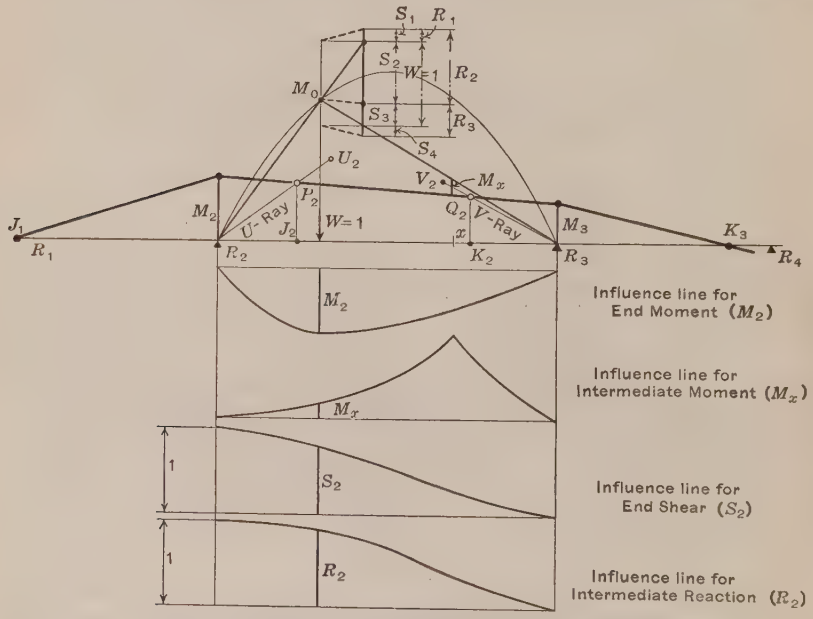


Fig. 25.—General Method for Constructing Influence Lines for Continuous Beams.

Fig. 25 shows the general procedure to be followed. A unit load, $W = 1$, is assumed to be acting at one of its successive positions on a continuous beam. The simple-beam moment graph is a triangle with a vertex, M_0, over the point of load; and the closing line is drawn through the conjugate points as indicated. The height of the closing line over any support indicates the value of the corresponding end moment, M_2 or M_3, and the vertical intercept between the M_0-triangle and the closing line at any intermediate section, x, gives the resultant moment, M_x, at the section. With the vertex, M_0, as a pole, a pole diagram is drawn with the unit load line, $W = 1$. A ray drawn parallel to the closing line of the loaded span yields intercepts representing the end shears, S_2 and S_3. Rays drawn parallel to the respective directions of the

closing line in the adjoining spans yield the shears, S_1 and S_4, in those spans. Summations of the shears for each support give the respective reactions, $R_1 = S_1$, $R_2 = S_1 + S_2$, and $R_3 = S_3 + S_4$.

The value of each moment, shear, or reaction is conveniently represented by a corresponding vertical intercept in the diagram. By plotting the successive values of one of these intercepts under successive positions of the moving load, the corresponding influence line is obtained. The influence lines obtained in this manner for an end moment, M_2, an intermediate moment, M_x, an end shear, S_2, and an intermediate reaction, R_2, are shown in Fig. 25.

In the unloaded spans, the outer conjugate points are at zero height, that is, they coincide with their projections on the base line. The projections of the conjugate points on the base line are called fixed points. In all spans to the left of the loaded span, the closing line passes through the left fixed points, J; in all spans to the right of the loaded span, the closing line passes through the right fixed points, K.

As the construction has to be repeated for successive assumed positions of the moving load, it is obviously desirable to reduce the necessary graphic operations to the minimum. Various graphic constructions for expediting the location of the conjugate points for this case of a single concentrated load are easily devised. The principle represented by Fig. 9 can be applied to advantage, because it dispenses with the necessity of drawing new pennant diagrams for successive load positions; the fixed points are first located, and these establish the verticals on which the conjugate points will lie for all positions of the moving load. Another useful principle (indicated in Figs. 16, 17, 18, and 25) is that the conjugate points will always lie on the U and V-rays (the point, P_2, on R_2 U_2, and the point, Q_2, on R_3 V_2) when the adjoining spans are unloaded. (That is true because the rays, R_2 U_2 and R_3 V_2, are sides of the respective pennant diagrams.) Hence, it is only necessary to locate the U and V-points; the rays drawn to these points from the respective ends of the span will intersect the conjugate verticals (through J and K) in the required conjugate points, P and Q, as indicated in Fig. 25. It may also be noted that the vertices, M_0, of the successive moment triangles in any span will lie on a parabola having the height, $\frac{l}{4}$, at mid-span.

The detailed application of the foregoing to the graphic construction of influence lines for a continuous beam is shown in Fig. 26. Fig. 26 (a) shows the graphic location of the fixed points, J and K, which fix the verticals on which the conjugate points, P and Q, will lie. Fig. 26 (b), (c), and (d) show the graphic location of the closing line for a load position in the first, second, and third spans, respectively. In this construction, the G-line is marked at one-third the distance from the span center, C, to the load point, W; diagonals to the horizontal through M_0 intersect the G-line at the required heights of the U-V points; and rays to the U-V points intersect the conjugate verticals in the required conjugate points, P and Q, which locate the closing line. Fig. 26 (e) shows the influence line for the end moment, M_2, obtained by plotting successive values of the corresponding intercept over the support,

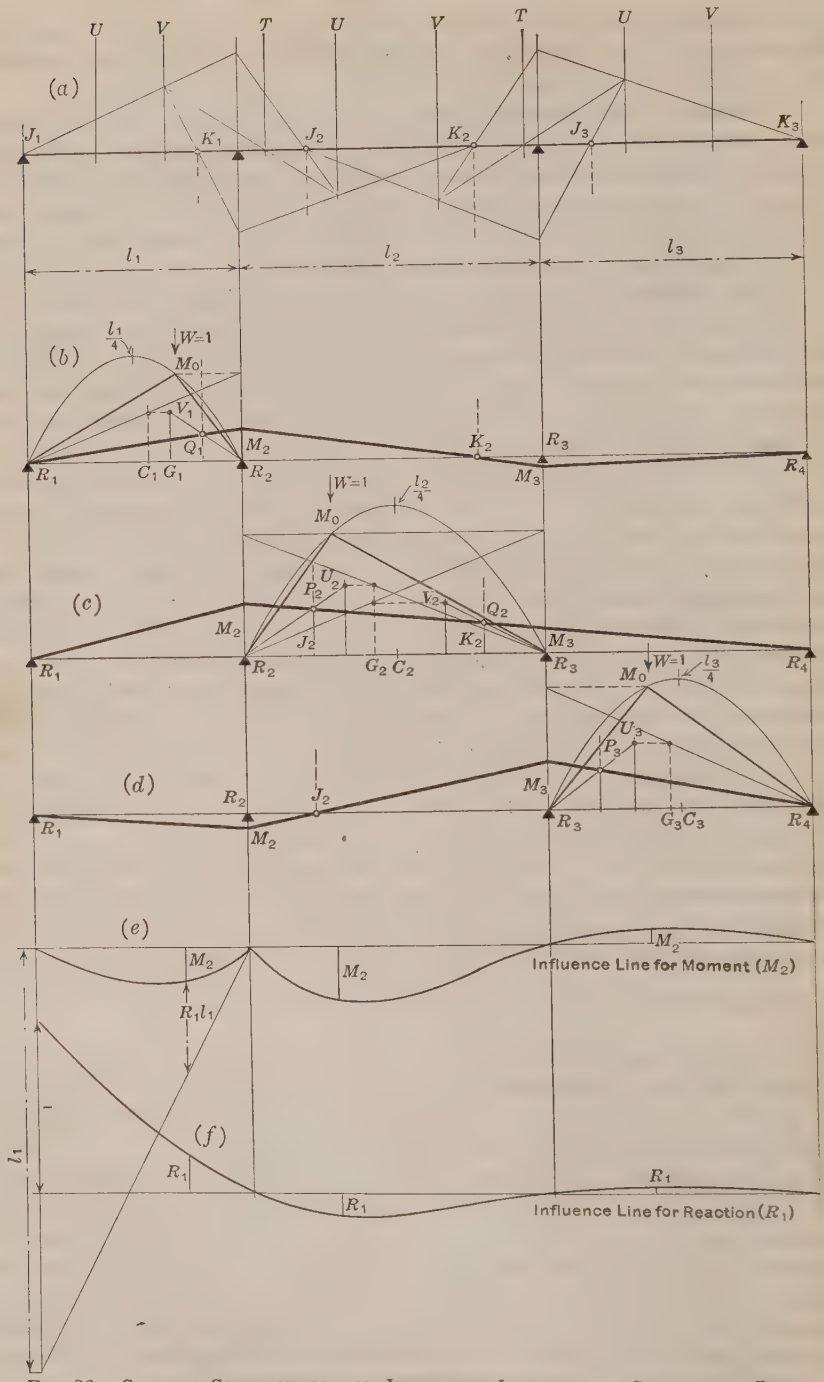


FIG. 26.—GRAPHIC CONSTRUCTION OF INFLUENCE LINES FOR A CONTINUOUS BEAM.

R_2. The M_2-influence line is also the influence line for the end reaction, R_1 (if the ordinates are divided by l_1); except that in the span, l_1, the ordinates have to be measured from a different base line, namely, a diagonal having the end ordinate, l_1. Fig. 26 (*f*) shows the influence line thus obtained for R_1.

In Fig. 26, influence lines for other moments, shears, or reactions could easily be plotted from Fig. 26 (*b*), (*c*), and (*d*), using the respective intercepts as indicated in Fig. 25.

The influence diagram for shear or bending moment at any intermediate point of the end span, l_1, may also be obtained simply by adding a straight line to the R_1-influence line or "elastic curve" (Fig. 26 (*f*)).*

17.—Application to Settlement of Supports

The Method of Conjugate Points can also be applied advantageously to problems involving the settlement or deflection of one or more supports of a continuous or restrained beam.

If the right end of a span, l_1, settled more than the left end by an amount, d_1, the slope change for the span is represented by $\phi_1 = \frac{d_1}{l_1}$. Similarly, for the span, l_2, the slope change is represented by $\phi_2 = \frac{d_2}{l_2}$.

Introducing this condition into the analysis by which Equation (1) was written, that fundamental equation takes the form of Equation (13) for the case of settlement of supports:

$$(M_1 + 2\,M_2)\,l_1 + (M_3 + 2\,M_2)\,l_2 = -\,6\,E\,I\,(\phi_1 - \phi_2) \ldots\ldots (13)$$

Equation (13) expresses the Theorem of Three Moments for settlement of supports, for the case of constant I. For the case of unequal values of I, Equation (13) takes the following form, corresponding to Equation (8):

$$(M_1 + 2\,M_2)\,\frac{l_1}{I_1} + (M_3 + 2\,M_2)\,\frac{l_2}{I_2} = -\,6\,E\,(\phi_1 - \phi_2) \ldots\ldots (14)$$

It is observed that the left-hand members of Equations (13) and (14) are identical with those of Equations (1) and (8), respectively. The right-hand members, representing the influence of the deflections, replace the corresponding members of Equations (1) and (8) which represented the influence of the loading.

It is apparent, therefore, that the graphic constructions by the Method of Conjugate Points heretofore presented may also be applied to the present problem, except that the heights of the U, V, and T-points must be made to satisfy the changed form of the load terms in the equations. The locations of the U, V, and T-lines remain unchanged.

It is easily shown that both Equations (13) and (14) will be satisfied by the foregoing constructions, if the height of the T-point is made equal to:

$$T_{1-2} = 2\,\frac{\phi_1 - \phi_2}{e_1 + e_2} \ldots\ldots (15)$$

* See "Continuous Bridges" in Hool and Kinne's "Movable and Long-Span Steel Bridges", N. Y., 1923.

in which e_1 and e_2 are the "flexibility constants" of the respective spans, defined by $e_1 = \frac{l_1}{E I_1}$ and $e_2 = \frac{l_2}{E I_2}$. These constants are the reciprocals of the rigidity constants defined by $r_1 = \frac{E I_1}{l_1}$ and $r_2 = \frac{E I_2}{l_2}$.

If the I-values are unequal, the modified T-line is located (as established in Section 7) by dividing the distance between the adjacent U-V lines in the inverse ratio, $e_2 : e_1$, or in the direct ratio, $r_1 : r_2$.

Several constructions are available for applying Equation (15) for fixing the height of the T-points. The following methods are submitted:

1.—Locate the T-points by laying off the heights:

$$U = -2r\phi;\ V = 2r\phi \qquad (16)$$

in each span, and then applying the usual construction. This is illustrated, in application to an actual problem, in Fig. 27. In order to show the respective treatments, one end of the continuous beam is assumed to be freely supported and the other end to be fixed. The values of the slope changes, ϕ, are scaled directly from the settlement diagram, as indicated.

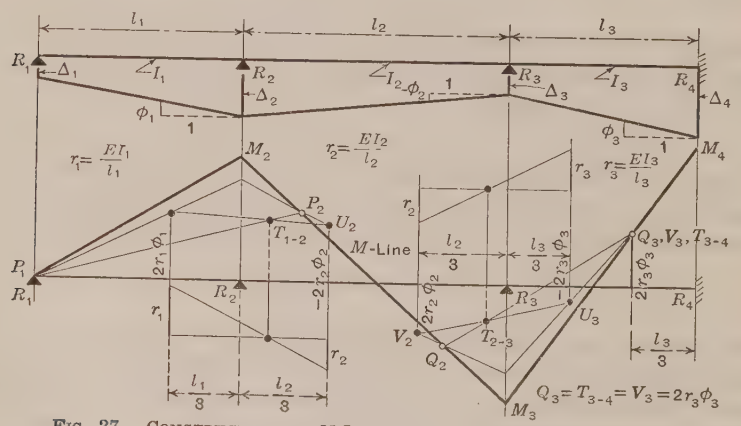


FIG. 27.—CONSTRUCTION OF M-LINE FOR SETTLEMENT OF SUPPORTS USING SLOPE CHANGES (ϕ).

2.—Locate each T-point directly by laying off its height as prescribed by Equation (15), namely,

$$T_{1-2} = 2\,\frac{\phi_1 - \phi_2}{e_1 + e_2}$$

This method is illustrated in Fig. 28. The values of the angle changes, $\phi_1 - \phi_2$, are scaled directly from the settlement diagram, as indicated.

3.—The device of transformed spans (as presented in Section 8) can be applied, using the values of U, V, and T defined by Equations (15) and (16) without change.

4.—If the successive spans are equal in length and in moment of inertia, the value of T reduces to:

$$T_{1-2} = \frac{2\,E\,I}{l^2}\,d_{1-2}$$

in which, d_{1-2} is the deflection of the support relative to the two adjoining supports. If straight lines are drawn joining the alternate supports, R_1 and R_3, in the settlement diagram, the distance of the intermediate support, R_2, below such chords is the value of the relative deflection, d_{1-2}. The heights of the T-points may be laid off equal to these values of d_{1-2}, d_{2-3}, etc., in which case the graphic results for the moments must be multiplied by the constant, $2\,\frac{E\,I}{l^2}$. (This method is illustrated in Fig. 29.)

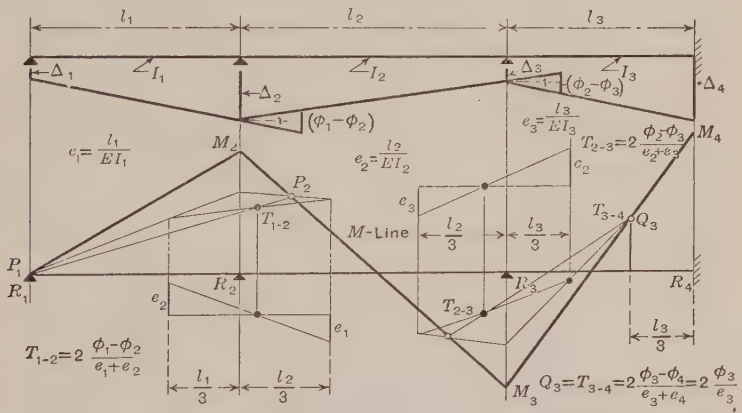


FIG. 28.—CONSTRUCTION OF M-LINE FOR SETTLEMENT OF SUPPORTS USING ANGLE CHANGES $(\phi_1 - \phi_2)$.

18.—APPLICATION TO GRAPHIC METHODS FOR SECONDARY STRESSES

Bridge engineers have felt the need for practical methods of quickly ascertaining approximate values of secondary stresses in trusses. The standard methods of secondary stress analysis are, at the best, long and tedious; they require the formulation and solution of a long string of simultaneous equations, covering the moment relations at all the panel points of a structure, before the secondary stresses at a single selected point can be determined. The engineer, in practice, is satisfied to know the approximate magnitude of the secondary stresses at a few indicated points of a structure; the academic methods yield a degree of precision which is not warranted by the accuracy of the underlying assumptions nor by practical requirements.

The constructions and principles presented in the foregoing Sections can be applied in devising short-cut, graphic methods for the determination of secondary stresses; the results will be sufficiently accurate for the usual requirements of the bridge engineer. Three methods for the graphic evaluation of secondary stresses by conjugate points will be presented, that offer three successively increasing degrees of accuracy of results:

1.—The first method is an approximate method, neglecting the influence of members of comparatively small rigidity. It is illustrated in Fig. 29 in application to the top chord of a truss, with the influence of the web members regarded as negligible. The known vertical deflections of the panel points are plotted in a deflection diagram. As the successive members are equal in length and in moment of inertia, the construction of Section 17, Method 4,

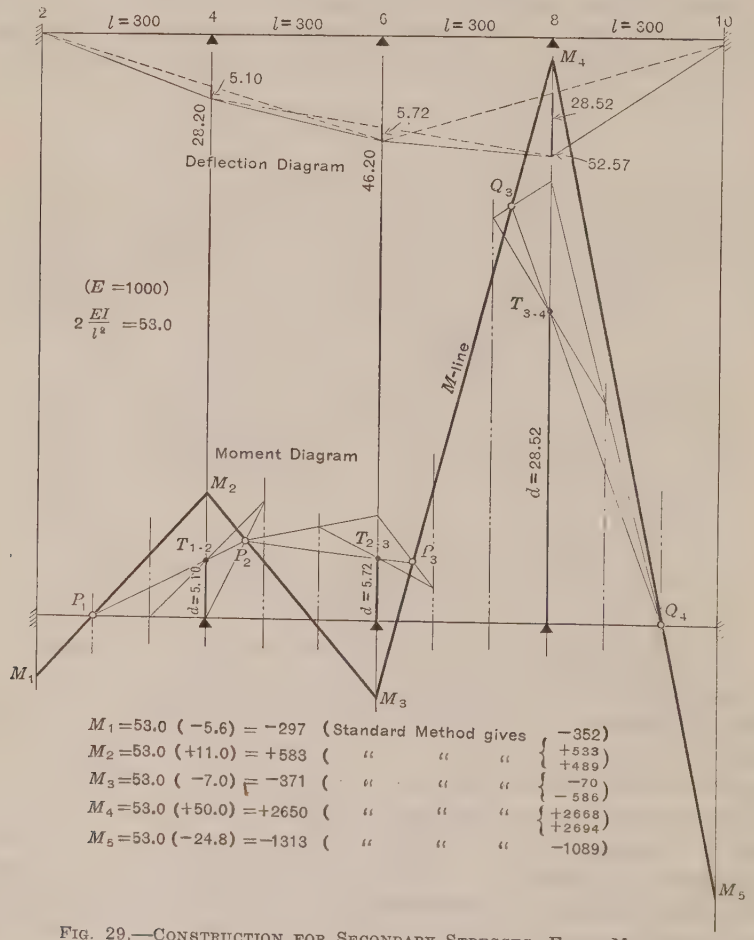


FIG. 29.—CONSTRUCTION FOR SECONDARY STRESSES, FIRST METHOD. (NEGLECTING WEB MEMBERS).

can be applied. The deflection of each intermediate panel point below the chord joining the adjacent panel points is scaled from the deflection diagram and is plotted as the height of the T-point on the corresponding reaction line of the moment diagram. Drawing pennant diagrams through these T-points, the required conjugate points are located as shown, and the closing line drawn

through these conjugate points represents the required secondary bending moments throughout the length of the chord. The ends of the chord are assumed to be restrained, which approximates the actual condition, so that the M-line passes through the one-third points of the end members.

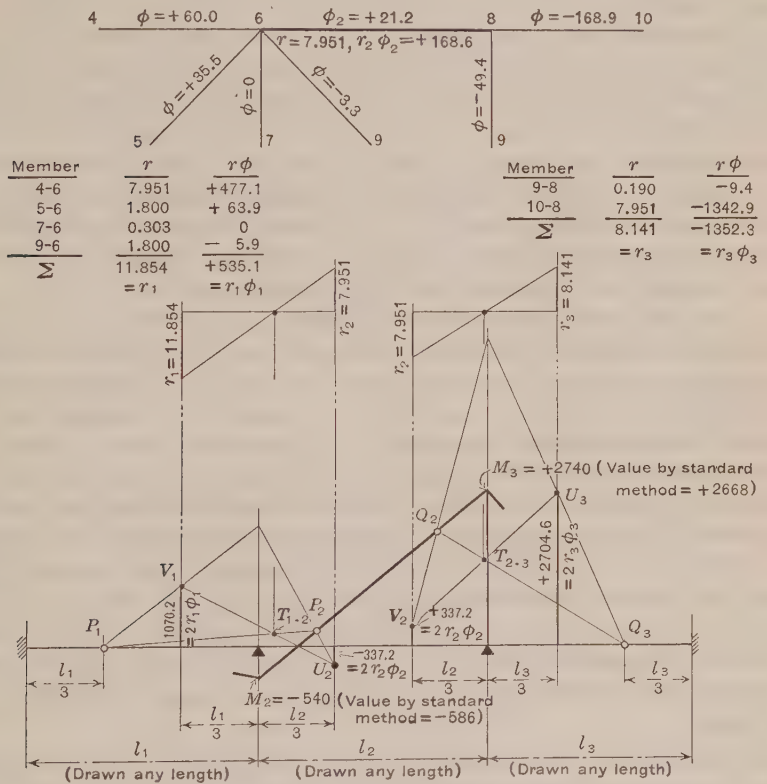


FIG. 30.—CONSTRUCTION FOR SECONDARY STRESSES, SECOND METHOD. (ALLOWING FOR WEB MEMBERS.)

2.—The second method takes into account the influence of the contiguous web members, but neglects the effect of the unknown moments at the outer ends of the contiguous members. It is based on a combination of the principles presented in Sections 7, 9, and 17. It is illustrated in Fig. 30 in application to a top chord member (6-8) of a truss. All truss members meeting the given member are taken into consideration. Their rigidities, $r = \frac{E\,I}{l}$ and their rotations, $\phi = \frac{d}{l}$ (determined from a Williot diagram which need not include more than the few members concerned) are noted in the diagram. The four members meeting the left end of the chord (6-8) are replaced by a single equivalent member, using the principle of the Composition of Rigidities;

the rigidity, r_1, of the equivalent member is the sum of the rigidities, r, of the component members, and the value of r_1, ϕ_1, for the equivalent member is the sum of the values of $r\,\phi$ for the component members. Similarly, the two members meeting the right end of the chord (6-8) are replaced by a single equivalent member of rigidity, r_3, and rotation, ϕ_3. The chord (6-8) itself has a rigidity, r_2, and a rotation, ϕ_2. This chord may now be considered as the middle span, l_2, of a three-span continuous beam, the side spans being formed by the left and right "equivalent members", l_1 and l_3, respectively. The spans, l_1, l_2, l_3, may be drawn to any respective lengths. The values of $2\,r\,\phi$ are then plotted on the $\frac{1}{3}$-lines to give the U and V-points; the straight lines joining these points locate the points, T_{1-2} and T_{2-3}, on the T-lines (which are located by the ratios, $r_1 : r_2$, and $r_2 : r_3$, as shown). Pennant diagrams through the two T-points give the conjugate points, P_2, Q_2, through which the required M-line is drawn. The initial conjugate points, P_1 and Q_3, are taken at the one-third points to comply with the assumption of restrained ends.

3.—The third method also takes into account all the contiguous members; in addition, it allows for the effect of any known moments at the outer ends of the contiguous members. It is illustrated in Fig. 31 in application to a chord member (4-6) of a truss. This is a member of comparatively low secondary stress adjoining a member of high secondary stress (namely, the member (6-8), investigated in Fig. 30). The neglect of the large bending moment, M_4, transmitted from the adjoining member would produce an error in the results, relatively large though numerically unimportant. The diagram shows the method of allowing for such end moments when known. These known end moments are plotted as M_1 and M_4 in the diagram. The M-line is then constructed exactly as in the preceding method (Fig. 30) except that the initial conjugate points, P_1 and Q_3, are taken at the points marking M_1 and M_4, respectively, as these are known points through which the M-line must pass.

For convenience of comparison, the foregoing graphic solutions (Figs. 29, 30, 31) have been applied to the truss problem used by C. V. von Abo, Jun. Am. Soc. C. E., in his paper on "Secondary Stresses in Bridges."* The comparisons of the respective graphic results with those determined by Mr. von Abo, who used the standard methods, are noted in the diagrams and indicate the following conclusions.

1.—The first graphic method (Fig. 29) furnishes a good general idea of the character and magnitude of the secondary stresses in any selected chain of principal members in a truss. The errors, expressed as a percentage of the maximum secondary stress in the chain of members, range from 1 to 11 per cent.

2.—The second graphic method (Fig. 30) gives fairly accurate results, particularly if applied to those members of a truss where the secondary stresses are a maximum. The two secondary stresses found in Fig. 30 differ from the

* *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 1.

respective correct values by less than 8% and 3%; or by less than 2% and 3%, respectively, when expressed in percentage of the greater secondary stress.

3.—The third graphic method (Fig. 31) can be applied to yield results as accurate as may be desired, if the transmitted end moments are known. In Fig. 31, with the end moments transmitted by the contiguous chord members allowed for, but with the end moments transmitted by the web members neglected, the results differ from the correct values by only 1% and 2%, respectively, of the maximum secondary stress in the adjacent member.

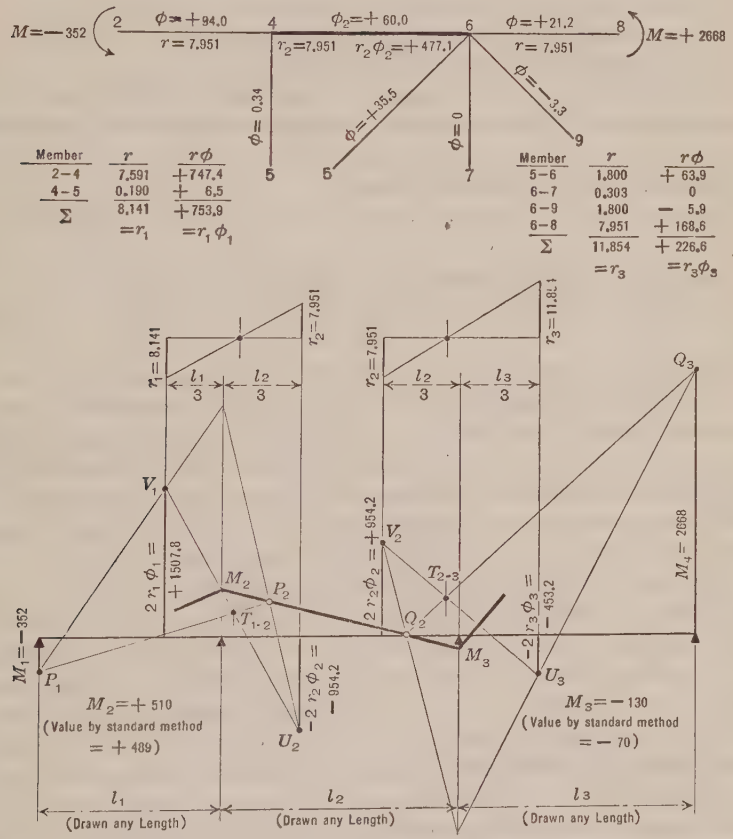


FIG. 31.—CONSTRUCTION FOR SECONDARY STRESSES, THIRD METHOD. (ALLOWING FOR WEB MEMBERS AND FOR END MOMENTS.)

In practical application, a member (such as 6-8, Fig. 30) in which maximum secondary stresses may be anticipated (from considerations of truss form and loading) would first be investigated by Method 2, and the results introduced as end moments for the investigation of successive adjoining members by Method 3. (Neglecting M_1 in Fig. 31 would not alter the results materially.)

With such procedure, fairly accurate values of the secondary stresses throughout the structure can be determined.

If approximate results suffice, the secondary stresses can be quickly estimated by Method 1 (Fig. 29).

Conclusion

The essential features of this paper are:

1.—The expression of the three-moment relation in a new, simplified form.

2.—The concept of conjugate points as a key to the geometrical solution of the three-moment equation.

3.—The general graphic construction for conjugate points.

4.—The modification for beams with restrained ends.

5.—The modification for unsymmetrical loading.

6.—Methods for treating the case of unequal moments of inertia.

7.—Extension of these methods to the graphic solution of indeterminate frames.

8.—Simplification for two-span beams.

9.—Extension of the graphic method to beams with varying moment of inertia.

10.—Application of the method to settlement of supports.

11.—Application of the method to the graphic solution of secondary stresses.

The foregoing methods have been devised by the writers as original solutions of the respective problems. That the resulting constructions would develop certain points of similarity to other solutions of the same problems is naturally to be expected. Nevertheless, after comparison with existing literature on the subject of continuous beams, the writers believe that their principal concepts and constructions are substantially new and offer sufficient advantages over previously published methods to justify presentation.

The Method of Conjugate Points, as devised and developed by the writers, is found to have a number of features in common with the older method of fixed points.* Some of these points of contact have been indicated in the paper. The two methods, however, are developed from different concepts; the procedure of fixed points has certain limitations in practical application; and the method of conjugate points is believed to be of greater flexibility and general usefulness. The latter method embraces the former as a special case.

The application of the concepts presented in this paper can be extended to the solution of many other problems of structural design. In the newer methods of stress analysis, there is a consistent trend toward simplification; the older, cumbersome, analytical procedures, frequently involving the solution of differential equations, are gradually being superseded by more attractive methods based on simple geometrical relations. As a contribution toward this objective, this paper is submitted to the profession.

* See "Continuous Bridges", in Hool and Kinne's "Movable and Long-Span Bridges", N. Y., 1923; also "Der kontinuierliche Balken", by Prof. W. Ritter, Zurich, 1900; and "Die Methode der Festpunkte", by Dr. Ernst Suter, Berlin, 1923.

APPENDIX

DEMONSTRATIONS OF THE FORMULAS AND CONSTRUCTIONS OF THE METHOD OF CONJUGATE POINTS

This Appendix gives the demonstrations of the principal working formulas and graphic constructions used in the paper. It has not been deemed necessary to include a detailed derivation of the special forms of the Equation of Three Moments, as the method of writing these equations has been sufficiently indicated in the text; moreover, these formulas are easily checked from the expressions usually given for the Theorem of Three Moments.

The sections of this Appendix are numbered to indicate the sections of the main paper to which they relate.

2.—Derivation of the Governing Equations for Conjugate Points

Two points, P_1 and P_2, located, respectively, in consecutive spans, l_1 and l_2, are conjugate points if any M-line drawn through them will satisfy the Equation of Three Moments which defines the true closing line. Consider any two M-lines drawn through such two points (Fig. 3). For the case of symmetrical loads and constant I, each of these lines must satisfy Equation (1).

From the geometry of Fig. 3:

$$\Delta M_1 = -\Delta M_2 \times \frac{l_1 - x_1}{x_1}\ ;\ \Delta M_3 = -\Delta M_2 \times \frac{l_2 - x_2}{x_2}$$

Substituting these relations in Equation (1):

$$\left(-\Delta M_2 \times \frac{l_1 - x_1}{x_1} + 2\,\Delta M_2\right) l_1 + \left(-\Delta M_2 \times \frac{l_2 - x_2}{x_2} + 2\,\Delta M_2\right) l_2 = 0$$

This expression reduces to:

$$\left(3 - \frac{l_1}{x_1}\right) l_1 = \left(\frac{l_2}{x_2} - 3\right) l_2 \qquad (2)$$

which is the equation fixing the abscissa relationship of the two conjugate points, P_1 and P_2.

Also, from the geometry of Fig. 3:

$$M_1 = M_2 - (M_2 - y_1)\frac{l_1}{x_1} = \left(1 - \frac{l_1}{x_1}\right) M_2 + \frac{l_1}{x_1}\, y_1$$

$$M_3 = M_2 - (M_2 - y_2)\frac{l_2}{x_2} = \left(1 - \frac{l_2}{x_2}\right) M_2 + \frac{l_2}{x_2}\, y_2$$

Substituting these two relations in Equation (1), and then subtracting the expressions of Equation (2), there results:

$$\frac{l_1^2\, y_1}{x_1} + \frac{l_2^2\, y_2}{x_2} = -3\,(A_1 + A_2) \qquad (3)$$

which defines the ordinate relationship of the two conjugate points, P_1 and P_2, in terms of the moment areas, A_1 and A_2.

4.—Proof of the Construction for Conjugate Points

When any three points lie on the same straight line, the height of the intermediate point is the weighted mean of the heights of the two outer points, the weight factor for each height being the horizontal distance between the two other points. This principle of collinear points will be frequently utilized in the following demonstrations.

Applying this principle to the construction shown in Fig. 4 for locating the T-point, there results:

$$\left(\frac{l_1}{3}+\frac{l_2}{3}\right) T_{1-2} = V_1\left(\frac{l_1}{3}\right) + U_2\left(\frac{l_2}{3}\right)$$

whence,

$$T_{1-2} = \frac{V_1\, l_1 + U_2\, l_2}{l_1 + l_2} = \frac{A_1 + A_2}{l_1 + l_2} \qquad \ldots\ldots(5)$$

The same result may also be written from the geometry of Fig. 4. Considering the triangles into which the diagram may be divided by the indicated dotted line, Fig. 4 yields:

$$T_{1-2} = \frac{l_1}{l_1 + l_2}\, V_1 + \frac{l_2}{l_1 + l_2}\, U_2 = \frac{A_1}{l_1 + l_2} + \frac{A_2}{l_1 + l_2} = \frac{A_1 + A_2}{l_1 + l_2} \ldots\ (5)$$

Equation (5) defines the height of the T-point as constructed.

Applying the same principle to the lines of Fig. 5, the following relations may be written:

As P_1, H_1, and B, are collinear:

$$h_1 = \frac{l_1}{3} \times \frac{y_1}{x_1} + \frac{b}{x_1}\left(x_1 - \frac{l_1}{3}\right) \qquad \ldots\ldots(a)$$

As B, P_2, and H_2, are collinear:

$$h_2 = \frac{l_2}{3} \times \frac{y_2}{x_2} + \frac{b}{x_2}\left(x_2 - \frac{l_2}{3}\right) \qquad \ldots\ldots(b)$$

As H_1, T_{1-2}, and H_2, are collinear:

$$h_1\, l_1 + h_2\, l_2 = T_{1-2}\,(l_1 + l_2) \qquad \ldots\ldots(c)$$

Substituting the values of h_1 and h_2 from Equations (a) and (b) in Equation (c):

$$\frac{l_1^2}{3} \times \frac{y_1}{x_1} + \frac{b\, l_1}{x_1}\left(x_1 - \frac{l_1}{3}\right) + \frac{l_2^2}{3} \times \frac{y_2}{x_2} + \frac{b\, l_2}{x_2}\left(x_2 - \frac{l_2}{3}\right) = T_{1-2}\,(l_1 + l_2) \quad \ldots(d)$$

By Equation (2), the terms containing b in Equation (d) cancel, leaving:

$$\frac{l_1^2}{3} \times \frac{y_1}{x_1} + \frac{l_2^2}{3} \times \frac{y_2}{x_2} = T_{1-2}\,(l_1 + l_2) = A_1 + A_2 \qquad \ldots\ldots(e)$$

This agrees with Equation (3). Hence the construction of Fig. 5 satisfies Equations (2) and (3) which govern the location of the conjugate points.

The change in sign of the right-hand member of Equation (e) represents the reversal of the M-diagram about the X-axis, as explained in Section 4.

A special case of Equation (*d*) obtains when y_1, y_2, and T_{1-2}, are all zero. In that case, the pennant diagram drops to the base line as represented in Fig. 9. On substituting these zero values, Equation (*d*) reduces to:

$$\frac{l_1}{x_1}\left(x_1 - \frac{l_1}{3}\right) + \frac{l_2}{x_2}\left(x_2 - \frac{l_2}{3}\right) = 0$$

which is identical with Equation (2). This is a check on the construction of Fig. 9.

If the right-hand member of Equation (1), representing the absolute or load term of the Three-Moment Formula, is denoted by N, Equation (5) shows that the height of the T-point must be:

$$T_{1-2} = -\frac{N}{3\,(l_1 + l_2)} \qquad (f)$$

in order that the construction may satisfy the Equation of Three Moments. If the T-points are plotted in accordance with Equation (*f*), the foregoing pennant-diagram construction will give the conjugate points, no matter what the value of N may be.

5.—Application to Beams with Restrained Ends

Applying the principle of collinear points to the construction of Fig. 12, if the M-line passes through U_1:

$$\frac{1}{3}M_2 + \frac{2}{3}M_1 = U_1 = \frac{A_1}{l_1}$$

This is identical with Equation (6); hence the construction of Fig. 12 satisfies Equation (6).

6.—Modification for Unsymmetrical Loading

By the construction of Fig. 14:

$$V_1 = \frac{g_1 l_1}{\frac{1}{2} l_1} \times \frac{A_1}{l_1} = 2\,g_1 \times \frac{A_1}{l_1} \qquad (a)$$

$$U_2 = \frac{g_2 l_2}{\frac{1}{2} l_2} \times \frac{A_2}{l_2} = 2\,g_2 \times \frac{A_2}{l_2} \qquad (b)$$

As V_1, T_{1-2}, and U_2, are collinear:

$$T_{1-2} = \frac{V_1 l_1 + U_2 l_2}{l_1 + l_2} \qquad (c)$$

Substituting Equations (*a*) and (*b*) in Equation (*c*):

$$T_{1-2} = 2\,\frac{g_1 A_1 + g_2 A_2}{l_1 + l_2} \qquad (d)$$

If the right-hand member of Equation (7) is denoted by N, Equation (*d*) yields:

$$T_{1-2} = -\frac{N}{3\,(l_1 + l_2)}$$

This relation is identical with Equation (*f*) of Section 4 of the Appendix; hence the Equation of Three Moments is satisfied.

7.—Modification for Unequal Moments of Inertia

The demonstrations for this case are parallel to those given in Sections 2 and 4 of the Appendix for the case of constant I.

The Equation of Three Moments for this case is Equation (8) instead of Equation (1). Consequently, Equations (2) and (3), which govern the location of the conjugate points, are modified as follows: As before, from the geometry of Fig. 3:

$$\Delta M_1 = -\Delta M_2 \times \frac{l_1 - x_1}{x_1}; \quad \Delta M_3 = -\Delta M_2 \times \frac{l_2 - x_2}{x_2}$$

Substituting these relations in Equation (8):

$$\left(-\Delta M_2 \times \frac{l_1 - x_1}{x_1} + 2\Delta M_2\right)\frac{l_1}{I_1} + \left(-\Delta M_2 \times \frac{l_2 - x_2}{x_2} + 2\Delta M_2\right)\frac{l_2}{I_2} = 0$$

This expression reduces to:

$$\left(3 - \frac{l_1}{x_1}\right)\frac{l_1}{I_1} = \left(\frac{l_2}{x_2} - 3\right)\frac{l_1}{I_2} \qquad (2')$$

which replaces Equation (2) for this case.

Also, from the geometry of Fig. 3:

$$M_1 = \left(1 - \frac{l_1}{x_1}\right) M_2 + \frac{l_1}{x_1} y_1$$

$$M_3 = \left(1 - \frac{l_2}{x_2}\right) M_2 + \frac{l_2}{x_2} y_2$$

Substituting these two relations in Equation (8) and then subtracting the expressions of Equation (2′), there results:

$$\frac{l_1^2 y_1}{I_1 x_1} + \frac{l_2^2 y_2}{I_2 x_2} = -3\left(\frac{A_1}{I_1} + \frac{A_2}{I_2}\right) \qquad (3')$$

which replaces Equation (3) for this case.

Equations (2′) and (3′) are now to be satisfied, instead of Equations (2) and (3), by the graphic construction for conjugate points. The only change introduced in the construction is the modified location of the T-line as shown in Fig. 15.

With this modified location, the height of the T-point will be (by the principle of collinear points applied to V_1, T_{1-2}, and U_2):

$$T_{1-2} = \frac{V_1\left(\frac{l_1}{I_1}\right) + U_2\left(\frac{l_2}{I_2}\right)}{\frac{l_1}{I_1} + \frac{l_2}{I_2}} = \frac{\frac{A_1}{I_1} + \frac{A_2}{I_2}}{\frac{l_1}{I_1} + \frac{l_2}{I_2}} \qquad (5')$$

which corresponds to Equation (5).

The geometrical relations of Fig. 5 may now be applied to this new location of the T-point.

As P_1, H_1, and B, are collinear:

$$h_1 = \frac{l_1}{3} \times \frac{y_1}{x_1} + \frac{b}{x_1}\left(x_1 - \frac{l_1}{3}\right) \ldots\ldots\ldots\ldots\ldots\ldots (a)$$

As B, P_2, and H_2, are collinear:

$$h_2 = \frac{l_2}{3} \times \frac{y_2}{x_2} + \frac{b}{x_2}\left(x_2 - \frac{l_2}{3}\right) \ldots\ldots\ldots\ldots\ldots\ldots (b)$$

As H_1, T_{1-2}, and H_2, are collinear:

$$h_1 \times \frac{l_1}{I_1} + h_2 \times \frac{l_2}{I_2} = T_{1-2}\left(\frac{l_1}{I_1} + \frac{l_2}{I_2}\right) \ldots\ldots\ldots\ldots (c)$$

Substituting the values of h_1 and h_2 from Equations (a) and (b) in Equation (c):

$$\frac{l_1^2}{3\,I_1} \times \frac{y_1}{x_1} + \frac{b\,l_1}{I_1\,x_1}\left(x_1 - \frac{l_1}{3}\right) + \frac{l_2^2}{3\,I_2} \times \frac{x_2}{x_2} + \frac{b\,l_2}{I_2\,x_2}\left(x_2 - \frac{l_2}{3}\right)$$

$$= T_{1-2}\left(\frac{l_1}{I_1} + \frac{l_2}{I_2}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (d)$$

By Equation (2′), the terms containing b in this Equation (d) cancel, leaving:

$$\frac{l_1^2}{3\,I_1} \times \frac{y_1}{x_1} + \frac{l_2^2}{3\,I_2} \times \frac{y_2}{x_2} = T_{1-2}\left(\frac{l_1}{I_1} + \frac{l_2}{I_2}\right) = \left(\frac{A_1}{I_1} + \frac{A_2}{I_2}\right) \ldots\ldots\ldots (e)$$

This agrees with Equation (3′). Hence the construction of Fig. 5, applied with the T-point located in accordance with Fig. 15, satisfies Equations (2′) and (3′) which prescribe the location of the conjugate points.

When $I_1 = I_2$, all the foregoing equations reduce to the simpler equations of Sections 2 and 4.

12.—Application to Beams with Varying Moment of Inertia

For this case, the Equation of Three Moments assumes its most general form, namely:

$$(C_1\,M_1 + D_1\,M_2)\,l_1 + (C_2\,M_2 + D_2\,M_3)\,l_2 = -\,(E_1\,A_1 + E_2\,A_2) \ldots (12)$$

The following demonstrations pertaining to the graphic solution of Equation (12) are parallel to the demonstrations given in connection with Equations (1) and (8). (Compare Sections 2, 4, and 7 of the Appendix.)

From the geometry of Fig. 3:

$$\Delta\,M_1 = -\,\Delta\,M_2\,\frac{l_1 - x_1}{x_1};\quad \Delta\,M_3 = -\,\Delta\,M_2\,\frac{l_2 - x_2}{x_2}$$

Substituting these relations in Equation (12), there results:

$$\left(-\,C_1\,\frac{l_1 - x_1}{x_1} + D_1\right)l_1 + \left(C_2 - D_2\,\frac{l_2 - x_2}{x_2}\right)l_2 = 0$$

This expression reduces to:

$$\left(C_1 + D_1 - \frac{l_1}{x_1}\,C_1\right)l_1 = -\left(C_2 + D_2 - \frac{l_2}{x_2}\,D_2\right)l_2 \ldots\ldots\ldots (2'')$$

which replaces Equation (2) as the general equation fixing the abscissa relationship of the two conjugate points, P_1 and P_2.

Also, from the geometry of Fig. 3:

$$M_1 = \left(1 - \frac{l_1}{x_1}\right) M_2 + \frac{l_1}{x_1} y_1$$

$$M_3 = \left(1 - \frac{l_2}{x_2}\right) M_2 + \frac{l_2}{x_2} y_2$$

Substituting these two relations in Equation (12) and then subtracting the expressions of Equation (2″), there results:

$$C_1 \frac{l_1^2 y_1}{x_1} + D_2 \frac{l_2^2 y_2}{x_2} = (E_1 A_1 + E_2 A_2) \ldots\ldots\ldots\ldots (3'')$$

which replaces Equation (3) as the general equation defining the ordinate relationship of the two conjugate points, P_1 and P_2.

Equations (2″) and (3″) are now to be satisfied, instead of Equations (2) and (3), by the graphic construction for conjugate points. The only changes introduced in the construction are in the more generalized locations of the U, V, and T-points as shown in Fig. 22.

With these modified locations, the heights of the U-V points are defined by:

$$V_1 = \frac{E_1}{C_1 + D_1} \times \frac{A_1}{l_1};\ U_2 = \frac{E_2}{C_2 + D_2} \times \frac{A_2}{l_2} \ldots\ldots\ldots\ldots (4'')$$

and the resulting height of the T-point will be (by the principle of collinear points applied to V_1, T_{1-2}, and U_2):

$$T_{1-2} = \frac{(C_1 D_1) l_1 \times V_1 + (C_2 + D_2) l_2 \times U_2}{(C_1 + D_1) l_1 + (C_2 + D_2) l_2}$$

$$= \frac{E_1 A_1 + E_2 A_2}{(C_1 + D_1) l_1 + (C_2 + D_2) l_2} \ldots\ldots\ldots\ldots\ldots\ldots (5'')$$

which is a generalized form of Equation (5).

The geometrical relations of Fig. 5 may now be applied to these new locations of the T-point and the U-V lines.

As P_1, H_1, and B, are collinear:

$$h_1 = \frac{C_1}{C_1 + D_1} \times \frac{l_1}{x_1} \times y_1 + b \left(1 - \frac{C_1}{C_1 + D_1} \times \frac{l_1}{x_1}\right) \ldots\ldots\ldots (a)$$

As B, P_2, and H_2, are collinear:

$$h_2 = \frac{C_2}{C_2 + D_2} \times \frac{l_2}{x_2} \times y_2 + b \left(1 - \frac{D_2}{C_2 + D_2} \times \frac{l_2}{x_2}\right) \ldots\ldots\ldots (b)$$

As H_1, T_{1-2}, and H_2, are collinear:

$$h_1 (C_1 + D_1) l_1 + h_2 (C_2 + D_2) l_2 = T_{1-2} [(C_1 + D_1) l_1 + (C_2 + D_2) l_2] \ldots (c)$$

Substituting the values of h_1 and h_2 from Equations (a) and (b) in Equation (c):

$$C_1 l_1^2 \frac{x_1}{y_1} + b \left[(C_1 + D_1) l_1 - C_1 \frac{l_1^2}{x_1}\right] + C_2 l_2^2 \frac{x_2}{y_2}$$

$$+ b \left[(C_2 + D_2) l_2 - D_2 \frac{l_2^2}{x_2}\right] = T_{1-2} [(C_1 + D_1) l_1 + (C_2 + D_2) l_2]$$

$$= (E_1 A_1 + E_2 A_2) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (d)$$

By Equation (2″), the terms containing b in this Equation (d) cancel, leaving:

$$C_1 l_1^2 \frac{y_1}{x_1} + C_2 l_2^2 \frac{y_2}{x_2} = (E_1 A_1 + E_2 A_2) \ldots\ldots\ldots\ldots\ldots (e)$$

This agrees with Equation (3″). Therefore, the construction of Fig. 5, applied with the U-V lines and the T-point located in accordance with Fig. 22, satisfies the generalized Equations (2″) and (3″) which prescribe the location of the conjugate points.

Instead of using the equational coefficients, C, D, and E, the beam constants, m, n, u, and v, and the load constants, f, g, may be used for plotting the U, V, and T-points.

By definition:

$$m = \frac{1}{l} \int \frac{l - x}{I\, l}\, d\, x = \frac{1}{l^2} \int \frac{l - x}{I}\, d\, x$$

$$n = \frac{1}{l} \int \frac{x}{I\, l}\, d\, x = \frac{1}{l^2} \int \frac{x}{I}\, d\, x$$

$$u = \frac{\int \frac{x\,(l - x)}{I\, l}\, d\, x}{l \int \frac{l - x}{I\, l}\, d\, x} = \frac{1}{m\, l^3} \int \frac{x\,(l - x)}{I}\, d\, x$$

$$v = \frac{\int \frac{x\,(l - x)}{I\, l}\, d\, x}{l \int \frac{x}{I\, l}\, d\, x} = \frac{1}{n\, l^3} \int \frac{x\,(l - x)}{I}\, d\, x = \frac{m}{n}\, u$$

$$f = \frac{\int \frac{y}{I}\, dx}{\int y\, dx} = \frac{1}{A} \int \frac{y}{I}\, dx$$

$$g = \frac{\int \frac{x\, y}{I}\, d\, x}{l \int \frac{y}{I}\, d\, x} = \frac{1}{A\, f\, l} \int \frac{x\, y}{I}\, d\, x$$

in which, y denotes the ordinates of the simple-beam moment area, and the integrations extend over the length of the span.

Hence, by definition:

$$C_1 = m_1 u_1 = \frac{1}{l_1^3} \int_0^{l_1} \frac{x\,(l_1 - x)}{I_1}\, d\, x = n_1 v_1$$

$$D_1 = n_1\,(1 - v_1) = n_1 - C_1$$

Therefore,

$$C_1 + D_1 = n_1$$

$$E_1 = f_1 g_1 = \frac{1}{A_1 l_1} \int_0^{l_1} \frac{x\, y_1}{I_1}\, d\, x$$

and,

$$D_2 = n_2 v_2 = \frac{1}{l_2^3} \int_0^{l_2} \frac{x (l_2 - x)}{I_2} d x = m_2 u_2$$

$$C_2 = m_2 (1 - u_2) = m_2 - D_2$$

Therefore,

$$C_2 + D_2 = m_2$$

$$E_2 = f_2 (1 - g_2) = \frac{1}{A_2 l_2} \int_0^{l_2} \frac{(l - x) y_2}{I_2} d x$$

From these expressions, the following simple relations may be written:

$$\frac{C_1}{C_1 + D_1} = v_1 \; ; \; \frac{D_1}{C_1 + D_1} = 1 - v_1$$

$$\frac{C_2}{C_2 + D_2} = 1 - u_2 \; ; \; \frac{D_2}{C_2 + D_2} = u_2$$

$$\frac{E_1}{C_1 + D_1} = \frac{f_1 g_1}{n_1} \; ; \; \frac{E_2}{C_2 + D_2} = \frac{f_2 (1 - g_2)}{m_2}$$

These relations explain the alternative values indicated in Fig. 22 for locating the U, V, and T-points.

For unsymmetrical conditions, Fig. 22 indicates how the U-V points may be located from the O-point by making the height of the latter:

$$O = \frac{f}{4 m u} \times \frac{A}{l} = \frac{f}{4 n v} \times \frac{A}{l}$$

The resulting heights of the U-V points, by this construction, are:

$$V_1 = \frac{v_1 l_1}{\frac{1}{2} l_1} \times \frac{g_1 l_1}{\frac{1}{2} l_1} \times \frac{f_1}{4 n_1 v_1} \times \frac{A_1}{l_1} = \frac{f_1 g_1}{n_1} \times \frac{A_1}{l_1} = \frac{E_1}{C_1 + D_1} \times \frac{A_1}{l_1}$$

$$U_2 = \frac{u_2 l_2}{\frac{1}{2} l_2} \times \frac{(1 - g_2) l_2}{\frac{1}{2} l_2} \times \frac{f_2}{4 m_2 u_2} \times \frac{A_2}{l_2}$$

$$= \frac{f_2 (1 - g_2)}{m_2} \times \frac{A_2}{l_2} = \frac{E_2}{C_2 + D_2} \times \frac{A_2}{l_2}$$

These values agree with Equation (4″), which is a check on the indicated construction.

17.—Application to Settlement of Supports

It was shown in Section 4 of the Appendix that if N denotes the right-hand member of a Three-Moment Equation for constant I, the equation will be satisfied by the conjugate point construction if:

$$T_{1-2} = -\frac{N}{3 (l_1 + l_2)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (f)$$

For settlement of supports, the Theorem of Three Moments is expressed by Equation (13); consequently, for this case:

$$N = -6\,E\,I\,(\phi_1 - \phi_2)$$

Hence, the necessary height of the T-point is:

$$T_{1\ 2} = 2\,E\,I\,\frac{\phi_1 - \phi_2}{l_1 + l_2} = 2\,\frac{\phi_1 - \phi_2}{e_1 + e_2}$$

which checks Equation (15).

For unequal values of I, it is shown in Section 7 of the Appendix (comparing Equation (5′) with Equation (8)) that the necessary height of the T-point is:

$$T_{1-2} = -\frac{N}{3\left(\frac{l_1}{I_1} + \frac{l_2}{I_2}\right)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (f')$$

For this case (settlement of supports, with unequal I), Equation (14) gives:

$$N = -6\,E\,(\phi_1 - \phi_2)$$

Substituting this value in Equation (f'), there results:

$$T_{1\ 2} = 2\,E\,\frac{\phi_1 - \phi_2}{\frac{l_1}{I_1} + \frac{l_2}{I_2}} = 2\,\frac{\phi_1 - \phi_2}{e_1 + e_2}$$

which again checks Equation (15).

The location of the T-line for settlement of supports is conveniently accomplished by applying the ratio, $\frac{r_1}{r_2} = \frac{e_2}{e_1}$, as shown in Figs. 27 and 28. If E is constant, this ratio is identical with the ratio, $\frac{l_2}{I_2} : \frac{l_1}{I_1}$, applied in Fig. 15; and if I is also constant, the latter ratio reduces to $l_2 : l_1$, applied in Fig. 4.

If the construction shown in Fig. 27 is used,

$$V_1 = 2\,r_1\,\phi_1;\ U_2 = -2\,r_2\,\phi_2.$$

As V_1, T_{1-2}, and U_2, are collinear:

$$T_{1-2} = \frac{2\,r_1\,\phi_1 \times r_2 - 2\,r_2\,\phi_2 \times r_1}{r_1 + r_2} = 2\,\frac{\phi_1 + \phi_2}{\frac{1}{r_1} + \frac{1}{r_2}} = 2\,\frac{\phi_1 - \phi_2}{e_1 + e_2}$$

which satisfies Equation (15).

DISCUSSION

F. E. Richart,* Assoc. M. Am. Soc. C. E. (by letter).—The method of analysis of indeterminate beams and frames given in this paper is excellent. The easy and rapid solutions presented should be welcomed by engineers as a means of bringing about a greater familiarity with, and consequently a greater use of, such structures. It appears, however, since no reference was made to it, that the authors were not aware of a method of graphical analysis quite similar to the one presented. Many of the essential constructions presented in this paper were published in 1883 by Fidler;† others were added by Ostenfeld‡ in 1913, or before.

Fidler in his original paper utilized the same properties of the simple-beam moment diagrams for the various spans of a continuous beam; provided for the location of the points, U_1, V_1, etc. (called by Fidler "characteristic points"), on the vertical through the one-third points of spans; fixed the location of the T-points on a line between adjacent U and V-points by the transposition of the one-third span lengths; and provided a means of locating the points here designated conjugate points and of drawing the negative moment lines, or M-lines, which constitute the solution of the problem. Fidler also devised constructions for the cases of unsymmetrical loadings, variations in depth of member along a span, and settlement of supports. He emphasized a physical concept of the construction that is not used by the authors, namely, that the distance from the characteristic points, U_1, V_1, to the M-line is proportional to the slopes or angular displacements of the elastic curve at the respective adjacent supports. This viewpoint is of particular value when the slope is known from conditions of restraint or of symmetry of loading.

Ostenfeld's graphical construction (corresponding to the "pennant diagrams" of the paper) replaced a simple calculation in Fidler's method by which the authors' "conjugate points", were located. Both the Fidler and Ostenfeld constructions were described by the writer some years ago in a paper§ wherein the method was extended to the analysis of a single column-and-girder frame, a hinged portal frame, and a frame consisting of four columns and three girders similar to that of Fig. 18. For the latter frame the method of replacing the several unloaded members meeting at a joint by a single "equivalent" member (designated by the authors as the Composition of Rigidities), and an equation for the length of the equivalent member, were given. The modification of the method for unequal moments of inertia by "transformed spans" was also shown, as well as a treatment of members having various degrees of restraint at the ends.

Since the analysis by Fidler and Ostenfeld, although derived from a different viewpoint, arrived at constructions practically identical with those of the authors, it may be of interest to consider the reasoning used in Fidler's

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† *Minutes of Proceedings*, Inst. C. E., Vol. LXXIV, 1883, p. 196, and "A Practical Treatise on Bridge Construction", by Thomas Claxton Fidler, Fourth Edition, Lond., 1909.

‡ "Teknisk Statik", by A. Ostenfeld, Vol. II, Second Edition, Copenhagen, 1913. Professor Ostenfeld added certain graphical constructions to Fidler's method in making a very general application to various forms of continuous beams.

§ "Graphical and Mechanical Analyses of Frames", by F. E. Richart and W. M. Wilson, *Engineering and Contracting*, Vol. 55, June, 1920, p. 700.

original demonstration. (Fidler made use of the effective depth of the girder rather than of the moment of inertia, but the substitution of the latter by Ostenfeld was an obvious step.) For convenience, the notation of the paper will be used wherever possible.

Consider, first, the simple beam of Fig. 32, having constant values of E and I and subject to two concentrated loads. The centroid of the moment diagram is at a distance, gl, from the end, R_1, and the area of the moment diagram is A. The slopes of the elastic curve at R_1 and R_2 are denoted by θ_1 and θ_2, respectively. By the well-known theorem of area moments the deflection at R_1 of the elastic curve from the line drawn tangent to the curve at R_2 is $l\,\theta_2$ and is equal to the statical moment of the $\frac{M}{E\,I}$ - diagram about R_1, or,

$$l\,\theta_2 = \frac{A\,g\,l}{E\,I}$$

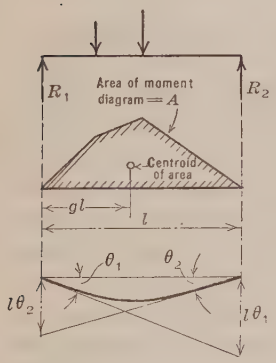

Fig. 32.

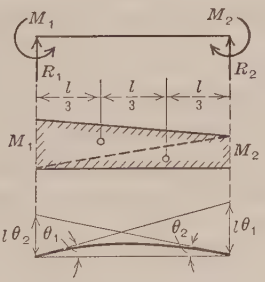

Fig. 33.

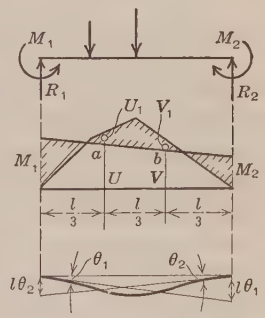

Fig. 34.

Now, consider the beam of Fig. 33 which is the same as that of Fig. 32, but is acted upon by external moments at the ends instead of by transverse loads. The moment diagram is a trapezoid varying in height from M_1 at R_1 to M_2 at R_2. Again, the deflection $l\,\theta_2$ at R_1 is equal to the statical moment of the $\frac{M}{E\,I}$ - diagram about R_1, or,

$$l\,\theta_2 = -\frac{M_1\,l^2}{6\,E\,I} - \frac{M_2\,l^2}{3\,E\,I}$$

In Fig. 34, the same beam is shown subjected simultaneously to the concentrated loads and end moments of both preceding examples; in other words, it represents a typical span of a continuous beam. The moment diagrams of Figs. 32 and 33, when superimposed, or added algebraically, produce the moment diagram shown in Fig. 34. From the foregoing theory, the deflection at R_1 is:

$$l\,\theta_2 = \frac{A\,g\,l}{E\,I} - \frac{M_1\,l^2}{6\,E\,I} - \frac{M_2\,l^2}{3\,E\,I} \qquad (17)$$

and the slope at R_2 multiplied by $\frac{2\,E\,I}{l}$ is:

$$\frac{2\,E\,I}{l}\,\theta_2 = \frac{2\,A\,g}{l} - \frac{M_1}{3} - \frac{2\,M_2}{3} \qquad (18)$$

In a similar way it may be shown that,

$$\frac{2\,E\,I}{l}\,\theta_1 = \frac{2\,A\,(1-g)}{l} - \frac{M_2}{3} - \frac{2\,M_1}{3} \qquad (19)$$

In Fidler's construction the span is divided into three equal parts (Fig. 34) and ordinates are erected at the points, U and V, making UU_1 equal to $\frac{2\,A\,(1-g)}{l}$ and $V\,V_1$ equal to $\frac{2\,A\,g}{l}$. From Equations (18) and (19) it is seen that the distances, $a\,U_1$ and $b\,V_1$, are thus made equal to $\frac{2\,E\,I}{l}\,\theta_1$ and $\frac{2\,E\,I}{l}\,\theta_2$, respectively. The points, U_1 and V_1, are called characteristic points and are the U and V-points of the paper, for the case of unsymmetrical loading shown in Fig. 14. For symmetrical loading, g is equal to $\frac{1}{2}$ and both the ordinates become equal to $\frac{A}{l}$, as is also shown in the paper. Since the distances, aU_1 and bV_1, are proportional to the slopes of the elastic curve at their respective adjacent supports, it is seen that, if the beam is fixed at R_1 and R_2, these distances are zero. If a partial restraint is offered at R_1 and R_2, the slopes at these points are inclined downward toward mid-span and the points, a and b, fall below U_1 and V_1. Conversely, if the end moments are sufficient to produce slopes at A and B that are inclined upward toward mid-span, a and b will lie above the characteristic points, U_1 and V_1.

Now, referring to Fig. 35, since adjacent spans of a continuous beam have the same slope at their common supports, it follows that the distances, aV_1 and bU_2, vary directly as the respective values of $\frac{E\,I}{l}$ for the two spans. If $\frac{E\,I}{l}$ is constant, aV_1 and bU_2 are equal and the closing M-line passes as far below the characteristic point, V_1, in one span as it does above the corresponding point, U_2, in the next span. If only EI is constant, aV_1 and bU_2 vary inversely as the values of l_1 and l_2, respectively. Fidler showed that with these limiting conditions the location of the M-line might be made by "cut-and-try" methods; but he also gave the following direct method of construction. The three-span continuous beam of Fig. 35 is considered to have a constant value of EI throughout with ends freely supported at R_1 and R_4. The simple-beam moment diagram for each span is first drawn and the characteristic points, U_1, V_1, etc., are located. The characteristic points, V_1, U_2 and V_2, U_3, are joined by straight lines and the points, T_{12} and T_{23}, located by transposing the one-third span distances.

The M-line is now to be drawn so that $aV_1 \times l_1$ equals $bU_2 \times l_2$; this is done by properly locating the points, P_2 and P_3, on the M-line. If the distance, $R_2\,W$, is divided so that $\frac{W\,J_2}{R_2\,J_2} = \frac{R_1\,V}{R_2\,R_3}$, and the point, P_2, is located vertically above J_2 on the line, R_1T_{12}, produced, then P_2 will lie on the required M-line. For,

assuming the required M-line, cP_2b, to be correctly located in the diagram, it follows that:

$$a\,f = \frac{R_1\,V}{R_1\,R_2}\,c\,h = \frac{R_1\,V}{R_1\,R_2}\,\frac{R_2\,R_3}{R_1\,V}\,b\,d = \frac{R_2\,R_3}{R_1\,R_2}\,b\,d$$

and

$$V_1\,f = U_2\,d\,\frac{R_2\,R_3}{R_1\,R_2}$$

whence,

$$a\,V_1 = V_1\,f - a\,f = \frac{R_2\,R_3}{R_1\,R_2}\,(U_2\,d - b\,d)$$

therefore,

$$a\,V_1 \times l_1 = b\,U_2 \times l_2$$

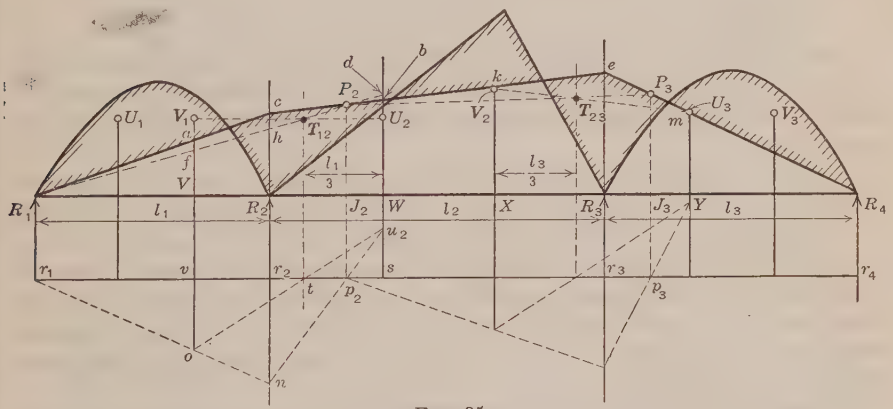

FIG. 35.

Proceeding to the next span the distance, R_3Y, must be divided so that $\frac{Y\,J_3}{R_3\,J_3} = \frac{J_2\,X}{J_2\,R_3} \times \frac{l_2}{l_3}$. With J_3 thus located, the point, P_3, will lie at the intersection of a vertical through J_3 and the line, P_2T_{23}, produced. By a proof similar to that of the preceding paragraph, it may be shown that this construction will result in making the distance, $V_2k \times l_2 = U_3m \times l_3$. This satisfies the requirement that the distance from the M-line to the characteristic points, V_2 and U_3, shall be inversely proportional to the respective span lengths.

There are now two known points on the M-line in the third span and one in each of the first and second spans, so that the M-line may be drawn, starting from R_4, through P_3 to e, thence through P_2 to c and from c to R_1. The resultant moment diagram is represented by the area between the simple-beam moment diagram and the M-line.

Obviously, the method might be applied to a beam having more than three spans by a repetition of the typical construction used in locating the point, J_3, thus locating a similar point, J_n, in each span.

It is seen that Fidler's method requires the division of the distances, R_2W and R_3Y, into two parts according to certain fixed proportions. A graphical method of accomplishing this division was devised by Ostenfeld.* Referring

* "Teknisk Statik". Vol. II. Second Edition. Copenhagen, 1913.

again to Fig. 35, it is required that the distance, R_2W, be divided so that $\frac{W J_2}{R_2 J_2} = \frac{R_1 V}{R_2 R_3}$. It is convenient to draw this construction entirely separate from the moment diagram as shown in the lower part of Fig. 35, all vertical R, U, V, and T-lines being produced to intersect the line, $r_1 r_4$. Starting at r_1 any line, $r_1 n$, is drawn to intersect the R_2-line. From the point, o, a second line through t intersects the U_2-line at u_2, and the third line, $u_2 n$, cuts the base line at p_2, which, when projected vertically upward to meet the line, $R_1 T_{12}$, locates the point, P_2, as desired.

The proof of the construction is as follows:

$$\frac{W J_2}{R_2 J_2} = \frac{u_2 s}{r_2 n} = \frac{u_2 s}{o v} \times \frac{R_1 V}{R_1 R_2} = \frac{R_1 R_2}{R_2 R_3} \times \frac{R_1 V}{R_1 R_2} = \frac{R_1 V}{R_2 R_3}$$

Similarly, starting from p_2, which is the projection of a known point on the M-line, a second pennant diagram is constructed to locate the point, p_3, which, when projected vertically upward to meet the line, $P_2 T_{23}$, locates the point, P_3.

It is seen that Ostenfeld's construction is identical with the "pennant diagram" of Fig. 9. The construction has not been interwoven with the other construction lines of the moment diagram as in the paper, and the writer feels that the separate diagram is preferable in many ways. Since it is independent of the distribution of loading, the Ostenfeld diagram may be drawn entirely as soon as the R, U, V, and T-lines are drawn. The construction can be checked by drawing a second diagram beginning at the right-hand end of the beam.

It has been noted that Fidler's derivations included applications of the method (similar to, but less detailed than, those of the paper) to the cases of settlement of supports and of a variation of moment of inertia along the beam. It is clear that applications may also be made for various conditions of restraint of the extreme ends of the continuous beam. If the ends of a beam are fixed as in Fig. 36, the distance from the adjacent characteristic point, U_1, to the M-line, which is equal to $\frac{2 E I}{l_1}$ times the end slope, is obviously zero, and the M-line goes through the point, U_1. It also follows that, since U_1 becomes the known point on the M-line, that the Ostenfeld diagram also begins at the U_1-line instead of at the reaction line through R_1.

For the case of a known partial restraint of the ends, as might be the case of a test beam on which observations of end slopes have been made, the distances from the M-line to the U and V-points adjacent to the ends of the beam may be calculated. Fig. 37 shows such a beam, for which the end slopes are θ_1, and θ_4. The distances, $U_1 U$ and $V_3 V$, are equal, respectively, to $\frac{2 E I \theta_1}{l_1}$ and $\frac{2 E I \theta_4}{l_3}$, and, as the slopes indicate a degree of restraint less than fixity, the M-line will pass below the points, U_1 and V_3. This beam also illustrates the variation in construction for the case of an unloaded span. The char-

acteristic points for the second span fall upon the base line, R_1R_4, and all other lines are drawn in the regular way. Since the known points on the M-line are directly below U_1 and V_3, the Ostenfeld diagram is started vertically below one or the other of these points as shown.

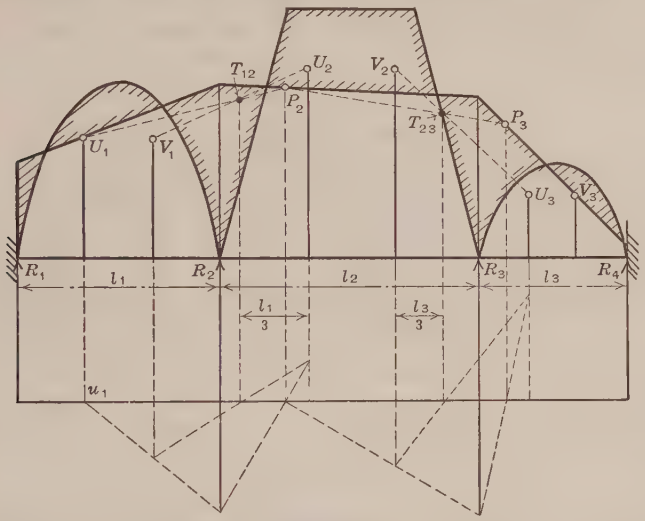

FIG. 36.

If the restraint at the end is due to a known moment such as that due to a cantilever bracket or projection, the M-line may obviously start with a negative cantilever moment at the first reaction, R_1, and similarly the Ostenfeld diagram will start from the reaction line, R_1. The construction is illustrated in Fig. 38.

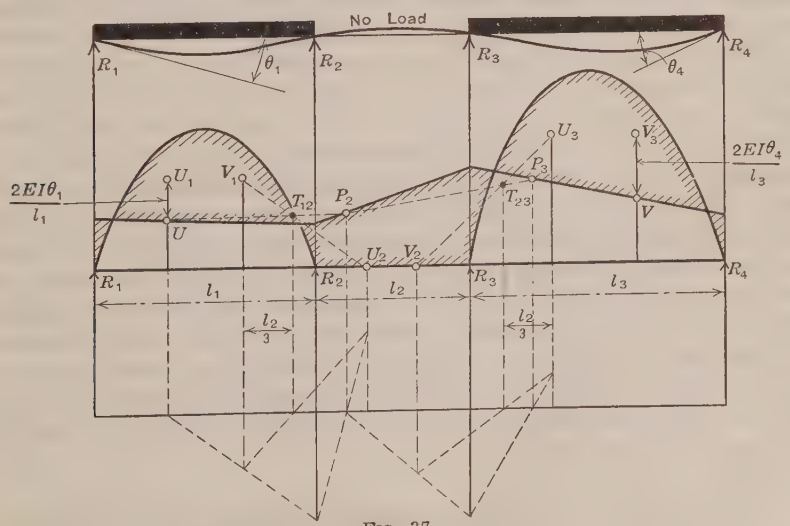

FIG. 37.

It is interesting that analyses proceeding from two distinct viewpoints should result in graphical constructions so nearly identical. The authors are to be commended on the treatment of secondary stresses and of variable moments of inertia of members. The tabulation of constants for beams having haunches of various shapes and sizes places the results of a great many analyses in a convenient form for reference, and gives a kind of information

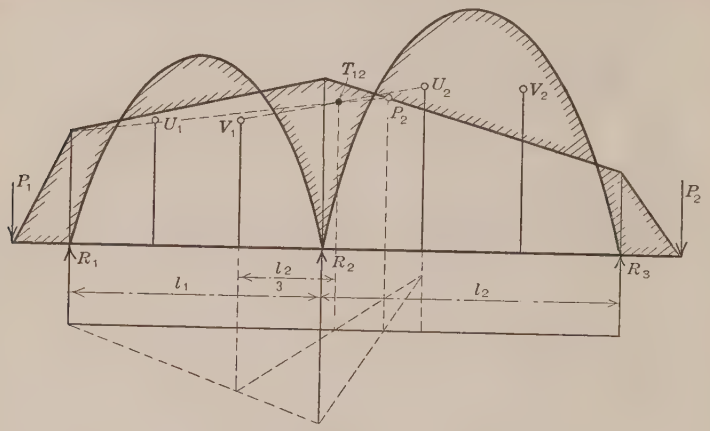

FIG. 38.

that is not found in most text and reference books. The application of the concept of conjugate points to continuous beam analysis gives a new and apparently useful line of attack on the problem, and the whole paper should be of distinct value in giving publicity to a rapid and simple solution of this particular class of statically indeterminate structures.

CHARLES S. WHITNEY,* M. AM. SOC. C. E. (by letter).—The authors have presented a very interesting graphical method for the analysis of continuous beams. As it has been proved that more economical reinforced concrete structures can be produced by designing them as elastic frames, any such method which simplifies the design is valuable. The writer has been using a very simple general analytical method† for locating the conjugate points which should also be of interest. Fundamentally, this method is probably the same as that of the authors, but its derivation, appearance, and use are different. It is a general method for the solution of continuous frames composed of straight members the ends of which do not move horizontally or vertically. The derivation is effected by considering a single member as a beam with elastic supports. The notation used is shown in the diagrams, Figs. 39 to 44, inclusive.

The beam, *A B*, is considered as elastically restrained in such a manner that a unit moment at the left support will produce a rotation of that support equal to ϵ_a and a unit moment at the right support will produce a rotation equal to ϵ_b. The total rotation of the left and right supports, therefore, will be $M_a \epsilon_a$ and $M_b \epsilon_b$, respectively.

* Cons. Engr., Milwaukee, Wis.

† See "Neuere Methoden zur Statik der Rahmentragwerke und der elastischen Bogentraeger," by A. Strassner, and "Methode der Festpunkte," by E. Suter.

Considering the beam as a simple span without restraint, the superimposed loading will produce an angular displacement of the left end equal to α_{ao} and of the right end equal to α_{bo}.

Still considering the beam as simply supported, the angular displacement at the left and right ends produced by a unit value of M_a are, respectively, α_a and β. The same values when $M_b = 1$ will be called β and α_b. By Maxwell's theorem of reciprocal displacements, the values of β are the same in both cases. If the beam has a uniform moment of inertia, the values of α_a and α_b will be equal.

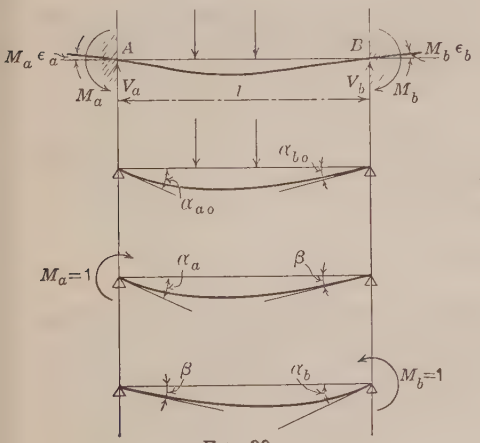

FIG. 39.

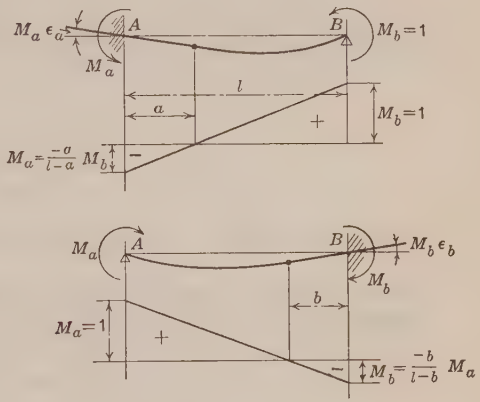

FIG. 40.

Using these displacements, the location of the conjugate points on the closing line of the bending moment diagram may easily be derived. Equating the angular displacements at the left support,

$$-M_a \varepsilon_a = \alpha_{ao} + M_a \alpha_a + M_b \beta$$

At the right support,

$$-M_b \varepsilon_b = \alpha_{bo} + M_b \alpha_b + M_a \beta$$

or,

$$\left.\begin{aligned} M_a(\alpha_a + \varepsilon_a) + M_b \beta &= -\alpha_{ao} \\ M_b(\alpha_b + \varepsilon_b) + M_a \beta &= -\alpha_{bo} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (20)$$

Assuming the beam as shown in Fig. 40 with the actual elastic support at the left end and a simple support at the right end, apply a unit moment, $M_b = 1$, at the right end, and determine the location of the point of inflection. This point is called the "fixed point" and lies in the same vertical line as the conjugate point.

From Equation (20),

$$M_a(\alpha_a + \varepsilon_a) + M_b \beta = -\alpha_{ao}$$

In this case, there is no superimposed load and $\alpha_{ao} = 0$. Hence,

$$M_a(\alpha_a + \varepsilon_a) = -M_b \beta$$

Placing,

$$M_a = \frac{-a}{l-a} M_b \quad \text{(see Fig. 40)}$$

$$\frac{a}{l-a} M_b (\alpha_a + \varepsilon_a) = M_b \beta$$

From which,

$$a = \frac{\beta l}{\alpha_a + \beta + \varepsilon_a} \quad \ldots\ldots (21)$$

Similarly,

$$b = \frac{\beta l}{\alpha_b + \beta + \varepsilon_b} \quad \ldots\ldots (22)$$

These values of a and b are the abscissas of the conjugate points. For unrestrained beams, the values of ϵ_a and ϵ_b are infinite and a and b are zero. For rigidly fixed ends, ϵ_a and ϵ_b are zero and

$$\left.\begin{aligned} a &= \frac{\beta l}{\alpha_a + \beta} \\ b &= \frac{\beta l}{\alpha_b + \beta} \end{aligned}\right\} \quad \ldots\ldots (23)$$

For prismatic beams, $\alpha_a = 2\beta$ and $a = b = \frac{l}{3}$.

The ordinates of the conjugate points will now be found. From Equation (21),

$$\frac{l-a}{l} = \frac{a}{l} \frac{\alpha_a + \varepsilon_a}{\beta} \quad \ldots\ldots (24)$$

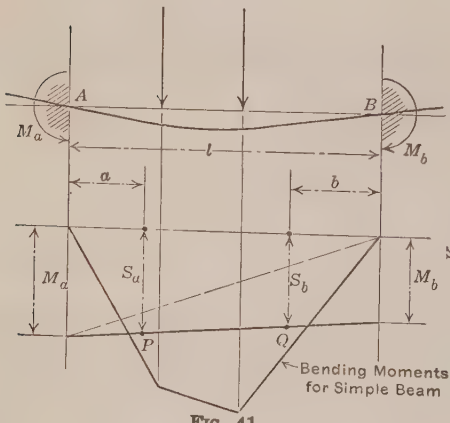

FIG. 41.

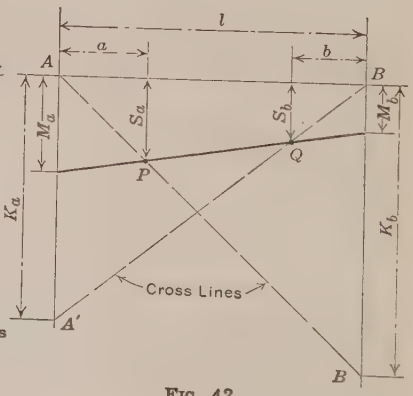

FIG. 42.

From the geometry of Fig. 41,

$$S_a = \frac{a}{l} M_b + \frac{l-a}{l} M_a$$

or, combining with Equation (24),

$$S_a = \frac{a}{l} M_b + \frac{a}{l} \frac{\alpha_a + \varepsilon_a}{\beta} M_a \quad \ldots\ldots (25)$$

From Equation (20),

$$M_a = - M_b \frac{\beta}{\alpha_a + \varepsilon_a} - \frac{\alpha_{ao}}{\alpha_a + \varepsilon_a}$$

Combining with Equation (25),

$$S_a = - \frac{a}{l} \frac{\alpha_{ao}}{\beta} \quad \ldots\ldots (26)$$

Similarly,

$$S_b = - \frac{b}{l} \frac{\alpha_{bo}}{\beta} \quad \ldots\ldots (27)$$

These are the ordinates of the conjugate points, P and Q.

The points, P and Q, lie on the "cross lines", $A\,B'$ and $B\,A'$, which are useful and can be located as follows. The intercept, $A\,A'$, called K_a, is seen in Fig. 42 to be equal to,

$$K_a = \frac{l}{b} S_b = - \frac{\alpha_{bo}}{\beta} \quad \ldots\ldots (28)$$

And, similarly,

$$K_b = - \frac{\alpha_{ao}}{\beta} \quad \ldots\ldots (29)$$

These cross lines are the loci of the conjugate points and depend for their location on the deformation of the unrestrained beam under the assumed loading. They are not affected therefore by the actual end conditions. The abscissas, a and b, are fixed by the end conditions and the elastic properties of the beam itself, but are independent of the actual loading.

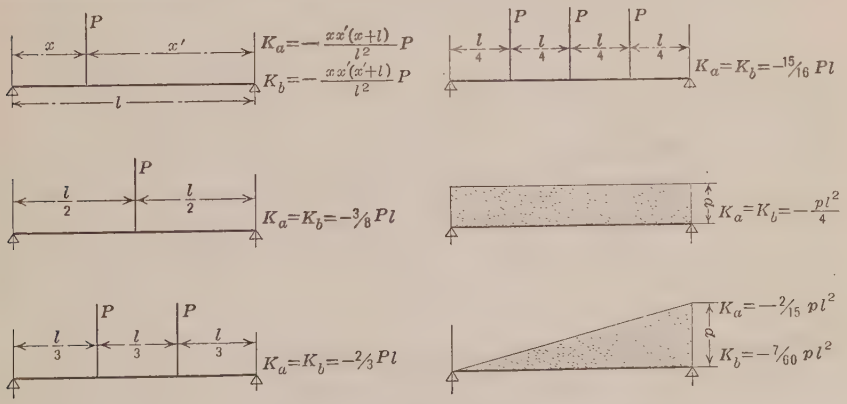

FIG. 42.

The values of α_a, α_b, β, α_{ao}, and α_{bo} can easily be found by the moment-area method or one of the other familiar methods. For beams of uniform moment of inertia,

$$\alpha_a = \alpha_b = \frac{l}{3\,E\,I}$$

$$\beta = \frac{l}{6\,E\,I}$$

For beams of variable moment of inertia, the values can be obtained by finding the reactions produced by loading a simple beam with the modified moment areas, as shown in Fig. 21.

The values of K_a and K_b for some common conditions of loading on beams of uniform moment of inertia are given in Fig. 43.

The value of ϵ_a is the angular displacement of the left support when $M_a = 1$. If the span under consideration is continuous with another beam to the left, ϵ_a will be equal to the angular displacement of the right end of this beam when $M_a = 1$. (See Fig. 44.)

$$\epsilon_a = \phi = \alpha_b' - \frac{a'}{l' - a'} \beta'$$

the values of α_b', a', l', and β' being the values for the span to the left of the support being considered.

If there is more than one member framing into the left end of the loaded beam, the value of ϵ_a is determined by the expression:

$$\frac{1}{\epsilon_a} = \frac{1}{\phi_1} + \frac{1}{\phi_2} + \frac{1}{\phi_3} + \ldots + \frac{1}{\phi_n}$$

in which, ϕ_1, ϕ_2, etc., are the angular displacements for each member separately when acted on by a unit moment at the end. When there is a series of spans, the values of ϵ_a and a are determined successively, starting with the the left-end span. The values of ϵ_b and b are determined in the same way from the left-end span, in which case,

$$\epsilon_b = \phi = \alpha_a'' - \frac{b''}{l'' - b''} \beta''$$

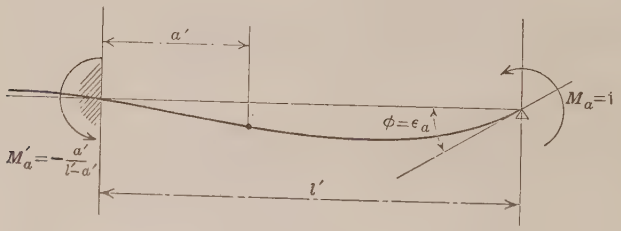

FIG. 44

For members with a uniform moment of inertia,

$$\phi = \frac{l}{6\,E\,I}\left(2 - \frac{a}{l - a}\right)$$

in which, the value of a is $\frac{l}{3}$ for the fixed end, zero for the hinged end, and lies between zero and $\frac{l}{3}$ for partial restraint.

Complete tables for the easy application of this method to continuous beams with haunches of various sizes at one or both ends are available.*

* "Neuere Methoden zur Statik der Rahmentragwerke und der elastischen Bogentraeger", by Strasser, and "Methode der Festpunkte", by Suter.

This method can be extended to cover the problems treated by the authors, and the writer is under the impression that it is quite as general as their graphical method.

One question in regard to the application of any such method to reinforced concrete structures which the writer would like to have discussed is the selection of the proper moment of inertia to use in the formulas. As the solutions depend only on the relative moment of inertia of the different members, it has been suggested that the moments of inertia of the full concrete section be used, neglecting the steel entirely. The writer believes that this will ordinarily give good results in the case of continuous beams, but it would appear that some modification would be required if some members were very much more heavily reinforced than others. In continuous T-beam construction, it is customary to use the moment of inertia of the full T-section for those parts of the beams subjected to negative bending as well as those with positive bending.

Edward Godfrey,* M. Am. Soc. C. E. (by letter).—This paper, as the "Introduction" sets forth, was written primarily to be of use to designers in reinforced concrete construction. A secondary use is mentioned, namely, in computing secondary stresses.

On several occasions the writer has emphasized the fact that concrete construction, as proved by tests, does not act in a true elastic manner nor even approximately so. He has also stated that there are many considerations, apart from the one stated by the authors, namely, tedious methods, which make it superfluous to consider secondary stresses.

In the writer's paper entitled "Some Mooted Questions in Reinforced Concrete Design,"† the following statements were made:

"Professor Gaetano Lanza has given some data on recorded deflections of reinforced concrete beams.‡ He has also worked out the theoretical deflections on various assumptions. An attempt to reconcile the observed deflections with one of the several methods of calculating stresses led him to the conclusion that:

" 'The observations made thus far are not sufficient to furnish the means for determining the actual distribution of the stresses, and hence for the deduction of reliable formulae for the computation of the direct stresses, shearing stresses, diagonal stresses, deflections, position of the neutral axis, etc., under a given load.'

* * * * * * *

"There are more than ample grounds for the contention that the ideal stresses worked out for a reinforced concrete structure are far from realization in this far from ideal material.

* * * * * * *

"The elastic theory stands or falls according as one is able or unable to calculate accurately the deflection of a reinforced concrete beam; and it is an impossibility to calculate this deflection even approximately. The tests cited by Professor Lanza show the utter disagreement in the matter of

* Structural Engr., Pittsburgh, Pa.

† *Transactions,* Am. Soc. C. E., Vol. LXX (1910), p. 54.

‡ "Stresses in Reinforced Concrete Beams," *Journal*, Am. Soc. Mech. Engrs., Mid-October, 1909.

deflections. Of those tested, two beams which were identical, showed results about 100% apart. A theory grounded on such shifting foundation does not deserve serious consideration."

The foregoing attack on the elastic theory as applied to reinforced concrete was not answered in any manner whatever in the long discussion which followed presentation of the paper.

In the discussion of a paper by A. C. Janni, M. Am. Soc. C. E., entitled "The Design of a Multiple-Arch System and Permissible Simplifications",* the writer made the following statements:†

"The modulus of elasticity of concrete varies considerably, even for specimens made of apparently the same kind of materials. It also varies for different intensities of stress, which means a variation in the same cross-section of a member in bending. The first of these inconsistencies could not be allowed for mathematically; the other, if the law of its change should be discovered, might yield to analysis by adding manyfold to the complexity of an already overburdened mathematical solution.

* * * * * * *

"Furthermore, steel reinforcement in a concrete arch contributes materially to the lack of true elasticity and the fact that it is unstressed in its original condition. Shrinkage in the concrete puts initial stress in the steel, and any calculations of equivalent moment of inertia based on unstressed steel and concrete are vitiated by this undisputed fact.

"The whole fabric of the method of designing reinforced concrete arches as elastic structures stands or falls according as the deflection of a reinforced concrete beam under test can or cannot be accurately determined by computation. Tests prove that this deflection cannot even be approximated by computation. Here, in a nutshell, is the proof that the elastic treatment of the reinforced concrete arch is improper."

So far as the writer knows these arguments have not been refuted.

Reinforced concrete arches designed on the elastic theory and continuous beams are on the same basis and what is said concerning the applicability of the theory to one has equal force as to the other. If neither is an elastic structure, the elastic theory does not have application.

A structure to be elastic in the mathematical sense must not only exhibit what is commonly understood as elasticity, but it must have a constant or nearly constant modulus of elasticity in all its parts. The theory of elasticity, particularly such phases as apply to elastic arches and continuous beams, can have no application unless the modulus of elasticity is a constant. When two specimens of concrete made in an identical manner show variations of 100% in deflections, this means that the modulus of elasticity in these two specimens varies by that amount or more; or else, what is still more significant, it indicates that that modulus may be different for different parts of the same member. That difference would have to be more than 100% in order to make the averages to differ by 100 per cent.

Many experiments demonstrate the fact that the modulus of elasticity of concrete is quite uncertain and varies even to the extent of several hundred per cent. Besides, many tests indicate that there is a variation for different

* *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 1142.

† *Loc. cit.*, p. 1189.

intensities of stress. The application of the elastic theory to a material such as this is a violation of the fundamentals of engineering mechanics.

So far as the writer knows no attempt has ever been made to reply to this objection to the elastic theory as applied to concrete and reinforced concrete. It is to be hoped that the authors will make serious reply to this objection. It is just as clearly a matter of exact analysis as any part of any solution of the problem of elastic concrete structures, and unless it can be controverted, every part of every such solution—a large total in engineering literature—is merely theoretical matter of no value.

On secondary stresses, with which the paper also deals, very complex mathematics has been introduced into the literature of engineering; and it is seriously questioned if any of this is of practical value. This highly theoretical matter has more than once been challenged as inappropriate and useless by the writer, chiefly on the ground that the malleability of steel vitiates all assumptions, and tests do not indicate any agreement between the theory and fact, because the theory referred to ignores malleability in steel.

In a seven-page discussion* of a paper on "Secondary Stresses in Bridges"† by Cecil Vivian von Abo, Jun. Am. Soc. C. E., the writer set forth reasons why it is unnecessary to consider secondary stresses in steel work. Mr. von Abo's answer‡ to this is, "Mr. Godfrey questions the definition of 'secondary stress' ".

Another paper§ further emphasizes the futility of the consideration of secondary stresses. A paragraph from that paper is pertinent:

"D. B. Steinman in his conclusions on the tests of the Hell Gate Arch says in *Transactions,* Am. Soc. C. E., Vol. LXXXII, p. 1071, that during erection, 'except for the smallest secondary stresses, the measured values were consistently lower than the calculated values. For the higher percentages of secondary stresses the measured values are only a small fraction of the calculated values.' Also, 'the actual secondary stresses will generally be lower than the calculated values. There is an automatic re-adjustment of strains within a structure in such direction as to relieve the secondary stresses.' There can be no doubt whatever that this 'automatic re-adjustment' is simply the working out of the effect of toughness or malleability of steel, a property absolutely ignored by the theorist."

If, in the most elaborate set of tests ever made to investigate secondary stresses, so striking a disagreement with theory as that indicated in the last paragraph was manifest, is it not legitimate to ask, "Why should consideration of secondary stresses have any place either in engineering design or literature?".

Felix H. Spitzer,‖ Assoc. M. Am. Soc. C. E. (by letter).—The graphic methods developed in the paper are simple, short, and easily checked and will soon replace the formulas with arbitrary coefficients in the design of continuous beams.

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* *Transactions,* Am. Soc. C. E., Vol. 89 (1926), p. 193.

† *Loc. cit.,* p. 1.

‡ *Loc. cit.,* p. 209.

§ *The Canadian Engineer,* May 19, 1925, p. 495.

‖ Cons. Structural Engr., San Francisco, Calif.

The writer has been familiar with the method previous to its publication through the courtesy of Mr. Nishkian and has used it in the design of several buildings, saving in the quantities of concrete and steel in the frame work as much as 20 per cent.

The designs resulting from this method call usually for larger negative moments and smaller positive moments than would have resulted from the use of building ordinance formulas. There takes place, to a certain extent, a shifting of the quantities from the center to the support. The negative or top steel becomes more important and its location has to be better insured. The general reduction of quantities cuts down the margin of carelessness and poor work.

For these reasons the writer recommends that the use of these graphic methods shall automatically call for better engineering field inspection than is given to present-day concrete building construction.

C. A. P. Turner,* M. Am. Soc. C. E. (by letter).—The authors follow the ordinary beam theory disregarding rhombic deformation. That the graphic solution outlined is simpler than available analytical methods, appears open to question. This might be true were only the methods of Clapeyron, as simplified by Merriman, available; but a series of three-moment equations of negative moments is more difficult to solve than a series of† two-moment equations of negative moments by which the problem may be attacked.

Half the sum of the moments over consecutive supports plus‡ the moment at the center of the span, equals the moment ordinate for the given loads at the center of that span considered as simply supported. From a series of these equations, is obtained the odd-and-even rule for center moments between supports, for negative moments over the supports, and for the shears at the support. For a uniform load and equal spans, these values may be written without computation.

In the solution for concentrated loads or unequal spans, the two-moment equations of negative moments permit the deduction of additional three-moment equations of center moments, and also give the relation that the negative moment over any interior support equals the numerical sum of the adjacent positive center moments between supports. Thus, this§ analytical method, as the writer views it, is simpler than the graphic.

Graphical analyses of continuous beams are in general based on Mohr's theorem. The general expression for the three-moment relation of Equation (1) appears to have been first published by the late Charles E. Greene, M. Am. Soc. C. E., in 1875.‖

The symmetry of the summation equations of beam theory permit two analogous but divergent graphical methods—that of fixed points determining the false polygon of deflection, as followed by Eddy,¶ and Church,** and that

* Cons. Engr., Minneapolis, Minn.

† "Concrete Steel Construction", by Eddy and Turner, Second Edition.

‡ Plus becomes minus in case of zero load on the span.

§ "Elasticity and Strength of Materials", by Turner, Section II.

‖ "Graphical Method for the Analysis of Bridge Trusses", Van Nostrand.

¶ "Researches in Graphical Statics", by Eddy, 1878.

** "Mechanics of Engineering", by Church, 1890

of characteristic points from which the closing line of the moment area may be found, as detailed by Fidler,* whose treatment follows almost identically the same lines as that of the authors and appears simpler in detail. Hence, a comparison by the authors of this method with their work would be of interest.

The authors' premise disregarding rhombic distortion involves the assumption that a plane normal section before bending remains plane after bending and normal to the axis of the beam; it assumes that the sum of the horizontal compressions balances the sum of the horizontal tensions, which, although true of the simply supported beam, is untrue for the continuous beam.

The equalizing of the horizontal tensions by the horizontal compressions takes place through horizontal shear, which changes sign at the neutral plane. The sum of the shears above the neutral plane constitutes the reaction to the sum of the horizontal compressions in simple flexure.

Under Newton's third law, action and reaction must be equal and opposite, but college textbooks and university professors teach in substance that in the mechanics of flexure, while action and reaction must be equal, the center of action of the positive shears need not for equilibrium be opposite the center of action of the compressions. In that way they erroneously locate the neutral plane at the center of gravity of the section along the length of the beam. This erroneous location of the neutral plane dates back 240 years to the investigation of Mariotte in which he assumed without proof, *on peut concevoir,* that there are as many fibers stretched as compressed.

Now, in order that the action of the compressions and the reaction of the shears may be opposite, the neutral plane must be depressed below the gravity axis toward the end of the simply supported beam a distance of nearly one-tenth the depth, following a curve of the same order as the slope curve for various loadings, whether central, uniform, or other.

In order that the material may obey Hooke's law there must be a shearing strain in proportion to the shearing force and the plane section instead of remaining plane and normal to the axis of the beam is rotated from it and assumes the form of a reverse third degree curve; since the shears are the summation of the moment stresses following a linear law the shear stress area is parabolic, bounded by a second degree curve, and consequently the summation of the shear areas which determines the deformation is a third degree curve.

Applying this reasoning to the restrained beam, if it were hinged at the points of inflection the neutral plane at the hinge of the suspended span would be nearly one-tenth the depth below the center of gravity of the section and the neutral plane for the cantilever portion would be one-tenth the depth above that center. Since the material is not separated, these diverse positions of the neutral plane, when hinged, are forced together by interference of the rotational shear strains represented by a couple of one-half the total intensity of the moment stresses having a lever arm of approximately one-fifth the depth of the beam. Hence, for the restrained steel beam under uniform load having a ratio of $\frac{L}{d} = 10$, the maximum tension would be

* "Treatise on Bridge Construction", 1887.

only three-quarters over the supports and one-half at mid-span of that computed from the erroneous location of the neutral plane following the common theory of flexure.

The author's proposed theory for the attainment of economic results in beam design is thus lacking in that degree of accuracy which permits the greatest economy. In this respect it offers little improvement over building codes, which likewise disregard shear distortion.

The criticism of the ordinary beam theory, because it disregards Hooke's law and Newton's third law as well, is applicable to the entire theory of secondary stresses. The bending of a prismatic beam is determined predominantly by the elongations and compressions parallel to the axis. In the column, on the other hand, before the stage of incipient failure has been reached, elongations are approximately normal to the axis and compressions parallel to it. Its change in shape is determined, therefore, predominantly by the very shear strains which are disregarded in the theory of secondary stress. This disregard of shear strain reaches its greatest absurdity in building codes and in the rules* of the Joint Committee on Standard Specifications for Concrete and Reinforced Concrete, for determining the resisting section of restrained and continuous beams under external shearing force.

The procedure is first to assume that shearing force produces no shear strain. Then, because shear strain causes shear failure, it is provided for by assuming a unit shear stress unrelated by proportion to the shear strain that causes failure—computing that shear stress on the basis of the external shear force divided by the cross-sectional area instead of calculating it from the accepted summation equations of beam theory.

If, on the side of an elastic prism, lines parallel to and lines normal to the axis are drawn so as to divide the surface into squares, and the prism is bent sharply, the squares at the center will be deformed into trapezoids. The squares at the end at mid-depth will assume a rhombic form. At the center under uniform load there is no external shear force. Yet a rotational deformation has taken place and the squares have been deformed into trapezoids in which both diagonals are shortened in the compression zone and both diagonals lengthened in the tension zone. Toward the end of the beam, where the rhombic deformation occurs, the diagonal sloping downward toward mid-span from the end of the beam is lengthened and the other diagonal is shortened in both tension and compression zones. These two distinctive deformations have different laws of summation and arise from divergent causes.

Trapezoidal deformation is produced by the external shear force and is proportional to the moment at any point in the beam. Rhombic deformation at the neutral plane on the other hand is proportional to the horizontal shear stress (or the sum of the moment areas), or to the slope of the beam at the point in question. Cracking of the concrete occurs where the sum of these two rotational deformations is a maximum. Hence, shear failure occurs in the simply supported deep concrete beam toward the end; in the shallow beam toward the center.

* *Proceedings*, Am. Soc. C. E., October, 1924, Papers and Discussions, p. 1153.

Since, in the stress mechanism of a concrete beam reinforced in the bottom, the steel resists the direct tension, checking or cracking of the concrete occurs in the shallow beam because of the sharpness of curvature requiring a limitation in the percentage of steel. In the deep beam cracking is caused predominantly by rhombic shear strain.

In a given beam for the same kind of loading, with the same percentage of steel, and the same working stress on the steel, the rhombic shear strain in the simple beam is proportional to the external shear force for a given distribution but not for different distributions of loading. In the continuous beam, on the contrary, because it is made up of cantilever and suspended span portions, the rhombic deformation is zero at the support instead of a maximum as in the simple beam and is a maximum at the point of contraflexure. Trapezoidal deformation, on the contrary, is a maximum at the support and at midspan for symmetrical loading.

From these divergent laws of summation there results a resistance to external shear force which may be three to four times as great in the continuous beam per unit of section as that developed in the simple beam, yet building code laws and the rules of the Joint Committee on Standard Specifications for Concrete and Reinforced Concrete, still err in requiring provision for shear strain in the continuous beam as if it were comparable in its mode of resistance to the simple beam.

It is to be regretted that the authors did not carry out their investigations on a stricter application of Hooke's law so that their theory would accord more closely to the conditions met in practical work.

The divergence from exact theory by the disregard of shear strain may be summarized as follows: The neutral plane in the simple beam passes through the center of gravity of the section only at the locus of maximum deflection. It dips downward toward the end of the beam in a curve of the same order as the slope curve. In the continuous beam, on the contrary, due to the interference of shear distortion, the neutral plane passes through the center of gravity of the section at the points of contraflexure only. It rises above the center of gravity at the supports and drops below it between supports, following in like manner a curve of the same order as the slope curve for the given applied loads. Shear distortion increases the deflection of the simple beam but reduces the deflection of the continuous beam just as in the case of the statically indeterminate arrangement of a load on two springs of different rigidities. The deflection is less for the indeterminate arrangement than for any statically determinate arrangement. In the statically indeterminate restrained or continuous beam the deflection is reduced not only by the divergent distribution of moment magnitudes but by interference of shear distortions as well.

It appears that heretofore no general definition has been formulated covering jointly what may be called the neutral plane in the homogeneous beam and in the reinforced concrete beam as well. The writer would define this plane as the locus of consecutive points of right shearing stress in vertical planes parallel to the axis where tension, compression, and shear have the same magnitude. This state of right shearing stress in the composite beam differs from

that in the homogeneous beam in that as the shears are equalized at the steel instead of at the neutral plane the states of right shearing stress at the neutral plane are rotated one-half an octant from that of the homogeneous beam, in keeping with the depression of the plane of equalization of the shears one-half the depth below the neutral plane. This state of facts indicates the error of the supposition that tension in the concrete offers material resistance to flexure and that the divergence in the newly cast beam between steel moment times the lever arm and the applied moments of the load must be attributed to the reaction of the residual stresses induced during the hardening of the concrete about the steel.

It is to be noted that the proportional reduction of the deflection caused by interference of shear distortion decreases as the square of $\frac{d}{L}$ decreases and in like manner does its influence in reducing the intensity of the tension over the support at the top and between the supports at the bottom.

The much greater rigidity of a column in flexure when loaded with the weight of several stories on the one hand as compared with the same column carrying the relatively light load of the deck structure on the other, is one of the surprising observations in testing concrete buildings. Here the ordinary beam theory as applied in the paper gives no indication of the phenomena, but it may be explained by the variation of the relative flexural resistance resulting from divergent stress areas at critical sections of the column.*

The bearings of the continuous beams in a concrete building are more or less fully restrained either by the rigidity of the columns into which they frame or by the torsional rigidity of the carrying beams and slabs through which continuous joists may extend. The three-moment relations thus become indeterminate, as the number of equations of condition is one less than the number of unknown moments over the supports, forcing the practical designer to use other relations in solving his problem.

The method of the authors is therefore applicable to continuous steel beams with hinged supports rather than to ordinary reinforced concrete construction. As the summation equations relating to trapezoidal and rhombic deformation follow divergent laws, it is simpler to treat each kind of deformation separately, determining the resultant state of stress by combining the two. The authors' analysis forms a correct basis for the preliminary computation of trapezoidal deformation. The resultant unit stress in tension is modified by combining the trapezoidal and rhombic deformations.† In the foregoing discussion it is to be kept in mind that the center moment is not the maximum positive moment but the positive moment at the center of span, which may be utilized as a kind of relation from which the maximum negative and positive moments may be determined.

Hardy Cross,‡ M. Am. Soc. C. E. (by letter).—This paper is an interesting contribution to American literature in this field. It is especially welcome to the writer because the authors emphasize that it really treats of geometry—

* "Elasticity and Strength of Materials," Section IV, p. 8.

† *Loc. cit.*, Section II.

‡ Prof., Structural Eng., Univ. of Illinois, Urbana, Ill.

the geometry of ideal distortions in structures. Since the relations are geometrical, they can be presented in almost unlimited variety either in purely geometrical form (graphically) or in algebraic form (analytically). The choice between these two methods is almost entirely one of individual taste and experience. The writer prefers analytical solutions for all the problems discussed and finds them more expeditious, but this does not mean that he discounts at all the value of the graphical constructions presented.

The paper is free from what seems to be a fault of frequent occurrence in the literature of indeterminate structures, exaggeration of the value and importance of exact influence lines. With uniform loads or where equivalent uniform loads (the equivalence being strictly correct only for a triangular influence line) are available or easily determined, influence lines are useful chiefly in determining load divides. In continuous girder problems one may determine by inspection what spans should be loaded, and uniform or equivalent uniform loads are sufficiently exact. It is only for points between the fixed points and the supports that divides for moment need to be found. In most cases, it is better to compute maximum moments directly rather than from influence lines.

The treatment of secondary stresses as due to bending moments in continuous girders on settling supports, either neglecting the web members or assuming pin connections at their far ends, is very interesting. The writer has at times indicated this method to his students, but without much faith in its general accuracy in the cases of secondary stresses which are worth computation. The truss used as an illustration by the authors is not such a case, and the writer hopes that they have available and will present further data on the degree of approximation of this method. It is probably not very close in such cases as arches, where the web is very stiff near the center.

The paper suggests many lines of study and comparison which space forbids developing. The writer has not investigated in detail the originality of the constructions as compared with those of Fidler or with later similar constructions which he is told are due to Ostenfeld. Indeed he does not consider the question very important. A contribution to American literature which is not merely uninterpreted translation—or, worst of all, undigested translation —has value in itself irrespective of comparisons with foreign literature; the literature of indeterminate structures has often been burdened by historical precedents which have limited it to so-called "classic", and very unusable, forms.

George Higgins,* M. Am. Soc. C. E. (by letter).—This paper is a valuable addition to engineering literature on the subject of continuous beams and, as the authors suggest, will prove helpful to designers of structures in reinforced concrete. Not only is the authors' method an excellent one, but their explanation is remarkably clear and their industry in applying the method of conjugate points to the solution of many different problems will earn the gratitude of many engineers.

* Cons. Engr., Rome, Italy.

In 1908 and 1909, the writer gave considerable study* to the problem of determining the best spacing of the posts, corresponding to different kinds of loading on 2-post trussed beams. Had he used the method of conjugate points, the simplification probably would have been considerable. The trussed beam problem differs from those given as examples by the authors† in that the ends are not fixed, but simply held down by the tie-rods when they tend to rise, and held up by the supports in all cases. In other words, negative bending moments occur. The writer has investigated several cases of partial uniform loading, and would be glad to communicate the results to other members.

E. G. Walker,‡ M. Am. Soc. C. E. (by letter).—The writer has read this paper with considerable interest. He believes it to mark a very definite advance in the practical design of continuous beams and girders. Although the Theorem of Three Moments and its more advanced generalizations have long been well known to mathematicians and engineers, practical application has always been on a restricted scale by reason of the great amount of arithmetical computation involved in applying these principles to practical design.

The graphical methods developed by the authors should go far toward increasing the use of more exact methods of calculation for constructions in which continuity of structure over points of support is involved. It may be legitimate to apply methods which require much computation in proportion to the results obtained to large and heavy structures involving a considerable expenditure (for example, long-span swing bridges, continuous girder bridges, etc.), but it is not economical to incur the cost involved in elaborate calculation to smaller structures, such as the framing of an ordinary reinforced concrete building.

The large advance which the paper makes toward enabling exact methods of computation to be applied in a relatively simple and easy manner, gives a great practical value to this Method of Conjugate Points. The writer regrets that he has not yet had an opportunity of using it in practice, but the principal conclusion which he has drawn from reading the paper is that the method is much more adaptable for ordinary office use than any other way of applying the Theorem of Three Moments to reinforced concrete design. It cannot compete in saving of time with the methods in general use for ordinary cases, in which empirical coefficients are used to calculate bending moments, but in structures of any appreciable magnitude the extra cost of the time involved in calculating by the authors' method should not be more than can be justified by increased accuracy of the results.

A great deal of the objection that has been taken in the past to the practical application of theories of continuity of structures has been based more on their complexities and lack of adaptability to the conditions of practical use rather than to valid opposition to the legitimacy of the basic assumptions used in developing the theories. Opponents of the application of exact statical methods have usually raised the objection that, in applying theories of continuity, conditions are assumed that are incorrect. In particular, it is fre-

* "Trussed Beams," *Transactions*, Australasian Assoc. for the Advancement of Science. Vol. 83.

† See Fig. 20, Cases 10–13, p. 21.

‡ Director, Maxted & Knott, Ltd., London, England.

quently asserted that stiff connections, such as the intersections of beams and columns in a reinforced concrete building, are not sufficiently rigid to justify the assumptions which are made in regard to transmission of stress through them. On this basis it is assumed apparently that it is necessary for such intersections to be infinitely rigid in order to make calculations based on statical principles nearer the truth than those made by imposing an empirical multiplier on the moments calculated on an admittedly imperfect basis. While he does not subscribe to this opinion, the writer, probably in accordance with a great many other engineers of similar views, frequently has used approximate methods rather than consume the time necessary for more exact computation, because in many instances the saving which might be effected would not be sufficient to justify the cost of time involved in the longer calculations. Methods which are justifiable on a long continuous-span bridge are quite the reverse when applied to continuous, 15-ft. span floor-beams. It is in the direction of making closer calculations worth while in smaller structures that the value of the paper lies.

The authors have presented in an exceptionally clear manner the principles of this new method and have indicated very fully how it is applied to practical cases. The application is certainly simpler than that of the older Theory of Fixed Points and brings the whole subject within the domain of practical application to every-day work. A great advantage which should not be overlooked is the fact that with relatively little additional computation exact solutions are possible for cases of complex distributions of load as well as for variations of the moments of inertia.

Even with the simplification which the authors have effected it would hardly appear practical, however, in the ordinary way to apply such methods as are developed in Table 1 to beams of the magnitude of the example which they have chosen. The writer, however, feels at a disadvantage in making much criticism on this point, because not having actually applied the methods of the paper to practical cases he is not qualified to judge how much of this computation can be saved by the use of the constants which the authors give in Table 2. The computations of Table 1, however, are not more extensive than are necessary in the case of an ordinary arched rib. No doubt there are many cases which occur, as there are instances in the writer's own experience, where such amount of calculation is quite justifiable even on relatively small structural members.

In conclusion, the writer hopes that it will be possible to develop the principles outlined in the paper so that less reliance may be placed on the use of empirical figures and considerably more use made of the Theorem of Three Moments in the design of continuous structures in reinforced concrete and steel, thus enabling fuller advantage to be taken of the strength characteristics of the materials.

Donald Derickson,* M. Am. Soc. C. E. (by letter).—This interesting paper is a notable contribution to the subject of structural mechanics. It offers the most expeditious method of which the writer is aware for the solution of a certain type of problem which arises in structural engineering and

* Prof. of Civ. Eng., Tulane Univ., New Orleans, La.

especially pertains to reinforced concrete design. It has been his experience that too often methods have been used in determining the bending moments in reinforced concrete beams, which do not even approximately relate to actual conditions, and the convenient coefficients of the building codes and reports are applied to spans and loadings for which they were never intended. This may be ascribed to the lack of time necessary for the proper computations, or to ignorance of the principles involved, either or both of which have no doubt wasted many a dollar and probably will continue to do so. This paper presents methods, the applications of which are so easily made as compared with some of the better known ways of computation that it should have a beneficial effect on practice.

The writer appreciates the paper and like Oliver Twist would ask for more. In Section 9,* "Application to Indeterminate Frames," it is stated:

"If the structure or loading is unsymmetrical, a small secondary correction must be introduced on account of the slight lateral displacement of the middle span; this displacement is equivalent to a yielding of the respective supports of the outer span."

An application of the principles involved in Section 17,† "Application to Settlement of Supports," to the solution of a problem based on assumed data would be of material assistance.

Can the methods of the paper be applied to a beam or girder in a building frame shown in Fig. 45?

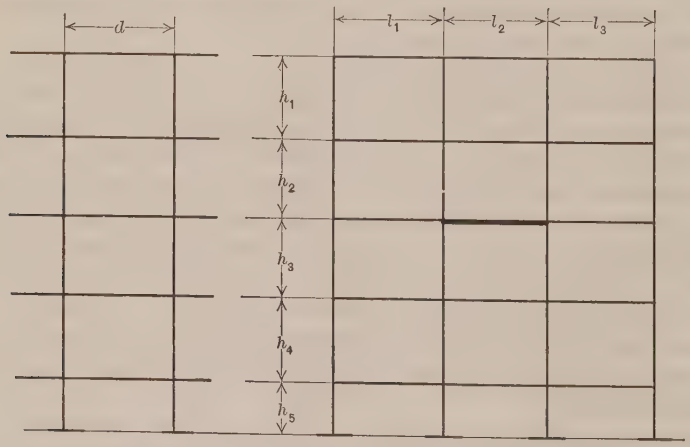


FIG. 45.

The ends of the columns, h_2 and h_3, are neither free nor fixed and the same applies to the ends of the beams, l_1 and l_3; accordingly, it would seem that there would be some uncertainty as to the values to be used for the lengths of the respective members in computing the nominal length of the imaginary beam which is to be substituted for the actual members located at the ends of the beam under consideration. It would also be interesting to know what allow-

* See p. 15.
† See p. 33.

ance should be made for the effect of the floor-slab in reinforced concrete construction, that is, what flange width should be used.

In the application of the principles of Section 17 to a reinforced concrete structure, it would be of value to know what the authors would recommend for the modulus of elasticity, or if such a value may not be stated directly, how may an equivalent value be found?

Quoting from a widely used textbook regarding the moments in continuous beams, "the exact determination of the maximum bending moments at the critical sections of a beam of several spans is a long and tedious operation." This strengthens the student in his disinclination to give the subject as deep consideration as it merits. The increasing tendency toward introducing business courses into the curricula of engineering colleges is another factor that militates against the allowance of reasonable time for the study of continuous beams and indeterminate frames. Accordingly, it is most gratifying to have this paper available for use in class work.

W. M. Wilson,* M. Am. Soc. C. E. (by letter).—For analyzing statically indeterminate flexural members the authors have presented some interesting graphical methods which they claim are much simpler than those previously available. In considering the relative merits of two methods the writer has observed that one's own "child" is invariably a favorite; also that a graphical method appeals to one type of mind whereas an algebraic method appeals to other types. Moreover, a method—even one that is rapid when completely mastered—only results in delay if the successive steps must be reviewed each time it is applied. The writer believes that the algebraic methods available are as short as the graphical method presented by the authors, that they have as great a range of application, and that they can be reviewed more quickly after a period of disuse. Moreover, they do not, as implied by the authors, involve the use of differential equations, as very simple algebra and arithmetic are all the mathematics that are necessary in their application.

The writer has developed formulas for determining the moments in many structures of the type analyzed in the paper.† These formulas are based on the fact that, for a flexural member restrained (not necessarily fixed) at the ends and subjected to intermediate loads, the moment at the ends is made up of two parts, (1) that resulting from the movement, rotation, or translation of one end relative to the other; and (2) that resulting from the loads if the ends were fixed. Inasmuch as the second part is independent of the remainder of the structure of which the member forms a part, its value having once been obtained for each of the many types of loadings is, thenceforth, always available.

For the purpose of comparison with the authors' graphical methods some of the cases treated analytically by the writer† and others will be noted briefly. Definitions and nomenclature for these are as follows:

a = distance from End A of a member to a load.
b = distance from End B of a member to a load.

* Research Prof., Structural Eng., Univ of Illinois, Urbana, Ill.

† *Bulletin 108*, Eng. Experiment Station, Univ of Illinois, by W. M. Wilson, F. E. Richart, and Camillo Weiss; "Stresses in Framed Structures," by Hool and Kinne, pp. 485 to 614.

d = deflection of one end of a member with respect to the other end, measured perpendicular to the initial position of the member.

e = eccentricity of the load.

h = vertical height of a structure.

k = error in the resisting moment due to neglect of the shearing strain.

l = length of a member.

m = change in the rate of loading in a unit distance.

n = ratio of K of the top member to K of the left-hand column for a four-sided frame.

p = ratio of K of the top member to K of the bottom member for a four-sided frame.

s = ratio of K of the top member to K of the right-hand column for a four-sided frame.

u = load per unit of length (variable).

w = uniformly distributed load per unit of length.

A = area of section of a member.

C_{AB} = resisting moment at End A of a member, AB, fixed at both ends and having both ends at the same level.

E = modulus of elasticity in tension and compression.

F = area of the moment diagram of a simple beam.

H_{AB} = resisting moment at End A of a member, AB, fixed at A and hinged at B and having both ends at the same level.

I = moment of inertia of section of a member.

K = ratio of moment of inertia of section to length of a member.

M = moment of an external couple.

M_{AB} = resisting moment acting at the end, A, of a member, AB.

M_{BA} = resisting moment acting at the end, B, of a member, AB.

P = concentrated load.

$R = \frac{d}{l}$ = ratio of the deflection of one end of a member (with respect to the other end) to the length of the member.

W = total distributed load on a member.

$\alpha = n^2 + 2pn + 2n + 3p$, for a symmetrical four-sided frame.

$\beta = 6n + p + 1$, for a symmetrical four-sided frame.

$\Delta = 22\ (pns + ps + ns + np) + 2\ (p^2s + ps^2 + pn^2 + p^2n + s^2 + s + n^2 + n) + 6\ (n^2s + ns^2 + p^2 + p)$, for a rectangular frame.

$\Delta_0 = 2\ [ns\ (4 + 3q + 4q^2) + (s^2 + s) + q^2\ (n^2 + n) + 3\ (q^2 s n^2 + s^2 n)]$ for a two-legged rectangular bent with unequal legs.

$\Delta_0 = 2\ (3ns^2 + 11\ ns + s^2 + s + 3n^2s + n^2 + n)$, for a two-legged rectangular bent.

θ = change in the slope of the tangent to the elastic curve of a member.

The signs of the quantities used in the equations are determined by the following rules:

(1) θ is positive (+) when the tangent to the elastic curve is turned in a clockwise direction.

(2) R is positive (+) when the member is deflected in a clockwise direction.

(3) The moment of the internal stresses on a section is positive (+) when the internal couple acts in a clockwise direction upon the part of the member considered.

(4) If the moment of the external forces on the member about the end at which the moment is to be determined is positive (+), the sign before the constant is minus (—); if the moment of the external forces about the end at which the moment is to be determined is negative (—), the sign before the constant is (+). With the external forces acting downward, (Fig. 46) for the moment at A, C_{AB} and H_{AB} are preceded by a minus (—) sign, but for the moment at B, C_{BA} and H_{BA} are preceded by a plus (+) sign.

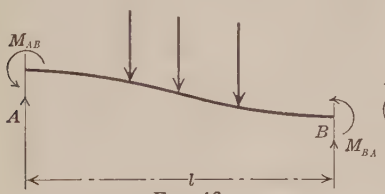


FIG. 46.

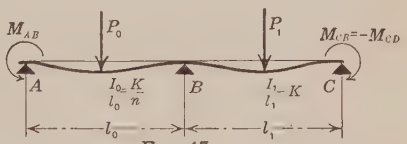


FIG. 47.

The moment at the ends of a member, AB (Fig. 46), is given by Equations (30) to (33):

$$M_{AB} = 2\,E\,K\,(2\,\theta_A + \theta_B - 3\,R) \mp C_{AB} \quad \ldots\ldots (30)$$

$$M_{BA} = 2\,E\,K\,(2\,\theta_B + \theta_A - 3\,R) \pm C_{BA} \quad \ldots\ldots (31)$$

If End B is hinged,

$$M_{AB} = E\,K\,(3\,\theta_A - 3\,R) \mp H_{AB} \quad \ldots\ldots (32)$$

If End A is hinged,

$$M_{BA} = E\,K\,(3\,\theta_B - 3\,R) \pm H_{BA} \quad \ldots\ldots (33)$$

The symbol, C, represents the moment that would be produced by the loads if the ends of the member were fixed, C_{AB} being the moment at End A and C_{BA} being the moment at End B. It is sometimes desirable for algebraic convenience, to introduce the moment produced by the given loads at the fixed end of a beam that is fixed at one end and hinged at the other. This moment has been represented by H.

The moment at the end of any flexural member, AB, can be represented by some one of these equations. Values of C and H for various loads are given in Tables 3 and 4, Table 3 being for unsymmetrical systems of loads and Table 4 being for loads symmetrical about the center of the member.

The analyses of indeterminate structures consist of the derivation of equations for the moments in statically indeterminate flexural members by the application of Equations (30) to (33) and the substitution of the values of C and H. It is unnecessary to reproduce the derivation of these equations.

The Equation of Three Moments for the continuous girder of Fig. 47 is:

$$n\,M_{AB} + 2\,M_{BC}\,(n+1) + M_{CD} = -2\,(n\,H_{BA} + H_{BC}) \quad \ldots\ldots (34)$$

TABLE 3.—VALUES OF CONSTANTS, C AND H, FOR DIFFERENT SYSTEMS OF LOADS.

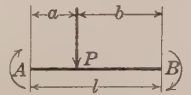
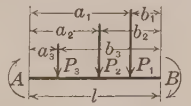
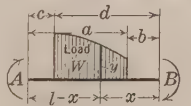
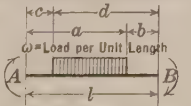
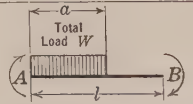
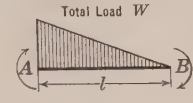
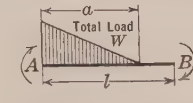
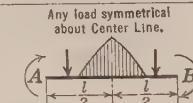
No.	Condition of loading.	$C_{AB.}$	$C_{BA.}$	$H_{AB.}$	$H_{BA.}$
1	No intermediate external loads	0	0	0	0
2	 Single concentrated load at any point	$\frac{P\,a\,b^2}{l^2}$	$\frac{P\,b\,a^2}{l^2}$	$\frac{P\,a\,b}{2\,l^2}(l+b)$	$\frac{P\,a\,b}{2\,l^2}(l+a)$
3	 Any number of concentrated loads	$\frac{1}{l^2}\sum P\,a\,b^2$	$\frac{1}{l^2}\sum P\,b\,a^2$	$\frac{1}{2\,l^2}\sum P\,a\,b\,(l+b)$	$\frac{1}{2\,l^2}\sum P\,a\,b\,(l+a)$
4	 Any distributed load over any portion at the member	$\frac{1}{l^2}\int_b^d y\,x^2\,(l-x)\,dx$	$\frac{1}{l^2}\int_b^d y\,x\,(l-x)^2\,dx$	$\frac{1}{2\,l^2}\int_b^d y\,x\,(l^2-x^2)\,dx$	$\frac{1}{2\,l^2}\int_b^d y\,x\,(l-x)\,(2\,l-x)\,dx$
5	 Uniform load over any portion of the member	$\frac{w}{12\,l^2}\left[d^3\,(4\,l-3d) - b^3\,(4\,l-3\,b)\right]$	$\frac{w}{12\,l^2}\left[a^3\,(4\,l-3\,a) - c^3\,(4\,l-3\,c)\right]$	$\frac{w}{8\,l^2}(d^2-b^2)\,(2\,l^2-b^2-d^2)$	$\frac{w}{8\,l^2}(a^2-c^2)\,(2\,l^2-a^2-c^2)$

TABLE 3.—(*Continued.*)

No.	Condition of loading.	C_{AB}.	C_{BA}.	H_{AB}.	H_{BA}.
6	 Uniform load over portion of member at one end	$\frac{W\,a}{12\,l^2}(3\,a^2 - 8\,a\,l + 6\,l^2)$	$\frac{W\,a^2}{12\,l^2}(4\,l - 3\,a)$	$\frac{W\,a}{8\,l^2}(2\,l - a)^2$	$\frac{W\,a}{8\,l^2}(2\,l^2 - a^2)$
7	Uniformly varying load over entire member	$\frac{l^2}{60}(5\,u + 3\,m\,l)$	$\frac{l^2}{60}(5\,u + 2\,m\,l)$	$\frac{l^2}{120}(15\,u + 8\,m\,l)$	$\frac{l^2}{120}(15\,u + 7\,m\,l)$
8	 Load varying uniformly from zero at one end to a maximum at the other end.	$\frac{W\,l}{10}$	$\frac{W\,l}{15}$	$\frac{2}{15}\,W\,l$	$\frac{7}{60}\,W\,l$
9	 Load varying uniformly from zero at any point to a maximum at one end.	$\frac{W\,a}{30\,l^2}(3\,a^2 - 10\,a\,l + 10\,l^2)$	$\frac{W\,a^2}{30\,l^2}(5\,l - 3\,a)$	$\frac{W\,a}{60\,l^2}(3\,a^2 - 15\,a\,l + 20\,l^2)$	$\frac{W\,a}{60\,l^2}(10\,l^2 - 3\,a^2)$
10	 Any load symmetrical about center of member.	$\frac{F}{l}$*	$\frac{F}{l}$*	$\frac{3}{2}\cdot\frac{F}{l}$*	$\frac{3}{2}\cdot\frac{F}{l}$*

* Values of $\frac{F}{l}$ for different loads are given in Table 4.

TABLE 4.—VALUES OF C AND H FOR LOADS SYMMETRICAL ABOUT THE CENTER OF THE MEMBER.

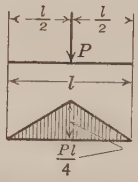
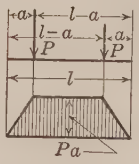
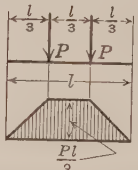
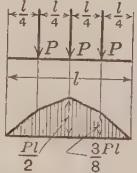
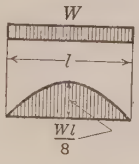
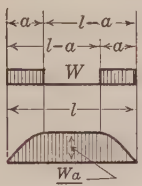
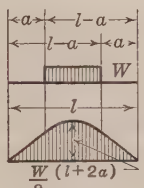
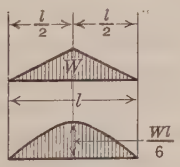
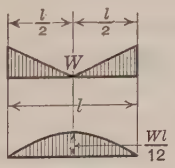
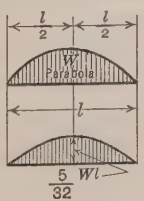
No.	Loading and Moment Diagram	$C_{AB}=C_{BA}=\frac{F}{l}$	$H_{AB}=H_{BA}=\frac{3}{2}\frac{F}{l}$
1	 Single load at the center.	$\frac{1}{8}Pl$	$\frac{3}{16}Pl$
2	 Two equal loads.	$\frac{Pa}{l}(l-a)$	$\frac{3}{2}\frac{Pa}{l}(l-a)$
3	 Equal loads at the third points.	$\frac{2}{9}Pl$	$\frac{1}{3}Pl$
4	 Equal loads at the quarter-points.	$\frac{5}{16}Pl$	$\frac{15}{32}Pl$
5	 Uniform load over entire span.	$\frac{1}{12}Wl$	$\frac{1}{8}Wl$

TABLE 4.—(*Continued.*)

No.	Loading and Moment Diagram	$C_{AB}=C_{BA}=\frac{F}{l}$	$H_{AB}=H_{BA}=\frac{3}{2}\frac{F}{l}$
6	 Equal uniform loads at the ends.	$\frac{Wa}{12l}(3l-2a)$	$\frac{Wa}{8l}(3l-2a)$
7	 Uniform load at the center.	$\frac{W}{12l}(l^2+2al-2a^2)$	$\frac{W}{8l}(l^2+2al-2a^2)$
8	 Load increasing uniformly from zero at the ends.	$\frac{5}{48}Wl$	$\frac{5}{32}Wl$
9	 Load increasing uniformly from zero at the center.	$\frac{1}{16}Wl$	$\frac{3}{32}Wl$
10	 Load varying as the ordinates of a parabola	$\frac{1}{10}Wl$	$\frac{3}{20}Wl$

Equation (34) is applicable to any two adjacent intermediate spans of any continuous girder having supports on the same level, no matter what the type of loading, length of span, or value of I, so long as I does not vary between the supports. The symbol, n, represents $\frac{I}{l}$ for the right-hand span divided by $\frac{I}{l}$ for the left-hand span. It should be noted that quantities corresponding to n and H must be computed in connection with the authors' graphical method before the diagrams can be constructed. With these quantities known, Equation (34) takes the form, $ax + by + cz = K$, in which, a, b, c, and K are numerical quantities. A similar equation can be written for each pair of adjacent spans and the values of the moments obtained by a process of elimination.

The Equation of Three Moments takes a somewhat different form when applied to two adjacent spans at one end of a girder, depending on the condition of restraint at the ends; it is still further modified if the supports are not on the same level. Forms of the Equation of Three Moments covering all possible combinations of loading, length of span, restraint of ends, settlement of supports, and changes in the moments of inertia at the supports are contained in Table 5. The P's in the diagrams accompanying Table 5 are symbolic of any system of loading, the value of H for any particular loading being given in Tables 3 and 4. The first term of the right-hand member of these equations is zero in all cases if the supports are on the same level. These equations make possible the solution of any problem in continuous girders except those involving members the I of which varies between supports. These latter problems could also be solved in a manner similar to that used by the authors, substituting algebraic for graphic solution of the equations.

Bulletin 108 of the Engineering Experiment Station of the University of Illinois, previously mentioned, contains equations giving directly the moment at each support of girders continuous over three supports and also for those continuous over four supports, the equations being applicable for all possible cases except that in which the I varies between supports. For girders continuous over more than four supports it is more convenient to write the Equation of Three Moments for each pair of adjacent spans, substituting the numerical values for the constants, H and n, and solving the resulting numerical equations by a process of elimination.

The authors have treated the two-legged rectangular bent carrying a vertical load on the top by substituting an equivalent three-span continuous girder. This method can be used only when the bent and loading are symmetrical about a vertical center line (Fig. 48), for which,

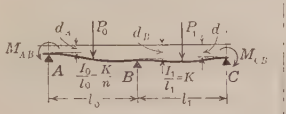
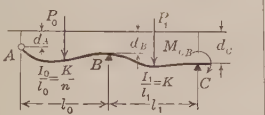
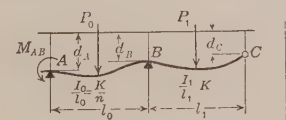
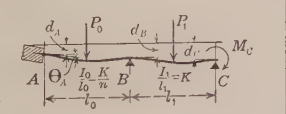
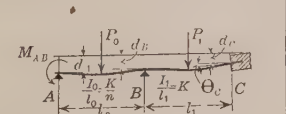
$$M_{AD} = -M_{BC} = \frac{3\,C_{AB}}{2\,n + 3} \qquad (35)$$

Certainly this solution is shorter than the graphical construction given by the authors (Fig. 16), especially since quantities corresponding to C_{AB} and n must be determined before the graphical construction is begun. The algebraic

TABLE 5.—CONTINUOUS GIRDERS—EQUATIONS OF THREE MOMENTS.

(Supports on Different Levels;* Any System of Vertical Loads.†

$K = \frac{I}{l}$ for right-hand span ; $\frac{K}{n} = \frac{I}{l}$ for left-hand span.)

No.	Portion of girder considered.	Equations of Three Moments.
(a)	 Intermediate spans.	$$nM_{AB} + 2M_{BC}(n+1) + M_{CD} = \frac{6EK}{l_0 l_1}[l_1(d_B - d_A) - l_0(d_C - d_B)] - [2H_{BC} + nH_{BA}]$$
(b)	 Two adjacent spans at left-hand end. End of girder hinged.	$$2M_{BC}(n+1) + M_{CD} = \frac{6EK}{l_0 l_1}[l_1(d_B - d_A) - l_0(d_C - d_B)] - 2[nH_{BA} + H_{BC}]$$
(c)	 Two adjacent spans at right-hand end. End of girder hinged.	$$nM_{AB} + 2M_{BC}(n+1) = \frac{6EK}{l_0 l_1}[l_1(d_B - d_A) - l_0(d_C - d_B)] - 2[nH_{BA} + H_{BC}]$$
(d)	 Two adjacent spans at left-hand end. End of girder restrained.	If M_{AB} is known : $$2M_{BC}(n+1) + M_{CD} = \frac{6EK}{l_0 l_1}[l_1(d_B - d_A) - l_0(d_C - d_B)] - 2[H_{BC} + nH_{BA}] - nM_{AB}$$ If θ_A is known : $$M_{BC}(4+3n) + 2M_{CD} = \frac{6EK}{l_0 l_1}[3l_1(d_B - d_A) - 2l_0(d_C - d_B)] + 2nH_{AB} - 4[H_{BC} + nH_{BA}] - 6EK\theta_A$$ If the girder is fixed at A, $\theta_A = 0$
(e)	 Two adjacent spans at right-hand end. End of girder restrained.	If M_{CB} is known : $$nM_{AB} + 2M_{BC}(n+1) = \frac{6EK}{l_0 l_1}[l_1(d_B - d_A) - l_0(d_C - d_B)] - 2[H_{BC} + nH_{BA}] + M_{CB}$$ If θ_C is known : $$2nM_{AB} + M_{BC}(4n+3) = \frac{6EK}{l_0 l_1}[2l_1(d_B - d_A) - 3l_0(d_C - d_B)] - 4[H_{BC} + nH_{BA}] + 2H_{CB} + 6EK\theta_C$$ If the girder is fixed at C, $\theta_C = 0$

* If there is no settlement of supports, let all values of d in these equations equal zero.

† If there are no loads on the girder except at supports, let all values of H in these equations equal zero.

method has the additional advantage that it can be applied when both the bent and loading are unsymmetrical (Fig. 49), for which,

$$M_{BC} = -\frac{3}{2}\left[\frac{C_{AB} + C_{BA}}{n + s + 3}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots (36)$$

$$M_{AD} = \frac{3}{2}\left[\frac{C_{AB} + C_{BA}}{n + s + 3}\right] = -M_{BC} \quad \ldots\ldots\ldots\ldots\ldots (37)$$

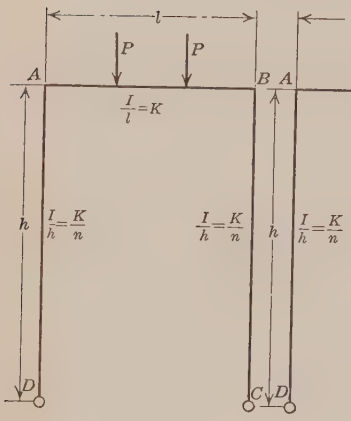


FIG. 48.

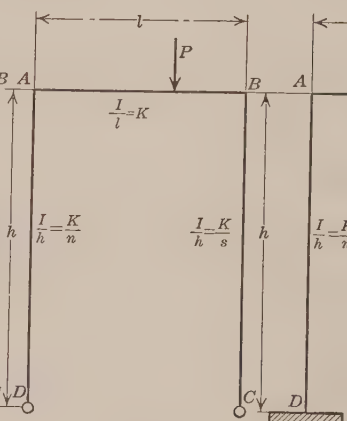


FIG. 49.

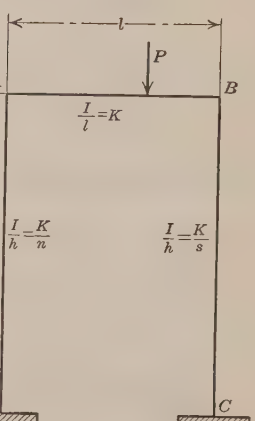


FIG. 50.

If the column bases are fixed and the bent and loading are symmetrical about a vertical center line (the case treated by the authors), $n = s$ and $C_{AB} = C_{BA} = \frac{F}{l}$. The equations for the moments then become:

$$M_{AB} = M_{BC} = -\frac{2}{(n+2)}\frac{F}{l} \quad \ldots\ldots\ldots\ldots\ldots (38)$$

$$M_{CB} = -M_{DA} = -\frac{1}{n+2}\frac{F}{l} \quad \ldots\ldots\ldots\ldots\ldots (39)$$

For the more general case with fixed bases where the bent is symmetrical but the loading is not (Fig. 50), a case for which the authors' method is not applicable,

$$M_{AB} = -\frac{1}{2}\left\{C_{BA}\left[\frac{2}{n+2} - \frac{1}{6n+1}\right] + C_{AB}\left[\frac{2}{n+2} + \frac{1}{6n+1}\right]\right\} \quad (40)$$

$$M_{BC} = -\frac{1}{2}\left\{C_{BA}\left[\frac{2}{n+2} + \frac{1}{6n+1}\right] + C_{AB}\left[\frac{2}{n+2} - \frac{1}{6n+1}\right]\right\} \quad (41)$$

$$M_{CB} = -\frac{1}{2}\left\{C_{BA}\left[\frac{1}{n+2} - \frac{1}{6n+1}\right] + C_{AB}\left[\frac{1}{n+2} + \frac{1}{6n+1}\right]\right\} \quad (42)$$

$$M_{DA} = \frac{1}{2}\left\{C_{BA}\left[\frac{1}{n+2} + \frac{1}{6n+1}\right] + C_{AB}\left[\frac{1}{n+2} - \frac{1}{6n+1}\right]\right\} \quad (43)$$

For the still more general case in which neither the bent nor the loading are symmetrical,

$$M_{AB} = -\frac{1}{\Delta_0}\left\{ C_{BA}\,(10\,n\,s + s^2) + C_{AB}\,(11\,n\,s + 2\,s^2 + 2\,s + 2\,n)\right\} . \quad (44)$$

$$M_{BC} = -\frac{1}{\Delta_0}\left\{ C_{BA}\,(11\,n\,s + 2\,n^2 + 2\,n + 2\,s) + C_{AB}\,(10\,n\,s + n^2)\right\} . \quad (45)$$

$$M_{CB} = -\frac{1}{\Delta_0}\left\{ C_{BA}\,(7\,n\,s - 2\,n^2 - 2\,n + s) + C_{AB}\,(8\,n\,s - n^2 + 3\,n)\right\} . . \quad (46)$$

$$M_{DA} = \frac{1}{\Delta_0}\left\{ C_{BA}\,(8\,n\,s - s^2 + 3\,s) + C_{AB}\,(7\,n\,s - 2s^2 - 2\,s + n)\right\} . . \quad (47)$$

Bulletin 108 of the Engineering Experiment Station of the University of Illinois, previously mentioned, also contains the treatment of the bent under many other conditions, such as horizontal loads on one or both legs, couples at one or both upper corners, and settlement, spread, and rotation of foundations. It also contains analyses of many similar structures not treated in the paper.

The writer welcomes the authors' contribution to American literature on the analysis of statically indeterminate structures because it gives publicity to a phase of structural engineering that too long has been slighted in the United States. In discussing the use of economic, statically indeterminate structures abroad, the authors state (page 2):

"* * * * but the development of the same class of structures in the United States appears to have been retarded by a prevailing impression that the required analysis is involved and tedious. The deterring influence of such an impression is generally under-estimated."

The writer is in complete accord with this statement. Too many American engineers prefer to spend $10 for construction work in order to avoid spending $1 for engineering work. And they wonder why engineers are poorly paid! So long as stress analysis is limited to the selection of data from a handbook, it will be on a plane with clerical work and the computer will receive, properly, a clerk's pay. The Medical Profession has pretty well rid itself of the quack with his pills for all diseases and it is time that the Engineering Profession rids itself of the "handbook artist" with coefficients for all conditions of restraint.

The term, "statically indeterminate," has been too long the "ghost in the graveyard" that frightens the student of structural engineering, but it is really nothing to frighten any one willing to do real "honest-to-goodness" studying. From statics to the analysis of statically indeterminate structures is no great step to the student who thoroughly understands statics. It probably takes longer to master the theory and it certainly takes longer to work numerical problems, but the individual steps are no greater; it is simply the difference between plowing a little field and plowing a big one.

The great amount of literature that is now being published, the large amount of time that the engineering schools are giving to the subject, and the extent to which practicing designers are using statically indeterminate structures and are computing secondary stresses, are evidence that the American engineer is becoming more and more convinced of the value of a careful analysis of structures.

S. M. Cotten,* Assoc. M. Am. Soc. C. E. (by letter).—In the sense that the authors have presented, in the technical language of the present day and as a comprehensive whole, the essential features of methods of analysis which heretofore have been available only in rather fragmentary and not readily accessible form, they have performed a great service to the bulk of the profession, and one that will undoubtedly receive its due appreciation.

There is, however, nothing essentially new in the final applied methods of analysis. Some old things are treated to new names, some established things are used without name or explanation, and the true major premise is proved as a secondary consequence. In the writer's opinion, these substitutions and re-arrangements have been to the detriment of both theory and presentation, and it would have been better for the authors to have accepted principles heretofore developed that were available for their purposes. The ultimate result would have been the same, and the development of it much more direct and tangible.

Relation of Authors' to Previous Methods.—The most peculiar feature of the paper is that the authors do not discuss the points, U and V, except to state that they are at a certain point on the span and that their height is given by a certain formula. The only theory or explanation given is that they represent the "influence of the applied loading", which is a somewhat indefinite statement in view of the fact that they appear to be the really essential element in the whole plan. Considerable space is devoted to the proof for the location and value of the point, T_{1-2}, all of which is predicated on the properties of the points, U and V. The "pennant diagram", in turn, has the point, T_{1-2}, as an essential element. Again, in the proof of the construction for the "conjugate points", the properties assigned to the U and V-points are essential to the demonstration, and it might be said that this is really a proof that the properties assigned to the points are correct. Because of the evident importance of these points, and the fact that they must possess certain definite properties in order to satisfy the authors' whole theory, the explanation given for them is decidedly inadequate. It would be interesting to know what suggested these points and their peculiar properties to the authors, and how their function may be defined, as related to the other parts of the authors' theory.

Viewed from the standpoint of methods previously developed and in use, the U and V-points are easily explained and their significance is known, for they are nothing other than Fidler's "characteristic points". These points were an original discovery of Fidler and the basis of his method of analysis, which in the writer's opinion is one of the really great contributions to the technique of engineering design.† In view of the publicity and the more or less current use of the method, it is quite remarkable that the authors should have developed the identical points in their method, without ever suspecting that their

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† Detailed descriptions and proofs of the method were published in *Minutes of Proceedings*, Inst., C. E., Vol. 74, 1883, p. 196, and in a "Practical Treatise on Bridge Construction," by T. Claxton Fidler. The method has since been treated by various writers, and was abstracted in a paper entitled "Graphical and Mechanical Analysis of Frames," by F. E. Richart, Assoc. M. Am. Soc. C. E., and W. M. Wilson, M. Am. Soc. C. E., *Engineering and Contracting*, June 23, 1920.

discovery had been antedated by about forty years, and that instead of the mere U and V designations their rightful name is Fidler's "characteristic points", which means something.

Another point of contact between the authors' method and those that have preceded, is the "pennant diagram", which has heretofore been known as "Ostenfeld's auxiliary diagram", as used in connection with Fidler's method. This diagram is described in the paper by Messrs. Richart and Wilson previously referred to, who state in a footnote that "Professor Ostenfeld devised this form of diagram in making a very general application of Fidler's method to various forms of continuous beams." The "pennant diagram" is identical with Ostenfeld's, both in appearance and function. The only point of departure is that under the latter name it was regarded merely as an adjunct to a major principle, whereas under the name of "pennant" it becomes a "prime mover", since it (with a little assistance from the U and V-points) determines the "conjugate points" on which the whole method hinges. Ostenfeld's diagram determines these same points, by what name if any, the writer does not know, but as a consequence and not as a cause.

It will be seen that in all essentials the authors' applied method is identical with that of Fidler plus Ostenfeld; the difference being that the authors start in with the effect, and, casually acquiring the "characteristic points", all unknown as such, at some stage of the process, prove the cause in terms of x, y, and z, *so* to speak, whereas, in Fidler's theory, this process is reversed, to its great improvement and clarity.

By definition, Fidler's "characteristic points" are such that the distance from the points to the moment closing line is a function of the slope of the beam at the support, and this he proved to hold true for the points when located according to the principles developed. Having these points, the solution of the problem consists simply in drawing moment closing lines that satisfy the requirements for distance from the points. This can be done by trial with relative ease if no other means are presented; that is, the solution is not dependent on "pennant diagrams", "conjugate points", or any other extraneous conception. Of course, a direct solution is preferable to one by trial, and this Ostenfeld's diagram provides. It is, however, nothing but a special form of an auxiliary geometrical construction that locates a "conjugate" point through which any line passing will satisfy the conditions imposed by Fidler's points. The same definite result may be accomplished by several other constructions, one of which will be given subsequently. Certainly, the conception of conjugate points is in no way essential to the problem, for the writer has been using them for several years without ever being impressed with the fact that they required a name and theory to support them. The purpose was equally well served when they were merely auxiliary points on the moment closing line.

In general, it appears to the writer that the authors' method of analysis is a clear case of "the tail wagging the dog"; that Fidler's points, by whatever name the authors may call them, are necessarily the actuating principle under cover of a mass of more or less incidental theory which principally serves to

prove the correctness of the points. Furthermore, when Fidler's points are adopted as the fundamental argument, there is available at once a definite, tangible, and self-sufficient law, which, if expressed in its general terms, applies without change to all the cases that the authors have considered. Also, as will be shown, the conception of the "characteristic points" leads directly to some very material short cuts, as compared with the authors' methods. It is for these reasons that the writer believes the purpose of the paper would have been better served had the authors informed themselves of Fidler's principles and used them as the foundation. This would have detracted nothing from the value and timeliness of the paper, for the strangest thing about Fidler's method, in view of its excellence, antiquity, and rather broad publication, is that so few engineers (relatively) are acquainted with it. Furthermore, the original material is generally difficult to obtain and to digest because of the peculiar form in which the subject is presented. Also, as far as the writer is aware, the method has never been carried to its logical and final development. Therefore, there was a real need for a paper such as the authors have presented, and this is in no wise diminished by the fact that the presentation would involve little of true originality or novelty.

Another peculiar coincidence as to methods and results is to be found in the tables of constants (Section 14*), for cases of varying moments of inertia. A very comprehensive set of tables of identical form, but different values, has been prepared and published by A. Strassner,† who developed a system quite different from that of Fidler. Superficially, there would appear to be no connection between the ideas and no possibility of using Strassner's tables in connection with Fidler's method. Certainly, Strassner himself indicates no such possibility; but due to the ingenuity of Walter Ruppel, Assoc. M. Am. Soc. C. E., assisted by Herman Schorer, Assoc. M. Am. Soc. C. E., both of whom were then (middle of 1924) associated with the writer on some rather intricate design, it was discovered that Strassner's tables could be adapted with relative ease to give the heights and lateral positions of Fidler's "characteristic points". Incidentally, these tables also give the fixing moments at the ends of the beam, as required in the solution of problems by the slope-deflection method. Mr. Ruppel worked out a complete set of formulas for transposing Strassner's tabular values into the quantities required in Fidler's or the slope-deflection method, an entirely original and very creditable achievement.

To return to the aforementioned similarity, the two tables given by the authors are identical in form, and even to interval of the arguments, with certain of Strassner's tables. The quantities which the authors denote as u and m, Strassner lists as, say, r and w, and the values are not alike. As Mr. Ruppel discovered, however, $u = \frac{w}{2r + w}$, and $m = \frac{2r + w}{6I}$; and this is verified by the authors' tables even to the last decimal place. The series of coincidences represented here is little short of marvellous.

* See p. 27.

† "Neuere Methoden," Vol. 1, 2d Edition, "Der durchlaufende Rahmen."

The authors refer very casually to the preparation of these tables, but the writer has no desire to undertake such a task. The time and effort required for original computations would be appalling, and out of all proportion to their value to the individual designing engineer. An alternative would be to accept Strassner's tables and modify them according to Mr. Ruppel's findings, which in itself is no mean undertaking. Fortunately, Mr. Ruppel has done this work and has presented it in his discussion of the paper,* so that each engineer will not be faced with the painful necessity of preparing his own original tables, as suggested by the authors.

Fidler's Method Needs Explanation.—As the writer has intimated, he is of the opinion that Fidler's Method of Characteristic Points is the peer of all methods for treating moments in restrained and continuous beams, and that it deserves a wider publicity and understanding than it now has. Unfortunately, as far as the writer is aware, there is no really acceptable and comprehensible treatment of the subject to which those interested might be referred. There are Fidler's own writings, of course, but these are not in current circulation and therefore are difficult to obtain. Although the writer has not read these books and so cannot speak from personal knowledge, he is informed, by engineers well qualified to judge, that the subject is there presented in such a peculiar and awkward manner as to be unnecessarily difficult to follow and digest. Furthermore, Fidler did not carry the development of his formulas and methods to cover conditions which now obtain in structural design, so that judging from his own treatment the method might be thought inadequate to meet the complex requirements of to-day. Aside from Fidler's own writings, the discussions of the subject, within the writer's knowledge, are exceedingly fragmentary and treat only of simple cases, without proof. The only one that has come to the writer's personal attention is the brief abstract by Richart and Wilson, previously referred to. It appears evident that the subject needs to be presented anew and in a comprehensive manner adapted to present-day uses.

Using Fidler's fundamental proposition as the argument, the writer has expanded his method to include the most general cases, and has shown that the basic equation controls throughout in unchanged form. In view of the situation as outlined, it seems appropriate to give here a complete description of this development. It is hoped that the presentation will succeed in demonstrating the inherent simplicity and excellence of Fidler's Method of Characteristic Points, and of making this method more generally available to the profession.

In so far as the writer is personally concerned, this discussion and the method of development is entirely original, but no claim is laid to the fundamental ideas on which it is based. As far as he is aware, Fidler's method has not heretofore been developed to cover the more complex cases herein treated, in which case the presentation will be fully justified. In any event it is certain that these broader applications of the method are not generally known.

* See p. 152.

Definitions and Conditions for Fidler's "Characteristic Points."—Before entering on the general equations relating to the "characteristic points", it will be advisable to discuss their more evident and tangible properties, particularly since fundamentally there is nothing to indicate the existence of such points, and the demonstration that they do exist must take the form of proving that properties observed for specific cases are also of general application. The conception of the characteristic point, in a general sense, is a mere hypothesis, and it must be demonstrated that this hypothesis is, in fact, a statement of the law of structural mechanics.

In a bending moment diagram, let the line connecting the plotted moments over the supports be called the moment closing line. Then, the "characteristic point" may be defined as a point such that the vertical intercept between the point and the moment closing line is a measure of and proportional to the slope of the beam over the support.

By the slope of the beam at or over the supports is meant the angular change in the direction of the elastic curve due to the stress in the beam. When the end of a beam is fully restrained, or "fixed," this angular change is zero by definition. Since the vertical distance between the characteristic point and the moment closing line is proportional to the angular change at the end of the beam, and since this angular change is zero when the end is fixed, the distance from the point to the line must also be zero; that is, the point lies on the line for this condition. Since, in general, the slopes at the two ends of the beam may be different, it is evident that there must be one characteristic point for each end of the beam. By definition, then, these two characteristic points will lie on the moment closing line when the two ends of the beam are fixed. That this conception has an important bearing on the practical application of the method will be shown subsequently. Likewise, by definition, considering the same beam under the same loading, but fixed at one end only and freely supported at the other, the characteristic point corresponding to the fixed end must lie on the moment closing line. The tendency to angular change at the ends of a beam is a function of both the loading and the properties of the beam itself, and since the position of the characteristic point is a function of the angular change this position must likewise be a function of the loading and beam properties. Keeping the beam and loading constant, it appears probable that, corresponding to the end of the beam considered fixed, there will be a characteristic point common to the two conditions considered. If so, it follows that this point must lie at the intersection of the corresponding moment closing lines. In fact, if there is a characteristic point, it can be nowhere else since no other position would satisfy the conditions imposed.

Consider now a beam of constant $E\,I$, with its ends fixed, and under uniform load. The end moments are $\frac{w\,L^2}{12}$. If one end only is fixed, the end moment is $\frac{w\,L^2}{8}$, or 1.5 times the preceding value. The moment diagrams, superimposed, are shown in Fig. 51, for negative moments only.

The line, $B'\,C'$, is the moment closing line when the beam is fixed at both ends, for which, let $M' = M'' = 1$. The lines, $B''\,C$ and $B\,C''$, are the corres-

ponding lines when the beam is fixed alternately at the one end and simply supported at the other, for which, M' or $M'' = 1.5$. It is evident that the line, $B'C'$, intersects the line, BC'', at a point, V, which by proportion is at two-thirds the height of the triangle, BCC'', and, therefore, at a horizontal distance of $\frac{1}{3}L$ from C. Likewise, U is a point $\frac{1}{3}L$ distant from B. Furthermore, it will be observed that the points, U and V, are in the same vertical plane as the centers of gravity of the corresponding triangular end moment areas. Also, when EI is constant, the $\frac{M}{EI}$-areas corresponding to the two end moments are likewise triangular, and their centers of gravity lie on the same vertical as those of the M-areas; therefore, in the case considered, on the same verticals as the points, U and V, which satisfy the conditions imposed for characteristic points.

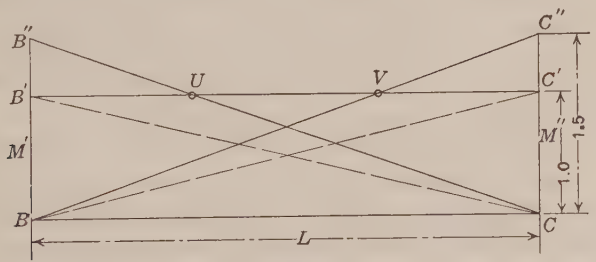

FIG. 51.

From the foregoing, certain general conditions have been established which the characteristic points must satisfy, and characteristic points have been established to satisfy a specific condition. Since there can necessarily be only one position of each characteristic point corresponding to a certain beam and loading, it follows that, if the hypothesis regarding the points is true, the general case must conform to the specific facts developed. It remains to demonstrate that such is the case; that a point located on the moment closing line, vertically above the center of gravity of the negative $\frac{M}{EI}$-area corresponding to one end of the beam, satisfies the definition of the characteristic point for that end.

Notation and Theorems.—The notation herein will be the same as that used by the authors, with the additions noted. It is assumed, unless otherwise stated, that the beam supports are at the same elevation and that the beam is horizontal. Also, that the moment diagrams are plotted with respect to a horizontal line between supports, which line will be called the "moment reference line" (sometimes called the "M. R. L."). It is also assumed that both positive and negative moments will be plotted normal to and above this line, as customary.

Let l = the span of the beam, center to center of supports.

U = the left characteristic point, or its height above the moment reference line (see Fig. 53).

V = the right characteristic point, or its height above the moment reference line (see Fig. 53).

U' and V' = points on the moment closing line, vertically above U and V, respectively. (It will be observed that U and V are the particular values of U' and V' when the moment closing line is that for the beam completely fixed at both ends.) The "moment closing line" will sometimes be abbreviated to "M. C. L."

d' = the vertical distance from the moment closing line to the left characteristic point, = U-U'.

d'' = the corresponding distance on the right side, = V-V'.

M' = the restraining moment at the left support.

M'' = the restraining moment at the right support. (Restraining moments will also be referred to as negative moments.)

A^0 = the area of the $\frac{M}{E\,I}$-diagram corresponding to the actual load, the beam being considered as simply supported. When $E\,I$ is constant, $A^0 = \frac{A}{E\,I}$, in which, A equals the area of the M-diagram.

A' = the area of the $\frac{M}{E\,I}$-diagram corresponding to a negative moment, $M = 1$, applied at the left end, the beam being considered as simply supported.

A'' = the same as A', except that $M = 1$ is applied at the right end.

(In the foregoing, if l is taken in feet, and M in foot-pounds, then E is in pounds per square foot and I is in feet to the fourth power.)

x^0 = the horizontal distance from the left support to the center of gravity of A^0.

x' = the horizontal distance from the left support to the center of gravity of A'.

x'' = the horizontal distance from the left support to the center of gravity of A''.

ϕ' = the slope of the beam at the left support.

ϕ'' = the slope of the beam at the right support. (By slope of beam is meant the angle, expressed in radians, through which the neutral axis is turned due to the stress in the beam, and is here considered as the angle which the tangent to the elastic curve makes with the horizontal.)

θ = the angle between the tangents to any two points on the elastic curve.

D = the deflection of any point on the elastic curve with respect to the position of the point in the unstressed beam; that is, the vertical displacement of the point due to stress.

y' = the vertical distance between a point on the elastic curve and the tangent to any other point on the curve.

The following theorems are used as the basis of the subsequent demonstration. These theorems are in no sense original, but they and their proofs

are here given because they have not generally been developed or used in American textbooks; they are given in one group for ease of reference, and because they are more or less interdependent.

Theorem No. 1.—The angle, θ, between the tangents to any two points on the elastic curve, is the $\frac{M}{E\,I}$-area between the points. Let the $\frac{M}{E\,I}$-area between the points be called a. Then, $\theta = a$.

Proof: This is simply a modified form of the usual integral expression as given in practically any text on the mechanics of materials, in the form of $\theta = \int \frac{M\,d\,x}{E\,I}$.*

Theorem No. 2.—The vertical displacement, y', of any point, c, on the elastic curve with respect to the tangent at any other point, b, is the statical moment of the $\frac{M}{E\,I}$-area between the points, taken about the point, c. That is, letting z equal the horizontal distance from c to the center of gravity of a, $y' = az$.

Proof: This is a modified form of the usual integral expression, $y' = \int \frac{M\,x\,d\,x}{E\,I}$.†

Theorem No. 3.—The angle, ϕ, which the elastic curve makes with the horizontal at the support is the reaction at that support when the beam is considered as a simple span supporting the $\frac{M}{E\,I}$-area treated as a load. That is, $\phi' = \frac{A^0\,(L - x^0)}{L}$, and $\phi'' = \frac{A^0\,x^0}{L}$.

Proof: (See Fig. 52.) Given a loaded beam, $B\,C$, the angle at B is ϕ', and from Theorem No. 2 the y' of C with respect to the tangent at B equals $A^0\,z = A^0\,(L - x^0)$. From the diagram, however, $y' = \phi'\,L$, also. Therefore, $\phi'\,L = A^0\,(L - x^0)$, and, hence, $\phi' = \frac{A^0\,(L - x^0)}{L}$, which is the expression for the reaction at B when the beam is loaded with the area, A^0.

Theorem No. 4.—The deflection, D, of any point, c, on the elastic curve, is the bending moment at the point when the beam is considered as a simple span supporting the $\frac{M}{E\,I}$-area treated as a load. That is, if the distance of c from the left support is called x, then, $D = \left[\frac{A^0\,(L - x^0)}{L}\right] x - a\,z = \phi'\,x - a\,z$, in which, a is the $\frac{M}{E\,I}$-area between the left support and the point, c, and z is the distance from c to the center of gravity of a.

Proof: (See Fig. 52.) The reaction at B is ϕ', from Theorem No. 3. From the diagram, the deflection, D, at c = Distance $c\,e$ = Distance $e\,f$ — Distance $c\,f$;

* For proof, if any is required, see Church's "Mechanics of Materials."

† See Church's "Mechanics of Materials."

but Distance $ef = \phi' x$, and from Theorem No. 2, Distance $cf = az$. Therefore, $D = \phi' x - az$, and since ϕ' is the reaction at B, this is the expression for the moment at c when the beam is loaded with the area, A^0.

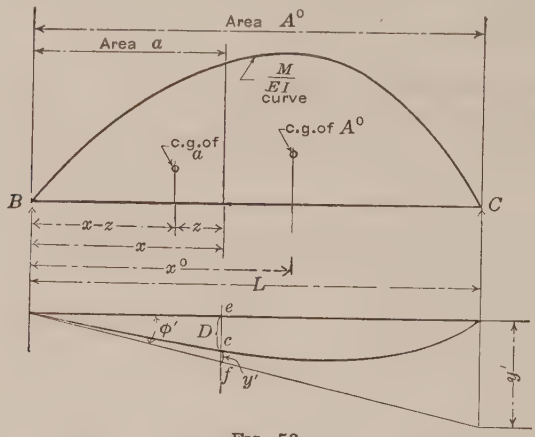

FIG. 52.

Theorem No. 5.—The angle, ϕ', at the left support, B, of a beam, due to a certain moment, M, applied at the right support, C, is equal to the angle, ϕ'', at C, when the same M is applied at B.

Proof: Consider the $\frac{M}{EI}$-area, A'. The moment at any point is equal to $\frac{(L-x)}{L}$, since $M = 1$ by definition, and x is measured from the left support. Corresponding to any value of x there is some particular value of I. Accordingly, I may be regarded as a function of x, and expressed as $f(x)$. Then,

$$A' = \frac{I}{E} \int_B^C \frac{\frac{L-x}{L}}{f(x)} dx$$

and from Theorem No. 2 the displacement of B with respect to the tangent at C is,

$$y' = \frac{I}{E} \int_B^C \frac{L-x}{L f(x)} x \, dx$$

but the angle at C equals

$$\phi'' = \frac{y'}{L} = \frac{I}{E} \int_B^C \frac{(L-x)x}{L^2 f(x)} dx \quad \ldots\ldots\ldots\ldots\ldots\ldots (48)$$

Consider now the $\frac{M}{EI}$-area, A''. The moment at any point is $\frac{x}{L}$, and,

$$A'' = \frac{I}{E} \int_C^B \frac{\frac{x}{L}}{f(x)} dx$$

The displacement of C with respect to the tangent at B equals

$$y'' = \frac{I}{E} \int_C^B \frac{x}{L f(x)} (L-x) \, dx$$

but the angle at B equals

$$\phi' = \frac{y''}{L} = \frac{I}{E}\int_C^B \frac{x\,(L-x)}{L^2\,f\,(x)}\,dx \qquad (49)$$

Equation (49) is identical with Equation (48), except as to sign, due to change of limits. Therefore, $\phi'' = -\phi'$ when the moment, $M = 1$, is alternately applied at B and C, respectively.

General Proof of the Theorem of "Characteristic Points."—Consider a beam of such character that $E\,I$ is a variable throughout. The ends are restrained by the moments, M' and M''. The values of A' and A'' are different; x' and x'' are apparently unrelated and may vary over wide limits. The conditions may be assumed as represented in Fig. 53. This drawing is purely diagrammatic and no attempt is made at consistency. The areas, A^0, A' and A'', may take practically any form, depending on the shape of the member and the loading. The magnitude of the negative $\frac{M}{E\,I}$ - loadings is $A'\ M'$ and $A''\ M''$, and the lateral position of the centers of gravity of these areas is the same as for A' and A''. These negative areas may be regarded as upward loads, and hence are shown below the line, opposite the positive $\frac{M}{E\,I}$ - area.

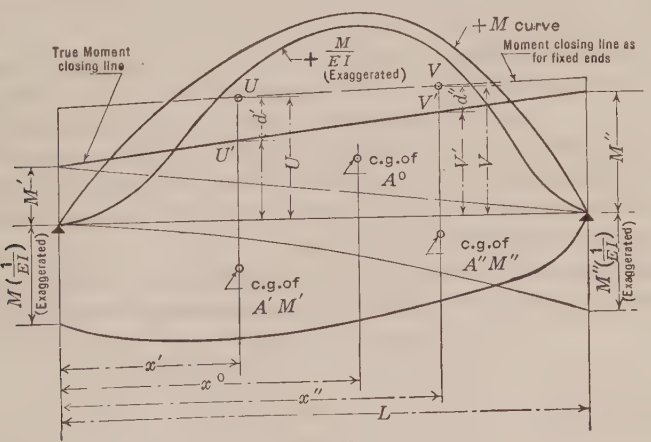

FIG. 53.

From Theorem No. 3,

$$\phi' = \frac{A^0\,(L-x^0)}{L} - \frac{M'\,A'\,(L-x')}{L} - \frac{M''\,A''\,(L-x'')}{L} \qquad (50)$$

$$\phi'' = \frac{A^0\,x^0}{L} - \frac{M'\,A'\,x'}{L} - \frac{M''\,A''\,x''}{L} \qquad (51)$$

From the geometry of Fig. 53:

$$U' \text{ (or } U) = \frac{M'\,(L-x')}{L} + \frac{M''\,x'}{L} \qquad (52)$$

$$V' \text{ (or } V) = \frac{M'\,(L-x'')}{L} + \frac{M''\,x''}{L} \qquad (53)$$

These are the four fundamental equations on which the following demonstration will be based.

The proof that the theorem of characteristic points applies to the general case is necessarily quite involved and peculiar. It is described in some detail, as otherwise the significance of the following four steps that are required will not be apparent.

(1).—It will be assumed that the theorem of "characteristic points" is still only a hypothesis, and that there are only those indications relative to the points which have so far been developed in this discussion. It will be further assumed that the hypothesis is true, and that the points will lie on the moment closing line as for completely fixed ends, and vertically above the center of gravity of the areas, A' and A'', respectively, as heretofore indicated.

(2).—A value will be found for the height of each characteristic point for a specific case that will permit this determination. If the theorem is true, these values must be the true values, since each point can have only one value for given conditions of beam and load.

(3).—The values as found will then be assumed to be true, and be substituted in a specific case involving both, which will lead to an identity between apparently unlike quantities, and to a valuable proportional relationship between others. These findings will be true only if the values as found for the characteristic points are true.

(4).—Assuming that the findings under Step (2) are true, Step (3) permits the substitution of terms in the original equations which puts them into the same form as those developed under Step (2). A comparison of these equations demonstrates that the required conditions for the characteristic point are satisfied. Therefore, the assumptions which led to this conclusion are verified, and the properties, as accepted tentatively for the characteristic points, are the true general properties.

Consider the beam fixed at the left end simply supported at the right end. Then, since both ϕ' and M'' equal zero, from Equation (50):

$$\frac{A^0\,(L - x^0)}{L} - \frac{M'\,A'\,(L - x')}{L} = 0 \qquad (54)$$

From Equation (52):

$$U = \frac{M'\,(L - x')}{L} \qquad (55)$$

From Equation (54):

$$\frac{M'\,(L - x')}{L} = \frac{A^0\,(L - x^0)}{L\,A'}$$

and the left-hand term of this equation equals the right-hand term of Equation (55). From this:

$$U = \frac{A^0\,(L - x^0)}{L\,A'} \qquad (56)$$

which will be accepted tentatively as true. It is not subject to proof at this stage.

Consider, now, the conditions as to end supports reversed, in which case, both ϕ'' and M' are equal to zero, and, from Equation (51),

$$\frac{A^0\,x^0}{L} - \frac{M''\,A''\,x''}{L} = 0 \qquad (57)$$

and from Equation (53):

$$V = \frac{M''\,x''}{L} \qquad (58)$$

From Equation (57):

$$\frac{M''\,x''}{L} = \frac{A^0\,x^0}{L\,A''}$$

from which, by comparison with Equation (58),

$$V = \frac{A^0\,x^0}{L\,A''} \qquad (59)$$

which also will be tentatively accepted as true.

To demonstrate that the values of U and V as given by Equations (56) and (59) are true, it will be necessary to prove that these values satisfy the general conditions imposed.

Now, consider the beam fixed at both ends, and assume that the values of U and V in Equations (56) and (59) are true for this condition also. Since $\phi'' = \phi' = 0$ from Equation (50), dividing all terms by A' and transposing:

$$\frac{M'\,(L - x')}{L} = \frac{A^0\,(L - x^0)}{A'\,L} - \frac{M''\,A''\,(L - x'')}{A'\,L} \qquad (60)$$

Adding $\frac{M''\,x'}{L}$ to both sides,

$$\frac{M'\,(L - x')}{L} + \frac{M''\,x'}{L} = \frac{A^0\,(L - x^0)}{A'\,L} - \frac{M''\,A''\,(L - x'')}{A'\,L} + \frac{M''\,x'}{L} \qquad (61)$$

From Equation (52) the left-hand side is equal to U, and from Equation (56) the first right-hand term is also equal to U. Therefore, the sum of the last two right-hand terms must equal zero, from which, multiplying these terms by A' and transposing:

$$\frac{M''\,A''\,(L - x'')}{L} = \frac{M''\,A'\,x'}{L} \qquad (62)$$

Substituting in Equation (50), the general equation becomes:

$$\phi' = \frac{A^0\,(L - x^0)}{L} - \frac{M'\,A'\,(L - x')}{L} - \frac{M''\,A'\,x'}{L} \qquad (63)$$

Now, $d' = U - U'$. Therefore, from Equations (56) and (52),

$$d' = \frac{A^0\,(L - x^0)}{L\,A'} - \frac{M'\,(L - x')}{L} - \frac{M''\,x'}{L} \qquad (64)$$

As Equation (64) is Equation (63) divided by A',

$$d' = \frac{\phi'}{A'}, \text{ or } \phi' = d'\,A' \qquad (65)$$

This shows that the distance from the assumed characteristic point to the moment closing line is a function of and proportional to the angle at the

adjacent support, which satisfies the definition of the characteristic point. Therefore, the assumed properties of the point are the true properties, and the theorem is verified for the point, U.

Considering now the point, V, from Equation (51), dividing all terms by A'', and transposing,

$$\frac{M''\,x''}{L} = \frac{A^0\,x^0}{L\,A''} - \frac{M'\,A'\,x'}{L\,A''} \qquad (66)$$

Adding $\frac{M'\,(L - x'')}{L}$ to both sides,

$$\frac{M''\,x''}{L} + \frac{M'\,(L - x'')}{L} = \frac{A^0\,x^0}{L\,A''} - \frac{M'\,A'\,x'}{L\,A''} + \frac{M'\,(L - x'')}{L} \qquad (67)$$

From Equation (53) the left-hand expression equals V, and from Equation (59) the first term of the right-hand side also equals V. Therefore, the sum of the last two right-hand terms equals zero. Multiplying each of these terms by A'' and transposing,

$$\frac{M'\,A'\,x'}{L} = \frac{M'\,A''\,(L - x'')}{L} \qquad (68)$$

Substituting in Equation (51), the general equation becomes:

$$\phi'' = \frac{A^0\,x^0}{L} - \frac{M'\,A''\,(L - x'')}{L} - \frac{M''\,A''\,x''}{L} \qquad (69)$$

Now, $d'' = V - V'$. Therefore, from Equations (59) and (53),

$$d'' = \frac{A^0\,x^0}{A''\,L} - \frac{M'\,(L - x'')}{L} - \frac{M''\,x''}{L} \qquad (70)$$

As Equation (70) is Equation (69) divided by A'',

$$d'' = \frac{\phi''}{A''}, \text{ or } \phi'' = d''\,A'' \qquad (71)$$

This gives the same proof for the point, V, as Equation (65) gave for the point, U, as noted previously. The theorem of characteristic points is thereby fully verified for the general case.

Dividing both sides of Equation (62) by M'' and multiplying by L,

$$A''\,(L - x'') = A'\,x' \qquad (72)$$

As $L - x''$ is the distance from the right end of the span to the center of gravity of A'', and x' is the distance from the left end to the center of gravity of A', Equation (72) gives the useful principle that the areas, A' and A'', are inversely proportional to the distances of their centers of gravity from the ends of the diagrams used as origins of moments. This provides an easy means of finding any one of four required values when the other three are known, or it provides a check on the accuracy of the results if the four values are determined independently.

The foregoing relationship was determined incidentally and was predicated on the truth of apparently unrelated assumptions. The proposition may be proved otherwise as follows:

From Theorem No. 5, $\phi' = \phi''$ when the moments, $M'' = 1$ and $M' = 1$, are applied alternately at the right and left ends, respectively, of the beam. Since ϕ' is the angle corresponding to M'', it is the left-end reaction of the

$\frac{M}{E\,I}$-area, A'', and for a like reason, ϕ'' is the right-end reaction of A'. Therefore, $\phi' = \frac{A''\,(L - x'')}{L}$ and $\phi'' = \frac{A'\,x'}{L}$. Since $\phi' = \phi''$, $A''\,(L - x'') = A'\,x'$, which is an independent verification of the conclusions drawn from Equation (61), and hence of the assumptions on which these conclusions were predicated.

To recapitulate, the following facts have been demonstrated:

(*a*) The theorem of Fidler's characteristic points is true for all conditions of beam and load.

(*b*) These points are located vertically above the centers of gravity of the areas, A' and A'', respectively.

(*c*) The height of the points is given by the following: $U = \frac{A^0\,(L - x^0)}{L\,A'}$ (Equation (56)), and $V = \frac{A^0\,x^0}{L\,A''}$ (Equation (59)), and these are points on the moment closing line, considering the beam as fixed at both ends.

(*d*) For any condition of end restraint, the vertical distances between the characteristic points and the moment closing line are: $d' = \frac{\phi'}{A'}$ (Equation (65)), and $d'' = \frac{\phi''}{A''}$ (Equation (71)), which show that these distances vary directly as the slope of the beam at the adjacent support, and inversely as A' or A''.

(*e*) The products, $A'\,x'$ and $A''\,(L - x'')$, are equal, or $A' : A'' = (L - x'') : x'$.

It is not required to evaluate d' or d'', as indicated in Equations (65) and (71). These equations have served their purpose in defining the fundamental relationships and proportions. By applying this law in the graphical construction to follow, the values of d' and d'' are automatically determined. Their actual values, however, are unimportant, since only their relative values enter into the solution of the problem.

It will be observed that, other things being equal, the height of the characteristic points is a function of the magnitude of the load on the span, and that their lateral position is a function of $\frac{1}{E\,I}$, and hence depends solely on the shape of the beam if E is constant. From this it will be seen that the "characteristic points" are indeed well named, being characteristic both of the loading on the beam and of the qualities of the beam itself.

"Characteristic Points" for Specific Conditions.—If the member is of varying $E\,I$, but symmetrical about its middle, $x' = L - x''$, and $A' = A''$. The characteristic points will then be symmetrically located on the span. If the load is also symmetrical about the center of the span, $U = V$, since $L - x^0 = x^0$.

If the member is of constant $E\,I$, the $\frac{M}{E\,I}$-areas are triangular and $A' = A'' = \frac{L}{2\,E\,I}$. Since the characteristic points are on the same vertical as the center

of gravity of these areas, $x' = \frac{L}{3}$ and $x'' = \frac{2L}{3}$. Also, substituting the values of A' and A'' in Equations (56) and (59),

$$U = \frac{2A(L - x^0)}{L^2} \quad \ldots\ldots (56a)$$

and

$$V = \frac{2Ax^0}{L^2} \quad \ldots\ldots (59a)$$

When, in addition, the load is symmetrical, these reduce to

$$U = V = \frac{A}{L} \quad \ldots\ldots (56a)$$

"Characteristic Points" for Beams of Constant E I, from the End That Fixes the Moments.—Except in cases of very simple loading the determination of the value of A, and of x^0 if the load is unsymmetrical, would require a considerable expenditure of time and effort. Fortunately, this usually can be avoided by utilizing the principle that the U and V-points lie on the moment closing line as for the beam with both ends fixed. Let M' equal the moment at the left end, and M'' that at the right end for this condition. Then

$$U = M' + \frac{(M'' - M')}{3}$$

and

$$V = M' + \frac{2(M'' - M')}{3}$$

Most engineering handbooks contain tables of end moments for fixed beams under various arrangements of load, from which, for most cases, U and V may be derived readily. A very comprehensive set of such tables appears in "Stresses in Framed Structures", Hool and Kinne, pages 486 to 490, inclusive, from which the moments, M' and M'', may be determined with relative ease for almost any type of loading. In these tables, $C_{AB} = M'$ and $C_{BA} = M''$. This suggested method of determining U and V is predicated, of course, on the possession of such tables and their application to the problem at hand. Otherwise the suggestion does not apply, since it would be more difficult to find the values of M' and M'' than to determine A and x^0. Likewise, it applies only when the member is of constant $E I$, since such tables do not cover any other case.

In these tables (Hool and Kinne) are also given values of H_{AB} and H_{BA}. The former is the moment at the left end when that end is fixed and the right end simply supported; the latter is the moment at the right end for the reverse condition. Consequently, $U = \frac{2}{3} H_{AB}$, and $V = \frac{2}{3} H_{BA}$. The values are the same as those just given, of course, and are somewhat easier to compute.

"Characteristic Points" for General Case.—If modified Strassner's tables are available, as the writer understands they will be, the values of U, V, x' and x'' may be derived with ease for beams of any shape and loaded in any

manner. The presentation of this phase is, however, outside the scope of this discussion.

Should such tables as described be not available, it will be necessary in all complex cases to determine the values of A^0, A', A'', x^0, x', and x'', in order to compute U and V. No great accuracy is required in this evaluation, and the graphical method will give a satisfactory solution. If the $\frac{M}{E\,I}$-diagrams are plotted to a convenient scale on a good quality of light cardboard or pressboard, the areas may first be taken off with a planimeter; then the diagrams may be cut out and their centers of gravity easily found by the method of successive suspensions, giving two intersecting lines each containing the center of gravity, which therefore will lie at the intersection.

Graphical Application of Theorem of Characteristic Points and Auxiliary Constructions.—It remains to make application of the foregoing facts in the graphical solution of moment distribution in continuous beams. Referring to Fig. 54, let V and U be the two characteristic points adjacent to and on opposite sides of one of the supports of a continuous beam. Let A'', d'', and ϕ'' refer to the right-hand properties of the left-hand span, and let A', d', and ϕ' refer to the left-hand properties of the right-hand span, according to the adopted notation.

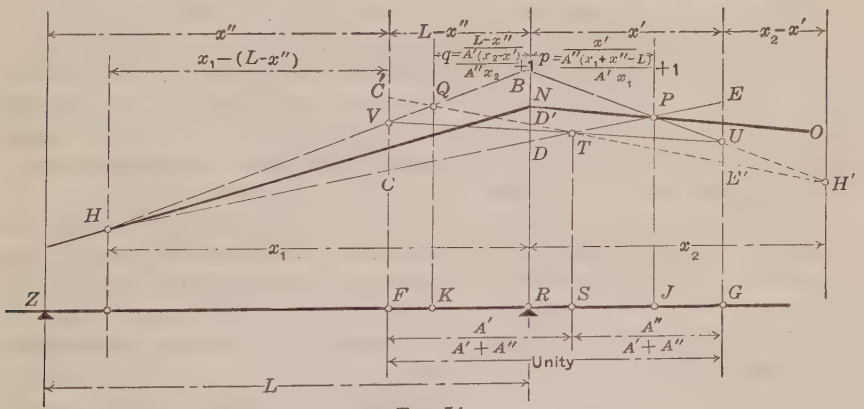

FIG. 54.

The slope of the beam over the support must be common to both spans; that is, $\phi'' = -\phi'$, or *vice versa* with respect to sign, since one span will slope upward and the other downward, at the support. From Equation (71), $\phi'' = d''\,A''$, and from Equation (65), $\phi' = d'\,A'$; but since $\phi'' = -\phi'$ $d''\,A'' = -d'\,A'$, from which, $d'' : A' = -d' : A''$. That is, considering the two characteristic points adjacent to and on opposite sides of a support, one of these is above and the other below the moment closing line, and the distances between the points and the line are inversely proportional to the areas, A' and A'', corresponding to the points. It is required to develop a graphical construction that will satisfy this condition.

In Fig. 54, let H be a known point on the moment closing line. Draw the line, $HVBU$. This line satisfies the required conditions, since it contains the

point, H, and $d'' = d' = 0$. (Because, if $d'' = 0$, $d'' A'' = 0$, and, hence, $-d' A' = 0$; but, as A' is not zero, $-d'$ equals zero.)

Draw the line, VU. It is required to establish a point on this line such that any other line drawn through it will satisfy the required proportions of d'' to d'. It has been shown that d'' is proportional to A', and d' to A''. Therefore, SF is proportional to A' and SG to A'', from which $SF : A' = FG : A' + A''$. Let $FG = 1$. Then, $SF = \frac{A'}{A' + A''}$. Similarly, $SG = \frac{A''}{A' + A''}$. Establish the point, S, accordingly and erect a vertical to intersect UV at the point, T, which is the required point, as above.

Draw the line, HTE, which satisfies the requirements, since it contains the point, H, and passes through T. This line intersects the line, BU, in the point, P.

Now, the distance, VC, is proportional to d'' and EU is proportional to d'; also, both VC and EU are proportional to BD. From the relationships of the triangles involved, it is apparent that any other line, such as ONH, passing through P and H, will maintain the same proportional values of d'' and d' as those given by the line, HTE. Therefore, the point, P, is a point on the moment closing line, since any line through this point will satisfy the required conditions thus far imposed.

Of all the possible lines that may be drawn through the known points, H and P, there is only one that will satisfy the conditions at other points in the beam. These conditions will be represented by other points, P, one for each additional span. Then, provided the degree of restraint at the outside ends of the beam is known, these several P-points determine the true moment closing line. The restraint at the outside ends must be known or assumed, in order to fix the starting and ending points of the diagram.

It is believed that the method of establishing the points, T and P, with A' and A'' as arguments, is the only possible method when the beam is non-symmetrical and of varying EI. The customary construction, as represented by Ostenfeld's auxiliary diagram (the authors' "pennant diagram"), has distances only as an argument, and thus requires the device of "transformed spans" when EI is not the same for every span. As in a continuous beam no one span of constant EI can be the equivalent of a non-symmetrical span of varying EI such diagrams are not applicable to this case. It may be stated in passing that such diagrams can be applied when the spans are of varying EI within themselves, but symmetrical, if the spans are laid off proportionally to their $A' = A''$. It would seem, however, that there is no merit in using the principle of "transformed spans" when the same result can be accomplished in a more simple and direct fashion.

A noteworthy feature of the foregoing method of establishing the point, T, with A' and A'' as the argument, is that it makes the construction entirely independent of the scale length of the spans. These may be laid out to any distance, regardless of their true relative lengths. Then, provided the characteristic points are located in their true relative position on the span and that A' and A'' are proportional to the true span lengths, the construction will give the correct results. No proof of this statement will be given

here, as it is practically self-evident, and can be easily verified by actual construction.

Let L_1 = the span to the left, and L_2 = the span to the right of the support. When EI is different for these spans, but constant throughout each, $A'' = \frac{L_1}{2\ E_1\ I_1}$ and $A' = \frac{L_2}{2\ E_2\ I_2}$. If $E_1 = E_2$, as is customary, A'' may be taken as $\frac{L_1}{I_1}$, and A' as $\frac{L_2}{I_2}$, since only relative values of these areas are required.

When EI is constant throughout each span of the beam, A' varies as L_2 and A'' varies as L_1. Therefore, d'' and d' vary inversely as the spans in which they occur, or $SF : L_2 = SG : L_1$; from which, $SF : L_2 = FG : L_1 + L_2$. In this case, however, $FG = \frac{(L_1 + L_2)}{3}$, from which,

$$SF = \frac{(L_1 + L_2)\ L_2}{3\ (L_1 + L_2)} = \frac{1}{3}\ L_2.$$

Similarly, $SG = \frac{1}{3}\ L_1$, which explains the location of the "transposition line" in the customary diagrams.

It will be observed that the construction herein described, except for the entirely general method of locating the lateral position of the T-point, is identical with that shown by the authors' Fig. 8. Also, the methods described under the authors' Section 7* are a modified form of the writer's general method, applied to a special case. The theory used by the writer to develop this method was entirely different, however, from that used by the authors and serves to demonstrate his assertion that the P (conjugate) points are merely incidental to, and derive all their significance from, the U and V- (Fidler's characteristic) points. Also, since the "Theorem of Three Moments in Simplified Form" (Section 1†), accounted only for the P-points, it is evident that it is the U and V-points which bridge the gap between the authors' theory and their graphical constructions. From this it appears that their failure to give any adequate explanation for, or theory to support, these points was a rather important omission.

Referring again to Fig. 54, if the known point on the moment closing line is H', to the right of the support instead of the left, a construction similar to that previously described will locate the point, Q, as indicated. This is also a point of the moment closing line, but in the span adjacent to the point, P.

A construction exactly as just described for locating the Q and P-points is inapplicable when the adjacent spans are unloaded. In an unloaded span, the U and V-points lie on the moment reference line, and hence will not generally permit the determination of the intersections required in the foregoing method. A modification of the construction to suit the condition of unloaded spans will now be developed.

* See p. 13.
† See p. 4.

As already shown, $V\,C$ is proportional to A', and $U\,E$ to A''. (See Fig. 54.) Since this construction deals with proportional relationships only, and VC may have any value, $V\,C$ may be made equal to A', and $U\,E$ equal to A''. Then, from the diagram,

$$B\,D = \frac{A'\,x_1}{x_1 + x'' - L}.$$

Therefore,

$$p : \frac{A'\,x'}{x_1 + x'' - L} = x' - p : A'',$$

from which,

$$p = \frac{x'}{\frac{A''\,(x_1 + x'' - L)}{A'\,x_1} + 1}$$

Also,

$$B\,D' = \frac{A''\,x_2}{x_2 - x'}.$$

Therefore,

$$q : \frac{A''\,x_2}{x_2 - x'} = L - x'' - q : A',$$

from which,

$$q = \frac{L - x''}{\frac{A'\,(x_2 - x')}{A''\,x_2} + 1}$$

These expressions for p and q are too cumbersome for practical use, and the purpose can be accomplished more simply than by performing the evaluation. They are given here to demonstrate analytically certain important facts. It will be observed that the formulas contain no load terms, also, that the distances, p and q, determine the points, J and K, respectively. This shows that the lateral position of these latter points is independent of the load, being a function of the properties of the beam only; from which it follows that for any given beam, irrespective of the conditions of loading, the points, P and Q, will lie vertically above, or will be coincident with, certain points, J and K, as for the beam without load. When the points are coincident that point is also a point of inflection, since the moment closing line will cross there. If there is a point of inflection in an unloaded beam it will be either at K or J, depending on the position of the loaded span with respect to the unloaded span considered. For unloaded interior spans to the left of a single loaded span, the inflection point is at J; for similar spans to the right, it is at K, as will be apparent from the subsequent diagram.

It will be seen that the points, K and J, possess considerable significance within themselves. They are the fixed points of Ritter's and of Strassner's method, and are the principal arguments in their constructions. These points are of prime importance in considering moving loads or the construction of influence lines, and much time and labor will be saved by the knowledge that the moment closing line always passes through certain predetermined points

to the right and left of the load, irrespective of the position, character, and magnitude of the latter.

A graphical method will now be developed for determining the K and J-points. Referring to Fig. 54, consider that the load in the left-hand span is removed, and that the known point, H, lies on the moment reference line, $Z\,G$, as it generally will in practice. There being no load in the span the height of the characteristic points is zero; that is, V becomes coincident with F, and, hence, Q with K and B with R. The situation will be as indicated in Fig. 55 (a), the other points being determined by the method previously described. This diagram is not in any sense essential to the following demonstration, but is given to indicate the transition between the two constructions, and to show that these are essential in principle.

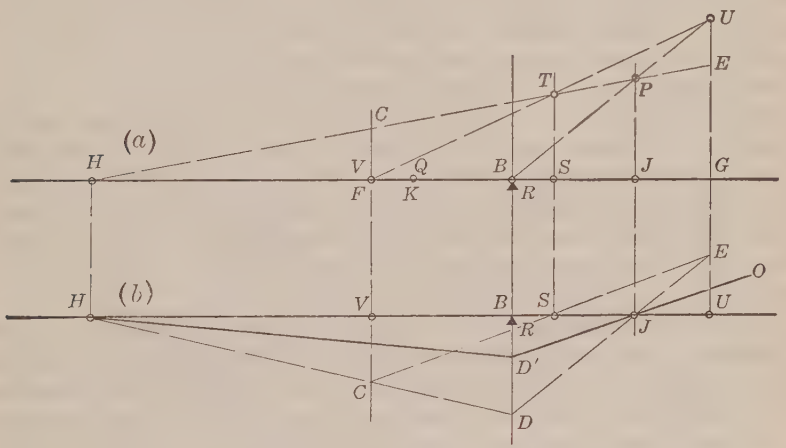

FIG. 55.

Consider now that the load in the right-hand span is also removed, there being some loaded span still farther to the right. Then, the points, T, P, and U, become coincident with S, J, and G, respectively, and all points in both spans lie in a straight line, as shown in Fig. 55 (b). Draw the verticals through the points, V and U. The point, S, being located as in Fig. 54, the line, $C\,E$, through this point will subtend on the above verticals the required proportional distances, $C\,V$ and $E\,U$. As proportionate values only are important, $V\,C$ may be made equal to A', and $E\,U$ equal to A''. Draw $H\,C$, produced, intersecting the vertical through the support at D, and draw $D\,E$, intersecting $R\,U$ in the point, J. Then $R\,D$ is proportional to $V\,C$; also to $E\,U$. As $V\,C = A'$ and $E\,U = A''$, $R\,D$ is proportional to both A' and A'', and it is evident from the triangles involved, that any other line as $O\,D'\,H$, through the point, J, will maintain the same proportional distances. Therefore, the point, J, is a point on the moment closing line, since any such line through it will satisfy the required proportions of d'' to d'. The point being also on the moment reference line, it is the inflection point for that span.

The point, S, is not essential to the location of J. Lay off $V\,C = A'$ and $E\,U = A''$. Produce $H\,C$ to D, and draw $D\,E$, intersecting $B\,U$ in the required

point, J. It is recommended, however, that the construction using the point, S, be adopted, thereby retaining the same methods throughout and also making the point available for other conditions.

With the exception of the manner of locating the point, S, the construction of Fig. 55 (b) is identical with the customary form of Ostenfeld's auxiliary diagram, which shows this diagram to be applicable to the general case, without transformation of spans, provided the point, S, is located with A' and A'' as arguments. It will be observed that this diagram establishes directly only the fixed point, J, but with the characteristic points as arguments, exactly as previously used to locate the point, P; also, that this latter point will lie at the intersection of a vertical through J with the line, $H\,T$, produced, or with the line, $B\,U$, as in Fig. 55 (a). This latter line is not adapted to the general construction, however, and is not recommended for use. If the known point is on the right, instead of on the left as in Fig. 55 (b), a reversed but otherwise similar construction locates the K-points.

The construction just described possesses the merit of being perfectly general and adapted without modification to any condition of beam and load. It requires a few more lines than the construction previously described, but has the advantage that many of these auxiliary lines may be placed entirely outside the principal diagram, resulting in a minimum confusion of lines and points. For these reasons the writer recommends this latter construction. In practice, it is evidently unnecessary to draw all the lines shown, since only intersections are generally required.

The practical application of the foregoing is shown in Fig. 56. Although for purposes of illustration it was necessary to assume some particular arrangement of loading, this diagram is essentially general in its nature, being applicable in principle to beams and loadings of any kind. It is assumed that the left end is freely supported, and the right end fixed. The moment at the left end being zero, the moment closing line will intersect the reference line at that end. The right end being fixed, the moment closing line will pass through the adjacent characteristic point, by definition.

The arrangement of Fig. 56 was chosen so as to show in one diagram, as far as possible, instances of every situation met in such problems. The auxiliary diagram for establishing the J-points is shown below and separate from the moment diagram, for the sake of clearness. It is not necessary to do this, but it will generally be advisable, in order to avoid a confusing jumble of lines and points.

The procedure in laying out such a diagram is, as follows:

1.—The moment reference and reaction lines are drawn, and the U and V-points located.

2.—The reference line for the auxiliary diagram is drawn, parallel to the M. R. L., and vertical lines are projected across it from the U, V, and R-points.

3.—The S-points are located on this auxiliary reference line.

4.—Begin at a point on the auxiliary reference line vertically below a known point on the M. C. L. in the first span to the left. If the end is simply supported, or if there is a known moment at the end support, as from a canti-

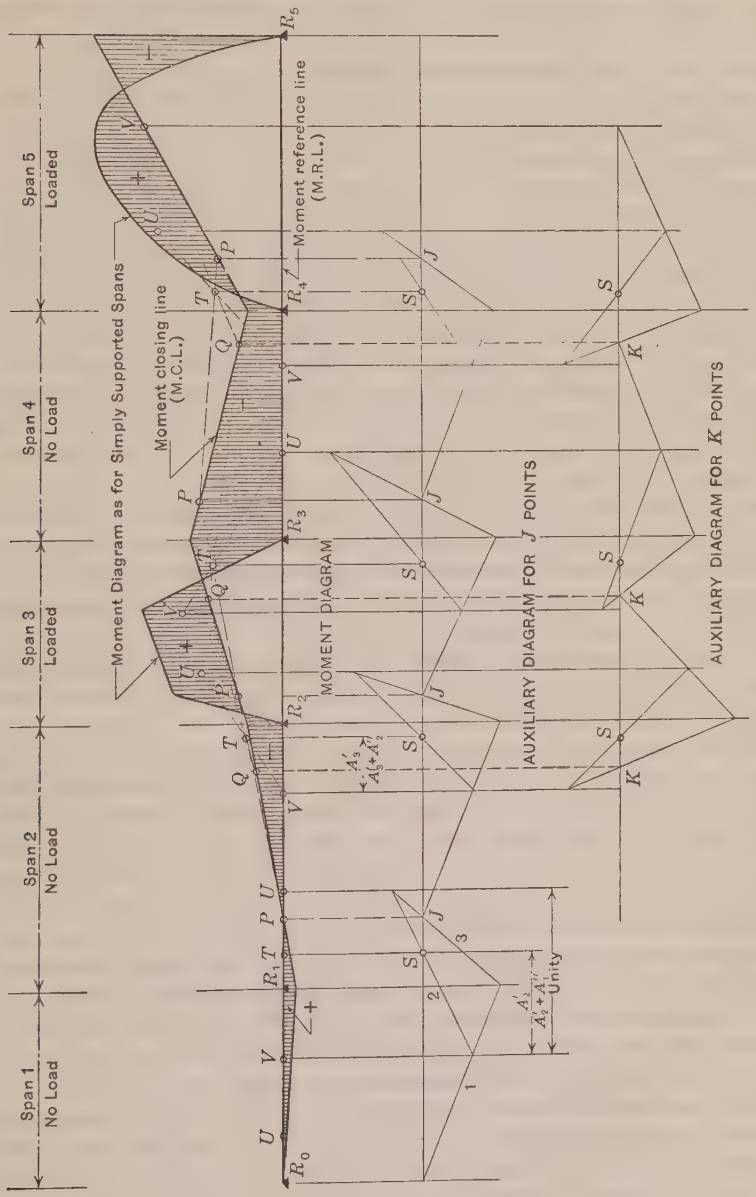
Span 1
No Load
Span 2
No Load
Span 3
Loaded
Span 4
No Load
Span 5
Loaded
Moment Diagram as for Simply Supported Spans
Moment closing line
(M.C.L.)
Moment reference line
(M.R.L.)
MOMENT DIAGRAM
AUXILIARY DIAGRAM FOR J POINTS
AUXILIARY DIAGRAM FOR K POINTS
Unity

FIG. 56.

lever projection, the point is below this support. It may be remarked that in so far as this discussion is concerned, a cantilever projection from the end span or spans has no influence except as it determines a known point on the M. C. L. over the end support. The beam is regarded as simply supported, but subjected to a known moment at this support, due to the cantilever. The cantilever itself may be considered as removed.

If the end is fixed, the point of beginning will lie vertically beneath the characteristic point adjacent to that end.

If the end is restrained, but in unknown degree, the relative fixity may be assumed, in the judgment of the designer, and a corresponding point located on the assumed M. C. L. above the support. Note, however, that it is one thing to assume a degree of fixity, and another to obtain it. Such procedure is pure guesswork, which at best can only roughly approximate the truth, and may be nowhere near it. Such assumption is only warranted when there is no rational method by which the restraint can be evaluated.

The point of beginning having been established, the J-points are then located for each span in succession, by the diagrams as indicated. It has been shown that the J-points are either on the moment closing line or vertically below the points, P, which are on that line. Therefore, each successive diagram begins at the last J-point established.

Note that to establish Points P' on the M. C. L. only, it is sufficient to begin the auxiliary diagrams vertically below any known point on the M. C. L. These points, however, will not be vertically above the "fixed points", J, as here defined. Since these points are important and useful, the diagram should be begun as previously described.

5.—Draw the lines in the moment diagram connecting the U and V-points adjacent to each support. Verticals produced from the S-points, intersect these lines in the required points, T.

6.—From the known point, P, on the left draw a line through the adjacent T-point to the right, for the corresponding span. The intersection of this line with a vertical produced from the corresponding J-point, determines another point, P. Beginning with this new point, the process is repeated until a point, P, is located in every span but the first, in which it must be known by condition.

7.—In the last span to the right the position of the point, P, and the end moment, or the condition of restraint, are both known, which determines the M. C. L. for that span. The intersection of this line with the vertical through the left support, is also a point on the M. C. L. for the span to the left. This point and the next point, P, to the left fix the M. C. L. for that span. The process is repeated until the line is drawn for every span.

It will be observed that if the points, P, are known, the Q-points are not required for the construction of the M. C. L. On the other hand, had the auxiliary diagrams been started from the right end the Q-points would have been established, and the points, P, would have been unnecessary. For ordinary cases the only merit in establishing both sets of points is that the combination serves as a check on the accuracy of the construction. In Fig. 56 both sets are shown, for purposes of demonstration.

When dealing with moving loads, or influence lines, both K and J-points have real value, and it will be advisable to establish both.

8.—After the M. C. L. is drawn, plot for each loaded span the positive moment curve due to the load on that span, considered as a simply supported beam. These diagrams will be plotted on the same side of the reference line, and to the same scale, of course, as the ordinates to the M. C. L.

9.—The moments in the loaded spans are given to scale by the ordinates between the M. C. L. and the positive moment curve; in the unloaded spans, by the ordinates between the M. C. L. and the reference line, as shown by the shaded areas in Fig. 56.

Application of the Method to Indeterminate Frames with Rigid Joints.—The authors' treatment of the three-member rectangular frames, as shown in Figs. 16 and 17, is correct in so far as simple bending is concerned. Of course, there is also direct stress in the vertical members, and these must be investigated as columns under bending and direct stress combined.

The statement that the construction shown is equally applicable to trapezoidal frames is true only if the frame and load is symmetrical about the vertical center line; that is, the legs must have the same inclination and the same EI, otherwise, the method may easily lead to the most absurd results. In general, it must be understood that this method, as described so far, is applicable only to such conditions of structure and loading as will satisfy the requirement that the supports be immovable, this being the fundamental premise of the theory. Consequently, the application of the method to indeterminate frames must be approached with judgment and caution.

The authors' statement (page 15), that an unsymmetrical load on such frames induces a condition equivalent to a yielding of the supports for the outer spans, is correct. They neglected to state, however, that lack of symmetry in the frame itself would result in the same thing, even if the load is symmetrical and the frame rectangular. They also neglected to tell how this yielding is to be evaluated, in order to treat it as a settlement of the support. It would be interesting to know how this is done, in connection with the methods described.

This yielding is nothing more, of course, than the deflection of the frame at that point, and can be computed by well-known methods. Before the deflection can be computed, however, the moments must be known, so that the problem will already have been solved, in so far as this discussion is concerned. It is the writer's opinion that when the load or frame is unsymmetrical, and the joints are not positively restrained against displacement, the problem is entirely outside the province of the methods described by the authors, or in this discussion. In such cases the writer would suggest that reliance for "shortcuts" be placed in some such work as Kleinlogel's "Rahmenformeln", which gives ready-for-use formulas for practically any frame and loading. The book is a German publication, but fortunately is arranged so that a knowledge of that language is not essential to its use. "Stresses in Framed Structures", previously mentioned, gives similar data, less comprehensive, but sufficient for most practical cases.

Referring to frames of the general order as shown in Fig. 18, there might as well be several members framing into a joint as the two there shown, of course. Irrespective of the number of such members the solution is effected in the same manner, by replacing the unloaded members by one member of equal resistance to end rotation, that is, of equal rigidity.

The unloaded members at the joint create a restraining moment at the end of the loaded member, this moment being developed by the rotation of the joint through some angle, ϕ. The end of each member at the joint must necessarily turn through this same angle, and the total moment developed will be contributed by the several members in proportion to their rigidities. The measure of the rigidity of a member, in this sense, is the angle, ϕ, through which the end will turn when a certain moment, M, is applied at that end.

Consider a member of any shape, simply supported at the right end, and a moment, $M'' = 1$, applied at that end. The left end may be restrained in any degree. The moment, M' at the left end may vary from zero, if freely supported, to $\frac{x'}{L - x'}$, if fixed. (See table of notation and Fig. 53 for the significance of the symbols.) In this latter case the moment line passes through the reference line opposite the center of gravity of A'. The area, A', is based on $M' = 1$. Let the $\frac{M}{EI}$-area due to the moment at the left end equal A'''. Then, $A''' = A'M'$. Let ϕ equal the angle at the right end. Then, from Theorem No. 3, when $M'' = 1$, $\phi = \frac{(A'' x'' - A''' x')}{L}$.

Evidently, the rigidity, or resistance, of a member varies inversely as ϕ. Let r = this resistance, when $M'' = 1$. Then, $r = \frac{1}{\phi}$,

Let $r_1, r_2, r_3, \ldots . r_n$, equal the resistances of the several unloaded members framing into a rigid joint, and let r equal the resistance of one member which would be equivalent to the group. Then, $r = r_1 + r_2 + r_3 + \ldots . r_n$. Let $\phi_1, \phi_2, \phi_3, \ldots . \phi_n$, equal the angles turned by the individual members when a moment, $M'' = 1$, is applied, and let ϕ equal the angle turned by the whole group at the joint when the same moment is there applied. Then, since $r = \frac{1}{\phi}$,

$$\frac{1}{\phi} = \frac{1}{\phi_1} + \frac{1}{\phi_2} + \frac{1}{\phi_3} + \ldots . \frac{1}{\phi_n}$$

from which,

$$\phi = \frac{1}{\frac{1}{\phi_1} + \frac{1}{\phi_2} + \frac{1}{\phi_3} + \ldots . \frac{1}{\phi_n}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (73)$$

That is, the effect of the several members is equivalent to that of any one member of such properties that it will develop the angle, ϕ, under the application of a moment, $M'' = 1$. Since a member of any character will suffice,

the simplest equivalent will naturally be adopted, this being a simply supported beam of constant $E\,I$. For this condition from Theorem No. 3,

$$\phi = \frac{A''\,x''}{L} = A'' \frac{\left(\frac{2}{3}\,L\right)}{L} = \frac{2\,A''}{3}$$

from which,

$$A'' = 1.5\,\phi \qquad (74)$$

For a beam of constant $E\,I$, $A'' = \frac{L}{2\,E\,I}$, and any combination of span and $E\,I$ may be used that will give the required A''; but it has been shown previously that when the point, S, is located according to the method herein described, the scale length of the spans is immaterial. Using the A'' given previously as one of the arguments in locating S, the equivalent span may be laid out to any scale. The characteristic points for this span will evidently be at the third points, since it is by condition a member of constant $E\,I$.

When a member is of constant $E\,I$, $\frac{x''}{L}$ is also a constant. Therefore, ϕ is directly proportional to A'', and if all the restraining members are of constant EI (not necessarily equal), with simply supported ends, Equation (73) may be changed to read:

$$A'' = \frac{1}{\frac{1}{A_1} + \frac{1}{A_2} + \frac{1}{A_3} + \cdots\cdots \frac{1}{A_n}} \qquad (75)$$

in which, A_1, A_2, etc., are the values of A'' for the various unloaded members, and A'' is the value corresponding to the equivalent member.

If E is constant for all members, as usual, Equation (75) reduces to:

$$A'' = \frac{1}{2\,E\left\{\frac{I_1}{L_1} + \frac{I_2}{L_2} + \frac{I_3}{L_3} + \cdots\cdots \frac{I_n}{L_n}\right\}} \qquad (76)$$

in which, I_1, L_1, etc., refer to the properties of the individual unloaded members.

If it is desired to express the length of the equivalent member to correspond with some particular value of I (call this I_4), then Equation (76) reduces to:

$$L = \frac{I_4}{\frac{I_1}{L_1} + \frac{I_2}{L_2} + \frac{I_3}{L_3} + \cdots\cdots \frac{I_n}{L_n}} \qquad (77)$$

which is a formula commonly used in effecting the "transformation of spans", and is equivalent to that given by the authors (page 17).

The moment at the joint having been found, using as an argument an equivalent member as here indicated, it is evident that this moment is to be distributed to the several component members in proportion to their values of r, or in inverse proportion to their values of ϕ, as already defined. If all

these members are of constant EI and simply supported, the moments will be distributed in inverse proportion to the values of A' for the several members, or in direct proportion to their values of $\frac{I}{L}$.

To comply with the method herein described, Equation (76) is best adapted for use when all the unloaded members are of constant EI. This formula is predicated on simply supported ends for these members, but it can be used when the ends of some of or all the members are fixed, provided these members are replaced by simply supported members of equal rigidity. The simplest way of effecting this transformation is by altering the span lengths.

Let L be the length of the simply supported member, and kL the length of the member with a fixed left end. The value of EI is the same for both. The coefficient, k, must be such that both members will develop the same angle, ϕ'', when a moment, $M'' = 1$, is applied at the right end.

From Theorem No. 3, for the simply supported member, $\phi = \frac{2A''}{3}$. For the member with fixed ends, $M' = \frac{M'' x'}{(L - x')} = 0.5$, since the inflection point is at the third point of the span. Expressing the functions for this member in terms of the length, L, the negative $\frac{M}{EI}$-area $= \frac{1}{2} kA' = \frac{1}{2} kA''$, since $A' = A''$, and the positive $\frac{M}{EI}$-area $= kA''$; but for a beam fixed at one end, $\phi = \theta$, taking the limits at the ends of the beam, and from Theorem No. 1,

$$\theta = kA'' - \frac{1}{2} kA'' = \frac{1}{2} kA'' = \phi''$$

This, however, must equal the angle as found for the simply supported beam, or $\frac{kA''}{2} = \frac{2A''}{3}$, from which, $k = \frac{4}{3}$. Conversely, the member with fixed ends is the equivalent in rigidity of a member three-fourths as long with simply supported ends. Therefore, when substituting in Equation (76) the lengths of any members fixed at the ends should be inserted at three-fourths their actual lengths.

This does not accord with the authors' statement, by inference (page 16), that the rigidity of a member with one end fixed is $\frac{3}{2}$ that of the same member with supported ends. It is believed that that statement is in error.

Application of the Method to Settlement of Supports.—Consider a loaded span, BC, of a continuous beam of length, L. The left end is designated B. The degree of restraint at the ends is unknown.

Assume that the support, B, settles a distance, D, with respect to the support, C. Evidently, the deflection of the beam at B with respect to C, is equal to that of C with respect to B, but of opposite sign. Let M' and M'' equal the end moments as before, at B and C, respectively, exclusive of the effect of the settlement, and let M_s' and M_s'' equal the corresponding end moments induced

by the settlement. For the case considered, M_s' will be positive and M_s'' negative, and the general expression for ϕ becomes:

$$\phi' = \frac{A^0\,(L - x^0)}{L} - \frac{M'\,A'\,(L - x')}{L} - \frac{M''\,A''\,(L - x'')}{L} + \frac{M_s'\,A'\,(L - x')}{L} - \frac{M_s''\,A''\,(L - x'')}{L} \quad \ldots\ldots (78)$$

$$\phi'' = \frac{A^0\,x^0}{L} - \frac{M'\,A'\,x'}{L} - \frac{M''\,A''\,x''}{L} + \frac{M_s'\,A'\,x'}{L} - \frac{M_s''\,A''\,x''}{L} \quad \ldots\ldots (79)$$

It has been previously proved that for beams of varying moment of inertia,

$$\frac{M''\,A''\,(L - x'')}{L} = \frac{M''\,A'\,x'}{L},$$

and,

$$\frac{M'\,A'\,x'}{L} = \frac{M'\,A''\,(L - x'')}{L}.$$

Substituting these values in Equations (78) and (79), and dividing by A' and A'', respectively, when $\phi' = \phi'' =$ zero,

$$\frac{A^0\,(L - x^0)}{A'\,L} - \left[\frac{M'\,(L - x')}{L} + \frac{M''\,x'}{L}\right] + \left[\frac{M_s'\,(L - x')}{L} - \frac{M_s''\,x'}{L}\right] = 0 \quad \ldots\ldots (80)$$

$$\frac{A^0\,x^0}{A''\,L} - \left[\frac{M'\,(L - x'')}{L} + \frac{M''\,x''}{L}\right] + \left[\frac{M_s'\,(L - x'')}{L} - \frac{M_s''\,x''}{L}\right] = 0 \ldots (81)$$

From Equation (52) and the conditions imposed, however, the left-hand bracketed term in Equation (80) is equal to U, the height of the characteristic point, exclusive of the effect of settlement. The right-hand term in brackets, which is identical in form except for sign, is the value of a characteristic point, U', corresponding to the settlement stresses.

Similarly, from Equation (53), the left-hand bracketed term of Equation (81) is equal to V, and the right-hand term to V', the designations corresponding to those given previously. Therefore:

From Equation (80),

$$\frac{A^0\,(L - x^0)}{A'\,L} = U - U' \quad \ldots\ldots (82)$$

From Equation (81):

$$\frac{A^0\,x^0}{A''\,L} = V - V' \quad \ldots\ldots (83)$$

The values of U' and V' are expressed in terms of the end moments, M_s' and M_s'', which must be evaluated on the basis of the deflection, D.

Since U' and V' are characteristic points, they lie by definition on the moment closing line connecting M_s' and M''_s when the ends of the beam are fixed, and, therefore, must be evaluated for this condition. Consider the beam as before, except that it is fixed at both ends, and there are no loads on the span. The support, B, settles with respect to C, giving rise to the positive moment, M_s', and the negative moment, M_s'', at the ends. The areas of the

$\frac{M}{E\,I}$-diagrams are $M_s'\,A'$ and $-M_s''\,A''$. From Theorem No. 1, the angle, θ, between the tangents at the points, B and C, is the area of the $\frac{M}{E\,I}$-diagram between the points, or $\theta = M_s'\,A' - M_s''\,A''$; but $\theta = 0$ by condition; therefore, $M_s'\,A' = M_s''\,A''$ from which, $M_s'' = \frac{M_s'\,A'}{A''}$.

Regarding the deflection of B with respect to C, the beam may be considered as a cantilever fixed at C, and subjected to a combination of downward pull and positive moment at B, resulting in the deflection, D. The pull at B generates a moment, $-M_s''$, at C, and the moment applied at B is M_s'. The corresponding $\frac{M}{E\,I}$-areas are $-M_s''\,A''$ and $M_s'\,A'$, the distances of their centers of gravity from B being x'' and x', respectively. Then, from Theorem No. 4, the value of the deflection will be:

$$D_B = -D_C = \phi''\,L + M_s'\,A'\,x' - M_s''\,A''\,x''$$

but $\phi'' =$ zero by condition, and it has been proved that $M_s'' = \frac{M_s'\,A'}{A''}$.

Therefore,

$$D_B = -D_C = M_s'\,A'\,x' - M_s'\,A'\,x'' = M_s'\,A'\,(x' - x''),$$

from which,

$$M_s' = \frac{D}{A'\,(x' - x'')}.$$

Similarly,

$$M'' = \frac{-D}{A''\,(x' - x'')}.$$

From Equation (52):

$$U' = \frac{D\,(L - x')}{L\,A'\,(x' - x'')} - \frac{D\,x'}{L\,A''\,(x' - x'')} \quad \ldots\ldots\ldots\ldots (84)$$

From Equation (53):

$$V' = \frac{D\,(L - x'')}{L\,A'\,(x' - x'')} - \frac{D\,x''}{L\,A''\,(x' - x'')} \quad \ldots\ldots\ldots\ldots (85)$$

From Equation (82):

$$U = \frac{A^0\,(L - x^0)}{A'\,L} + \frac{D\,(L - x')}{A'\,L\,(x' - x'')} - \frac{D\,x'}{A''\,L\,(x' - x'')} \quad \ldots\ldots (86)$$

From Equation (83):

$$V = \frac{A^0\,x^0}{A''\,L} + \frac{D\,(L - x'')}{L\,A'\,(x' - x'')} - \frac{D\,x''}{L\,A''\,(x' - x'')} \quad \ldots\ldots\ldots (87)$$

Equations (86) and (87) give the heights of the characteristic points for the combined condition of load and settlement. The second and third terms on the right-hand side are the heights of characteristic points as for the settlement alone; the first term on the right-hand side is the normal height, without settlement.

When the member is symmetrical about the center of the span, so that $A' = A''$, and $x'' = L - x'$, Equation (84) reduces to $\frac{-D}{A' L}$, and Equation (86) to:

$$U = \frac{A^0 (L - x^0)}{A' L} - \frac{D}{A' L} \quad \ldots\ldots (88)$$

Equation (85) reduces to $\frac{D}{A' L}$, and Equation (87) becomes:

$$V = \frac{A^0 x^0}{A' L} + \frac{D}{A' L} \quad \ldots\ldots (89)$$

When $E I$ is constant throughout the beam, $A' = A'' = \frac{L}{2 E I}$, and Equations (88) and (89) reduce to:

$$U = \frac{A (L - x^0)}{L^2} - \frac{2 E I D}{L^2} \quad \ldots\ldots (90)$$

$$V = \frac{A x^0}{L^2} + \frac{2 E I D}{L^2} \quad \ldots\ldots (91)$$

In general, the corrections to the normal heights of the characteristic points are subtractive for the two points immediately adjacent to the settled support, and additive to the other two. The settlement of a support effects the characteristic points only in the two spans common to the support.

Conclusion.—As previously stated, the writer lays no claim to originality as respects the fundamental conceptions on which this discussion is based. The entire credit for these belongs primarily to Fidler, and, secondarily, to Ostenfeld and others who have added in one respect or another to the structure for which Fidler laid the foundation. The writer accepts entire responsibility for the particular manner in which the concept is herein developed and expanded, and trusts that he has succeeded in adding something to the existing literature on the subject. The purpose throughout has been to develop and present formulas for the purely general case, and to lay stress on the true significance of these formulas and the graphical operations involved, rather than to deal with specific instances of limited application. This has necessarily resulted in some rather intricate and involved operations, which may severely test the forbearance of the reader. If the writer has failed to demonstrate the inherent simplicity and excellence of Fidler's Method of Characteristic Points, this is due only to the limitations of his ability to present the subject.

The writer wishes to express his appreciation of the assistance rendered by Mr. Ruppel and H. A. Schirmer, Jun. Am. Soc. C. E., in having given this text a critical and careful reading, and in making suggestions which have helped to determine the final form in which the discussion is presented.

A. T. Granger,* Assoc. M. Am. Soc. C. E. (by letter).—Two distinct subjects are presented in this paper—the simplified form of the Theorem of Three Moments, and the graphical solution of the theorem by the Method of Conjugate Points.

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The writer has been particularly interested in the simplified form and derivation of the three-moment equation. Various methods of deducing this equation have been published, but this derivation appears simpler and more easily understood than any previously proposed. The form given in Equation (7) can easily be converted to the familiar form by substituting for the right-hand members, $g_1 A_1$ and $g_2 A_2$, their values in terms of the loads, positions, and span lengths expressed in the ordinary notation.

The graphical solution of the equation by the Method of Conjugate Points possesses two distinct advantages over the analytical method, namely, the check on the correctness of the construction in Fig. 11, and the simplicity of the graphical method when applied to several spans.

A positive check on the correctness of the moments over the supports is advantageous in eliminating errors that would otherwise be carried through the succeeding calculations. Any one familiar with computations for continuous girders is aware that an error in these moments cannot be detected by any failure of the reactions, shears, or moments based thereon to check. The writer has frequently examined calculations which would satisfy all the laws of statics, but which were wrong throughout because of an error in solving the three-moment equation. Such an error is much less likely to occur when the moments are determined graphically.

Perhaps the principal advantage of the graphical method lies in its easy extension to cases involving more than two unknown moments. Although the actual solution of such cases in practice is comparatively infrequent, there are doubtless numerous instances where a more complete analysis would be made were it not for the labor involved. The complication of the analytical method increases rapidly with each additional unknown moment. The graphical method, on the contrary, is easily extensible to any number of spans and such extension offers no additional difficulty. This characteristic should commend it for use in analyzing complex structures.

Study of this paper led the writer to the conclusion that an alternative graphical solution utilizing the familiar equilibrium polygon would possess some advantages. He accordingly devised the following method of procedure, which is presented with the hope that it may prove interesting and of some value.

Fig. 57(*a*) represents a continuous beam loaded in any fashion. Although a three-span girder has been chosen for illustration, it will be obvious that the method developed is general and applicable to any number of spans. The simple beam moment diagrams, based on discontinuity, are plotted in Fig. 57(*b*), the moment areas, A_1, A_2, and A_3, determined, and the centers of gravity of these areas located, vertical lines being drawn through such centers of gravity. The areas A_1, A_2, and A_3, are now laid off in Fig. 57(*c*), as vertical forces to any convenient scale, a pole, O, chosen at a pole distance, H, and the rays drawn as shown. In Fig. 57(*d*), an equilibrium or string polygon, $L\ m\ n\ q\ Q$, is now constructed using these rays. If, instead of representing A_1, A_2, and A_3, as single forces in Fig. 57(*c*), these areas had been divided into a great number of minute areas, the corresponding equilibrium polygon in

Fig. 57(*d*), instead of the broken line, *L m n q Q*, would have been practically a smooth curve This curve represents the elastic curve of the beam (considered as three simple spans);* and straight lines connecting the points, *L*, *N*, *P*, and *Q*, where the polygon intersects the reaction verticals, will represent the unstrained position of the beam.

The correct elastic curve for the continuous beam may be obtained by combining with this simple beam curve a similar curve representing the

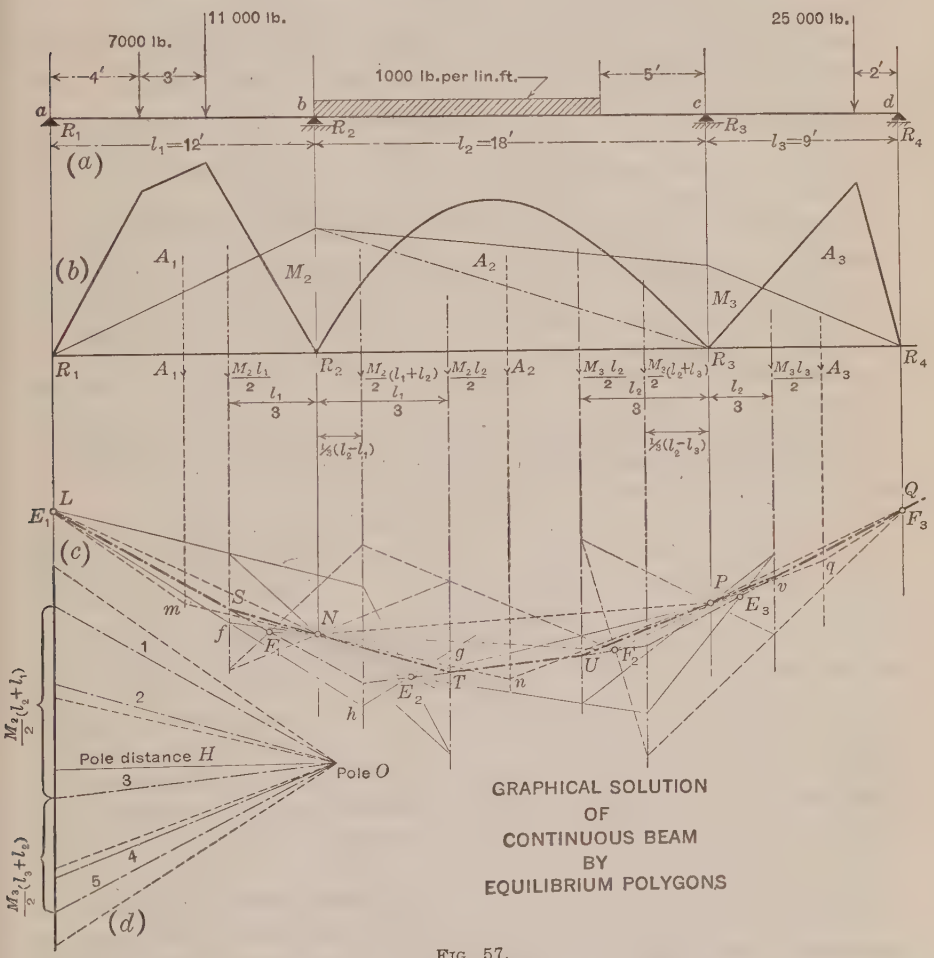

FIG. 57.

effect of the areas due to M_2 and M_3, the moments over the supports due to continuity. (See Fig. 57(*b*)). The net displacement of the beam at points *a*, *b*, *c*, and *d* (Fig. 57(*a*)), is obviously zero; and as it is zero due to the effect of the areas A_1, A_2, and A_3, alone, it must be zero due to the areas caused by M_2 and M_3 alone. In other words, inasmuch as the equilibrium polygon for

* "Modern Framed Structures", by Messrs. Johnson, Bryan and Turneaure, Pt. II, 1917 Edition, pp. 5-6.

A_1, A_2, and A_3, passes through $L, N, P,$ and Q, a similar polygon (using the same pole, O, and pole distance, H) drawn for $\frac{M_2 l_1}{2}$, $\frac{M_2 l_2}{2}$, $\frac{M_3 l_2}{2}$, and $\frac{M_3 l_3}{2}$, must also pass through L, N, P, and Q. This condition will fix the direction of the rays for these latter forces, and if these rays are then inserted in Fig. 57(c), the forces, $\frac{M_2 l_1}{2}$, etc., may be scaled, and the desired moments obtained directly.

As the polygon for A_1, A_2, and A_3, is needed only to locate $L, N, P,$ and Q, the sub-division of these areas discussed previously is unnecessary. To pass the equilibrium polygon for $\frac{M_2 l_1}{2}$, etc., through L, N, P, and Q, the procedure is as follows: Through L (Fig. 57(a)) draw any random ray, $L f$, to the line of action of $\frac{M_2 l_1}{2}$. From f draw a ray through N to g, on the line of action of $\frac{M_2 l_2}{2}$. The direction of the ray from g to the line of action of $\frac{M_3 l_2}{2}$ must be such that this ray will intersect $L f$ produced at h on the line of action of the resultant of $\frac{M_2 l_1}{2}$ and $\frac{M_2 l_2}{2}$. If this process be repeated using a second random direction for the initial ray, $g h$ will be intersected at a point, E_2, by the ray in this second construction corresponding to $g h$. For simplicity, it is well to let $L N$ be the second random direction.

No matter what direction is chosen for the initial ray, the gh line must pass through E_2. This is easily verified, and will be apparent from a consideration of the authors' "pennant diagrams." E_2 being located at a point through which the ray from $\frac{M_2 l_2}{2}$ to $\frac{M_3 l_2}{2}$ must pass, random rays can now be drawn through E_2 and by a process similar to the foregoing construction a point, E_3, may be located through which the ray from $\frac{M_3 l_3}{2}$ to Q must pass. The points, E_2 and E_3, suffice to locate the required polygon, but as a check the points, F_2 and F_1, should be similarly located by starting at Q and working in the reverse direction. The complete polygon, $L S T U V Q$, is shown in Fig. 57(d).

In Fig. 57(c), Rays 1, 2, 3, 4, and 5 are now drawn parallel, respectively, to $L S$, $S T$, $T U$, $U V$, and $V Q$. It is at once apparent that Rays 2 and 4 are not essential and that the intercept on the load line between Rays 1 and 3 is $M_2\left(\frac{l_1 + l_2}{2}\right)$, while that for Rays 3 and 5 is $M_3\left(\frac{l_2 + l_3}{2}\right)$. From these intercepts the values of M_2 and M_3 are obtained directly.

In case the ends are fixed there will be moments, M_1 and M_4, existing at a and d, and hence areas, $\frac{M_1 l_1}{2}$ and $\frac{M_4 l_3}{2}$, acting through the end third

points of the outer spans. As the end tangents must be horizontal, the ray from L to $\frac{M_1 l_1}{2}$ must coincide in direction with the simple beam ray, $L m$, and that from Q to $\frac{M_4 l_3}{2}$ with $Q q$. Aside from this, the procedure is similar to that just described.

It appears to the writer that this method possesses some distinct advantages. The procedure is identical for symmetrical and unsymmetrical moment areas, and in the case of fixed ends the proper procedure as just given, suggests itself almost spontaneously. No derivations or formulas are required, as it is unnecessary to make use of even the three-moment equation. The graphical procedure need not be memorized, as after the principle is once grasped the necessary constructions are almost self-evident.

It may not be inappropriate to suggest some conditions under which the Method of Conjugate Points does not seem to be advisable. The writer does not believe that any graphical method will be extensively used for cases in which the number of unknown moments does not exceed two, as there is a certain "irreducible minimum" of graphical work which will frequently take more time than a complete analytical solution. Methods of computation are to a considerable extent matters of individual taste, but the writer believes that most computers familiar with both the analytical and graphical methods could obtain reactions, moments, shears, and influence lines for a three-span girder, for instance, more quickly analytically than graphically. For more than two unknown moments, however, the advantage of graphics appears to be considerable.

The object of the authors, as stated in their "Introduction," of stimulating more careful analysis of indeterminate structures, is unquestionably commendable. Progress in structural design can only be made through the gradual elimination of the unknown. There exists in some quarters too great a tendency to decry thorough calculation of such structures, on the ground that the entire subject is enveloped in such a haze of uncertainty as to defy analysis. It will not do to lose sight of the fact that those who are best acquainted with the so-called "exact" methods of calculation should be best qualified to decide what short cuts are permissible, and to what degree of refinement the analysis need be carried. For calling attention to the importance of this subject, and for their valuable addition to its literature, the authors should receive the thanks of the profession.

JOHN J. GOULD,* ASSOC. M. AM. SOC. C. E. (by letter).—The graphical solution of continuous beams and frames as presented by the authors has brought to engineers a considerable simplification in structural design.

In the past few years, in Europe, extensive attention has been given to the graphical solution of reinforced concrete frames. Among others, the books published by Strassner† have treated this subject in a very thorough and complete manner.

* Structural Engr., L. H. Nishkian, San Francisco, Calif.

† "Berechnung statisch unbestimmter Systeme," Bd. I-II; "Neuere Methoden," Bd. I-II.

To those not familiar with German literature, a short description of the principles and methods used by Strassner as compared with those of the authors, may demonstrate the value of the new method. Strassner applied to the design of frames Mohr's principles* that:

(1) The angular deflection of the elastic line at the supports is equal to the reaction of the beam loaded by its moment-area divided by EI; and

(2) The ordinates of the elastic line are equal to the bending moment of the beam loaded by the moment-area divided by EI.

Strassner, using Ritter's graphical method of finding the points of inflection of the elastic line (of unloaded spans), determines, by means of these points and Mohr's principles, the negative moments of the actual loaded beam. The application of these principles may be best illustrated by the following example:

Fig. 58 shows a frame with a single concentrated load. The dotted lines indicate deflection lines which are assumed for a preliminary determination of moments and shears.

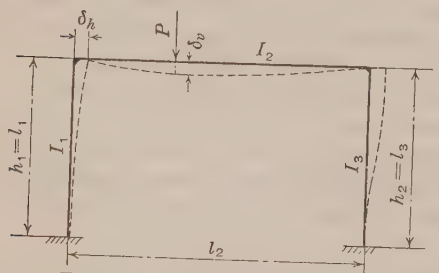

FIG. 58.—FRAME WITH SINGLE CONCENTRATED LOAD.

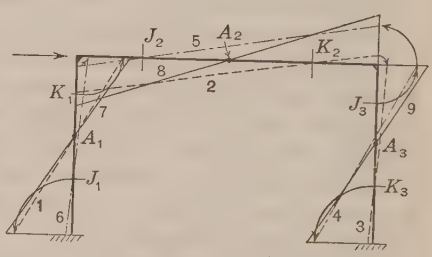

FIG. 59.—DETERMINATION OF A-POINTS.

Fig. 59 represents the moments resulting from a unit horizontal load acting at the upper left corner of the frame and is used for correcting the "preliminary" moments and shears as shown on Figs. 61 and 62, which are self-explanatory.

Fig. 59 also shows how the A-points, the points of zero moment, are obtained. Point J_1 is projected to the left and Point K_1 to the right. Then Line 1 is drawn, followed by Lines 2 and 3. Point K_3 is projected to the left and Point J_3 to the right. Then Line 4 is drawn, followed by Lines 5 and 6. Lines 7, 8, and 9 represent the sum of Lines 1 to 6, inclusive.

Fig. 60 shows how the points of inflection—fixed points—or J and K-points are obtained. After locating the V and U-lines at the third points of the span and the T-line, as illustrated, Line 1 is drawn at any angle. Then Lines 2, 3, 4..... 10 are successively drawn in the order named.

Due to the unsymmetrical load conditions the shears, S_1 and S_2, obtained from the moments shown on Fig. 61 are not equal. For these conditions the frame would not be in equilibrium and a horizontal displacement equal to δ_h (Fig. 58) would take place. Fig. 63 illustrates how Strassner corrects, by

* A detailed demonstration of these principles has been given by Charles S. Whitney, M. Am. Soc. C. E., see pp. 56 *et seq.*

a very simple method, these moments (as shown on Fig. 61), making thereby $S_1 = S_2$ (Fig. 62).

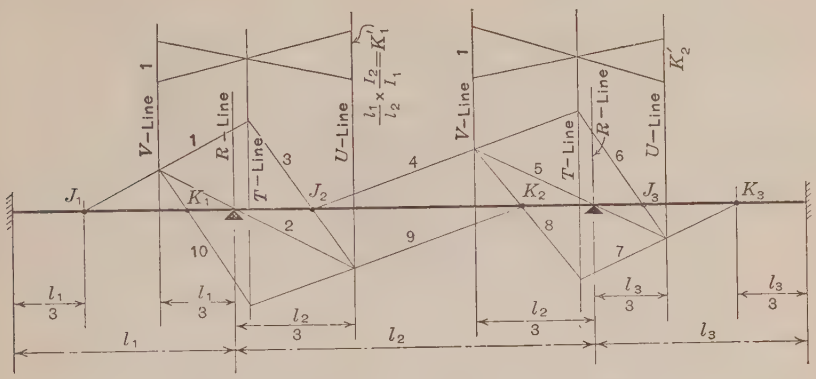

FIG. 60.—DETERMINATION OF J AND K-POINTS.

Draw any horizontal line, 1 (Fig. 63). Then draw parallel to Lines M_1 and M_2 (see Fig. 61) Lines M_1' and M_2'. Locate Point G perpendicular to and in the center of Line 1. Then Lines 2 and 3 give the new inclinations of the column moment lines based on the condition that the shears, S_1 and S_2, are equal.

Fig. 64 shows how the final moments (full lines) are obtained. The dotted lines represent the preliminary moments (as shown on Fig. 61). Project Point A_1 to Point A_1' and Point A_3 and Point A_3', intersecting thereby the dotted lines. Then draw parallel to Line 2 (Fig. 63), Line 2′ in Fig. 64, passing through Point A_1'. Draw parallel to Line 3 (Fig. 63), Line 3′ in Fig. 64, passing through Point A_3'. These final moments fulfill the condition that $S_1 = S_2$.

In practical design, however, this correct method of determining moments and shears usually may be neglected. In this case the preliminary design as illustrated on Figs. 60, 61, and 62, may be considered as being sufficiently accurate. Comparing the authors' method with that of Strassner, it is clear that the new method leads more quickly, in many cases, to the desired result.

Fig. 65 shows the design of the frame illustrated in Fig. 16, as made by Strassner's method. The authors' method has been applied by the writer to the design of continuous beams of six and eight spans and the moments have been obtained in an astonishingly short time..

The simple and scientific methods, as presented by the authors, or as published by Strassner, for designing reinforced concrete frames should lead to more uniform structural design throughout the United States. The present arbitrary coefficients for the design of continuous beams and frames in concrete, as specified in most of the building codes, are irrational and involve an unnecessary and wasteful financial burden. Scientific design of a concrete structure not only is more economical, but also often safer. Numerous tests have confirmed plainly the fundamental assumptions of the science.*

* "Concrete—Plain and Reinforced," by Taylor, Thompson, and Smulski, 1925, p. 64, etc.; also, "Vorlesungen über Eisenbeton," by E. Probst.

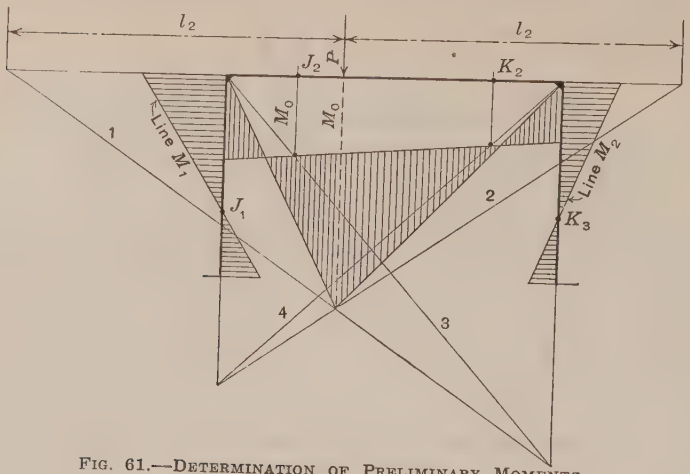

FIG. 61.—DETERMINATION OF PRELIMINARY MOMENTS.

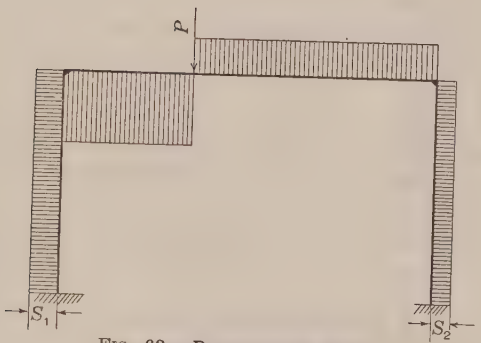

FIG. 62.—PRELIMINARY SHEARS.

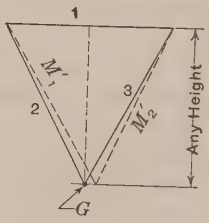

FIG. 63.

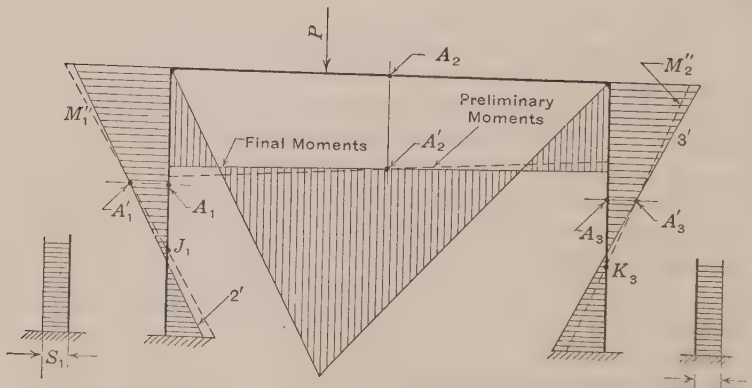

FIG. 64.—DETERMINATION OF FINAL MOMENTS AND SHEARS.

An interesting comparative design between a 6-story reinforced concrete building in Los Angeles, Calif., and the same building as it would have been built in San Francisco, Calif., has been made by the writer. The Building Code of San Francisco gives the engineer the privilege of using scientific principles, specifying in Part VIII, "Design in General, Restrained Beams", that:

"All continuous and restrained slabs, beams, and girders shall be designed to resist, at all supports, the full moments and between all supports, $\frac{4}{5}$ of the moments resulting from all spans being fully loaded. In no case, however, shall the resisting moments between supports be less than bending moments produced by any condition of partial loading."

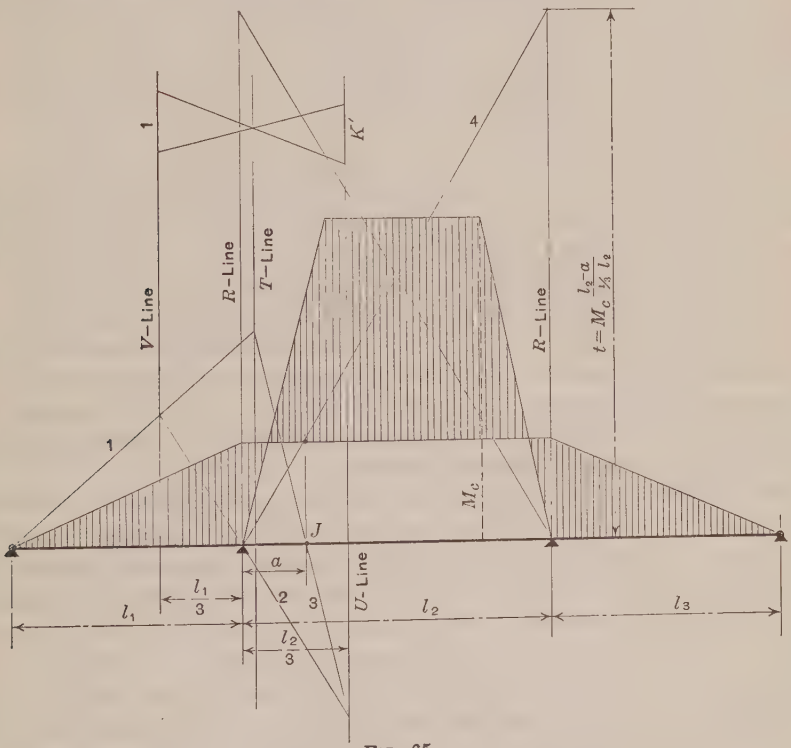

FIG. 65.

For the building in question girders 30 in. deep had to be used in Los Angeles, whereas in San Francisco girders 24 in. deep would have been sufficient to provide for the actual stresses. From the economic point of view a reduction of 3 ft. in building height in this case, as well as greater safety and a material additional saving in cost, justify plainly a thoroughly scientific and rational design.

The writer believes that the authors' method, or any other quick method, is of primary importance for a sane and economic development of reinforced concrete construction. The scientific value of the method merits the attention of university courses throughout the United States.

RICHARD G. DOERFLING,* M. AM. SOC. C. E. (by letter).—The writer has read this paper with much interest and is pleased to note that the Society has given free scope to papers like those by Cecil Vivian von Abo, Jun. Am. Soc. C. E.,† Charles S. Whitney, M. Am. Soc. C. E.,‡ and this one.

For the most part the methods demonstrated in these papers were invented and developed from 30 to 60 years ago in Switzerland, Austria, and Germany, where they have become the working tools of structural engineers. Although they occupy considerable space in German technical literature, they are not readily accessible in English. In fact, what little has entered English literature has come in rather timidly and fragmentarily, and remained almost unknown after facing some stern critics who boldly doubt the existence of secondary stresses, do not agree that a concrete arch should be treated as an elastic structure, and would remedy continuous beams by replacing the elusive fixed points with substantial joints that can be seen and inspected, and oiled if necessary.

A perplexing question is frequently raised as to "what is the best and shortest method of doing a certain thing", which rather involves the personal equation. The best method of course for any one person, and also the shortest and safest method for him, is the one he understands best and feels sure about, be it graphical, analytical, or a combination of these methods.

This paper furnishes useful graphical methods for many problems of continuous beams and frames. The authors have preferred the roundabout way of representing the methods demonstrated as analytical deductions from the geometry of graphics again translated into graphics, while the simpler and clearer proof may be furnished by the geometry of the construction directly, in most cases by similar triangles or ratios.

The "conjugate points" of the authors are readily recognized, by those acquainted with the literature on the subject, as the projections, F', of the fixed points, F, upon the M-polygon. The name and designation only are new, and the derivation is in reversed order—rather a retrograde step in regard to clearness in demonstrations as well as in application. The multiplicity of points and lines in the body of the diagram is confusing (see Figs. 11 or 13, for instance), and in practical application the lines will frequently intersect at angles too acute for precise determination of other points and lines required. On the other hand a random construction for the fixed points outside the body of the diagram obviates both these objections; the intersection angles can be chosen at will.

Arbitrary coefficients in building ordinances giving moments 50% greater or less than the true moments for uniformly distributed loads are not altogether wrong, considering the fact that the live load on adjacent floor-panels may in reality vary from zero to the maximum, upsetting all fine-spun computations for uniform loading on continuous beams and slabs. The sharp peak moments

* Cons. Engr., San Francisco, Calif.

† "Secondary Stresses in Bridges," *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 1.

‡ "Design of Symmetrical Concrete Arches," *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 931.

over concrete columns and piers could be safely rounded off parabolically to one-half within the width of the column (because these supports are distributed and not knife-edges), while the moments at mid-span should be increased to guard against unbalanced loads.

Although the more laborious means of influence lines show most clearly how the live load on continuous beams must be distributed for maximum moment at any point, the next best method, and one very appropriate for more important work in building construction, is to determine the moments with the load on one span only, and add the results, either graphically or arithmetically. This can be done by very simple methods—simple in demonstration as well as in application, with little tax on the memory. In the belief that a presentation of these methods will shed more light on the important elements and characteristics of the whole subject, the writer offers the following contribution as a supplement to the paper, and, incidentally only, makes his demonstration bear out his critical contention. Some repetition will be unavoidable.

(*1*).—*Elastic Weights.*—As an introduction it seems pertinent to discuss briefly the subject of "elastic weights".

The maximum bending moment, M, caused by a load, W, is directly proportional to W and to the distance, l, at which W acts.

The maximum deflection, D, caused by a bending moment, M, is directly proportional to M and to the distance, l, at which M acts, and inversely proportional to the stiffness, $E\,I$, of the structural member under consideration.

From this analogy $M = l\,W$ and $D = l\,\frac{M}{E\,I}$. The term, $\frac{M}{E\,I}$, is called the elastic weight, and deflections and deflection diagrams are determined (like moments and moment diagrams) by using the moment area as a load area and dividing by $E\,I$. Assume a force diagram and an equilibrium polygon constructed with a pole distance equal to $E\,I$; then D at any point is the intercept of the equilibrium polygon at that point.

In general: Using the load area,

$$M = \text{intercept} \times \text{pole distance}$$

and using the moment area as a load area,

$$D = \frac{\text{intercept} \times \text{pole distance}}{E\,I}$$

In order to gain a working acquaintance with the fundamental idea of elastic weights, deflection formulas may be derived by its use for the following four familiar cases:

Cantilever with a load, W, at the end (Fig. 66 (*a*)):

Maximum moment $= W\,l$

Moment area $= W\,l \times \frac{l}{2} = \frac{W\,l^2}{2}$

Maximum deflection $= \frac{\frac{W\,l^2}{2} \times \frac{2\,l}{3}}{E\,I} = \frac{W\,l^3}{3\,E\,I}$

Cantilever with a load, W, uniformly distributed (Fig. 66 (*b*)):

$$\text{Maximum moment} = \frac{W\,l}{2}$$

$$\text{Moment area} = \frac{W\,l}{2} \times \frac{l}{3} = \frac{W\,l^2}{6}$$

$$\text{Maximum deflection} = \frac{\frac{W\,l^2}{6} \times \frac{3\,l}{4}}{E\,I} = \frac{W\,l^3}{8\,E\,I}$$

Simple beam with a load, W, at the center (Fig. 66 (*c*)):

$$\text{Maximum moment} = \frac{W\,l}{4}$$

$$\text{One-half the moment area} = \frac{W\,l}{4} \times \frac{l}{4} = \frac{W\,l^2}{16}$$

$$\text{Maximum deflection} = \frac{\frac{W\,l^2}{16} \times \frac{l}{3}}{E\,I} = \frac{W\,l^3}{48\,E\,I}$$

Simple beam with a load, W, uniformly distributed (Fig. 66 (*d*)):

$$\text{Maximum moment} = \frac{W\,l}{8}$$

$$\text{One-half the moment area} = \frac{W\,l}{8} \times \frac{l}{3} = \frac{W\,l^2}{24}$$

$$\text{Maximum deflection} = \frac{\frac{W\,l^2}{24} \times \frac{5\,l}{16}}{E\,I} = \frac{5\,W\,l^3}{384\,E\,I}$$

The few problems now to be solved may readily be extended to continuous beams of any number of supports. Incidentally, these problems will introduce the essential terms and characteristics of the subject in tangible order and reason.

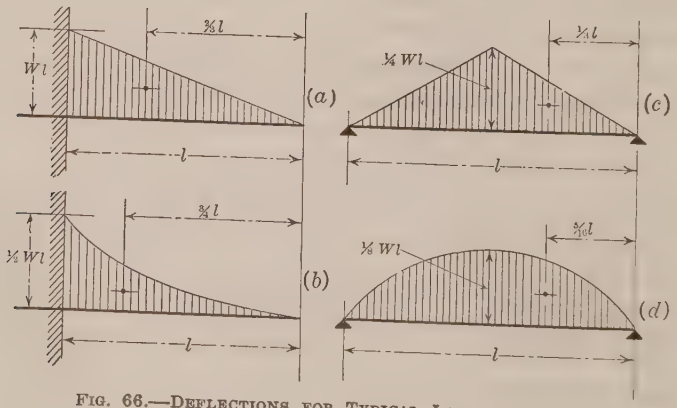

FIG. 66.—DEFLECTIONS FOR TYPICAL LOADING BY THE METHOD OF ELASTIC WEIGHTS.

2.—Fixed Points.—Referring to Fig. 67 (*a*), let the span, l_1, be loaded while the span, l_2, carries no load; required the moment, M, over mid-support.

Without knowing the value of M, the form of the moment diagram is known, because it must consist of the moment diagram, $A\ N\ D$, for a simple beam, $A\ B$, minus the moment triangle, $A\ D\ C$, due to the reaction, B. The resulting moment diagram would be the load area for a deflection polygon. For the problem under consideration it will be sufficient to know of such deflection polygon only the tangents at the supports, the "pier tangents". For determining these tangents the load area, Fig. 67 (a), is considered to be com-

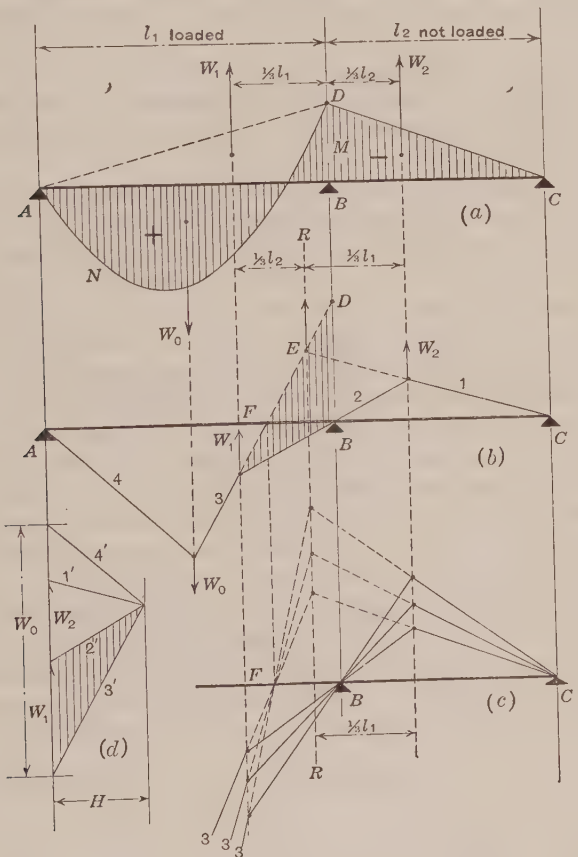

FIG. 67.—TWO-SPAN BEAM, ONE SPAN LOADED.

posed of three parts, the area, $A\ N\ D = W_0$, termed the "simple moment area", and the triangles, $A\ B\ D = W_1$ and $C\ B\ D = W_2$. These three areas are used as elastic weights acting at the gravity lines of the areas. The first is positive, causing downward bending, and the other two are negative, causing upward bending. The numerical value of W_0, the simple moment area, may be computed, and it is seen that the ratio of W_1 and W_2 remains constant whatever the value of M, for

$$W_1 = \frac{M\ l}{2} \text{ and } W_2 = \frac{M\ l_2}{2}$$

and, therefore,

$$\frac{W_1}{W_2} = \frac{l_1}{l_2}$$

The lines of action of W_1 and W_2 are verticals, distant $\frac{l_1}{3}$ and $\frac{l_2}{3}$ from the axes of the piers, the "pier verticals", and hence termed the "third verticals". The resultant of W_1 and W_2 is also a vertical, at a distance from W_1 and W_2 inversely proportional to these values, namely, $\frac{l_2}{3}$ from W_1 and $\frac{l_1}{3}$ from W_2. Hence, it would be suggestive to term this the "resultant vertical" and designate it by R, as in Fig. 67 (b), which shows an equilibrium polygon of W_0, W_1, and W_2. Supposing that the beam under bending is held to the supports, this polygon is obtained, as follows:

The pier tangent to C is drawn at random and from the point where it intersects W_2 the pier tangent to B is drawn intersecting W_1. The third line of the equilibrium polygon must necessarily intersect the first line on the resultant vertical, R, hence its position is given after the first and second lines have been drawn, and from the point where this third line intersects W_0 the pier tangent to A is drawn, completing the polygon. Notice the important fact that the third line, $E\ W_1\ W_0$, is found entirely independently of W_0. Another fact of fundamental importance is that this third line intersects the axis of the beam at a fixed point, F, no matter in what direction the first line was drawn. This is readily seen by referring to Fig. 67 (c) which shows that for any number of random lines through C the intercepts of such first lines and all following lines on any of the verticals are proportional to each other, hence, all third lines must go through F, a fixed point, dependent only on the ratio of the spans to each other, but independent of the loads.

From the equilibrium polygon, Fig. 67 (b), inversely to the general rule, the force diagram, Fig. 67 (d), may now be constructed by drawing on the known value of W_0 as a load line the rays, 4′ and 3′, parallel to the lines, 4 and 3, of the equilibrium polygon, and from the pole thus located, the rays, 2′ and 1′, parallel to the pier tangents, 2 and 1, cutting off on the load line the numerical values of W_1 and W_2. Then, as $W_1 = \frac{M\,l_1}{2}$, $W_2 = \frac{M\,l_2}{2}$, $M = \frac{2\ W_1}{l_1}$, and the check value, $M = \frac{2\ W_2}{l_2}$.

3.—Two-Span Beam.—For the practical application of the problem just demonstrated a further simplification may be introduced, derived directly from Figs. 67 (b) and (d). Notice that the shaded parts of these two related diagrams are similar triangles; hence

$$W_1 : H = B\ D : \frac{l_1}{3}$$

or,

$$\frac{l_1}{2}\ M : H = B\ D : \frac{l_1}{3}$$

or, after constructing a force triangle with $\frac{W_0}{\frac{l_1}{2}}$ as a load line and $\frac{l_1}{3}$ as the pole distance,

$$M : \frac{l_1}{3} = B : \frac{l_1}{3}$$

showing that an equilibrium triangle, $A\,W_0\,D$, derived from this force triangle and drawn though F would cut off the correct value of M on the pier vertical. Fig. 68 (*a*) shows this construction for a concentrated load, and Fig. 68 (*b*) for a uniformly distributed load, on the first span. The fixed point, F, is located as before, but by a separate construction, and the equilibrium triangles are drawn this time on $A\,B$ as a base. Hence, a straight line from A through F' (the vertical projection of F on the equilibrium polygon) will cut off the correct value of M on the pier vertical as before, except that it is below B. If the equilibrium line be drawn through F, the desired value of M would be cut off above B as before.

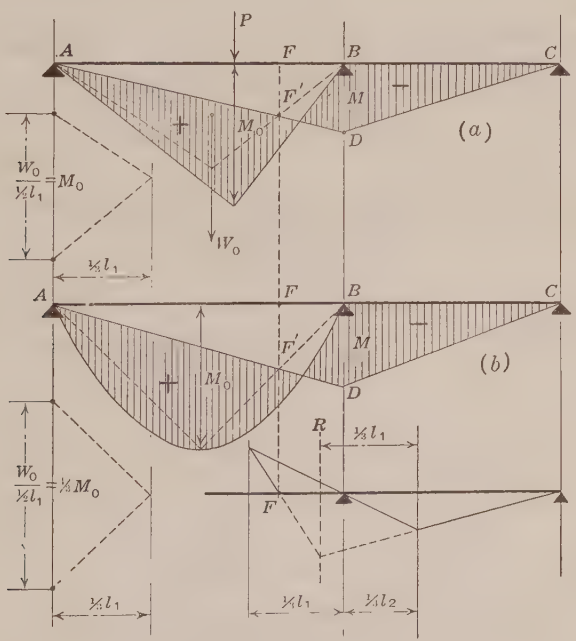

FIG. 68.—TWO-SPAN BEAM WITH CONCENTRATED OR UNIFORM LOAD ON ONE SPAN.

From Fig. 68 (*b*) it will be readily seen that for a uniformly distributed load on one span the peak of the equilibrium triangle coincides with the apex of the parabola for uniform load, so that for this case the moment diagram may be drawn at once as indicated, without the use of the force triangle.

4.—Pier Depression.—Fig. 69 shows a convenient graphical determination of the pier moment, M, due to an actual or assumed depression of the mid-

support. Consider the beam devoid of all loads including its own weight, but held to the piers, while the support, B, is lowered $\frac{1}{2}$ in. Without knowing the value of M caused by such a depression, the form of the moment diagram is known, for it must be a triangle, $A\ D\ C$, as indicated. Each half of the total load triangle is used as an elastic weight for determining the pier tangents, giving,

$$W_1 = \frac{M\ l_1}{2};\quad W_2 = \frac{M\ l_2}{2};\quad \text{and}\quad \frac{W_1}{W_2} = \frac{l_1}{l_2}$$

Draw the "third vertical" and the "resulting vertical", R, as before and construct the equilibrium polygon due to W_1 and W_2. Since the polygon must pass through the contact points of the three supports while its first and third lines must intersect on the resultant vertical, only one such polygon is possible. It may be drawn by trial or constructed by using the fixed point, F, and its projection, F', of either one or both spans as indicated. Now, if the depression, $B\ B'$, was plotted to full size ($\frac{1}{2}$ in. in this case) and the spans to a scale of $\frac{1}{240}$, the pole distance of the force diagram to be constructed from the equilibrium polygon is known; it must equal $\frac{E\ I}{240}$ in order to correspond to full-sized deflections. Laying off this distance to any convenient scale from a vertical

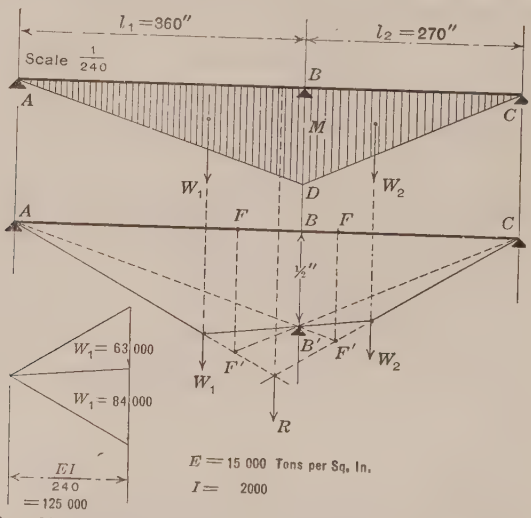

FIG. 69.—TWO-SPAN BEAM WITH MID-PIER DEPRESSION.

and drawing through the pole thus located three rays parallel to the three pier tangents, the intercept of these rays on the vertical will constitute the load line, giving the correct values of W_1 and W_2, from which, $M = \frac{2\ W_1}{l_1}$, and the check value, $M = \frac{2\ W_2}{l_2}$, may be computed. Assuming numerical values of $E\ I$, l_1, and l_2 as given in Fig. 69, the pole distance will equal 125 000 in-tons,

giving W_1 and W_2 equal, respectively, to 84 000 and 63 000 in-tons, and, hence,

$$M = \frac{2 \times 84\,000}{360} = 467 \text{ in-tons}$$

and the check value,

$$M = \frac{2 \times 63\,000}{270} = 467 \text{ in-tons}$$

The problem may be solved otherwise by considering that the force at B which will deflect the simple beam, $A\,C$, $\frac{1}{2}$ in. equals the reaction due to a pier depression of $\frac{1}{2}$ in.

5.—Three-Span Beam, Mid-Span Loaded.—Let it be required to determine the pier moments, M' and M'', Fig. 70 (a), when the mid-span only is loaded.

Without knowing the moments, the form of the moment diagram is known, because it must consist of the "simple moment area" for the mid-span plus a moment triangle for each end span, as shown. In drawing the equilibrium polygon of the pier tangents consider the moment diagram a load area furnishing the five elastic weights, W_0, W_1, W_2' W_2'', and W_3. The simple moment area, W_0, is known, or may be computed; it causes downward bending. The other four elastic weights are unknown because M' and M'' are unknown, but since they are areas of triangles their lines of action must be at the "third verticals" and it is self-evident that they cause upward bending as indicated.

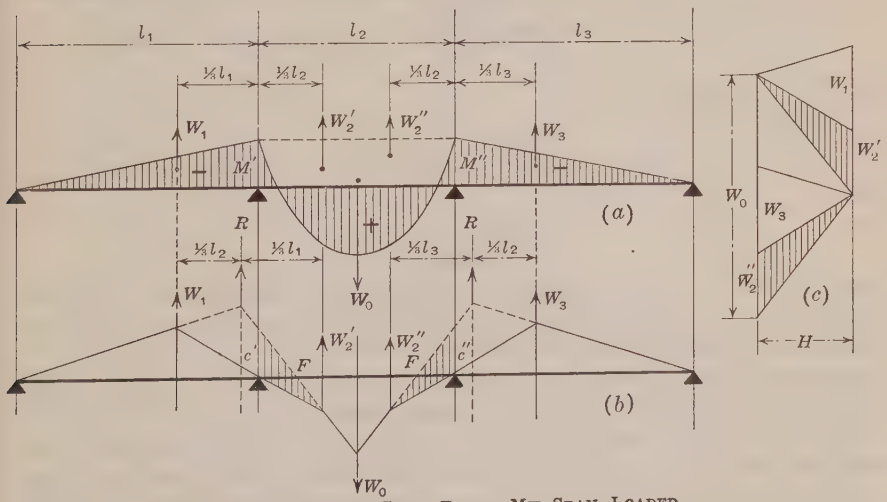

FIG. 70.—THREE-SPAN BEAM, MID-SPAN LOADED.

Now, analogous to Fig. 67 (b), draw the first pier tangent of Fig. 70 (b) at random, and from the point where it intersects W_1 draw the second pier tangent through the second support to the point where it intersects W_2'. The third line of the equilibrium polygon must go through the point just located, and since it must also intersect the first line on the "resultant vertical", R, its position is given regardless of the value, W_0. A reference to Fig. 67 (c) will show the important characteristic of the third line, namely, that it will pass through a fixed point, F, of the beam axis no matter in what direction

the first line was drawn. Now, locate the other fixed point of the span, l_2, by a separate random construction, as in Fig. 68 (*b*), and complete Fig. 70 (*b*). From the equilibrium polygon just drawn a force diagram is constructed, Fig. 70 (*c*), by drawing six rays parallel to the corresponding six lines of the equilibrium polygon. Since the value, W_0, is known, the value of the other four elastic weights are now given to the scale of W_0, and, because $W_1 = \frac{M' l_1}{2}$ and $W_2' = \frac{M' l_2}{2}$,

$$M' = \frac{2 W_1}{l_1} = \frac{2 W_2'}{l_2}$$

and, similarly,

$$M'' = \frac{2 W_2''}{l_2} = \frac{2 W_3}{l_3}$$

For the practical application of this problem note from the similarity of the shaded triangles of the equilibrium polygon and the force diagram that

$$\frac{l_2}{2} M' : H = C' : \frac{l_2}{3}$$

and,

$$\frac{l_2}{3} M'' : H = C'' : \frac{l_2}{3}$$

or, after constructing a force triangle as for Fig. 68 (*a*) with $\frac{W_0}{\frac{l_2}{2}}$ as a load line and $\frac{l_2}{3}$ as a pole distance,

$$M' : \frac{l_2}{3} = C' : \frac{l_2}{3}$$

and

$$M'' : \frac{l_2}{3} = C'' : \frac{l_2}{3}$$

showing that the two lines of the equilibrium polygon drawn through the fixed points, *F*, and parallel to the rays of this force triangle will cut off the correct values of M' and M'' on the pier verticals.

6.—Cross-Lines for Mid-Span.—From this, the simple construction of Fig. 71 (*a*), analogous to Fig. 68 (*a*) is self-explanatory.

For a uniformly distributed load on mid-span the force triangle is not required, for, analogous to Fig. 68 (*b*), the two lines of the equilibrium polygon intersect at the apex of the parabola for uniform load, giving Fig. 71 (*b*).

Indeed, the solution of the problem, namely, to determine M' and M'', may be made still more general and more simple, giving the same construction for either concentrated or uniformly distributed loading on the mid-span, and requiring no force triangle, by considering the following relation of similar triangles in Fig. 71 (*a*):

$$\text{Distance, } A\,A' : g' = \frac{W_0}{\frac{l_2}{2}} : \frac{l_2}{3}$$

or,

$$\text{Distance, } A\ A' = \frac{\frac{g'\ W_0}{l_2^{\,2}}}{6}$$

in which, g' is the distance to the gravity line of the simple moment area, W_0. The cut-off on the other vertical in accordance with this is $B\ B' = \frac{\frac{g''\ W_0}{l_2^{\,2}}}{6}$. The lines, $A\ B'$ and $B\ A'$, are termed the cross-lines.

The Sum Total of the Foregoing Demonstrations.—To determine M' and M'' for any loading on the mid-span, draw the "cross-lines" cutting the pier verticals at points distant $\frac{\frac{g'\ W_0}{l_2^{\,2}}}{6}$ and $\frac{\frac{g''\ W_0}{l_2^{\,2}}}{6}$ below A and B, respectively; project the two fixed points, F, downward on these cross-lines; then, the straight line, $F'\ F'$, will cut off the values of M' and M'' on the pier verticals.

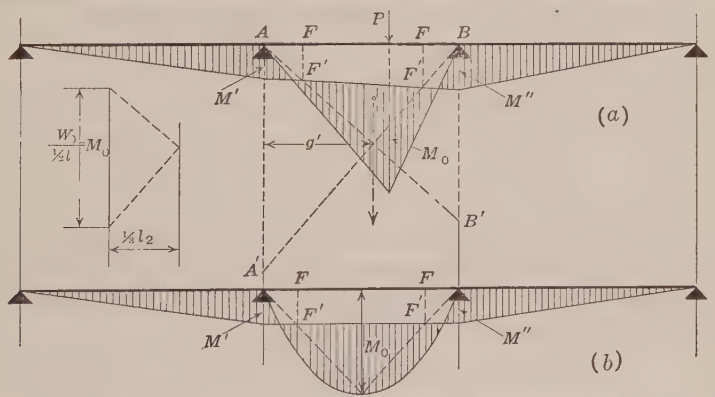

FIG. 71.—THREE-SPAN BEAM WITH CONCENTRATED OR UNIFORM LOAD ON MID-SPAN.

7.—Three-Span Beam, End Span Loaded.—Figs. 72 (*a*), (*b*), and (*c*), analogous to Figs. 70 (*a*), (*b*), and (*c*), illustrate the method for finding the pier moments, M' and M'', for loads on one end span. All these diagrams of course are drawn on the assumption that the structure subjected to deformation is held to its supports. Again, without knowing the value of M' and M'', the form of the moment diagram must be as illustrated in Fig. 72 (*a*). The reaction at M'' is downward, for the beam would rise from the support if not held to it. The elastic weight, W_0, the simple beam area, is known, and the unknown elastic weights, W_1, W_2', W_2'', and W_3, since they represent triangular areas, must act at the "third verticals".

In constructing the equilibrium polygon of the pier tangents begin at the right support and draw the first line at random. From the point where it intersects W_3 draw the second line through the support to an intersection with W_2''. The third line is now given, for it must intersect the first line on the "resultant vertical", R, and from the point where this third line intersects W_2'

another pier tangent is drawn to intersect W_1. Since the fifth and third lines must intersect the resultant vertical of W_1 and W_2', the fifth line is now given, and from the point where it intersects W_0 the last pier tangent is drawn, completing the polygon. Referring to Fig. 67 (*c*) again, the important fact will be evident that the third and fifth lines, termed the "medial lines", in order to distinguish them from the pier tangents, will traverse fixed points, *F*, for any random first line.

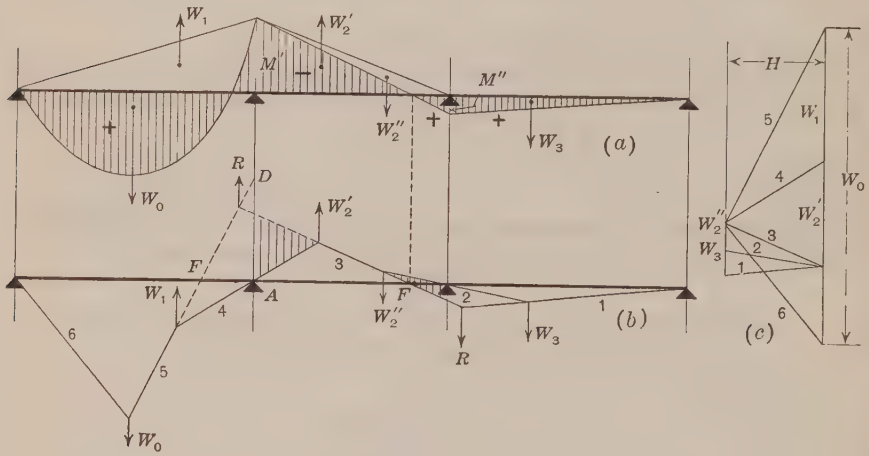

FIG. 72.—THREE-SPAN BEAM, END SPAN LOADED.

From Fig. 72 (*b*) the force diagram, Fig. 72 (*c*), is now derived by drawing its six rays parallel to the corresponding six lines of the equilibrium polygon, giving as vertical intercepts the values of W_1, W_2', W_2'', and W_3 to the scale of W_0. These, in turn, yield the values of,

$$M' = \frac{2\ W_1}{l_1} = \frac{2\ W_2'}{l_2}$$

and, similarly,

$$M'' = \frac{2\ W_2''}{l_2} = \frac{2\ W_3}{l_3}$$

8.—Cross-Lines for End Span.—For the end span loaded, reasoning identically the same as for the mid-span loaded, leads to constructions like Figs. 71 (*a*) and (*b*) and, finally, to the more general method of cross-lines, Fig. 73 (*a*).

It has been shown that the cut-off by a medial line of the loaded span on the pier vertical has a definite relation to the pier moments. It is further important to remember that the cut-offs by the medial lines of any unloaded span on the pier verticals are proportional to the pier moments. This latter fact is apparent when noting that the two shaded triangles of Fig. 72 (*b*) are similar to the corresponding two triangles on $W_2' = \frac{M'\ l_2}{2}$ and on $W_2'' = \frac{M''\ l_2}{2}$, Fig. 72 (*c*); hence, it follows that the moment line of an unloaded span, like the medial line of such a span, must traverse the fixed point, *F*. Since the pier moments decrease from the loaded span toward each end of the structure,

the point, F, through which the moment line passes is necessarily always the second one in that direction. Hence, referring again to Fig. 73 (a), it is only necessary to determine M', for the moment line from that point to the end must pass through the fixed points, F. The end support is the last fixed point.

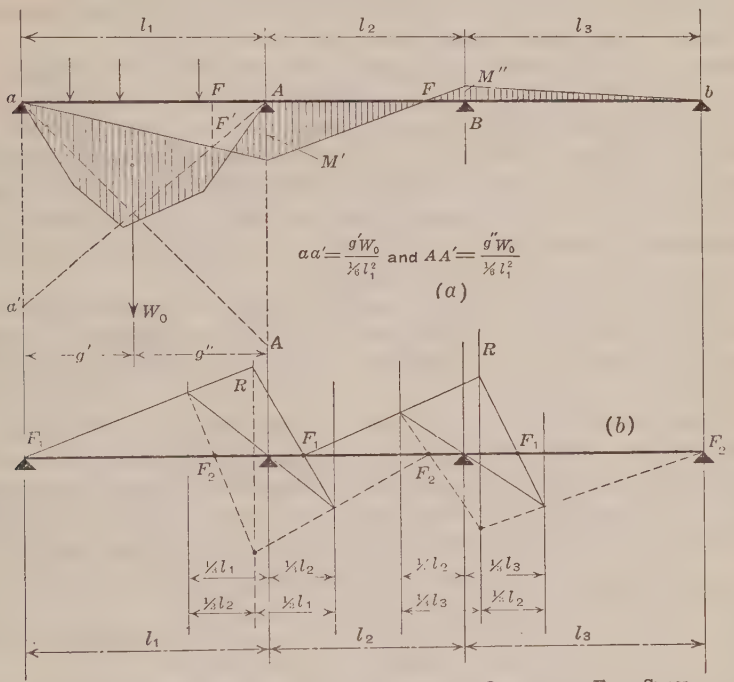

FIG. 73.—THREE-SPAN BEAM CONCENTRATED LOADS ON END SPAN.

It is self-evident that this reasoning applies to continuous beams of any number of supports. If the loaded span is not an end span it is only necessary to determine the two pier moments to the right and left of the loaded span by "cross-lines" and complete the moment diagram by drawing the remaining moment lines through the fixed point, F. A glance at Fig. 67 (b) and Fig. 68 (b), and an examination of Fig. 72 (b) will show that by a random construction like Fig. 73 (b), the fixed points, F, may be located for any number of spans. In Fig. 73 (b), proceeding from left to right, the full lines locate the points, F_1, and, from the other direction, the dotted lines locate the points, F_2. For equal spans, the resultant verticals coincide, of course, with the pier verticals.

9.—Continuous Beams of Any Number of Spans.—To recapitulate and show the general procedure for continuous beams of any number of spans, equal or unequal, a beam of five equal spans (Fig. 74) is chosen for simplicity of illustration. First, the fixed points should be located by a separate construction, Fig. 74 (a). The M-polygons are then drawn for loads on one span only; Fig. 74 (b) shows this for the second span, and the full lines of Fig. 74 (c) for the first span. The dotted lines of Fig. 74 (c) show the com-

plete moment diagram for which the M-line, $A\ B'\ C'\ D'\ E'\ F$, is obtained by adding the pier moments for one span loading; but it should be noted again, that the moments for partial loading may be greater. It will be seen that the maximum moment for any span occurs when this span and the alternate spans on either side of it are loaded; and the maximum moment for any pier occurs when the spans to the right and left of the pier and the alternate spans on either side of them are loaded.

Suppose the simple moment diagrams were constructed by means of force diagrams, Fig. 74 (d); then lines through the poles of these force diagrams, parallel to the corresponding lines of the M-polygon, $A\ B'\ C'\ D'\ E'\ F$, will cut off the reactions, $A\ B\ C\ D\ E\ F$, on the load line as shown. Having the reactions, the shear diagram, Fig. 74 (e), can be obtained. If it is desired to refer all moments to a horizontal base line, Fig. 74 (f), the diagram may be constructed by using the poles, O', of Fig. 74 (d). This would afford an acute check on the accuracy of the whole work.

It will be seen from Figs. 74 (b) and (c) that the moments for one span loaded diminish rapidly toward the ends, and it seems sufficient in practice to extend the investigation for maximum moments over a radius of two adjacent spans only.

10.—One-Span Beam, Fixed Ends.—The methods demonstrated may be applied of course to the most simple case, the one span beam fixed at both ends. To obtain the moments, draw the equilibrium polygon of end tangents (Fig. 75 (a)), and from it the force diagram, which, in turn, will yield the moments, M_1 and M_2, analogous to Fig. 67 (b). Since the "medial lines" must pass through the fixed points, these latter are seen to be distant $\frac{l}{3}$ from the ends, and may be used in constructing the moment diagram by "cross-lines", Fig. 75 (b), analogous to Figs. 71 (a) and 73 (a).

Diagrams like this are well adapted for deriving formulas for typical loading. For the simple case of a uniformly distributed load, p per ft., no construction is necessary. The simple moment is $\frac{p\ l^2}{8}$, hence, the parabolic area,

$$W_0 = \frac{p\ l^2}{8} \times \frac{2\ l}{3} = \frac{p l^3}{12}$$

and since $W_1 = W_2 = \frac{1}{2}\ W_0$ in this case,

$$M_1 = M_2 = \frac{\frac{1}{2}\ W_0}{\frac{l}{2}} = \frac{p\ l^2}{12}$$

leaving a moment for mid-span of $\frac{p\ l^2}{24}$ as shown in Fig. 75 (c).

11.—Consideration of Variable Moment of Inertia.—If I varies from span to span only, being uniform throughout each span, the constructions remain the same except for the location of the resultant verticals, the "T-lines" of the authors, as explained by Fig. 15, or by transformed spans as will be treated in Fig. 78 (b).

The procedure for the case of variable moments of inertia throughout, although not differing from the foregoing demonstrations in theory, becomes more involved in practical application. The elastic weights, $\frac{M}{E\,I}$, reduce to

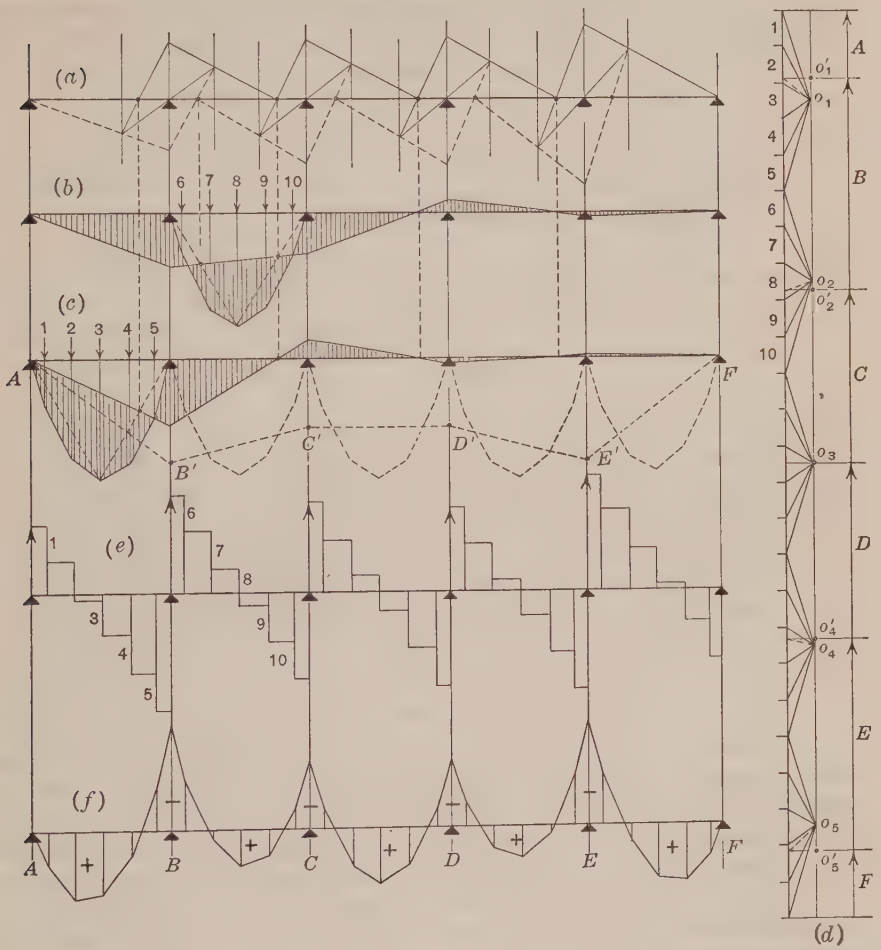

FIG. 74.—TYPICAL CONSTRUCTIONS FOR CONTINUOUS BEAMS OF ANY NUMBER OF SPANS.

M for a constant, $E\,I$, and to $\frac{M}{I}$ for a constant, E, only; but in order to avoid small fractional figures, and to make all reduction factors nearly equal to unity for convenience in application, the elastic weights are made $\frac{M\,C}{I}$, C being a constant equal to the greatest, the smallest, or any average I occurring most frequently. Hence, all ordinates of the simple moment areas as well as of the triangular M-areas must be multiplied by $\frac{C}{I}$ in order to obtain the "modified"

elastic load areas. Then the gravity lines of these new areas must be determined, being the lines of action of the new elastic weights, W_0, W_1, W_2, etc. Note that the third lines, being no longer gravity lines of triangles, have shifted slightly, but the resultant vertical is located as before, that is, by transposing the distances of W_1 and W_2 from the pier verticals. The fixed points, F, are now located as before, using the new gravity lines, and the remainder of the procedure is the same as for beams of uniform I.

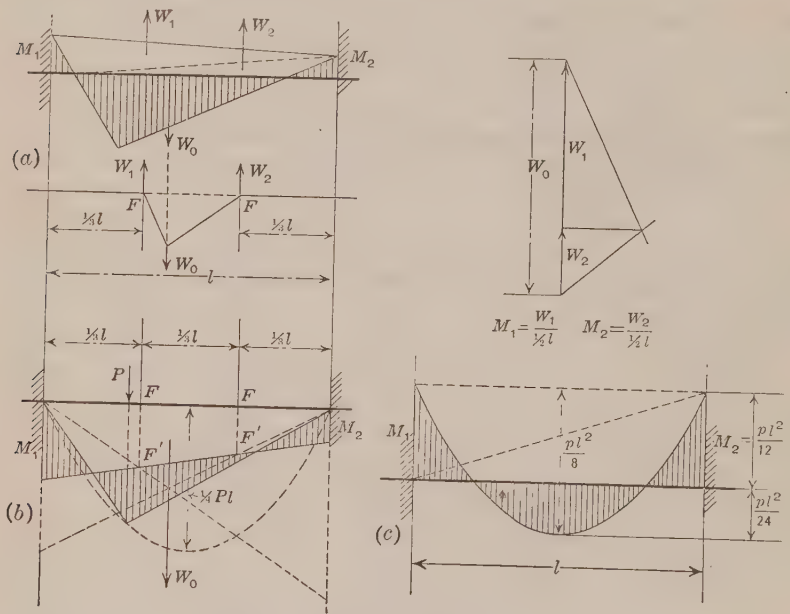

FIG. 75.—BEAMS FIXED BOTH ENDS.

If the value of I for a continuous beam is varied without increasing its depth, by changing the reinforcement or adding cover-plates, the error in moments is not more than 6%, while the error in shear and reactions is still less. By using a little care in selecting a fair average value of I as a uniform I, the errors will be very small indeed. A small settlement of a support, or the small unavoidable errors in fitting the supports to the unstrained shape of a continuous steel girder, may cause errors much greater than those just mentioned.

Again, as to the applicability of the elastic theory to steel structures there is no question; the measured deflections agree well with those determined by theory, and almost exactly if the shear deformation is taken into consideration. In concrete structures, however, E also is a variable quantity, varying within wide limits, not only with the amount of stress, but even with the sense of stress (tension and compression), and with the age and composition of the concrete. In concrete arches the variation is much less than in beams, because the amount of stress is fairly uniform and of one sense, the bending moments only alter the compressive stresses, and tension is practically absent, whereas in concrete beams the stress varies from zero to a maximum plus

or minus. In trying to find an answer to the question, "what is the E and I of a reinforced concrete beam that will yield a deflection within reasonable agreement with the measured deflection", a great refinement in the analysis of reinforced concrete beams and slabs must seem illusory.

12.—Another Method of Constructing the M-Polygon.—A practical and useful line relation is obtained as follows: The intercepts of the moment diagram on the "third verticals", Fig. 76 (*a*), are:

$$T_1'' = \frac{1}{3} M_1 + \frac{2}{3} M_2 \text{ and } T_2' = \frac{2}{3} M_2 + \frac{1}{3} M_3$$

Hence,

$$T_1'' l_1 + T_2' l_2 = \frac{1}{3} [M_1 l_1 + 2 M_2 (l_1 + l_2) + M_3 l_2] = \frac{1}{3} N_2$$

equals one-third the load term, N_2, because the expression in the parentheses is the "three-moment equation". Drawing now the line, mn (Fig. 76 (*b*)), the intercept on the "resultant vertical" is,

$$R_2 = \frac{T_1'' l_1 + T_2' l_2}{l_1 + l_2} = \frac{N_2}{3 (l_1 + l_2)} \quad \ldots\ldots\ldots\ldots (92)$$

By means of these R-intercepts on the resultant verticals, together with the fixed points, F, the pier moment polygon may be drawn for any loading whatever, as illustrated by Fig. 76 (*c*). First plot the values, R_1, R_2, R_3, and R_4, on the resultant verticals as indicated by the heavy ordinates. Then draw the auxiliary polygon, F_0, F_1', F_2', F_3', F_4', and F_5, each line through the top of each R-ordinate until it intersects the vertical through the fixed point, F. The intersections, F_1', F_2', etc., are points of the required pier moment polygon, and the last line, F_4' F_5, is at once a line of such polygon and will cut off the moment, M_4, on the vertical through Pier 4. The full-line polygon traced backwardly through the points, F', will complete the pier moment polygon as shown.

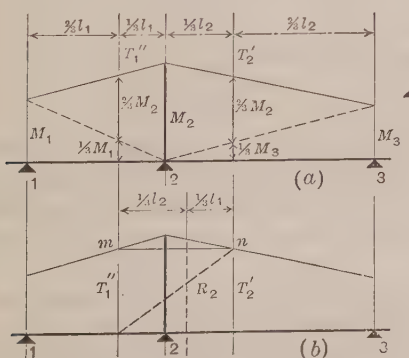

FIG. 76.—THE PIER MOMENT POLYGON DETERMINED BY R-ORDINATES.

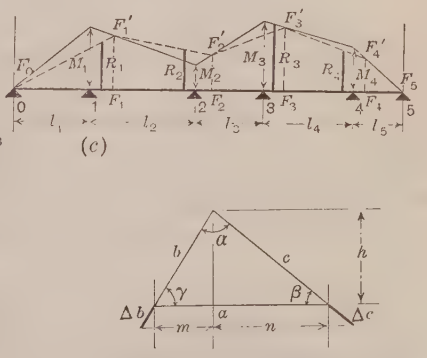

FIG. 77.—THE ANGULAR DEFORMATION OF A STRUCTURAL TRIANGLE.

An acute check on the accuracy of the drawing may be obtained by repeating the construction in the opposite direction, using the same R-intercepts, but the second fixed points in each span (not shown), beginning the auxiliary polygon at F_5.

For a uniformly distributed load, p, per linear foot, the simple moment area $= \frac{1}{8} p l^2$. $\frac{2}{3} l = \frac{1}{12} p l^3$, and its reaction at the pier $= \frac{1}{24} p l^3$. Hence, the load term of the three-moment equation for Pier 2, since both sides of the equation were multiplied by 6 EI (see the authors' derivation, page 5) is,

$$N_2 = \frac{1}{4} p (l_1^3 + l_2^3)$$

which gives an R-intercept of,

$$R_2 = \frac{p}{12} \left(\frac{l_1^3 + l_2^3}{l_1 + l_2} \right)$$

For a pier depression, d_2, the pier reaction is,

$$\frac{d_2}{l_1} + \frac{d_2}{l_2} = d_2 \frac{l_1 + l_2}{l_1 l_2}$$

Hence, as before,

$$N_2 = 6 E I \frac{d_2 (l_1 + l_2)}{l_1 l_2}$$

and,

$$R_2 = \frac{2 E I d_2}{l_1 l_2}$$

13.—Application to Secondary Stresses.—The basis of the subject of secondary stress is the deformation of a structural triangle. If the three sides of a triangle, Fig. 77, change their lengths, a, b, and c, by the amounts, Δa, Δb, and Δc, respectively, the change of the angle, α, is

$$\Delta \alpha = \frac{\Delta a}{h} - \frac{\Delta b \cos \gamma}{h} - \frac{\Delta c \cos \beta}{h}$$

$$= \frac{\Delta a}{h} \cdot \frac{m + n}{a} - \frac{\Delta b}{h} \cdot \frac{m}{b} - \frac{\Delta c}{h} \cdot \frac{n}{c}$$

$$= \left(\frac{\Delta a}{a} - \frac{\Delta b}{b} \right) \frac{m}{h} + \left(\frac{\Delta a}{a} - \frac{\Delta c}{c} \right) \frac{n}{h}$$

Since the unit strain, $\frac{\Delta a}{a}$, etc., equals the unit stress, $\frac{S}{E}$,

$$E \Delta \alpha = (S_a - S_b) \cot \gamma + (S_a - S_c) \cot \beta$$

This equation is readily remembered by noting its law of formation: That for each angle of a triangle the equation is the sum of two terms, each the unit stress in the opposite side minus that in the adjacent side, times the cotangent of the included angle. The check for each triangle is $\Delta\alpha + \Delta\beta + \Delta\gamma = 0$.

Let Fig. 78 (a) represent a truss having a top chord of continuous construction between the pins, 0 and 5. Considering chord deformation only, the problem is simplified by regarding the top chord as a continuous beam on the panel points as supports. The moment reaction for any panel point (its elastic weight) is equal to its angle of deformation; hence, for Panel Point 2:

$$\frac{\frac{1}{3} M_1 + \frac{2}{3} M_2}{E I_2} \cdot \frac{l_2}{2} + \frac{\frac{2}{3} M_2 + \frac{1}{3} M_3}{E I_3} \cdot \frac{l_3}{2} = \Delta \phi_2$$

or,

$$M_1 \frac{l_2}{I_2} + 2 M_2 \left(\frac{l_2}{I_2} + \frac{l_3}{I_3} \right) + M_3 \frac{l_3}{I_3} = 6 E \Delta \phi_2$$

Multiplying this equation by a constant, I_c, in order to obtain convenient numerical values, and introducing the abbreviation,

$$l_2 \frac{I_c}{I_2} = l_2' \text{ and } l_3 \frac{I_c}{I_3} = l_3'$$

gives,

$$M_1 l_2' + 2M_2 (l_2' + l_3') + M_3 l_3' = 6 E I_c \Delta \phi_2$$

which again is seen to be the three-moment equation, in this case with the load term, $N_2 = 6 E I_c \Delta \phi_2$. Hence by Equation (92),

$$R_2 = \frac{2 E I_c \Delta \phi_2}{l_2' + l_3'} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (93)$$

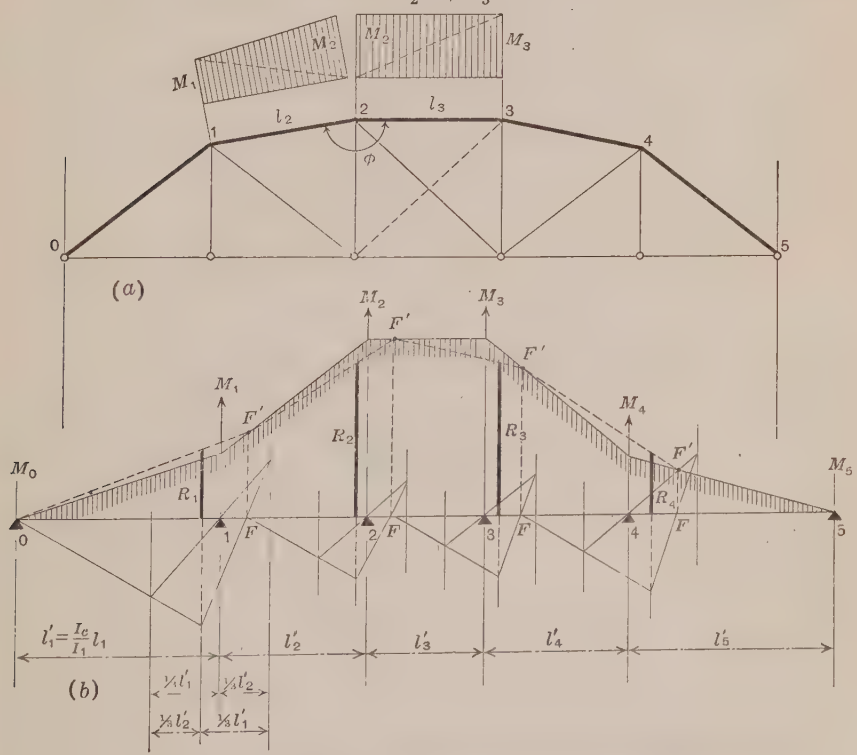

FIG. 78.—SECONDARY STRESSES IN A CONTINUOUS TOP CHORD.

Fig. 78 (*b*) shows the construction for the upper chord, 0-5, of Fig. 78 (*a*) and is typical for all such cases where the two end moments are zero. For a symmetrical structure symmetrically loaded the *M*-polygon will appear in symmetrical form and thus check itself. For any unsymmetrical conditions a check construction may be had by using the same *R*-intercepts but the second fixed point in each span, and beginning the auxiliary *F'*-polygon at Point 5, exactly as explained for Fig. 76 (*c*).

Note, as appears at a glance from Fig. 78 (*b*), that the intercepts, R, alone give a fair approximation of the moments, M_1, M_2, M_3, etc., entirely sufficient for a practical estimate of the secondary stresses to be expected. Further, since l_n' is inversely proportional to I_n, the greater the moment of inertia, I, the greater the values of R and hence of M, pointing to the advisability of making the depth of any member as small as practicable, consistent with other considerations.

14.—Circumferential Chords of Trusses.—If for a circumferential upper and lower chord no moment is zero but the structure is symmetrical, the determination of the panel-point moments does not differ greatly from the problem just demonstrated. The following three cases will cover such problems:

I.—The axis of symmetry contains no panel point (Fig. 79 (*a*));
II.—The axis of symmerty contains two panel points (Fig. 79 (*b*)); and,
III.—The axis of symmetry contains one panel point (Fig. 79 (*c*)).

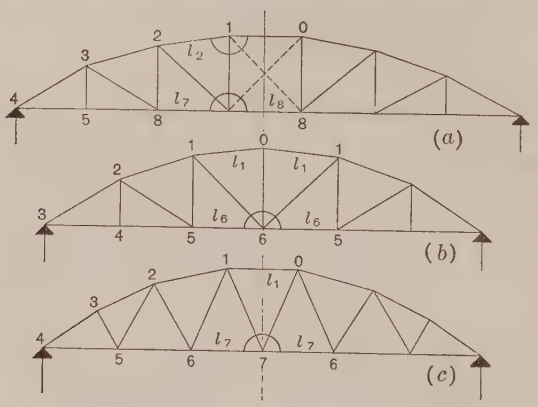

FIG. 79.—THREE TYPICAL CASES OF SECONDARY STRESSES IN CONTINUOUS TOP AND BOTTOM CHORDS.

Case I.—The enumeration of the panel points for the auxiliary continuous beam is given in Fig. 79 (*a*). The three-moment equation for the moments, M_0, M_1, and M_2, is:

$$M_0\, l_1' + 2M_1\,(l_1' + l_2') + M_2\, l_2' = N_1$$

but, since $M_0 = M_1$, the equation may be written:

$$2\, M_1 \left(\frac{3\, l_1'}{2} + l_2'\right) + M_2\, l_2' = N_1 \ldots\ldots\ldots\ldots\ldots\ldots (94)$$

Similarly, the three-moment equation for M_6, M_7, and M_8, or the seventh equation, is:

$$M_6\, l_7' + 2\, M_7\,(l_7' + l_8') + M_8\, l_8' = N_7$$

and, since $M_8 = M_7$, it may be written:

$$M_6\, l_7' + 2\, M_7 \left(l_7' + \frac{3\, l_8'}{2}\right) = N_7 \ldots\ldots\ldots\ldots\ldots\ldots (95)$$

Equations (94) and (95) show that the auxiliary continuous beam, 0-8, has freely supported ends, because the two end terms, $M_0\, l_1'$ and $M_8\, l_8'$, respectively,

have vanished from these two equations. Hence, the M-polygon will be analogous to Fig. 78 (b), except that the first and last spans are, respectively:

$$\frac{3\,l_1'}{2} = \frac{3\,l_1}{2}\frac{I_c}{I_1}, \text{ and } \frac{3\,l_8'}{2} = \frac{3\,l_8}{2}\frac{I_c}{I_8}$$

from which,

$$R_1 = \frac{2\,E\,I_c\,\Delta\,\phi_1}{\frac{3\,l_1'}{2} + l_2'}, \text{ and } R_7 = \frac{2\,E\,I_c\,\Delta\,\phi_7}{l_7' + \frac{3\,l_8'}{2}}$$

Case II.—The three-moment equation for Point 0 is:

$$M_1\,l_1' + 2M_0\,(l_1' + l_1') + M_1\,l_1' = N_0$$

divided by 3 it gives,

$$\frac{2}{3}\,M_0 + \frac{1}{3}\,M_1 = \frac{N_0}{3\,(l_1' + l_1')} = R_0$$

and, similarly,

$$\frac{1}{3}\,M_5 + \frac{2}{3}\,M_6 = \frac{N_6}{3\,(l_6' + l_6')} = R_6$$

If now Fig. 80 represents the moment polygon desired, its intercepts, FF', on the first and last "third verticals" are:

$$F\,F' = \frac{2}{3}\,M_0 + \frac{1}{3}\,M_1 = R_0$$

and,

$$F\,F' = \frac{1}{3}\,M_5 + \frac{2}{3}\,M_6 = R_6$$

Hence, two points, F', of the M-polygon are known, and it is shown at once that the two points, F, are the first and last fixed points of the continuous beam, 0-6. Starting with these fixed points, all other fixed points may be located as before, allowing the auxiliary F'-polygon to be drawn, as in Fig. 78 (b), and, in turn, the M-polygon desired.

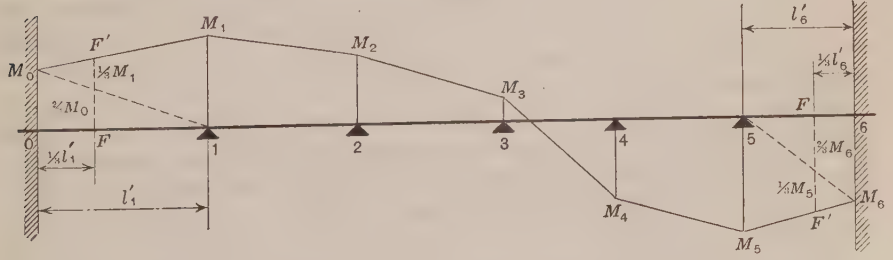

FIG. 80.—THE M-POLYGON FOR CASE 2, FIG. 79(b).

Note, incidentally, the additional proof furnished, that for a continuous beam fixed at both ends the first and last fixed points are distant one-third of the end span from the ends.

Case III.—This is evidently a combination of Cases I and II. The auxiliary continuous beam is freely supported at Point 0, corresponding to Fig.

78 (*b*), while the other end at Point 7 is fixed, corresponding to Fig. 80. Hence, for the first span,

$$l_1' = \frac{3\ l_1}{2}\ \frac{I_c}{I_1}$$

while for all the other spans,

$$l' = l\ \frac{I_c}{I}$$

The second fixed point of the end span is $\frac{l_7'}{3}$ from the end, and its ordinate is,

$$R_7 = \frac{2\ E\ I_c\ \Delta\ \phi_7}{l_7' + l_7''}$$

For a complete determination of secondary stresses, including web members, methods similar to that just demonstrated could be used by considering the three members of each structural triangle a continuous beam on four supports, the first coinciding with the last, but such methods become cumbersome, and the computation of secondary stresses by the method of normal equations* is superior for such cases.

General.—In conclusion, the writer wishes to add a few words regarding statically indeterminate problems in general.

The whole subject can be demonstrated, in an elementary way, within wide limits of practical application, with little tax on the memory, that is, with few elements as a base, namely, similar triangles, force diagram, and equilibrium polygon, the elastic weight, and Maxwell's "theorem of reciprocal deformation".

This statement will seem less radical when it is considered that in constructing an influence line (a line of deflection) one has determined the numerical values of strings of differentials; in determining the area of a graph one has similarly performed numerical integration. The whole process is really graphical differentiation and integration, vastly clearer and more tangible than the analytical process, within the scaling accuracy of the drawing, of course. If more accuracy is required the ordinates of any influence line may be computed by continued addition of elastic panel shear, in the same simple way as in determining bending moments by continued addition of ordinary panel shear.

Much time and clearness in demonstration could be gained by treating reinforced concrete structures, solid steel web structures, and framed structures of one type under one heading. The analysis is the same for all three, except the small difference in determining the elastic weights and the accuracy desired.

R. McC. Beanfield,† Assoc. M. Am. Soc. C. E. (by letter).—The practical application of graphical analysis to the solution of problems involving continuous beams and indeterminate frames is now rather conspicuous by reason of its limited use by the profession in general. It is the exception

* "Secondary Stresses in Bridges," by Cecil Vivian von Abo, Jun. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 1.

† Structural and Mech. Engr., Los Angeles, Calif.

rather than the rule for the average designing engineer to make a complete and proper stress analysis of statically indeterminate problems, particularly in reinforced concrete construction, due to the laborious, slow, and cumbersome analytical methods, and to the difficulties of obtaining a proper check. In general, if engineers had a more practical working knowledge of graphical analysis applied to statically indeterminate structures, as suggested by the authors, they would have confidence in their work resulting in safer and more economical structures.

The writer has been using, for a number of years, a system of graphical analysis which is an adaptation, in general, of Professor W. Ritter's method,* and is somewhat simpler than that of the authors. Professor Ritter's graphical analysis seems to be little known or understood by American engineers, possibly due to the fact that a proper translation into English has never been published. Prior to discussing the advantages of Ritter's graphical method, a complete graphical solution by that method, using the authors' example and their notations wherever possible, will be given.

Construct the R, U, V, T-lines and the simple beam moment diagrams for the various spans as described by the authors. (Fig. 81 (b) and (a), respectively.) In Fig. 81 (b) starting from J_1 draw any straight line that will cut the V or trisecant line and the first T or reversed third line in the points, e and f, respectively. Draw the straight line, $e\,R_2$, which, prolonged, will cut the U-line in the second span in the point, g. Join g and f by a straight line intersecting the base line in the point, J_2, called a fixed point in the second span.

Starting from the fixed point, J_2, repeat the same process, thus obtaining the second fixed point, J_3, in the third span. The fixed points, k_1 and k_2, are found in a similar manner starting from k_3, Fig. 81 (b). Draw any straight line, $k_3\ m$, cutting the T or reversed third line at n. The intersection of the straight line, nh, with the base line at k_2 defines the other fixed point for the second span. Starting from k_2, by similar construction, the fixed point, k_1, can be obtained. The proof of this will be given subsequently. From each of the fixed points erect verticals, locating J and k-points in Fig. 81 (a).

In case of a symmetrical arrangement of spans relative to the center of the continuous girder, the fixed points are located symmetrically relative to the center of the continuous girder. It should be noted that the position of the fixed points is dependent only on the relation of the spans to each other and not in any way on the loading of the spans. After the fixed points are located, the bending moment of each load on all supports is determined, considering only one span to be loaded at a time, usually the left-hand span. The algebraic summation of the moments (intercepts) at the supports due to the separate loads will give the resultant moments due to the entire system of loads on the continuous girder. The final closing line is drawn through these points, or final intercepts at the supports.

The bending moments for a uniformly loaded span in a continuous girder are found graphically, as follows:

* "Anwendungen der Graphischen Statik," by C. Culmann; "Der Kontinuierliche Balken," by W. Ritter, 1900.

Construct the simple beam moment diagram (parabola, Fig. 82 (*a*)). Connect the point, *S*, at the apex of the parabola with the points of supports of the loaded span. The intersection of these lines with the fixed lines (*i* and *k*-lines) give the points, i_1' and k_2', which are points in the closing line of the moment area of the girder. (See Fig. 82 (*a*) and (*b*).) When more than one span is loaded, treat each span separately and summarize the results.

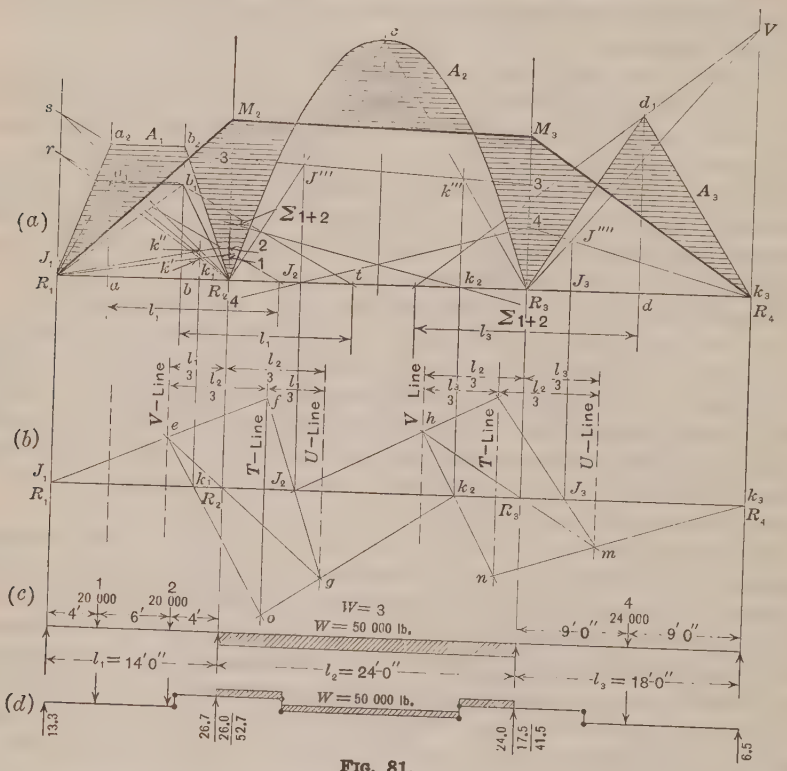

FIG. 81.

For a single concentration or moving load the moments or their intercepts on the reaction verticals are found, as follows: Refer to Fig. 83 (the proof will be given subsequently). Lay off on the base line the length, l_2, on each side of the load. From the points thus found draw lines through the vertex, *S*, of the simple beam moment area, *bSc*. These lines intersect the support perpendiculars in *b''* and *c''*. From the points, *b''* and *c''*, draw lines to *c* and *b*, respectively. The respective intersections of *cb''* and *bc''* with verticals through the fixed points, i_2 and k_2, are points on the closing lines, which, extended, intersect the verticals through the supports at b_1 and c_1. The intercepts, bb_1 and cc_1, are the moments measured to the scale of moments, at the supports. The moment closing line for the load, *P*, is extended from b_1 to *a* and from c_1 through the fixed point, *k*, to d_1, and continued to *e*. The moment closing line indicates the moment effects on the other spans for the concentrated load, *P*, on the second span.

The zero moment or inflection point, i, lies between the support and the fixed perpendicular, $i_2\ i_1'$, and the inflection point, k, between the fixed line, $k_2\ k_2'$, and the support. From this it follows that in a middle span as a single load moves from left to right the inflection points, i and k, in the bending diagram move in the same direction, but always remain inside the loaded span, l_2, and outside the fixed distance, $i_2\ k_2$.

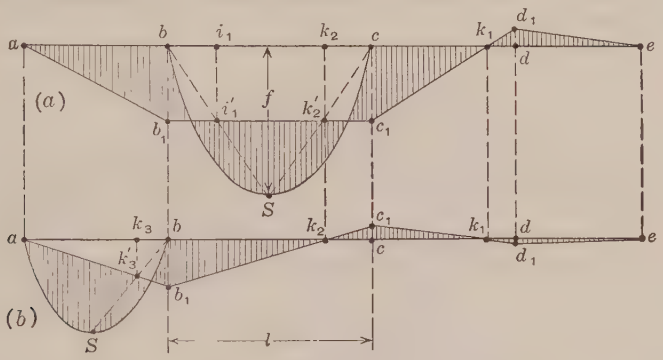

FIG. 82.

To determine the moments due to the combined effects of the various loads refer to Fig. 81 (*a*) (the authors' example) which is a continuous girder with freely supported ends. From similar construction to that shown in Fig. 83, for a single concentrated load draw the closing lines, $R_1\ k'1$, and $R_1\ k''2$, which intersect the vertical through the support, R_2, with segments or intercepts, R_2-1 and R_2-2, respectively, measured to the scale of moments. These intercepts measure the moments at the supports due to Loads 1 and 2, respectively. The summation of these intercepts or segments equals the segment, R_2-Σ 1 + 2. Through Σ 1 + 2 on R_2 produce a line passing through the fixed point, k_2, and intersecting R_3 at Σ 1 + 2. The segment, R_3-Σ 1 + 2, equals the moment, measured to scale of moments, due to Loads 1 and 2 at R_3.

The closing line for the uniform load on the second span is 3, $j'''\ k'''$ 3. The segments, R_2-3 and R_3-3, are the moments at these supports induced by the uniform load, 3, on the second span.

Similarly, the concentrated load, 4, produces moments at R_3 equal to the segment, R_3-4, and at R_2 equal to the segment, R_2-4, respectively. All segments above the base line, $R_1\ R_4$, are positive and those below, are negative. An algebraic summation of the segments or intercepts at each support line, such as R_2-M_2 and R_3-M_3, give the M-points through which the final closing line, $R_1\ M_2\ M_3\ R_4$, can be drawn. The writer suggests that the summation of the moment intercepts on the support or R-lines can be easily and accurately accomplished by a good pair of dividers.

The difference between the ordinates of the simple beam moment diagrams and those of the diagram, $R_1\ M_2\ M_3\ R_4$, will give the resultant moment diagram shown shaded in Fig. 81 (*a*). All areas above the closing line, $R_1\ M_2\ M_3\ R_4$, give positive moments and those below, negative moments. The intersection of the final closing line with the simple beam curves gives the points of inflection.

Fig. 84 (*a*) shows the solution of the authors' example assuming the ends of the continuous girder to be fixed. The graphical construction is identical with that of Fig. 81 (*a*) and (*b*) except that the initial fixed points are located on the first and last trisecant V or U-lines instead of on the extreme ends of the girder. The moment closing lines are located by passing through the proper intersections on the fixed point or extreme trisecant verticals, J_1 or k_3, as shown in Fig. 84 (*a*).

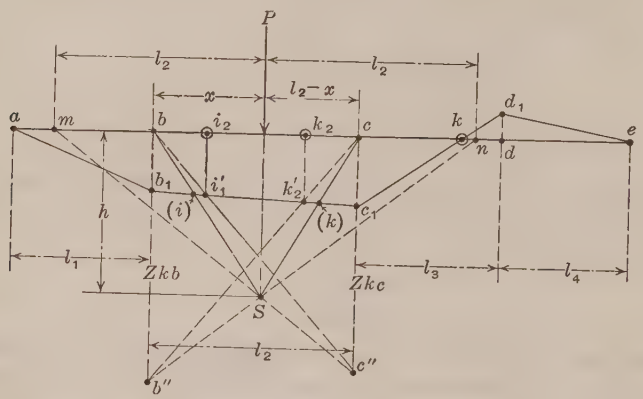

FIG. 83.

In a continuous girder with partly restrained end conditions the fixed point is located between R_1 and $\frac{L}{3}$ in proportion to the relative degree of fixity considered. The moment closing lines are then drawn passing through the proper intersections on the fixed points, otherwise the procedure is similar to that shown in Fig. 81 (*a*).

The advantages of Professor Ritter's method when compared with the authors' system using conjugate or characteristic points are as follows:

(*a*).—Elimination of the computation for moment areas and of dividing them by the length of span; also of the computations of the centers of gravity of the moment areas, which for some conditions of loading become rather complicated and laborious. The elimination of computations lessens the possibilities of errors.

(*b*).—In the method advocated by the writer the influence of each load relative to the effects of moments on the entire continuous girder can be readily ascertained and visualized without additional operations or separate diagrams. This advantage is particularly valuable when investigating a continuous girder for live loads on alternate spans or for unbalanced loading, also for moving loads and the construction of influence lines for them. For the general solution and the construction of the influence lines the methods of Professor Ritter* are doubtless much simpler and shorter than those advocated by the authors.

* "Taschenbuch für Bauingenieur," E. H. Max Foerster, Fourth Edition, pp. 361-363.

(*c*).—Practical examples of the authors' graphical system involving the use of conjugate points discloses the fact that for certain conditions of loading and for varying spans the "pennant diagrams" become so flattened and the angles so acute that the intersections locating the conjugate points are very difficult to ascertain.

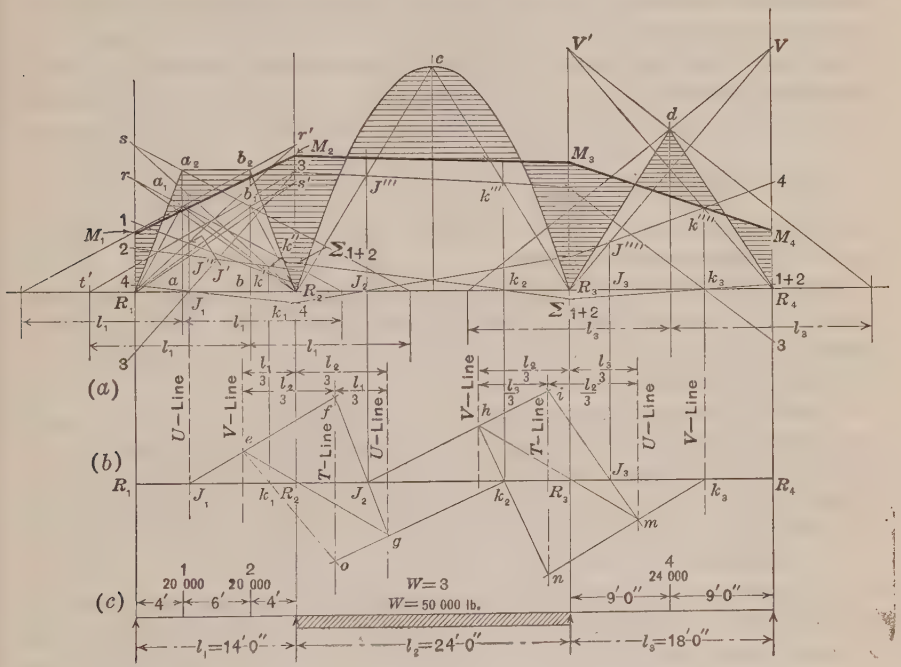

FIG. 84.

For the determination of shears and reactions, the following method will be found advantageous. (Fig. 81 (*c*) and (*d*)). The continuous girder may be considered as a series of cantilevers over the supports, with simple suspended spans hung at the points of contraflexure. The total reaction of any support would then be the total load on the cantilever plus the reactions from the adjacent suspended spans. The point of zero shear is located by constructing an ordinate through the point of maximum positive bending moment.

In the design of continuous beams proper consideration should be given to the maximum positive moments due to partial or unbalanced loading of the spans. For ordinary conditions the writer suggests the following rule: Design for full moments at the supports and for four-thirds of the positive moments between the supports for all spans fully loaded. Where unusual conditions of loading occur, the positive moment between the supports must be carefully investigated as it may exceed four-thirds of the positive moment for all spans fully loaded. The graphical analysis suggested by the writer provides for quick and easy investigation for partial loading for the moment effects on all spans.

The suggestions given in the paper for solving problems involving members with varying moments of inertia and statically indeterminate frames can be applied in like manner to the graphical analysis advocated by the writer. The authors' graphical construction as an aid in the solution of secondary stress problems is rather ingenious and worthy of much consideration to those interested in investigations of secondary stresses.

It is unfortunate that the building codes of many cities lack proper regulations relative to the design of statically indeterminate structures, particularly continuous beams, in reinforced concrete construction. The "rule-of-thumb" methods for computing the bending moments of continuous beams on the usual basis of $\frac{WL}{8}$, $\frac{WL}{10}$, or $\frac{WL}{12}$, disregarding lengths of spans, conditions of loading, and degree of restraint, are illogical and inaccurate, and result in uneconomical structures.

It is hoped that this excellent paper and other discussions suggesting methods of reducing laborious mathematical work will be an incentive to engineers to give more consideration to problems involving continuity and statically indeterminate structures.

*Proof of Graphical Analysis of Continuous Beams.**—Consider a continuous beam over five supports, and assume the second span to be loaded by the loads, 1, 2, and 3 (Fig. 85 (*a*)). The reaction, *d*, is then negative and the moment over the support, *d*, is positive. The moment areas are, as follows (Fig. 85 (*b*)):

In the second span the true moment area equals the positive moment area of the simple beam, *bc*, reduced by the trapezoid, *bb' cc'*, which is divided into two triangles by the line, *bc'*. In the third span the moment area is equal to the difference between the two triangles, *c' cd* and *c' d d'*. In both end spans the moment area is always a triangle (for loads on the interior spans only).

1.—In the center of gravity of the triangles and of the simple moment area of the loaded span, Fig. 85 (*b*), assume the elastic forces, W_1 W_2 W_3 . . ., applied in a positive or negative direction, and draw for these forces the elastic line. The elastic line of a beam is obtained by considering the moment area as a load and drawing the funicular and string polygons. (Fig. 85 (*c*) and (*e*)). From the shape of the elastic line the following important relations are obtained: In the unloaded spans the points of application of the forces, *W*, lie on the third points (Fig. 85 (*b*), centers of gravity of triangles); the intersection, b_2, of the component of the outer lines of the bending diagram (which touch W_1 and W_2) lies on a perpendicular which is derived by interchanging the third line, $\frac{l_1}{3}$ and $\frac{l_2}{3}$, of the respective spans (Fig. 85(*c*)). This perpendicular is called the reversed third line. In the same manner, W_4 and W_5 intersect in the point, c_2. Thus, the different angles or corners of the bending diagrams lie on the third lines except the corner at W_3, and

* "Taschenbuch für Bauingenieur," E. H. Max Foerster, Fourth Edition, pp. 361-363; "Anwendungen der Graphischen Statik," C. Culmann; "Der Kontinuierliche Balken," W. Ritter, 1900.

the components of W_1, W_2, W_3, W_5, and W_6, fall in the reversed third lines no matter how the second span is loaded.

2.—Fig. 85 (*d*) shows the left-hand part of the bending diagrams enlarged from Fig. 85 (*c*). The corners of the triangle, 1-2-b_2, lie each on a fixed perpendicular and two of its sides run through fixed points; therefore, the third side must also go through a certain fixed point that is in the same relation of the two other fixed points. This condition holds for the other spans and thus it can be proved that in every span of a continuous beam there are in the span line two fixed points, i_1 and k_2, which depend only on the relation of the spans to each other, and not on the loading of the spans.

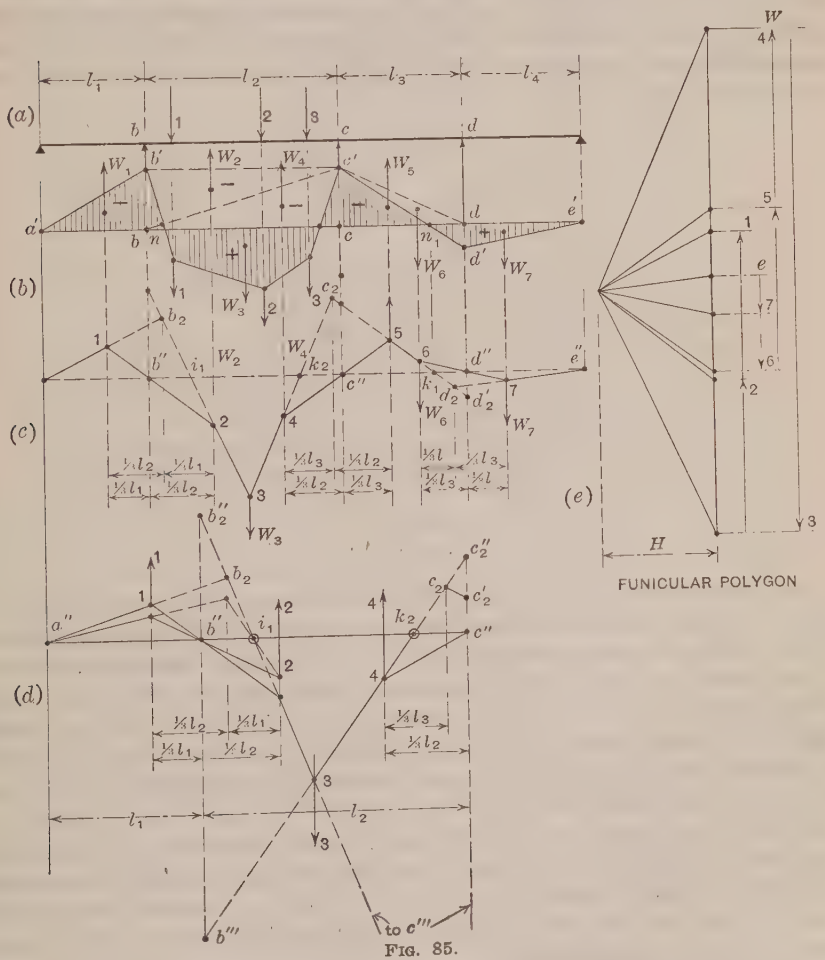

FIG. 85.

The moment in any point, i_1, is zero for all spans to the right, and the moment in any point, k_2, is zero for all spans to the left. The left support, a'', is thereby the i or fixed left point for the first span; the right support is the k or fixed point for the last span. Thus, in the indicated perpendiculars

running through the fixed points (the fixed lines), lie the zero moment points and the turning points of the bending diagram.

3.—The fixed points (Fig. 86): Draw the third lines in all spans and from the left support, a, draw any inclined line by which the second third line of span, l_1, and the reversed third line of the second span are intersected at the points, 1 and b_2. Then draw any line through the points, 1 and b, and produce it to its intersection (Point 2) with the first third line of the second span. The intersection of the line connecting the points, 2 and b_2, with the horizontal will establish i_1 the first fixed i-point of the second span. Starting, now, from i_1 repeat the operation and establish the point, i_2, and thence the point, i_3. The fixed k-points are k_1, k_2, etc., found by starting from the right reaction, e, and proceeding in the same manner toward the left. In spans symmetrical about the center line of the girder, the i and k-points lie also symmetrical about this line.

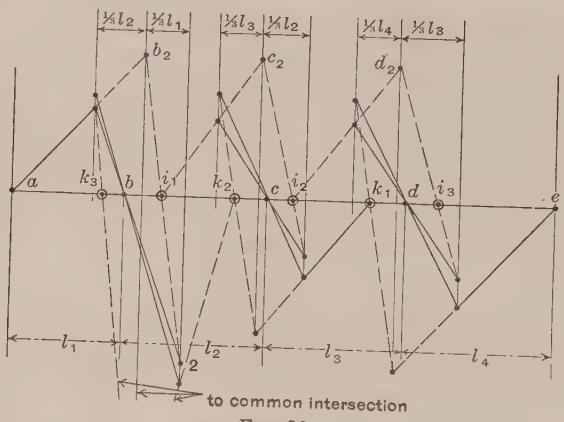

FIG. 86.

4.—Cross-lines and moment areas: The connecting lines between the points, b_2'' and c''', and of the points, c_2'' and b''' (Fig. 85 (d)) the intersection of which, Point 3, is on the center-of-gravity line of the simple moment areas of the loaded span, are called cross-lines. After the fixed points are established, calculate the area and the center of gravity of the simple moment areas in the loaded span. The area, multiplied by the distance of the center of gravity from the support perpendicular and divided by one-sixth of the square of the loaded span, gives the length, $b'''\ b_2''$ and $c'''\ c_2''$. These lines are drawn from b and c in Fig. 87 and give the cross-lines, bc''' and cb'''. They cross the fixed lines in i' and k' and the prolongation of $i'\ k'$ intersects the support perpendiculars at the points, b_1 and c_1. Connect these points, b_1 and c_1, with the fixed points of the other spans. This gives the total moment areas for the continuous beam.

For a single concentration or moving load, P, in the second span (Fig. 83), construct the simple beam moment diagram, $b\ S\ c$. The area of triangle, $b\ S\ c$, equals $\frac{h\ l_2}{2}$. Its center of gravity is $\frac{1}{3}\ (l_2 + x)$ from the support vertical,

$b\ b''$. The static moment of this area about b is $\frac{1}{6}\ l_2\ h\ (l_2 + x)$. Dividing this statical moment by $\frac{l_2^{\ 2}}{6}$, the value of the intercept, $b\ b''$, is found to be $\frac{h}{l_2}$ $(l_2 + x)$.

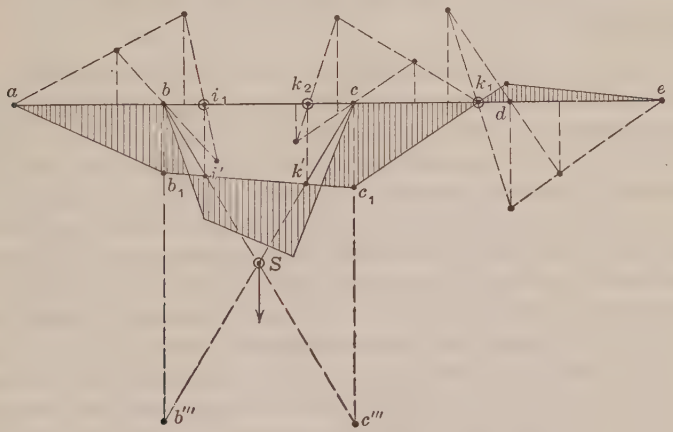

FIG. 87.

The value, $\frac{l_2^{\ 2}}{6}$, used to obtain the intercept, $b\ b''$, is found as follows: Call r the ratio of any moment area referred to that of a particular span considered as standard. Let H be an assumed pole distance for a funicular polygon wherein the moment areas are considered as loads. (Fig. 85 (*e*)).

Assume the following relation: $r\ H = \frac{l_2^{\ 2}}{6}$. The selection of this equation involves an important relation in the following construction in Fig. 85 (*c*). The intersections of the sides, 2-3 and 3-4, of the string polygon in the second or loaded span with the support perpendiculars produce intercepts which, measured to the scale of moments, equal the moments at the supports.

In Fig. 85 (*b*) the load, W_2, represents the area of the triangle, $b'\ b\ c'$, and $\frac{b'\ b}{2} \times \frac{l_2}{r}$ is equal to the area of $b'\ b\ c'$. The lever arm of the load, W_2, relative to the left support, is $\frac{l_2}{3}$. The static moment, therefore, $= \frac{b'\ b}{6}\ \frac{l_2^{\ 2}}{r} = M$.

Accordingly, by the theory of parallel forces, the statical moment, M, is equal to the product of the pole distance, H, and the intercept, $b_2''\ b''$ (Fig. 85 (*d*)),

$$M = H \times b_2''\ b''$$

Equating the two expressions and substituting $r\ H$ for $\frac{l_2^{\ 2}}{6}$, the following relation is obtained:

$$H \times b_2''\ b'' = \frac{b'\ b}{6}\ \frac{l_2^{\ 2}}{r}$$

or, $b_2''\ b''$ in Fig. 85 (*d*) is equal to $b'\ b$ in Fig. 85 (*b*) and, likewise, $c_2''\ c''$ in Fig. 85 (*d*) is equal to $c'\ c$ in Fig. 85 (*b*).

The intercept can be easily found graphically by laying off in Fig. 83 the distance, l_2, to the right of the point of concentration giving the point, *n*, and drawing the line in nSb''. By similar triangles is obtained the relation, $b\ b'' : h :: l_2 + x : l_2$. The remainder of the construction is found as previously described.

WALTER RUPPEL,* ASSOC. M. AM. SOC. C. E. (by letter).—The graphical method for the determination of the moments in restrained and continuous beams, presented by the authors, is indeed an excellent one. It is unfortunate, however, that under the impression that they had discovered something new, they engaged in the great amount of labor required to develop the method independently and succeeded only in doing what Fidler† published more than forty years ago and Ostenfeld‡ simplified more than ten years ago. With the ingenuity the authors have displayed, they could certainly have added a great deal more to the advancement of methods of design had they been aware of the earlier work and commenced where Fidler and Ostenfeld left off. The paper is nevertheless a valuable addition to engineering literature in that it collects in one article what has heretofore been available in a number of more or less inaccessible publications. The writer's attention was first called to the method by the paper§ by F. E. Richart, Assoc. M. Am. Soc. C. E., and W. M. Wilson, M. Am. Soc. C. E., and he has used it to good advantage since then, particularly in a form extended to the treatment of continuous beams of varying moment of inertia, in the design of parts of the Estuary Subway, Oakland, Calif. The writer will not dwell on the similarity of the authors' method with the earlier ones. That has been well covered by Professor Richart and Mr. Cotten in their discussions in which, in addition, they point out a number of geometric conceptions which clarify the method remarkably.

SOLUTION OF FRAMES

Trapezoidal Frame.—The authors touch lightly on the solution of frames in which there is a displacement of the joints such as would occur in the structure shown in Fig. 16 if subjected to a horizontal load. The "small secondary correction" of which they speak may be greater than the stresses before correction, if, for example, the horizontal load were acting alone at one of the upper joints. The only stresses then, neglecting dead load, are those due to the displacement of the joints and there is no limit to their possible magnitude. The authors refer the reader to Section 17‖, but there they fail to state in what manner this problem may be solved since the deflection of the joints is unknown. This problem puzzled the writer for some time and he has not been

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† *Minutes of Proceedings*, Inst. C. E., Vol. LXXIV, 1883, p. 196, and "A Practical Treatise on Bridge Construction," by T. Claxton Fidler.

‡ "Teknisk Statik," Vol. 2, Second Edition, by A. Ostenfeld, Copenhagen, Denmark.

§ *Engineering and Contracting*, June 23, 1920.

‖ See p. 48.

alone in the quandary. Possibly the authors will explain their method if it is simpler or different from the one suggested here.

If the members of the frame are of uniform moment of inertia the use of a formula as suggested by Mr. Cotten is of course the simplest method, but if such a formula is not available or if the members are of varying moment of inertia some other method must be used. The writer suggests the following solution for a trapezoidal frame the joints of which will be displaced due to either unsymmetrical loading, or unsymmetrical frame, or both.

Consider the frame, $A\ B\ C\ D$ (Fig. 88), subject to any loading due to the forces, $P_1, P_2, P_3 \ldots P_n$. Assume a reaction, R, at the joint, C, of known direction and of such magnitude (as yet unknown) as will prevent the displacement of the joints of the frame. The moments in the frame under this assumption may be determined by Fidler's method. The internal moments being known, the magnitude of the reaction, R, can be computed. Now, remove the loading from the frame and supply the force, R', acting in the opposite direction to that assumed for R in the first step and force the point, C, to deflect a unit distance in the direction of R'. The deflection of B and C, perpendicular to the various members, may be computed or determined graphically. The moments can now be found by Fidler's method as applied to the settlement of supports as explained by Mr. Cotten. The computer is warned to consider the signs of the deflections properly. Now, determine the magnitude of R' in the same manner as for R. It is evident that the true displacement of the point, C, is equal to $\frac{R}{R'}$ times the assumed deflection due to R and that the true moments in the frame are equal to those found in the first step plus $\frac{R}{R'}$ times those found in the second step; all with due regard to signs.

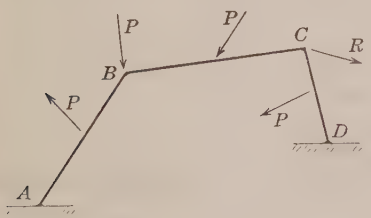


Fig. 88.

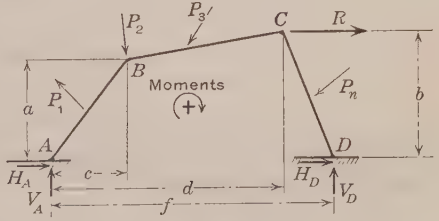


Fig. 89.

This process needs no proof; it should be practically self-evident. What has actually been done is to apply two equal and opposite loads at one of the upper panel points, applying first one with the actual loading on the frame and then the other acting alone, to solve the two cases independently and to add the stresses produced. The stresses due to the equal and opposite loads at the upper joint cancel, leaving the true stresses.

In order to avoid the labor of solving the equations necessary to determine R and R' in each problem, the writer has solved these equations in general

terms so that the reactions at the legs and R can be evaluated easily by substitution. It is to be noted that in the general case following, that A and D need not be on the same level. The horizontal and vertical components of the reactions at A and B can be determined by resolving the components, H_A, V_A, and H_B, V_B, into others of any desired direction either graphically or analytically. (Fig. 89.)

Assume R to be parallel to the line connecting A and D.

Let H = the component, parallel to $A\,D$, of all external loads on the frame.

V = the component perpendicular to $A\,D$, of all external loads on the frame.

M_B = the summation of the external and internal moments about and to the left of B, except those due to the reactions, H_A and V_A.

M_C = the summation of the external and internal moments about and to the left of C, except those due to the reactions, H_A and V_A.

M_D = the summation of the external and internal moments about and to the left of D, except those due to the reactions, H_A, V_A, and R.

The equations of equilibrium are:

$$H_A + H_D + H + R = 0$$
$$V_A + V_D + V = 0$$
$$M_B + V_A\,c - H_A\,a = 0$$
$$M_C + V_A\,d - H_A\,b = 0$$
$$M_D + V_A\,f + R\,b = 0$$

and the solution for the general case is:

$$V_A = \frac{M_C\,a - M_B\,b}{b\,c - a\,d}$$

$$V_D = -V_A - V$$

$$R = \frac{-V_A\,f - M_D}{b}$$

$$H_A = \frac{M_C\,c - M_B\,d}{b\,c - a\,d}$$

$$H_D = -H - H_A - R$$

When the frame is symmetrical, $a = b$ and $f - d = c$, in which case the foregoing values reduce to:

$$V_A = \frac{M_C - M_B}{c - d}$$

$$V_D = -V_A - V$$

$$R = \frac{-V_A\,f - M_D}{a}$$

$$H_A = \frac{M_C\,c - M_B\,d}{a\,(c - d)}$$

$$H_D = -H - H_A - R$$

When the frame is rectangular, $a = b$, $f = d$, and $c = 0$, in which case the values still further simplify to:

$$V_A = \frac{M_B - M_C}{d}$$

$$V_D = -V_A - V$$

$$R = \frac{M_C - M_B - M_D}{a}$$

$$H_A = \frac{M_B}{a}$$

$$H_D = -H - H_A - R$$

The Unsymmetrical Closed Frame.—The method outlined is also applicable to the unsymmetrical closed frame of four members under unsymmetrical loading. The solution is identical, except that Ostenfeld's auxiliary diagram must be solved by trial since no point on the moment closing line is known from which to start the construction. Let $A\,D$ be the fourth member of the frame (otherwise similar to Figs 88 and 89) and assume that the points, A and D, are not displaced. Actually they may move in space but it may be assumed that any pair of points do not move with respect to each other. The reactions, H_A, V_A, and H_D, V_D, may not actually exist. However, they must be supplied in the two solutions, but if the external loads balance, which they must do if there are no reactions and transposition is not to take place, their values in the two solutions will be equal and opposite.

The moment closing line is determined as follows: Make a cut at Joint D and lay the frame out as a continuous beam. Assume a moment at the one severed end and draw the moment closing line. It is known that the moments at the two severed ends must be equal. If they do not come out equal in the construction, the assumed value was in error. Adjust it and try again. The second or third trial should be successful. It is to be noted that only the P-points (see Fig. 92) must be found anew for each trial, the U, V, and T-points are fixed. In computing R and R', the moments, M_B, M_C, and M_D, are to be summed up to the severed end.

The Symmetrical Closed Frame, Symmetrically Loaded.—When the closed frame is symmetrical and symmetrically loaded as is the case for most box culverts not designed for moving loads, the solution is much simplified. It is known that the joints will not be displaced, and, therefore, this part of the solution is unnecessary. It is also known that the moment closing line is symmetrical so that only one-half the box need be considered in the solution. The determination of the moment closing line by trial is aided by the fact that it is parallel to the neutral axis in two of the members.

Haunched Beams

Introduction.—The authors have shown how symmetrical beams of varying moment of inertia may be treated by Fidler's method and Mr. Cotten in his discussion, has explained how both the symmetrical and the unsymmetrical case may be solved. In continuous systems, haunched beams have numerous

advantages over beams of uniform moment of inertia. These advantages are mainly economic and esthetic.

Effect of Haunches on Bending Moments.—In continuous beams of uniform moment of inertia, the cross-section is usually determined by the bending moment and the shear at the support. When no haunches are provided there is a considerable waste of material at the center. Realizing this, it has often been the practice to reduce the section at the center by haunching the beam. Usually, however, it has not been the custom to take into account the effect of the variation of moment of inertia on the distribution of the bending moments. By most methods it is difficult to determine the bending moments in beams of varying moment of inertia and it has not been realized generally that there are any great changes in the bending moments due to this variation.

On a restrained beam haunches decrease the center bending moment and increase the bending moment at the support as compared with a beam of constant cross-section. In some instances the difference may become quite large. Consider a beam under uniform load, having fixed ends, straight haunches one-fourth the length of the span, and a depth at the supports one and one-half times that of the uniform portion. The center moment is only 74% of that for a beam of uniform section, whereas the end moment is 13% greater. The writer has an actual case in mind, namely, that of a reinforced concrete retaining wall* about 30 ft. high, continuous at the base with a floor-slab and simply supported at the top. The effect of tapering the wall from 5 ft. at the base to 2 ft. at the top, was to increase the moment at the base by 25% over that for a wall of uniform thickness, but still effecting a considerable saving in the upper part of the wall. Had the base been completely fixed the increase in moment would have been almost 33 per cent. Although neither of these structures would have failed had the increased moment due to the haunch been disregarded, it is readily seen that the actual factor of safety may be considerably less than was intended and yet there is a waste of material at other points in the structure. The action of continuous haunched beams, the effect on the distribution of moments, and the savings in an approach trestle to the Pulaski Bridge over the Salmon River at Pulaski, N. Y., has been described† by E. H. Harder, Assoc. M. Am. Soc. C. E.

Advantages of Haunched Beams.—Haunches better accommodate the beam to its shearing stresses and in a concrete beam often eliminate the necessity of extremely heavy shear reinforcement. The added depth at the support also decreases the bond stresses, making the use of larger bars possible. The neutral axis of a haunched beam is actually curved and the induced thrust provides added safety. The ability to carry shear is also increased due to this inclination.‡ Another advantage of haunches lies in the reduction of the high stresses existing at sharp re-entrant angles as at the corners of a box culvert. Theoretically, it has been shown and tests have proven that extremely

* Outside substructure wall, Alameda Portal Building, Estuary Subway, Oakland, Calif.

† "Building a Concrete Arch Around an Old Iron Bridge," *Engineering News-Record*, Vol. 92, No. 8, February 21, 1924.

‡ See "Der Eisenbetonbau," by Prof. Dr. E. Mörsch, Fifth Edition, Vol. I, Pt. 2, p. 16. and also "Concrete Engineer's Handbook," by Hool and Johnson, p. 314.

high stresses occur at sharp re-entrant angles due to the abrupt curvature of the neutral axis at such points.*

Saving in Material.—The possible saving in material due to haunches varies considerably with the loading and the conditions of restraint. In the beam referred to previously with a depth of haunch one and one-half times that at the center, the saving in material is about 10 per cent. For this loading and condition of restraint the most economical depth of haunch is about twice the center depth for which the saving in material is about 14% over a uniform beam. There is an additional saving due to the reduction in the dead load of the beam itself and due to the concentration of the material in the beam nearer the supports.

Increased Costs Due to Haunches.—Against these savings must be balanced the increased cost of formwork, if the structural material is concrete, or the increased cost of fabrication, if it is steel. In the case of concrete the increase in form cost due to a straight haunch is small. In steel there is usually no economy until built-up members must be used.

Improved Methods of Design.—For reasons of economy, the advantages of haunched beams were earlier recognized in Europe than in the United States, mainly because of the difference in the relation of cost of labor to that of material. In Europe, principally in Germany, the increasing use of haunched beams led to the development of shorter methods for their design and also to the preparation of tables to expedite the computations. Such a set of tables has been prepared by A. Strassner.† These tables are directly applicable to Strassner's own method for designing continuous structures, a semi-analytical-graphical method of very general application, but probably not as simple as Fidler's method for conditions to which the latter is applicable. Strassner's tables give coefficients from which the angular change of the elastic curve at the supports of the simply supported beam may be determined, when subjected to the moment, 1, at the support or to certain classes of loading. When these tables were first brought to the writer's attention, he developed a number of simple formulas by which the values in Strassner's tables could be converted so as to be directly applicable to Fidler's method and to the slope deflection method,‡ when this method is extended to beams of variable moment of inertia. With the aid of Strassner's tables and these formulas the writer has now computed a new set of tables which give coefficients directly useful in simplifying Fidler's method. These tables (Tables 6 to 23, inclusive), and several others to be introduced later, are presented as a part of this discussion (see Appendix). In part, these new tables are similar to, but much more extensive than, the table presented by the authors (Table 2).

Strassner computed his tables for three types of haunch which have been referred to in the recomputed tables as Case I, II, and III. These three types,

* See "Analysis and Tests of Rigidly Connected Reinforced Concrete Beams," by Mikishi Abe, M. Am. Soc. C. E., *Bulletin 107*, Eng. Experiment Station, Univ. of Illinois, p. 99; "Elastizität und Festigkeit," by C. v. Bach; and "Drang und Zwang, Eine Höhere Festigkeitslehre für Ingenieure," by Dr.-Ing. Aug. Föppl and Dr. Ludwig Föppl, Vol. I, Second Edition, p. 248, Berlin, 1924.

† "Neuere Methoden," Vol. I, Second Edition, Berlin, 1921.

‡ *Bulletin 108*, Eng. Experiment Station, Univ. of Illinois.

together with a fourth, introduced later in this discussion, will either cover or closely approximate almost any shape that is likely to be found in practice.

Nomenclature.—In the nomenclature to be used as far as practicable the authors' terms have been followed. Others are as used by Mr. Cotten in his discussion. Some additional terms have been introduced by the writer.

Let,

I = minimum moment of inertia of the member.

I' = maximum moment of inertia of the member (at the support).

I_z = moment of inertia of the haunch at the distance, z, from the beginning of the haunch.

h_z = depth of haunch corresponding to I_z.

a = ratio of length of haunch to span of beam.

$b = \frac{I}{I'}$.

c = an exponent indicative of the degree of curvature of the haunch in Case IV.

E = modulus of elasticity.

W = any concentrated load, the total uniform load, or the total triangular load on the member.

l = span of member.

$u = \frac{x'}{l}$ = the distance from the left support to the left characteristic point when $l = 1$.

$v = 1 - \frac{x''}{l}$ = distance from the right support to the right characteristic point when $l = 1$.

$p = A' E I$ = area of the $\frac{M}{E I_z}$-diagram for $M = 1$ at the left support of the beam of the unit span having E and $I = 1$.

$q = A'' E I$ = area of the $\frac{M}{E I_z}$-diagram for $M = 1$ at the right support of the beam of the unit span having E and $I = 1$.

$s = \frac{U}{l W}$ = height of the left characteristic point when l and $W = 1$.

$t = \frac{V}{l W}$ = height of the right characteristic point when l and $W = 1$.

Shapes of Haunches.—

Case I.—Sharply Curved Haunch (Fig. 90).—

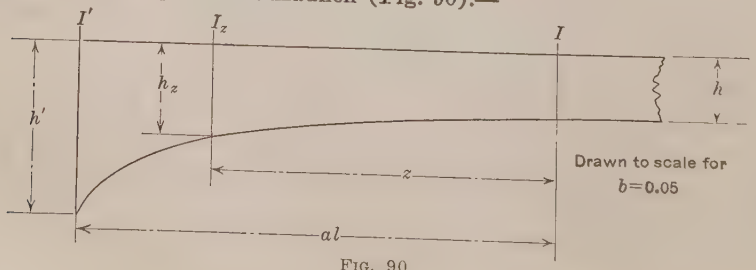


FIG. 90.

$$I_z = \frac{I}{1 - \frac{(1-b)\,z}{a\,l}}$$

$$h_z = \frac{h}{\sqrt[3]{1 - \frac{(1-b)\,z}{a\,l}}}$$

As shown in Fig. 90, this haunch curves at an increasingly rapid rate as the support is reached. It is not quite tangent to the uniform part of the beam, but in practice may be made so if desired. The preceding equations are given so that the shape may be reproduced or be fitted to a given haunch.

Case II.—Straight Haunch.—This type of haunch decreases uniformly from the support to a point toward the center of the span.

Case III.—Parabolic Haunch.—This is a parabola of the second degree; the vertex is at the beginning of the haunch and the curve is tangent to the uniform part of the beam at the vertex.

Case IV.—Sharply Curved Haunches of Varying Degrees of Sharpness (Fig. 91).—Professor Max Ritter,* one of the first to develop simpler methods for treating beams of variable moment of inertia, introduced a formula for the variation of the moment of inertia, which, when substituted in the $\frac{M}{E\,I}$-equation, gave functions that could be readily integrated. It is the general formula of which Case I is the special condition for the value of the exponent, c, equal to 1. (See Fig. 91.)

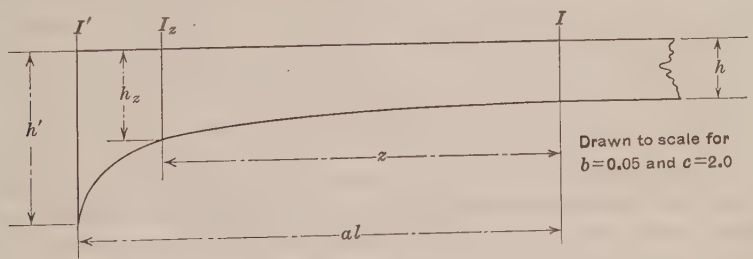


FIG. 91.

$$I_z = \frac{I}{1 - \frac{(1-b)\,z^c}{a\,l}}$$

$$h_z = \frac{h}{\sqrt[3]{1 - \frac{(1-b)\,z^c}{a\,l}}}$$

The exponent, c, which determines the degree of sharpness of the curvature of the haunch usually has a value between 0.5 and 5.0 in practical problems and by varying it between these limits, the haunch can be adapted to a great variety of shapes. As c increases, the haunch curves more sharply at the support.

* *Schweizerische Bauzeitung*, Vol. LIII, Nos. 18 and 19, 1914.

Use of the Tables.—In order to determine the moments in a continuous beam of varying or uniform moment of inertia, it has been shown by the authors and by Mr. Cotten in his discussion, that the position of several points and lines must be determined, namely, the height and lateral position of the U and V-points (Fidler's characteristic points) and the lateral position of the T-lines (one of the lines of Ostenfeld's auxiliary diagram).

For the four shapes of haunch (Cases I to IV), these points and lines may be located with the aid of Tables 6 to 32, inclusive. As shown by the diagrams at the head of the various tables, they are applicable to the symmetrical beam with haunches at both supports and also to the beam with a haunch at only one support.

The lateral position of the U and V-points is determined from one of Tables 6 to 11, inclusive, and Table 28, with the aid of the following relation:

Distance of the U-line from the left support $= u\ l$.
Distance of the V-line from the right support $= v\ l$.

As shown in Mr. Cotten's discussion, to determine the lateral position of the T-line it is necessary to know the area of the $\frac{M}{E\,I}$-diagram when the moment, 1, is applied successively at the left and right support. This is also found by using the coefficients given in Tables 6 to 11, inclusive, and Table 28, in the following formulas: The area of the $\frac{M}{E\,I}$-diagram for M equals 1 at the:

	Cotten.	Ruppel.
Left support	$= A'$	$= \frac{p\,l}{I\,E}$
Right support	$= A''$	$= \frac{q\,l}{I\,E}$

It will be noticed that these functions are independent of the loading on the beam and hence the coefficients, u, v, p, and q, obtained from the tables have been called "beam coefficients", being dependent only on the shape of the beam.

The height of the U and V-points is dependent on the loading as well as on the shape of the beam. Coefficients for a variety of loadings have been computed and are tabulated under the heading of "Load Coefficients" in Tables 12 to 27, inclusive, and Table 29. The heights of the U and V-points are obtained by substitution in the following formulas:

$$U = \Sigma\ (W\ s)\ l;\ \text{and}\ V = \Sigma\ (W\ t)\ l$$

The Σ-sign is used to indicate that the load must be divided into parts to which the tables for "load coefficients" are applicable.

Tables 12 to 17, inclusive, give the values of the "load coefficients", s and t, for a concentration at any twelfth point of the span. Tables 18 to 23, inclusive, and Table 29 give the values of the coefficients for a uniform load over the entire span. Tables 24 to 27, inclusive, give the coefficients for a triangular load over the whole span. Coefficients for other loadings, such as

uniform or triangular loads partially distributed, etc., may be found approximately within 1% by dividing the load into a number of concentrations and then using Tables 12 to 17, inclusive, or exactly as follows: Multiply the coefficients for a unit concentration at each twelfth point by the intensity of the actual load at that point; plot these products at their respective twelfth points and draw a curve through them, as shown in Fig. 93 or Fig. 94. The area under the curve times 12, divided by the product of the span times the load considered, is the coefficient for that load. The factor, 12, is introduced to take care of the horizontal scale of the diagram. The area under the curve may be obtained by planimeter or other method. Simpson's rule or any one of Cotes' formulas* will give the area rapidly and accurately.

For Case IV, "load coefficients" are given only for a uniform load over the whole span. The coefficients for other types of loading may be found with considerable accuracy by using the values given for Case I(a) and multiplying them by the ratio of the distances from the support to the characteristic point in Case IV to that for Case I(a). This relation is exactly true for a uniform load and approximately true for other loadings, departing more from the truth as the loading ceases to approximate a uniform one and as the values of b and c depart from unity. For $b = 0.03$ and $c = 5.0$, the error in the coefficient for a central concentration is 2.1% and will probably not exceed this figure for any practical beam and combination of loading. The approximate method is certainly exact enough for all preliminary designs and may be checked by an exact method when the final design is adopted if the loading is very abnormal.

When values for special loadings or beams are determined, it is suggested that they be computed for, or reduced to, coefficients for a unit load on the span of unity with I and E equal to unity. These coefficients will then be in the same form as those given in the writer's tables and may be tabulated for future use in other work.

The tabular interval is so chosen that intermediate values of the coefficients may be obtained by direct interpolation with sufficient accuracy for any practical problem. If greater accuracy is desired, a curve may be drawn through several values of the coefficient on either side of the desired one and the value read from the curve.

Tables 24 to 27, inclusive, of triangular load coefficients are new; their equivalent does not appear in Strassner's tables. Tables 28 and 29 were computed by the writer from formulas developed with the aid of the work of Max Ritter previously referred to.

Three other tables which the writer has found useful as an aid in the solution of continuous beams of uniform moment of inertia have been added. Table 30 gives the "load coefficients" for a partial uniform load in various positions on the span. This table was originally prepared by Mr. Cotten and has been published† in a slightly different form. It is here included in the revised form at the request of Mr. Cotten.

* See "Methods of Approximate Integration", by Willis Whited, M. Am. Soc. C. E., *Engineering News*, Vol. 73, No. 17, April 29, 1915.

† *Engineering and Contracting*, Vol. 43, No. 15, April 14, 1915.

Table 31 gives load coefficients for triangular loads, with a maximum intensity at the support and extending over various parts of the span. Table 32 gives load coefficients for trapezoidal loads and for uniform and triangular loads at the upper and lower limit. In addition are given coefficients for plotting the simple beam moment diagram for these loadings. Tables 31 and 32 are particularly useful as an aid in the application of Fidler's method to structures under hydraulic load or its equivalent.

Accuracy of the Tables.—Due to the limitations of Strassner's original tables and to the mathematical operations of the recomputations by the writer, the tabular values will be found to have slight errors occasionally in the last figure. In Tables 12 to 18, inclusive, this may amount to two or three units in the last place. In the other tables, the error will usually not exceed one unit in the last place and will seldom be that large. The tables have been carefully checked to detect errors in both the original tables and in the writer's computations. All values have been plotted in two or more curves and, for many, first differences have been plotted in addition. A large number of values were computed from first principles to check points that failed to plot in a smooth curve and also to detect constant errors which the curves might not reveal. It is believed and hoped that if any errors remain, they are small and inconsequential.

The preparation of the tables has involved a considerable amount of labor and the writer hopes that the work may be of benefit to other engineers. With the aid of these tables the solution of a haunched beam involves little if any additional work over that required for a beam of uniform moment of inertia. They should lead to the more general use of haunched beams.

Application of the Elastic Theory to Haunched Concrete Beams.—Some may doubt whether the ordinary elastic theory is applicable to reinforced concrete haunched beams in which the moment of inertia and modulus of elasticity are open to question. However, if continuous concrete beams of uniform moment of inertia can be proportioned according to this theory, it may also be used for beams of variable moment of inertia. Tests have shown that if the moment of inertia of a concrete beam be computed using the effective depth and if the steel be proportioned in accordance with the computed moments, the actual stresses will be closely in accord with the theoretical ones.* It is true that occasionally tests have shown wide deviations of the actual stresses from the theoretical ones, but most of the evidence seems to point to the fact that the ordinary elastic theory is applicable, within safe limits, to reinforced concrete structures when the steel is properly proportioned and care is taken to obtain a uniform concrete throughout the structure. The latter requires not only uniform proportions of cement and aggregate, but proper mixing, uniform water content, and uniformity of curing as well.

Illustrative Example.—It now remains to solve an actual problem to illustrate the use of the tables. For comparison, consider the beam shown by

* For further discussion of this subject and for the results of a large number of independent tests reference is made to, "Der Eisenbetonbau," by Prof. E. Mörsch, Third Edition, p. 210; "Neuere Methoden," by A. Strassner, Vol. I, Second Edition, p. 73, and "Analysis and Tests of Rigidly Connected Reinforced Concrete Frames," by Mikishi Abe, M. Am. Soc. C. E., *Bulletin 107*, Eng. Experiment Station, Univ. of Illinois.

the authors in Fig. 23, but add an unsymmetrical beam loaded as shown in Fig. 92, continuous at the left end and simply supported at the right end. This will completely illustrate the problem and the tables.

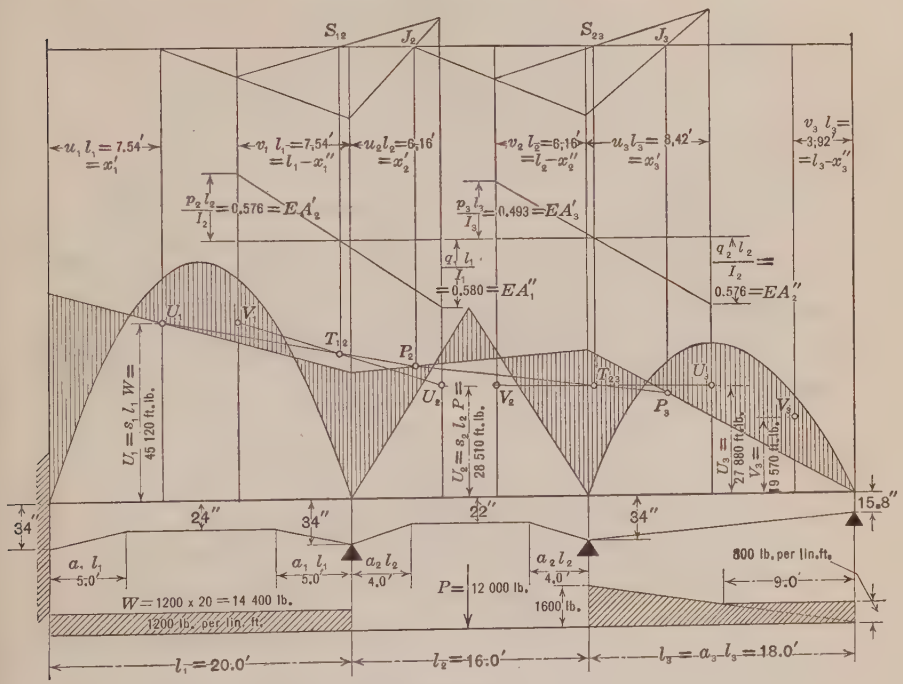


FIG. 92.

It is to be noted that the method used for determining the lateral position of the *T*-line, in Fig. 92, is the graphical solution of the equations given by Mr. Cotten in his solution of the general case. The computed values of *E A'* and *E A''* have been multiplied by 1 000 for use in this diagram.

Span 1.—

$$I_1' = 34^3 = 39\,304$$

$$I_1 = 24^3 = 13\,824$$

$$b_1 = \frac{I_1}{I_1'} = \frac{13\,824}{39\,304} = 0.3517$$

$$a_1 = \frac{5}{20} = 0.25$$

From Table 8:

$$u_1 = v_1 = 0.377 \quad \cdots \quad u_1 l_1 = v_1 l_1 = 0.377 \times 20.0 = 7.54 \text{ ft.}$$

$$p_1 = q_1 = 0.401 \quad \cdots \quad E A_1' = E A_1'' = \frac{p_1 l_1}{I_1} = \frac{0.401 \times 20.0}{13\,824} = 0.000580$$

From Table 20:

$$s_1 = t_1 = 0.0940 \quad \cdots \quad U_1 = V_1 = 0.0940 \times 20 \times 20 \times 1\,200 = 45\,120 \text{ ft-lb.}$$

Span 2.—

$$I_2' = 34^3 = 39\ 304$$
$$I_2 = 22^3 = 10\ 648$$
$$b_2 = \frac{I_2}{I_2'} = \frac{10\ 648}{39\ 304} = 0.2709$$
$$a_2 = \frac{4}{16} = 0.25$$

From Table 8:

$$u_2 = v_2 = 0.385 \therefore u_2\, l_2 = v_2\, l_2 = 0.385 \times 16.0 = 6.16 \text{ ft.}$$

$$p_2 = q_2 = 0.383 \therefore E\, A_2' = E\, A_2'' = \frac{p_2\, l_2}{I_2} = \frac{0.383 \times 16.0}{10\ 648} = 0.000576$$

From Table 14:

$$s_2 = t_2 = 0.1485 \therefore U_2 = V_2 = 0.1485 \times 16 \times 12\ 000 = 28\ 510 \text{ ft-lb.}$$

Span 3.—

$$I_3 = 15.8^3 = 3\ 944$$
$$I_3' = 34^3 = 39\ 304$$
$$b_3 = \frac{I_3}{I_3'} = \frac{3\ 944}{39\ 304} = 0.100$$
$$a_3 = \frac{18}{18} = 1.00$$

From Table 9:

$$u_3 = 0.468 \therefore u_3\, l_3 = 0.468 \times 18.0 = 8.42 \text{ ft.}$$
$$v_3 = 0.218 \therefore v_3\, l_3 = 0.218 \times 18.0 = 3.92 \text{ ft.}$$
$$p_3 = 0.108 \therefore E\, A_3' = \frac{p_3\, l_3}{I_3} = \frac{0.108 \times 18.0}{3\ 944} = 0.000493$$
$$q_3 = 0.232 \therefore E\, A_3'' = \frac{q_3\, l_3}{I_3} = \frac{0.232 \times 18.0}{3\ 944} = 0.001059$$

From Table 26, for the total triangular load:

$$s_3' = 0.0910 \therefore U_3' = 0.0910 \times 18 \times \frac{1\ 600 \times 18}{2} = 23\ 590 \text{ ft-lb.}$$

$$t_3' = 0.0583 \therefore V_3' = 0.0583 \times 18 \times \frac{1\ 600 \times 18}{2} = 15\ 110 \text{ ft-lb.}$$

To find the values of U_3'' and V_3'' for the partial triangular load it will be necessary to compute the coefficient with the aid of Table 15 according to the method previously outlined on pages 160 and 161. For convenience in computing the coefficient assume a span of 12 and a load of 18. Then to determine s_3'' by the exact method (Fig. 93):

$$\text{Area under the curve} = 1.192 \text{ ; } \therefore s_3'' = \frac{1.192}{18} = 0.0662$$

By the approximate method:

$$s_3'' = \frac{0.1367 \times 0.5 \times 0.25 + 0.1335 + 0.1213 \times 2 + 0.1007 \times 3 + 0.0726 \times 4}{18}$$

$$+ \frac{0.0384 \times 5 + 0.0096 \times 0.5 \times 5.75}{18} = 0.0669, \text{ about } 1\% \text{ too large.}$$

$$U_3'' = 0.0662 \times 18 \times \frac{800 \times 9}{2} = 4\,290 \text{ ft-lb.}$$

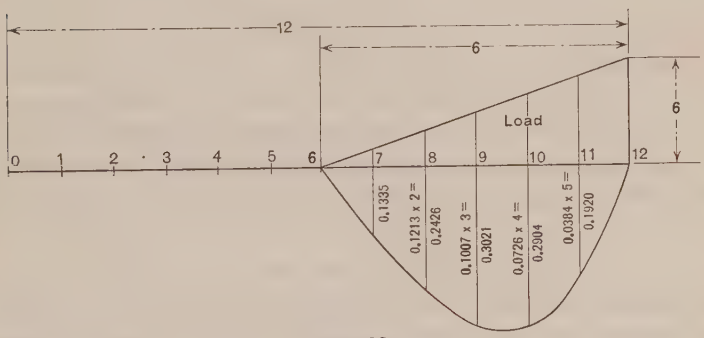


Fig. 93.

To determine t''_3, by the exact method (Fig. 94):

$$\text{Area under the curve} = 1.239 \quad \therefore \quad t_3'' = \frac{1.239}{18} = 0.0688.$$

By the approximate method:

$$t_3'' = \frac{0.0965 \times 0.5 \times 0.25 + 0.1022 + 0.1042 \times 2 + 0.0990 \times 3 + 0.0829 \times 4}{18}$$

$$+ \frac{0.0520 \times 5 + 0.0130 \times 0.5 \times 5.75}{18} = 0.0694, \text{ about } 1\% \text{ too large.}$$

$$V_3'' = 0.0688 \times 18 \times \frac{800 \times 9}{2} = 4\,460 \text{ ft-lb.}$$

$$U_3 = U_3' + U_3'' = 23\,590 + 4\,290 = 27\,880 \text{ ft-lb.}$$
$$V_3 = V_3' + V_3'' = 15\,110 + 4\,460 = 19\,570 \text{ ft-lb.}$$

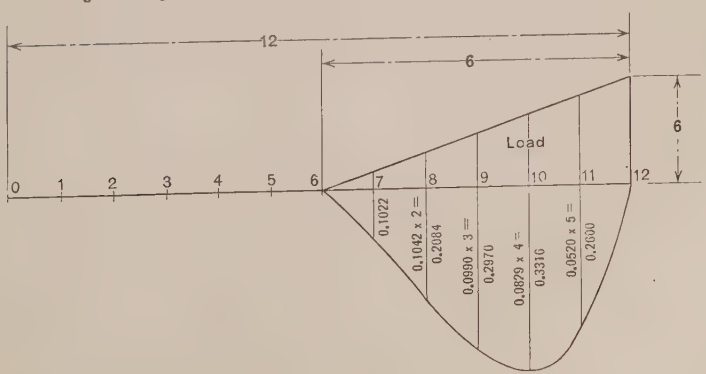


Fig. 94.

All these values are indicated in their proper positions in Fig. 92 and the moment closing line drawn as explained by the authors in their paper and by Mr. Cotten in his discussion.

Conclusion

The writer has endeavored to present in this discussion:

1.—A direct solution of trapezoidal frames with deflecting joints, by Fidler's method.

2.—A solution by trial for closed quadrangular frames, with or without deflecting joints.

3.—The effects and advantages of haunches on continuous beams.

4.—A set of tables and an explanation of their use to facilitate the solution of haunched beams and beams of uniform moment of inertia.

He hopes he has not failed in attempting to present the material in such a manner that it may be readily understood and used to advantage by the profession.

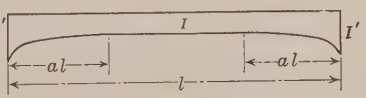
Acknowledgments

The use the writer has made of Strassner's tables in the preparation of his own, has already been referred to. Without those tables, the labor would have been many times as great as it has been and the computation of such an extensive set of tables would probably not have been undertaken. The writer wishes to acknowledge the valuable assistance he has received from Mr. Cotten and Herman Schorer, Assoc. M. Am. Soc. C. E., to the former for suggestions and assistance in checking the text of the discussion, and to the latter for suggestions and for bringing to the writer's attention and making available many of the foreign publications.

APPENDIX

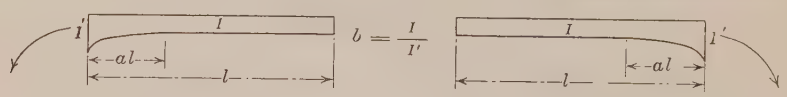
TABLE 6.—Beam Coefficients for Beams of Varying Moment of Inertia. Case I(*a*).—Symmetrical, Sharply Curved Haunches.

$b = \frac{I}{I'}$



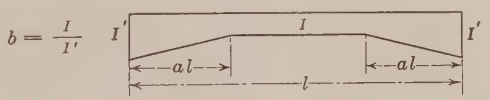
Values of *a*.		Values of *b*.													
		1.00	0.60	0.30	0.20	0.15	0.12	0.10	0.08	0.06	0.05	0.04	0.03	0.02	
0.50	*u*	0.333	0.354	0.378	0.389	0.395	0.399	0.402	0.404	0.407	0.409	0.410	0.412	0.413	*v*
	p	0.500	0.400	0.325	0.300	0.288	0.280	0.275	0.270	0.265	0.263	0.260	0.257	0.255	*q*
0.40	*u*	0.333	0.356	0.380	0.390	0.395	0.398	0.401	0.403	0.406	0.407	0.408	0.410	0.411	*v*
	p	0.500	0.420	0.360	0.340	0.330	0.324	0.320	0.316	0.312	0.310	0.308	0.306	0.304	*q*
0.35	*u*	0.333	0.356	0.379	0.388	0.393	0.396	0.398	0.400	0.403	0.404	0.405	0.406	0.407	*v*
	p	0.500	0.430	0.378	0.360	0.351	0.346	0.342	0.339	0.335	0.334	0.332	0.330	0.328	*q*
0.30	*u*	0.333	0.356	0.377	0.385	0.389	0.392	0.394	0.396	0.397	0.399	0.400	0.400	0.401	*v*
	p	0.500	0.440	0.395	0.380	0.373	0.368	0.365	0.362	0.359	0.358	0.356	0.355	0.353	*q*
0.25	*u*	0.333	0.354	0.373	0.380	0.384	0.386	0.388	0.389	0.391	0.392	0.392	0.393	0.394	*v*
	p	0.500	0.450	0.413	0.400	0.394	0.390	0.388	0.385	0.382	0.381	0.380	0.379	0.378	*q*
0.20	*u*	0.333	0.352	0.368	0.374	0.377	0.379	0.380	0.382	0.383	0.384	0.384	0.385	0.385	*v*
	p	0.500	0.460	0.430	0.420	0.415	0.412	0.410	0.408	0.406	0.405	0.404	0.403	0.402	*q*
0.15	*u*	0.333	0.349	0.362	0.366	0.369	0.370	0.371	0.372	0.373	0.373	0.374	0.374	0.375	*v*
	p	0.500	0.470	0.448	0.440	0.436	0.434	0.432	0.431	0.430	0.429	0.428	0.427	0.426	*q*

TABLE 7.—Beam Coefficients for Beams of Varying Moment of Inertia. Case I(*b*).—Unsymmetrical, Sharply Curved Haunch.



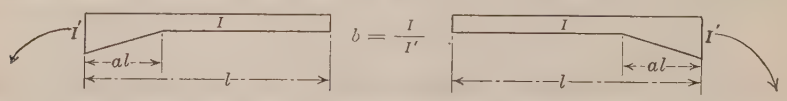
Values of *a*.	Haunch at left support.	Values of *b*. 1.00	0.60	0.30	0.20	0.15	0.12	0.10	0.08	0.06	0.05	0.04	0.03	0.02	Haunch at right support.
1.00	*u*	0.333	0.364	0.406	0.429	0.442	0.452	0.458	0.466	0.473	0.477	0.481	0.485	0.490	*v*
	v	0.333	0.308	0.283	0.273	0.267	0.264	0.262	0.260	0.257	0.256	0.255	0.254	0.252	*u*
	p	0.500	0.367	0.267	0.233	0.217	0.207	0.200	0.193	0.187	0.184	0.180	0.177	0.173	*q*
	q	0.500	0.433	0.383	0.367	0.358	0.353	0.350	0.347	0.343	0.342	0.340	0.338	0.337	*p*
0.50	*u*	0.333	0.370	0.409	0.425	0.434	0.440	0.443	0.448	0.451	0.454	0.456	0.457	0.460	*v*
	v	0.333	0.319	0.308	0.304	0.302	0.300	0.300	0.299	0.298	0.298	0.297	0.297	0.296	*u*
	p	0.500	0.416	0.354	0.333	0.323	0.316	0.312	0.308	0.304	0.302	0.300	0.298	0.296	*q*
	q	0.500	0.484	0.471	0.467	0.461	0.463	0.462	0.462	0.461	0.460	0.460	0.460	0.459	*p*
0.40	*u*	0.333	0.367	0.401	0.414	0.421	0.425	0.429	0.432	0.435	0.436	0.438	0.440	0.442	*v*
	v	0.333	0.323	0.315	0.313	0.311	0.310	0.310	0.309	0.309	0.308	0.308	0.308	0.308	*u*
	p	0.500	0.431	0.379	0.361	0.352	0.348	0.344	0.340	0.337	0.335	0.334	0.332	0.330	*q*
	q	0.500	0.490	0.481	0.479	0.477	0.476	0.476	0.475	0.475	0.475	0.474	0.474	0.474	*p*
0.35	*u*	0.333	0.365	0.395	0.407	0.413	0.417	0.420	0.422	0.425	0.427	0.428	0.430	0.431	*v*
	v	0.333	0.325	0.319	0.317	0.316	0.315	0.315	0.314	0.314	0.313	0.313	0.313	0.313	*u*
	p	0.500	0.438	0.392	0.376	0.369	0.364	0.361	0.358	0.355	0.353	0.352	0.350	0.348	*q*
	q	0.500	0.492	0.486	0.484	0.483	0.482	0.482	0.481	0.481	0.481	0.480	0.480	0.480	*p*
0.30	*u*	0.333	0.362	0.389	0.399	0.405	0.408	0.410	0.412	0.415	0.416	0.417	0.418	0.419	*v*
	v	0.333	0.327	0.322	0.321	0.320	0.319	0.319	0.319	0.318	0.318	0.318	0.318	0.318	*u*
	p	0.500	0.446	0.405	0.392	0.385	0.381	0.378	0.376	0.373	0.372	0.370	0.369	0.368	*q*
	q	0.500	0.494	0.489	0.488	0.487	0.487	0.486	0.486	0.486	0.486	0.486	0.485	0.485	*p*
0.25	*u*	0.333	0.359	0.382	0.390	0.395	0.397	0.399	0.401	0.403	0.404	0.405	0.406	0.407	*v*
	v	0.333	0.329	0.325	0.324	0.324	0.323	0.323	0.323	0.323	0.322	0.322	0.322	0.322	*u*
	p	0.500	0.454	0.420	0.408	0.403	0.399	0.397	0.395	0.392	0.391	0.390	0.389	0.388	*q*
	q	0.500	0.496	0.493	0.492	0.491	0.491	0.490	0.490	0.490	0.490	0.490	0.490	0.490	*p*
0.20	*u*	0.333	0.355	0.374	0.381	0.384	0.386	0.388	0.389	0.390	0.391	0.392	0.393	0.394	*v*
	v	0.333	0.330	0.328	0.327	0.327	0.327	0.327	0.326	0.326	0.326	0.326	0.326	0.326	*u*
	p	0.500	0.463	0.435	0.425	0.420	0.418	0.416	0.414	0.412	0.411	0.410	0.410	0.409	*q*
	q	0.500	0.497	0.496	0.494	0.494	0.494	0.491	0.494	0.494	0.494	0.494	0.494	0.494	*p*
0.15	*u*	0.333	0.351	0.365	0.370	0.373	0.374	0.375	0.376	0.377	0.378	0.378	0.379	0.379	*v*
	v	0.333	0.332	0.330	0.330	0.330	0.330	0.329	0.329	0.329	0.329	0.329	0.329	0.329	*u*
	p	0.500	0.472	0.450	0.443	0.439	0.437	0.436	0.434	0.433	0.432	0.432	0.431	0.430	*q*
	q	0.500	0.499	0.497	0.497	0.497	0.497	0.496	0.496	0.496	0.496	0.496	0.496	0.496	*p*

TABLE 8.—Beam Coefficients for Beams of Varying Moment of Inertia. Case II(*a*).—Symmetrical, Straight Haunches.



Values of *a*.		Values of *b*.													
		1.00	0.60	0.30	0.20	0.15	0.12	0.10	0.08	0.06	0.05	0.04	0.03	0.02	
0.50	*u*	0.333	0.354	0.380	0.394	0.403	0.410	0.416	0.422	0.430	0.434	0.439	0.446	0.455	*v*
	p	0.500	0.389	0.279	0.232	0.203	0.184	0.170	0.154	0.136	0.126	0.115	0.102	0.086	*q*
0.40	*u*	0.333	0.358	0.387	0.402	0.412	0.420	0.425	0.431	0.440	0.444	0.449	0.455	0.462	*v*
	p	0.500	0.411	0.324	0.286	0.263	0.248	0.236	0.223	0.209	0.201	0.192	0.182	0.169	*q*
0.35	*u*	0.333	0.358	0.387	0.402	0.412	0.419	0.424	0.430	0.437	0.441	0.446	0.450	0.457	*v*
	p	0.500	0.422	0.346	0.312	0.292	0.279	0.269	0.258	0.245	0.238	0.230	0.221	0.210	*q*
0.30	*u*	0.333	0.357	0.385	0.400	0.408	0.415	0.419	0.425	0.431	0.434	0.438	0.443	0.448	*v*
	p	0.500	0.433	0.368	0.339	0.322	0.310	0.302	0.292	0.282	0.276	0.269	0.261	0.252	*q*
0.25	*u*	0.333	0.356	0.382	0.394	0.402	0.407	0.412	0.416	0.422	0.425	0.428	0.432	0.436	*v*
	p	0.500	0.444	0.390	0.366	0.352	0.342	0.335	0.327	0.318	0.313	0.307	0.301	0.293	*q*
0.20	*u*	0.333	0.354	0.376	0.387	0.393	0.398	0.401	0.405	0.409	0.412	0.415	0.418	0.422	*v*
	p	0.500	0.455	0.412	0.392	0.381	0.374	0.368	0.362	0.354	0.350	0.346	0.341	0.334	*q*
0.15	*u*	0.333	0.350	0.368	0.377	0.382	0.385	0.388	0.391	0.394	0.396	0.398	0.401	0.404	*v*
	p	0.500	0.466	0.434	0.420	0.411	0.405	0.401	0.396	0.391	0.388	0.384	0.380	0.376	*q*

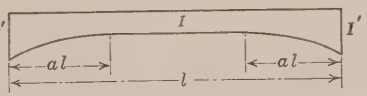
TABLE 9.—Beam Coefficients for Beams of Varying Moment of Inertia. Case II(*b*).—Unsymmetrical, Straight Haunch.



Values of *a*.	Haunch at left support.	Values of *b*. 1.00	0.60	0.30	0.20	0.15	0.12	0.10	0.08	0.06	0.05	0.04	0.03	0.02	Haunch at right support.
1.00	*u*	0.333	0.362	0.403	0.427	0.443	0.458	0.468	0.482	0.500	0.510	0.523	0.540	0.566	*v*
	v	0.333	0.306	0.269	0.250	0.236	0.226	0.218	0.207	0.196	0.188	0.179	0.168	0.153	*u*
	p	0.500	0.356	0.224	0.171	0.141	0.122	0.108	0.093	0.077	0.068	0.059	0.048	0.037	*q*
	q	0.500	0.422	0.335	0.292	0.266	0.247	0.232	0.215	0.196	0.184	0.171	0.156	0.136	*p*
0.50	*u*	0.333	0.373	0.425	0.455	0.475	0.490	0.501	0.515	0.531	0.540	0.551	0.564	0.580	*v*
	v	0.333	0.317	0.297	0.288	0.282	0.277	0.274	0.270	0.266	0.263	0.260	0.256	0.252	*u*
	p	0.500	0.408	0.321	0.284	0.262	0.248	0.237	0.225	0.212	0.205	0.197	0.188	0.177	*q*
	q	0.500	0.480	0.459	0.448	0.441	0.437	0.433	0.429	0.424	0.421	0.418	0.414	0.409	*p*
0.40	*u*	0.333	0.370	0.417	0.441	0.458	0.470	0.479	0.489	0.502	0.509	0.517	0.527	0.538	*v*
	v	0.333	0.321	0.308	0.301	0.297	0.294	0.292	0.289	0.286	0.284	0.282	0.280	0.277	*u*
	p	0.500	0.423	0.350	0.319	0.300	0.288	0.279	0.269	0.258	0.251	0.244	0.237	0.227	*q*
	q	0.500	0.487	0.474	0.467	0.462	0.460	0.457	0.454	0.451	0.450	0.447	0.445	0.442	*p*
0.35	*u*	0.333	0.368	0.410	0.433	0.447	0.456	0.465	0.474	0.485	0.491	0.498	0.505	0.515	*v*
	v	0.333	0.324	0.313	0.308	0.305	0.302	0.300	0.298	0.296	0.295	0.293	0.291	0.288	*u*
	p	0.500	0.432	0.366	0.338	0.321	0.310	0.302	0.293	0.283	0.277	0.271	0.264	0.255	*q*
	q	0.500	0.490	0.480	0.474	0.471	0.469	0.467	0.465	0.463	0.461	0.460	0.458	0.455	*p*
0.30	*u*	0.333	0.365	0.403	0.422	0.435	0.443	0.450	0.457	0.466	0.471	0.477	0.483	0.492	*v*
	v	0.333	0.326	0.318	0.314	0.312	0.310	0.308	0.307	0.305	0.304	0.303	0.301	0.299	*u*
	p	0.500	0.440	0.382	0.358	0.343	0.333	0.326	0.318	0.309	0.304	0.299	0.292	0.284	*q*
	q	0.500	0.493	0.485	0.481	0.479	0.477	0.476	0.474	0.473	0.472	0.470	0.469	0.467	*p*
0.25	*u*	0.333	0.362	0.394	0.410	0.420	0.427	0.433	0.439	0.446	0.450	0.455	0.460	0.466	*v*
	v	0.333	0.328	0.322	0.319	0.318	0.316	0.315	0.314	0.312	0.312	0.311	0.310	0.309	*u*
	p	0.500	0.449	0.400	0.379	0.366	0.358	0.352	0.345	0.337	0.333	0.328	0.322	0.316	*q*
	q	0.500	0.495	0.490	0.487	0.485	0.484	0.483	0.482	0.481	0.480	0.480	0.478	0.477	*p*
0.20	*u*	0.333	0.357	0.384	0.397	0.405	0.411	0.415	0.420	0.425	0.428	0.432	0.436	0.441	*v*
	v	0.333	0.330	0.326	0.324	0.323	0.322	0.321	0.321	0.319	0.319	0.318	0.318	0.317	*u*
	p	0.500	0.458	0.418	0.401	0.391	0.384	0.379	0.373	0.366	0.363	0.359	0.354	0.349	*q*
	q	0.500	0.497	0.494	0.492	0.491	0.490	0.489	0.489	0.488	0.488	0.487	0.486	0.486	*p*
0.15	*u*	0.333	0.352	0.373	0.383	0.389	0.393	0.396	0.399	0.403	0.406	0.408	0.411	0.414	*v*
	v	0.333	0.331	0.329	0.328	0.327	0.326	0.326	0.326	0.325	0.325	0.325	0.324	0.324	*u*
	p	0.500	0.468	0.438	0.424	0.416	0.411	0.407	0.403	0.398	0.395	0.392	0.388	0.384	*q*
	q	0.500	0.498	0.496	0.495	0.495	0.494	0.494	0.494	0.493	0.493	0.493	0.492	0.492	*p*

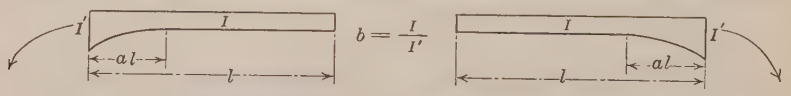
TABLE 10.—Beam Coefficients for Beams of Varying Moment of Inertia. Case III(a).—Symmetrical, Parabolic Haunches.

$b = \frac{I}{I'}$



Values of a.		Values of b.													
		1.00	0.60	0.30	0.20	0.15	0.12	0.10	0.08	0.06	0.05	0.04	0.03	0.02	
0.50	u	0.333	0.355	0.382	0.395	0.405	0.411	0.416	0.422	0.429	0.433	0.438	0.444	0.451	v
	p	0.500	0.424	0.345	0.308	0.286	0.270	0.257	0.243	0.227	0.218	0.206	0.194	0.177	q
0.40	u	0.333	0.355	0.380	0.393	0.402	0.408	0.413	0.418	0.425	0.428	0.433	0.438	0.444	v
	p	0.500	0.439	0.376	0.347	0.328	0.316	0.306	0.295	0.282	0.274	0.266	0.255	0.242	q
0.35	u	0.333	0.354	0.378	0.390	0.399	0.404	0.408	0.413	0.419	0.423	0.426	0.431	0.438	v
	p	0.500	0.447	0.392	0.366	0.350	0.338	0.330	0.320	0.309	0.302	0.295	0.286	0.274	q
0.30	u	0.333	0.353	0.375	0.386	0.393	0.398	0.403	0.407	0.412	0.415	0.419	0.423	0.428	v
	p	0.500	0.455	0.407	0.385	0.371	0.362	0.354	0.346	0.336	0.331	0.324	0.316	0.306	q
0.25	u	0.333	0.351	0.371	0.381	0.387	0.392	0.395	0.398	0.403	0.406	0.409	0.413	0.417	v
	p	0.500	0.462	0.422	0.404	0.393	0.385	0.378	0.372	0.364	0.359	0.353	0.347	0.339	q
0.20	u	0.333	0.348	0.366	0.374	0.379	0.383	0.385	0.389	0.393	0.395	0.397	0.400	0.404	v
	p	0.500	0.470	0.438	0.423	0.414	0.408	0.403	0.398	0.391	0.387	0.383	0.378	0.371	q
0.15	u	0.333	0.346	0.359	0.366	0.370	0.373	0.375	0.377	0.380	0.382	0.384	0.386	0.390	v
	p	0.500	0.477	0.454	0.442	0.436	0.431	0.427	0.423	0.418	0.415	0.412	0.408	0.403	q

TABLE 11.—Beam Coefficients for Beams of Varying Moment of Inertia. Case III(*b*).—Unsymmetrical Parabolic Haunch.

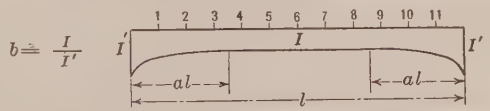


Values of *a*.	Haunch at left support.	Values of *b*. 1.00	0.60	0.30	0.20	0.15	0.12	0.10	0.08	0.06	0.05	0.04	0.03	0.02	Haunch at right support.
1.00	*u*	0.333	0.368	0.416	0.444	0.464	0.479	0.492	0.507	0.526	0.538	0.552	0.571	0.598	*v*
	v	0.333	0.311	0.283	0.267	0.257	0.249	0.242	0.235	0.225	0.219	0.212	0.204	0.192	*u*
	p	0.500	0.389	0.279	0.232	0.203	0.184	0.170	0.154	0.136	0.126	0.115	0.102	0.086	*q*
	q	0.500	0.459	0.411	0.385	0.367	0.355	0.344	0.333	0.318	0.309	0.298	0.286	0.268	*p*
0.50	*u*	0.333	0.366	0.406	0.428	0.443	0.454	0.462	0.472	0.484	0.492	0.500	0.511	0.524	*v*
	v	0.333	0.324	0.312	0.306	0.302	0.299	0.297	0.294	0.291	0.289	0.286	0.283	0.279	*u*
	p	0.500	0.434	0.367	0.337	0.318	0.306	0.296	0.285	0.273	0.266	0.257	0.247	0.235	*q*
	q	0.500	0.490	0.478	0.471	0.467	0.464	0.461	0.458	0.454	0.452	0.450	0.446	0.442	*p*
0.40	*u*	0.333	0.362	0.397	0.415	0.427	0.436	0.443	0.451	0.461	0.466	0.473	0.481	0.492	*v*
	v	0.333	0.327	0.319	0.315	0.312	0.310	0.308	0.306	0.304	0.303	0.301	0.299	0.296	*u*
	p	0.500	0.446	0.390	0.365	0.350	0.339	0.331	0.322	0.311	0.305	0.297	0.289	0.279	*q*
	q	0.500	0.494	0.486	0.482	0.479	0.477	0.475	0.473	0.471	0.469	0.468	0.466	0.463	*p*
0.35	*u*	0.333	0.360	0.391	0.408	0.418	0.426	0.432	0.439	0.447	0.452	0.458	0.465	0.474	*v*
	v	0.333	0.328	0.322	0.319	0.316	0.315	0.313	0.312	0.310	0.309	0.308	0.306	0.304	*u*
	p	0.500	0.452	0.402	0.380	0.366	0.356	0.349	0.341	0.332	0.326	0.319	0.312	0.303	*q*
	q	0.500	0.495	0.489	0.486	0.484	0.482	0.481	0.480	0.478	0.476	0.475	0.474	0.472	*p*
0.30	*u*	0.333	0.357	0.385	0.399	0.408	0.415	0.420	0.426	0.433	0.437	0.442	0.448	0.455	*v*
	v	0.333	0.329	0.325	0.322	0.320	0.319	0.318	0.317	0.316	0.315	0.314	0.313	0.311	*u*
	p	0.500	0.458	0.415	0.395	0.383	0.375	0.368	0.361	0.353	0.348	0.342	0.336	0.327	*q*
	q	0.500	0.496	0.492	0.490	0.488	0.487	0.486	0.485	0.484	0.483	0.482	0.480	0.479	*p*
0.25	*u*	0.333	0.354	0.378	0.390	0.397	0.403	0.407	0.412	0.418	0.421	0.425	0.430	0.436	*v*
	v	0.333	0.330	0.327	0.325	0.324	0.323	0.322	0.322	0.321	0.320	0.319	0.318	0.317	*u*
	p	0.500	0.464	0.428	0.411	0.401	0.394	0.388	0.382	0.375	0.371	0.366	0.360	0.353	*q*
	q	0.500	0.497	0.494	0.493	0.492	0.491	0.490	0.490	0.489	0.488	0.487	0.487	0.486	*p*
0.20	*u*	0.333	0.351	0.370	0.380	0.386	0.390	0.394	0.398	0.402	0.405	0.408	0.412	0.417	*v*
	v	0.333	0.331	0.329	0.328	0.327	0.327	0.326	0.326	0.325	0.325	0.324	0.324	0.323	*u*
	p	0.500	0.471	0.441	0.428	0.420	0.414	0.409	0.404	0.398	0.395	0.391	0.386	0.380	*q*
	q	0.500	0.498	0.496	0.496	0.495	0.494	0.494	0.493	0.493	0.492	0.492	0.491	0.491	*p*
0.15	*u*	0.333	0.347	0.362	0.369	0.374	0.377	0.379	0.382	0.386	0.388	0.390	0.393	0.396	*v*
	v	0.333	0.332	0.331	0.330	0.330	0.330	0.329	0.329	0.328	0.328	0.328	0.327	0.327	*u*
	p	0.500	0.478	0.456	0.445	0.439	0.434	0.431	0.427	0.422	0.420	0.416	0.413	0.408	*q*
	q	0.500	0.499	0.498	0.497	0.497	0.497	0.496	0.496	0.496	0.496	0.496	0.495	0.495	*p*

TABLE 12.—Load Coefficients for Beams of Varying Moment of Inertia.

Concentrated Unit Load.

Case I(*a*).—Symmetrical, Sharply Curved Haunches.

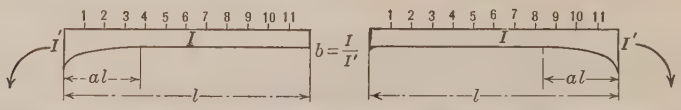


Values of:		Values of *s*, Read Down. Concentrated unit load at Point:										
a	*b*	1	2	3	4	5	6	7	8	9	10	11
....	1.00	0.0488	0.0849	0.1094	0.1235	0.1283	0.1250	0.1148	0.0988	0.0781	0.0540	0.0276
0.50	0.20	0.0482	0.0894	0.1217	0.1435	0.1540	0.1528	0.1408	0.1202	0.0937	0.0638	0.0323
	0.10	0.0478	0.0904	0.1241	0.1478	0.1598	0.1590	0.1466	0.1249	0.0972	0.0663	0.0333
	0.05	0.0478	0.0911	0.1258	0.1504	0.1630	0.1626	0.1495	0.1275	0.0993	0.0674	0.0339
	0.03	0.0478	0.0914	0.1265	0.1516	0.1643	0.1643	0.1512	0.1289	0.1001	0.0680	0.0342
0.40	0.20	0.0480	0.0901	0.1224	0.1443	0.1540	0.1525	0.1408	0.1201	0.0940	0.0643	0.0324
	0.10	0.0481	0.0910	0.1247	0.1484	0.1592	0.1580	0.1456	0.1243	0.0970	0.0662	0.0333
	0.05	0.0480	0.0915	0.1261	0.1505	0.1619	0.1607	0.1485	0.1265	0.0989	0.0671	0.0338
	0.03	0.0479	0.0918	0.1270	0.1516	0.1630	0.1622	0.1495	0.1278	0.0995	0.0676	0.0340
0.35	0.20	0.0485	0.0904	0.1226	0.1439	0.1529	0.1509	0.1393	0.1195	0.0935	0.0640	0.0322
	0.10	0.0486	0.0912	0.1250	0.1477	0.1577	0.1557	0.1438	0.1230	0.0964	0.0657	0.0331
	0.05	0.0484	0.0920	0.1263	0.1497	0.1598	0.1582	0.1461	0.1251	0.0981	0.0666	0.0335
	0.03	0.0483	0.0925	0.1270	0.1505	0.1611	0.1594	0.1473	0.1262	0.0986	0.0669	0.0337
0.30	0.20	0.0489	0.0909	0.1228	0.1433	0.1513	0.1486	0.1371	0.1178	0.0928	0.0632	0.0320
	0.10	0.0488	0.0917	0.1252	0.1464	0.1551	0.1527	0.1409	0.1212	0.0949	0.0649	0.0327
	0.05	0.0486	0.0925	0.1264	0.1483	0.1570	0.1551	0.1431	0.1228	0.0965	0.0658	0.0331
	0.03	0.0488	0.0925	0.1269	0.1489	0.1582	0.1558	0.1437	0.1237	0.0969	0.0661	0.0332
0.25	0.20	0.0491	0.0913	0.1225	0.1415	0.1487	0.1460	0.1346	0.1156	0.0913	0.0624	0.0316
	0.10	0.0493	0.0923	0.1245	0.1443	0.1516	0.1493	0.1377	0.1183	0.0935	0.0640	0.0322
	0.05	0.0493	0.0928	0.1257	0.1457	0.1535	0.1508	0.1390	0.1198	0.0944	0.0646	0.0325
	0.03	0.0492	0.0929	0.1259	0.1464	0.1543	0.1519	0.1401	0.1204	0.0948	0.0649	0.0327
a	*b*	11	10	9	8	7	6	5	4	3	2	1
Values of:		Concentrated unit load at Point: Values of *t*, Read Up.										

TABLE 13.—Load Coefficients for Beams of Varying Moment of Inertia.

Concentrated Unit Load.

Case I(*b*).—Unsymmetrical, Sharply Curved Haunch.

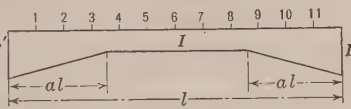


Values of:		Haunch at left support.	Concentrated Unit Load at Point:											Haunch at right support.
a.	*b.*		1	2	3	4	5	6	7	8	9	10	11	
....	1.00	*s*	0.0488	0.0849	0.1094	0.1235	0.1283	0.1250	0.1148	0.0988	0.0781	0.0540	0.0276	*t*
		t	0.0276	0.0540	0.0781	0.0988	0.1148	0.1250	0.1283	0.1235	0.1094	0.0849	0.0488	*s*
1.00	0.20	*s*	0.0441	0.0819	0.1106	0.1307	0.1414	0.1427	0.1359	0.1200	0.0973	0.0681	0.0356	*t*
		t	0.0226	0.0450	0.0660	0.0854	0.1017	0.1137	0.1200	0.1181	0.1088	0.0870	0.0515	*s*
	0.10	*s*	0.0431	0.0807	0.1109	0.1329	0.1453	0.1485	0.1421	0.1265	0.1031	0.0729	0.0380	*t*
		t	0.0217	0.0432	0.0639	0.0828	0.0993	0.1116	0.1184	0.1181	0.1085	0.0872	0.0521	*s*
	0.05	*s*	0.0420	0.0797	0.1111	0.1340	0.1478	0.1511	0.1459	0.1306	0.1068	0.0758	0.0396	*t*
		t	0.0212	0.0425	0.0627	0.0817	0.0980	0.1108	0.1178	0.1178	0.1085	0.0874	0.0525	*s*
	0.03	*s*	0.0417	0.0795	0.1112	0.1345	0.1490	0.1528	0.1480	0.1325	0.1082	0.0767	0.0403	*t*
		t	0.0210	0.0421	0.0624	0.0812	0.0974	0.1104	0.1174	0.1177	0.1085	0.0875	0.0525	*s*
0.50	0.20	*s*	0.0455	0.0846	0.1156	0.1373	0.1488	0.1500	0.1411	0.1232	0.0986	0.0684	0.0353	*t*
		t	0.0252	0.0501	0.0738	0.0950	0.1126	0.1251	0.1299	0.1263	0.1126	0.0880	0.0507	*s*
	0.10	*s*	0.0448	0.0847	0.1170	0.1401	0.1529	0.1551	0.1463	0.1281	0.1024	0.0714	0.0368	*t*
		t	0.0249	0.0495	0.0731	0.0944	0.1124	0.1250	0.1303	0.1268	0.1133	0.0884	0.0512	*s*
	0.05	*s*	0.0445	0.0848	0.1175	0.1420	0.1553	0.1579	0.1492	0.1306	0.1048	0.0730	0.0376	*t*
		t	0.0247	0.0494	0.0729	0.0943	0.1123	0.1250	0.1303	0.1271	0.1134	0.0887	0.0512	*s*
	0.03	*s*	0.0444	0.0846	0.1180	0.1423	0.1563	0.1588	0.1501	0.1318	0.1057	0.0737	0.0380	*t*
		t	0.0246	0.0492	0.0727	0.0942	0.1122	0.1250	0.1303	0.1271	0.1136	0.0887	0.0513	*s*
0.40	0.20	*s*	0.0464	0.0862	0.1180	0.1396	0.1499	0.1495	0.1392	0.1209	0.0965	0.0671	0.0344	*t*
		t	0.0260	0.0516	0.0757	0.0972	0.1148	0.1263	0.1304	0.1260	0.1119	0.0872	0.0503	*s*
	0.10	*s*	0.0459	0.0869	0.1196	0.1423	0.1539	0.1539	0.1436	0.1252	0.0994	0.0694	0.0356	*t*
		t	0.0257	0.0512	0.0753	0.0970	0.1146	0.1264	0.1304	0.1263	0.1125	0.0874	0.0505	*s*
	0.05	*s*	0.0458	0.0871	0.1204	0.1440	0.1558	0.1562	0.1458	0.1270	0.1012	0.0707	0.0362	*t*
		t	0.0256	0.0511	0.0752	0.0968	0.1147	0.1264	0.1307	0.1264	0.1125	0.0876	0.0506	*s*
	0.03	*s*	0.0457	0.0871	0.1209	0.1447	0.1570	0.1570	0.1469	0.1280	0.1020	0.0708	0.0365	*t*
		t	0.0256	0.0508	0.0752	0.0970	0.1149	0.1265	0.1309	0.1266	0.1127	0.0878	0.0506	*s*
0.35	0.20	*s*	0.0468	0.0875	0.1193	0.1401	0.1494	0.1482	0.1376	0.1193	0.0949	0.0660	0.0338	*t*
		t	0.0263	0.0523	0.0767	0.0982	0.1153	0.1264	0.1302	0.1258	0.1115	0.0868	0.0501	*s*
	0.10	*s*	0.0466	0.0882	0.1209	0.1432	0.1538	0.1524	0.1415	0.1230	0.0978	0.0680	0.0349	*t*
		t	0.0261	0.0519	0.0764	0.0982	0.1154	0.1265	0.1302	0.1258	0.1119	0.0871	0.0501	*s*
	0.05	*s*	0.0465	0.0883	0.1221	0.1447	0.1554	0.1545	0.1438	0.1246	0.0994	0.0691	0.0354	*t*
		t	0.0260	0.0517	0.0764	0.0981	0.1154	0.1265	0.1304	0.1260	0.1119	0.0872	0.0502	*s*
	0.03	*s*	0.0464	0.0885	0.1224	0.1452	0.1559	0.1550	0.1447	0.1254	0.1001	0.0696	0.0356	*t*
		t	0.0260	0.0516	0.0763	0.0980	0.1155	0.1265	0.1305	0.1261	0.1120	0.0873	0.0502	*s*
0.30	0.20	*s*	0.0479	0.0886	0.1201	0.1401	0.1485	0.1465	0.1357	0.1173	0.0930	0.0647	0.0331	*t*
		t	0.0266	0.0529	0.0773	0.0988	0.1154	0.1263	0.1298	0.1253	0.1112	0.0862	0.0497	*s*
	0.10	*s*	0.0476	0.0894	0.1218	0.1431	0.1517	0.1501	0.1390	0.1205	0.0955	0.0664	0.0340	*t*
		t	0.0265	0.0526	0.0772	0.0989	0.1154	0.1263	0.1301	0.1254	0.1113	0.0864	0.0497	*s*
	0.05	*s*	0.0474	0.0897	0.1230	0.1442	0.1533	0.1517	0.1409	0.1217	0.0968	0.0673	0.0345	*t*
		t	0.0264	0.0525	0.0772	0.0989	0.1154	0.1264	0.1301	0.1256	0.1114	0.0865	0.0499	*s*
	0.03	*s*	0.0473	0.0899	0.1234	0.1451	0.1543	0.1526	0.1418	0.1225	0.0974	0.0677	0.0347	*t*
		t	0.0264	0.0525	0.0772	0.0989	0.1154	0.1265	0.1304	0.1256	0.1115	0.0868	0.0499	*s*

TABLE 13.—(*Continued.*)

Values of:		Haunch at left support.	Concentrated Unit Load at Point:											Haunch at right support.
a.	*b.*		1	2	3	4	5	6	7	8	9	10	11	
0.25	0.20	*s*	0.0484	0.0897	0.1206	0.1393	0.1467	0.1440	0.1331	0.1151	0.0913	0.0632	0.0324	*t*
		t	0.0269	0.0533	0.0778	0.0992	0.1154	0.1261	0.1296	0.1248	0.1108	0.0859	0.0495	*s*
	0.10	*s*	0.0483	0.0907	0.1222	0.1418	0.1498	0.1470	0.1362	0.1174	0.0935	0.0647	0.0332	*t*
		t	0.0268	0.0533	0.0778	0.0992	0.1156	0.1263	0.1299	0.1250	0.1108	0.0860	0.0495	*s*
	0.05	*s*	0.0483	0.0909	0.1233	0.1431	0.1511	0.1487	0.1374	0.1188	0.0946	0.0655	0.0335	*t*
		t	0.0268	0.0532	0.0778	0.0992	0.1157	0.1264	0.1299	0.1251	0.1109	0.0861	0.0495	*s*
	0.03	*s*	0.0483	0.0909	0.1238	0.1437	0.1518	0.1494	0.1380	0.1193	0.0950	0.0658	0.0337	*t*
		t	0.0267	0.0532	0.0778	0.0992	0.1157	0.1264	0.1299	0.1253	0.1111	0.0861	0.0496	*s*

TABLE 14.—Load Coefficients for Beams of Varying Moment of Inertia.
Concentrated Unit Load.
Case II(*a*).—Symmetrical, Straight Haunches.



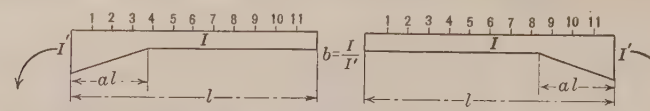
$l = \frac{I}{I'}$

Values of:		Values of *s*, Read Down.										
		Concentrated unit load at Point.										
a.	*b.*	1	2	3	4	5	6	7	8	9	10	11
....	1.00	0.0488	0.0849	0.1094	0.1235	0.1283	0.1250	0.1148	0.0988	0.0781	0.0540	0.0276
0.50	0.20	0.0473	0.0879	0.1206	0.1443	0.1573	0.1577	0.1447	0.1226	0.0951	0.0650	0.0327
	0.10	0.0466	0.0881	0.1235	0.1509	0.1679	0.1709	0.1563	0.1318	0.1014	0.0686	0.0345
	0.05	0.0456	0.0876	0.1250	0.1558	0.1770	0.1827	0.1666	0.1393	0.1067	0.0720	0.0360
	0.03	0.0450	0.0874	0.1262	0.1592	0.1833	0.1908	0.1739	0.1440	0.1101	0.0740	0.0370
0.40	0.20	0.0471	0.0885	0.1227	0.1476	0.1613	0.1609	0.1484	0.1259	0.0977	0.0664	0.0334
	0.10	0.0463	0.0888	0.1254	0.1552	0.1726	0.1739	0.1607	0.1356	0.1041	0.0706	0.0353
	0.05	0.0453	0.0884	0.1274	0.1607	0.1825	0.1856	0.1714	0.1434	0.1097	0.0737	0.0369
	0.03	0.0445	0.0877	0.1282	0.1636	0.1882	0.1923	0.1777	0.1482	0.1127	0.0754	0.0377
0.35	0.20	0.0474	0.0892	0.1234	0.1483	0.1604	0.1596	0.1471	0.1258	0.0977	0.0663	0.0334
	0.10	0.0466	0.0894	0.1271	0.1560	0.1712	0.1712	0.1585	0.1348	0.1038	0.0704	0.0352
	0.05	0.0458	0.0890	0.1291	0.1617	0.1798	0.1811	0.1674	0.1419	0.1088	0.0731	0.0366
	0.03	0.0450	0.0887	0.1299	0.1644	0.1851	0.1869	0.1729	0.1463	0.1117	0.0747	0.0374
0.30	0.20	0.0475	0.0899	0.1246	0.1482	0.1586	0.1570	0.1450	0.1242	0.0971	0.0659	0.0332
	0.10	0.0470	0.0906	0.1283	0.1556	0.1682	0.1669	0.1548	0.1325	0.1028	0.0696	0.0348
	0.05	0.0465	0.0908	0.1308	0.1612	0.1755	0.1751	0.1620	0.1386	0.1073	0.0721	0.0361
	0.03	0.0461	0.0904	0.1316	0.1644	0.1799	0.1804	0.1671	0.1427	0.1099	0.0736	0.0368
0.25	0.20	0.0485	0.0911	0.1254	0.1467	0.1554	0.1530	0.1412	0.1215	0.0954	0.0651	0.0327
	0.10	0.0478	0.0923	0.1293	0.1532	0.1635	0.1619	0.1495	0.1285	0.1005	0.0684	0.0342
	0.05	0.0472	0.0923	0.1321	0.1585	0.1700	0.1683	0.1555	0.1338	0.1045	0.0705	0.0353
	0.03	0.0471	0.0924	0.1335	0.1611	0.1732	0.1719	0.1594	0.1369	0.1067	0.0717	0.0358
a	*b.*	11	10	9	8	7	6	5	4	3	2	1
		Concentrated unit load at Point.										
Values of:		Values of *t*, Read Up.										

TABLE 15.—Load Coefficients for Beams of Varying Moment of Inertia.

Concentrated Unit Load.

Case II(*b*).—Unsymmetrical, Straight Haunch.



Values of:		Haunch at left support.	Concentrated Unit Load at Point.											Haunch at right support.
a	*b*		1	2	3	4	5	6	7	8	9	10	11	
....	1.00	*s*	0.0488	0.0849	0.1094	0.1235	0.1283	0.1250	0.1148	0.0988	0.0781	0.0540	0.0276	*t*
		t	0.0276	0.0540	0.0781	0.0988	0.1148	0.1250	0.1283	0.1235	0.1094	0.0849	0.0488	*s*
1.00	0.20	*s*	0.0440	0.0790	0.1063	0.1247	0.1349	0.1370	0.1306	0.1165	0.0952	0.0675	0.0350	*t*
		t	0.0205	0.0412	0.0604	0.0782	0.0931	0.1046	0.1114	0.1119	0.1039	0.0849	0.0517	*s*
	0.10	*s*	0.0412	0.0754	0.1026	0.1222	0.1339	0.1377	0.1335	0.1213	0.1007	0.0726	0.0384	*t*
		t	0.0181	0.0359	0.0531	0.0689	0.0831	0.0946	0.1022	0.1042	0.0990	0.0829	0.0520	*s*
	0.05	*s*	0.0382	0.0709	0.0979	0.1178	0.1310	0.1371	0.1351	0.1244	0.1050	0.0775	0.0413	*t*
		t	0.0156	0.0312	0.0461	0.0603	0.0733	0.0842	0.0921	0.0957	0.0925	0.0795	0.0513	*s*
	0.03	*s*	0.0361	0.0674	0.0933	0.1138	0.1273	0.1354	0.1349	0.1257	0.1079	0.0804	0.0432	*t*
		t	0.0140	0.0281	0.0414	0.0542	0.0665	0.0766	0.0845	0.0889	0.0872	0.0766	0.0510	*s*
0.50	0.20	*s*	0.0428	0.0800	0.1110	0.1346	0.1496	0.1542	0.1469	0.1301	0.1046	0.0732	0.0377	*t*
		t	0.0239	0.0475	0.0702	0.0916	0.1100	0.1235	0.1298	0.1273	0.1140	0.0895	0.0518	*s*
	0.10	*s*	0.0401	0.0766	0.1087	0.1347	0.1537	0.1625	0.1577	0.1407	0.1142	0.0801	0.0416	*t*
		t	0.0228	0.0455	0.0677	0.0885	0.1077	0.1226	0.1299	0.1280	0.1154	0.0907	0.0526	*s*
	0.05	*s*	0.0372	0.0729	0.1053	0.1333	0.1555	0.1684	0.1663	0.1501	0.1225	0.0864	0.0443	*t*
		t	0.0218	0.0436	0.0652	0.0860	0.1054	0.1212	0.1296	0.1286	0.1162	0.0918	0.0534	*s*
	0.03	*s*	0.0355	0.0699	0.1020	0.1314	0.1556	0.1714	0.1708	0.1550	0.1274	0.0896	0.0462	*t*
		t	0.0213	0.0425	0.0638	0.0843	0.1038	0.1202	0.1294	0.1286	0.1166	0.0922	0.0538	*s*
0.40	0.20	*s*	0.0441	0.0830	0.1152	0.1399	0.1536	0.1554	0.1461	0.1280	0.1024	0.0711	0.0366	*t*
		t	0.0250	0.0497	0.0735	0.0955	0.1139	0.1260	0.1308	0.1269	0.1133	0.0883	0.0509	*s*
	0.10	*s*	0.0421	0.0809	0.1149	0.1432	0.1609	0.1647	0.1561	0.1374	0.1106	0.0771	0.0397	*t*
		t	0.0242	0.0485	0.0718	0.0940	0.1133	0.1261	0.1314	0.1279	0.1142	0.0894	0.0517	*s*
	0.05	*s*	0.0402	0.0784	0.1135	0.1440	0.1654	0.1715	0.1644	0.1451	0.1171	0.0819	0.0422	*t*
		t	0.0236	0.0472	0.0705	0.0927	0.1123	0.1260	0.1317	0.1285	0.1149	0.0899	0.0520	*s*
	0.03	*s*	0.0390	0.0769	0.1122	0.1438	0.1675	0.1754	0.1686	0.1496	0.1206	0.0848	0.0437	*t*
		t	0.0232	0.0465	0.0697	0.0919	0.1118	0.1258	0.1319	0.1288	0.1157	0.0905	0.0524	*s*
0.35	0.20	*s*	0.0450	0.0848	0.1181	0.1419	0.1545	0.1545	0.1445	0.1259	0.1004	0.0697	0.0359	*t*
		t	0.0256	0.0508	0.0752	0.0973	0.1149	0.1266	0.1309	0.1266	0.1127	0.0878	0.0505	*s*
	0.10	*s*	0.0433	0.0833	0.1186	0.1465	0.1619	0.1638	0.1540	0.1344	0.1079	0.0749	0.0386	*t*
		t	0.0249	0.0498	0.0739	0.0964	0.1147	0.1267	0.1315	0.1276	0.1135	0.0886	0.0511	*s*
	0.05	*s*	0.0417	0.0815	0.1183	0.1487	0.1668	0.1703	0.1609	0.1413	0.1134	0.0790	0.0407	*t*
		t	0.0244	0.0489	0.0730	0.0957	0.1146	0.1269	0.1319	0.1281	0.1143	0.0892	0.0515	*s*
	0.03	*s*	0.0409	0.0804	0.1173	0.1496	0.1693	0.1739	0.1648	0.1450	0.1167	0.0814	0.0419	*t*
		t	0.0241	0.0483	0.0721	0.0951	0.1143	0.1268	0.1321	0.1286	0.1146	0.0896	0.0518	*s*
0.30	0.20	*s*	0.0460	0.0866	0.1203	0.1435	0.1537	0.1528	0.1423	0.1233	0.0984	0.0684	0.0350	*t*
		t	0.0260	0.0518	0.0765	0.0985	0.1154	0.1267	0.1305	0.1261	0.1120	0.0872	0.0502	*s*
	0.10	*s*	0.0445	0.0859	0.1222	0.1485	0.1611	0.1611	0.1507	0.1309	0.1044	0.0729	0.0373	*t*
		t	0.0256	0.0511	0.0758	0.0980	0.1155	0.1269	0.1313	0.1269	0.1128	0.0878	0.0505	*s*
	0.05	*s*	0.0433	0.0847	0.1226	0.1516	0.1657	0.1671	0.1567	0.1365	0.1092	0.0758	0.0391	*t*
		t	0.0252	0.0504	0.0750	0.0978	0.1157	0.1272	0.1315	0.1275	0.1132	0.0883	0.0507	*s*
	0.03	*s*	0.0425	0.0841	0.1227	0.1532	0.1686	0.1701	0.1599	0.1401	0.1121	0.0778	0.0401	*t*
		t	0.0250	0.0500	0.0747	0.0976	0.1156	0.1274	0.1319	0.1280	0.1138	0.0885	0.0512	*s*

TABLE 15.—(*Continued.*)

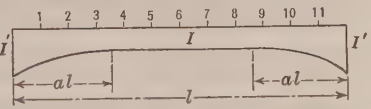
Values of:		Haunch at left support.	Concentrated Unit Load at Point:											Haunch at right support.
a	*b*		1	2	3	4	5	6	7	8	9	10	11	
0·25	0.20	*s*	0.0472	0.0887	0.1223	0.1433	0.1523	0.1502	0.1392	0.1207	0.0956	0.0665	0.0341	*t*
		t	0.0265	0.0527	0.0776	0.0990	0.1159	0.1264	0.1302	0.1254	0.1114	0.0865	0.0498	*s*
	0.10	*s*	0.0464	0.0888	0.1248	0.1486	0.1586	0.1573	0.1464	0.1269	0.1009	0.0702	0.0360	*t*
		t	0.0262	0.0523	0.0772	0.0990	0.1160	0.1270	0.1308	0.1261	0.1119	0.0870	0.0501	*s*
	0.05	*s*	0.0450	0.0882	0.1265	0.1517	0.1630	0.1625	0.1513	0.1315	0.1049	0.0729	0.0374	*t*
		t	0.0259	0.0518	0.0768	0.0989	0.1161	0.1273	0.1310	0.1267	0.1123	0.0874	0.0502	*s*
	0.03	*s*	0.0446	0.0879	0.1270	0.1537	0.1657	0.1657	0.1546	0.1344	0.1072	0.0746	0.0382	*t*
		t	0.0257	0.0515	0.0766	0.0989	0.1163	0.1275	0.1315	0.1268	0.1126	0.0878	0.0505	*s*

TABLE 16.—Load Coefficients for Beams of Varying Moment of Inertia.

Concentrated Unit Load.

Case III(*a*).—Symmetrical, Parabolic Haunches.

$b = \frac{I}{I'}$



Values of *s*, Read Down.

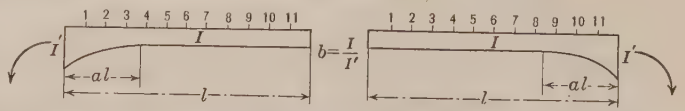
Values of:		Concentrated unit load at Point:										
a	*b*	1	2	3	4	5	6	7	8	9	10	11
....	1.00	0.0488	0.0849	0.1094	0.1235	0.1283	0.1250	0.1148	0.0988	0.0781	0.0540	0.0276
0.50	0.20	0.0478	0.0893	0.1229	0.1459	0.1569	0.1561	0.1439	0.1225	0.0957	0.0652	0.0328
	0.10	0.0474	0.0898	0.1260	0.1526	0.1672	0.1676	0.1543	0.1310	0.1015	0.0690	0.0345
	0.05	0.0464	0.0897	0.1283	0.1586	0.1760	0.1777	0.1638	0.1383	0.1066	0.0719	0.0360
	0.03	0.0457	0.0892	0.1291	0.1615	0.1814	0.1841	0.1695	0.1428	0.1096	0.0736	0.0368
0.40	0.20	0.0480	0.0905	0.1235	0.1459	0.1554	0.1538	0.1420	0.1215	0.0952	0.0649	0.0326
	0.10	0.0474	0.0908	0.1275	0.1526	0.1650	0.1634	0.1514	0.1295	0.1002	0.0685	0.0342
	0.05	0.0467	0.0908	0.1293	0.1580	0.1726	0.1721	0.1593	0.1357	0.1053	0.0711	0.0355
	0.03	0.0464	0.0906	0.1305	0.1616	0.1773	0.1778	0.1647	0.1401	0.1082	0.0727	0.0363
0.35	0.20	0.0484	0.0906	0.1238	0.1448	0.1538	0.1515	0.1398	0.1199	0.0941	0.0644	0.0324
	0.10	0.0480	0.0915	0.1276	0.1515	0.1621	0.1605	0.1482	0.1270	0.0992	0.0674	0.0339
	0.05	0.0473	0.0917	0.1302	0.1568	0.1690	0.1682	0.1555	0.1331	0.1035	0.0702	0.0351
	0.03	0.0470	0.0919	0.1311	0.1600	0.1734	0.1730	0.1600	0.1368	0.1061	0.0716	0.0358
0.30	0.20	0.0486	0.0911	0.1236	0.1436	0.1514	0.1490	0.1371	0.1181	0.0931	0.0637	0.0320
	0.10	0.0483	0.0922	0.1276	0.1498	0.1590	0.1566	0.1445	0.1244	0.0978	0.0664	0.0334
	0.05	0.0480	0.0926	0.1304	0.1545	0.1649	0.1632	0.1508	0.1296	0.1013	0.0689	0.0345
	0.03	0.0479	0.0931	0.1321	0.1575	0.1689	0.1672	0.1545	0.1329	0.1037	0.0708	0.0351
0.25	0.20	0.0495	0.0917	0.1229	0.1416	0.1484	0.1458	0.1344	0.1157	0.0913	0.0624	0.0316
	0.10	0.0490	0.0932	0.1268	0.1469	0.1548	0.1525	0.1406	0.1209	0.0952	0.0652	0.0328
	0.05	0.0487	0.0938	0.1295	0.1515	0.1600	0.1576	0.1454	0.1255	0.0987	0.0674	0.0337
	0.03	0.0483	0.0941	0.1313	0.1540	0.1631	0.1610	0.1486	0.1280	0.1007	0.0685	0.0343
a	*b*	11	10	9	8	7	6	5	4	3	2	1
Values of:		Concentrated unit load at Point:										

Values of *t*, Read Up.

TABLE 17.—Load Coefficients for Beams of Varying Moment of Inertia.

Concentrated Unit Load.

Case III(*b*).—Unsymmetrical, Parabolic Haunch.



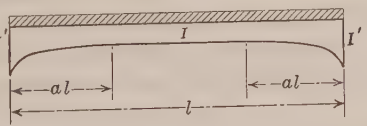
Values of:		Haunch at left support.	Concentrated Unit Load at Point:											Haunch at right support.
a.	*b.*		1	2	3	4	5	6	7	8	9	10	11	
....	1.00	*s*	0.0488	0.0849	0.1091	0.1235	0.1283	0.1250	0.1148	0.0988	0.0781	0.0540	0.0276	*t*
		t	0.0276	0.0540	0.0781	0.0988	0.1148	0.1250	0.1283	0.1235	0.1094	0.0849	0.0488	*s*
1.00	0.20	*s*	0.0435	0.0798	0.1087	0.1295	0.1415	0.1442	0.1384	0.1233	0.1003	0.0705	0.0364	*t*
		t	0.0222	0.0441	0.0649	0.0842	0.1007	0.1133	0.1202	0.1200	0.1101	0.0882	0.0524	*s*
	0.10	*s*	0.0403	0.0757	0.1047	0.1274	0.1421	0.1480	0.1446	0.1313	0.1087	0.0777	0.0403	*t*
		t	0.0201	0.0402	0.0596	0.0778	0.0938	0.1071	0.1154	0.1168	0.1088	0.0887	0.0533	*s*
	0.05	*s*	0.0371	0.0704	0.0995	0.1230	0.1403	0.1495	0.1484	0.1376	0.1156	0.0839	0.0441	*t*
		t	0.0182	0.0364	0.0542	0.0711	0.0869	0.0998	0.1093	0.1123	0.1066	0.0886	0.0542	*s*
	0.03	*s*	0.0348	0.0668	0.0948	0.1194	0.1371	0.1485	0.1497	0.1405	0.1200	0.0880	0.0468	*t*
		t	0.0169	0.0338	0.0505	0.0664	0.0815	0.0943	0.1043	0.1088	0.1045	0.0880	0.0546	*s*
0.50	0.20	*s*	0.0454	0.0852	0.1174	0.1401	0.1516	0.1525	0.1426	0.1246	0.0994	0.0690	0.0355	*t*
		t	0.0251	0.0505	0.0744	0.0961	0.1139	0.1258	0.1304	0.1265	0.1127	0.0879	0.0505	*s*
	0.10	*s*	0.0434	0.0832	0.1173	0.1428	0.1580	0.1608	0.1520	0.1331	0.1067	0.0744	0.0383	*t*
		t	0.0246	0.0493	0.0730	0.0946	0.1130	0.1258	0.1308	0.1273	0.1136	0.0887	0.0513	*s*
	0.05	*s*	0.0418	0.0806	0.1160	0.1445	0.1622	0.1671	0.1592	0.1406	0.1130	0.0791	0.0408	*t*
		t	0.0240	0.0479	0.0713	0.0932	0.1123	0.1255	0.1312	0.1281	0.1146	0.0898	0.0519	*s*
	0.03	*s*	0.0404	0.0792	0.1144	0.1440	0.1645	0.1711	0.1640	0.1451	0.1170	0.0822	0.0424	*t*
		t	0.0235	0.0470	0.0702	0.0923	0.1115	0.1254	0.1313	0.1285	0.1152	0.0903	0.0524	*s*
0.40	0.20	*s*	0.0465	0.0871	0.1195	0.1415	0.1514	0.1506	0.1398	0.1216	0.0967	0.0672	0.0344	*t*
		t	0.0261	0.0519	0.0761	0.0978	0.1151	0.1265	0.1305	0.1260	0.1119	0.0871	0.0500	*s*
	0.10	*s*	0.0452	0.0863	0.1213	0.1461	0.1581	0.1585	0.1479	0.1289	0.1027	0.0717	0.0368	*t*
		t	0.0256	0.0512	0.0752	0.0974	0.1153	0.1267	0.1310	0.1267	0.1128	0.0879	0.0506	*s*
	0.05	*s*	0.0438	0.0853	0.1212	0.1487	0.1631	0.1645	0.1547	0.1352	0.1081	0.0750	0.0387	*t*
		t	0.0251	0.0502	0.0744	0.0968	0.1150	0.1268	0.1313	0.1274	0.1135	0.0884	0.0508	*s*
	0.03	*s*	0.0428	0.0842	0.1208	0.1502	0.1665	0.1685	0.1588	0.1391	0.1112	0.0775	0.0399	*t*
		t	0.0248	0.0496	0.0738	0.0965	0.1147	0.1269	0.1317	0.1278	0.1138	0.0887	0.0511	*s*
0.35	0.20	*s*	0.0473	0.0884	0.1206	0.1414	0.1508	0.1488	0.1382	0.1198	0.0950	0.0660	0.0338	*t*
		t	0.0265	0.0526	0.0771	0.0985	0.1155	0.1265	0.1303	0.1256	0.1115	0.0867	0.0497	*s*
	0.10	*s*	0.0462	0.0881	0.1226	0.1463	0.1571	0.1563	0.1455	0.1261	0.1006	0.0699	0.0358	*t*
		t	0.0260	0.0520	0.0764	0.0983	0.1156	0.1265	0.1306	0.1262	0.1121	0.0872	0.0501	*s*
	0.05	*s*	0.0448	0.0873	0.1239	0.1496	0.1623	0.1623	0.1515	0.1316	0.1052	0.0732	0.0375	*t*
		t	0.0256	0.0513	0.0760	0.0980	0.1155	0.1269	0.1309	0.1266	0.1127	0.0878	0.0506	*s*
	0.03	*s*	0.0441	0.0861	0.1241	0.1515	0.1650	0.1654	0.1552	0.1352	0.1078	0.0748	0.0386	*t*
		t	0.0254	0.0508	0.0756	0.0979	0.1157	0.1270	0.1313	0.1270	0.1133	0.0882	0.0508	*s*
0.30	0.20	*s*	0.0479	0.0893	0.1212	0.1412	0.1492	0.1468	0.1356	0.1172	0.0933	0.0646	0.0331	*t*
		t	0.0267	0.0531	0.0776	0.0989	0.1156	0.1263	0.1300	0.1253	0.1111	0.0863	0.0496	*s*
	0.10	*s*	0.0470	0.0899	0.1239	0.1457	0.1549	0.1533	0.1423	0.1230	0.0978	0.0680	0.0349	*t*
		t	0.0264	0.0525	0.0773	0.0990	0.1158	0.1266	0.1305	0.1257	0.1117	0.0866	0.0500	*s*
	0.05	*s*	0.0463	0.0896	0.1255	0.1495	0.1600	0.1587	0.1473	0.1281	0.1019	0.0708	0.0363	*t*
		t	0.0261	0.0522	0.0771	0.0988	0.1158	0.1268	0.1306	0.1262	0.1120	0.0872	0.0501	*s*
	0.03	*s*	0.0452	0.0891	0.1262	0.1513	0.1625	0.1616	0.1508	0.1307	0.1043	0.0725	0.0372	*t*
		t	0.0259	0.0519	0.0769	0.0988	0.1159	0.1272	0.1309	0.1266	0.1122	0.0878	0.0503	*s*

TABLE 17.—(*Continued.*)

Values of:		Haunch at left support.	Concentrated Unit Load at Point.											Haunch at right support.
a.	b.		1	2	3	4	5	6	7	8	9	10	11	
0.25	0.20	s	0.0487	0.0904	0.1212	0.1399	0.1470	0.1442	0.1333	0.1150	0.0912	0.0631	0.0324	t
		t	0.0270	0.0537	0.0780	0.0992	0.1154	0.1262	0.1294	0.1249	0.1106	0.0859	0.0494	s
	0.10	s	0.0480	0.0912	0.1242	0.1441	0.1523	0.1498	0.1388	0.1197	0.0953	0.0660	0.0338	t
		t	0.0268	0.0532	0.0780	0.0993	0.1158	0.1264	0.1300	0.1251	0.1109	0.0861	0.0497	s
	0.05	s	0.0476	0.0914	0.1263	0.1478	0.1566	0.1545	0.1432	0.1238	0.0981	0.0682	0.0350	t
		t	0.0266	0.0531	0.0778	0.0993	0.1158	0.1267	0.1302	0.1254	0.1113	0.0864	0.0499	s
	0.03	s	0.0469	0.0916	0.1273	0.1497	0.1592	0.1575	0.1458	0.1265	0.1002	0.0697	0.0357	t
		t	0.0264	0.0529	0.0777	0.0994	0.1159	0.1268	0.1306	0.1258	0.1115	0.0866	0.0500	s

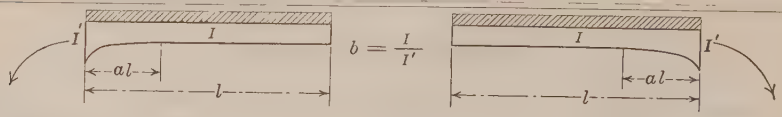
TABLE 18.—Load Coefficients for Beams of Varying Moment of Inertia.
Total Load 1, Uniformly Distributed.
Case I(a).—Symmetrical, Sharply Curved Haunches.

$b = \frac{I}{I'}$



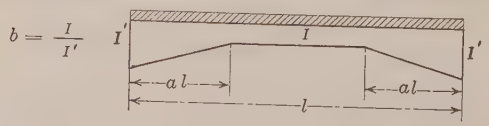
Values of a:		Values of b:												
		1.00	0.60	0.30	0.20	0.15	0.12	0.10	0.08	0.06	0.05	0.04	0.03	0.02
0.50	s	0.0833	0.0885	0.0946	0.0972	0.0987	0.0997	0.1004	0.1011	0.1018	0.1022	0.1026	0.1030	0.1033
0.40	s	0.0833	0.0891	0.0950	0.0975	0.0987	0.0996	0.1002	0.1008	0.1014	0.1017	0.1020	0.1024	0.1027
0.35	s	0.0833	0.0891	0.0948	0.0970	0.0982	0.0990	0.0996	0.1000	0.1006	0.1009	0.1012	0.1014	0.1018
0.30	s	0.0833	0.0889	0.0942	0.0963	0.0973	0.0980	0.0984	0.0989	0.0994	0.0996	0.0999	0.1001	0.1003
0.25	s	0.0833	0.0885	0.0932	0.0951	0.0960	0.0966	0.0970	0.0973	0.0977	0.0979	0.0981	0.0984	0.0986
0.20	s	0.0833	0.0880	0.0920	0.0934	0.0942	0.0947	0.0950	0.0954	0.0956	0.0959	0.0960	0.0962	0.0963
0.15	s	0.0833	0.0872	0.0904	0.0915	0.0922	0.0925	0.0928	0.0930	0.0932	0.0934	0.0934	0.0936	0.0937

TABLE 19.—Load Coefficients for Beams of Varying Moment of Inertia. Total Load 1, Uniformly Distributed. Case I(*b*).—Unsymmetrical, Sharply Curved Haunch.



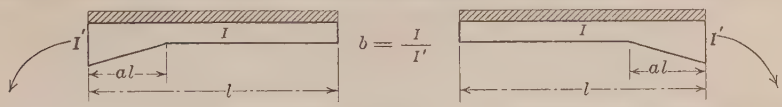
Values of *a*:	Haunch at left support.	Values of *b*: 1.00	0.60	0.30	0.20	0.15	0.12	0.10	0.08	0.06	0.05	0.04	0.03	0.02	Haunch at right support.
1.00	*s*	0.0833	0.0864	0.0906	0.0929	0.0941	0.0952	0.0958	0.0966	0.0974	0.0976	0.0981	0.0985	0.0991	*t*
	t	0.0833	0.0808	0.0783	0.0772	0.0767	0.0764	0.0762	0.0759	0.0757	0.0756	0.0755	0.0753	0.0752	*s*
0.50	*s*	0.0833	0.0885	0.0939	0.0963	0.0976	0.0984	0.0988	0.0995	0.1000	0.1002	0.1006	0.1008	0.1012	*t*
	t	0.0833	0.0832	0.0831	0.0830	0.0830	0.0830	0.0830	0.0830	0.0830	0.0830	0.0830	0.0830	0.0830	*s*
0.40	*s*	0.0833	0.0887	0.0940	0.0962	0.0973	0.0979	0.0985	0.0990	0.0996	0.0997	0.1000	0.1003	0.1006	*t*
	t	0.0833	0.0835	0.0836	0.0837	0.0837	0.0837	0.0837	0.0837	0.0838	0.0838	0.0838	0.0838	0.0838	*s*
0.35	*s*	0.0833	0.0887	0.0938	0.0958	0.0969	0.0975	0.0979	0.0984	0.0988	0.0991	0.0994	0.0996	0.0999	*t*
	t	0.0833	0.0836	0.0837	0.0838	0.0838	0.0838	0.0839	0.0839	0.0839	0.0839	0.0839	0.0839	0.0839	*s*
0.30	*s*	0.0833	0.0885	0.0934	0.0952	0.0962	0.0968	0.0971	0.0975	0.0980	0.0981	0.0984	0.0986	0.0987	*t*
	t	0.0833	0.0836	0.0838	0.0838	0.0839	0.0839	0.0840	0.0840	0.0840	0.0840	0.0840	0.0840	0.0841	*s*
0.25	*s*	0.0833	0.0882	0.0926	0.0941	0.0951	0.0956	0.0960	0.0963	0.0966	0.0969	0.0970	0.0971	0.0973	*t*
	t	0.0833	0.0836	0.0838	0.0839	0.0839	0.0839	0.0839	0.0840	0.0840	0.0840	0.0840	0.0840	0.0840	*s*
0.20	*s*	0.0833	0.0877	0.0915	0.0929	0.0936	0.0940	0.0943	0.0946	0.0949	0.0950	0.0953	0.0954	0.0956	*t*
	t	0.0833	0.0836	0.0837	0.0838	0.0838	0.0838	0.0839	0.0839	0.0839	0.0839	0.0839	0.0839	0.0838	*s*

TABLE 20.—Load Coefficients for Beams of Varying Moment of Inertia. Total Load 1, Uniformly Distributed. Case II(*a*).—Symmetrical, Straight Haunches.



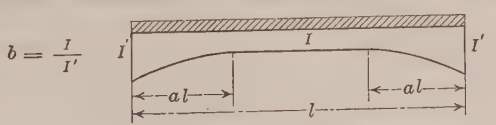
Values of *a*:		Values of *b*: 1.00	0.60	0.30	0.20	0.15	0.12	0.10	0.08	0.06	0.05	0.04	0.03	0.02	
0.50	*s*	0.0833	0.0885	0.0951	0.0986	0.1008	0.1025	0.1039	0.1055	0.1074	0.1085	0.1098	0.1115	0.1136	*t*
0.40	*s*	0.0833	0.0894	0.0967	0.1006	0.1031	0.1049	0.1063	0.1078	0.1099	0.1110	0.1123	0.1136	0.1154	*t*
0.35	*s*	0.0833	0.0894	0.0968	0.1005	0.1030	0.1048	0.1060	0.1075	0.1093	0.1102	0.1114	0.1126	0.1142	*t*
0.30	*s*	0.0833	0.0893	0.0963	0.0999	0.1020	0.1037	0.1048	0.1061	0.1077	0.1086	0.1096	0.1108	0.1120	*t*
0.25	*s*	0.0833	0.0890	0.0954	0.0986	0.1006	0.1018	0.1030	0.1040	0.1054	0.1062	0.1070	0.1080	0.1091	*t*
0.20	*s*	0.0833	0.0884	0.0940	0.0967	0.0984	0.0995	0.1003	0.1013	0.1024	0.1030	0.1037	0.1044	0.1054	*t*
0.15	*s*	0.0833	0.0876	0.0921	0.0942	0.0955	0.0964	0.0970	0.0978	0.0986	0.0991	0.0996	0.1002	0.1010	*t*

TABLE 21.—Load Coefficients for Beams of Varying Moment of Inertia. Total Load 1, Uniformly Distributed. Case II(*b*).—Unsymmetrical, Straight Haunch.



Values of a:	Haunch at left support.	Values of b: 1.00	0.60	0.30	0.20	0.15	0.12	0.10	0.08	0.06	0.05	0.04	0.03	0.02	Haunch at right support.
1.00	*s*	0.0833	0.0858	0.0884	0.0899	0.0905	0.0908	0.0910	0.0910	0.0909	0.0905	0.0900	0.0894	0.0885	*t*
	t	0.0833	0.0805	0.0755	0.0722	0.0701	0.0683	0.0666	0.0645	0.0622	0.0607	0.0588	0.0563	0.0527	*s*
0.50	*s*	0.0833	0.0885	0.0947	0.0976	0.0993	0.1006	0.1014	0.1025	0.1035	0.1040	0.1047	0.1051	0.1058	*t*
	t	0.0833	0.0831	0.0826	0.0822	0.0819	0.0817	0.0815	0.0812	0.0810	0.0808	0.0805	0.0803	0.0799	*s*
0.40	*s*	0.0833	0.0889	0.0955	0.0985	0.1006	0.1018	0.1028	0.1038	0.1051	0.1057	0.1065	0.1073	0.1082	*t*
	t	0.0833	0.0835	0.0835	0.0834	0.0834	0.0833	0.0833	0.0832	0.0832	0.0831	0.0830	0.0830	0.0828	*s*
0.35	*s*	0.0833	0.0890	0.0954	0.0985	0.1004	0.1016	0.1028	0.1038	0.1052	0.1057	0.1065	0.1073	0.1082	*t*
	t	0.0833	0.0835	0.0837	0.0838	0.0838	0.0838	0.0838	0.0838	0.0838	0.0838	0.0838	0.0837	0.0836	*s*
0.30	*s*	0.0833	0.0889	0.0951	0.0981	0.1000	0.1012	0.1021	0.1032	0.1044	0.1048	0.1057	0.1065	0.1075	*t*
	t	0.0833	0.0836	0.0839	0.0840	0.0841	0.0841	0.0841	0.0841	0.0841	0.0842	0.0842	0.0842	0 0842	*s*
0.25	*s*	0.0833	0.0886	0.0944	0.0972	0.0988	0.0999	0.1009	0.1018	0.1028	0.1034	0.1042	0.1049	0.1057	*t*
	t	0.0833	0.0836	0.0839	0.0840	0.0841	0.0842	0.0842	0.0842	0.0842	0.0843	0.0843	0.0843	0.0844	*s*
0.20	*s*	0.0833	0.0881	0.0933	0.0956	0.0972	0.0981	0.0988	0.0998	0.1007	0.1012	0.1017	0.1024	0.1031	*t*
	t	0.0833	0.0836	0.0838	0.0839	0.0840	0 0841	0.0841	0.0841	0.0841	0.0841	0.0842	0.0842	0.0843	*s*

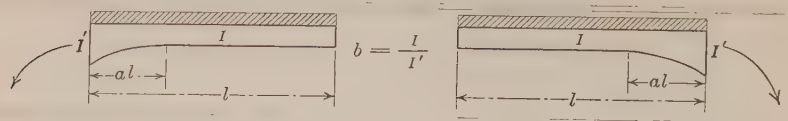
TABLE 22.—Load Coefficients for Beams of Varying Moment of Inertia. Total Load 1, Uniformly Distributed. Case III(*a*).—Symmetrical, Parabolic Haunches.



Values of a:		Values of b: 1.00	0.60	0.30	0.20	0.15	0.12	0.10	0.08	0.06	0.05	0.04	0.03	0.02	
0.50	*s*	0.0833	0.0888	0.0954	0.0988	0.1012	0.1028	0.1040	0.1055	0.1073	0.1084	0.1096	0.1109	0.1126	*t*
0.40	*s*	0.0833	0.0887	0.0951	0.0983	0.1005	0.1021	0.1031	0.1044	0.1062	0.1070	0.1081	0.1095	0.1110	*t*
0.35	*s*	0.0833	0.0885	0.0946	0.0976	0.0997	0.1010	0.1021	0.1034	0.1048	0.1056	0.1066	0.1078	0.1094	*t*
0.30	*s*	0.0833	0.0882	0.0938	0.0965	0.0983	0.0996	0.1006	0.1017	0.1031	0.1038	0.1047	0.1058	0.1071	*t*
0.25	*s*	0.0833	0.0878	0.0928	0.0952	0.0968	0.0979	0.0988	0.0996	0.1008	0.1015	0.1023	0.1032	0.1043	*t*
0.20	*s*	0.0833	0.0871	0.0914	0.0935	0.0948	0.0957	0.0964	0.0972	0.0982	0.0987	0.0994	0.1001	0.1011	*t*
0.15	*s*	0.0833	0.0864	0.0898	0.0914	0.0925	0.0932	0.0938	0.0944	0.0951	0.0955	0.0961	0.0966	0.0974	*t*

TABLE 23.—Load Coefficients for Beams of Varying Moment of Inertia. Total Load 1, Uniformly Distributed.

Case III(*b*).—Unsymmetrical, Parabolic Haunch.



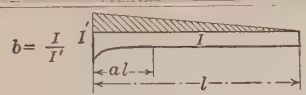
Values of *a*:	Haunch at left support.	Values of *b*: 1.00	0.60	0.30	0.20	0.15	0.12	0.10	0.08	0.06	0.05	0.04	0.03	0.02	Haunch at right support.
1.00	*s*	0.0833	0.0878	0.0916	0.0935	0.0945	0.0952	0.0956	0.0961	0.0962	0.0962	0.0962	0.0961	0.0955	*t*
	t	0.0833	0.0813	0.0792	0.0773	0.0761	0.0750	0.0741	0.0729	0.0715	0.0704	0.0692	0.0676	0.0653	*s*
0.50	*s*	0.0833	0.0886	0.0946	0.0974	0.0992	0.1005	0.1014	0.1024	0.1036	0.1043	0.1051	0.1060	0.1068	*t*
	t	0.0833	0.0835	0.0835	0.0834	0.0834	0.0833	0.0832	0.0831	0.0831	0.0830	0.0829	0.0828	0.0827	*s*
0.40	*s*	0.0833	0.0884	0.0941	0.0969	0.0987	0.1000	0.1008	0.1019	0.1032	0.1038	0.1046	0.1055	0.1066	*t*
	t	0.0833	0.0836	0.0837	0.0838	0.0839	0.0839	0.0839	0.0839	0.0839	0.0839	0.0839	0.0839	0.0838	*s*
0.35	*s*	0.0833	0.0882	0.0937	0.0964	0.0980	0.0992	0.1000	0.1011	0.1023	0.1030	0.1037	0.1046	0.1056	*t*
	t	0.0833	0.0836	0.0838	0.0839	0.0840	0.0840	0.0840	0.0840	0.0841	0.0841	0.0841	0.0842	0.0842	*s*
0.30	*s*	0.0833	0.0879	0.0930	0.0955	0.0970	0.0982	0.0990	0.0999	0.1011	0.1018	0.1024	0.1032	0.1042	*t*
	t	0.0833	0.0836	0.0838	0.0839	0.0840	0.0840	0.0840	0.0840	0.0841	0.0841	0.0842	0.0842	0.0842	*s*
0.25	*s*	0.0833	0.0875	0.0922	0.0944	0.0957	0.0967	0.0975	0.0984	0.0993	0.0999	0.1006	0.1014	0.1023	*t*
	t	0.0833	0.0835	0.0837	0.0838	0.0839	0.0840	0.0840	0.0841	0.0841	0.0841	0.0842	0.0842	0.0842	*s*
0.20	*s*	0.0833	0.0869	0.0910	0.0930	0.0940	0.0949	0.0955	0.0963	0.0973	0.0977	0.0982	0.0990	0.0998	*t*
	t	0.0833	0.0835	0.0837	0.0837	0.0838	0.0839	0.0839	0.0839	0.0839	0.0840	0.0840	0.0841	0.0841	*s*

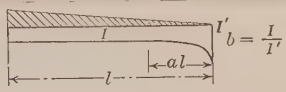
TABLE 24.—Load Coefficients for Beams of Varying Moment of Inertia. Total Triangular Load 1.

All Cases.—Symmetrical Beam. $b = \frac{I}{I'}$.

Values of *a*:	Load maximum at left support.	Uniform Moment of Inertia.	Case I (*a*).—Symmetrical, Sharply Curved Haunches.				Case II (*a*).—Symmetrical, Straight Haunches.				Case III (*a*).—Symmetrical, Parabolic Haunches.				Load maximum at right support.
		b	Values of *b*:				Values of *b*:				Values of *b*:				
		1.00	0.20	0.10	0.05	0.03	0.20	0.10	0.05	0.03	0.20	0.10	0.05	0.03	
0.50	*s*	0.0889	0.1019	0.1047	0.1064	0.1072	0.1028	0.1075	0.1114	0.1141	0.1033	0.1079	0.1118	0.1139	*t*
	t	0.0778	0.0926	0.0959	0.0978	0.0987	0.0943	0.1003	0.1054	0.1089	0.0944	0.1000	0.1049	0.1078	*s*
0.40	*s*	0.0889	0.1022	0.1047	0.1062	0.1068	0.1047	0.1097	0.1138	0.1160	0.1030	0.1074	0.1107	0.1129	*t*
	t	0.0778	0.0928	0.0957	0.0973	0.0980	0.0964	0.1028	0.1082	0.1112	0.0937	0.0989	0.1032	0.1060	*s*
0.35	*s*	0.0889	0.1018	0.1042	0.1055	0.1061	0.1047	0.1096	0.1133	0.1153	0.1024	0.1065	0.1097	0.1117	*t*
	t	0.0778	0.0922	0.0949	0.0963	0.0969	0.0962	0.1022	0.1069	0.1097	0.0927	0.0975	0.1016	0.1040	*s*
0.30	*s*	0.0889	0.1013	0.1034	0.1045	0.1050	0.1043	0.1088	0.1122	0.1141	0.1015	0.1053	0.1083	0.1102	*t*
	t	0.0778	0.0918	0.0936	0.0948	0.0953	0.0954	0.1008	0.1049	0.1075	0.0915	0.0959	0.0998	0.1015	*s*
0.25	*s*	0.0889	0.1003	0.1021	0.1030	0.1034	0.1034	0.1075	0.1104	0.1120	0.1004	0.1038	0.1064	0.1079	*t*
	t	0.0778	0.0999	0.0919	0.0929	0.0933	0.0938	0.0985	0.1021	0.1040	0.0900	0.0937	0.0967	0.0985	*s*

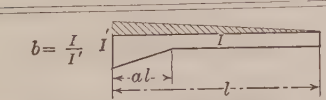
TABLE 25.—Load Coefficients for Beams of Varying Moment of Inertia.
Total Triangular Load 1.
Case I(*b*).—Unsymmetrical, Sharply Curved Haunch.

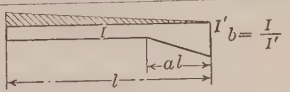




Values of *a*:	Load Maximum at Haunch.							Load Zero at Haunch.						
	Haunch at left support.	Values of *b*: 1.00	0.20	0.10	0.05	0.03	Haunch at right support.	Haunch at left support.	Values of *b*: 1.00	0.20	0.10	0.05	0.03	Haunch at right support.
1.00	*s*	0.0889	0.0953	0.0973	0.0984	0.0989	*t*	*s*	0.0778	0.0906	0.0945	0.0970	0.0980	*t*
	t	0.0778	0.0697	0.0682	0.0675	0.0672	*s*	*t*	0.0889	0.0848	0.0841	0.0838	0.0836	*s*
0.50	*s*	0.0889	0.0992	0.1012	0.1025	0.1029	*t*	*s*	0.0778	0.0934	0.0965	0.0982	0.0989	*t*
	t	0.0778	0.0762	0.0760	0.0759	0.0758	*s*	*t*	0.0889	0.0899	0.0901	0.0901	0.0901	*s*
0.40	*s*	0.0889	0.0998	0.1018	0.1028	0.1033	*t*	*s*	0.0778	0.0927	0.0954	0.0967	0.0973	*t*
	t	0.0778	0.0773	0.0771	0.0771	0.0771	*s*	*t*	0.0889	0.0901	0.0902	0.0903	0.0904	*s*
0.35	*s*	0.0889	0.0998	0.1017	0.1028	0.1031	*t*	*s*	0.0778	0.0919	0.0944	0.0956	0.0962	*t*
	t	0.0778	0.0776	0.0776	0.0775	0.0775	*s*	*t*	0.0889	0.0901	0.0902	0.0902	0.0903	*s*
0.30	*s*	0.0889	0.0996	0.1014	0.1023	0.1027	*t*	*s*	0.0778	0.0908	0.0930	0.0940	0.0946	*t*
	t	0.0778	0.0778	0.0778	0.0778	0.0778	*s*	*t*	0.0889	0.0899	0.0899	0.0900	0.0902	*s*
0.25	*s*	0.0889	0.0990	0.1006	0.1015	0.1018	*t*	*s*	0.0778	0.0894	0.0913	0.0922	0.0926	*t*
	t	0.0778	0.0780	0.0780	0.0780	0.0781	*s*	*t*	0.0889	0.0897	0.0898	0.0898	0.0899	*s*

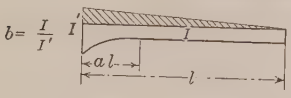
TABLE 26.—Load Coefficients for Beams of Varying Moment of Inertia.
Total Triangular Load 1.
Case II(*b*).—Unsymmetrical, Straight Haunch.

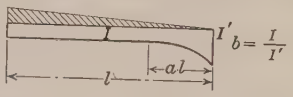




Values of *a*:	Load Maximum at Haunch.							Load Zero at Haunch.						
	Haunch at left support.	Values of *b*: 1.00	0.20	0.10	0.05	0.03	Haunch at right support.	Haunch at left support.	Values of *b*: 1.00	0.20	0.10	0.05	0.03	Haunch at right support.
1.00	*s*	0.0889	0.0919	0.0910	0.0892	0.0871	*t*	*s*	0.0778	0.0877	0.0901	0.0914	0.0917	*t*
	t	0.0778	0.0645	0.0583	0.0520	0.0476	*s*	*t*	0.0889	0.0803	0.0751	0.0694	0.0650	*s*
0.50	*s*	0.0889	0.0988	0.1007	0.1014	0.1015	*t*	*s*	0.0778	0.0964	0.1023	0.1067	0.1090	*t*
	t	0.0778	0.0746	0.0733	0.0720	0.0712	*s*	*t*	0.0889	0.0898	0.0897	0.0896	0.0894	*s*
0.40	*s*	0.0889	0.1007	0.1037	0.1054	0.1062	*t*	*s*	0.0778	0.0963	0.1021	0.1064	0.1088	*t*
	t	0.0778	0.0765	0.0759	0.0753	0.0750	*s*	*t*	0.0889	0.0903	0.0907	0.0907	0.0908	*s*
0.35	*s*	0.0889	0.1014	0.1046	0.1065	0.1074	*t*	*s*	0.0778	0.0957	0.1011	0.1051	0.1073	*t*
	t	0.0778	0.0772	0.0768	0.0766	0.0763	*s*	*t*	0.0889	0.0904	0.0908	0.0910	0.0911	*s*
0.30	*s*	0.0889	0.1016	0.1049	0.1068	0.1079	*t*	*s*	0.0778	0.0946	0.0996	0.1030	0.1051	*t*
	t	0.0778	0.0777	0.0775	0.0774	0.0773	*s*	*t*	0.0889	0.0903	0.0906	0.0909	0.0911	*s*
0.25	*s*	0.0889	0.1014	0.1046	0.1066	0.1078	*t*	*s*	0.0778	0.0931	0.0971	0.1004	0.1023	*t*
	t	0.0778	0.0780	0.0780	0.0780	0.0780	*s*	*t*	0.0889	0.0901	0.0904	0.0906	0.0908	*s*

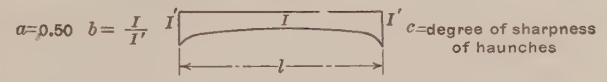
TABLE 27.—Load Coefficients for Beams of Varying Moment of Inertia.
Total Triangular Load 1.
Case III(*b*).—Unsymmetrical, Parabolic Haunch.





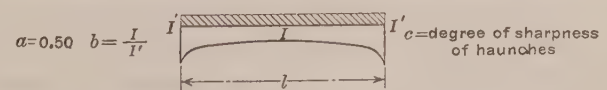
Values of *a*:		Load Maximum at Haunch.						Load Zero at Haunch.					
	Haunch at left support.	Values of *b*: 1.00	0.20	0.10	0.05	0.03	Haunch at right support.	Haunch at left support.	Values of *b*: 1.00	0.20	0.10	0.05	0.03	Haunch at right support.
1.00	*s*	0.0889	0.0952	0.0952	0.0939	0.0922	*t*	*s*	0.0778	0.0920	0.0961	0.0989	0.1000	*t*
	t	0.0778	0.0693	0.0653	0.0610	0.0579	*s*	*t*	0.0889	0.0852	0.0828	0.0797	0.0772	*s*
0.50	*s*	0.0889	0.1005	0.1031	0.1048	0.1055	*t*	*s*	0.0778	0.0946	0.0997	0.1039	0.1064	*t*
	t	0.0778	0.0767	0.0761	0.0755	0.0751	*s*	*t*	0.0889	0.0901	0.0904	0.0906	0.0907	*s*
0.40	*s*	0.0889	0.1007	0.1037	0.1058	0.1069	*t*	*s*	0.0778	0.0932	0.0980	0.1018	0.1042	*t*
	t	0.0778	0.0775	0.0774	0.0770	0.0769	*s*	*t*	0.0889	0.0901	0.0905	0.0907	0.0909	*s*
0.35	*s*	0.0889	0.1006	0.1036	0.1057	0.1069	*t*	*s*	0.0778	0.0923	0.0967	0.1002	0.1023	*t*
	t	0.0778	0.0779	0.0777	0.0775	0.0775	*s*	*t*	0.0889	0.0901	0.0903	0.0906	0.0908	*s*
0.30	*s*	0.0889	0.1001	0.1030	0.1053	0.1063	*t*	*s*	0.0778	0.0910	0.0949	0.0982	0.1000	*t*
	t	0.0778	0.0780	0.0780	0.0779	0.0779	*s*	*t*	0.0889	0.0899	0.0902	0.0904	0.0905	*s*
0.25	*s*	0.0889	0.0994	0.1021	0.1043	0.1054	*t*	*s*	0.0778	0.0895	0.0929	0.0957	0.0974	*t*
	t	0.0778	0.0780	0.0781	0.0781	0.0781	*s*	*t*	0.0889	0.0897	0.0899	0.0901	0.0903	*s*

TABLE 28.—Beam Coefficients for Beams of Varying Moment of Inertia. Case IV.—Symmetrical, Sharply Curved Haunches, of Varying Degrees of Sharpness.



Values of c:		Values of b: 1.00	0.60	0.30	0.20	0.15	0.12	0.10	0.08	0.06	0.05	0.04	0.03	0.02	
0.5	u	0.333	0.351	0.375	0.388	0.396	0.401	0.405	0.409	0.413	0.416	0.418	0.420	0.423	v
	p	0.500	0.367	0.267	0.233	0.217	0.207	0.200	0.193	0.187	0.183	0.180	0.177	0.173	q
1.0	u	0.333	0.354	0.378	0.389	0.395	0.399	0.402	0.404	0.407	0.409	0.410	0.412	0.413	v
	p	0.500	0.400	0.325	0.300	0.288	0.280	0.275	0.270	0.265	0.263	0.260	0.257	0.255	q
2.0	u	0.333	0.354	0.374	0.382	0.386	0.389	0.390	0.392	0.394	0.395	0.396	0.397	0.398	v
	p	0.500	0.433	0.383	0.367	0.358	0.353	0.350	0.347	0.343	0.342	0.340	0.338	0.337	q
3.0	u	0.333	0.352	0.369	0.375	0.378	0.380	0.382	0.383	0.385	0.385	0.386	0.387	0.387	v
	p	0.500	0.450	0.412	0.400	0.394	0.390	0.388	0.385	0.382	0.381	0.380	0.379	0.378	q
5.0	u	0.333	0.348	0.361	0.365	0.368	0.369	0.370	0.371	0.372	0.373	0.373	0.374	0.374	v
	p	0.500	0.467	0.442	0.433	0.429	0.427	0.425	0.423	0.422	0.421	0.420	0.419	0.418	q
10.0	u	0.333	0.343	0.351	0.353	0.355	0.356	0.356	0.357	0.357	0.358	0.358	0.358	0.358	v
	p	0.500	0.482	0.468	0.464	0.461	0.460	0.459	0.458	0.457	0.457	0.456	0.456	0.455	q

TABLE 29.—Load Coefficients for Beams of Varying Moment of Inertia. Total Load 1, Uniformly Distributed.* Case IV.—Symmetrical, Sharply Curved Haunches, of Varying Degrees of Sharpness.



Values of c:		Values of b: 1.00	0.60	0.30	0.20	0.15	0.12	0.10	0.08	0.06	0.05	0.04	0.03	0.02	
0.5	s	0.0833	0.0877	0.0938	0.0969	0.0989	0.1002	0.1012	0.1022	0.1033	0.1039	0.1045	0.1051	0.1058	t
1.0	s	0.0833	0.0885	0.0946	0.0972	0.0987	0.0997	0.1004	0.1011	0.1018	0.1022	0.1026	0.1030	0.1034	t
2.0	s	0.0833	0.0885	0.0935	0.0955	0.0965	0.0972	0.0976	0.0981	0.0985	0.0988	0.0990	0.0993	0.0995	t
3.0	s	0.0833	0.0880	0.0922	0.0938	0.0946	0.0951	0.0954	0.0958	0.0961	0.0963	0.0965	0.0967	0.0969	t
5.0	s	0.0833	0.0871	0.0902	0.0913	0.0919	0.0923	0.0925	0.0928	0.0930	0.0931	0.0933	0.0934	0.0935	t
10.0	s	0.0833	0.0858	0.0877	0.0884	0.0887	0.0889	0.0890	0.0892	0.0893	0.0894	0.0895	0.0895	0.0896	t

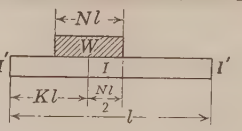
* To obtain load coefficients for other loadings, see p. 161.

TABLE 30.—Load Coefficients for Beams of Uniform Moment of Inertia. Total Load 1. Uniformly Distributed Over Various Parts of the Span. Uniform Moment of Inertia.

$b = \frac{I}{I'}, = 1.000$

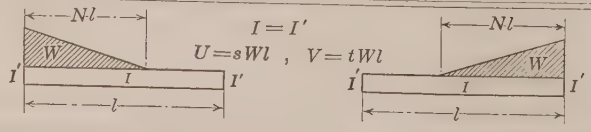
$u = v = 0.333$, $p = q = 0.500$

$U = sWl$, $V = tWl$



s	Values of K:	Values of N: 0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	Values of K:	t
Values of s, read right.	0.05	0.0301	.....	.....	.....	.....	.....	.....	.....	.....	.....	0.95	Values of t, read left.
	0.10	0.0562	0.0540	.....	.....	.....	.....	.....	.....	.....	.....	0.90	
	0.15	0.0779	0.0758	0.0722	.....	.....	.....	.....	.....	.....	.....	0.85	
	0.20	0.0953	0.0933	0.0900	0.0853	.....	.....	.....	.....	.....	.....	0.80	
	0.25	0.1088	0.1069	0.1037	0.0994	0.0938	.....	.....	.....	.....	.....	0.75	
	0.30	0.1184	0.1167	0.1138	0.1097	0.1044	0.0980	.....	.....	.....	.....	0.70	
	0.35	0.1246	0.1230	0.1202	0.1165	0.1116	0.1056	0.0986	.....	.....	.....	0.65	
	0.40	0.1275	0.1260	0.1235	0.1200	0.1155	0.1100	0.1035	0.0960	.....	.....	0.60	
	0.45	0.1274	0.1260	0.1238	0.1205	0.1164	0.1114	0.1054	0.0985	0.0903	.....	0.55	
	0.50	0.1246	0.1233	0.1212	0.1183	0.1146	0.1100	0.1046	0.0983	0.0912	0.0833	0.50	
	0.55	0.1192	0.1181	0.1162	0.1136	0.1102	0.1061	0.1012	0.0956	0.0892	.....	0.45	
	0.60	0.1117	0.1107	0.1090	0.1067	0.1037	0.1000	0.0957	0.0907	.....	.....	0.40	
	0.65	0.1021	0.1012	0.0998	0.0977	0.0951	0.0919	0.0881	.....	.....	.....	0.35	
	0.70	0.0908	0.0900	0.0887	0.0870	0.0848	0.0820	.....	.....	.....	.....	0.30	
	0.75	0.0779	0.0773	0.0762	0.0748	0.0729	.....	.....	.....	.....	.....	0.25	
	0.80	0.0638	0.0633	0.0625	0.0613	.....	.....	.....	.....	.....	.....	0.20	
	0.85	0.0488	0.0484	0.0477	.....	.....	.....	.....	.....	.....	.....	0.15	
	0.90	0.0329	0.0327	.....	.....	.....	.....	.....	.....	.....	.....	0.10	
	0.95	0.0166	.....	.....	.....	.....	.....	.....	.....	.....	.....	0.05	
s	Values of K:	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	Values of K:	t
		Values of N:											

TABLE 31.—Load Coefficients for Beams of Uniform Moment of Inertia. Total Triangular Load 1. Maximum at Support and Extending over Various Parts of Span. Uniform Moment of Inertia.



Load maximum at left support.	Values of N: 0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	Load maximum at right support.
s	0.0206	0.0381	0.0526	0.0643	0.0736	0.0805	0.0853	0.0881	0.0893	0.0889	t
t	0.0111	0.0220	0.0324	0.0423	0.0514	0.0595	0.0668	0.0718	0.0757	0.0778	s

TABLE 32.—Moment Coefficients for Simply Supported Beam and Load Coefficients for Beams of Uniform Moment of Inertia.

Total Load 1. Uniform, Trapezoidal, and Triangular Distribution. Uniform Moment of Inertia.

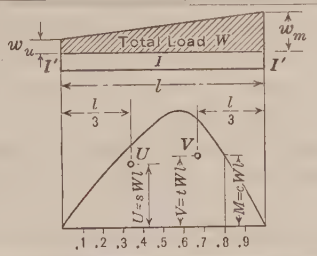
$\frac{w_m}{w_u} = \left\{ \begin{array}{l} \text{ratio of max. to min.} \\ \text{intensity of load} \end{array} \right.$

$b = \frac{I}{I'}$, = one

$u = v = 0.333$

$p = q = 0.500$

$U = sWl$, $V = tWl$



Load Maximum at Right Support, Read Down.

Type of loading:	Values of $\frac{W_m}{W_u}$:	Coefficient for simple beam moment, at Point.									Load coefficient.	
		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	s.	t.
Uniform.	1.0	0.0450	0.0800	0.1050	0.1200	0.1250	0.1200	0.1050	0.0800	0.0450	0.0833	0.0833
Trapezoidal.	1.1	0.0444	0.0792	0.1044	0.1196	0.1250	0.1204	0.1056	0.0808	0.0456	0.0830	0.0836
	1.2	0.0439	0.0785	0.1038	0.1192	0.1250	0.1208	0.1062	0.0815	0.0461	0.0828	0.0839
	1.3	0.0434	0.0779	0.1031	0.1190	0.1250	0.1210	0.1068	0.0821	0.0465	0.0826	0.0840
	1.4	0.0430	0.0772	0.1026	0.1186	0.1250	0.1214	0.1072	0.0826	0.0470	0.0824	0.0842
	1.5	0.0425	0.0768	0.1021	0.1184	0.1250	0.1216	0.1078	0.0832	0.0474	0.0821	0.0845
	1.6	0.0421	0.0762	0.1018	0.1181	0.1250	0.1219	0.1082	0.0838	0.0478	0.0820	0.0846
	1.8	0.0415	0.0754	0.1010	0.1178	0.1250	0.1222	0.1090	0.0846	0.0484	0.0818	0.0849
	2.0	0.0410	0.0746	0.1002	0.1174	0.1250	0.1226	0.1096	0.0854	0.0490	0.0815	0.0851
	2.25	0.0404	0.0739	0.0996	0.1169	0.1250	0.1230	0.1104	0.0861	0.0496	0.0811	0.0855
	2.50	0.0399	0.0731	0.0990	0.1165	0.1250	0.1234	0.1110	0.0869	0.0501	0.0809	0.0858
	2.75	0.0394	0.0725	0.0985	0.1162	0.1250	0.1238	0.1115	0.0875	0.0506	0.0806	0.0860
	3.0	0.0390	0.0720	0.0980	0.1160	0.1250	0.1240	0.1120	0.0880	0.0510	0.0805	0.0861
	3.5	0.0384	0.0711	0.0972	0.1155	0.1250	0.1244	0.1128	0.0889	0.0516	0.0802	0.0864
	4.0	0.0378	0.0704	0.0966	0.1152	0.1250	0.1248	0.1134	0.0896	0.0521	0.0800	0.0866
	4.5	0.0374	0.0698	0.0961	0.1149	0.1250	0.1251	0.1139	0.0901	0.0526	0.0798	0.0869
	5.0	0.0370	0.0694	0.0956	0.1146	0.1250	0.1254	0.1144	0.0906	0.0530	0.0796	0.0870
	6.0	0.0364	0.0685	0.0950	0.1142	0.1250	0.1258	0.1150	0.0915	0.0535	0.0794	0.0874
	7.0	0.0360	0.0680	0.0945	0.1140	0.1250	0.1260	0.1155	0.0920	0.0540	0.0791	0.0875
	8.0	0.0356	0.0675	0.0941	0.1138	0.1250	0.1262	0.1159	0.0925	0.0542	0.0790	0.0876
	9.0	0.0354	0.0671	9.0938	0.1136	0.1250	0.1264	0.1161	0.0928	0.0546	0.0789	0.0878
	10.0	0.0351	0.0669	0.0935	0.1135	0.1250	0.1265	0.1165	0.0931	0.0548	0.0788	0.0879
	15.0	0.0345	0.0660	0.0928	0.1130	0.1250	0.1270	0.1172	0.0940	0.0555	0.0785	0.0881
	20.0	0.0341	0.0655	0.0924	0.1128	0.1250	0.1272	0.1176	0.0945	0.0559	0.0784	0.0884
	40.0	0.0335	0.0648	0.0916	0.1124	0.1250	0.1276	0.1184	0.0952	0.0564	0.0780	0.0886
Triangle.	Infinity.	0.0330	0.0640	0.0910	0.1120	0.1250	0.1280	0.1190	0.0960	0.0570	0.0778	0.0889
		0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	t.	s.
Type of loading:	Values of $\frac{W_m}{W_u}$:	Coefficient for simple beam moment at Point.									Load coefficient.	

Load Maximum at Left Support, Read Up.

JULES D. GEDO,* ESQ. (by letter).—The authors' method is excellent so far as dead load is concerned. As to the live loads, the method does not differ much from that of fixed points. As soon as a span is not loaded the conjugate points fall into the beam and are called fixed points.

The writer has developed a method which allows the use of tabulations. These can be made for every possible case, no matter how many spans there are, whether they are equal or not, or whether the moments of inertia vary from span to span. This would seem to be impossible, because the cases to be covered are innumerable.

The writer's method is based on the fact that every multiple-span continuous beam can be conceived of as an aggregate of 2 and 3-span beams. According to the paper, to determine influence lines every span is loaded separately. Consider, for instance, Span *L-R* in Fig. 95.

If the two moments, M_L, and M_R, over the supports of the loaded span are found, the moment diagram is determined inside and outside the span. Outside the loaded span the moments spread through the farther fixed points of the successive spans. To determine these two moments is a laborious task. Instead of determining them consider Figs. 96 and 97.

Fig. 96 shows how to develop Fig. 97 from Fig. 95. Fig. 97 represents a 3-span beam. This 3-span beam is equivalent to the mutiple-span beam for the case when Span *L-R* is loaded. In other words if Span *L-R* is loaded as a span of the multiple-span beam, and if it is loaded the same way as the middle span of the 3-span beam, in both cases the same moment diagram for Span *L-R* will result. This may be shown statically as follows: The problem of an n-span beam is to determine the $n + 1$ components of the load. This problem is reduced at once by recognizing that the resultant of R_G and R_E (reactions in Supports *G* and *E*) must pass through F_l''. As is shown in the paper the moments in the fixed points are zero, wherever the loads may be (certainly, the loads must be outside the span to which the fixed points belong and if it is a left fixed point, the load must be to the right).

How is it possible to get at F_l'' a moment that equals zero? Left from F_l'' are two reactions, R_G and R_E. If their moment relative to F_l'' is zero (wherever the load may be between *L* and *R*) it shows that their resultant passes through F_l''. Hence it is only necessary to find n-reactions instead of $n + 1$. For similar reasons the resultant of R_G, R_E, and R_D goes through F_l' and the resultant of R_C, R_B, and R_A is in F_r'. Therefore, only four reactions must be found instead of $n + 1$. Furthermore, the moment at *L* is the same as if it were considered as the moment of R_G, R_E, and R_D, or as the moment of the resultant of R_G, R_E, and R_D. Hence, the problem of the n-span beam is reduced to that of a 3-span beam.

Considering the non-vertical dotted lines in Fig. 96 as funicular polygons for the reactions acting in the vertical dotted lines, it may be noted that the first and last rays are horizontal, which shows easily where the resultants of the reactions are. Thus, the fixed points are more than imaginary hinges. They play statically the same rôle as the cable of a suspension bridge, or the line of resistance of an arch.

* Structural Steel Detailer, Am. Bridge Co., Gary, Ind.

The foregoing statements show that for every loaded span there may be substituted a 3-span beam. The middle span of this equivalent 3-span beam is identical with the loaded span of the multiple-span beam. The outside supports of the 3-span beam are fixed points of the n-span beam, namely, the farther fixed points of the neighboring spans. If there is only one neighboring span to the loaded span, that is, if a side span is loaded, there results a 2-span beam.

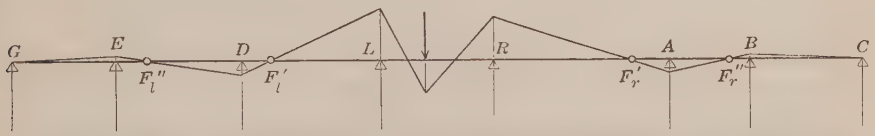


FIG. 95.

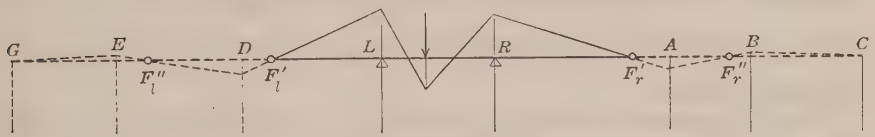


FIG. 96.

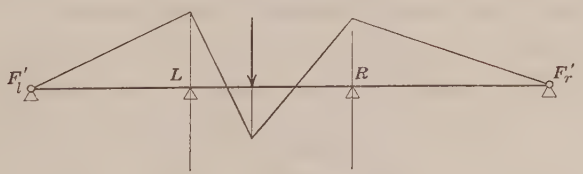


FIG. 97.

Therefore, to analyze an n-span beam, it is necessary to compute n-2 3-span beams and two 2-span beams. Since as far as calculation is concerned there are none but 3 and 2-span beams, furthermore, since in these 2 and 3-span beams only one span is loaded, it is both worth while and easy to prepare tables for them.

The 3-span beams may be studied with respect to their fixed points. To compute i, in Fig. 98, two relations are known. The first is given by the Theorem of Three Moments:

$$2\,M_L\,(l_l + l_m) + M_R\,l_m = 0 \qquad (96)$$

Note that Span L-R is not loaded (a loaded span has no fixed points).

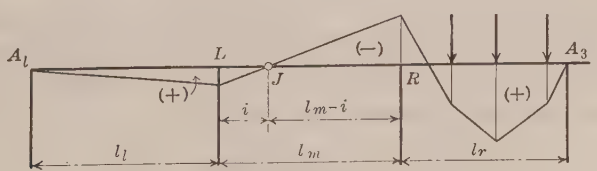


FIG. 98.

The second relation is given by the geometry of Fig. 98:

$$M_L : (-M_R) = i : (l_m - i) \qquad (97)$$

From Equations (96) and (97),

$$i = \frac{l_m^2}{l_m + 2\,(l_m + l_l)} \qquad (98)$$

or,

$$i = \frac{l_m}{1 + \dfrac{2\,(l_m + l_l)}{l_m}} \qquad (99)$$

or,

$$i = \frac{l_m}{3 + 2\,\dfrac{l_l}{l_m}} \qquad (100)$$

It is easy to remember Equation (100). The members, 3 and 2, are to be remembered from the fact that a multiple-span continuous beam, as already proved, is an aggregate of 3 and 2-span beams. Furthermore, if $l_l = 0$, $i = \frac{l_m}{3}$* (restrained beams).

It will be shown now how it is possible to prepare tables for the 3-span beams. In Fig. 99 load Span L-R (the middle span) with the load of unity. For this case according to the Theorem of Three Moments:

$$2\,M_L\,(l_l + l_m) + M_R\,l_m = -\,\alpha\,(1 - \alpha)\,(2 - \alpha)\,l_m^2 \qquad (101)$$

$$M_L\,l_m + 2\,M_R\,(l_m + l_r) = -\,\alpha\,(1 - \alpha)\,(1 + \alpha)\,l_m^2 \qquad (102)$$

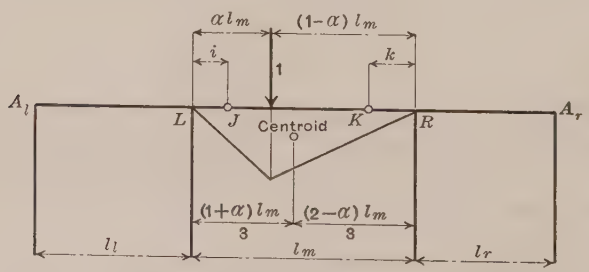


FIG. 99.

Dividing Equation (101) by $l_m + 2\,(l_l + l_m)$, and Equation (102) by $l_m + 2\,(l_r + l_m)$,

$$\frac{2\,M_L\,(l_l + l_m)}{l_m + 2\,(l_l + l_m)} + \frac{M_R\,l_m}{l_m + 2\,(l_l + l_m)} = -\,\frac{\alpha\,(1 - \alpha)\,(2 - \alpha)\,l_m^2}{l_m + 2\,(l_l + l_m)} \qquad (103)$$

$$\frac{M_L\,l_m}{l_m + 2\,(l_r + l_m)} + \frac{2\,M_R\,(l_m + l_r)}{l_m + 2\,(l_r + l_m)} = -\,\frac{\alpha\,(1 - \alpha)\,(1 + \alpha)\,l_m^2}{l_m + 2\,(l_r + l_m)} \qquad (104)$$

Introducing the two relations:

$$\gamma_l = \frac{i}{l_m} = \frac{l_m}{l_m + 2\,(l_m + l_l)} = \frac{1}{1 + \dfrac{2\,(l_m + l_l)}{l_m}}$$

* To determine the fixed points analytically for any given multiple-span beams, see Müller-Breslau or Strassner.

and correspondingly,

$$\gamma_r = \frac{k}{l_m} = \frac{l_m}{l_m + 2\,(l_m + l_r)} = \frac{1}{1 + \dfrac{2\,(l_m + l_r)}{l_m}}$$

Equations (103) and (104) become,

$$(1 - \gamma_l)\, M_L + \gamma_l\, M_R = -\,\alpha\,(1 - \alpha)\,(2 - \alpha)\,\gamma_l\, l_m \ldots\ldots (105)$$

$$\gamma_r\, M_L + (1 - \gamma_r)\, M_R = -\,\alpha\,(1 - \alpha)\,(1 + \alpha)\,\gamma_r\, l_m \ldots\ldots (106)^*$$

Assuming $M_R = \beta\, M_L$, then,

$$M_L\,(1 - \gamma_l + \beta\,\gamma_l) = -\,\alpha\,(1 - \alpha)\,(2 - \alpha)\,\gamma_l\, l_m \ldots\ldots (107)$$

$$M_L\,[\gamma_r + (1 - \gamma_r)\,\beta] = -\,\alpha\,(1 - \alpha)\,(1 + \alpha)\,\gamma_r\, l_m \ldots\ldots (108)$$

Taking $M_L = A_l\, l_l$ (see Fig. 99), then,

$$A_l = -\,\frac{\alpha(1 - \alpha)\,(2 - \alpha)}{1 - \gamma_l + \beta\,\gamma_l} \times \frac{2\,\gamma_l^2}{1 - 3\,\gamma_l} \ldots\ldots (109)$$

Dividing Equation (107) by Equation (108) and taking,

$$\frac{\gamma_l}{\gamma_r} = \delta$$

then,

$$\beta = \frac{3\,\gamma_l - (1 + \alpha)}{3\,\gamma_l - \delta\,(2 - \alpha)} \ldots\ldots (110)$$

or,

$$A_l = \frac{-\,6\,\gamma_l^3\,\alpha\,(1 - \alpha) + 2\,\delta\,\gamma_l^2\,\alpha\,(1 - \alpha)\,(2 - \alpha)}{[\gamma_l - \delta\,(1 - \gamma_l)]\,(1 - 3\,\gamma_l)} \ldots\ldots (111)$$

From Fig. 99,

$$A_r = \frac{M_R}{l_r} = \frac{\beta\, M_L}{l_r} = \frac{\beta\, A_l\, l_l}{l_r} \ldots\ldots (112)$$

but,

$$\frac{l_l}{l_r} = \frac{\dfrac{l_l}{l_m}}{\dfrac{l_r}{l_m}} = \frac{\dfrac{1 - 3\,\gamma_l}{2\,\gamma_l}}{\dfrac{1 - 3\,\gamma_r}{2\,\gamma_r}} = \frac{1 - 3\,\gamma_l}{\delta - 3\,\gamma_l} \ldots\ldots (113)$$

so that,

$$A_r = A_l \times \frac{3\,\gamma_l - (1 + \alpha)}{3\,\gamma_l - \delta\,(2 - \alpha)} \times \frac{1 - 3\,\gamma_l}{\delta - 3\,\gamma_l} \ldots\ldots (114)$$

As soon as A_l and A_r are known, the problem is statically determinate and the middle reactions, at points, L and R (see Fig. 99—these reactions are likewise termed L and R), are:

$$L = 1 - \alpha + \frac{1 - \gamma_l}{2\,\gamma_l} \times A_l - \frac{\delta - 3\,\gamma_l}{2\,\gamma_l} \times A_r \ldots\ldots (115)$$

$$R = \alpha - \frac{1 - 3\,\gamma_l}{2\,\gamma_l} \times A_l + \frac{\delta - \gamma_l}{2\,\gamma_l} \times A_r \ldots\ldots (116)$$

* The writer did not want to exploit the geometrical meaning of Equations (105) and (106).

By tabulating Equations (111), (114), (115), and (116), the designer can determine the influence lines for A_l, A_r, L, and R. All the designer has to do is to determine (1) the fixed points of the multiple-span beam; (2) the fixed points of the 3-span beam with the formulas, $i = \dfrac{l_m}{3 + 2\dfrac{l_l}{l_m}}$ and $k = \dfrac{l_m}{3 + 2\dfrac{l_r}{l_m}}$; and (3) the relations, $\gamma_l = \dfrac{i}{l_m}$ and $\delta = \dfrac{\gamma_l}{\gamma_r} = \dfrac{i}{k}$. For these γ_l and δ-values the tabulations will give the corresponding A_l, A_r, L, and R. By knowing these four influence lines the designer can answer every possible question relative to the n-span beam. This will be shown later by a definite problem.

Generally, the fixed points of the 3-span beam are not identical with the fixed points of the n-span beam. This, of course, involves extra work, because tables could be prepared for the relations, $l_l : l_m : l_r = 1 : x : y$. However, the fixed points of the 3-span beam, which are secondary fixed points of the n-span beam, play an important and beautiful rôle as will be shown.

For the 2-span beams call the loaded span l_λ and the unloaded span l_u. The middle reaction is called A_m, that for loaded span, A_λ, and for the unloaded span, A_u. The secondary fixed point in the loaded span is determined by the formula:

$$i = \frac{l_\lambda}{3 + 2\dfrac{l_u}{l_\lambda}};$$

then,

$$\gamma = \frac{i}{l_\lambda} = \frac{l_\lambda}{3 + 2\dfrac{l_u}{l_\lambda}}$$

and so if by measuring α from A_m to A_λ,

$$A_u = -\alpha(1-\alpha)(2-\alpha)\frac{\gamma^2}{(1-\gamma)(1-3\gamma)} \quad \ldots\ldots\ldots (117)$$

$$A_m = \alpha + \alpha(1-\alpha)(2-\alpha)\frac{\gamma}{2(1-3\gamma)} \quad \ldots\ldots\ldots (118)$$

$$A_\lambda = (1-\alpha) - \alpha(1-\alpha)(2-\alpha)\frac{\gamma}{2(1-\gamma)} \quad \ldots\ldots\ldots (119)$$

These three equations will be expressed in tabular form.

Example.—Fig. 100 shows a 4-span beam having spans $l_1 = 5$ (unit lengths), $l_2 = 9$, $l_3 = 6$, and $l_4 = 5$. Fig. 100 shows how to determine the fixed points graphically. The writer determined the fixed points analytically and used the graphic construction as a check. Consider first the case when l_1 is loaded. The equivalent 2-span beam is shown in Fig. 100. The spans are: $l_\lambda = 5$ and $l_{lu} = 6.832$.

To determine the secondary fixed point,

$$i = \frac{5}{3 + 2\dfrac{6.832}{5}} = 0.872$$

$$\gamma = \frac{i}{l_\lambda} \quad \frac{0.872}{5} = 0.174$$

For $\gamma = 0.174$, the values are as given in Table 33.

TABLE 33.—VALUES FOR $\gamma = 0.174$.

γ	α	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.174	A_u (−)	0.000	0.013	0.023	0.028	0.030	0.028	0.026	0.021	0.015	0.008	0.000
	A_m (+)	0.000	0.131	0.253	0.365	0.470	0.568	0.661	0.750	0.835	0.918	1.000
	A_λ (+)	1.000	0.882	0.770	0.663	0.560	0.460	0.365	0.271	0.180	0.090	0.000
	$A_m - A_u$ (+)	0.000	0.118	0.230	0.337	0.440	0.540	0.635	0.729	0.820	0.910	1.000
	$\frac{A_u}{A_m - A_u}$	$\frac{0}{0}$	0.111	0.097	0.081	0.068	0.054	0.041	0.029	0.018	0.008	0.000

If l_2 is loaded, the equivalent 3-span beam has the spans, $l_l = 5$, $l_m = 9$, and $l_r = 4.714$ (see Fig. 100). l means left, m, middle, and r, right.

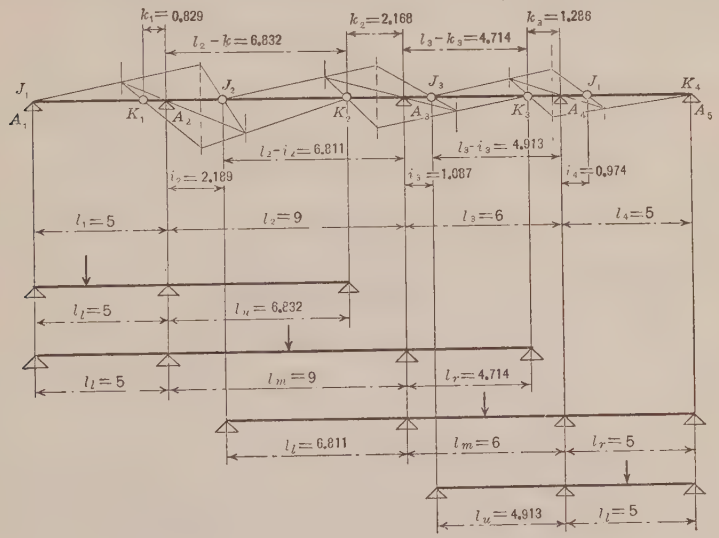


FIG. 100.

The secondary fixed points may then be found as follows: i falls into the fixed point of the multiple-span beam because l_2 is preceded in both cases by $l = 5$, hence,

$$i = 2.189;$$

$$k = \frac{9}{3 + 2\,\frac{4.714}{9}} = 2.223$$

$$\gamma_l = \frac{2.189}{9} = 0.243$$

and,

$$\delta = \frac{i}{k}\,\frac{2.189}{2.223} = 0.985$$

For $\delta = 0.985$, the values are as given in Table 34.

TABLE 34.—VALUES FOR $\delta = 0.985$.

γ_l	α	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.00
0.243	A_l (−)	0.000	0.089	0.145	0.172	0.176	0.162	0.135	0.100	0.063	0.028	0.000
	A_r (−)	0.000	0.031	0.069	0.110	0.148	0.177	0.192	0.187	0.157	0.097	0.000
	L (+)	1.000	1.023	0.990	0.911	0.797	0.660	0.509	0.358	0.215	0.092	0.000
	R (+)	0.000	0.097	0.225	0.372	0.528	0.679	0.818	0.930	1.005	1.033	1.000
	$L - A_l$ (+)	1.000	0.934	0.844	0.738	0.620	0.498	0.374	0.258	0.152	0.064	0.000
	$R - A_r$ (+)	0.000	0.066	0.156	0.262	0.380	0.502	0.626	0.742	0.848	0.936	1.000
	$\frac{A_l}{L - A_l}$	0.000	0.096	0.171	0.234	0.285	0.326	0.362	0.390	0.414	0.432	$\frac{0}{0}$
	$\frac{A_r}{R - A_r}$	$\frac{0}{0}$	0.465	0.444	0.420	0.389	0.353	0.307	0.252	0.185	0.103	0.000

If l_3 is loaded, the spans of the equivalent 3-span beams are: $l_l = 6.811$; $l_m = 6$; and $l_r = 5$.

The secondary fixed points are then,

$$i = \frac{6}{3 + 2\,\frac{6.811}{6}} = 1.138$$

k (identical with the fixed point of the 4-span) $= 1.286$

$$\gamma_l = \frac{i}{l_m} = \frac{1.138}{6} = 0.190$$

$$\delta = \frac{i}{k} = \frac{1.138}{1.286} = 0.885$$

For $\delta = 0.885$, the values are as given in Table 35.

TABLE 35.—VALUES FOR $\delta = 0.885$.

γ_l	α	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.190	A_l (−)	0.000	0.032	0.052	0.062	0.065	0.060	0.051	0.039	0.025	0.011	0.000
	A_r (−)	0.000	0.021	0.044	0.067	0.086	0.101	0.107	0.103	0.085	0.051	0.000
	L (+)	1.000	0.951	0.875	0.778	0.667	0.544	0.420	0.298	0.182	0.082	0.000
	R (+)	0.000	0.102	0.221	0.351	0.485	0.616	0.738	0.844	0.928	0.980	1.000
	$L - A_l$ (+)	1.000	0.919	0.823	0.716	0.601	0.485	0.369	0.259	0.157	0.071	0.000
	$R - A_r$ (+)	0.000	0.081	0.177	0.284	0.399	0.515	0.631	0.741	0.843	0.929	1.000
	$\frac{A_l}{L - A_l}$	0.000	0.035	0.063	0.087	0.107	0.124	0.138	0.150	0.160	0.163	0
	$\frac{R}{R - A_r}$	$\frac{0}{0}$	0.255	0.247	0.234	0.216	0.196	0.170	0.139	0.101	0.055	0.000

If l_4 is loaded, the spans of the equivalent 2-span beam are: $l_u = 4.913$, and $l_\lambda = 5$.

The secondary fixed point in l_λ is,

$$i = \frac{5}{3 + 2\,\frac{4.913}{5}} = 1.007$$

$$\lambda = \frac{1.007}{5} = 0.201$$

For $\gamma = 0.201$, the values are as given in Table 36.

For the sake of convenience the writer tabulated $L - A_l$ and $R - A_r$ for the 3-span beams and $A_m - A_u$ for the 2-span beams. These values are needed for

TABLE 36.—VALUES FOR $\lambda = 0.201$.

γ.	α.	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.201	A_u (−)	0.000	0.023	0.037	0.045	0.049	0.048	0.043	0.035	0.025	0.013	0.000
	A_m (+).....	0.000	0.145	0.273	0.390	0.497	0.595	0.685	0.769	0.849	0.926	1.000
	A_λ (+).....	1.000	0.878	0.764	0.655	0.552	0.453	0.358	0.266	0.176	0.087	0.000
	$A_m - A_u$ (+)	0.000	0.122	0.236	0.345	0.448	0.547	0.642	0.734	0.824	0.913	1.000
	$\frac{A_u}{A_m - A_u}$	$\frac{0}{0}$	0.182	0.156	0.133	0.110	0.088	0.070	0.048	0.030	0.014	0.000

the determination of the moments inside the loaded span. The values, $\frac{A_l}{L - A_l}$, $\frac{R}{R - A_r}$, and $\frac{A_u}{A_m - A_u}$, will facilitate the work greatly. Multiplying $\frac{A_l}{L - A_l}$ by l_l gives the point of application of $L - A_l$. The distance, $l_l \frac{A_l}{L - A_l}$, is to be measured from L to the right. Similarly, the distance, $l_r \frac{A_r}{R - A_r}$, is the point of application of the resultant of R and A_r, and is to be measured from R to the left. The distance, $l_\lambda \frac{A_u}{A_m - A_u}$, has similar meaning and is to be scaled from A_m to the loaded span. These distances may be scaled vertically as shown in Fig. 101.

For the inside spans of the multiple-span beam there are two curves. To get the lower one, for instance for l_2, multiply the values, $\frac{A_l}{L - A_l}$, by $l_l = 5$. Thus, this curve is the influence line for the location of the resultant of L and A_l. The influence line for the location of the resultant of R and A_r is the upper curve which has been developed by multiplying $\frac{A_r}{L - A_r}$ by $l_r = 4.714$. The vertical distances between the lower and upper curves show simple beams the left reaction of which is $L - A_l$ and the right reaction, $R - A_r$. For every position of the load there is a simple beam. The position of the load on these simple beams is shown by the straight line drawn at 45 degrees. These simple beams change their positions and their lengths.

If $\alpha = 0$, the left end of this beam is at Point L. As the load moves to the right, the left end of the beam (point of application of the resultant of forces, L and A_l) is moving to the right too. Finally, the load arrives in Point R. In this case, $\alpha = 1$. Tables 34 and 35 show the value,

$$\frac{A_l}{L - A_l} = \frac{0}{0 - 0} = \frac{0}{0}$$

Without using higher mathematics, it can be shown statically that $\frac{0}{0}$ in this case equals i_2 (the index, 2, indicating that the second span of the n-span beam is loaded), or, in other words, when the load arrives in the right middle support of the equivalent 3-span beam the point of application of the resultant of the two left reactions, A_l and L, is in the left fixed point of the equivalent 3-span beam (secondary fixed point of the n-span beam). Similarly, if the load is in Point L, the right end of the simple beam (point of application of the right reactions, R and A_r) is in the right fixed point of the 3-span beam.

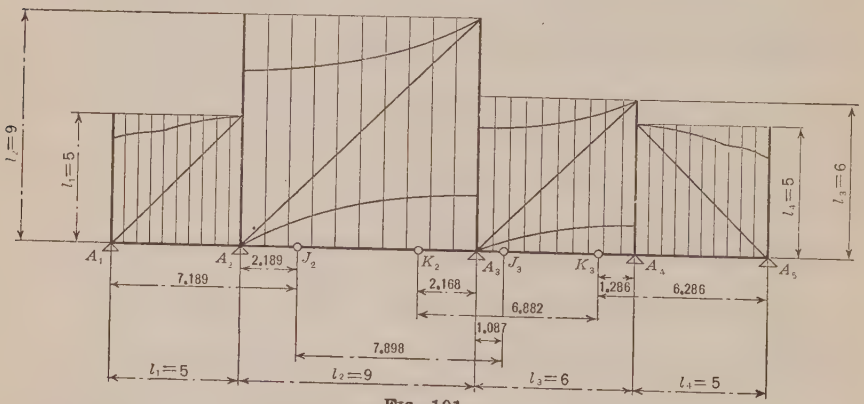


FIG. 101.

To prove this, imagine that the load arrived at Point L from the left (Fig. 102 (a) to (c)). From the theory of the fixed points the moments will pass through Point K (right fixed point of the middle span). Fig. 102 (b) and (c) shows the resulting 2-span beam. As the load is approaching Point L, K remains the right support of the 2-span beam; and, finally, if the load reaches Point L the left span of the 2-span beam may be omitted as being not stressed and thus the right end of the simple beam is at K or at the right fixed point of the middle span of the equivalent 3-span beam.

The influence lines of Fig. 101 will facilitate the work greatly inside the span. The tabulations give the reactions of the simple beam; Fig. 101 shows the distances of any section from the reactions and from the load if it is desired to determine influence lines for moments inside the span.

Outside the loaded span the influence line for A_l is the influence line for every moment, reaction, and shear left of the loaded span. Note, however, that every quantity has its own multiplier; and, correspondingly, on the right from the loaded span, A_r, similar relations hold.

To determine, for instance, the influence line for A_1 (Fig. 101)—as long as the load is between A_1 and A_2 the influence line for A_1 is identical with A_λ as given in Table 33. If the load is between A_2 and A_3 the influence line for A_1 is identical with A_l, as given in Table 34. If the load is between A_3 and A_4 the influence line for A_1 is that for A_l, as given in Table 35, multi-

plied by $-\frac{2.189}{7.189}$ (if l_3 is loaded, the outside left reaction is in I_2; this outside left reaction, A_l, is the resultant of A_1 and A_2). If the load is between A_4 and A_5, the influence line for A_1 is that for A_u, as given in Table 36, multiplied by $-\left(\frac{1.087}{7.898}\right) \times \left(-\frac{2.189}{7.189}\right)$.

As soon as the fixed points are found and the tabulations made, everything else may be computed by slide-rule.

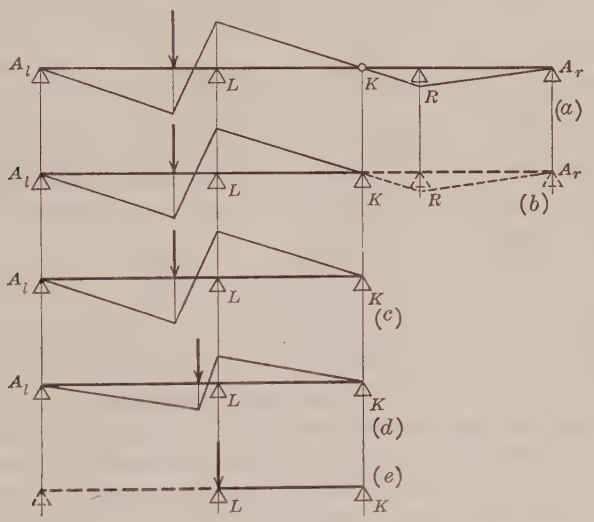


FIG. 102.

Somewhat more complicated is the computation for variable moments of inertia. If the moment of inertia does not change inside the span, in order to use the tables, determine the fixed points of the n-span beam n times. If the moment of inertia of the loaded span be I_l, and that of any other span, I_p, multiply every span length by the corresponding $\frac{I_l}{I_p}$. Thus may be obtained for every loaded span a different n-span beam. After the n-span is found the method is the same as before.

For uniformly distributed loads the functions may simply be integrated. For instance, by remodeling Equation (111):

$$A_1 = -C_1 \alpha (1 - \alpha) + C_2 \alpha (1 - \alpha) (2 - \alpha)$$

and for a uniformly distributed load extending from L to α,

$$A_l = C_1 \int_0^\alpha \alpha (1 - \alpha)\, d\alpha + C_2 \int_0^\alpha \alpha (1 - \alpha) (2 - \alpha)\, d_\alpha$$
$$= C_1 \left(\frac{\alpha^2}{2} - \frac{\alpha^3}{3}\right) + C_2 \left(\alpha^2 - \frac{2}{3}\alpha^3 + \frac{\alpha^4}{4}\right)$$

which values might be tabulated.

If the load extends from L to R,

$$A_l = C_1 \left[\frac{\alpha^2}{2} - \frac{\alpha^3}{3}\right]_0^1 + C_2 \left[\alpha^2 - \frac{2\,\alpha^3}{3} + \frac{\alpha^4}{4}\right]_0^1$$

or,

$$A_l = -\frac{C_1}{6} + \frac{7}{12} C_2$$

The values, A_l, thus obtained are to be multiplied, of course, by the load.

If all the spans are equal, the influence lines may be taken from the tabulations without any previous work, because in this case the fixed points and, consequently, the equivalent beams are known in advance.

Conclusions.—

(1) There never will be a better method for dead loads than that of the authors, which is built up on the broadest mathematical vision.

(2) The writer claims originality in discovering the fact that every n-span beam is an aggregate of n-2 three-span and 2 two-span beams; and in introducing the secondary fixed points and the influence lines for the location of the equivalent simple beams.

(3) The writer's original problem was: Where are the points of application of certain groups of reactions of an n-span beam?

(4) The writer believes that this question is important for every kind of continuous structures, such as multi-story rigid frames, Vierendeel girders, etc.

L. H. Nishkian* and D. B. Steinman,† Members, Am. Soc. C. E. (by letter).—The generous number and quality of the discussions elicited by this paper have been gratifying to the writers as an evidence of the interest evoked. That the paper has filled an educational need is indicated by the fact that it has already been adopted as a required text in engineering classes and by the requests that have been received that it be made available in pamphlet form for the use of students. That the method has filled a practical need is evidenced by its adoption by engineers in their office practice.

The discussions have supplemented the paper in fulfilling its main purpose, namely, to make available to engineers short and easily applied methods for the design of continuous beams and frames; further, they have brought forward interesting comparisons between European methods and those developed by the writers. The European methods, however, were not available in current American literature and were known to few engineers. The discussions added to the original paper make up a valuable symposium on the various methods for the analysis of indeterminate beams and frames.

The writers believe that their method of conjugate points offers advantages, in simplicity of development, presentation, and application, over the various European methods given in the discussions.

Graphical scientific methods are unquestionably superior to the procedures that have been generally available to the busy designer, whether he used analytical methods or empirical coefficients of building codes. In directing attention to the availability of such graphical methods, and in presenting the

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most practical graphic method they have been able to develop, the writers feel that they have accomplished a useful purpose.

Professor Richart contributes an outline of Fidler's method for the analysis of continuous beams, and points to its similarity, in certain elements, to the writers' method of conjugate points. That various constructions devised for the solution of the same problem would develop points of resemblance or correspondence is naturally to be expected. The principal distinguishing feature is generally the method of analysis and the viewpoint from which the construction is developed. From this standpoint, the concept of conjugate points is quite different from Fidler's method and from the other methods brought out in the discussion.

The physical concept pointed out by Professor Richart, namely, that the distances of the U and V-points from the M-line are proportional to the slopes of the elastic curves at the respective supports, was not developed by the writers. This interesting and useful fact is apparently the basis for Fidler's method.

In addition to Professor Richart's discussion of the Fidler method, Mr. Cotten has given a detailed presentation of the same method. Mr. Doerfling and Mr. Beanfield have likewise presented the older European graphical methods.

Mr. Doerfling and Mr. Beanfield as well as Professor Richart suggest that it is preferable to locate first the projections of the conjugate points on the base line ("fixed points") rather than to locate the conjugate points directly. The solution of hundreds of problems by the method of conjugate points has not led the writers to that conclusion. In those cases where the usual pennant diagram does flatten out too much, it is a simple matter to draw one at random in order to locate the conjugate point or the vertical on which it lies. This flexibility of the writers' pennant diagram construction is illustrated in Figs. 6, 7, 8, and 9.

Mr. Whitney has presented an analytical method for the solution of continuous beams. To those who prefer such a method to the graphical, this should prove valuable. In some intricate problems a combination of the algebraic and graphical methods with Fidler's physical concept of the U and V-points should be of great assistance.

Mr. Godfrey's discussion appears to go somewhat at a tangent; but the admitting of some truth in his statements as applied to present-day concrete does not necessarily lead to discarding the Theorem of Three Moments. It is still the best average obtainable. If two apparently identical specimens showed a variation of 100% in important structural properties, the logical conclusion should be not to use that material at all. Mr. Godfrey does not recommend that, judging from his statements. The writers do not believe that concrete made under the present knowledge of the art will result in a 100% variation in its elastic properties.

The writers' attention has been directed, through the courtesy of Mr. Gould, to a report of tests made in Germany on full-sized continuous beams of reinforced concrete. The results of these tests have been reported by

Dr. Ing. E. Probst,* who states that the relative deflections of the test beams in the different spans varied only 3 to 5% from the theoretical relative deflections calculated for the same spans assuming homogeneous sections. Since the bending moments depend on the relative deflections, these percentages would express the error, from all causes, in the calculated bending moments. These findings from actual tests should effectively answer the objections raised by Mr. Godfrey, as well as those raised by Mr. Turner, against the practical applicability of the standard theories of continuous beam analysis.

Mr. Godfrey likewise attacks the theory of secondary stresses. The deviations of measured secondary stresses from calculated secondary stresses, cited by him, hardly bear out his argument that secondary stresses should be ignored; rather, they bear out the writers' contention that extreme refinement in the determination of secondary stresses is unwarranted. This consideration provides the justification for the writers' effort, in this paper, to introduce the use of short-cut graphic methods for the approximate evaluation of secondary stresses.

Mr. Spitzer feels that the "margin of carelessness and poor work" is cut down by more accurate methods of design. Although favoring a much more extensive and thorough inspection of concrete structures than is practiced at present, the writers do not feel that more accurate methods of design will reduce the safety factor, but rather that they produce safer structures at less cost.

Mr. Turner raises the question of "rhombic distortion", apparently referring to shear deflections. Undoubtedly it would be possible to devise some graphical method for designing continuous beams with due allowances for shear distortions. It would have practical value only in short deep beams. For beams of the ordinary ratios of length to depth, the shear correction would be an unjustifiable refinement.

In discussing two-moment equations for concentrated loads or unequal spans, Mr. Turner states that: "* * * the negative moment over any interior support equals the numerical sum of the adjacent positive center moments between supports". This cannot be generally true, therefore something else must be meant. Mr. Turner also states that: "Shear distortion increases the deflection of the simple beam but reduces the deflection of the continuous beam". This statement does not sound reasonable. The work equation does not permit one to believe that any distortion reduces the deflection. The only conclusion to be drawn from the analogy of the springs is simply that the statically indeterminate or continuous beam deflects less than the simple beam, which of course is true. The writers do not believe that, in the usual structural problem, any further refinement in calculating moments is justifiable.

Professor Cross senses the real purpose of this paper, namely, to present the subject to American engineers in a concise form developed along original lines from primary principles as distinguished from a translation or compilation of existing literature in other languages.

* "Vorlesungen über Eisenbeton," pp. 443 to 495.

The problem of the two-post trussed beam mentioned by Mr. Higgins is much more involved than appears at first glance. Consideration must be given to the end moment due to eccentricity and to settlement of supports due to sagging. It readily yields to solution by the graphic methods presented in the paper.

As expected by Mr. Walker, the application of the writers' graphic methods to more than forty structures has shown that little, if any, more time is required to make a correct design than by the use of empirical coefficients. If all the loads were uniform and all the spans equal, the use of arbitrary coefficients might give a rational design; but for any other condition a satisfactory and economical design will not result. Mr. Walker's criticism, that the application of exact methods to haunched beams of the magnitude of the beam given as an example is hardly justifiable, is well taken. Unless there are a large number of identical beams, such refinement may be unwarranted. The tables presented by Mr. Ruppel should prove helpful in applying the graphic method to the solution of any important problem involving varying moments of inertia.

The multiple-story problem presented by Professor Derickson will naturally be involved since so many elements have to be considered. For practical purposes, the moments in l_2 (Fig. 45) can be obtained closely enough by assuming the far ends of h_2, h_3, l_1, and l_3 as fixed.

Concerning the moment of inertia of reinforced concrete beams, it is the writers' practice to locate the neutral axis and then calculate the I (moment of inertia) of the compression concrete and all the steel about the neutral axis, multiplying the steel area by $n = \frac{E\,s}{E\,c}$ (usually taken as 15). This moment of inertia can be used in the formula, $M = K_c \frac{I_c}{d_c}$, in which, K_c = the extreme fiber stress in the concrete, and d_c = the distance to that fiber from the neutral axis.

In calculating the deflection of concrete beams, it is the writers' practice to substitute for I in the usual deflection formulas, its equivalent, $\frac{M \times d}{K}$. It is evident that the deflection of a given beam is dependent only on the extreme fiber stress and the distance to the neutral axis. This is equivalent to taking twice the moment of inertia of the compression area about the neutral axis and using it in the usual deflection formulas.

Since in a continuous beam the moment of inertia is involved only in so far as it affects the deflection of the beam, the foregoing method would be applicable, that is, to locate the neutral axis and obtain comparative moments of inertia by considering the compression area only. In locating the neutral axis it will be more accurate of course to consider everything that may affect its position as, for instance, tension in the concrete. For computing deflections, the writers believe the tension in the concrete is quite important and cannot be neglected. For calculating comparative moments of inertia, the tensile resistance of the concrete may be neglected without appreciable error.

In the design of the usual concrete beams of uniform depth, the writers generally assume that I is constant throughout the span. When adjoining spans have different moments of inertia, it is the writers' practice to compute the comparative moments of inertia at the common support; if the stresses used are the same in the two beams, it is sufficiently accurate to use the depths of the beams without locating the neutral axis.

The value of E to be used depends directly on what I is used. For computing moments, E is eliminated since it is uniform throughout. For computing deflections or moments due to settlement of supports, the writers use twice the moment of inertia of the compression area about the neutral axis, assuming $E_c = 2\,000\,000$ lb. per sq. in. for the usual grade of concrete.

Professor Wilson's discussion outlines still another method of attacking statically indeterminate structures. To one who has once fully digested the method, it is probably as rapid as any other and, for the more complicated cases, it may be more rapid. All algebraic methods, however rapid, lack one important advantage possessed by most graphical methods, namely, that they show at a glance the complete answer with moments at all points in the beam. In graphic methods, automatic checking against errors is provided, and visual inspection and review are facilitated.

Mr. Cotten has performed a useful service in making available to American engineers the elements of the Fidler method which he states "have been available only in rather fragmentary and not readily accessible form". However, he seems to be so steeped in the Fidler method that any other viewpoint appears to him to be upside down. The method of conjugate points as presented by the writers is based on the concept that if a point on the M-line is known in one span of a continuous beam, then a point can be located in each span. Everything that follows is merely a geometric process of locating these points. U, V, and T-points are employed only because their use appears to give the shortest method of obtaining the result. It is entirely possible to locate the conjugate points without the use of the U and V-points. As a matter of fact, that is what was done originally, and the use of the U, V, and T-points was developed later. If Mr. Cotten will realize that the writers' method is based on an entirely different concept from that of Fidler, then everything will not appear reversed to him. The writers do not feel it necessary to answer in any more detail the claim made by Mr. Cotten and Mr. Ruppel that the method of conjugate points "is identical" with that of Fidler.

Professor Granger's method of solving for moments by equilibrium polygons treating moment areas as loads, is quite ingenious and interesting. He uses the basic principles from which Equation (7) is developed. A study of Fig. 57 reveals many of the constructions of the method of conjugate points and shows the physical meaning to the various points, lines, and constructions used in that method.

Professor Granger feels that for two and three spans the analytic methods are shorter. If the moments at only one or two points are required, that may be true. The writers believe, however, that the shorter methods given in Fig. 20 for such conditions are fully as quick as any analytic method.

Mr. Gould has explained Strassner's method for making the secondary correction for a frame with unsymmetrical loads which the writers stated (in Section 9) would be necessary for exact results. As Professor Derickson and Mr. Ruppel have suggested that a solution for this secondary correction be given, the writers have added Figs. 103 to 106, inclusive, which indicate the procedure by the method of conjugate points. The basis for this solution is the locating of the points of inflection, H_1, H_2, and H_3, in the spans l_1, l_2, and l_3, respectively, for a horizontal load at A, and then rotating the M-line about these points of inflection such an amount as to make the shears, S_1 and S_2, across the respective side posts equal in magnitude. These shears are represented by the slopes of the M-line in the respective end spans.

Fig. 103 shows the construction as applied to a symmetrical portal frame hinged at the base. In this case, the point of inflection, H_2, for a horizontal load at A, is located at the center line of the middle span; and the M-line is simply rotated about that point into a horizontal position. The corrected M-line will then have symmetrical slopes in the side spans, representing equal (but opposite) values of the shears across the vertical posts.

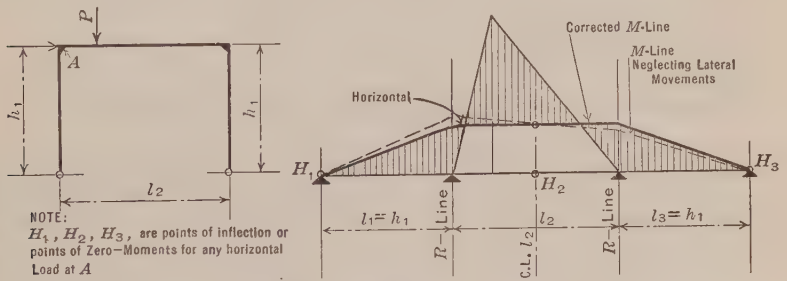

FIG. 103.—MOMENTS IN PORTAL FRAME, HINGED AT BASE, UNDER UNSYMMETRICAL LOADING.

Fig. 104 shows the construction as applied to a symmetrical portal frame with the posts fixed at the base. For any horizontal load at A, the point of inflection, H_2, in the middle span is located at the center line; and the symmetrical points of inflection, H_1 and H_3, in the side spans are located graphically, utilizing the principle of Fig. 27. This construction for locating H_1 and H_3 is shown complete in the lower diagram of Fig. 104. The M-line, in the upper diagram, is then rotated about the inflection points, H_1, H_2 and H_3, to a position yielding equal shears (slopes) in the side spans. There are various simple methods of accomplishing this; the graphic method illustrated consists in drawing Line 3 across the middle span, with end heights equal to the heights of H_1 and H_3, respectively; a line through H_2 parallel to Line 3 will then give the corrected M-line.

Fig. 105 shows the construction as applied to an unsymmetrical portal frame hinged at the base. For the location of the point of inflection, H_2, the principle of Fig. 27 is again utilized, and the construction is shown complete in the lower diagram of Fig. 105. In this case, the effect of any horizontal displacement of the top of the portal is represented by laying off

V_1 and U_3 to any heights inversely proportional to the squares of the respective spans, l_1 and l_3. The M-line thus obtained locates the inflection point, H_2. The M-line in the upper diagram is then rotated about H_2 to a position yielding equal shears (slopes) in the side spans. In Fig. 105, this is accomplished graphically by marking a pole (intersection of Lines 3 and 4), any ray from which will cut the reaction verticals at heights proportional to the respective side spans; a ray, 5, drawn from this pole through H_2 gives the required corrected M-line.

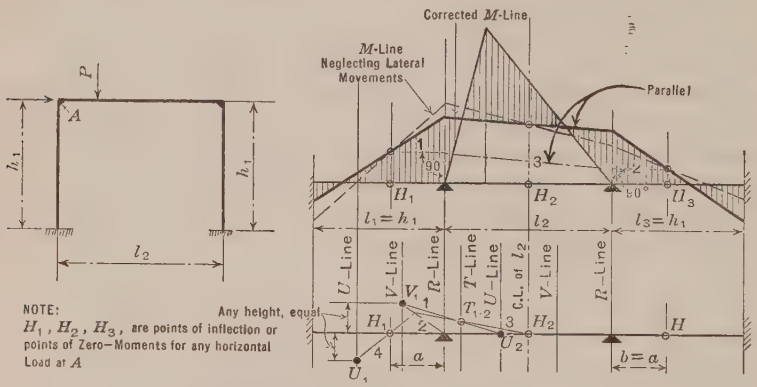

FIG. 104.—MOMENTS IN PORTAL FRAME, FIXED AT BASE, UNDER UNSYMMETRICAL LOADING.

Fig. 106 shows the construction as applied to an unsymmetrical portal frame with the posts fixed at the base. The points of inflection, H_1, H_2, and H_3, for any horizontal load at A are located by the principle of Fig. 27, and the construction, shown complete in the lower diagram of Fig. 106, is identical, except for the fixed ends, with the corresponding construction in Fig. 105. The inflection points, H_1, H_2, and H_3, are then projected up to the M-line in the upper diagram of Fig. 106, and the M-line is then rotated about these inflection points to a position yielding equal shears (slopes) in the side spans. In Fig. 106, this is accomplished graphically by a combination of the principles used in Figs. 104 and 105. In this case, the auxiliary pole (intersection of Lines 3 and 6) is one from which any ray cuts the reaction verticals at heights above H_1 and H_3, respectively, proportional to the horizontal distances of H_1 and H_3; a ray, 7, drawn from this pole through H_2 gives the required corrected M-line.

These constructions (Figs. 103 to 106, inclusive) show how easily the method of conjugate points solves the problem of unsymmetrical loading on portal frames, and serve to illustrate the flexibility and wide range of applicability of the graphic method.

Mr. Doerfling and Mr. Beanfield have explained in considerable detail the European methods of obtaining moments in continuous beams. Mr. Doerfling's discussion is quite complete and covers practically every problem treated in the original paper. It is regrettable that he had not found time to submit his discussion as a separate paper many years ago. Much progress would have resulted.

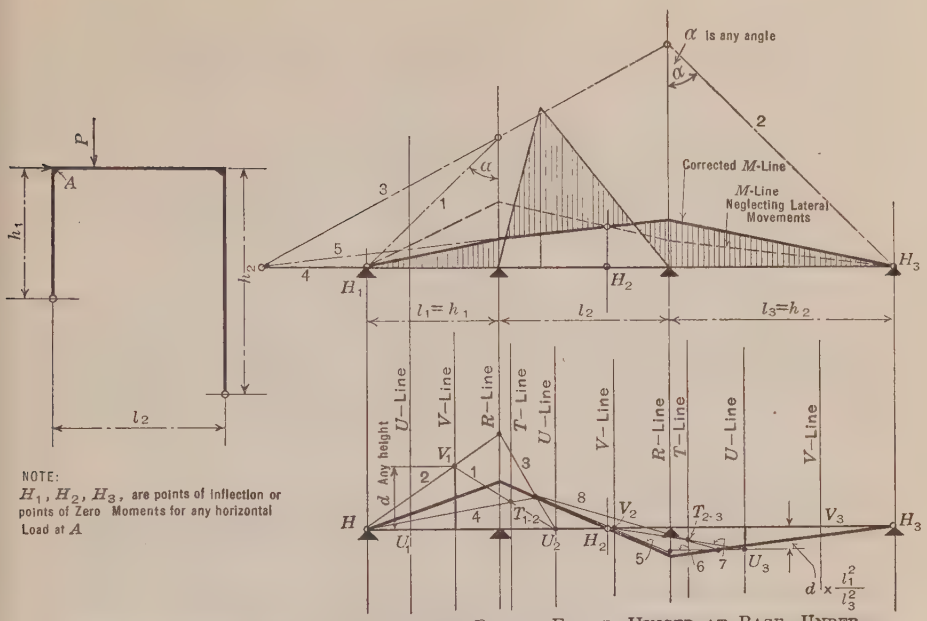

FIG. 105.—MOMENTS IN UNSYMMETRICAL PORTAL FRAME, HINGED AT BASE, UNDER UNSYMMETRICAL LOADING.

Mr. Beanfield's contention that the Ritter method is simpler than the method of conjugate points, does not seem to be borne out by the statements of Mr. Gould who is also familiar with the Ritter method.

The three advantages of the Ritter method given by Mr. Beanfield under the headings (*a*), (*b*), and (*c*), may be considered as follows:

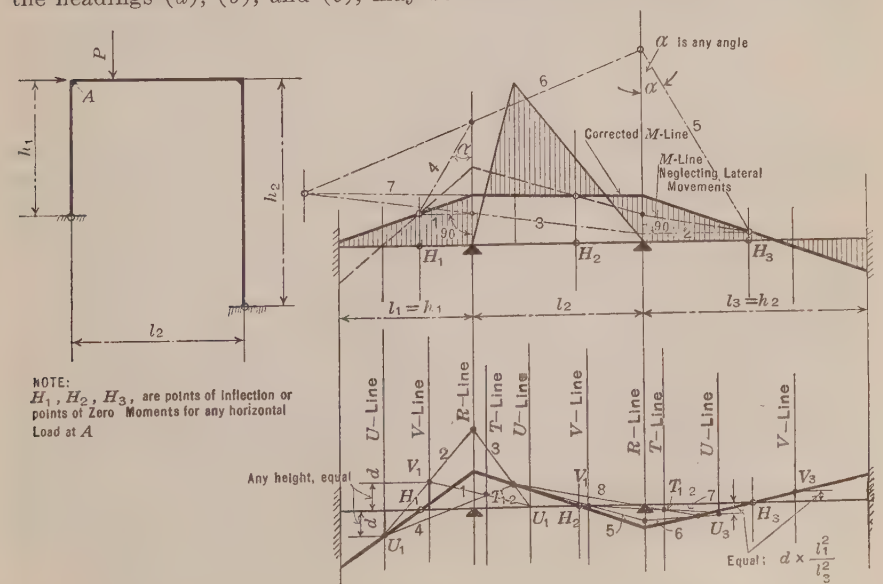

FIG. 106.—MOMENTS IN UNSYMMETRICAL PORTAL FRAME, FIXED AT BASE, UNDER UNSYMMETRICAL LOADING.

(*a*) In the writers' method for triangular and parabolic moment areas, no computations are necessary, as the required answer is one-half and two-thirds of the respective heights. For unsymmetrical loads, computations for the center of gravity of the moment area also become necessary in Ritter's method.

(*b*) In general, it is of no advantage (but rather a marked disadvantage) to compute the moments produced by loading each span separately. Where it is an advantage, it can be done by the method of conjugate points as well.

(*c*) Point (*c*) has already been answered in the discussion of pennant diagrams.

Mr. Ruppel has presented an extremely valuable set of tables, which have been transformed from those published by Strassner, and constructed along the lines suggested by the writers in Section 14. Although referring to Fidler's method, Mr. Ruppel's tables are directly applicable in the method of conjugate points as well as in other graphical methods. With the aid of these tables, practically any problem in continuous beams with varying moments of inertia may be solved readily.

Mr. Gedo has made a commendable attempt to resolve the problem of multiple-span continuous beams into component elements of 2-span and 3-span beams. It is regrettable that his basic theorem proves to be fallacious on close examination. Although the fixed points outside the loaded span (Fig. 95) are points of zero bending moment, they are not necessarily points of zero deflection; consequently Mr. Gedo is in error in assuming that the moment diagram will remain unchanged when reaction supports are placed under the outer fixed points. Similarly, although it is true, as Mr. Gedo demonstrates, that a fixed point in an unloaded span is the point of application of the resultant of all the external reactions beyond it, it does not follow that that resultant is identical with the reaction which would be developed at the fixed point if it were made the end of the continuous beam. Consequently, the reduced 3-span beam (Fig. 97) will not yield the same moment-diagram as the complete multiple-span continuous beam (Fig. 95). The necessity of locating new fixed points for the reduced continuous beam is further confirmation. Moreover, even if Mr. Gedo's basic postulate had been correct, the procedure he has developed appears unduly indirect and laborious. Once the fixed points are located, it is unnecessary to consider 2-span or 3-span beams; all bending moments and influence lines may be found by considering each span independently, as illustrated by the writers in Fig. 26. Mr. Gedo's discussion, nevertheless, contains some useful elements which will be of interest to students of the subject.

In conclusion, the writers wish to express their grateful appreciation to those who have so generously contributed discussions. These contributions have greatly amplified the scope and enhanced the usefulness of the paper. The writers trust that it and the discussions, rendering available expeditious methods of analysis, will serve the intended purpose of directing engineers to the more extensive use of advantageous forms of construction and of methods of analysis inadequately utilized in the past.

AMERICAN SOCIETY OF CIVIL ENGINEERS
INSTITUTED 1852

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Paper No. 1599

RELATION OF DEPTH TO CURVATURE OF CHANNELS*

By H. C. Ripley,† M. Am. Soc. C. E.

With Discussion by Messrs. E. G. Walker, Hubert Engels, Olin H. Landreth, Leonard C. Jordan, Lewis M. Haupt, B. Malaval, David A. Molitor, Edward N. Chisolm, Jr., P. N. Fawcett, and H. C. Ripley.

Synopsis

The object of this paper is to record the results of some of the writer's investigations into the law of river hydraulics. As a result of these investigations, two empiric formulas have been devised by means of which the cross-profile of a channel may be computed. Some characteristics of the law were also disclosed, which are believed to be the most important discoveries in that law in more than fifty years. In conclusion, some practical applications of these formulas are given which furnish the solution to problems that heretofore have been considered not susceptible of solution.

Introduction

It is common knowledge with those who are familiar with river conditions, that a greater depth is found in bends of channels than in the straight reaches and that the elevation of the water surface on the concave side of the channel is higher than that on the convex side.

The writer first observed these conditions about 1872, when he was making a survey of the Red River in Louisiana for the removal of the Great Raft which obstructed about 100 miles of that stream. Since that time he has never ceased in his efforts to determine the cause of the greater depth in bends, and the relation of that depth to the curvature of the channel.

Many other engineers have also been interested in the same problem. By the use of a colored liquid, a French engineer found that in bends the water descended at the concave side of the channel, crossed over at the bottom, and came up on the convex side. An English engineer has demonstrated the helicoidal movement of water in bends, and another French engineer, M. R. H.

* Presented at the meeting of February 3, 1926.
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Gockinga, has devised the following formula (Formula (1)), for determining the difference in elevation of the water surface on opposite sides of the channel in bends:

$$Y = 0.235\ V^2 \log \left(1 - \frac{X}{R}\right) \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which,

V = the longitudinal velocity, in meters per second;
R = the radius of curvature; and
X and Y = the co-ordinates of a point on the surface referred to a system of rectangular axes through the intersection of the axis of the channel and the water surface. That is, X equals one-half the width of the channel and Y equals one-half the difference in elevation on opposite sides of the channel.

For a width of 200 m., a radius of curvature of 500 m., and a depth sufficient to give a velocity of 1 m. per sec., with a longitudinal slope of 1 in 10 000, M. Gockinga found a cross-slope twice as great as the longitudinal slope.

The accuracy of Formula (1) has been confirmed by observations on the Mississippi River where differences of head of about 1 ft. have been noted on opposite banks of bends with mean mid-section longitudinal slopes of about 0.4 ft. per mile. These results seem to indicate conclusively that the increased depth in bends is caused by the helicoidal movement of the water induced by the centrifugal force of the current on the concave side of the channel.

The only attempts to determine the relation of depth to curvature, known to the writer, were made by M. Fargue, a Frenchman, and the late Henry Mitchell, M. Am. Soc. C. E., of the United States Coast and Geodetic Survey. M. Fargue devised a formula for determining the radius of curvature necessary to produce a given maximum depth, which he designated as "Fargue's law of greatest depth". His formula is:

$$C = 0.03\ H^3 - 0.23\ H^2 + 0.78\ H - 0.76 \ldots\ldots\ldots\ldots (2)$$

in which,

C = the reciprocal of the radius of curvature, in kilometers; and
H = the low-water depth at the deepest point of the channel, in meters.

It is evident that Formula (2) is only applicable to the particular stream for which it was devised. Mr. Mitchell's formula is for determining the cross-profile of the Delaware River at Philadelphia, Pa., supposing it to be perfectly canalized so as to be reduced to the minimum width.*

His formula is:

$$Y = 33\,\frac{1\,100^2 - X^2}{1\,100^2} + \frac{289}{R}\left(\frac{1\,100^2 - X^2}{1\,100^2}\right) 2\,X \ldots\ldots\ldots (3)$$

in which,

R = the radius of curvature varying from 16 200 ft. to infinity; and
X and Y = the co-ordinates of the cross-profile, the origin being at the surface of the water in the center of the channel, and at a

* U. S. Coast Survey Rept., 1878, p. 125.

place where the width of the channel is 2 200 ft. and the depth 33 ft.

It will thus be seen that Formula (3) is quite limited in its application both as to curvature and character of channel.

The Writer's Formulas

The writer's formulas are modifications of the Mitchell formula so as to make them of general application to all sizes of streams and to all degrees of curvature, and to every character of channel whether that channel occupies the entire waterway or only a part of it; or whether it is across an outer bar created artificially by the action of a single curved jetty or by straight or curved jetties in pairs.

These formulas are as follows:

$$Y = D\left(1 - \frac{X^2}{W^2}\right) + D\,\frac{17.52}{R}\left(1 - \frac{X^2}{W^2}\right)X \quad \ldots\ldots(4)$$

$$Y = P\left(1 - \frac{X^2}{W^2}\right) + P\,\frac{26.28}{R}\left(1 - \frac{X^2}{W^2}\right)X \quad \ldots\ldots(5)$$

in which,

D = the mean depth of the channel, in feet, multiplied by 1.445;
P = the mean depth of the channel, in feet, multiplied by 1.65;
W = the half width of the channel, in feet;
R = the radius of curvature, in feet, on the concave side of the channel; and
X and Y = the co-ordinates of the cross-profile of the channel, the origin being in the center of the channel at the surface of the water.

$$Y = D\left(1 - \frac{X^2}{W^2}\right) + D\,\frac{5.34}{R}\left(1 - \frac{X^2}{W^2}\right)X \quad \ldots\ldots(6)$$

$$Y = P\left(1 - \frac{X^2}{W^2}\right) + P\,\frac{8.01}{R}\left(1 - \frac{X^2}{W^2}\right)X \quad \ldots\ldots(7)$$

in which,

D = the mean depth of the channel, in meters, multiplied by 1.445;
P = the mean depth of the channel, in meters, multiplied by 1.65;
W = the half-width of the channel, in meters;
R = the radius of curvature, in meters, on the concave side of the channel; and
X and Y = the co-ordinates of the cross-profile of the channel, the origin being in the center of the channel at the surface of the water.

Notations.—

1.—Formula (4) is applicable to all streams of whatever size where the channel occupies the entire width of the waterway, with certain restrictions to be indicated subsequently.

2.—Formula (5) is applicable to channels not occupying the entire width of the waterway and to channels at entrances created by the action of a single curved jetty.

3.—Formula (6) is the same as Formula (4) adapted to measurements in meters.

4.—Formula (7) is the same as Formula (5) adapted to measurements in meters.

5.—The algebraic sign of X must be recognized in the solution of the formulas.

6.—Whenever the radius of curvature is less than 40 times the square root of the area of the channel, no further deepening of the channel results from the increased curvature. Hence, in such cases, the value to be given to R in the formula must be equal to $40\sqrt{\text{area}}$.

7.—Whenever the radius of curvature is greater than about 50 times the square root of the area, the shape of the cross-profile does not conform strictly to that due to curvature. This result is presumably on account of the small helicoidal action when the curvature is slight, allowing other influences to predominate. It is believed, however, that where there are no other influences, the formula will almost exactly reproduce the actual cross-profile as shown in Fig. 5 (A), although the radius of curvature is 54.65 times the square root of the area $(54.65\sqrt{\text{area}})$.

8.—Formulas (5) and (7) are not applicable to straight channels nor to curved ones where the radius of curvature exceeds about 110 times the square root of the area of the channel. (The exact figure has not been determined.)

9.—On a "cross-over bar", where the channel is neither on a curve, nor in a straight reach, the maximum depth is about 14½% less than its computed value.

10.—Formulas (5) and (7) give a width of channel at mean depth about 20% greater than the actual width, hence, the width of the channel at mean depth (a little more or less), as given by these formulas, must be reduced by 20% to obtain its exact value.

Two important corollaries result from these findings: (1) That every artificial channel, whether it be a flume, an open aqueduct, a ditch, or a drainage canal, to discharge the greatest volume with a given slope, should have the form determined by Formula (4), both in straight and curved reaches; and (2) that bends in streams, the radii of curvature of which are greater than 40 times the square root of the area of the cross-section, are constructive bends and tend to stability of channel both in position and depth; whereas those bends the radii of curvature of which are less than 40 times the square root of the area of the cross-section are destructive bends, tending to cause a shifting of the position of the channel and to form cut-offs and crevasses.

Test of the Formulas.

In order to test the accuracy of the results from the use of these formulas and to show their general application to all sizes of streams and to all degrees of curvature, the writer has computed a large number of cross-profiles of channels ranging in size from the Brazos River with a cross-sectional area of 5 000 sq. ft., a width of 336 ft., and a radius of curvature of 1 250 ft., to the Mississippi River with a cross-sectional area of 153 000 sq. ft., a width of 3 000 ft., and a radius of curvature of 36 600 ft.

These tests embrace the following streams: (1) Brazos River, Texas; (2) Mississippi River; (3) Aransas Pass, Texas (jetty channel); (4) Rio Grande do Sul, Brazil (Canal Norte); and (5) Southwest Pass, Mississippi River.

A comparison of the results of these computations with data on the actual channels is given in Tables 1 to 25, inclusive, and on Figs. 1 to 8, inclusive. In addition to these twenty-five tables and eight diagrams, Tables 26 to 29, inclusive, and Figs. 9 and 10 give some theoretical results which will be explained subsequently.

TABLE 1.—Brazos River (Fig. 1 (*A*)).

Co-Ordinates, in Feet.			General remarks.
Computed.		Actual.	
X.	*Y.*	*Y.*	
255	0.00	0.00	Location, 3½ miles from the mouth.
250	0.60	0.60	Area of section, 5 293 sq. ft.
225	3.30	5.00	Width, 510 ft.
200	5.80	7.40	Mean depth, 10.38 ft.
175	7.90	9.60	Maximum depth, 15 ft.
150	9.80	11.60	Radius of curvature, infinite.
125	11.40	11.60	Value of D in Formula (4) = 15.
120	12.70	12.50	Area of the computed section is 3.9% less than that of the actual section.
75	13.70	13.00	The maximum depth of the computed section is the same as that of the actual section.
50	14.40	13.80	Computed from Formula (4).
25	14.90	14 50	The computed cross-profiles are shown on all diagrams in broken lines, and the actual ones in solid lines.
0	15.00	15.00	
— 25	14.90	14.50	
— 50	14.40	14.00	
— 75	13.70	13.70	
—100	12.70	12.60	
—125	11.40	11.40	
—150	9.80	10.30	
—175	7.90	9.20	
—200	5.80	7.50	
—225	3.30	4.40	
—250	0.60	0.60	
—255	0.00	0.00	

TABLE 2.—Brazos River (Fig. 1 (*B*)).

Co-Ordinates, in Feet.			General remarks.
Computed.		Actual.	
X.	*Y.*	*Y.*	
195	0.00	0.00	Computed from Formula (4).
175	7.99	10.00	Location, 4¾ miles from the mouth.
150	15.53	18.00	Area of section, 5 384 sq. ft.
125	20.63	21.00	Width, 390 ft.
100	23.58	23.50	Mean depth, 13.81 ft.
75	24.70	24.25	Maximum depth, 25 ft.
50	24.26	22.50	Radius of curvature, 2 900 ft.
25	22.31	20.50	Value of D in Formula (4) = 19.95.
0	19.95	17.50	Area of computed section is 4% less than that of the actual section.
— 25	16.93	15.50	This is Cross-Section No. 12 of the Official Survey of 1878.
— 50	13.00	13.00	
— 75	9.30	11.00	
—100	5.82	9.00	
—125	2.87	6.50	
—150	0.77	4.50	
—175	— 0.23	2.30	
—195	0.00	0.00	

TABLE 3.—Brazos River (Fig. 1 (*C*)).

Co-Ordinates, in Feet.			General remarks.
Computed.		Actual.	
X.	*Y.*	*Y.*	
190	0.00	0.00	Computed from Formula (4).
175	5.91	11.50	Location, 13 miles from the mouth.
150	13.59	20.00	Area of section, 4 896 sq. ft.
125	18.52	22.00	Width, 380 ft.
100	21.87	21.00	Mean depth, 12.88 ft.
75	23.08	20.00	Maximum depth, 22.00 ft.
50	22.74	20.00	Radius of curvature, 1 900 ft.
25	21.15	17.00	Value of *R* in Formula (4) is 2 800 ft., which is 40 times the square root of the area; this being the minimum radius of curvature that will produce increased depth of channel.
0	18.61	16.00	
— 25	15.43	15.00	
— 50	11.90	12.00	Value of *D* in Formula (4) = 18.61.
— 75	8.34	9.50	Area of computed section is 3% less than that of the actual section.
—100	5.03	7.00	
—125	2.30	5.00	This is Cross-Section No. 5 of the Official Survey of 1878.
—150	0.43	4.00	
—175	— 0.27	2.50	
—190	0.00	0.00	

TABLE 4.—Brazos River (Fig. 1 (*D*)).

Co-Ordinates, in Feet.			General remarks.
Computed.		Actual.	
X.	*Y.*	*Y.*	
168	0.00	0.00	Computed from Formula (4).
150	8.64	5.50	Location, 13½ miles from the mouth.
125	17.50	14.50	Area of section, 5 179 sq. ft.
100	23.13	19.60	Width, 336 ft.
75	25.97	23.60	Mean depth, 15.41 ft.
50	26.48	25.00	Maximum depth, 25 ft.
25	25.09	23.50	Radius of curvature, 1 250 ft.
0	22.27	21.50	Value of *R* in Formula (4) = 2 879 ft., which is 40 times the square root of the area, which is the minimum radius of curvature to produce increased depth.
— 25	18.47	19.50	
— 50	14.12	18.60	
— 75	9.69	16.30	Value of *D* in Formula (4) = 22.27.
—100	5.63	13.60	Area of computed section is 4% less than that of the actual section, and the maximum depth is 6% greater.
—125	2.38	10.80	
—150	0.40	6.90	
—168	0.00	0.00	

TABLE 5.—Brazos River (Fig. 1 (*E*)).

Co-Ordinates, in Feet.			General remarks.
Computed.		Actual.	
X.	*Y.*	*Y.*	
162	0.00	0.00	Computed from Formula (4).
150	4.90	4.00	Location, 13¾ miles from the mouth.
125	12.93	10.00	Area of section, 4 407 sq. ft.
100	18.25	14.50	Width, 324 ft.
75	21.24	16.00	Mean depth, 13.60 ft.
50	22.23	17.50	Maximum depth, 20 ft.
25	21.59	19.50	Radius of curvature, 3 500 ft., which is $52\frac{1}{2}\sqrt{\text{area}}$. (See Notation 7.)
0	19.65	20.00	
— 25	16.79	19.00	Value of *R*, in Formula (4) = 3 500 ft.
— 50	13.33	17.30	Value of *D*, in Formula (4) = 19.65 ft.
— 75	9.64	15.90	Area of computed section is 4% less than that of the actual section and the computed maximum depth is 11.11% more than the actual depth.
—100	6.07	13.00	
—125	2.97	9.00	
—150	0.76	3.40	
—162	0.00	0.00	

TABLE 6.—Brazos River (Fig. 2 (*A*)).

Co-Ordinates, in Feet.				General remarks.
Computed.			Actual.	
X.	*Y.*	*Y* + 5 ft.	*Y.*	
185	0.00	5.00	5.00	Computed from Formula (5). Location, 2 miles from the mouth.
175	4.72	9.72	14.00	
150	14.14	19.14	23.50	Total area of section, 375 ft. wide, 6 068 sq. ft.
125	20.30	25.30	30.00	Area of section, 5 ft. below water surface, 4 237 sq. ft.
100	23.74	28.74	25.00	
75	25.32	30.32	21.00	Width, 5 ft. below water surface, 370 ft.
50	24.03	29.03	19.50	Mean depth, 5 ft. below water surface, 11.45 ft.
25	21.73	26.73	18.00	Maximum depth, 30 ft.
0	18.32	23.32	15.60	Value of *P* in Formula (5) = 18.32.
— 25	14.25	19.25	14.50	Radius of curvature, 1 900 ft.
— 50	9.93	14.93	13.60	Value of *R* in Formula (5) = 3 116 ft., which is 40 times the square root of total area. (See Notation 6.)
— 75	6.24	11.24	13.50	
—100	2.20	7.20	13.50	
—125	—0.38	4.62	11.60	Area of computed section is 3.4 % more than that of the actual section and the maximum depth is 1% more. Channel occupies 65% of width of waterway.
—150	—1.20	3.80	8.60	
—175	—0.88	4.12	5.60	
—185	0.00	5.00	5.00	

TABLE 7.—Mississippi River (Fig. 3 (*A*)).

Co-Ordinates, in Feet.			General remarks.
Computed.		Actual.	
X.	*Y.*	*Y.*	
1 450	0.00	0.00	Computed from Formula (4).
1 400	4.53	10.00	Location, 33° 58′ 30″ N. lat.
1 200	21.08	32.00	Area of section, 134 275 sq. ft.
1 000	35.09	42.00	Width, 2 900 ft.
800	46.54	45.00	Mean depth, 46.30 ft.
600	55.36	47.00	Maximum depth, 66 ft.
400	61.81	53.00	Radius of curvature, infinity.
200	65.64	60.00	Value of R in Formula (4) = infinity.
0	66.91	64.00	Value of D in Formula (4) = 66.91.
— 200	65.64	65.00	Area of computed section is 5% less than that of the actual section and the maximum depth is 2.9% more.
— 400	61.81	59.00	
— 600	55.30	52.00	
— 800	46.54	50.00	
—1 000	35.09	47.00	
—1 200	21.08	40.00	
—1 400	4.53	15.00	
—1 450	0.00	0.00	

TABLE 8.—Mississippi River (Fig. 3 (*B*)).

Co-Ordinates. in Feet.			General remarks.
Computed.		Actual.	
X.	*Y.*	*Y.*	
1 500	0.00	0.00	Computed from Formula (4).
1 400	15.85	17.00	Location, 33° 52′ 00″ N. lat.
1 200	41.62	49.00	Area of section, 152 850 sq. ft.
1 000	60.48	82.00	Width, 3 000 ft.
800	72.85	81.00	Mean depth, 50.95 ft.
600	79.60	78.50	Maximum depth, 82 ft.
400	81.48	77.00	Radius of curvature, 36 600 ft.
200	79.23	75.00	Value of R in Formula (4) = 36 600 ft.
0	73.62	67.50	Value of D in Formula (4) = 73.62.
— 200	65.39	57.00	Area of computed section is 4% less than that of the actual section and the maximum depth is 0.6% less.
— 400	55.30	50.00	
— 600	44.08	43.00	
— 800	32.51	37.00	
—1 000	21.32	27.00	
—1 200	11.38	15.00	
—1 400	3.13	10.00	
—1 500	0.00	0.00	

TABLE 9.—Mississippi River (Fig. 4 (*A*)).

Co-Ordinates, in Feet.				General remarks.
Computed.			Actual.	
X.	*Y.*	*Y'.*	*Y.*	
1 120	0.00	0.00	0.00	Location, 30° 10′ 53″ N. lat.; 91° 08′ 45″ W. long.
1 000	107.71	53.09	18.00	Area of section, 206 670 sq. ft.
800	222.29	115.63	60.00	Width, 2 240 ft.
600	266.74	150.01	146.00	Mean depth, 92.26 ft.
400	256.45	161.14	172.00	Maximum depth, 178 ft.
200	216.84	153.94	150.00	Radius of curvature, 5 800 ft.
0	133.32	133.32	136.60	Column *Y* is computed for a value of *R* in Formula (4) of 5 800 ft. shown in the diagram by dotted line.
— 200	51.30	104.20	122.00	
— 400	—24.11	71.50	100.00	
— 600	—76.62	40.11	52.00	Column *Y'* is computed for a value of *R* in Formula (4) of 18 184 ft., which is equal to $40\sqrt{\text{area}}$.
— 800	—91.69	14.97	23.00	
—1 000	—53.63	0.99	10.00	Value of *D* in Formula (4) = 133.32 ft.
—1 120	0.00	0.00	0.00	Area of computed section is 4.88% less than the actual section and the computed maximum depth is 6.4% less than the actual depth.

TABLE 10.—Mississippi River (Fig. 4 (*B*)).

Co-Ordinates, in Feet.			General remarks.
Computed.		Actual.	
X.	*Y.*	*Y.*	
1 170	0.00	0.00	Location, 30° 00′ 07″ N. lat.; 90° 27′ 58″ W. long.
1 000	48.21	36.00	Area of section, 148 010 sq. ft.
800	85.94	88.00	Width, 2 340 ft.
600	106.06	108.00	Mean depth, 63.25 ft.
400	111.63	111.00	Maximum depth, 111 ft.
200	105.68	97.00	Radius of curvature, 18 300 ft., equal to $47.57\sqrt{\text{area}}$.
0	91.40	74.00	Value of *R* in Formula (4) = 18 300 ft.
— 200	71.70	65.00	Value of *D* in Formula (4) = 91.40 ft.
— 400	49.81	55.00	Area of computed section is 4.35% less than the actual area, and the computed maximum depth is 0.57% more than the actual depth.
— 600	28.66	44.00	
— 800	11.40	38.00	
—1 000	1.05	24.00	
—1 170	0.00	0.00	

TABLE 11.—Mississippi River (Fig. 4 (*C*)).

Co-Ordinates, in Feet.			General remarks.
Computed.		Actual.	
X.	*Y.*	*Y.*	
1 493	0.00	0.00	Location, 30° 11′ 44″ N. lat.; 91° 01′ 40″ W. long.
1 400	21.96	60.00	Area of section, 191 669 sq. ft.
1 200	62.42	106.00	Width, 2 986 ft.
1 000	89.54	102.00	Mean depth, 64.18 ft.
800	105.86	96.00	Maximum depth, 106 ft.
600	112.81	91.00	Radius of curvature, 23 300 ft., equal to $53.8\sqrt{\text{area}}$.
400	111.95	85.00	Value of R in Formula (4) = 23 300 ft.
200	104.77	80.00	Value of D in Formula (4) = 92.75 ft.
0	92.75	71.00	Area of the computed section is 4.04% less than the actual section, and the computed maximum depth is 6.42% more than the actual depth.
— 200	74.41	60.00	
— 400	60.22	52.00	
— 600	42.73	48.00	
— 800	26.38	48.00	
—1 000	12.74	43.00	
—1 200	3.24	28.00	
—1 400	—0.58	6.00	
—1 493	0.00	0.00	

TABLE 12.—Mississippi River (Fig. 5 (*A*)).

Co-Ordinates, in Feet.			General remarks.
Computed.		Actual.	
X.	*Y.*	*Y.*	
1 165	0.00	0.00	Location, 29° 58′ 14″ N. lat.; 90° 15′ 35″ W. long.
1 000	41.01	58.00	Area of section, 133 932 sq. ft.
800	74.65	73.00	Width, 2 330 ft.
600	93.11	96.00	Mean depth, 57.48 ft.
400	98.94	92.00	Maximum depth, 96 ft.
200	94.73	90.00	Radius of curvature, 20 000 ft.
0	83.06	80.00	Value of R, in Formula (4) = 20 000 ft., equal to $54.65\sqrt{\text{area}}$.
— 200	66.49	68.00	Value of D, in Formula (4) = 83.06 ft.
— 400	47.60	47.00	Area of computed section is 4.35% less than the actual section, and the computed maximum depth is 3.06% more than the actual depth.
— 600	28.95	30.00	
— 800	13.13	26.00	
—1 000	2.71	20.00	
—1 165	0.00	0.00	

TABLE 13.—Mississippi River (Fig. 5 (*B*)).

Co-Ordinates, in Feet.			General remarks.
Computed.		Actual.	
X.	*Y.*	*Y.*	
1 000	0.00	0.00	Location, 30° 03′ 04″ N. lat.; 90° 35′ 14″ W. long.
1 000	31.06	28.00	Area of section, 155 500 sq. ft.
800	77.06	114.00	Width, 2 200 ft.
600	104.12	104.00	Mean depth, 70.60 ft.
400	115.29	93.00	Maximum depth, 114 ft.
200	113.61	84.00	Radius of curvature, 23 300 ft.
0	102.14	82.00	Value of R in Formula (4) = 23 300 ft., equal to 59.09 $\sqrt{\text{area}}$.
— 200	83.91	68.00	Value of D in Formula (4) = 102.14 ft.
— 400	61.97	65.00	Area of computed section is 3.3% less than the actual section,
— 600	39.38	61.00	and the computed maximum depth is 1.75% more than the
— 800	19.18	56.00	actual depth.
—1 000	4.40	34.00	
—1 100	0.00	0.00	

TABLE 14.—Mississippi River (Fig. 5 (*C*)).

Co-Ordinates, in Feet.			General remarks.
Computed.		Actual.	
X.	*Y.*	*Y.*	
1 240	0.00	0.00	Location, 30° 01′ 28″ N. lat.; 90° 42′ 20″ W. long.
1 200	10.00	22.00	Area of section, 165 720 sq. ft.
1 000	51.52	76.00	Width, 2 480 ft.
800	80.10	90.00	Mean depth, 66.82 ft.
600	97.29	80.00	Maximum depth, 90 ft.
400	104.72	76.00	Radius of curvature, 33 300 ft.
200	103.95	73.00	Value of R in Formula (4) = 33 300 ft. equal to 81.80 $\sqrt{\text{area}}$.
0	96.56	70.00	Value of D in Formula (4) = 96.56 ft.
—200	84.15	67.00	Area of computed section is 4.3% less than the actual section
—400	68.30	60.00	and the computed maximum depth is 16.36% more than the
—600	50.61	52.00	actual depth.
—800	32.64	52.00	
—1 000	16.00	50.00	
—1 200	2.26	14.00	
—1 240	0.00	0.00	

TABLE 15.—Aransas Pass (Fig. 2 (*B*)).

Co-Ordinates, in Feet.				General remarks.
Computed.			Actual.	
X.	*Y.*	*Y* + 6.	*Y.*	
250	0.00	6.00	6.00	Computed from Formula (5).
200	10.58	16.58	24.50	Location, jetty channel.
150	16.60	22.60	26.00	Total area of section, 510 ft. wide, 7 775 sq. ft.
100	18.91	24.91	23.30	Area of section, 6 ft. below water surface, 4 745 sq. ft.
50	18.32	24.32	20.00	
0	15.66	21.66	16.00	Width, 6 ft. below water surface, 500 ft.
— 50	11.74	17.74	12.70	Mean depth, 6 ft. below water surface, 9.49 ft.
—100	7.39	13.39	10.30	Maximum depth, 26 ft.
—200	0.70	6.70	7.50	Radius of curvature, 6 000 ft.
—250	0.00	6.00	6.00	Value of *P* in Formula (5) = 15.66.
				Area of computed section is 5.4% more than that of the actual section and the maximum depth is 4.2% less.

TABLE 16.—Rio Grande do Sul, Brazil (Fig. 6 (*A*)).

Co-Ordinates, in Meters.			General remarks.
Computed.		Actual.	
X.	*Y.*	*Y.*	
388	0.00	0.00	Computed from Formula (6).
350	3.48	5.00	Location, Canal Norte, 2 800 m. above zero of jetties.
300	7.50	11.20	Area of section, 10 022 sq. m.
250	10.91	13.20	Width, 776 m.
200	13.70	14.00	Mean depth, 12 915 m.
150	15.87	14.40	Maximum depth, 18.4 m.
100	17.42	14.80	Radius of curvature = infinity.
50	18.35	15.60	Value of *D*, in Formula (6) = 18.66.
0	18.66	17.20	Value of *W*, in Formula (6) = 388 m.
— 50	18.35	18.40	Area of computed section is 1.5% less than that of the actual section and the maximum depth is 1.4% more.
—100	17.42	18.40	
—150	15.87	17.60	Channel occupies the entire waterway.
—200	13.70	15.80	
—250	10.91	13.20	
—300	7.50	10.00	
—350	3.48	3.00	
—388	0.00	0.00	

TABLE 17.—Rio Grande do Sul, Brazil (Fig. 6 (*B*)).

Co-Ordinates, in Meters.				General remarks.
Computed.			Actual.	
X	*Y*.	*Y*+1	*Y*.	
690	0.00	1.00	1.00	Computed from Formula (7).
600	6.09	7.09	13.20	Location, 5 500 m. above the origin of the jetties.
500	10.82	11.82	15.90	Total area of section, 11 233 sq. m.
400	13.67	14.67	15.60	Area of section, 1 m. below water surface, 9 853 sq. m.
300	14.91	15.91	13.60	Width, 1 m. below water surface, 1 380 m.
200	14.82	15.82	12.00	Mean depth of section, 1 m. below water surface, 7.14 m.
100	13.69	14.69	10.30	Maximum depth, 16 m.
0	11.78	12.78	7.30	Radius of curvature, 4 285 m.
—100	9.37	10.37	5.00	Value of *P* in Formula (7) = 11.78.
—200	6.76	7.76	4.50	Area of computed section is 16% more than that of the actual section and the maximum depth is the same.
—300	4.19	5.19	4.45	Channel occupies 52% of the width of the waterway.
—400	1.97	2.97	4.25	
—500	0.36	1.36	3.00	
—600	—0.35	0.65	1.80	
—690	0.00	1.00	1.00	

TABLE 18.—Rio Grande do Sul, Brazil (Fig. 6 (*C*)).

Co-Ordinates, in Meters.				General remarks.
Computed.			Actual.	
X.	*Y*.	*Y*+2.	*Y*.	
580	0.00	2.00	2.00	Computed from Formula (7).
500	4.62	6.62	10.30	Location, 6 500 m. above origin of jetties.
400	8.82	10.82	14.80	Total area of section, 11 109 sq. m.
300	11.53	13.53	15.60	Area of section, 2 m. below water surface, 8 769 sq. m.
200	12.92	14.92	14.80	Width, 2 m. below water surface, 1 160 m.
100	13.15	15.15	13.60	Mean depth of section, 2 m. below water surface, 7.55 m.
0	12.46	14.46	11.50	Maximum depth, 15.60 m. Value of *P* in Formula (7) = 12.46.
—100	11.03	13.03	7.80	Radius of curvature, 9 144 m.
—200	8.96	10.96	6.80	Area of computed section is 6.8% more than that of the actual section and the maximum depth is 2.9% less.
—300	6.73	8.73	6.60	Channel occupies 41% of the waterway.
—400	4.24	6.24	5.70	
—500	1.78	3.78	3.10	
—580	0.00	2.00	2.00	

TABLE 19.—Rio Grande do Sul, Brazil (Fig. 6 (*D*)).

Co-Ordinates, in Meters.				General remarks.
Computed.			Actual.	
X.	*Y.*	*Y* + 1.	*Y.*	
640	0.00	1.00	1.00	Computed from Formula (7.)
600	2.77	3.77	4.60	Location, 1 000 m. above Cocoruto.
500	8.13	9.13	11.30	Total area of section, 1 290 m. wide, 9 584 sq. m.
400	11.48	12.48	14.80	Area of section, 1 m. below water surface, 8 294 sq. m.
300	13.11	14.11	16.40	Width, 1 m. below water surface, 1 280 m.
200	13.13	14.13	14.50	Mean depth, 1 m. below water surface, 6.48 m.
100	12.42	13.42	8.40	Maximum depth, 16.40 m.
0	10.69	11.69	6.50	Radius of curvature, 4 200 m.
—100	8.44	9.44	5.30	Value of *P* in Formula (7) = 10.69.
—200	5.97	6.97	4.60	Area of computed section is 8.2% more than that of the actual section and the maximum depth is 13.4% less.
—300	3.57	4.57	3.70	
—400	1.54	2.54	3.10	
—500	0.19	1.19	2.40	
—600	—0.19	0.81	1.50	Channel occupies 52% of the width of the waterway.
—640	0.00	1.00	1.00	

TABLE 20.—Southwest Pass, Mississippi River (Fig. 7 (*A*)).

Co-Ordinates, in Feet.			General remarks.
Computed.		Actual.	
X.	*Y.*	*Y.*	
600	0.00	0.00	Computed from Formula (4).
500	22.89	39.00	Location, 21 960 ft. from Head of Passes (Ockerson Section 48).
400	41.61	50.00	Area, 62 200 sq. ft.
300	56.18	56.50	Width, 1 200 ft.
200	66.58	58.50	Mean depth, 51.83 ft.
100	72.82	59.50	Maximum depth, 64 ft.
0	74.90	61.50	Value of *D* in Formula (4) = 74.90.
—100	72.82	62.30	Radius of curvature = infinity.
—200	66.58	62.50	Area of computed section is 4.3% less than that of the actual section and the maximum depth is 17% more.
—300	56.18	61.00	
—400	41.61	57.50	This section is on a "cross-over" bar (see Notation 9 *re* formulas for computing cross-profiles of channels).
—500	22.89	42.00	
—600	0.00	0.00	

TABLE 21.—Southwest Pass, Mississippi River (Fig. 7 (*B*)).

Co-Ordinates, in Feet.			General remarks.
Computed.		Actual.	
X.	*Y.*	*Y.*	
710	0.00	0.00	Computed from Formula (4).
700	8.53	0.50	Location, 27 445 ft. from Head of Passes (Ockerson Section 59).
600	33.52	38.00	Area, 64 140 sq. ft.
500	54.73	72.00	Width, 1 420 ft.
400	68.20	71.50	Mean depth, 45.17 ft.
300	74.97	64.00	Maximum depth, 78 ft.
200	76.04	66.60	Value of *D*, in Formula (4), = 65.27.
100	72.47	60.00	Radius of curvature, 13 200 ft.
0	65.27	57.50	Area of computed section is 4% less than that of the actual section and the maximum depth is 4% more.
—100	55.49	53.30	
—200	44.14	49.00	
—300	32.27	42.00	
—400	20.90	36.80	
—500	11.07	31.00	
—600	3.80	22.00	
—700	0.13	0.70	
—710	0.00	0.00	

TABLE 22.—Southwest Pass, Mississippi River (Fig. 8 (*A*)).

Co-Ordinates, in Feet.			General remarks.
Computed.		Actual.	
X.	*Y.*	*Y.*	
762.5	0.00	0.00	Computed from Formula (4).
700	21.30	20.00	Location, 37 600 ft. from Head of Passes (Ockerson Section 80).
600	47.91	80.00	Area, 67 280 sq. ft.
500	65.81	91.00	Width, 1 525 ft.
400	76.19	83.50	Mean depth, 44.12 ft.
300	80.10	77.00	Maximum depth, 91 ft.
200	78.65	67.50	Value of *D* in Formula (4) = 63.75.
100	72.81	56.50	Radius of curvature, 10 800 ft.
0	63.75	46.00	Area of computed section is 4.9% less than that of the actual section and the maximum depth is 12% less.
—100	52.49	38.60	
—200	40.07	32.60	
—300	27.66	27.00	
—400	16.23	22.50	
—500	6.87	18.60	
—600	0.65	16.00	
—700	— 1.46	10.50	
—762.5	0.00	0.00	

TABLE 23.—Southwest Pass, Mississippi River (Fig. 8 (*B*)).

Co-Ordinates, in Feet.			General remarks.
Computed.		Actual.	
X.	*Y.*	*Y.*	
600	0.00	0.00	Computed from Formula (4).
500	23.31	38.50	Location, 44 475 ft. from Head of Passes.
400	42.37	50.40	Area, 63 340 sq. ft.
300	57.20	56.50	Width, 1 200 ft.
200	67.80	63.50	Mean depth, 52.78 ft.
100	74.15	64.80	Maximum depth, 73 ft.
0	76.27	73.00	Radius of curvature = infinity.
—100	74.15	67.00	Value of *D* in Formula (4) = 76.27.
—200	67.80	65.50	Area of computed section is 4.33% less than that of the actual
—300	57.20	64.30	section and the maximum depth is 4.5% more.
—400	42.37	56.40	
—500	23.31	30.50	
—600	0.00	0.00	

TABLE 24.—Southwest Pass, Mississippi River (Fig. 8 (*C*)).

Co-Ordinates, in Feet.			General remarks.
Computed.		Actual.	
X.	*Y.*	*Y.*	
920	0.00	0.00	Computed from Formula (4).
900	3.86	10.00	Location, 53 605 ft. from Head of Passes (Ockerson Section 113).
800	20.88	33.00	Area, 67 540 sq. ft.
700	34.34	40.00	Width, 1 840 ft.
600	44.53	45.00	Mean depth, 36.71 ft.
500	51.72	51.50	Maximum depth, 55 ft.
400	56.23	55.00	Value of *D* in Formula (4) = 53.04.
300	58.33	54.60	Radius of curvature, 22 800 ft.
200	58.30	50.00	Area of computed section is 4% less than that of the actual
100	56.44	47.40	section and the maximum depth is 6.1% more.
0	53.04	42.00	
—100	48.38	45.50	
—200	42.76	42.60	
—300	36.47	43.00	
—400	29.79	36.50	
—500	23.00	33.30	
—600	16.43	30.70	
—700	10.32	27.00	
—800	4.98	20.00	
—900	0.70	5.00	
—920	0.00	0.00	

TABLE 25.—Southwest Pass, Mississippi River (Fig. 8 (*D*)).

Co-Ordinates, in Feet.			General remarks.
Computed.		Actual.	
X.	*Y.*	*Y.*	
1 395	0.00	0.00	Computed from Formula (4).
1 200	19.92	21.00	Location, 63 940 ft. from Head of Passes (Ockerson Section 134).
1 000	34.28	28.30	Area, 76 990 sq. ft.
800	43.21	40.00	Width, 2 790 ft.
600	47.47	46.00	Mean depth, 27.60 ft.
400	47.84	43.30	Maximum depth, 46 ft.
200	45.05	39.30	Value of *D* in Formula (4), = 39.87.
0	39.87	37.50	Radius of curvature, 22 800 ft.
— 200	33.05	34.50	Area of computed section is 4.2% less than that of the actual
— 400	25.34	30.30	section and the maximum depth is 4% more.
— 600	17.51	27.50	
— 800	10.31	23.60	
—1 000	4.48	20.50	
—1 200	0.80	11.50	
—1 395	0.00	0.00	

TABLE 26.—Rio Grande do Sul, Brazil (Fig. 9 (*A*)).

Co-Ordinates, in Meters.				General remarks.
	Computed.			
X.	I.	II.	III.	
400	4.00	4.00	4.00	Computed from Formulas (6) and (7).
350	6.88	8.07	9.13	Location, outer bar (theoretical).
300	9.37	11.10	13.09	Assumed normal depth, 4 m.
250	11.48	13.48	15.97	Assumed area of cross-section, 10 000 sq. m.
200	13.21	15.18	17.89	Curve I is the cross-profile of the channel between
150	14.55	16.24	18.95	parallel straight jetties, 800 m. apart.
100	15.51	16.74	19.26	Curve II is the cross-profile of the channel between
50	16.09	16.74	18.92	parallel curved jetties, 800 m. apart, with a radius
0	16.28	16.74	18.02	of curvature of 5 000 m.
— 50	16.09	15.44	16.70	Curve III is the cross-profile of the channel for a
—100	15.51	14.28	15.04	single curved jetty with a radius of curvature of
—150	14.55	12.86	13.15	5 000 m.
—200	13.21	11.24	11.15	Areas of Curves I and II are each 2.8% less and the
—250	11.49	9.48	9.13	area of Curve III is 6.5% more than the assumed
—300	9.37	7.64	7.19	area of the channel.
—350	6.88	5.69	5.45	
—400	4.00	4.00	4.00	

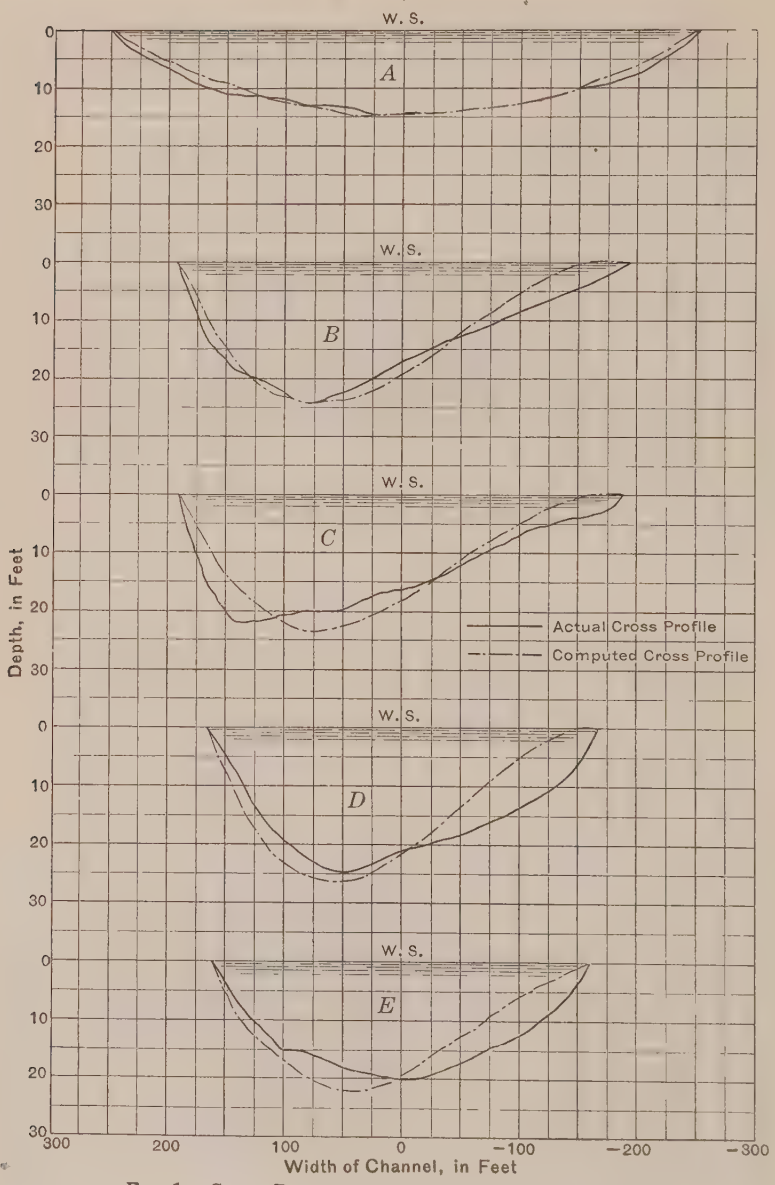

FIG. 1.—CROSS-PROFILES OF CHANNELS, BRAZOS RIVER.

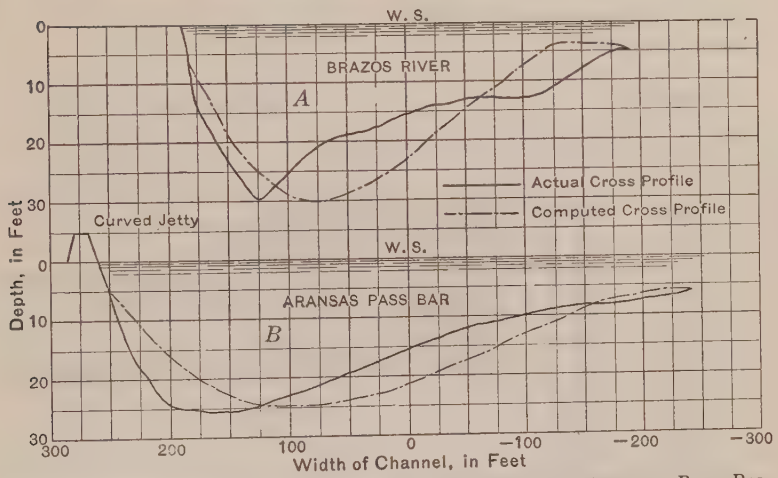

FIG. 2.—CROSS-PROFILES OF CHANNELS, BRAZOS RIVER AND ARANSAS PASS BAR.

TABLE 27.—SOUTHWEST PASS, MISSISSIPPI RIVER (FIG. 10 (*A*)).

CO-ORDINATES, IN FEET.				General remarks.
Computed.				
X.	IV.	V.	VI.	
600	10.00	10.00	10.00	Computed from Formulas (4) and (5).
500	27.89	35.15	44.00	Location, outer bar (theoretical).
400	44.34	54.90	67.30	Assumed normal depth on Bar, 10 ft.
300	56.36	67.05	81.25	Assumed area of cross-section, 63 340 sq. ft.
200	64.95	73.40	87.22	Area of cross-section, 10 ft. below water surface, 51 340 sq. ft.
100	70.10	74.73	86.54	Assumed width of channel, 1 200 ft.
0	71.82	71.82	80.59	Curve IV is the cross-profile of channel between parallel straight jetties, 1 200 ft. apart.
−100	70.10	65.47	70.72	Curve V is the cross-profile of channel between parallel curved jetties, 1 200 ft. apart, with a radius of curvature of 22 800 ft.
−200	64.95	56.50	58.28	Curve VI is the cross-profile of channel for a single curved jetty with a radius of curvature of 22 800 ft.
−300	56.36	45.67	44.63	Areas of Curves IV and V are each 3.8% less than the assumed area of the section and the area of Curve VI is 7.4% more.
−400	44.34	33.78	31.14	
−500	27.89	20.63	19.14	
−600	10.00	10.00	10.00	

TABLE 28.—Southwest Pass, Mississippi River (Fig. 10 (*B*)).

Co-Ordinates, in Feet.			General remarks.
Computed.			
X.	*Y.*	*Y* + 10.	
750	0.00	10.00	Computed from Formula (5).
700	11.70	21.70	Assumed normal depth on bar, 10 ft.
600	30.75	40.75	Location, outer bar (theoretical).
500	44.47	54.47	Assumed area of cross-section, 63 340 sq. ft.
400	53.42	63.42	Area of cross-section, 10 ft. below water surface, 48 340 sq. ft.
360	58.20	68.20	Width, 10 ft. below water surface, 1 500 ft.
200	59.38	69.38	Mean depth, 10 ft. below water surface, 32.23 ft.
100	57.50	67.50	Maximum depth, 69.38 ft.
0	53.17	63.17	Radius of curvature, 26 000 ft.
—100	46.94	56.94	Value of *P* in Formula (5) = 53.17.
—200	39.40	49.40	Width of channel at the 35-ft. depth (1 005 ft. less 20%) = 804 ft.
—300	31.12	41.12	
—400	22.66	32.66	
—500	14.61	24.61	
—600	7.53	17.53	
—700	2.00	12.00	
—750	0.00	10.00	

TABLE 29.—Rio Grande do Sul, Brazil (Fig. 9 (*B*)).

Co-Ordinates, in Meters.			General remarks.
Computed.			
X.	*Y.*	*Y* + 4.	
400	0.00	4.00	Computed from Formula (7).
350	4.42	8.42	Location, outer bar (theoretical).
300	7.77	11.77	Assumed normal depth on bar, 4 m.
250	10.44	14.44	Assumed area of cross section, 10 000 sq. m.
200	12.38	16.38	Area, 4 m. below water surface, 6 800 sq. m.
150	13.68	17.68	Width, 4 m. below water surface, 800 m.
100	14.38	18.38	Mean depth, 4 m. below water surface, 8.50 m.
50	14.41	18.41	Maximum depth, 18.41 m.
0	14.02	18.02	Radius of curvature, 9 000 m.
— 50	13.19	17.19	Value of *P* in Formula (7) = 14.02.
—100	11.97	15.97	Width of channel at the 10-m. depth (610 m. less 20%) = 488 m.
—150	10.44	14.44	
—200	8.64	12.64	
—250	6.64	10.64	
—300	4.49	8.49	
—350	2.14	6.14	
—400	0.00	4.00	

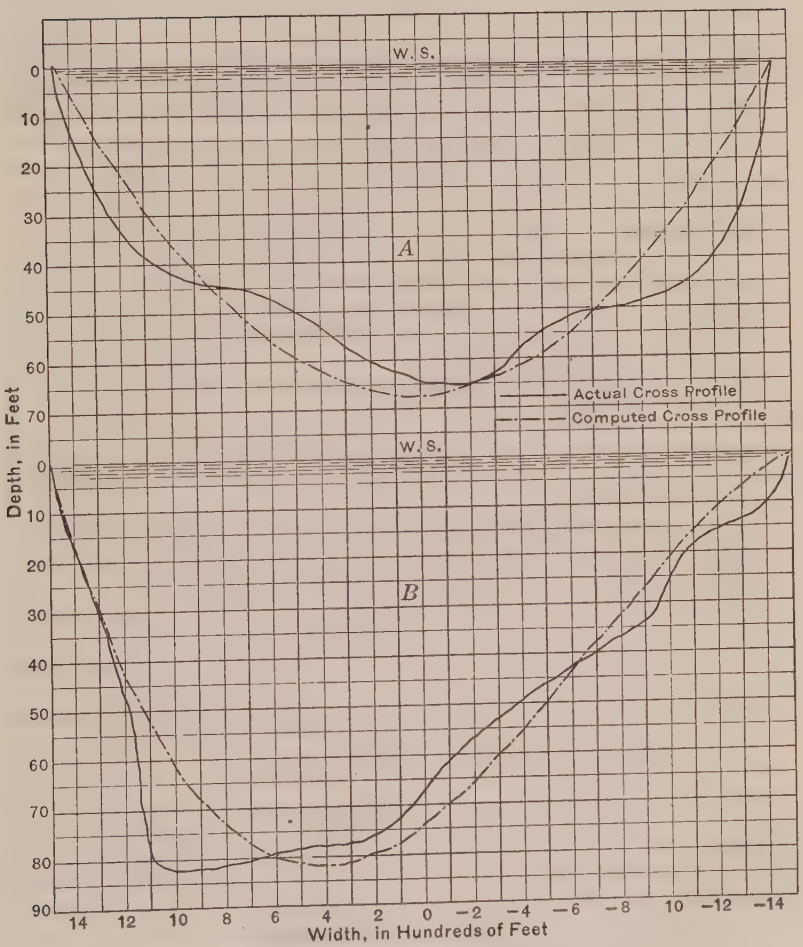

FIG. 3.—CROSS-PROFILES OF CHANNELS, MISSISSIPPI RIVER.

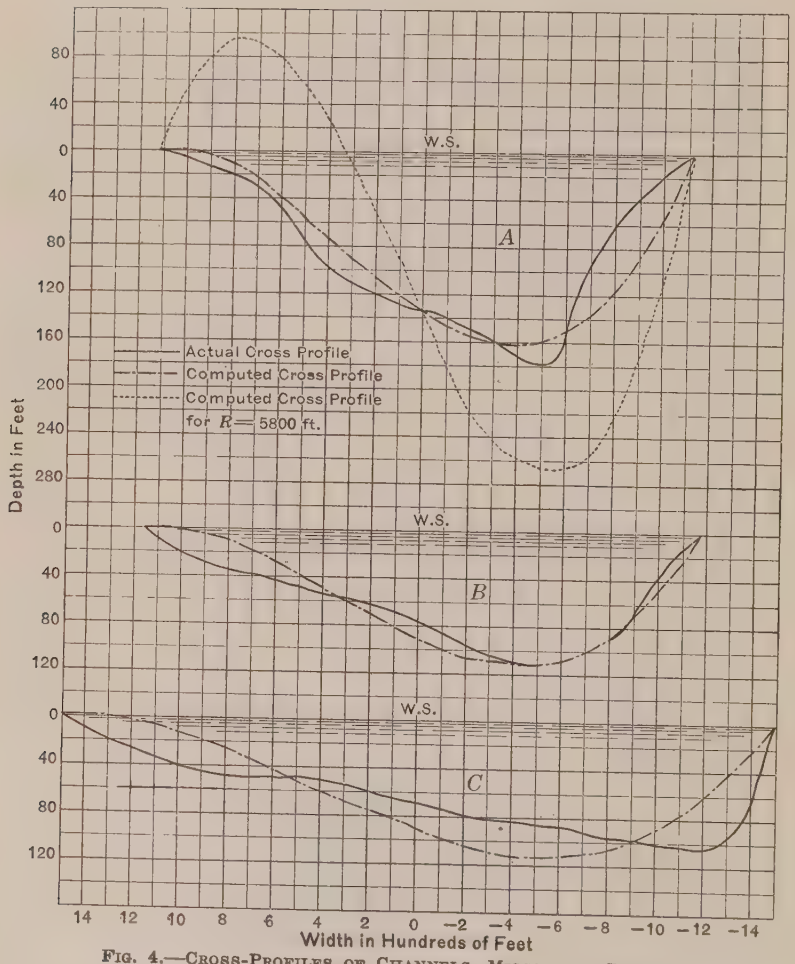

FIG. 4.—CROSS-PROFILES OF CHANNELS, MISSISSIPPI RIVER.

The explanatory remarks given in the tables furnish every essential detail necessary to a complete understanding of the results; so that further explanation would seem to be unnecessary. It should be stated, however, that in the use of Formulas (5) and (7), where the channel occupies only part of the waterway and, therefore, has only one bank, it is necessary to define the limit of the channel by the establishment of a second bank. This is accomplished by depressing the water surface until a depth is reached that will furnish a second bank and, at the same time, contain the entire actual channel. The co-ordinates of the cross-profile are then computed from this depressed water surface and the amount of depression is afterward added, as shown in Tables 6, 15, 17, 18, 19, 28 and 29. These formulas have enabled the writer to dis-

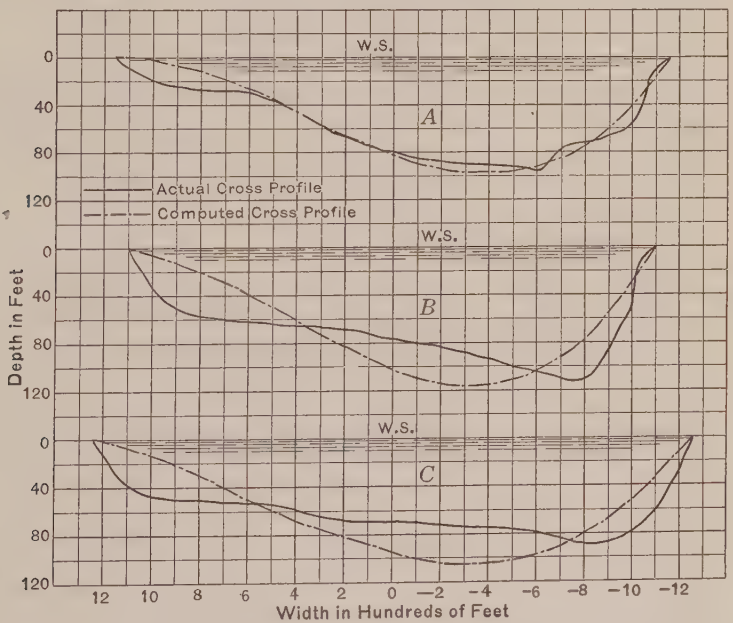

FIG. 5.—CROSS-PROFILES OF CHANNELS, MISSISSIPPI RIVER.

cover some characteristics in the law of river hydraulics not heretofore known, and it is believed they are the most important discoveries made in this law in more than fifty years. These discoveries may be enumerated as follows:

First.—The increasing curvature in bends increases the depth only to a certain point. This point is reached when the radius of curvature is equal to 40 times the square root of the area of the cross-section $(40\sqrt{\text{area}})$. This characteristic is illustrated in Tables 3, 4, and 9, and in Fig. 1 (*C*), Fig. 1 (*D*), and Fig. 4 (*A*).

Second.—Where the channel occupies the entire waterway, and the radius of curvature exceeds about 50 times the square root of the area of the cross-section $(50\sqrt{\text{area}})$, the cross-profile may not conform strictly to that due to curvature; hence, the efficiency of the formula to reproduce results is restricted to those channels having a radius of curvature not exceeding this amount.

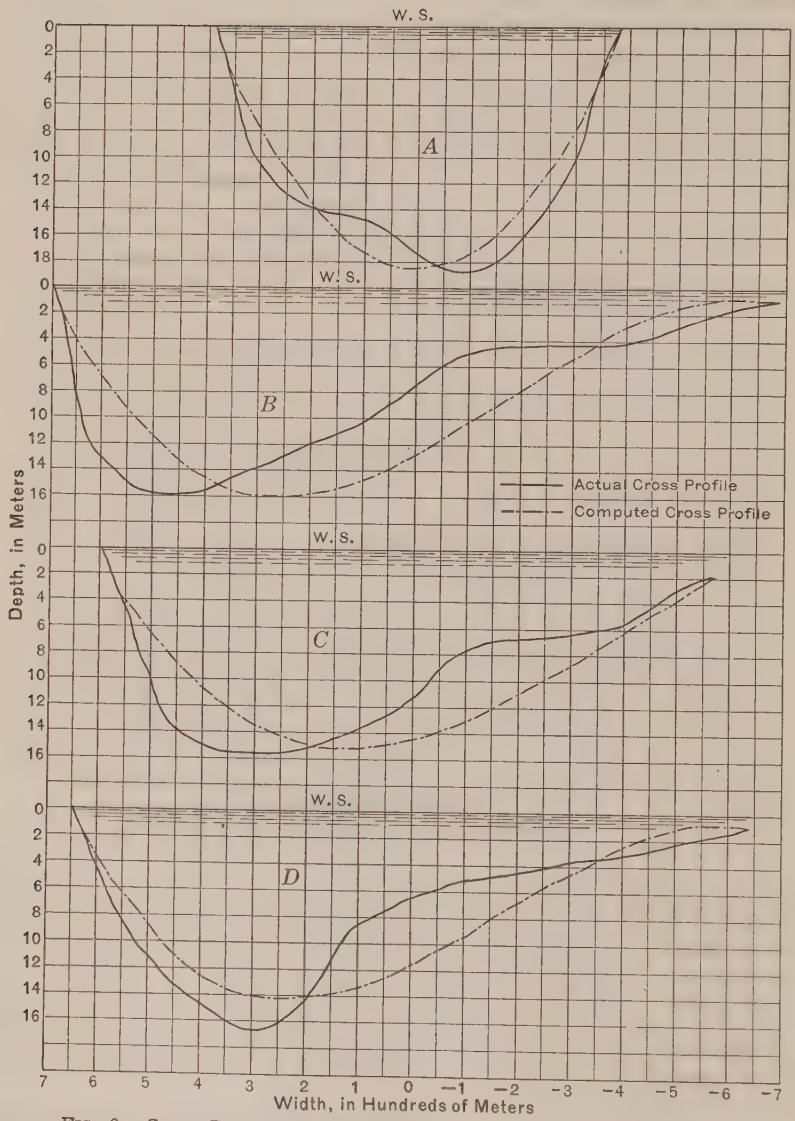

FIG. 6.—CROSS-PROFILES OF CHANNELS, RIO GRANDE DO SUL, BRAZIL.

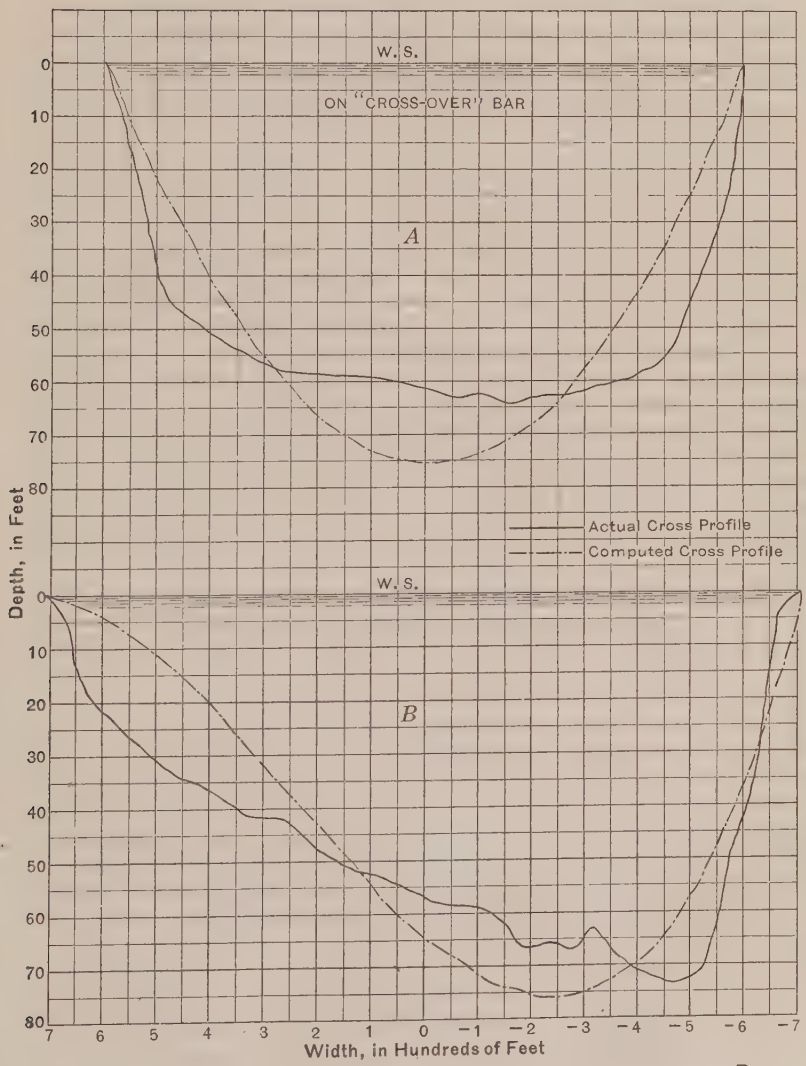

FIG. 7.—CROSS-PROFILES OF CHANNELS, SOUTHWEST PASS, MISSISSIPPI RIVER.

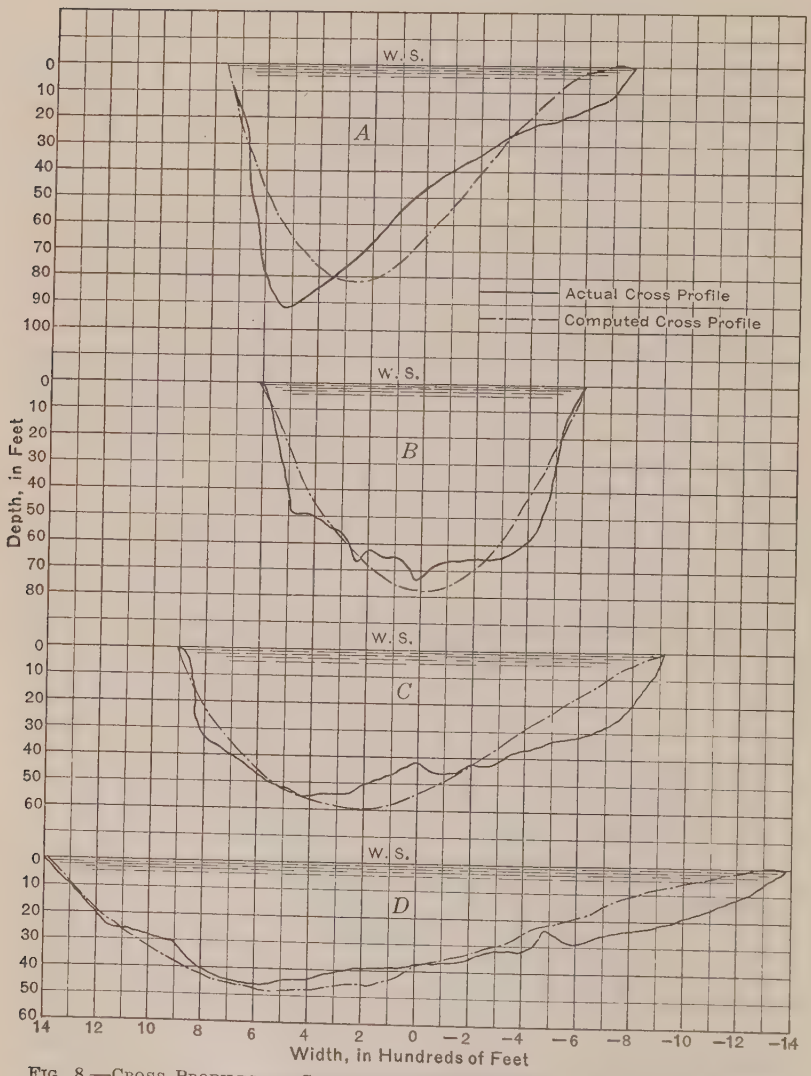

FIG. 8.—CROSS-PROFILES OF CHANNELS, SOUTHWEST PASS, MISSISSIPPI RIVER.

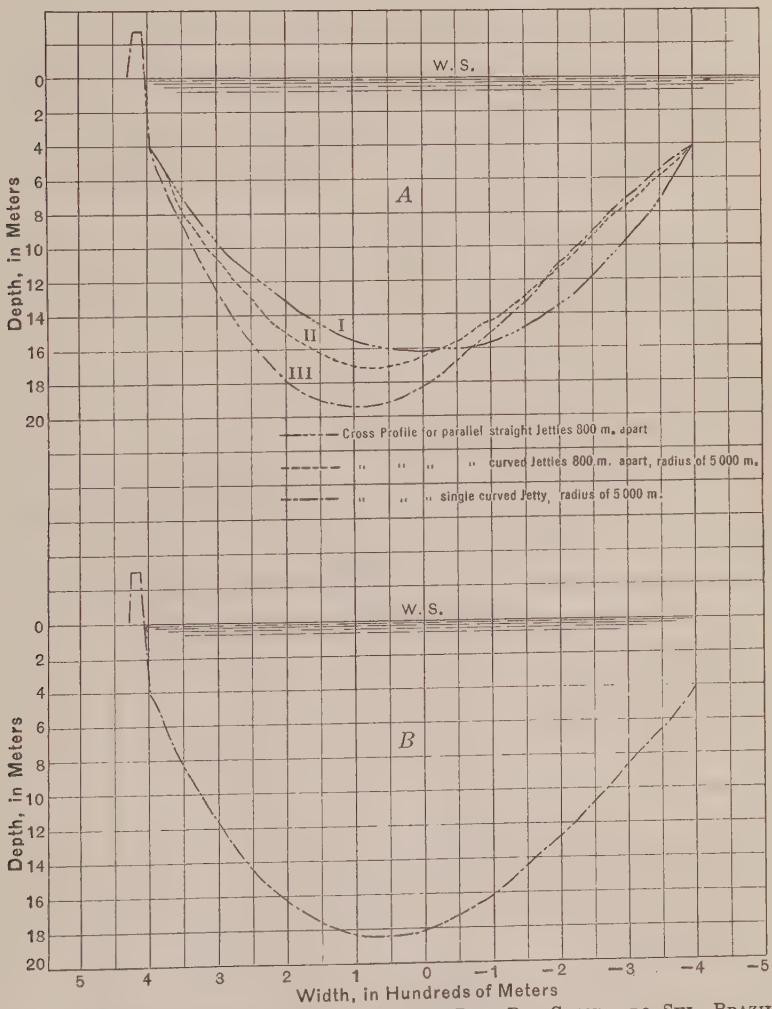

FIG. 9.—CROSS-PROFILES OF CHANNELS, OUTER BAR, RIO GRANDE DO SUL, BRAZIL.

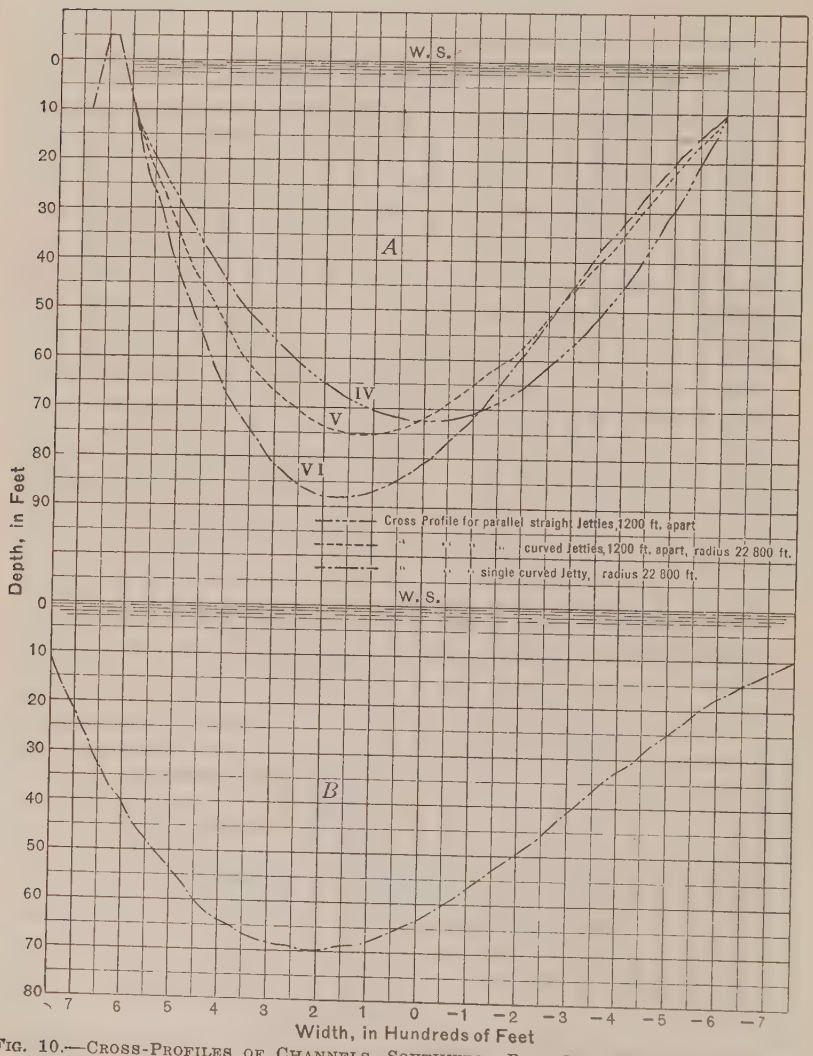
W. S.
A
IV
V
VI
Cross Profile for parallel straight Jetties, 1200 ft. apart
" " " " curved Jetties, 1200 ft. apart, radius 22 800 ft.
" " " single curved Jetty, radius 22 800 ft.
W. S.
B
Depth, in Feet
Width, in Hundreds of Feet
0 10 20 30 40 50 60 70 80 90
7 6 5 4 3 2 1 0 −1 −2 −3 −4 −5 −6 −7

FIG. 10.—CROSS-PROFILES OF CHANNELS, SOUTHWEST PASS BAR, MISSISSIPPI RIVER.

These irregularities are illustrated in Table 5, Fig. 1 (*E*), Table 11, Fig. 4 (*C*), Table 12, Fig. 5 (*A*), Table 13, Fig. 5 (*B*), and Table 14, Fig. 5 (*C*). It appears, therefore, that in channels occupying the entire waterway, any curvature greater than that due to a radius of curvature equal to $40\sqrt{\text{area}}$ is ineffectual in producing increased depths; and any curvature less than that due to a radius of curvature equal to about $50\sqrt{\text{area}}$ may have irregularities which the formula cannot reproduce.

Third.—A single curved jetty will produce a deeper channel than two parallel jetties of the same curvature. This feature is illustrated in Tables 26 and 27 and Fig. 9 (*A*) and Fig. 10 (*A*). This is probably due to the greater facility with which the material excavated is dispersed and also may account for the fact that a single curved jetty will produce a channel across an outer bar without causing bar advance.

General Remarks on Results from Use of Formulas

In straight reaches the maximum depth is practically the same as its computed value. The width at mean depth is somewhat greater than its computed value.

In bends, where the channel occupies the entire waterway, the maximum depth is practically the same as its computed value with the single exception of Fig. 8 (*A*), but its position is generally somewhat closer to the concave side of the channel, with the single exception of Fig. 1 (*D*), and its width at mean depth is about the same or greater than its computed value. Where the channel does not occupy the entire width of the waterway, the maximum depth is generally greater and its position closer to the concave side of the channel than given by the formula. The width of the channel at mean depth is uniformly less than that given by the formula.

Where the channel occupies the entire waterway, the computed areas of all sections are uniformly about 4% less than the actual areas; whereas the computed value of the areas of all sections, where the channel does not occupy the entire waterway, is uniformly greater than the actual areas by an average of 8 per cent.

Practical Use of Formulas

The accuracy of the formulas in reproducing the cross-profiles of channels under various conditions makes it possible to compute the cross-profile of a channel across an outer bar when improved by the construction of jetties of different forms and degrees of curvature. This has been done for two important entrances, namely, Rio Grande do Sul and Southwest Pass, Mississippi River, and the results are given in Tables 26 and 27 and shown on Fig. 9 (*A*) and Fig. 10 (*A*). These computations are made for parallel straight jetties, parallel curved jetties, and a single curved jetty of the same curvature. The superiority of the single curved jetty thus becomes very apparent not only over the parallel straight jetties but over the parallel curved jetties as well.

Inasmuch as there is a great excess in the depth beyond that required for navigation, it would be better in either case to give to the jetty a greater

radius of curvature. This has been done and the results are given in Tables 29 and 28 and shown on Fig. 9 (*B*) and Fig. 10 (*B*), respectively.

For Rio Grande do Sul, the initial width is the same as in Fig. 9 (*A*), and the radius of curvature has a length of 9 000 m. This gives a channel with a maximum depth of 18.41 m. and a width of channel, at the 10-m. depth, of 610 m., which being reduced 20%, in accordance with Notation 10 (page 210) will give an actual width of 488 m.

For Southwest Pass, the initial width is taken at 1 500 ft. and the radius of curvature at 26 000 ft. This gives a maximum depth of 69.38 ft., and a width of channel, at the 35-ft. depth, of 1 005 ft., which, being reduced by 20% in accordance with Notation 10, will give an actual width of 804 ft. This width is only 4% less than is found in the Pass itself in a stretch of 3 500 ft., from Sections 74 to 81, where the average width between the 35-ft. curves is 837 ft. and the minimum width is 765 ft., and the channel will be more nearly straight than in the Pass itself for much of its length.

In the use of these formulas, it may sometimes be desirable to determine the maximum depth without making repeated computations for that purpose. By making the differential coefficient of the formulas equal to zero and solving for X, a value is determined which, if substituted in the formula, gives the maximum value of Y at once. Thus, taking Formula (4):

$$Y = D\left(1 - \frac{X^2}{W^2}\right) + D\,\frac{17.52}{R}\left(1 - \frac{X^2}{W^2}\right)X$$

$$\frac{d\,Y}{d\,X} = -\,2\,D\,\frac{X}{W^2} + D\,\frac{17.52}{R} - 3\,D\,\frac{17.52}{R}\,\frac{X^2}{W^2} = 0$$

Letting $\frac{17.52}{R} = C$, and dividing by D:

$$-\,\frac{2\,X}{W^2} + C - \frac{3\,C\,X^2}{W^2} = 0$$

which by simple transformation becomes:

$$X^2 + \frac{2\,X}{3\,C} = \frac{1}{3}\,W^2$$

whence,

$$X + \frac{1}{3\,C} = \pm\sqrt{\frac{W^2}{3} + \frac{1}{9\,C^2}}$$

and,

$$X = \pm\sqrt{\frac{W^2}{3} + \frac{1}{9\,C^2}} - \frac{1}{3\,C}$$

By restoring the value of C:

$$X = \pm\sqrt{\frac{W^2}{3} + \frac{1}{9}\left(\frac{R}{17.52}\right)^2} - \frac{1}{3}\,\frac{R}{17.52}$$

which becomes:

$$X = \pm\sqrt{\frac{W^2}{3} + \left(\frac{R}{52.56}\right)^2} - \frac{R}{52.56} \qquad (a)$$

In like manner, Equations (*b*), (*c*), and (*d*), are found, which, together with Equation (*a*), are, as follows:

$$X = \pm \sqrt{\frac{W^2}{3} + \left(\frac{R}{52.56}\right)^2} - \frac{R}{52.56} \quad \ldots\ldots\ldots\ldots\ldots\ldots (a)$$

$$X = \pm \sqrt{\frac{W^2}{3} + \left(\frac{R}{78.84}\right)^2} - \frac{R}{78.84} \quad \ldots\ldots\ldots\ldots\ldots\ldots (b)$$

$$X = \pm \sqrt{\frac{W^2}{3} + \left(\frac{R}{16.02}\right)^2} - \frac{R}{16.02} \quad \ldots\ldots\ldots\ldots\ldots\ldots (c)$$

$$X = \pm \sqrt{\frac{W^2}{3} + \left(\frac{R}{24.03}\right)^2} - \frac{R}{24.03} \quad \ldots\ldots\ldots\ldots\ldots\ldots (d)$$

Equations (*a*), (*b*), (*c*) and (*d*) are to be used with Formulas (4), (5), (6) and (7), respectively. The positive value of X will give the maximum value to Y in every case. A minimum value of X will generally not occur and, therefore, need not be considered.

Some important applications of Formulas (5) and (7) have already been given for determining the cross-profiles of channels on the outer bars of Southwest Pass, Mississippi River, and at Rio Grande do Sul, Brazil. Numerous other applications of these formulas will be found in the improvement of the channels in streams. For a cross-over bar where greater depth is desirable, Formula (4) furnishes a ready means for calculating the contraction necessary to give the required depth. In Formula (4), D represents the central depth which is 17% greater than the actual depth on a cross-over bar. (See Table 4.) To find the contraction necessary, reduce W in Formula (4) until the value of D, reduced by 17%, equals the depth required.

Analyzing Formula (4), it will be seen that D is the central depth, W is the half-width of the stream, and R is the radius of curvature. Hence, if the central depth, the width, and radius of curvature are known, it is possible to compute the mean depth, the maximum depth, and the area of the cross-section; for the mean depth is equal to D divided by 1.445, the area is equal to the mean depth multiplied by the width, and the maximum depth is found by substituting the value of X obtained from Equation (*a*) in Formula (4) and solving for Y.

Some interesting facts are thus determined in regard to the Amazon River which, at a point about 300 miles from its mouth, has a central depth of 360 ft., a width of 18 240 ft., and a radius of curvature of 273 600 ft. From these data the computation gives a mean depth of 249 ft., an area of 4 544 200 sq. ft., and a maximum depth of 387 ft.

The Mississippi River, as shown in Table 8, has a central depth of 67.5 ft., a width of 3 000 ft., a radius of curvature of 36 600 ft., a mean depth of 51 ft., an area of 152 850 sq. ft., and a maximum depth of 82 ft. That is, the Amazon has 6 times the width, 30 times the area of cross-section, 5 times the mean depth, and 4.7 times the maximum depth of the Mississippi River.

The relative discharge of these two great rivers is unknown, but some idea of it may be formed from the fact that the Amazon at flood shows river water

in the ocean 250 miles from its mouth, whereas the Mississippi River at flood shows river water in the Gulf of Mexico only 25 miles.

Conclusions

These formulas and the discoveries enumerated will be of the greatest value in the regularization of rivers and in the improvement of outer bar channels. They furnish a proof that the jetty channel at Aransas Pass, Texas, was caused by the action of the single curved jetty alone and that any effect produced by the straight jetty to the south of it has been detrimental. They also furnish the means by which the cross-profile of any proposed channel across an outer bar may be computed with remarkable accuracy, heretofore impossible of determination. They show that there is a limit to the effective curvature of a channel and that any attempt to exceed this limit will be fruitless.

Professor Mitchell has shown,* in his investigations of the estuary of the Delaware River embracing a length of 52 miles from Philadelphia, Pa., down, "that the mean depth is the same in all sections and is equal to 18.64 ft. from all soundings for 46 nautical miles".

Now, as the maximum depth in the straight reaches of any stream is equal to the mean depth multiplied by a constant, and the maximum depth in bends is equal to the mean depth multiplied by the same constant plus the effect due to curvature, it follows that the maximum depth in all straight reaches will be the same. Any attempt, therefore, to deepen the channel by dredging will disturb this persistent tendency to uniformity of depth. Hence, it would appear that it is not practicable to secure any permanent improvement in the navigable channel of that river by dredging. It is evident, therefore, that the logical method would be to convert the straight reaches into channels of suitable curvature by means of training walls and to let Nature do the rest. In this way equality of mean depth would be preserved and the increased depth secured would remain permanent forever.

* U. S. Coast and Geodetic Survey, Report, 1883, pp. 239-245.

DISCUSSION

E. G. WALKER,* M. AM. SOC C. E. (by letter).—It would seem that the author has developed his formulas into needlessly long statements. Both Equations (4) and (5) may be written in general terms:

$$Y = D\left(1 - \frac{X^2}{W^2}\right) + D\frac{a}{R}\left(1 - \frac{X^2}{W^2}\right)X$$

$$= D\left(1 - \frac{X^2}{W^2}\right)\left(1 + \frac{a}{R}X\right)$$

Putting $x = \frac{X}{W}$ and $r = \frac{R}{W}$

$$\frac{X}{R} = \frac{X}{W} \times \frac{W}{R} = \frac{x}{r}$$

Therefore,

$$y = \frac{Y}{D} = (1 - x^2)\left(1 + \frac{a}{r}x\right) \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

It follows from Equation (8) that as the curvature, $\frac{1}{R}$, to which $\frac{a}{r}$ is directly proportional, is increased, the value of y for a given value of x is increased; that is, increased curvature makes increased depth. According to Equation (8) there is no limit to this, a conclusion which is at variance with the first result given by the author on page 229.

Examination of the cross-sections plotted in the paper shows that in the great majority of the curved channels the actual erosion on the outer bank and accretion on the inner are more than is indicated by the profile plotted from the author's formulas. On the other hand there is a very close agreement indeed between the calculated and actual maximum depths and their positions relative to the center line of the cross-section.

These considerations lead to the suggestion that it might be possible to amend the formulas by the introduction of additional terms which would make a still closer approximation between the calculated curve and the actual profile, but which would not have appreciable effect on the calculated maximum depth or area of cross-section. No doubt the author has tried a number of approximations before arriving at his formulas, but, unfortunately, he gives little clue to the reasoning which has led him to adopt the particular expressions given. It is a little difficult, therefore, to estimate the possibilities of getting a closer approximation.

DR. HUBERT ENGELS† (by letter).—The practical value of Mr. Ripley's formulas lies principally in the possibility of determining the "cross-profiles" of "outer bar channels". The investigations of the mouth of the Rio Grande do Sul and the Southwest Pass of the Mississippi with the aid of these formulas have shown the superiority of a single curved jetty, not only over two straight parallel jetties, but also over two parallel curved jetties. This is new and important knowledge.

* Director, Maxted & Knott, Ltd., London, England.

† Prof. Emeritus of Hydraulics, Technische Hochschule, Dresden, Germany.

The writer is fully in accord with the author that his formulas are "of the greatest value in the regularization of rivers and in the improvement of outer bar channels". He cannot agree with him, however, that any attempt to deepen "straight reaches" in estuaries by dredging would destroy the "tendency to uniformity of depth" persisting there; that it, therefore, would not seem practicable to secure "permanent improvement in the navigable channel" by dredging, nor that it would be much more logical "to convert the straight reaches into channels of suitable curvature by means of training walls and to let Nature do the rest". Certainly, dredging alone is no method of regulation, as it attacks only the result without removing the cause. Nevertheless, one should not leave the intended deepening of the channel by works to the influence of the works alone, but should bring it about as far as possible by dredging; if the works (as, for instance, "training walls") are properly placed, their influence consists in the maintenance of the dredged channel.

If, in accordance with the author's proposal, a straight channel in estuaries were transformed into a curved one, the force of the tide wave would be lessened, a part of its energy wasted, and the quantity of water flowing in and out decreased, so that the scouring force of the stream would be weakened. Therefore, the retention of straight channels in estuaries is unobjectionable if these channels are so bordered by "training walls" that the scouring force of the river remains sufficiently great. Above the tidal region it is generally desirable to give the river such bends that the current is in a state of inertia. Aside from this objection, however, the paper is to be welcomed with gratitude as a valuable advance in the still dark field of river hydraulics. In addition to M. Fargue and the late Mr. Mitchell, referred to by the author, M. Boussinesq* has also studied "the relation of depth to curvature".

Olin H. Landreth,† M. Am. Soc. C. E.—The author purports to have developed and presents Equations (4), (5), (6), and (7), for the determination of the form of transverse profile of beds of rivers of different widths, depths, and radii of curvature. Before discussing these equations or the author's conclusions, it appears necessary to draw attention to the fact that water flowing in a channel is not different from other material in that it follows the fundamental laws of mechanics and of motion. If it moves in a straight line at a given linear velocity it possesses kinetic energy which, if disturbed, may be converted into kinetic energy in another direction, into pressure, or into both. If the water is constrained to move in a circular path at the same linear or tangential velocity it possesses the same kinetic energy in the tangential direction and, in addition, it exerts pressure radially outward due to its centrifugal force, which is nothing more than the radial component of its kinetic energy in the curved path.

Although the stream, as a whole, is constrained to follow the curved channel, like a railroad train running around a curve, the individual particles of water do not follow this simple curved path, as the author points out. Years

* "Essai sur la théorie des eaux courantes," Paris, Mémoires présentés par divers savants a l'Académie des Sciences, de l'Institut de France, Tome 23, 1877; Supplement 24 (1877).

† Cons. Engr., New York, N. Y.

ago a French engineer, by using filaments of colored liquid, found that in channel bends the particles of water on the surface passed to the concave side, descended down the bank, crossed over the bottom, and arose to the surface again on the convex side of the stream, thus, when taken in conjunction with their longitudinal motion, following a helicoidal path. The speaker, more than thirty-five years ago, suspected and experimentally showed this helicoidal motion to exist in a number of curves on the Cumberland River, in Tennessee; he used small blocks of wood, loaded to be of the same density as water, and painted white so as to be visible at considerable depths.

This radially outward flow of water particles in their helicoidal paths causes them to accumulate when they reach the outer or concave bank and thus to raise the surface of the stream to a higher elevation than at the inner bank. In other words, the stream surface has a transverse slope rising from the inner to the outer bank. This difference of elevation of the water surface at the two banks is given quantitatively by the Gockinga formula, the author's Equation (1). It will be noted that this transverse slope varies directly as the square of the longitudinal velocity of the stream, a relation which later will be seen to be significant.

It is this centrifugal force, and this helicoidal movement of the water particles, which enable a stream flowing in a curved channel, through either alluvial or at least erodable material forming its banks, to erode its outer or concave bank, to increase its channel depth near its outer or concave bank, to transfer the eroded material to the opposite inner or convex bank, and thus as a whole to change its channel location. Centrifugal force varies directly as the square of the linear velocity and inversely as the radius of curvature of the stream.

With the water particles moving in such helicoidal paths, it is easy to see just how the mechanical agencies operate to produce the forms of channel cross-section which are found to exist under such conditions: A particle of erodable bank material, whether humus, silt, sand, gravel, or cobble, on the outer or concave bank of the channel has two forces which tend to erode it from its bank setting and carry it to the bottom. These are (*a*) its natural gravity; and (*b*) the downward component of friction and of impact of the successive water particles which flow past it. These two agencies concur to loosen and to carry particles down to the bottom. On the bottom the radially inward friction and impact of the water particles are usually adequate without the assistance of gravity (which is now ineffective) to transfer the eroded bank material radially (or more properly, diagonally) inward toward, or partly up, the inner bank, and if the velocity component of the water particles in the diagonally inward direction is sufficient to erode the bottom, without the assistance of gravity, the channel will be deepened near the concave or outer bank. When the water particles, however, begin to flow toward the surface, their erosive power, which is now diagonally upward, finds itself in opposition to the downward force of gravity, so that after a certain point is reached not only are the water particles unable to loosen the bank material, but they are generally unable even to transport the loose eroded material far up the ascend-

ing path on the inner bank, and as a consequence they deposit the material on that bank, or sometimes before the bank is reached, forming a bar extending diagonally out into or across the river. This causes the transverse slope of that bank to be much flatter and more irregular than the slope of the outer bank which is sometimes nearly vertical and, as a consequence, unstable and subject to slides.

For any given stream, channel, and degree of curvature, the eroding and transferring power of the water particles depends entirely on their velocity in their helicoidal paths. If the stream is a slow, sluggish one, this velocity will be slow and the eroding and transferring power will be slight, no matter what the curvature may be, and as a consequence the maximum depth of the channel will be less than if the velocity were greater. Also the location of the deepest part of the channel will not be so near the outer bank as if the velocity were greater. In other words, not only the eroding power but also the form of the transverse profile is clearly dependent on, and, therefore, a function of, the linear velocity of the flow.

One would expect, therefore, to find in the author's equations, which purport to determine the shape of cross-section, a term to indicate either the linear velocity or the longitudinal slope of the stream. An examination, however, of Equations (4), (5), (6), and (7) will show that nowhere in them does a term indicating the linear velocity or longitudinal slope of the stream appear.

This omission would seem to be a fatal defect, since the equations in question as given would predict the same depths and the same form of transverse cross-section for the channel of a stream, whether it had a flowing velocity of 0.1 ft. per sec., or 10 ft. per sec., which every engineer conversant with river hydraulics knows is not the case. This omission and defect doubtless explains why some of the author's calculated or predicted curves of transverse profile do not approach more closely to the profiles as actually observed, as in Figs. 1(*E*), 2(*B*), 3(*B*), 4(*C*), 5(*C*), and 6(*C*), none of which would appear to present strong proof of the correctness of the equations from which they were produced.

Leonard C. Jordan,* M. Am. Soc. C. E.—Having little direct knowledge of this subject, the speaker has been somewhat amazed to find a mathematical determination of a matter which is usually regarded as indefinite and independent of rules and formulas. The cross-sections of the streams noted in the paper seem to serve as reliable proofs of the dependability of the formula. The matters of grade and velocity have already been mentioned. The speaker fails to see just how they are included in the formula. Likewise, the factor of material through which the stream runs surely is given no consideration in the mathematical prediction of the shape of channel. Is it to be assumed that even in solid rock the stream, if given sufficient time, would wear its channel to the calculated shape?

On the other hand, what about the application of the formula to those sandy bedded streams of the Western prairies? In this case there are at least

* New Rochelle, N. Y.

two conditions to consider. After a flood has subsided the sand is left in a shape which possibly is in agreement with the formula for some definite depth of water, probably for the greatest depth possible without any considerable erosion.

During the violent floods to which these streams are subject there is an entirely different condition, one which probably is no more obedient to any formula than it is to the desires of the engineer who is trying to cross the river with his camp wagons and is wondering what type of bridge and foundations will be required for his new railroad line. When, due to flood conditions, the stream attains sufficient velocity, it whips up the sand of its bed until the mixture of water and sand flows several times as deep as the dry-weather channel. Further flood waters will add greatly to the velocity, erosion, and depth until the channel extends downward to unknown distances.

An example of such floods is the case of the South Platte River in Denver, Colo., which went on a "rampage," tore out a railroad bridge, and carried away a locomotive. After the waters had subsided to the usual small trickle, the sandy bottom appeared precisely as before, but the locomotive was never seen again. It had gone with the sand and had found a place down in the bottom of the deep flood channel where it could wait until the next flood carried it farther. Residents of Denver vouch for the truth of this story.

These freak conditions are well known to men who have had sad experiences with them. They most certainly throw serious obstacles in the way of a complete and universal application of the formula. The question naturally arises: Can it be possible that the channel conforms to the formula at all stages from the water-and-sand, deep-flood, condition to the shape observed in dry weather?

The author is to be congratulated on the boldness of his delving into the realms of the supposedly unknown factors governing river erosion during floods. His paper is certain to benefit those of the profession who are engaged in the problems of stream control, bridge construction, and dredging.

Lewis M. Haupt,* M. Am. Soc. C. E. (by letter).—This remarkable analysis is of great practical importance to the welfare of humanity and is the climax of time-honored research in the development of commercial channels. It solves the problem of the requisite canalization of streams to secure and maintain ample channels at least cost, without dredging for maintenance.

Hitherto, reliance has been placed mainly on dredging and jetties or revetments regardless of the reactions of the directing banks, as their relations to the depths and erosion were not formulated, whereas by the application of the author's equations, accurate and relatively permanent results may be obtained for securing and maintaining the requisite channels in navigable waters by reaction.

When the many variable properties of water are considered, it would seem impossible to include them in a single algebraic equation applicable to ever-changing conditions. Water is at times a liquid, a solid, or a gas, depending

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on temperature, and in violation of physical laws, it is lighter when solid than fluid, and in air it moves with high velocities, causing drifts of both rock and earth, thus creating alluvial plains and obstructing channels. Moreover, it is incompressible so that when driven by high winds it is extremely destructive. These properties would seem to render abortive any attempts to reduce its energies to formulas for their utilization; and yet the tests to which the Ripley formulas have been applied under variable conditions indicate their great practical value as aids to navigation as well as to the protection of banks from erosion.

It is also apparent that if the paths of rivers were straight between parallel banks there would be no reaction and the channels would have to be adjusted to the ever-varying volumes from floods and tributaries while the variable quantities of sediment ejected into such artificial channels, at junctions, would soon create bars and render the stream unnavigable without constant dredging.

The efficiency of the reaction principle in creating automatic channels was recognized by the author when he was in local charge of the construction of jetties in the Texas District. He designed a curved jetty for the Galveston entrance (which was not built) and, later, he contracted for the reaction breakwater at Aransas Pass, which has opened and maintained that channel to date, thus fully demonstrating its utility, as shown in Fig. 2(*B*). Similar proposals were made for the bars at the Passes of the Mississippi in 1902 (not built), and for the Rio Grande do Sul, Brazil, in 1887, but more conservative counsels prevailed with disastrous results and great expense (see Fig. 6 (*A*), (*B*), and (*C*), and Fig. 7 (*A*) and (*B*), and Fig. 10 (*A*) and (*B*).

From the citations made in the paper and the results of years of experience at great expense, it is evident that a relatively permanent deep-water channel could have been obtained at these two localities with less than one-half the cost and without dredging; and there would be no advance of the bars seaward, as is demonstrated by the curved jetty at Aransas Pass opened in 1907 and in use ever since, at an inlet which had been abandoned for many years.

It appears that where there is a prevailing littoral drift and large tidal basins it is practicable, by the application of these formulas, to create permanent navigable channels of any reasonable depths at less than one-half the cost for construction, and practically nothing for maintenance. The author is entitled to the highest commendation for this contribution to engineering knowledge.

B. Malaval,* Esq. (by letter).—The writer, after carefully studying the methods of application of the author's formulas, has tried to verify them by applying them to different rivers, as he considers they will have great value in the regulation of the rivers of France, especially in the estuaries.

For some sections of the Lower Seine at its entrance into the estuary, and also at various places on Rio Grande do Sul, there is approximate agreement between the real and calculated sections. In the estuary of the Seine, a deep channel follows the south shore, independent of the great central channel of the estuary, called the Honfleur Channel.

* *ex*-Director, French Company, Port of Rio Grande do Sul, Brazil, Paris, France.

The concave shore has a radius of 7 000 m. and, without any work except what Nature has done, gives a maximum depth of 11.25 m. at high water of neap tides, whereas that calculated from Equation (7) gives a maximum depth of 10.86 m. The cross-profile calculated from the author's formula follows almost everywhere the real one. For a point 1 km. away, with the same radius of curvature and the same maximum depth, the calculation gives a depth of 10.90 m., with a cross-profile following the real one and in a remarkable manner.

The writer is convinced that a project for the improvement of the Seine in which a dike is used to prolong the natural curve of 7 000-m. radius down stream would remove the obstacles to navigation from this side. Still it would be necessary to study the placing of another dike up stream in accordance with the principles that Mr. Ripley has established, in order to create a passage connecting the existing works.

The cross-profiles (Fig. 6 and Fig. 9), calculated from maps of Rio Grande do Sul of different dates, give also the most satisfactory results; one gives a depth of 12.53 m. instead of 13 m. in a curve of 7 500-m. radius, and a second gives another close approximation to the actual.

The writer is in perfect accord with the author when he states that it is not practicable to secure a permanent improvement in the navigable channel of a river by dredging. At Rio Grande do Sul, all the dredges accomplished absolutely nothing. How could it be otherwise when at times on the crest of the bar, with velocities of 2 m. per sec., erosions of several thousand cubic meters per hour occur?

It would be interesting to know how to avoid the cross-over bar in sinuous parts of rivers. Probably it would be necessary to confine the stream between two dikes for a length at least equal to the width of the river and return finally to the single curved dike.

In straight reaches it suffices to narrow the stream by spur jetties to obtain the given depth by Mr. Ripley's Equation (4) when R = infinity.

The studies made at Rio Grande do Sul and the absence of rules for determining the curve of jetties as a function of the depth to be obtained, emphasize the importance of the formulas which Mr. Ripley has determined, and prove that they will render the greatest service to engineers who are engaged in the improvement of rivers.

David A. Molitor,* M. Am. Soc. C. E. (by letter).—The paper discusses a subject about which relatively little is known. Such information as may have been published at various times is not readily accessible.

No very exact results can be expected from formulas of the kind treated by the author, chiefly because they depend on conditions which can scarcely be duplicated in any two rivers. As shown in the paper, they appear to accord with flow conditions in alluvial bottoms, but may be greatly in error when applied to other sand or clay bottoms, and would probably be inapplicable to channels in rock, as for the Niagara and Ottawa Rivers.

* Cons. Engr., Detroit, Mich.

Hence any statement as to the percentage of error resulting from the use of the formulas can apply only to the particular examples given, without in any way reflecting a general measure of accuracy to be expected in other untried cases. However, approximate solutions to such problems are all that can reasonably be expected, so that the formulas may often serve a useful purpose, especially where errors as high as 20% may be unimportant.

The author presents formulas for two conditions of flow. The first formula (Equations (4) and (5)) is Mitchell's formula, Equation (3),* devised for the Delaware River in 1878, in which the author substituted literal dimensions for the numerical values. He then states that the formula has become general in its application to streams of all sizes and characters of channel. One would naturally expect that such a formula necessitates different values for the numerical coefficient, 17.52, to fit the variations in the bottoms of different rivers. Even if this formula does give results according well with observed cross-sections of several large rivers for values of R greater than $40\sqrt{A}$, that fact is undoubtedly attributable to a uniformity in the alluvial bottoms of the particular rivers considered. Other results obtained from the use of the formula may not agree with observed cross-sections and some decided modifications of the numerical constant would be required.

The second formula (Equation (6) or Equation (7)), is the author's modification of Mitchell's Equation (3) made applicable to curved channels occupying only part of a waterway—such channels as may result at high-water stages when the stream spreads out beyond its normal banks sometimes to great distances. Under such conditions the contracted channel becomes the only important subject to be considered. The author then employs the expedient of depressing the water to some lower level approaching the normal width of the stream, and deals with a reduced area of average depth, D_a, and depth at center, $P = 1.65\ D_a$. A new numerical constant, 26.28, is now introduced to fit this condition of a partial flow area.

Here, again, it is to be expected that the two numerical factors, 1.65 and 26.28, may require considerable modifications depending on the amount by which the water level is depressed below the abnormal high level. The same would apply when the bottom material differs from that cited in the paper.

Another expedient might have been employed, by restricting the excessive width to approximately a normal cross-section between elevated banks, retaining the actual high-water level and solving the problem by the use of the previous formulas with $D = 1.445\ D_a$. This would certainly fit the hydraulic conditions better than depressing the high-water level.

The formulas might have been expressed in more workable shape with all limitations and special cases collected together for ready reference. The writer begs leave to submit such a compilation of the author's data with a view of enhancing the value of the paper and assisting the reader.

The fact that such a complex problem with so many variable factors left unconsidered, or possibly covered by numerical coefficients, should be amenable to treatment by any formula of whatever kind, seems quite remarkable.

* U. S. Coast Survey Rept., 1878, p. 125.

The subject-matter of the paper is, therefore, of considerable value to the profession and the author's efforts deserve hearty appreciation.

Streams with Curved Channels Occupying the Entire Widths of Their Waterways (Fig. 11).—

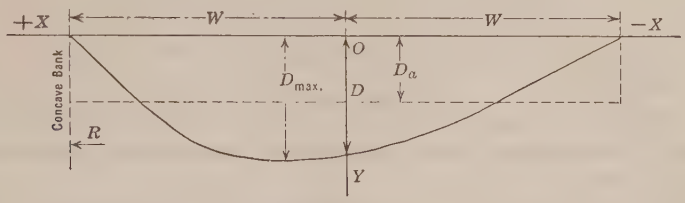

FIG. 11.

Let A = the area of cross-section, in square feet.

D_a = the mean depth, in feet = $\frac{A}{2\,w}$.

D = the depth at center of stream, in feet = 1.445 D_a.

R = the radius of curvature of concave bank of stream, in feet.

Mitchell's formula expressed in terms of these dimensions becomes:

$$Y = D\left(1 - \frac{X^2}{W^2}\right) + \frac{17.5\,D}{R}\left(1 - \frac{X^2}{W^2}\right) X$$

$$= D\left(1 - \frac{X^2}{W^2}\right)\left(1 + \frac{17.5\,X}{R}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

In this formula, Y becomes maximum, or $Y = D_{max.}$, when,

$$X = \pm\sqrt{\frac{W^2}{3} + \left(\frac{R}{52.56}\right)^2} - \frac{R}{52.56} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9a)$$

For dimensions, in meters, the constant, 17.5, in Equation (9) becomes 5.34 and the constant, 52.56, in Equation (9*a*) becomes 16.02.

Limitations.—When $R < 40\sqrt{A}$, there will be no further deepening of the channel and hence the minimum, $R = 40\sqrt{A}$, will govern.

Areas computed by Equation (9) are about 4% too small, and on cross-over bars the maximum depth is actually about 15% less than that given by the formula.

Streams with Curved Channels Occupying Only Parts of Their Waterways; also Channels Alongside Single Curved Jetties (Fig. 12).—

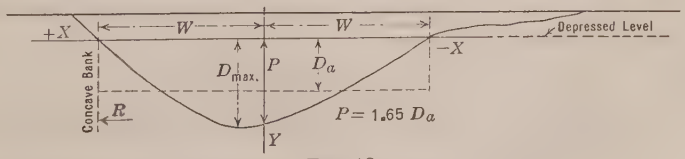

FIG. 12.

$$Y = P\left(1 - \frac{X^2}{W^2}\right) + \frac{26.28\,P}{R}\left(1 - \frac{X^2}{W^2}\right) X$$

$$= P\left(1 - \frac{X^2}{W^2}\right)\left(1 + \frac{26.28\,X}{R}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

In this formula, Y becomes maximum, or $Y = D_{max.}$, when,

$$X = \pm\sqrt{\frac{W^2}{3} + \left(\frac{R}{78.8}\right)^2} - \frac{R}{78.8} \quad \ldots\ldots\ldots\ldots\ldots\ldots (10a)$$

For dimensions, in meters, use 8.01 for 26.28 and 24.0 for 78.8.

Limitations.—when $R < 40\sqrt{A}$, there will be no further deepening of the channel. Equations (10) and (10*a*) are good for values of $R > 40\sqrt{A}$ to $R < 110\sqrt{A}$.

The width of the channel at mean depth as given by Equation (10) is 20% too large. The values of A, by Equation (10) are generally 8% in excess of the actual values.

Edward N. Chisolm, Jr.,* M. Am. Soc. C. E. (by letter).—A study has been made of the relation of depth to curvature of channels to ascertain whether the author's formulas and conclusions are applicable to the Mississippi River from Cairo, Ill., to Fort Jackson, La. The physical characteristics differ materially along this stretch, but in general terms the river may be divided into two reaches; Cairo to Red River, La., 772 miles in length, and Red River to Fort Jackson, 269 miles in length. Above Red River the stream is non-tidal, but below this point the slope and current are affected by the tide in the Gulf of Mexico.

It is common knowledge with those familiar with this stretch of river that a greater depth is found in the bend of the channels than in the straight reaches and that the water surface on the concave side of the channel is higher than that on the convex side. It is also well known that the water in the bends descends at the concave side of the channel, crosses over at the bottom, and comes up on the convex side, causing excessive erosion of the alluvial bank. Observations confirm the accuracy of the author's formulas where differences of head of about 1 ft. have been noted on opposite banks of bends with mean mid-section longitudinal slopes of about 0.4 ft. per mile. The results seem to indicate that the increased depth in bends is caused by the helicoidal movement of the water induced by the centrifugal force of the current on the concave side of the channel.

The modification of the Mitchell formula so as to make it of general application to all sizes of streams, to all degrees of curvature, and to every character of channel is worthy of note, and further consideration will be given the derived Formulas (4) and (5) to test the accuracy with which the various sections along the Mississippi may be computed, and the constant to be applied, if necessary.

Notation 6 (page 210), states, in part, as follows:

"Whenever the radius of curvature is less than 40 times the square root of the area of the channel, no further deepening of the channel results from the increased curvature."

This seems to conflict with a recent comparison of cross-section elements made along this stretch of river. Practically all the prominent bends have a radius of less than 40 $\sqrt{\text{area}}$, or 18 000 to 20 000 ft., and the tabulated results

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show that some sections decreased and others increased, depending somewhat on the time the surveys were made, the stage of water, and whether the bend was sounded on a rising or on a falling stage.

The author's Corollary (2) (page 209), has been given consideration and practical applications were made to test its accuracy at various points along the river with the following results.

At College Point, La., 907 miles below Cairo, Section 996 of the cross-section elements was selected. Soundings taken in 1898 showed an area of 270 480 sq. ft. and again in 1921, an area of 229 900 sq. ft., which would indicate that a radius of about 20 000 ft. was necessary at this point to produce a non-destructive bend. The actual radius of the concave side of this bend is 4 300 ft. (scaled), or about one-fifth the radius computed by the formula for a bend classed as destructive. There is a slight erosion, but the bend has changed very little in a number of years and can hardly be classed as destructive.

At 81-Mile Point, 883 miles below Cairo, Section 849 of the cross-section elements was selected. Areas taken in 1897 and 1921 were 229 720 sq. ft. and 266 300 sq. ft., respectively. These areas would indicate that a radius of about 20 000 ft. was required, whereas the radius scaled from the chart is 3 300 ft., or about one-sixth the computed radius. The bend is not classed as destructive.

At Waverly Point, near Natchez, Miss., 705 miles below Cairo, Section 2415 had an area of 203 972 sq. ft. in 1895 and 254 730 sq. ft. in 1913, which would give a computed radius of 18 000 ft. to 20 000 ft., whereas the actual radius is 7 000 ft., or about one-third the computed radius, and the bend is not actually destructive.

At Marengo Bend, above Natchez, 696 miles below Cairo, Section 2388 had an area of 160 139 sq. ft. in 1895 and 193 620 sq. ft. in 1913, which would indicate a radius of 16 000 to 17 000 ft. necessary for a non-destructive bend. In this case, the concave side of the bend scaled 10 000 ft., or about five-eighths the computed radius, and the bend is very destructive.

At Giles Bend, 691 miles below Cairo, Section 2371 shows an area of 221 717 sq. ft. in 1895 and 195 893 sq. ft. in 1913, which would require a radius of about 18 000 ft. to form a stable bend. In this instance, the actual radius is about 13 000 ft. (scaled) ; this is considerably less than 40 $\sqrt{\text{area}}$ in 1895 and somewhat smaller in 1913 than in 1895-96, but the mean depth on this section increased from 40.0 to 45.7 ft. at bank-full stage and from 33.8 to 50.6 ft. at low water. The maximum depth also increased at low water from 62.0 to 105.2 ft. This does not agree with the statement that "no further deepening of the channel results from increased curvature."

At Kempe Bend, 673 miles below Cairo, Section 2304 had an area of 317 142 sq. ft. in 1895 and 269 796 sq. ft. in 1913, which would give a computed radius of about 21 000 ft. The actual radius is 9 000 ft., or about one-half the computed radius, and the bend is actually destructive.

At Hard Times Bend, 643 miles below Cairo, Section 2174, the areas were 188 891 sq. ft. in 1895 and 206 856 sq. ft. in 1913, which would give a computed radius of about 18 000 ft. The actual radius is 7 000 ft. (scaled) and

the bend is destructive. At bank-full stage the mean depth was 30.8 ft. in 1895 and 32.8 ft. in 1913. At low water, the maximum depth was 65.0 ft. in 1895 and 54.2 ft. in 1913.

At Island 16, which is 117 miles below Cairo, Section 441 was selected with an area of 199 150 sq. ft. in 1902 and 155 358 sq. ft. in 1912, which would give a computed radius of about 16 000 ft. In this instance the actual radius is about 16 000 ft. and the bend is not destructive and agrees with the formula. At bank-full stage the mean depths were 34.5 ft. in 1902 and 26.6 ft. in 1912. The maximum depths at low water increased from 35.9 ft. in 1902 to 49.7 ft. in 1912.

Near Island No. 8, 47 miles below Cairo, Section 173 was selected with an area of 218 199 sq. ft. in 1902 and 216 933 sq. ft. in 1911, which would give a computed radius of about 18 000 ft. for a non-destructive bend. The actual radius is about 17 000 ft. (scaled) and somewhat less than 40 $\sqrt{\text{area}}$, and the bend is slightly destructive at some points and quite destructive at others. At bank-full stage, the mean depths were 28.6 ft. in 1902 and 31.1 ft. in 1911 and, at low water, the maximum depths were 46.0 ft. in 1902 and 47.9 ft. in 1911.

The localities used to test the application of the formula were selected at convenient points along the river and the results show that bends at College Point, 81-Mile Point, and Waverly Point, are not classed as destructive bends, whereas the bends at Marengo Bend, Giles Bend, and Hard Times Bend, are classed as destructive. In all the localities mentioned the radius of the bends was considerably less than 40 $\sqrt{\text{area}}$.

At Island 16 and at Island 8 the radius of the bends was about 40 $\sqrt{\text{area}}$. In one instance the bend was not destructive; in the other, parts of the bend were destructive and parts were not destructive.

Under the heading, "Conclusions" (page 238), the statement is made that "the maximum depth in the straight reaches of any stream is equal to the mean depth multiplied by a constant, and the maximum depth in bends is equal to the mean depth multiplied by the same constant plus the effect due to curvature". In order to verify the statement, the maximum and mean depths were taken from the computations of cross-section elements, Cairo to Red River and Red River to Fort Jackson.

A comparison of Stretches Nos. 14, 119, 143, 204, and 286, comprising 57 sections, shows that the average maximum depth is 1.734 greater than the average mean depth on the straight reaches at low water, whereas a comparison of Stretches Nos. 8, 127, 128, 156, 197, 199, and 313, comprising 170 sections, shows that the average maximum depth is 1.841 greater than the average mean depth in the bends at low water. The results on this stretch—Cairo to Red River—appear to agree with the statement that the constant to be applied in the bends is greater than the constant to be applied in the straight reaches, but judging from the various constants found on the individual bends and straight reaches, the results are rather an arithmetical coincidence instead of a law. The constant on the straight reach varied from 1.387 to 2.196 and on the bends from 1.519 to 2.019, which indicates a range of 25% or more in the

constant to be applied in either case and a rather broad range to compute accurate results.

A comparison of Stretches Nos. 23, 37, 50, 95, and 111, comprising 110 sections, shows that the average maximum depth is 1.490 greater than the average mean depth on the straight reaches at low water, whereas a comparison of Stretches Nos. 9, 38, 51, 96, and 141, comprising 98 sections, shows that the average maximum depth is 1.744 greater than the average mean depth on the bends at low water.

The results on this stretch—Red River to Fort Jackson—seem to agree more uniformly with the formula and with each other. The constant in the bends is greater and the individual results are greater in general terms. The constant on the straight stretches varied from 1.362 to 1.627 and on the bends from 1.595 to 1.954, which shows more uniformity than on the non-tidal river above Red River.

The practical application of the results shows that there is a constant between the average maximum depth and the average mean depth, but the constant varies 25% on the river above Red River, which would not produce results of a high degree of accuracy. On the stretch below Red River, the constant is more uniform, and the results would be more accurate in its application.

P. N. Fawcett,* Assoc. M. Am. Soc. C. E. (by letter).—Mr. Ripley is to be congratulated on the thorough manner in which he has attacked a most important subject in connection with the hydraulics of rivers. His Formula (4) is identical with Mr. Mitchell's Equation (3) for the Delaware River at Philadelphia, if D be taken as 33 and W as 1 100.

With the author's formula, using two sections of equal average depth, one 4 000 ft. wide and the other 2 000 ft. wide, the former with a radius of 40 000 ft. and the latter with a radius of 20 000 ft., so that the ratio of $\frac{R}{W}$ is the same in both cases, the depth will be the same at equal proportionate distances from the center line.

From the author's assumptions in Formulas (4) and (5) in which $\frac{P}{D} = \frac{1.65}{1.445}$, and the constant in the second part of the formula is in the proportion, $\frac{26.28}{17.52}$, it naturally follows that any depth calculated by Formula (5) with one training wall will be greater than the depths calculated from Formula (4) with two training walls. Reverse the formulas and the single training wall will produce by calculation a less depth than the double training walls. This does not prove, however, that such results will follow, as stated by the author.

It is obvious that with a greater constant, greater depths will be obtained by calculations and *vice versa,* but there has been no allowance made in the formula for the "greater facility with which the material excavated is dispersed" so that it is difficult to understand how greater depths are claimed to be obtained from this formula on this account.

* Engr.-in-Chf., Lower Liao River Conservancy, Newchwang, Manchuria, North China.

As regards the statement that a single training wall does not cause a bar advance, such a statement would appear to be contrary to reason. The material eroded from within the bar must be carried seaward whether the confinement of the currents causing the erosion is obtained by one training wall or two walls, and this material (other things, such as littoral current, being the same in both cases) must cause a bar advance.

Cases in which bars obstructing the mouths of rivers are being trained by one training wall only, unassisted by dredging, are extremely rare. In fact, the writer is unable to quote any cases. Under these circumstances doubtless some of the particulars of the bar of the Liao River, in Manchuria, will be of interest.

The bar of the Liao River is being trained by one training wall constructed first, after which it will be seen whether a second wall will be required. It is highly probable that a second training wall will never be constructed, reliance being placed on one wall, assisted by dredging, if necessary. At present (July, 1926), this wall has been built to a length of 35 000 ft. It still requires a further 6 000 ft. to be constructed. Its top is about 3 ft. above low-water level only, as any further height requires heavy block stone on account of the ice effect during the winter months.

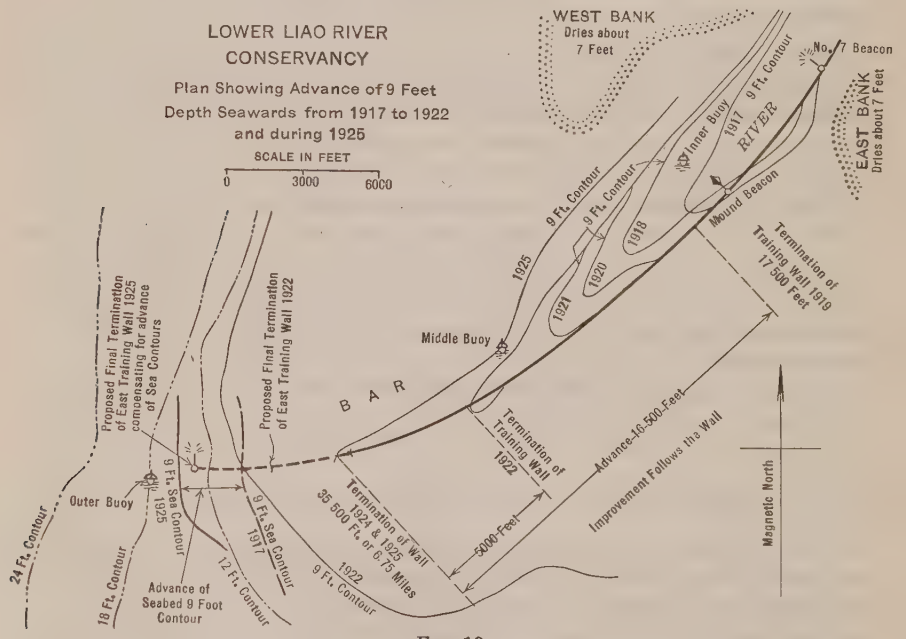

FIG. 13.

This wall is producing a considerable improvement as will be seen from Fig. 13. It should be noted, however, that the bar is advancing the 9-ft. sea contour, which has been moved seaward 3 000 ft. since 1917 and other contours *pro rata*. Although this advance is not serious, it is nevertheless a fact and disproves Mr. Ripley's theory that a single training wall does not cause a bar advance.

A few particulars of this port will be of advantage in considering the situation. Spring tides rise 13 ft. Flood and ebb currents attain about 3½ knots across the bar and up to 4½ knots in the river proper. The ebb runs across the wall S. 40 W. to S. 60 W. and the flood in a direction N. 34 E. to N. 40 E. The prevailing winds are northeast during the winter months while the port is closed by ice, and southwest during the summer months.

Thus, it will be seen that the ebb tide hugs the training wall throughout its entire length, there being full play for centrifugal force, so that the situation is entirely favorable to the success of a single wall. The soil is the light loess of the North China plains, so fine that 50% will pass through a sieve of 144 meshes per sq. cm. When compacted on the bed of the bar it forms a hard impenetrable surface and becomes extremely difficult to erode or to dredge by suction dredges.

H. C. Ripley,* M. Am. Soc. C. E. (by letter).—The writer wishes to express his appreciation of the many helpful criticisms of his paper and he is especially pleased with the numerous responses from foreign countries. England, France, Germany, and China have contributed to this discussion.

The interest shown by Mr. Walker, although it is in the mathematical feature only, is gratifying. While recognizing that his Equation (8) is more concise than Formulas (4) and (5), it is considered that what is gained in conciseness is lost in ease of solution. The value of these formulas is for computing the co-ordinates of the cross-profile of a given channel or its maximum depth by the use of Equations (*a*) or (*b*). Equation (8) in its present form cannot be used for these purposes for it does not contain the elements necessary to those ends, as Y and X in Equation (8) are not the co-ordinates of the curve to be reproduced and it does not contain D, W, and R, all of which are in immediate demand when one undertakes to compute the co-ordinates of the curve.

However, the writer quite agrees with Mr. Walker in the possibility of modifying the formulas so as to get closer approximations to the actual curves. Since submitting his paper, he has given some further study to this feature of the subject.

The basis for the formulas submitted is that of a conic section and the first term of the second member of Formulas (4) and (5) is that of a parabola the focal distance of which from the apex is equal to $\frac{W^2}{4\,D}$, and in Formulas (5) and (7) this distance is equal to $\frac{W^2}{4\,P}$ and with the origin at a point on the axis at a distance from the apex equal to D and P, respectively.

In a straight channel, the equation of the arc of a circle the chord of which is 2 W, the middle ordinate, D, and the radius, $\frac{D^2 + W^2}{2\,D}$, gives almost iden-

* Cons. Engr., Detroit, Mich.

tical results with Formula (4). The equation of a circle with these conditions is:

$$Y = \pm \sqrt{\left(\frac{D^2 + W^2}{2\,D}\right) - X^2} - \frac{D^2 + W^2}{2\,D} + D \quad \ldots\ldots\ldots\ldots (11)$$

Both Equation (11) and Formula (4) give the area of the cross-section about 4% less than the actual area. With the ellipse, however, an equation may be obtained which will give a computed area the same as the actual area and, at the same time, a computed cross-profile more closely approximating the actual. This equation is determined as follows.

The standard equation of the ellipse the semi-transverse axis of which is a, and the semi-conjugate axis, b, with the origin at their intersection, is:

$$x^2\,b^2 + y^2\,a^2 = a^2\,b^2$$

With the origin removed to a point distant from the center equal to $-k$, and the X-axis parallel to the transverse axis and the Y-axis passing through the center,

$$x^2\,b^2 + (y - k)^2\,a^2 = a^2\,b^2$$

In this case, $b^2 = (D + k)^2$ and $a^2 = W^2\left(1 + \frac{k^2}{D^2 + 2\,D\,k}\right)$, for when $y = 0$, $x = W$. Substituting these values of a^2 and b^2 and reducing,

$$y = \pm \sqrt{(D^2 + 2\,D\,k)\left(1 - \frac{x^2}{W^2}\right) + k^2} + k$$

By making $k = 1.9\,D$,

$$y = \pm D \sqrt{\left(4.8\left(1 - \frac{x^2}{W^2}\right) + 3.61\right)} - 1.9$$

Limiting the equation to that part of the ellipse below the X-axis, giving y a positive value, and providing for curvature, the complete equation becomes:

$$y = D \sqrt{\left(4.8\left(1 - \frac{x^2}{W^2}\right) + 3.61\right)} - 1.9 + D\,\frac{17.52}{R} \sqrt{\left(4.8\left(1 - \frac{x^2}{W^2}\right) + 3.61\right)} - 1.9\,x \quad \ldots\ldots\ldots\ldots (12)$$

For measurements, in meters,

$$y = D \sqrt{\left(4.8\left(1 - \frac{x^2}{W^2}\right) + 3.61\right)} - 1.9 + D\,\frac{5.34}{R} \sqrt{\left(4.8\left(1 - \frac{x^2}{W^2}\right) + 3.61\right)} - 1.9\,x \quad \ldots\ldots\ldots\ldots (13)$$

Equations (12) and (13) give the area of the cross-section exactly and the cross-profile a little nearer to the actual than Formulas (4) and (5). It cannot be doubted that subsequent investigators will be able to devise an equation that will give a still closer approximation to the actual cross-profile. This is a fruitful field for future study. The writer believes, however, that the formulas in their present shape, together with the notations, will give results sufficient

for all practical purposes and will enable the engineer to design his works with a confidence in their success never heretofore experienced.

Table 30 gives the co-ordinates of the cross-profile of the Brazos River in a straight reach computed from Formula (4), a parabolic curve, from Equation (11), the arc of a circle, and from Equation (12), the curve of an ellipse (see Table 1 and Fig. 1 (*A*)), together with the co-ordinates of the actual cross-profile.

Table 31 gives the co-ordinates of the cross-profile of the Brazos River in a curve computed from Formula (4) and Equation (12), together with the co-ordinates of the actual cross-profile.

TABLE 30.—BRAZOS RIVER.

Co-Ordinates, in Feet.					General remarks.
Computed.				Actual.	
X.	*A.*	*B.*	*C.*		
255	0.00	0.00	0.00	0.00	Location 3½ miles from the mouth.
250	0.60	0.60	0.73	0.60	
225	3.30	3.32	3.93	5.00	Area of section. 5 293 sq. ft.
200	5.80	5.72	6.54	7.40	Width. 510 ft.
175	7.90	7.90	8.70	9.60	Mean depth. 10.38 ft.
150	9.80	9.82	10.47	11.60	Maximum depth, 15 ft.
125	11.40	11.50	11.91	11.60	Radius of curvature infinite.
100	12.70	12.70	13.05	12.50	
75	13.70	13.70	13.91	13.00	Value of *D* in Formula (4) = 15.
50	14.40	14.40	14.52	13.80	
25	14.90	14.85	14.88	14.50	Column *A* is computed from Formula (4), a parabola, has an area 3.9% less than the actual area.
0	15.00	15.00	15.00	15.00	
— 25	14.90	14.85	14.88	14.50	
— 50	14.40	14.40	14.52	14.00	
— 75	13.70	13.70	13.91	13.70	Column *B* is computed from Equation (11), an arc of circle, has an area 4.4% less than the actual.
—100	12.70	12.70	13.05	12.60	
—125	11.40	11.40	11.91	11.40	
—150	9.80	9.82	10.47	10.30	
—175	7.90	7.90	8.70	9.20	Column *C* is computed from Equation (12), an elliptic curve, has an area the same as the actual area.
—200	5.80	5.72	6.54	7.50	
—225	3.30	3.32	3.93	4.40	
—250	0.60	0.60	0.73	0.60	
—255	0.00	0.00	0.00	0.00	

It is an interesting fact that the inexorable laws of Nature which control the orbits of the heavenly bodies, compelling them to revolve in curves of conic sections, also control the shape of the cross-profile of flowing streams, giving it the curve of a conic section, and just as the orbit of the moon around the earth is modified by the influence of the sun and planets, so the cross-profile of channels is modified by the effect of curvature, which can be computed, and other influences in a lesser degree, but more difficult to determine and impracticable to compute.

Professor Engels' discussion is important and the Society is to be congratulated in receiving a contribution from so eminent an authority. He is in general agreement with the writer's conclusions; but as to the conversion of straight reaches into curved channels in estuaries, he regards such practice as questionable, since "the force of the tide wave would be lessened, a part of its energy wasted, and the quantity of water flowing in and out decreased, so

that the scouring force of the stream would be weakened". The writer regards a suitable retardation of the tidal flow as essential to the development of its scouring force as it allows an accumulation of head so necessary to produce deepening. Although it may diminish the actual tidal flow in and out, that flow is effected by a greater water slope. In the estuary of the Delaware River the mean depth is the same in the upper reaches as in the lower ones, although the tidal flow is much less.

TABLE 31.—Brazos River.

Co-Ordinates, in Feet.				General remarks.
Computed.			Actual.	
X.	*A.*	*B.*		
195	0.00	0.00	0.00	Location, 4¾ miles from mouth.
175	7.99	9.51	10.00	Area of section, 5 384 sq. ft.
150	15.53	17.31	18.00	Width, 390 ft.
125	20.63	22.31	21.00	Mean depth, 13.81 ft.
100	23.58	24.75	23.50	Maximum depth, 25 ft.
75	24.70	25.36	24.25	Radius of curvature. 2 900 ft.
50	24.26	24.55	22.50	Value of *D*, in Formula (4), 19.95.
25	22.31	22.65	20.50	Column *A* is computed from Formula (4), has an area of section 4% less than the actual section.
0	19.95	19.95	17.50	
— 25	16.93	16.71	15.50	
— 50	13.00	13.16	13.00	Column *B* is computed from Equation (12), an elliptic curve, has an area of section the same as the actual section.
— 75	9.30	9.55	11.00	
—100	5.82	6.11	9.00	
—125	2.87	3.11	6.50	
—150	0.77	1.05	4.50	This is Cross-Section No. 12 of the Official Survey of 1878.
—175	—0.23	0.26	2.30	
—195	0.00	0.00	0.00	

That the uniformity of depth in straight reaches would be disturbed by dredging operations, which Nature would proceed at once to restore, would seem to be self-evident. However, after regularization by permanent works, dredging may be desirable and even necessary to secure the required channel depth, especially where the resistance to erosion is very great. It must be understood that the uniformity of mean depth can exist only in estuaries. As far as the writer knows it is true only of the Delaware River, although he thinks it must be true of all similar estuaries.

The writer is pleased with Mr. Jordan's inquiring disposition and constructive criticism. Doubtless more questions can be asked on this subject than the writer is able to answer; however, he may be able to clear up some of the difficulties that Mr. Jordan encounters.

To be sure the bed of a stream must be amenable to erosion else it cannot develop a shape in conformity with the forces acting upon it. When this is true, the shape of the cross-profile and the area of the cross-section are the result of the volume of flow, the velocity of the current, the curvature of the channel, and the resistance to erosion. It will thus be seen that the formula, by including the half width and a factor of the mean depth of the channel, does include, if not the actual velocity, the actual effect the velocity produces on any given stream. The curvature is the variable element which is provided

for by its radius. The lack of uniformity in the resistance to erosion gives the cross-profile an irregularity which it would not have otherwise. A greater resistance to erosion over the entire bed of the stream causes an increased water slope and greater velocity to discharge a given volume of water through the smaller cross-section which results. Thus the factors of velocity and resistance to erosion combined give to the channel its cross-sectional area, and, therefore, the formula really contains these two factors.

The flood problem is one that still requires much study. In a sediment-bearing stream, like the Mississippi River, the channel during floods is deepened in bends and shoaled in straight reaches. When the flood subsides, the channel shoals in the bends and deepens in the straight reaches. During this transition period it is possible that the formula cannot keep pace with these changes. This condition pertains to that part of the river that transports both fine and coarse material, the former in suspension and the latter by pushing it along the bottom. In Southwest Pass, where only the fine material is transported, the channel remains practically uniform in size and shape. The material carried in suspension passes through without deposits until it approaches the mouth and the bar is formed. The material thus deposited is of quite uniform resistance to erosion and, hence, by confining the channel at the mouth to a suitable width, its shape across the bar may be computed for any given radius of curvature to a remarkable degree of exactness.

In the writer's opinion this is the most important and at the same time the most reliable use to which the formula can be put. Its value and reliability for this purpose is in no way lessened by any uncertainty about its application to other parts of the river in time of floods.

The writer is indebted to Mr. Landreth for a comprehensive and lucid description of the actual movement of the particles of water in the bend of a stream and the manner in which those particles under the influence of velocity and curvature are able to erode the material of the banks and bed, which gives the cross-profile the shape that it assumes under these influences. However, like Mr. Jordan, he does not find in the formula any factor representing the linear velocity of the water or the slope of its surface which are the active forces that produce the results.

This subject has already been touched upon in replying to Mr. Jordan. It may be added, however, that to introduce into the formula the elements of velocity or slope would necessitate an adjustment for the size of the stream, for the degree of roughness of its bed, and for the resistance to erosion—an insurmountable task. The difficulties and complications of such a method have been well illustrated by Boussinesq.* Whereas by including in the formula only the area of the cross-section and the width of the channel the effect of all these factors is combined and there is only the relation of depth to curvature to consider. It must be evident, therefore, that if a velocity of 0.1 ft. per sec. will not develop a channel of the same depth and shape as one of

* "Essai sur la théorie des eaux courantes", Paris, Mémoires présentés par divers savants à l'Académie des Sciences, de l'Institut de France, Tome 23, 1877; Supplement 24 (1877).

10 ft. per sec., neither will the formula give the same depth and shape in the two cases.

The writer makes no pretension that his formulas are perfect as is evident by his effort to improve them in the earlier part of this discussion, and he welcomes any suggestions which may lead to their improvement.

In Notation 7 (page 210), the writer has pointed out some of the discrepancies cited by Mr. Landreth and has attributed them to the small helicoidal action when the curvature is slight, allowing other influences to predominate. In other words, it is the actual profile and not the formula that is to blame, if it is allowable to make so discourteous a remark about Nature's performances.

Professor Haupt has devoted so many years to the study of this problem that what he says comes from one well qualified to express a mature judgment on the subject. The writer, therefore, greatly appreciates the very favorable criticism that he makes.

Any one who has followed Professor Haupt in his extensive writings on this and allied subjects and has noted the many proposals he has made for the improvement of outer bar channels in this country and abroad, will understand his appreciation of the value of a discovery which enables an engineer to calculate quite exactly beforehand what he may expect from any given work.

It is also gratifying to get so favorable an endorsement from one as well qualified to express an opinion on the subject as Mr. Malaval. As a member of the Society of Civil Engineers of France, as Ingénieur des Ponts et Chaussées, as Director of the French Company of the Port of Rio Grande do Sul (Brazil), and as Professor a l'Ecole des Travaux Publics, his standing in France is indicated; and as the author of a most excellent work, "The History of the Port and Bar Works at Rio Grande do Sul Brazil", his qualification as an analyst and his scientific instinct are clearly shown.

Captain Chisolm has brought into this discussion some most valuable information. Being in possession of the wonderful accumulation of data of the Mississippi River Commission, acquired by years of painstaking surveys and computations, he has been able to test the writer's formulas in a most comprehensive manner. These tests cover a stretch of the river extending from Cairo, Ill., to Fort Jackson, La., a distance of more than 1 000 miles. Although many of these tests confirm the writer's conclusions, some of them would indicate a failure to do so. These apparent disagreements will now be given consideration.

Captain Chisolm, referring to Corollary (2), finds, at Collehe Point, La., at 81-Mile Point, and at Waverly Point, Miss., that in each case the radius of curvature is less than 40 $\sqrt{\text{area}}$ and yet these bends are not destructive.

A corollary is an evident conclusion from what has preceded. If the depth in a bend does not increase with an increased curvature beyond a certain amount, it is reasonable to conclude that the force which failed to increase the depth will be exerted laterally. That it does not actually produce destruc-

tion may be attributed to the fact that resistance to erosion is greater than the eroding force. That it does so in many cases is evidence that the tendency is there.

At four bends, namely, Giles Bend (Section 2371), Hard Times Bend (Section 2174), Island 16 (Section 441), and near Island No. 8 (Section 173), Captain Chisolm regards the conditions such that they disagree with the writer's conclusions as expressed in Notation 6. Especially is this deemed to be true in reference to Section 2371 of which he says: "This does not agree with the statement that 'no further deepening of the channel results from increased curvature'."

The conclusion in Notation 6 is too well established in the paper to be lightly rejected. It will be desirable, therefore, to examine into the conditions which exist in these bends to see whether they warrant Captain Chisolm's conclusion. It is found that these conditions are wholly exceptional in character and produce effects quite different from those found in ordinary bends. In each of these bends, except at Hard Times Bend, the conditions are complicated by an island, dividing the stream into two channels of unequal widths and depths. At Hard Times Bend, the conditions are complicated by a tributary entering on the concave side of the river just at the point of Section 2174. Any conclusion drawn in regard to this section that does not take into account the effect of this tributary stream is not reliable.

As the island complications at the three other bends are similar and as Section 2371 has the additional complication of artificial protection of its concave bank against erosion, this section will be used in the further discussion of this feature.

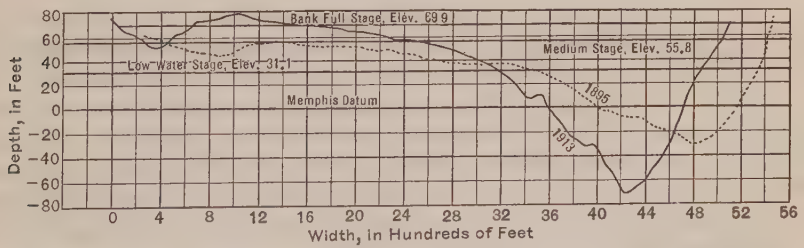

FIG. 14.—CROSS-PROFILES, MISSISSIPPI RIVER, AT STATION 2371.

Fig. 14 shows the cross-profile of Section 2371 for the years 1895 and 1913 and Table 32 gives its cross-sectional elements, as compiled by the Mississippi River Commission. These profiles, however, do not represent the actual condition at the time and stage of the river when the soundings were taken. In a river like the Mississippi, subject to erosion and accretion, the channel adjusts itself to the changed conditions as the water in the river rises and falls. This cannot be shown in a diagram embracing all stages of the water. The different planes of elevation and computed areas, however, are given for the different stages, namely, low water, medium, and bank-full stage, as if no change took place in the process of shifting from one stage to another. These cross-profiles, however, serve a useful purpose in showing the relative width

and depth of the two channels into which the stream is divided at the two periods (1895 and 1913), and the way in which the flow in the smaller channel is transferred to the larger one as the water falls from a higher to a lower stage. Thus, the width of the two channels at bank-full stage was 5 540 ft. in 1895 and 4 290 ft. in 1913. At medium stage, these widths were, respectively, 5 080 and 2 890 ft., and at low water, 1 940 ft. in 1895, and 1 820 ft. in 1913. These variations in width plainly show that the great difference in the maximum depth on the two dates was mainly due, not to the difference in curvature, but to the great difference in width. However, to give the matter further elucidation, the co-ordinates of the cross-profile of this section for 1895 and 1913 have been computed and the curves plotted. These co-ordinates are given in Tables 33 and 34 and the curves are shown on Fig. 15 (*A*) and (*B*).

TABLE 32.—Cross-Section Elements of Section 2371, Mississippi River, for the Years 1895 and 1913.

Stage.	Areas, in Square Feet.				Widths, in Feet.		Mean Depths, in Feet.		Maximum Depth, in Feet.	
	1895.		1913.		1895.	1913.	1895.	1913.	1895.	1913.
Low water..	65 650	65 650	92 100	92 100	1 940	1 820	33.8	50.6	62.0	105.2
Medium.....	80 300	145 950	54 640	146 740	5 089	2 890	38.7	50.8	....	.....
Bank-full...	75 767	221 717	149 153	195 893	5 540	4 290	40.0	45.7	....	.....

An inspection of Tables 33 and 34 shows that the actual maximum depth exceeds the computed depth in both cases. This excess is 6.5% for 1895 and 9.3% for 1913. These results bring into question the truth of Notation 6 and the validity of Formula (4); they would certainly do so if there were no other influence except that due to curvature. An inspection of Fig. 14, however, shows that as the water declines, the flow in the smaller channel is gradually shifted to the main channel, and that when the bottom of the smaller channel is reached the entire flow of the river passes through the one channel.

When the width of a stream is diminished, the channel deepens and when the volume of flow is increased without increasing the width, the same result follows. This is exactly what happens in this case. The addition of this influence to the effect of the bank protection, which prevents the normal development of the channel, gives two influences independent of the curvature, which make Captain Chisolm's conclusion in regard to Notation 6 exceedingly questionable. He must have entirely overlooked the fact of the bank protection in this bend, which was continued each year from 1897 to 1914 to prevent its movement, else he could not have classed this as a non-destructive bend.

Allusion has already been made to the fact that a cross-profile of a stream is not correctly represented when a change is made from the normal section, that is, the one in which the soundings were taken, to another stage, by the manipulation of the water surface. It will now be shown how the ratio of the

maximum depth to the mean depth is affected by a reduction of the water surface to a lower plane.

TABLE 33.—Mississippi River at Station 2371 (1913) (Fig. 15 (*A*)).

Co-Ordinates, in Feet.			General remarks.
Computed.		Actual.	
X.	*Y.*	*Y.*	
1 130	0.00	0.00	Computed from Formula (4).
1 000	38.56	20.00	Location, Mississippi River at Station 2371.
800	78.86	60.00	Area of section, 123 925 sq. ft.
600	100.37	96.00	Width, 2 260 ft.
400	103.79	114.00	Mean depth, 54.83 ft.
200	95.85	96.00	Maximum depth, 115.8 ft.
9	79.23	81.00	Radius of curvature, 6 660 ft. equal to $18.92\sqrt{\text{area}}$.
— 200	57.65	63.00	
— 400	34.81	43.00	Value of R in Formula (4) = 14 081 ft. equal to $40\sqrt{\text{area}}$.
— 600	15.41	27.00	Computed maximum depth, 105 ft., which is 9.3% less than the actual maximum depth.
— 800	0.18	16.00	
—1 000	—4.20	6.00	The stage of the water at the time these soundings were taken, was 47.5 ft. above Memphis Datum and 16.4 ft. above the plane indicated on Fig. 14 as low water.
—1 130	0.00	0.00	

TABLE 34.—Mississippi River at Station 2371 (1895) (Fig. 15 (*B*)).

Co-Ordinates, in Feet.			General remarks.
Computed.		Actual.	
X.	*Y.*	*Y.*	
970	0.00	0.00	Computed from Formula (4),
800	28.62	22.00	Location, Section 2371, 691 miles below Cairo.
600	48.94	59.00	Area, 65 270 sq. ft.
400	57.33	60.00	Width, 1 940 ft.
200	56.39	46.00	Mean depth, 33.64 ft.
0	48.62	42.00	Maximum depth, 62 ft.
—200	36.71	37.00	Radius of curvature, 16 670 ft.
—400	23.37	34.00	Value of R in Formula (4) = 16 670, equal to $65\sqrt{\text{area}}$.
—600	11.10	20.00	Value of D in Formula (4) = 48.62.
—800	2.48	9.00	W = 970 ft.
—970	0.00	0.00	Computed maximum depth, 58 ft. which is 6.5% less than the actual maximum depth.
			The stage of the water at time these soundings were taken was 31.1 ft. above Memphis Datum: which is the plane indicated on Fig. 14 as low water.

In Fig. 1 (*A*), Brazos River, the normal ratio of the maximum depth to the mean depth is 1.445. By lowering the normal plane 5 ft., this ratio becomes 1.498. By lowering the normal plane 10 ft., the ratio becomes 1.663.

In Fig. 3 (*A*), Mississippi River, the normal ratio is 1.426. By lowering the normal plane 10 ft., the ratio becomes 1.507 and by lowering it 20 ft., the ratio becomes 1.577.

In Fig. 6 (*A*), Rio Grande do Sul, Brazil, the normal ratio is 1.425. By lowering the normal plane 4 m., the ratio becomes 1.429. By lowering it 8 m., the ratio becomes 1.528, and by lowering it 12 m., the ratio becomes 1.688.

It will thus be seen that, in every instance, the reduction of the normal water surface to a lower plane increases the ratio of the maximum depth to

the mean depth, and the greater the reduction, the greater the increase in this ratio.

Now it will be noted that, in making his tests of this ratio, Captain Chisolm invariably reduces his sections to the plane of low water and therefore he obtains a greater ratio than the normal section would give. When he speaks of this range varying as much as 25% on the reach above Red River, it simply shows that a varying amount of reduction was made in this stretch of river to make his sections conform to that of low water, and in no way discredits the practically constant ratio in the normal section. It will also be noted that in his tests of this ratio in bends, he makes no allowance for the effect due to curvature, which alone causes as wide a variation in the ratio as exists in the curvatures, one from another.

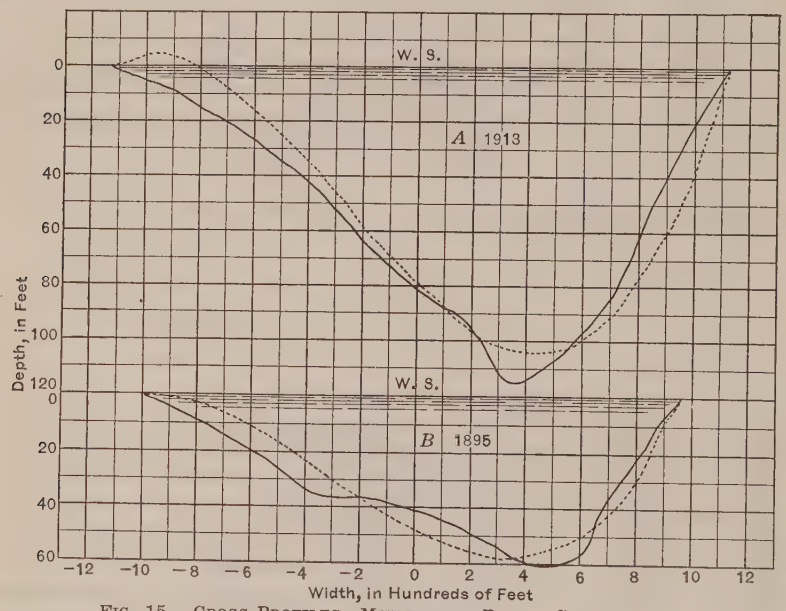

FIG. 15.—CROSS-PROFILES, MISSISSIPPI RIVER, SECTION 2371.

It will thus be apparent that the changes in the ratio caused by the manipulation of the water surface from one plane to another will cast no doubt about the truth of the practically constant ratio which exists in all normal sections.

The writer appreciates Mr. Molitor's constructive criticism and especially that feature of it where he tries to simplify and clarify the writer's formulas. His Figs. 11 and 12 add much to their clearness. All Mr. Molitor's conclusions, however, cannot be accepted. For example, he considers that the numerical constants in the formulas must be varied to suit the differences in the character of the material through which the stream flows.

It seems to be difficult for the hydraulician to realize that the area of the cross-section of a stream is an expression of the result of the factors which produce it, namely, the volume of flow, the force of the current, and the resistance to erosion. A change in any one of these factors, while the others remain

constant, will enlarge or reduce the area of the cross-section and hence the varying area expresses the varying value of these factors. This area being incorporated in the formula as has been previously explained, no variation in the numerical constants is required to provide for these changes.

Of course, a rock formation or other non-erodable material is out of the question in this discussion for no practicable current can give shape to the cross-profile of such a channel, in accordance with the law of channel development, and nothing short of artificial excavation will suffice to create the desired channel.

As regards the expedient used in the solution of Formulas (5) and (7), the writer has been unable to find any method which gives as good results as the one used. In the writer's judgment no principle in hydraulics is violated by its use.

Mr. Fawcett's discussion is particularly interesting and important because he is dealing with a real live problem in process of solution. As he seems to be somewhat at sea about many features of the question, it may be well to state briefly some of the fundamental principles that enter into it.

He has to deal with a river (the Liao River of China) of which the cross-sectional area is about 58 100 sq. ft., the width 2 700 ft., and the maximum depth about 30 ft. at low water—a river having a tidal range of 13 ft. at spring tide, a current velocity as great as 4½ knots per hour, and a bar at its mouth extending more than 8 miles into the sea, with a depth of only 6 ft. at low water on its crest.

This river debouches at the north end of the Gulf of Liao-tung, a protected body of water extending in a southwest by south direction 300 miles, including the Gulf of Pe-Chi-Li, and having a width of 75 miles. The southwest by south winds, therefore, will produce the heaviest waves, which will strike the jetty end on, a direction most favorable for its resistance.

As stated by Mr. Fawcett an attempt is being made to deepen the channel across the bar by means of a single curved training wall, of which 35 000 ft. has already been constructed to a height of about 3 ft. above low water, and about 6 000 ft. more has to be constructed to complete its projected length. This work is progressing from the shore end seaward and there has been an advance of the bar of about 3 000 ft. in the 8 years since its commencement. From this fact Mr. Fawcett concludes that a single curved jetty will cause bar advance; although the writer maintains that it will not do so when given the proper order of construction and carried to a height above ordinary high water.

A paper entitled "The Economic Location of Jetties"* gives the rules for the proper construction of a single curved jetty so as to prevent bar advance. These rules require that the foundation be constructed first to the full length of the proposed jetty and then that the structure be completed to its full height, working from the outer end toward the shore. It is also shown in that paper that the single curved jetty at Aransas Pass, Texas, built in accord with these rules caused no bar advance during the 14 years of its construction, nor after its completion.

* *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), pp. 979–986 and 1103–1127.

With a single curved jetty, the centrifugal force constrains the current to follow the line of the wall, and the helicoidal action causes the deepening of the channel. When the jetty is submerged, however, these forces cannot function and hence any attempt to compute the results to be expected from such a wall will be impossible, as Mr. Fawcett has found out for himself. He lays the blame to the formula, however, instead of to the submerged wall. With the jetty built to full height, it will create a channel across the bar of equal cross-sectional area to the channel in the river itself, and it is possible to compute its shape and size with a close approximation to the actual. This computation has been made; the results are given in Table 35 and Fig. 16. This diagram shows a channel width of 900 ft. at the 30-ft. depth at low tide.

TABLE 35.—Liao River, Manchuria, North China (Fig. 16).

Co-Ordinates, in Feet.			General remarks.
Computed.			
X.	*Y.*	*Y* + 6.	
1 350	0.00	6.00	Computed from Formula (5).
1 200	9.42	15.42	Location, outer bar (theoretical).
1 000	18.79	24.79	Area of cross-section of river at mean low tide, 58 100 sq. ft.
800	24.95	30.95	Width of river, 2 700 ft.
600	28.26	34.26	Area of section, 6 ft. below water surface, 41 900 sq. ft.
400	29.21	35.21	Mean depth of section, 6 ft. below water surface, 15.52 ft.
200	28.18	34.18	Value of P = 25.61.
0	25.61	31.61	Radius of curvature of jetty under construction, 42 000 ft.
— 200	21.92	27.92	Value of W in Formula (5) = 1 350 ft.
— 400	17.51	23.51	
— 600	12.84	18.84	
— 800	8.30	14.30	
—1 000	4.33	10.33	
—1 200	1.34	7.34	
—1 350	0.00	6.00	

The basis for this computation is the area of the cross-section of the river at a point 1 000 ft. above the inner end of the jetty. This area may not be quite exact, as the map from which it was calculated is of small scale and indicates only a few soundings. However, it will suffice to show the method to be pursued.

It seems hardly necessary to state that the quantities, D and P, are not interchangeable in Formulas (4) and (5). One of them was devised to reproduce the cross-profile for a channel occupying the entire waterway and the other for a channel occupying only part of the waterway. To interchange these constants would make both formulas untrue.

In a curved channel the helicoidal movement causes the material excavated to be carried from the concave to the convex side of the channel. It seems evident that with only one jetty the material excavated can be more easily dispersed laterally than with a second jetty resisting the lateral flow. This will explain the fact of a single curved jetty producing a better channel than two parallel curved jetties of the same curvature.

The material excavated by a single curved jetty is not carried seaward, but is dispersed laterally causing deposits along the convex side of the channel. These deposits are continually being dissipated by wave action and sea currents.

The minimum radius to be used in Formulas (5) and (7) must not be less than 40 times the square root of the total area of the section. (See "General Remarks" in Table 6.)

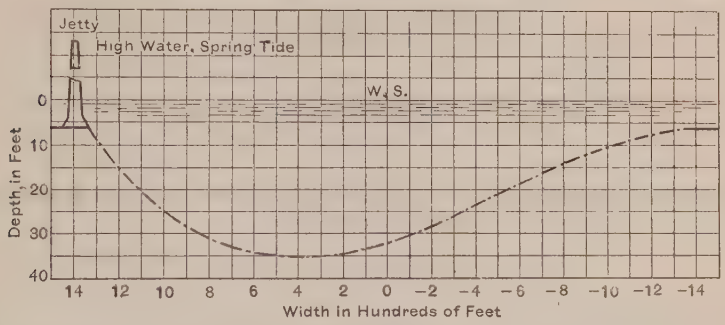

FIG. 16.—COMPUTED CROSS-PROFILE OF CHANNEL ON BAR, LIAO RIVER, CHINA.

In Notation 8 (page 210), it is stated that:

"Formulas (5) and (7) are not applicable to straight channels nor to curved ones where the radius of curvature exceeds about 110 times the square root of the area of the channel. (The exact figure has not been determined.)"

At the time this paper was presented this was the greatest radius actually tested. Subsequent consideration of the matter, however, has convinced the writer that the maximum limit is much greater than this. Indeed he believes the limit is only reached when the centrifugal force is insufficient to constrain the current to follow the curve of the jetty or the velocity is insufficient to produce any effectual helicoidal movement.

Mr. Fawcett is to be congratulated for the excellent location given to the jetty and for the prospect that this work may prove to be a remarkable success without the necessity of a second jetty or of a resort to dredging. The increased curvature indicated for the outer portion of the jetty is objectionable. In order to secure and maintain a uniform channel, one radius of curvature should be maintained throughout. He may also be assured that any further material advance of the bar can be prevented by constructing the foundation at once to the proposed outer end of the jetty and then building up the work to full height from the outer end toward the shore.

In concluding this discussion, the writer wishes to acknowledge his indebtedness to the Chief of Engineers, U. S. Army, to the Director of the U. S. Coast and Geodetic Survey, and to the Secretary of the Mississippi River Commission, for valuable information and data furnished him. He would also like to call the attention of the members of the Society to the number of eminent scientists who have joined in this discussion.

AMERICAN SOCIETY OF CIVIL ENGINEERS
INSTITUTED 1852

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Paper No. 1600

EVAPORATION ON UNITED STATES RECLAMATION PROJECTS*

By Ivan E. Houk,† M. Am. Soc. C. E.

With Discussion by Messrs. H. S. Kleinschmidt, Robert Follansbee, R. I. Meeker, B. E. Torpen, C. S. Jarvis, Reuben B. Sleight, Lynn Perry, Ralph L. Parshall, S. T. Harding, Charles H. Lee, C. M. O'Neil, J. B. Lippincott, E. F. Chandler, and Ivan E. Houk.

Synopsis

It is roughly estimated that on Federal irrigation projects alone, 1 000 000 acre-ft. of water is lost each year by evaporation from reservoirs. Much of this loss is unavoidable, but in some instances substantial economies can be effected if the nature and extent of the losses are thoroughly studied.

Reclamation Bureau engineers have been collecting and studying evaporation data for many years. Floating or land pans, sometimes both, have been maintained on projects where large storage reservoirs are necessary parts of the works, the primary object, of course, being to determine as accurately as possible the actual evaporation losses from the reservoir surfaces. Although variations in type of installation and equipment sometimes have been necessary in order to meet local conditions, the general design for land-pan installations during recent years has followed the specifications adopted by the United States Weather Bureau for its Class A stations. The Weather Bureau has co-operated with the Reclamation Bureau, in many instances, in furnishing equipment as well as in publishing daily and monthly records. Records published

Note.—The subject of this paper is one of ten selected by the Special Committee on Irrigation Hydraulics for study and research. The paper was prepared in co-operation with E. B. Debler, M. Am. Soc. C. E., at the suggestion of the Committee in order to make available for general use much valuable data heretofore only found in the files of the U. S. Bureau of Reclamation.

* Published in January, 1926, *Proceedings*.

† Engr., U. S. Bureau of Reclamation, Denver, Colo.

in the "Climatological Data for the United States", issued by the Weather Bureau, are mostly for land-pan stations, but a few floating-pan records are included therein. Daily and monthly records at several stations included in the tabulations accompanying this paper have not been published.

This paper summarizes the evaporation investigations of the Reclamation Bureau,* the more important features of which are briefly described as follows:

1.—Mean monthly evaporation records, with corresponding temperature, wind, and relative humidity data, whenever available, are given for all stations on Federal irrigation projects.

2.—Maximum and minimum values of monthly and annual evaporation are given for selected stations.

3.—The necessity for considering meteorological conditions in applying evaporation data is shown by a diagrammatic comparison of records (Fig. 4) at the two Yuma stations.

4.—The evaporation from a land pan, 14 in. deep and 3 ft. square, set in the ground with earth banked around the sides, was found to be only 72% of the evaporation from a Class A land pan, 10 in. deep and 4 ft. in diameter, set on the ground with no earth around the sides.

5.—The evaporation from a land pan on the ground at the edge of an alfalfa field was found to be only 62% of the evaporation on a tower 11 ft. above the ground.

6.—Evaporation from a floating Class A pan mounted on a raft was found to be 67% of the evaporation from a similar pan on land.

7.—Sloshing troubles in floating pans have been partly eliminated by installing a sheet-iron honeycomb in the lower part of the pan, thus dividing it into compartments.

8.—Measurements of ice evaporation at Saco, Mont., showed daily rates of from 0.010 to 0.070 in., the mean temperatures varying from about 9.6 to 31.1° Fahr.

Mean Monthly Records.—Records of mean monthly evaporation at all stations on Federal irrigation projects, whether maintained by the Bureau of Reclamation or by other agencies, are summarized in Table 1. In addition to the actual data, pertinent descriptive data regarding location, equipment, exposure, records, and parties maintaining the stations, are given in the table. Stations in the same section of the country have been kept together as far as possible. The locations of the stations are indicated on the key map, Fig. 1, by the numbers enclosed in circles, these numbers being the same as those noted above the station descriptions in Table 1.

In order to avoid useless distinctions, the name, Reclamation Bureau, has been used in Table 1 to designate the former Reclamation Service as well as the present organization. Acknowledgments for use of unpublished records are due the irrigation officials of the Office of Public Roads and Rural Engineering, United States Department of Agriculture; also the California-Oregon Power Company.

* See, also, "Records of Evaporation Obtained at 23 Different Stations in Various Parts of the United States," *Engineering News*, June 16, 1910.

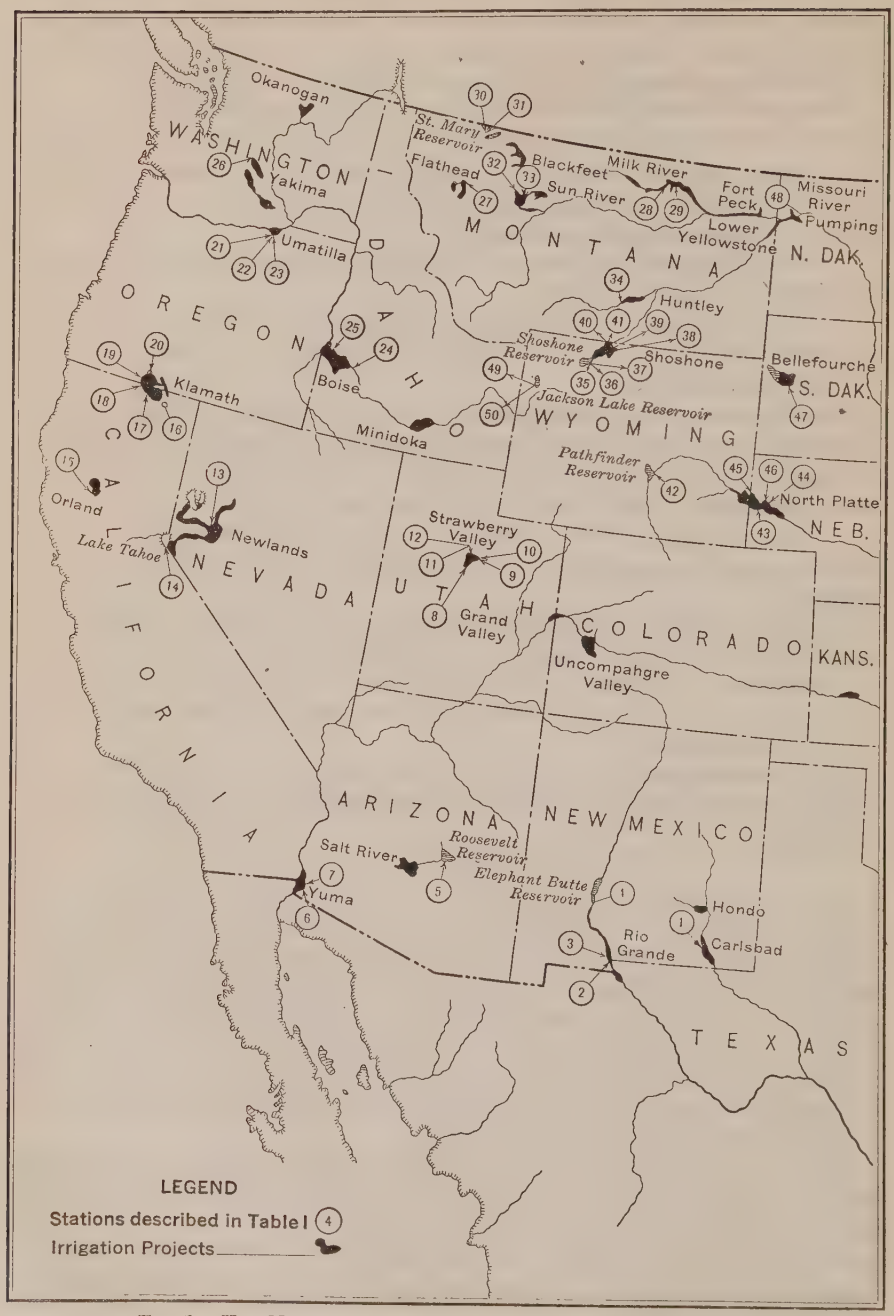

FIG. 1.—KEY MAP SHOWING LOCATION OF EVAPORATION STATIONS.

TABLE 1.—Evaporation on Reclamation Bureau Projects.

	No.	1.		2.		3.			4.			5.		
Descriptive data	Station	Avalon Reservoir		Agricultural College		Mesilla Park*			Elephant Butte*			Roosevelt*		
	State	New Mexico		New Mexico		New Mexico			New Mexico			Arizona		
	Project	Carlsbad		Rio Grande		Rio Grande			Rio Grande			Salt River		
	Exposure	Floating		Land		Land			Land			Land		
	Pan	19 in. by 3 ft., square.		3 ft., cubical		10 in. by 4 ft., circular			10 in. by 4 ft., circular			10 in. by 4 ft., circular		
	Elevation	3188		.....		3863			4475			2175		
	Records	1914 to 1923		1908 to 1913		1918 to 1923			1917 to 1923			1916 to 1923		
	Maintained by	Reclamation Bureau		.....		Agricultural College			Reclamation Bureau			Salt River Valley Water Users' Assoc.		
	Remarks	No records for 1916 and 1917, Temperature measured at Carlsbad. Pan anchored near concrete wall.		Pan set 3 ft. in ground. Water kept within 6 in. of top.		Pan on cultivated land, about 1 ft. above ground. Small trees near-by. Anemometer about 2 ft. above ground.			Pan on desert hill near reservoir, about 1 ft. above ground. Anemometer about 2 ft. above ground.			Pan on steep slope about 300 ft. from lake shore, and about 1 mile east of dam.		
Mean records	Month.	Evaporation, in inches.	Temperature, in degrees Fahrenheit.	Evaporation, in inches.	Temperature, in degrees Fahrenheit.	Evaporation, in inches.	Temperature, in degrees Fahrenheit.	Wind, in miles per hour.	Evaporation, in inches.	Temperature, in degrees Fahrenheit.	Wind, in miles per hour.	Evaporation, in inches.	Temperature, in degrees Fahrenheit.	Wind, in miles per hour.
	January	2.09	43.9	4.68	43.5	3.06	41.3	3.72	2.82	41.1	3.81	2.31	48.0	1.55
	February	3.10	48.6	3.89	44.8	4.26	45.1	4.88	3.98	46.6	4.43	3.03	51.6	1.69
	March	5.33	53.5	6.56	52.7	7.82	51.3	4.88	8.05	51.6	5.31	5.46	57.3	2.07
	April	7.15	61.7	6.90	58.9	10.03	58.4	6.69	11.03	59.4	5.92	7.88	64.3	2.21
	May	7.99	70.2	10.79	67.6	12.19	68.0	3.84	14.15	68.8	5.84	11.08	73.4	2.07
	June	8.41	77.5	10.46	76.7	12.49	76.6	4.78	14.29	78.6	4.71	13.36	84.1	1.87
	July	9.97	80.2	10.62	79.5	12.23	79.1	4.22	12.55	80.4	4.03	12.52	86.9	2.13
	August	9.14	78.9	10.09	78.6	10.08	77.5	3.36	10.88	78.1	3.63	10.45	85.3	1.62
	September	8.31	73.3	7.51	71.5	7.61	71.8	3.07	9.08	73.4	4.02	8.73	80.2	1.54
	October	6.25	62.9	6.41	60.2	6.13	61.4	3.37	7.90	62.5	4.59	6.04	69.3	1.38
	November	3.16	51.4	4.82	49.8	3.56	47.2	3.53	4.05	49.9	3.97	4.05	57.2	1.25
	December	2.47	43.4	3.83	40.0	2.61	41.7	3.49	3.02	42.9	3.98	2.30	49.5	1.37
	Totals and means	73.37	62.1	86.56	60.3	92.07	60.0	4.15	101.80	61.1	4.52	87.21	67.3	1.73

* Class A Station.

TABLE 1.—(*Continued.*)

	No.	6.			7.				8.		9.	
Descriptive data	Station	Yuma Citrus*			Yuma Evaporation*				Spanish Fork		West End Tunnel	
	State	Arizona			Arizona				Utah		Utah	
	Project	Yuma			Yuma				Strawberry Valley		Strawberry Valley	
	Exposure	Land			Land				Land		Land	
	Pan	10 in. by 4 ft., circular			10 in. by 4 ft., circular				.....		17 in. by 3 ft., square	
	Elevation	181			127				4 700		7 500	
	Records	1921 to 1923			1917 to 1923				1911 to 1915		1910 to 1915	
	Maintained by	University of Arizona			University of Arizona				Reclamation Bureau		Reclamation Bureau	
	Remarks	Pan 8 miles southwest of Yuma, on desert mesa			Pan in alfalfa field about 1 mile west of Yuma, on nearly level mesa. Relative humidity taken in Yuma				.....		Pan set 15 in. in ground	
Mean records	Month.	Evaporation, in inches.	Temperature, in degrees Fahrenheit.	Wind, in miles per hour.	Evaporation, in inches.	Temperature, in degrees Fahrenheit.	Wind, in miles per hour.	Relative humidity, percentage.	Evaporation, in inches.	Temperature, in degrees Fahrenheit.	Evaporation, in inches.	Temperature, in degrees Fahrenheit.
	January	4.23	53.2	2.41	3.08	51.1	1.44	47.0	....	30.0	....	21.8
	February	6.08	58.5	3.28	3.95	54.6	1.66	46.2	....	32.8	....	20.9
	March	8.71	61.8	3.52	5.82	58.7	1.87	40.1	....	41.4	....	31.4
	April	11.02	66.3	4.01	7.60	64.0	2.13	37.4	4.05	50.5	....	38.9
	May	14.32	75.9	3.70	8.25	69.9	1.40	38.6	5.17	57.1	4.55	47.0
	June	15.90	83.8	3.38	9.00	78.7	0.96	37.4	5.89	65.2	5.23	56.9
	July	17.53	84.9	4.05	10.42	87.0	1.31	46.4	7.67	72.6	5.20	61.8
	August	14.93	90.5	3.28	9.46	85.7	1.40	52.1	9.57	73.9	5.05	61.9
	September	11.85	84.8	2.57	7.30	79.9	1.18	56.1	6.41	62.9	3.44	52.9
	October	8.76	72.4	1.91	5.22	68.7	0.99	46.7	3.68	54.3	2.32	41.8
	November	5.50	61.3	1.99	3.16	57.6	1.02	45.5	....	41.0	....	32.6
	December	3.98	55.2	2.27	2.48	52.6	1.17	51.5	....	28.1	....	22.6
	Totals and means	122.81	70.7	3.03	75.81	67.4	1.38	44.9	42.44	50.8	25.79	40.9

*Class A Station.

TABLE 1.—(*Continued*).

	No.	10.		11.		12.			13.			14.		
Descriptive data	Station	East End Tunnel		Provo		Provo*			Fallon Experiment Farm			Lake Tahoe		
	State	Utah		Utah		Utah			Nevada			California		
	Project	Strawberry Valley		Strawberry Valley		Strawberry Valley			Newlands			Newlands		
	Exposure	Land		Land		Land			Land			Floating		
	Pan	17 in. by 3 ft., square		17 in. by 3 ft., square		10 in. by 4 ft., circular			2 by 6 ft., circular			10 in. by 4 ft., circular		
	Elevation	7600		4600		4600			3960			6230		
	Records	1905, 1914, and 1915		1910 to 1917		1918 to 1923			1908 to 1923			1916 to 1923		
	Maintained by	Reclamation Bureau		Reclamation Bureau		Reclamation Bureau			U. S. Department of Agriculture			Reclamation Bureau		
	Remarks	No temperature records for 1905. Pan set 15 in. in ground		Pan set 15 in. in ground		Pan in center of 0.38-acre field in irrigated district			Wind records began in 1912. Pan set 21 in. in ground			Pan submerged about 7 in. in river about 125 ft. from shore and 60 ft. from dam. Partly sheltered from wind		
Mean records	Month.	Evaporation, in inches.	Temperature, in degrees Fahrenheit.	Evaporation, in inches.	Temperature, in degrees Fahrenheit.	Evaporation in inches.	Temperature, in degrees Fahrenheit.	Wind, in miles per hour.	Evaporation, in inches.	Temperature, in degrees Fahrenheit.	Wind, in miles per hour.	Evaporation, in inches.	Temperature, in degrees Fahrenheit.	Wind, in miles per hour.
	January	....	13.4	....	25.9	....	27.9	....	1.12	29.9	2.75	1.58	26.0	3.04
	February	....	16.2	....	31.4	....	31.1	....	1.78	34.7	3.42	....	28.4	2.60
	March	....	24.1	....	41.3	....	40.2	1.80	4.08	43.3	4.47	3.37	30.5	2.96
	April	0.39	34.9	3.81	49.0	3.78	45.6	1.95	6.22	49.6	5.29	3.08	36.6	2.87
	May	1.32	44.2	4.29	55.6	6.06	56.4	1.62	8.28	56.7	4.84	3.75	44.4	2.51
	June	5.16	49.2	5.11	64.3	6.84	65.5	0.92	9.76	65.5	3.61	4.44	53.4	2.34
	July	5.52	57.2	5.14	70.8	7.12	71.7	0.84	10.65	73.4	2.81	5.69	60.5	2.50
	August	4.65	56.5	4.88	69.4	6.07	69.3	0.85	9.56	71.2	2.49	5.91	59.5	2.41
	September	2.60	47.2	3.42	60.3	4.18	61.4	0.84	6.40	61.5	2.60	4.61	53.5	2.28
	October	1.66	40.4	1.94	49.1	2.26	49.1	0.91	3.83	50.2	2.35	3.43	43.7	2.70
	November	0.21	29.5	0.87	39.3	0.94	38.4	0.73	1.91	39.9	2.37	2.62	35.3	2.74
	December	....	....	....	27.9	....	29.8	....	0.76	31.5	2.38	2.02	29.5	2.89
	Totals and means	21.51	34.4	29.46	48.7	37.25	48.9	1.16	64.30	50.6	3.24	40.50	41.8	2.65

*Class A Station.

TABLE 1.—(*Continued*).

	No.	15.		16.		17.		18.		19.		
Descriptive data	Station	East Park		Clear Lake		Adam's Boat Landing		Coppocks		A-Canal, Klamath Falls		
	State	California		California		Oregon		California		Oregon		
	Project	Orland		Klamath		Klamath		Klamath		Klamath		
	Exposure	Floating		Floating		Floating		Floating		.....		
	Pan	10 in. by 4 ft., circular		.....		10 in. by 4 ft., circular		10 in. by 4 ft., circular		.....		
	Elevation	1130 to 1200		4350		4670		4035		4100		
	Records	1911 to 1923		1911 to 1913		1913 to 1919		1916 to 1923		July, 1917, to June, 1918		
	Maintained by	Reclamation Bureau		Reclamation Bureau		Reclamation Bureau		Reclamation Bureau		Reclamation Bureau		
	Remarks	Anchored near reservoir shore. No wind obstructions		.....		Temperature taken at Merrill		Temperature taken at Merrill		Wind records for 1912 only		
Mean records	Month.	Evaporation, in inches.	Temperature, in degrees Fahrenheit.	Evaporation, in inches.	Temperature, in degrees Fahrenheit.	Evaporation, in inches.	Temperature, in degrees Fahrenheit.	Evaporation, in inches.	Temperature, in degrees Fahrenheit.	Evaporation, in inches.	Temperature, in degrees Fahrenheit.	Wind, in miles per hour.
	January	1.13	43.3	....	....	....	....	....	....	....	35.0	5.24
	February	1.64	46.3	....	....	....	....	....	....	....	32.8	5.89
	March	2.79	50.0	....	....	....	....	....	....	....	39.4	7.92
	April	4.75	55.2	2.04	42.4	....	....	....	....	....	45.0	7.69
	May	7.22	62.5	3.48	49.7	5.85	59.6	6.32	49.7	5.17	51.7	7.80
	June	8.87	70.6	5.60	58.4	6.34	58.1	7.69	58.4	6.61	68.2	7.62
	July	10.54	76.5	7.68	66.0	7.71	63.6	7.61	64.1	9.08	72.2	7.05
	August	9.85	74.7	7.80	64.4	6.49	63.3	6.67	62.4	7.55	70.0	5.90
	September	7.21	68.4	5.88	51.8	4.48	56.9	4.47	57.6	4.61	60.2	6.32
	October	4.47	58.9	3.60	43.3	3.05	47.8	2.86	48.5	....	55.9	6.86
	November	2.38	49.3	1.44	36.9	....	....	1.00	....	....	40.6	5.03
	December	1.54	42.8	....	....	....	....	....	....	....	36.7	5.78
	Totals and means	62.39	58.2	37.52	34.4	33.92	56.7	36.62	56.8	33.02	50.6	6.59

TABLE 1.—(*Continued.*)

	No.	20.		21.			22.			23.		
Descriptive data	Station	Klamath Falls		Cold Springs			Cold Springs			Hermiston Experiment Farm		
	State	Oregon		Oregon			Oregon			Oregon		
	Project	Klamath		Umatilla			Umatilla			Umatilla		
	Exposure	Floating		Floating			Land			Land		
	Pan	22 by 42 in., circular		.....			10 in. by 4 ft., circular			2 by 6 ft., circular		
	Elevation	4100		600			623			450		
	Records	1921 to 1924		1909 to 1913			1914 to 1923			1912 to 1923		
	Maintained by	Calif.—Ore. Power Co.		Reclamation Bureau			Reclamation Bureau			U. S. Department of Agriculture		
	Remarks	In river about 500 ft. below main canal intake		Temperature taken at Hermiston			Temperature taken at Hermiston. Anemometer 17 ft. above ground. Pan near edge of reservoir resting on ground. Some brush and willows near-by			Anemometer 3 ft. above ground		
	Month.	Evaporation, in inches.	Temperature, in degrees Fahrenheit.	Evaporation, in inches.	Temperature, in degrees Fahrenheit.	Wind, in miles per hour.	Evaporation, in inches.	Temperature, in degrees Fahrenheit.	Wind, in miles per hour.	Evaporation, in inches.	Temperature, in degrees Fahrenheit.	Wind, in miles per hour.
Mean records	January	3.07	27.0	1.01	30.4	5.69	1.13	31.6	5.86	1.28	31.2	2.68
	February	2.52	32.8	1.12	35.7	6.42	0.94	36.5	6.02	0.90	34.8	2.53
	March	1.82	37.5	2.72	44.1	8.40	3.20	45.6	8.49	2.20	43.8	4.13
	April	2.82	44.8	5.86	52.7	9.93	4.21	52.2	8.62	3.96	51.1	4.34
	May	4.73	55.0	7.01	61.1	9.25	6.47	58.9	8.57	5.41	58.7	3.68
	June	6.51	61.5	8.90	65.6	10.29	7.90	67.2	8.65	7.22	67.2	3.72
	July	8.55	69.0	12.44	72.5	9.14	9.13	73.2	8.18	8.36	73.8	3.39
	August	7.49	66.0	9.40	69.3	7.93	8.29	71.6	6.71	6.63	72.6	2.45
	September	6.10	59.8	6.87	61.8	7.93	5.44	62.3	6.70	4.42	62.3	2.31
	October	4.33	49.0	3.80	50.0	6.57	3.23	52.1	5.63	2.49	51.4	2.15
	November	2.96	39.7	1.75	43.6	7.00	1.37	40.2	5.73	0.84	40.3	1.71
	December	....	30.0	1.08	32.9	6.07	0.71	31.3	5.72	0.27	32.0	2.34
	Totals and means	50.90	47.7	61.96	51.6	7.88	52.02	51.9	7.07	43.98	51.6	2.95

TABLE 1.—(*Continued*).

	No.	24.			25.				26.			27.	
Descriptive data	Station	Arrowrock*			Deer Flat*				Lake Kachess*			Ninepipe Reservoir	
	State	Idaho			Idaho				Washington			Montana	
	Project	Boise			Boise				Yakima			Flathead	
	Exposure	Land			Land				Land			Floating and land	
	Pan	10 in. by 4 ft., circular			10 in. by 4 ft., circular				10 in. by 4 ft., circular			10 in. by 5 ft., circular	
	Elevation	3220			2510				2270			3003	
	Records	1916 to 1923			1916 to 1923				1917 to 1923			1914 to 1923	
	Maintained by	Reclamation Bureau			Reclamation Bureau				Reclamation Bureau			Reclamation Bureau	
	Remarks	Pan on bench in hillside, about 30 ft. above maximum water surface in reservoir			Pan on farm lane, surrounded by irrigated fields, about 800 ft. west of lower embankment. Relative humidity data taken at Boise. Anemometer about 2 ft. above ground				Pan about 100 yd. from high-water line. Watchman's house, and a few trees near-by			Project now under Indian Service. Temperature taken at St. Ignatius. Pan moved several times	
Mean records	Month.	Evaporation, in inches.	Temperature, in degrees Fahrenheit.	Wind, in miles per hour.	Evaporation, in inches.	Temperature, in degrees Fahrenheit.	Wind, in miles per hour.	Relative humidity, percentage.	Evaporation, in inches.	Temperature, in degrees Fahrenheit.	Wind, in miles per hour.	Evaporation, in inches.	Temperature, in degrees Fahrenheit.
	January	....	26.6	....	....	26.3	2.62	73.5	....	27.3	....	....	23.8
	February	....	29.8	....	....	31.2	2.72	69.4	....	28.1	....	....	26.2
	March	....	37.6	....	....	41.3	5.00	59.0	....	33.5	....	....	36.0
	April	3.45	46.7	1.00	5.52	49.7	4.92	54.9	....	40.2	....	....	44.7
	May	5.76	55.4	1.04	5.65	57.7	3.47	52.8	3.30	47.2	2.41	4.88	51.6
	June	7.62	65.7	0.88	6.81	65.1	1.98	48.3	4.90	56.1	2.26	5.76	59.7
	July	10.19	74.9	0.87	7.99	73.0	1.31	37.8	6.11	63.5	2.28	6.03	66.7
	August	9.15	72.7	0.83	7.16	70.6	0.97	39.4	4.40	63.3	1.92	6.27	65.5
	September	5.79	63.4	0.85	4.95	59.5	1.44	47.0	2.14	55.4	1.36	3.97	55.9
	October	2.37	50.0	0.79	2.89	50.2	1.90	58.9	....	46.5	....	1.62	46.3
	November	....	37.7	....	1.09	38.7	2.34	66.0	....	35.4	....	....	33.7
	December	....	27.8	....	0.85	31.7	3.26	74.2	....	28.7	....	....	25.3
	Totals and means	44.33	49.0	0.89	42.91	49.6	2.66	56.8	20.85	43.8	2.05	28.53	44.6

* Class A Station.

TABLE 1.—(*Continued*).

	No.	28.	29.					30.			31.	
Descriptive data	Station	Nelson Reservoir	Nelson Reservoir					Sherburne Reservoir			Sherburne Reservoir	
	State	Montana	Montana					Montana			Montana	
	Project	Milk River	Milk River					Milk River			Milk River	
	Exposure	Floating	Floating and land					.....			Land	
	Pan	18 in. by 3 ft., square	10 in. by 4 ft., circular					.....			10 in. by 4 ft., circular	
	Elevation	2215	2215					4865			4865	
	Records	1917 to 1920	1921 to 1923					1915 to 1918			1922 to 1923	
	Maintained by	Reclamation Bureau	Reclamation Bureau					Reclamation Bureau			Reclamation Bureau	
	Remarks	Temperature taken at Malta. Pan on raft in lake, partly submerged	Wind records began in 1922. Floating pan anchored about 350 yd. from shore. Land pan on small knoll 100 yd. from shore. No wind obstructions					No wind records for 1918			Pan about 1 000 ft. from shore. No wind obstructions	
	Month.	Evaporation, in inches.	Temperature, in degrees Fahrenheit.	Evaporation, in Inches. Floating.	Land.	Ratio.	Temperature, in degrees Fahrenheit.	Wind, in miles per hour.	Evaporation, in inches.	Temperature, in degrees Fahrenheit.	Wind, in miles per hour.	Evaporation, in inches.	Temperature, in degrees Fahrenheit.
Mean records	January	....	....	....	....	....	....	....	....	16.4	8.02	....	....
	February	....	....	....	....	....	....	....	....	14.8	7.92	....	....
	March	....	....	....	....	....	....	....	....	25.3	11.08	....	....
	April	....	....	....	....	....	....	....	....	32.9	4.97	....	....
	May	5.52	57.8	5.26	8.82	0.60	54.5	6.64	3.75	42.8	7.23	....	47.8
	June	6.26	66.9	6.37	9.57	0.67	65.9	5.80	5.56	53.6	6.93	4.90	55.4
	July	7.94	72.1	7.82	11.18	0.70	69.6	4.65	6.46	60.8	6.63	5.16	59.8
	August	8.20	69.5	8.67	11.36	0.76	64.2	4.69	4.97	57.4	5.43	4.86	59.4
	September	4.58	57.6	5.31	7.70	0.64	58.7	5.01	2.92	50.5	7.87	3.70	53.
	October	3.27	42.1	2.78	4.26	0.74	47.9	5.32	3.08	39.9	13.05	....	45.0
	November	....	....	....	....	....	34.9	....	....	32.5	13.53	....	....
	December	....	....	....	....	....	19.6	....	....	16.6	11.87	....	....
	Totals and means	35.77	61.0	36.21	52.89	0.68	51.9	5.35	26.74	37.0	8.71	18.62	53.4

TABLE 1.—*(Continued).*

	Number.	32.			33.			34.			35.	
Descriptive data	Station	Sun River Diversion			Willow Creek			Osborn Experiment Farm			Shoshone Dam	
	State	Montana			Montana			Montana			Wyoming	
	Project	Sun River			Sun River			Huntley			Shoshone	
	Exposure	Floating			Floating			Land			Land	
	Pan	10 in. by 4 ft., circular			10 in. by 4 ft., circular			2 by 6 ft., circular			10 in. by 4 ft., circular	
	Elevation	4474			4180			3000			5890	
	Records	1919 to 1923			1919 to 1923			1911 to 1923			1915 to 1924	
	Maintained by	Reclamation Bureau			Reclamation Bureau			U. S. Department of Agriculture			Reclamation Bureau	
	Remarks	Some of the earlier records are questionable. Pan near north bank. No wind obstructions. Sides of pan partly submerged			Pan about 200 ft. from dam, sides partly submerged			Temperature taken at Huntley, 1911 to 1918; at Osborn, January, February, and March, 1919; and at Ballantine since March, 1919. Pan set 22 in. in ground, in open field			Pan on ground 1916 to 1918; 6 in. in ground in 1915; on roof, 1919 to 1922; and on ground, 1923 and 1924	
Mean records	Month.	Evaporation, in inches.	Temperature, in degrees Fahrenheit.	Wind, in miles per hour.	Evaporation, in inches.	Temperature, in degrees Fahrenheit.	Wind, in miles per hour.	Evaporation, in inches.	Temperature, in degrees Fahrenheit.	Wind, in miles per hour.	Evaporation, in inches.	Temperature, in degrees Fahrenheit.
	January	....	26.2	....	....	26.1	....	....	21.2	5.42	0.87	26.2
	February	....	22.6	....	....	23.0	....	....	23.0	4.68	1.25	29.6
	March	....	29.0	....	....	30.8	....	....	34.4	4.79	1.62	34.5
	April	5.69	39.0	....	3.93	39.4	9.71	3.33	45.5	5.20	3.44	42.5
	May	6.54	48.9	9.07	4.38	50.2	7.50	4.72	54.5	4.96	4.85	51.0
	June	4.95	56.5	8.23	5.45	60.6	6.35	6.01	64.9	3.70	7.46	62.0
	July	8.23	67.7	7.99	7.08	67.7	5.61	7.49	71.2	3.25	8.22	69.7
	August	6.01	63.4	8.15	7.83	66.0	5.42	6.80	69.1	3.20	7.88	68.2
	September	4.19	53.7	10.97	4.70	57.8	6.57	4.50	58.7	3.54	5.43	59.2
	October	2.79	43.4	12.33	3.87	45.8	6.57	....	46.9	3.88	2.97	48.5
	November	....	31.6	....	....	33.4	....	....	34.4	4.06	0.50	38.4
	December	....	23.8	....	...	24.0	....	....	24.8	4.60	....	28.3
	Totals and means	38.40	42.2	9.46	37.24	43.7	6.82	32.85	45.7	4.27	44.49	46.5

TABLE 1.—(*Continued.*)

	No.	36.		37.		38.		39.		40.	
Descriptive data	Station	Shoshone Dam		Shoshone Dam		Powell		Powell		Ralston	
	State	Wyoming		Wyoming		Wyoming		Wyoming		Wyoming	
	Project	Shoshone		Shoshone		Shoshone		Shoshone		Shoshone	
	Exposure	Land		Land		Land		Land		Land	
	Pan	10 in. by 4 ft., circular		10 in. by 4 ft., circular		8 in. by 3 ft., circular		4 ft., circular		3 ft., circular	
	Elevation	5390		5390		4390		4390		4600	
	Records	1915		1919 to 1922		1911 to 1919		1911 to 1919		1911 to 1916	
	Maintained by	Reclamation Bureau		Reclamation Bureau		Reclamation Bureau		Reclamation Bureau		Reclamation Bureau	
	Remarks	Pan set 6 in. in ground. These values included in No. 35		Pan on shed roof 10 ft. above ground. These values included in No. 35		Pan on tower 11′ ft. above ground		Pan on edge of alfalfa field		Temperature taken at Powell. Pan on bank of reservoir. No wind obstructions	
Mean records	Month.	Evaporation, in inches.	Temperature, in degrees Fahrenheit.	Evaporation, in inches.	Temperature, in degrees Fahrenheit.	Evaporation, in inches.	Temperature, in degrees Fahrenheit.	Evaporation, in inches.	Temperature, in degrees Fahrenheit.	Evaporation, in inches.	Temperature, in degrees Fahrenheit.
	January	0.87	29.0	....	29.8	....	17.6	....	17.6	....	17.9
	February	1.25	35.0	....	28.4	....	22.8	....	22.8	....	23.9
	March	1.62	34.6	....	35.3	1.20	32.7	1.01	32.7	....	32.8
	April	4.00	52.6	....	41.4	7.52	44.2	3.99	44.2	....	46.1
	May	3.94	49.2	5.44	52.6	7.68	52.2	5.74	52.2	6.00	52.4
	June	5.19	55.8	8.48	64.6	10.21	62.8	6.55	62.8	8.42	62.1
	July	6.00	62.4	8.79	70.3	10.34	68.2	6.49	68.2	9.23	67.5
	August	4.63	68.2	8.91	70.2	9.49	66.3	6.01	66.3	8.71	66.4
	September	2.63	55.0	7.22	60.3	6.30	56.7	3.29	56.7	5.87	56.1
	October	3.19	54.3	3.16	48.9	3.88	43.1	1.94	43.1	3.00	43.7
	November	0.50	39.2	....	35.9	....	32.3	....	32.3	....	30.0
	December	....	31.2	....	28.4	....	20.1	....	20.1	....	19.8
	Totals and means	33.82	47.2	42.00	47.2	56.62	43.2	35.02	43.2	41.23	43.5

TABLE 1.—(*Continued.*)

	No.	41.		42.		43.			44.			45.		
Descriptive data	Station	Ralston		Pathfinder Reservoir		Mitchell Experiment Farm			Lake Minatare			Dutch Flats Camp		
	State	Wyoming		Wyoming		Nebraska			Nebraska			Nebraska		
	Project	Shoshone		North Platte		North Platte			North Platte			North Platte		
	Exposure	Floating		Land		Land			Land			Land		
	Pan	3 ft., circular		10 in. by 4 ft., circular		2 by 6 ft., circular			10 in. by 4 ft., circular			10 in. by 4 ft., circular		
	Elevation	4590		5900		4080			4120			4110		
	Records	1911 to 1914		1914 to 1923		1911 to 1923			1918 and 1919			1909		
	Maintained by	Reclamation Bureau		Reclamation Bureau		U. S Department of Agriculture			Reclamation Bureau			Reclamation Bureau		
	Remarks	Temperature taken at Powell. Pan floated on reservoir		Pan set on concrete foundation, sides exposed. No wind obstructions. About 1000 ft. from reservoir		Pan set 22 in. in ground. In open cultivated field prior to 1922, when it was moved ¼ mile east. Now partly sheltered from wind			Pan on wood foundation just above ground, partly sheltered from wind; 800 ft. from shore			Pan on wood foundation just above ground, partly sheltered from wind		
Mean records	Month.	Evaporation, in inches.	Temperature, in degrees Fahrenheit.	Evaporation, in inches.	Temperature, in degrees Fahrenheit.	Evaporation, in inches.	Temperature, in degrees Fahrenheit.	Wind, in miles per hour.	Evaporation, in inches.	Temperature, in degrees Fahrenheit.	Wind, in miles per hour.	Evaporation, in inches.	Temperature, in degrees Fahrenheit.	Wind, in miles per hour.
	January	....	21.2	....	22.8	....	24.3	6.55	....	24.3	9.33	....	....	....
	February	....	22.1	....	24.4	....	25.3	6.26	....	25.8	8.69	....	....	....
	March	....	31.3	....	32.9	....	34.5	7.50	....	37.7	9.37	....	....	....
	April	....	44.7	....	40.2	4.13	43.6	8.07	4.13	41.2	12.57	4.50	40.6	10.28
	May	4.00	53.7	6.60	51.2	6.23	53.9	7.76	7.68	55.5	9.17	6.25	50.0	12.16
	June	4.48	64.2	9.60	62.7	7.18	65.5	5.92	9.12	67.2	8.08	8.05	59.5	7.96
	July	4.68	67.7	11.03	70.5	8.02	71.5	4.36	9.65	73.9	7.74	10.95	69.2	6.37
	August	9.43	66.4	9.85	67.9	6.95	69.1	3.91	8.83	72.2	7.41	9.39	71.8	4.34
	September	4.74	56.9	7.40	59.0	5.24	59.8	4.29	4.48	61.4	5.61	7.44	60.1	6.29
	October	....	43.3	4.16	46.7	2.57	47.5	5.07	2.72	47.2	6.97	5.59	51.6	8.71
	November	....	34.4	....	35.4	....	35.5	5.06	....	30.0	7.97	....	....	....
	December	....	20.2	....	24.3	....	24.4	5.53	....	24.6	7.45	....	....	....
	Totals and means.	27.33	43.8	48.64	44.8	40.32	46.2	5.86	46.61	46.7	8.36	52.17	57.5	8.01

TABLE 1.—(*Continued*).

	No.	46.			47.			48.			49.		50.	
Descriptive data	Station	Sunflower Camp			Newell Experiment Farm			Williston Experiment Station			Jackson Lake		Jackson Lake	
	State	Nebraska			South Dakota			North Dakota			Wyoming		Wyoming	
	Project	North Platte			Belle Fourche			North Dakota Pumping Project			Minidoka		Minidoka	
	Exposure	Land			Land			.....			Land		Land	
	Pan	10 in. by 4 ft., circular			2 by 6 ft., circular			.....			14in. by 3 ft., square		10 in. by 4 ft., circular	
	Elevation	4070			2880			.....			6770		6770	
	Records	1910 to 1917			1908 to 1923			1918 and 1919			1922 to 1924		1924	
	Maintained by	Reclamation Bureau			U. S. Dept. of Agriculture.			.....			Reclamation Bureau		Reclamation Bureau	
	Remarks	Pan on wood foundation just above ground, partly sheltered from wind			Pan set 21 in. in ground; surrounded by cultivated fields. Anemometer 2 ft. above ground			.....			Pan set in ground, earth banked around sides to water surface. Water kept 12 in. deep		Pan set on ground, no earth around sides	
Mean records	Month.	Evaporation, in inches.	Temperature, in degrees Fahrenheit.	Wind, in miles per hour.	Evaporation, in inches.	Temperature, in degrees Fahrenheit.	Wind, in miles per hour.	Evaporation, in inches.	Temperature, in degrees Fahrenheit.	Wind, in miles per hour.	Evaporation, in inches.	Temperature, in degrees Fahrenheit.	Evaporation, in inches.	Temperature, in degrees Fahrenheit.
	January	0.92	22.9	8.57	....	16.8	7.20	....	12.3	8.55	....	....	....	....
	February	0.84	25.9	7.32	....	18.4	6.55	....	4.2	7.80	....	....	....	....
	March	2.06	31.8	9.46	....	30.9	7.50	....	26.0	8.37	....	....	....	....
	April	4.48	47.1	10.25	3.99	43.8	8.05	4.06	42.8	10.60	....	....	....	....
	May	6.35	53.5	9.60	5.57	53.6	8.26	5.98	54.0	10.09	....	....	....	....
	June	7.94	64.4	8.96	6.88	65.6	6.67	7.21	67.3	10.40	5.49	51.7	....	51.2
	July	8.21	70.9	6.66	8.06	71.8	5.43	8.57	68.6	8.34	5.61	58.0	7.56	56.8
	August	6.78	68.4	6.53	7.02	69.2	5.08	6.42	67.7	8.29	4.75	56.5	8.60	54.1
	September	5.39	59.7	6.63	5.03	59.4	5.58	3.73	55.4	8.15	3.11	49.0	4.90	47.2
	October	3.11	47.7	6.95	....	45.9	6.70	....	39.4	8.03	....	....	....	....
	November	1.71	38.6	6.96	....	33.0	6.00	....	22.1	8.17	....	....	....	....
	December	....	23.2	6.90	....	20.1	8.40	....	16.0	7.65	....	....	....	....
	Totals and means	47.79	46.2	7.90	36.55	44.0	6.78	35.97	39.6	8.70	18.96	....	21.06	....

Mean monthly evaporation records, in inches, are given in Table 1, under "Descriptive Data". Following the evaporation records, wherever the data were available, are columns giving mean monthly temperatures, in degrees Fahrenheit, mean monthly wind velocities, in miles per hour, and mean monthly relative humidities, in percentages, these being the principal meteorological factors that affect evaporation rates. Unfortunately, relative humidity data were available in only one or two instances. Years of record are given inclusively in all cases. Sizes of pans, for all stations except Station No. 2, are noted by giving the vertical depth first, then the distance across, then the shape of the cross-section, as "19 in. by 3 ft., square."

Evaporation records, like most observational data, are difficult to maintain continuously for long periods of time. Unforeseen interruptions occur despite the best laid plans. Animals break through the fence and drink the water, waves splash over the edges of the pan, the pan develops a leak, or some other happening of similar consequence vitiates the records. Although the data for some of the stations, principally Class A installations, are practically continuous, records at others are not. As a general rule, interruptions are less frequent at land pans than at floating pans. They are also less frequent at the warmer stations where freezing seldom or never occurs. At the northern stations it is, of course, necessary to discontinue the records during the winter months. Temperature, wind, and relative humidity records are seldom interrupted, except when the stations are dismantled for the winter. Sometimes these records are maintained during the cold weather and only the evaporation observations discontinued.

In compiling the data in Table 1, questionable records were discarded. Records were not interpolated except in cases where only a few days' observations were missing. Ordinarily, if more than a week's data were missing in any one month the remaining records for that month were disregarded in calculating the averages. Consequently, the mean monthly values for a given station may not be entirely comparable as regards length of record. To illustrate, the mean evaporation for June may be calculated from five records whereas the mean evaporation for July may be based on six, seven, or some other number of observations.

Range in Evaporation.—The range in annual evaporation at selected stations is indicated by the data in Table 2. Maximum and minimum records, their differences, and the years in which they occurred, are given for the stations that had the greater numbers of full years of record. Quantities are given in inches, also in percentages of mean values. It will be noted that the total range varied from 8.55 in. at the East Park Reservoir to 25.47 in. at Cold Springs, the corresponding percentages being 13.7 and 49.0, respectively.

Similar data showing variations of monthly evaporation at the floating-pan station at the East Park Reservoir and at the land-pan station at Elephant Butte are given in Table 3, these stations having been chosen because their records contain practically no interruptions, thanks to the efficient supervision of R. C. E. Weber and L. M. Lawson, M. Am. Soc. C. E., Project Superintendents. The difference between maximum and minimum records for individual months vary from 1.14 to 4.20 in. at East Park, and from 2.34

TABLE 2.—RANGE IN ANNUAL EVAPORATION AT SELECTED STATIONS.

Station.		Full years of record.	Mean annual evaporation, in inches.	Maximum Annual Evaporation.			Minimum Annual Evaporation.			Total Range.	
				Year.	Amount:		Year.	Amount:		In inches.	In percentage of mean.
No.	Name.				In inches.	In percentage of mean.		In inches.	In percentage of mean.		
1	Avalon Reservoir	6	73.37	1922	82.51	112.5	1921	65.75	89.6	16.76	22.9
3	Mesilla Park	5	92.07	1922	98.03	106.5	1923	84.74	92.0	13.29	14.5
4	Elephant Butte	7	101.80	1917	109.70	107.8	1919	90.19	88.6	19.51	19.2
5	Roosevelt	6	87.21	1917	94.64	108.4	1920	82.47	94.6	12.17	13.8
6	Yuma Citrus	3	122.81	1921	135.70	110.5	1923	113.69	92.6	22.01	17.9
7	Yuma Evaporation	5	75.81	1918	88.00	116.1	1920	70.31	92.7	17.69	23.4
13	Fallon Experiment Farm	8	64.30	1914	72.80	113.2	1913	60.94	94.8	11.86	18.4
15	East Park Reservoir	12	62.39	1923	67.01	107.4	1915	58.46	93.7	8.55	13.7
22	Cold Springs	6	52.02	1914	61.99	119.2	1923	36.52	70.2	25.47	49.0

to 4.40 in. at Elephant Butte. Fig. 2 is a view of the floating pan at East Park Reservoir. Fig. 3 shows the land-pan station at Mesilla Park, Rio Grande Project, maintained by the New Mexico Agricultural College. In compiling the data it was noted that records of daily evaporation as great as 0.8 in. have occurred at the Yuma and Roosevelt Stations.

Effect of Meteorological Conditions.—An unusual example of the variations in evaporation caused by variations in meteorological conditions is furnished by the records at two Yuma Stations. These are both Class A installations, maintained by the University of Arizona. Records are published by the U. S. Weather Bureau. The stations are less than 10 miles apart, and the mean annual temperatures differ by only 3.3 degrees. Consequently, one would expect the records to be quite similar. However, the annual evaporation measured at the Yuma Citrus Station is about 70% more than that observed at the Yuma Evaporation Station. The diagrams in Fig. 4, platted from the data in Table 1, would indicate that differences in wind velocity were responsible for this variation, the velocity at the Citrus Station being more than twice as great as at the Evaporation Station. Although the greater wind velocity undoubtedly accounts for part of the increased evaporation, of greater importance is the much lower relative humidity at the Citrus Station. The Yuma Citrus Station is situated on a desert mesa whereas the Yuma Evaporation Station is in an irrigated alfalfa field. This situation illustrates the necessity of observing relative humidity data at evaporation stations. It also shows that a careful study of meteorological conditions must be made before evaporation data obtained in one locality can be used elsewhere, even if the new location is only a few miles distant.

TABLE 3.—Range in Monthly Evaporation at the East Park Floating-Pan Station, and the Elephant Butte Land-Pan Station.

Month.	East Park Evaporation, in Inches.				Elephant Butte Evaporation, in Inches.			
	Mean.	Maximum.	Minimum.	Range.	Mean.	Maximum.	Minimum.	Range.
January	1.13	1.70	0.56	1.14	2.82	3.85	1.40	2.45
February	1.64	2.92	0.92	2.00	3.98	5.64	2.90	2.74
March	2.79	4.36	1.20	3.16	8.05	9.64	6.76	2.88
April	4.75	5.84	3.52	2.32	11.03	12.93	9.23	3.70
May	7.22	8.56	4.36	4.20	14.15	15.71	12.58	3.13
June	8.87	9.88	7.69	2.19	14.29	17.06	12.66	4.40
July	10.54	11.42	9.24	2.18	12.55	13.85	10.24	3.61
August	9.85	10.60	9.18	1.42	10.88	12.19	9.10	3.09
September	7.21	8.30	5.64	2.66	9.08	10.17	7.83	2.34
October	4.47	5.52	2.50	3.02	7.90	9.41	6.72	2.69
November	2.38	3.52	1.60	1.92	4.05	5.34	2.77	2.57
December	1.54	2.20	1.00	1.20	3.02	3.96	0.99	2.97

Size and Installation of Pans.—The stations included in Table 1 are not consistent as regards kinds of pans and methods of installation. Several different sizes and shapes are represented, as are also several different styles of mounting. Fortunately, direct comparisons of records at differently designed installations are possible in one or two instances.

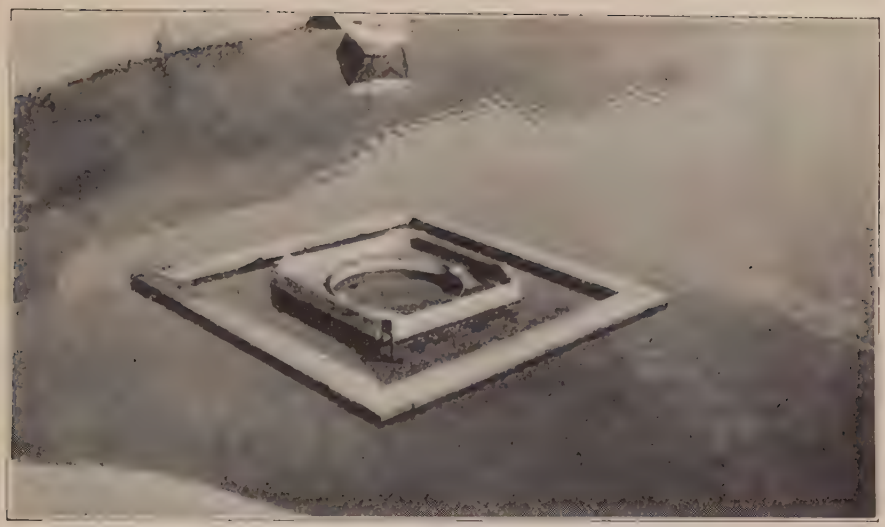

FIG. 2.—FLOATING EVAPORATION PAN AT THE EAST PARK RESERVOIR, ORLAND PROJECT.

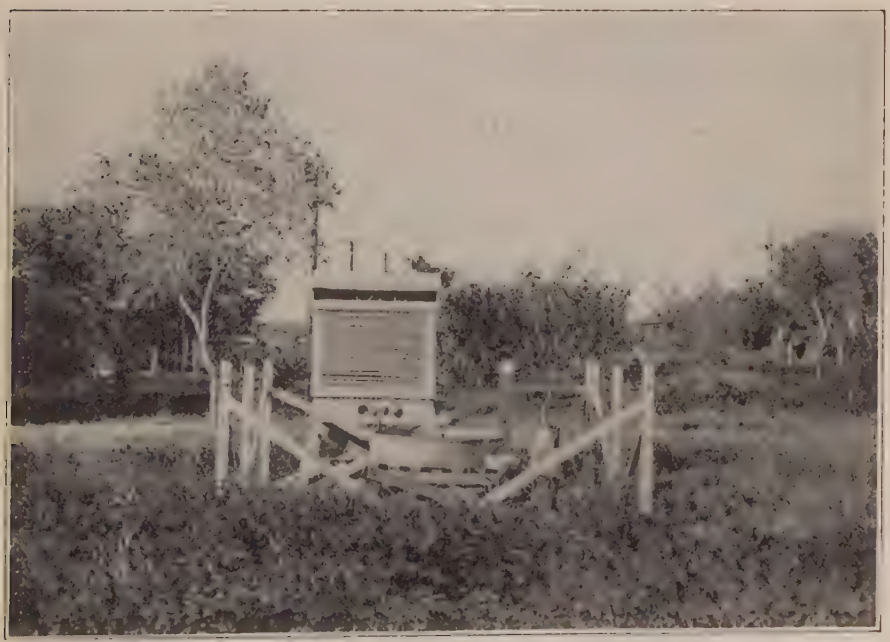

FIG. 3.—LAND-PAN STATION AT MESILLA PARK, RIO GRANDE PROJECT, MAINTAINED BY THE NEW MEXICO AGRICULTURAL COLLEGE.

Two land pans were maintained at Jackson Lake Reservoir, in Wyoming, during the summer of 1924. One pan, 14 in. deep and 3 ft. square, was set in the ground with earth banked around the sides to the level of the water which was kept about 2 in. below the top. The other—a Class A pan, 10 in. deep and 4 ft. in diameter—was set on the ground with no earth placed around the sides. The evaporation from the former was only 72% of that measured at the latter, the monthly percentages varying from 69 to 75.

At Powell, Wyo., on the Shoshone Project, two land pans were maintained during the summer seasons from 1911 to 1919, inclusive. One pan, 8 in. deep and 3 ft. in diameter, was mounted on a tower 11 ft. above the ground. The other, 4 ft. in diameter, depth not given, was situated on the ground at the edge of an alfalfa field. The evaporation from the latter was only 62% that from the former. Individual monthly percentages varied from 50 to 84.

Floating and Land-Pan Evaporation.—Floating pans and land pans were maintained at the Nelson Reservoir, Milk River Project, Montana, from 1921 to 1923, inclusive. Both pans were the Standard U. S. Weather Bureau Class A type, 10 in. deep and 4 ft. in diameter. Pan No. 29 in Table 1 gives the mean records and the ratios of the floating-pan averages to the land-pan averages. The ratios vary from 0.60 to 0.76, averaging 0.68.

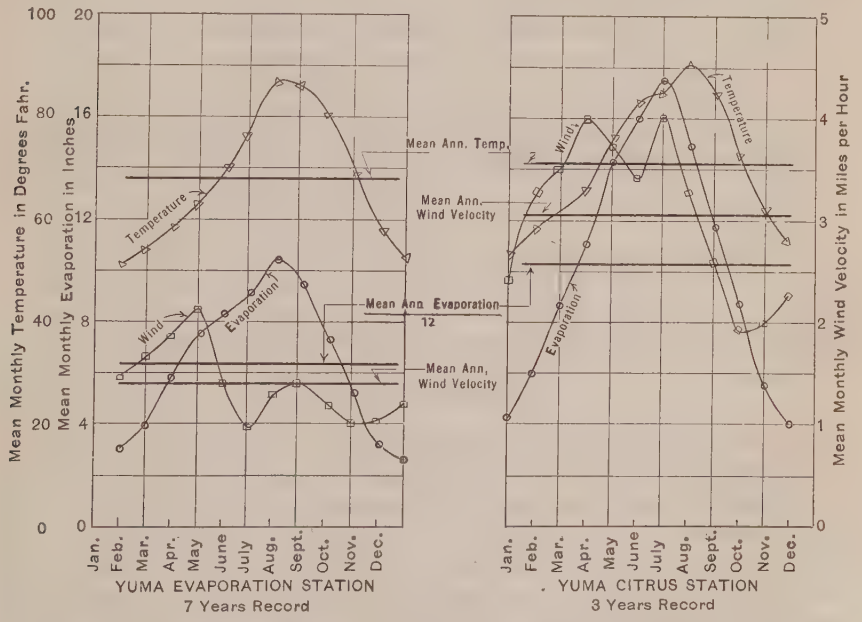
Mean Monthly Temperature in Degrees Fahr.
Mean Monthly Evaporation in Inches
Mean Monthly Wind Velocity in Miles per Hour
Mean Ann. Temp.
Mean Ann. Wind Velocity
Mean Ann. Evaporation 12
Mean Ann. Wind Velocity
Temperature
Wind
Evaporation
Jan. Feb. Mar. Apr. May June July Aug. Sept. Oct. Nov. Dec.
YUMA EVAPORATION STATION
7 Years Record
YUMA CITRUS STATION
3 Years Record

FIG. 4.—COMPARISON OF RECORDS AT THE YUMA EVAPORATION STATIONS.

Similar comparative records were maintained at the Ralston Reservoir, Shoshone Project, Wyoming, from 1911 to 1914, inclusive, using circular pans 3 ft. in diameter, depths not known. The ratios of floating-pan averages to land-pan averages varied from 0.50 to 1.09, averaging 0.71. These values do not appear in Table 1 because the land-pan data given therein are for 1911 to 1916, inclusive.

Attempts to obtain similar comparisons at the Deer Flat and Arrowrock Stations on the Boise Project, Idaho, were made during the summer of 1916, but owing to sloshing troubles no very satisfactory information was secured. Sloshing troubles at the floating-pan station at Nelson Reservoir were partly eliminated by installing a No. 22 gauge, galvanized, sheet-iron honeycomb in the lower part of the pan, dividing it into compartments approximately 12 in. square and 6 in. deep.

Ice Evaporation.—Ice evaporation was measured at Saco, on the Milk River Project, Montana, during the winter of 1922 and 1923. G. E. Stratton, M. Am. Soc. C. E., Superintendent, reports as follows:

"An attempt was made to get some rough data on the winter evaporation. For this purpose, a pan, 14.85 in. in diameter (in which 1 oz. of water equals 0.01 in. in depth) and 20 in. in depth, was installed on a platform scale, at Saco. Some trouble was experienced with the pan bursting during thawing and freezing weather, but in general it was maintained during the winter months."

The results of Mr. Stratton's experiments are given in Table 4. It will be noticed that the average daily rates vary from 0.010 in., in January, when the mean temperature was 9.6° Fahr., to 0.070 in., in November, when the mean temperature was 31.1° Fahr.

TABLE 4.—Ice Evaporation on the Milk River Project, Montana, During the Winter of 1922 and 1923.

Month.	Number of days observed.	Total evaporation, in inches	Average Daily Rate.		Equivalent, in inches per month.	Mean* temperature, in degrees Fahrenheit.
			In inches.	In feet.		
1922:						
November	16	1.12	0.070	0.0059	2.10	31.1
December	13	0.17	0.013	0.0011	0.40	28.1
1923:						
January	20	0.20	0.010	0.0009	0.31	9.6
February	28	0.41	0.015	0.0012	0.41	9.8
March	10	0.56	0.056	0.0047	1.73	26.6

* For period of observations.

Practical Value of the Data.—Generally speaking, the mean data in Table 1 are not well adapted to theoretical studies of evaporation laws, or to testing the various formulas which have been prepared for calculating such losses. This is because of the varied types of equipment and installation represented, as well as of the interruptions in the records. Some of the longer, more continuous records at the warmer stations, such as those obtained at East Park, Elephant Butte, Roosevelt, and Yuma, may be suited to such purposes; although they are not as desirable as specially conducted experiments carried on by scientifically trained observers. However, it is believed that the data in Table 1 will be found of great practical value, not only to irrigation engineers, but also to those engaged on municipal water supply works, water power developments, or any other of the many engineering problems which require the use of evaporation records.

DISCUSSION

H. S. KLEINSCHMIDT,* M. AM. SOC. C. E. (by letter).—As indicated by the author, the question of evaporation is of great importance to those interested in reservoirs in the western part of the United States. There has always been considerable uncertainty and difference of opinion on this subject, and the author is to be complimented on having assembled his information in such good form. As stated by him, the obtaining of dependable information has been attended by many circumstances affecting the reliability of the records. To one not familiar with this subject it might seem that it should be a simple matter but it is far from simple.

No mention is made of precipitation. The writer believes that definite information should be furnished in this connection. The question at once arises whether the data submitted are for actual total evaporation, or for total evaporation less precipitation. This question has frequently arisen in the writer's experience, some engineers being in the habit of speaking of evaporation with precipitation deducted, and others without taking it into account.

Also, information is desirable as to method of handling pans, that is, whether water is added daily, weekly, monthly, etc., to keep the water level up to a fixed mark. As the keeping of records for a floating pan is much more difficult than for a land pan the comparative values between floating and land pans are especially valuable.

ROBERT FOLLANSBEE,† M. AM. SOC. C. E. (by letter).—This paper presents a mass of valuable information on a subject that is of especial importance to hydraulic engineers in the Western States, and in regard to which existing literature is meager. The measured evaporation from ice is the only record of that kind the writer has seen.

The records as the author has given them represent evaporation measured in pans of different sizes, either set in the ground, resting on top of it, or floating in ponds and reservoirs. In the form given, the records are not all comparable, nor do any of them represent directly the evaporation from reservoir surfaces, the phase of evaporation that chiefly concerns the hydraulic engineer.

TABLE 5.—FACTORS TO REDUCE OBSERVED EVAPORATION TO RESERVOIR EQUIVALENTS.

Size of pan.	Reduction factor.
Class A Station, 4 ft., circular, 10 in. deep, on ground	0.66
3 by 3 ft. by 18 in., floating	0.91
3 by 3 by 3 ft., set in ground	0.80
6 ft., circular, 2 ft. deep, set in ground	0.90
Class A Station, floating	0.92
3½ ft. circular, 22 in. deep, floating	0.90

Fortunately, data are available for the reduction of these records to what have been called "reservoir equivalents" or the corresponding evaporation from

* Cons. Engr., Salt Lake City, Utah.

† Dist. Engr., U. S. Geological Survey, Denver, Colo.

the surface of a large body of water, under the same conditions of temperature, relative humidity, and wind velocity. The most comprehensive experiments to determine the proper reduction factors for pans of various diameter, depth, and immediate surroundings were those made by the Office of Public Roads

TABLE 6.—Mean Monthly Evaporation, in Inches, Reduced to Reservoir Equivalents.

Month.	Mesilla Park.*	Elephant Butte.	Roosevelt.	Yuma Citrus.	Yuma Evaporation.	Provo.
January	1.98	1.94	1.54	2.89	2.14	(0.68)
February	2.88	2.92	2.17	4.01	2.80	(0.77)
March	5.10	5.30	3.57	5.51	3.99	(2.40)
April	6.37	7.12	5.08	7.19	5.10	2.84
May	7.48	8.91	7.14	9.45	5.70	3.74
June	7.95	9.30	8.56	10.63	6.20	4.16
July	7.42	8.08	8.29	11.76	7.20	4.46
August	6.49	7.13	7.05	10.28	7.10	3.81
September	5.20	6.11	5.80	8.00	5.57	2.74
October	4.04	5.24	3.88	5.53	3.77	1.61
November	2.52	2.67	2.44	3.58	2.41	0.83
December	1.70	2.02	1.49	2.45	1.83	(0.60)
Total	59.13	66.74	57.01	81.28	53.81	28.64

* Called "Agricultural College" in Weather Bureau reports.

and Rural Engineering in Denver, Colo., from November, 1915, to November, 1916.* The comparison between evaporation from a Class A pan and a 12-ft. pan has been checked by experiments made by the U. S. Geological Survey in Escalante Valley, near Milford, Utah, during 1925.

The factors of Table 5, applicable to most of the author's records, have either been taken directly, or deduced, from the Denver experiments.

It will be noted from Table 5 that the factors for the Class A pan, floating, and set in the ground, show a relation comparable with that cited by the author at Nelson Reservoir, Montana.

The writer has recently made a study of the evaporation records in, and adjacent to, the Colorado River Basin and has reduced to reservoir equivalents a number of the records presented by the author (Table 1), who used data to 1923, inclusive, in determining his mean monthly values. In the table of reservoir equivalents, Table 6, the means are based on all records to 1925, inclusive. The addition of two years more makes very little change in the mean monthly temperatures and wind velocities as originally given, and they are not repeated.

In addition to the Reclamation records, the writer has compiled records (Table 7), from nine evaporation stations in the Southwest, maintained by various organizations, giving the mean monthly reservoir equivalents of these records, together with temperature and wind velocities.

On an average 50% of the evaporation at the stations noted in Table 7 occurs during the 4-month period from June to September.

* *Journal of Agricultural Research*, Vol. 10, July 30, 1917, pp. 209–242.

TABLE 7.—MEAN MONTHLY RESERVOIR EQUIVALENT FOR EVAPORATION STATIONS IN AND ADJACENT TO COLORADO RIVER BASIN.*

Month.	Reservoir equivalent, in inches.	Temperature, in degrees Fahrenheit.	Wind velocity, in miles per hour.	Month.	Reservoir equivalent, in inches.	Temperature, in degrees Fahrenheit.	Wind velocity, in miles per hour.
	WAGONWHEEL GAP, COLO., 1920-1924.				SANTA FE, N. MEX., 1913-1914; 1916-1925.		
Jan.	(0.85)	15	1.7	Jan.	1.13	29	2.8
Feb.	(0.77)	18	1.8	Feb.	1.50	34	3.1
Mar.	(1.21)	22	2.2	Mar.	2.75	38	3.6
Apr.	(1.95)	29	2.4	Apr.	4.27	46	3.9
May	(2.83)	42	2.6	May	5.94	56	3.5
June	3.36	53	2.6	June	7.01	66	3.0
July	3.04	55	2.2	July	5.80	69	2.1
Aug.	2.36	52	1.8	Aug.	5.14	67	1.6
Sept.	2.10	46	1.7	Sept.	4.48	61	1.8
Oct.	1.35	35	1.7	Oct.	3.35	50	2.2
Nov.	(1.17)	24	1.5	Nov.	1.98	40	2.4
Dec.	(0.78)	17	1.4	Dec.	1.07	30	2.5
Year.	21.77	34	2.0	Year.	44.42	49	2.7
	MYTON, UTAH, 1918-1925.				FARMINGTON, N. MEX., 1915-1925.		
Jan.	(0.43)	14	(3.1)	Jan.	.69	38	....
Feb.	(0.65)	23	(3.6)	Feb.	1.33	41	....
Mar.	(1.68)	36	4.3	Mar.	2.88	48	....
Apr.	4.14	46	4.1	Apr.	4.90	54	....
May	5.92	57	3.4	May	6.34	63	....
June	6.75	66	2.9	June	6.90	71	....
July	6.50	72	2.4	July	7.24	74	....
Aug.	5.55	70	2.4	Aug.	5.62	72	....
Sept.	4.15	61	2.6	Sept.	4.37	64	....
Oct.	2.48	48	2.5	Oct.	3.20	54	....
Nov.	(1.20)	34	2.1	Nov.	1.76	44	....
Dec.	(0.40)	20	(1.6)	Dec.	.87	38	....
Year.	39.85	46	3.0	Year.	46.10	55	....
	PIUTE DAM, UTAH, 1918-1925.				WILLCOX, ARIZ., 1917-1925.		
Jan.	(0.94)	26	(3.4)	Jan.	2.25	41	3.8
Feb.	(1.03)	31	(3.5)	Feb.	3.11	44	3.9
Mar.	(1.79)	37	(3.8)	Mar.	5.12	48	4.4
Apr.	4.84	44	3.9	Apr.	6.67	55	4.5
May	6.08	56	3.6	May	7.78	63	3.5
June	7.81	64	3.4	June	8.06	72	2.8
July	7.11	71	2.8	July	7.02	76	2.5
Aug.	6.31	68	2.8	Aug.	5.95	73	1.8
Sept.	5.20	60	2.9	Sept.	5.35	69	2.3
Oct.	3.35	47	3.0	Oct.	4.44	58	2.3
Nov.	1.44	36	2.9	Nov.	3.08	48	3.0
Dec.	(0.80)	28	(3.0)	Dec.	2.22	41	3.4
Year.	46.70	47	3.2	Year.	61.05	57	3.2

* Unpublished records furnished through courtesy of U. S. Weather Bureau and U. S. Forest Service.

Wagonwheel Gap: Mean of records for two Class A Weather Bureau Stations, on near-by slopes, one having a northern exposure, and the other a southern exposure. Myton: Class A Station. Santa Fé: Records for 1913-14 taken by floating pan, 3 by 3 ft. by 18 in., in reservoir, 1 mile west of city. Class A Station established in open space on edge of city in 1916. Farmington: Floating pan, 3 by 3 ft. by 18 in., on slough near city. Piute Dam: Class A Station in Sevier River bottom, 8 miles south of Marysvale. Piute Reservoir, 500 ft. south, and Sevier River, 200 ft. southeast. Mesa: Class A Station in alfalfa field, 1 mile west of Mesa, in Salt River Valley. Lees Ferry: Class A Station in canyon of Colorado River, 10 miles south of Utah line. Walls of canyon distant 100 and 200 yd., respectively; river, 400 to 600 ft. wide and 140 ft. distant from pan. Willcox: Class A Station in alfalfa field, 3 miles northwest of town, in north central part of Sulphur Springs Valley, which has nearly level floor, 9 miles wide. Deming: Floating pan, 3 by 3 ft. by 18 in., in pond of considerable size. Yuma Reservoir: Floating pan, 3 by 3 ft. by 18 in., on railroad reservoir in Yuma. Values in parentheses were estimated.

TABLE 7.—(*Continued*).

Month.	Reservoir equivalent in inches.	Temperature, in degrees, Fahrenheit.	Wind velocity, in miles per hour.
Mesa, Ariz., 1917-1925.			
Jan.	1.85	49	1.6
Feb.	2.44	54	1.9
Mar.	3.82	57	2.0
Apr.	5.31	63	2.2
May	6.99	72	1.9
June	7.53	82	1.7
July	6.91	86	1.5
Aug.	5.51	84	1.1
Sept.	4.32	78	0.9
Oct.	3.20	68	1.0
Nov.	2.18	57	1.4
Dec.	1.62	50	1.5
Year	51.68	66	1.6
Lees Ferry, Ariz., 1922-1925.			
Jan.	1.15	33	1.4
Feb.	2.07	43	1.4
Mar.	3.84	50	2.9
Apr.	5.21	59	2.8
May	7.79	72	2.7
June	8.72	80	2.4
July	8.87	86	2.1
Aug.	7.04	81	1.9
Sept.	5.76	74	1.7
Oct.	4.65	60	1.9
Nov.	1.96	47	1.5
Dec.	1.32	37	1.8
Year	58.38	60	2.0

Month.	Reservoir equivalent in inches.	Temperature, in degrees, Fahrenheit.	Wind velocity, in miles per hour.
Deming, N. Mex., 1915-1925.			
Jan.	2.60	43	....
Feb.	3.12	46	....
Mar.	5.48	53	....
Apr.	6.78	60	....
May	7.39	66	....
June	7.97	70	....
July	6.03	78	....
Aug.	5.42	72	....
Sept.	5.42	69	....
Oct.	4.84	63	....
Nov.	3.49	51	....
Dec.	2.66	45	....
Year	61.20	59	....
Yuma Reservoir, Arizona, 1903.			
Jan.	3.02	..	....
Feb.	3.17	..	....
Mar.	4.80	..	....
Apr.	6.56	..	....
May	8.95	..	....
June	9.33	..	....
July	9.78	..	....
Aug.	9.73	..	....
Sept.	8.24	..	....
Oct.	5.27	..	....
Nov.	3.46	..	....
Dec.	3.08	..	...
Year	75.39	..	....

The evaporation at the higher altitudes is influenced greatly by the slope on which the records are taken. The Wagonwheel Gap records (Elevation 9 610) are the combined results of measurements made on a slope having a northern exposure and one having a southern exposure. The former receives the direct rays of the sun for a shorter period than the latter, and the resulting difference in temperature and relative humidity is strikingly shown by the evaporation on the two slopes during the period from June to October, for which it is possible to measure evaporation without interference from freezing. Table 8 shows the mean monthly values, each covering records of five years, for the two slopes.

Reservoirs of any considerable size at the higher altitudes are usually surrounded by slopes of both exposures, and the evaporation from the water surfaces will approximate the mean of the values for each slope.

In the arid Southwest the factors influencing evaporation for any given month have a relatively small variation from year to year, and this causes a small variation in the total evaporation for each year. Table 9 shows the percentage of mean evaporation measured each year at Farmington, N. Mex., and Mesa, Ariz.

TABLE 8.—Evaporation (Reservoir Equivalent) on Slopes of Northern and Southern Exposure at Wagonwheel Gap, Colo.

Month.	Reservoir Equivalent, in Inches.	
	Northern exposure.	Southern exposure.
June	2.91	3.82
July	2.39	3.69
August	1.59	3.12
September	1.23	2.96
October	0.45	1.89
Total	8.57	15.48

The author calls attention to the need for observations of relative humidity data at evaporation stations. In none of the records given by him are strictly comparable relative humidity records available, as the only ones presented were taken some distance away under different conditions. Being at regular U. S. Weather Bureau stations, they were presumably taken at a point some distance above the ground where the air is dryer than close to the evaporating water surfaces.

TABLE 9.—Variation in Annual Evaporation at Farmington, N. Mex., and Mesa, Ariz.

Year.	Evaporation, in inches.	Percentage of mean.	Year.	Evaporation, in inches.	Percentage of mean.
Farmington, N. Mex.			Mesa, Ariz.		
1915	39.11	85	....	.....	...
1916	43.34	94	....	.....	...
1917	45.69	99	1917	41.21	86
1918	44.18	96	1918	51.52	100
1919	48.57	106	1919	51.86	101
1920	45.01	98	1920	54.76	106
1921	45.58	99	1921	58.63	114
1922	47.21	102	1922	51.27	99
1923	47.04	102	1923	52.85	103
1924	51.71	112	1924	54.97	106
1925	52.62	114	1925	46.13	89

The writer agrees thoroughly with the statement that a careful study of meteorological conditions must be made before evaporation data obtained in one locality can be used elsewhere. Unfortunately, records of wind velocity obtained at regular U. S. Weather Bureau stations are not directly comparable with wind velocities recorded at the Class A Weather Bureau evaporation stations. The former are usually taken on the roofs of buildings from 35 to 60 ft. above the ground and the latter are taken 2 or 3 ft. above the ground. Table 10 shows a few comparisons between wind velocities in the two classes of records.

The chief factors influencing the rate of evaporation are relative humidity, temperature, and wind velocity, and these factors are so interdependent that it is impossible to make, with any considerable degree of accuracy, a comparison between evaporation in different localities, based on a single factor.

TABLE 10.—Relation of Wind Velocities at Different Elevations.

Station.	Wind Velocity, in Miles per Hour.	
	Near ground.	Regular Weather Bureau.
Santa Fe, N. Mex.	2.7	7.1
Yuma, Ariz., evaporation	1.4	5.4
Mesa, Ariz.	1.6	5.2
Roosevelt, Ariz.	1.7	5.2
Agricultural College	2.4	7.5

Since the rate of evaporation depends primarily on the capacity of the air to absorb additional moisture which, in turn, is measured by the relative humidity, it is possible that if records of the humidity, taken close to the water surface, were available, a relationship between it and the rate of evaporation might be found. Comparable records of relative humidity, however, are practically non-existent.

The interdependence of temperature, relative humidity, and wind velocities is well illustrated by the three Yuma records. While the mean annual temperature at the different stations varied by only 4°, the reservoir equivalent of the observed evaporation varied from 53.8 to 81.3 in. Although records of relative humidity are not available, it is known that they must have been considerably higher at the Evaporation Station in an alfalfa field near the river than at the railroad reservoir, and still higher than at the Citrus Station situated on a barren mesa having a wind velocity 85% greater.

R. I. Meeker,* M. Am. Soc. C. E. (by letter).—The compilation on evaporation given by Mr. Houk fills a large void in engineering literature, and his extensive tabular data should find wide use, when reduced by the proper coefficients to open-water conditions. One valuable feature of the evaporation records on Reclamation projects is their general standardization; another important feature is the duration of the records, largely ranging from five to sixteen years. Hereafter there can be little excuse in the Western United States for the evaporation vagaries resorted to in the Lake Conchos studies of 1915,† and the Southeastern Oregon Duty of Water Analysis in 1918.‡

It is to be regretted that a few additional pages were not devoted to the relation of evaporation data to open-water surfaces, together with the neces-

* Cons. Engr., Interstate River Compacts, State of Colorado, Denver, Colo.

† "A Study of the Depth of Annual Evaporation from Lake Conchos, Mexico," by Edwin Duryea, Jr., M. Am. Soc. C. E., and H. L. Haehl, Assoc. M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXX (1916), p. 1829.

‡ "Determination of the Duty of Water by Analytical Experiment," by W. C. Hammatt, M. Am. Soc. C. E., *Transactions*, Vol. LXXXIII (1919–20), p. 200.

sary coefficients for reducing various types of pans and tanks, to a common basis. While such material is meager, it is correspondingly valuable. There is a pressing need for additional field laboratory studies to check and confirm the present incomplete data on coefficients of reduction, for pans and tanks of various types, to open-water surface conditions.

Especially is a check needed on the Sleight data secured at the Denver Field Laboratory in 1916.* Such check work should be done in a different climatic area. It is especially desirable that the relation of data secured by Class A land pans be checked, and that the relation of Class A floating pans be determined for open-water conditions.

Coefficients of Reduction for Evaporation Records to Open-Water Surfaces.—During 1916, R. B. Sleight, Assoc. M. Am. Soc. C. E., Irrigation Engineer, U. S. Department of Agriculture, conducted an elaborate series of field laboratory studies of evaporation from water surfaces in evaporation pans and tanks of various shapes and sizes, and under diverse conditions. These field studies were made at Denver, Colo., and are, as far as the writer knows, the most comprehensive data extant in this field of endeavor. The field laboratory was situated in open prairie country, and the investigations covered a period of one year.†

Table 11 contains coefficients of reduction for evaporation records to open-water surfaces, taken therefrom. In three instances coefficients of reduction have been interpolated by the writer from the Sleight data and are properly noted. Mr. Sleight qualifies‡ the use of his coefficients of reduction as follows:

"Data on such tanks may be safely extended to large open water surfaces under exactly the same conditions of wind, air temperatures, and relative humidity, by multiplying the evaporation depth by the various coefficients."

Confirmatory Data on Coefficients of Reduction for Evaporation Records Secured in Tanks and Pans.—Confirmatory data on coefficients of reduction for various types of evaporation pans and tanks are meager. Table 1 contains records of an evaporation station on Nelson Reservoir, Milk River Project, Montana, which offers a rough confirmation of the Sleight coefficient of 0.66 for 4-ft. circular land pans, 10 in. deep.

At the Nelson Reservoir Station simultaneous records were secured in both land and floating pans of the same size (4-ft. circular, 10 in. deep) for a period of 6 months, covering 3 years. The Nelson Reservoir data give a coefficient of 0.68 for the reduction of records secured in a land pan 4 ft. in diameter to floating pans of the same dimensions. This figure is only roughly comparable to Sleight's coefficient of 0.66 because the floating-pan record is not the equivalent of an open-water surface, but is still subject to a slight reduction to represent open-water evaporation.

Evaporation Data Reduced for Application to Open-Water Surfaces.—Table 12 contains evaporation data reduced for application to open-water surfaces. It is derived from the evaporation records submitted by Mr. Houk

* "Evaporation from the Surfaces of Water and River Bed Materials," by R. B. Sleight, Assoc. M. Am. Soc. C. E., *Journal of Agricultural Research*, Vol. X, No. 5, July 30, 1917.

† *Journal of Agricultural Research*, Vol. X, No. 5, July 30, 1917.

‡ *Loc. cit.*, p. 237.

and from the evaporation records submitted by the writer in Table 13. Columns (4) and (7) contain the important data used in reduction to open-water surfaces. Column (4) gives average yearly records of evaporation, transposed from depths, in inches, to depths, in feet. Column (7) gives the coefficient of reduction, and Column (8) gives the evaporation values for open-water surfaces, and is the result of applying the coefficients in Column (7) to Column (4).

TABLE 11.—Coefficients of Reduction, Evaporation Records to Open-Water Surfaces.*

Land Tanks and Pans.

Depth of tank or pan, in feet.	Depth of water in tank or pan, in feet.	Dimensions of tank or pan, in feet.	Coefficient of reduction to open-water surfaces.	Remarks.
		Circular:		
3.0	2.75	2.0, diameter	0.77	Tank set in ground to depth 2.75 ft.
3.0	2.75	4.0, diameter	0.84	Tank set in ground to depth 2.75 ft.
3.0	2.75	6.0, diameter	0.90	Tank set in ground to depth 2.75 ft.
3.0	2.75	9.0, diameter	0.98	Tank set in ground to depth 2.75 ft.
3.0	2.75	12.0, diameter	0.99	Tank set in ground to depth 2.75 ft.
2.5	2.25	6.0, diameter	0.89‡	Tank set in ground to depth 2.17 ft.
2.0	1.75	6.0, diameter	0.88	Tank set in ground to depth 1.75 ft.
0.83†	0.62	4.0, diameter	0.66	Pan set on timbers on ground. This pan is used at Class A evaporation stations, U. S. Weather Bureau.
		Cubical:		
3.0	2.75	3 by 3, square	0.80	Tank set in ground to depth of 2.75 ft. Fort Collins type.

Floating Pans.

		Square:		
1.5	1.25	3 by 3, square	0.91	Pan set in water and protected by raft to reduce wave action. This is U. S. Geological Survey floating standard.
		Circular:		
0.83†	0.58	4.0, diameter	0.92±‡	Pan set in water; submerged to a depth of 7 in.
1.83	1.50±	3.5, diameter	0.91‡	.....

* Data from Denver Field Laboratory located at Elevation 5346, open prairie land. Long-time climatic records at U. S. Weather Bureau in down-town section of Denver, located on roofs of office buildings, are as follows:

Mean annual temperature, in degrees Fahrenheit	50.1	53 years record
Wind, in miles per hour	7.4	" " "
Relative humidity, percentage	53	" " "
Mean annual precipitation, in inches	14.27	" " "

† 10-in. depth.

‡ Approximate interpolation by writer from Sleight data.

At the risk of incurring criticism because of the paucity of reliable information concerning proper evaporation pan coefficients, the writer has prepared Table 12 primarily as a step in the correlation of evaporation records with mean annual temperatures. Records from Mr. Houk's paper have been used only where there are complete yearly records for a period of several years.

Table 12 contains the information used in Fig. 5. This diagram is a correlation of evaporation with mean annual temperatures and contains "guide

lines" for the determination of yearly evaporation values from open-water surfaces. It should be clearly understood that the "guide lines" represent average conditions, and that the plotting of individual years with mean annual temperatures would show a wider dispersion. The heavy straight lines represent approximately average conditions of evaporation for various mean annual temperatures. To the right and left of these heavy lines are lighter parallel lines which roughly set the limit of maximum and minimum values. A study of Table 12 will show that the minimum line is controlled by data at stations where evaporation values are largely reduced because of special protection either artificial or natural. The maximum and minimum lines define graphically an evaporation belt.

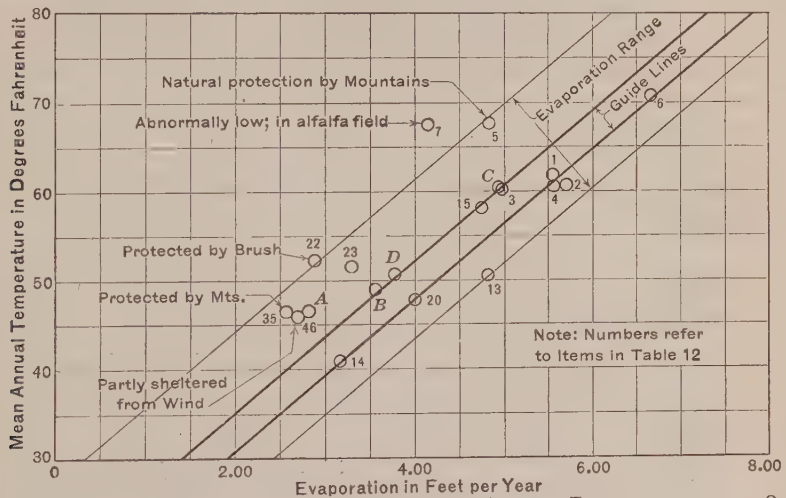

FIG. 5.—CORRELATION OF EVAPORATION WITH MEAN ANNUAL TEMPERATURES FOR OPEN WATER SURFACE IN WESTERN UNITED STATES.

The upper end of the heavy "guide lines" is controlled by data for mean annual temperatures between 60 and 70°, and are applicable to Imperial Valley conditions. The lower end of the heavy "guide lines" is controlled by evaporation values for mean annual temperatures between 40 and 50 degrees. The evaporation values taken from Fig. 5 are gross values, and in applying them to reservoir studies, yearly quantities, in feet, should be reduced by subtraction of yearly precipitation values in order to determine net evaporation values for the water surface under consideration.

The straight line relation, defined by the "guide line", is not offered as a panacea for any and all evaporation determinations where no records are available. Fig. 5 is offered as a guide for the determination of maximum and minimum limits as correlated with mean annual temperatures, and furnishes the ground for a decision as to the probable average yearly values, assuming, of course, that consideration be given to local conditions obtaining at the reservoirs or bodies of water where evaporation values are desired. Where time and other considerations preclude the actual determination by field laboratory methods of evaporation values, it may aid to define limits and set an approximate figure based on mean annual temperatures.

TABLE 12.—Evaporation Summary with Conversion to Open-Water Surface.*

Number.	Station and location.	Mean annual temperature, in degrees Fahrenheit.	Average annual evaporation, in feet.	Number of years recorded.	Pan or tank, description.	Reduction coefficient.	Annual evaporation, open-water surface, in feet.	Wind, in miles per hour.	Remarks.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1	Avalon Reservoir, Carlsbad Project, New Mexico	62.1	6.11	10	Floating, 19 in. by 3 by 3 ft.	0.91	5.56	....	Pan anchored near concrete wall.
2	Agricultural College, Las Cruces, New Mexico	60.3	7.21	7	Land, 3 by 3 by 3 ft.	0.80	5.77	....	Pan set 3 ft. in ground. Water kept within 6 in. of top.
3	Mesilla Park Agricultural College, New Mexico	60.0	7.67	6	Land, 10 in. by 4 ft. (circular)	0.66	5.06	4.15	Pan on cultivated land on campus. Small trees near-by.
4	Elephant Butte, Rio Grande Project, New Mexico	61.1	8.48	7	Land, 10 in. by 4 ft. (circular)	0.66	5.60	4.52	Pan on desert hill near reservoir.
5	Roosevelt, Salt River Project, Arizona	67.3	7.27	8	Land, 10 in. by 4 ft. (circular)	0.66	4.80	1.73	Rugged mountains enclose lake on south and west. Pan on steep slope about 300 ft. from lake shore; 1 mile east of dam.
6	Yuma Citrus, Yuma Project, Arizona	70.7	10.23	3	Land, 10 in. by 4 ft. (circular)	0.66	6.75	3.03	Pan 8 miles southwest of Yuma, on Desert Mesa.
7	Yuma Evaporation, Yuma Project, Arizona	67.4	6.32	7	Land, 10 in. by 4 ft. (circular)	0.66	4.17	1.38	Pan in alfalfa field about 1 mile west of Yuma on nearly level mesa. Relative humidity, 44.9%, taken at Yuma.
13	Fallon Experiment Farm, Newlands Project, Nevada	50.6	5.36	16	Land, 24 in. by 6 ft. (circular)	0.90	4.82	3.24	Pan set 21 in. in ground. Wind records commenced in 1912
14	Lake Tahoe, Newlands Project, California	41.8	3.54	8	Floating, 10 in. by 4 ft. (circular)	0.92	3.26	2.65	Pan submerged 7 in. in river, 125 ft. from shore and 60 ft. from dam. Partly sheltered from wind.
15	East Park, Orland Project, California	58.2	5.20	13	Floating, 10 in by 4 ft. (circular)	0.92	4.78	....	Anchored near reservoir shore. No wind obstructions.
20	Klamath Falls, Klamath Project, Oregon	47.7	4.41	4	Floating, 22 in. by 3.5 ft. (circular)	0.91	4.01	....	In river about 500 ft. below Main Canal Intake.
22	Cold Springs, Umatilla Project, Oregon	51.9	4.34	10	Land, 10 in. by 4 ft. (circular)	0.66	2.86	7.07	Temperature at Hermiston. Anemometer 17 ft. above ground. Pan near edge of reservoir on ground. Some brush and willows near-by.
23	Hermiston Experiment Farm, Umatilla Project, Oregon	51.6	3.66	12	Land, 24 in. by 6 ft. (circular)	0.90	3.29	2.95	Anemometer 3 ft. above ground.

*Column (8) gives evaporation values for open-water surfaces. The values in Columns (3) and (8) are used on Fig. 5.

TABLE 12.—(*Continued.*)

Number.	Station and location.	Mean annual temperature, in degrees Fahrenheit.	Average annual evaporation, in feet.	Number of years recorded.	Pan or tank description.	Reduction coefficient.	Annual evaporation, open-water surface, in feet.	Wind, in miles per hour.	Remarks.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
35	Shoshone Dam, Shoshone Project, Wyoming.....	46.5	3.89	10	Land, 10 in. by 4 ft. (circular)	0.66	2.57	....	Pan on ground, 1916 to 1918; 16 in. in ground, 1915; on roof, 1919 to 1922; and on ground, 1923 and 1924.
46	Sunflower Camp, North Platte Project, Nebraska.....	46.2	4.07	8	Land, 10 in. by 4 ft. (circular)	0.66	2.69	7.90	Pan on wood foundation just above ground, partly sheltered from wind.
A	Fort Collins Agricultural College, Colorado.....	46.5	3.50	39	Land, 3 by 3 ft.....	0.80	2.80	6.10	From Table 13. Relative humidity, 67.5 per cent. Tank set in ground with 4-in top rim exposed. Water carried at ground surface. Anemometer 60 ft. above ground.
B	Santa Fe Field, U. S. Weather Bureau, New Mexico.....	48.8	5.38	9	Land, 10 in. by 4 ft. (circular)	0.66	3.55	2.68	Pan in open field in northwestern part of Santa Fe. Relative humidity, 54.5 per cent.
C	Deming, State Engineer, New Mexico.....	60.6	5.52	11	Floating, 11 in. by 3 by 3 ft.....	0.91	5.02	....	At Cobbs Ranch; temperature of water in pan, 60.2°; outside of pan, 59.3 degrees.
D	Farmington, State Engineer, New Mexico.....	51. *	4.20	10	Floating, 18 in. by 3 by 3 ft.....	0.91	3.82	....	Pan in slough on Wahls Ranch. Temperature of water in pan, 55.1°; outside of pan, 55.1°.

* Mean of 20 years' records at Bloomfield and Fruitland.

TABLE 13.—Evaporation Records, Plains and Plateau Areas, Western United States.

	A.				B.				C.			
Station	Fort Collins				Santa Fe Field				Deming			
State	Colorado				New Mexico				New Mexico			
Exposure	Land				Land				Floating			
Pan or tank	3 ft., cubical				10 in. by 4 ft., circular				18 in. by 3 ft., square			
Elevation	5 000				6 975				4 300			
Records	1887 to 1925, inclusive				1917 to 1925, inclusive				1914 to 1924, inclusive			
Maintained by	Colorado Agricultural College				U. S. Weather Bureau				State Engineer, New Mexico			
Remarks	Tank set in ground with 4 in. of top rim exposed; water kept at ground surface. Tank located on campus; 4 different locations in 39 years. Anemometer located on ground at evaporation tank. Records, 1911-25.				Class A station located in open field (unirrigated) in northwestern part of Santa Fe. Relative humidity taken at Weather Bureau Station ½ mile southeast Anemometer just above evaporation pan 16 in. above ground.				Pan located on small body of water at Cobb's Ranch. Air temperatures taken at Deming by U. S. Weather Bureau.			
Month.	Evaporation, in inches.	Temperature, in degrees Fahrenheit.	Wind, in miles per hour.	Relative humidity, percentage.	Evaporation, in inches.	Temperature, in degrees Fahrenheit.	Wind, in miles per hour.	Relative humidity, percentage.	Evaporation, in inches.	Temperature of water in pan, in degrees Fahrenheit.	Temperature of water outside of pan, in degrees Fahrenheit.	Temperature, in degrees Fahrenheit.
January	1.41	25.9	2.1	71.5	1.62	28.8	2.9	64.2	2.68	44.4	43.6	43.9
February	1.51	27.4	2.0	72.4	2.00	33.7	3.1	62.3	3.41	48.5	47.7	47.5
March	2.63	35.7	2.6	66.7	3.82	38.2	3.6	54.6	5.94	53.0	52.4	50.9
April	4.29	45.6	2.8	60.2	6.07	45.5	3.9	47.8	7.14	58.2	57.5	58.0
May	4.92	54.0	2.0	63.1	8.78	55.5	3.5	43.9	8.19	65.2	65.1	66.5
June	5.61	63.4	1.6	63.6	10.28	65.6	2.9	41.7	8.78	71.7	71.0	76.6
July	5.78	68.4	0.85	66.5	8.83	69.1	1.9	53.4	7.14	73.5	72.2	78.5
August	5.33	67.3	0.8	67.1	7.97	67.3	1.6	56.6	5.58	73.5	72.5	77.2
September	4.38	59.2	0.95	67.8	6.50	61.3	1.8	53.6	5.52	70.7	69.7	71.0
October	3.25	47.5	1.3	68.4	4.60	50.1	2.1	53.9	5.55	65.1	63.9	62.3
November	1.58	35.8	1.5	70.4	2.56	39.1	2.4	57.0	3.74	53.2	52.0	51.8
December	1.29	27.7	1.8	72.8	1.52	31.1	2.4	65.2	2.62	44.8	44.0	43.4
Totals and means	41.98	46.5	1.7	67.5	64.55	48.8	2.68	54.5	66.29	60.2	59.3	60.6

TABLE 13.—(*Continued.*)

	D.	E.	F.
Station	Farmington	North Platte	Garden City
State	New Mexico	Nebraska	Kansas
Exposure	Floating	Land	Land
Pan or tank	18 in. by 3 ft., square	30 in. by 6 ft., circular	30 in. by 6 ft., circular
Elevation	5 300 ±	3 000	2 900
Records	1915 to 1924, inclusive	1907 to 1925, inclusive	1908 to 1925, inclusive
Maintained by	State Engineer of New Mexico	University of Nebraska	Kansas State Agricultural College.
Remarks	Pan located in slough at Wahls Ranch	Tank located on tableland near Experiment Station. Set in ground to average height of water surface. Water kept to within 2 to 4 in. of top of tank. Wind velocities at substation.	Tank located at Experiment Station, 5 miles northeast of Garden City on open prairie. Set in ground with lip 4 in. above Water surface, kept at ground level. Anemometer, 2 ft. above ground at evaporation tank.

Month.	Evaporation, in inches.	Temperature of water in pan, in degrees Fahrenheit.	Temperature of water outside of pan, in degrees Fahrenheit.	Temperature, in degrees Fahrenheit.	Evaporation, in inches.	Temperature, in degrees Fahrenheit.	Wind, in miles per hour.	Relative humidity, percentage.	Evaporation, in inches.	Temperature, in degrees Fahrenheit.	Wind, in miles per hour, 1908-24.	Relative humidity, percentage.
January	0.86	35.6	36.4	....	....	....	....	....	....	28.6	7.1	....
February	1.41	41.0	41.0	....	....	....	....	....	....	34.1	7.9	....
March	3.25	47.0	47.1	....	....	....	....	....	....	42.4	9.7	....
April	5.30	54.7	54.3	....	5.22	47.9	8.6	....	6.37	51.4	11.0	....
May	6.90	64.7	63.9	....	6.39	57.1	8.3	....	8.28	62.0	10.3	....
June	7.67	71.6	71.0	....	7.71	68.4	7.3	....	9.88	72.8	9.2	....
July	7.56	75.9	75.5	....	8.93	74.0	6.0	....	10.97	78.1	7.8	....
August	6.17	72.1	72.4	....	7.73	72.2	5.6	....	9.50	76.7	6.8	....
September	4.74	63.2	63.0	....	5.90	63.7	6.2	....	7.66	68.4	9.7	....
October	3.57	54.5	54.0	....	....	....	....	....	....	55.1	7.9	....
November	1.94	43.3	43.8	....	....	....	....	....	....	46.0	7.3	....
December	0.99	37.6	38.4	....	....	....	....	....	....	29.4	7.2	....
Totals and means	50.36	55.1	55.1	51.0*	41.88	63.9	7.0	....	52.66	53.8	8.5	....

* Annual temperature is average of 20-year periods at Bloomfield, 12 miles east, and Fruitland, 12 miles west, of Farmington.

A study of evaporation data in the paper, and elsewhere in engineering literature, shows the danger of making blind applications of existing records to other areas without due consideration of local climatic and physical conditions in both the areas of origin and application. Intelligent interpretation of evaporation data requires careful study of the conditions under which evaporation observations have been made.

This discussion is intended as a preliminary effort at correlation of evaporation records in the Western United States. It is hoped that Fig. 5 may serve as an engineering tool and guide to further efforts and studies along this line of engineering research. Table 13 is submitted as additional information to supplement Table 1. Messrs. Houk and Debler have performed a notable task in compiling evaporation records for forty-six stations; they deserve the thanks of the Engineering Profession.

B. E. TORPEN,* M. AM. SOC. C. E. (by letter).—This subject is of general interest in the West. From all the data collected it is hoped that definite rules and instructions will be formulated for the use of, say, three standard types of evaporation pan installations, namely, floating, on the ground, and 10 ft. above the ground, and that a factor will be specified for use with each type in determining the probable evaporation from a storage reservoir.

TABLE 14.—EVAPORATION FROM CUSHMAN STORAGE RESERVOIR.

Station: Lake Cushman.
State: Washington.
Project: Cushman Power Project.
Exposure: Floating.
Dimensions of Pan: 10 in. by 4 ft., circular.
Elevation: 760.
Records: 1925 (Water Year) October, 1924 to September, 1925.
Maintained by: City of Tacoma.
Remarks: No wind obstructions; pan kept half full; 6-in. stilling well in center of pan.

MEAN RECORDS.

Month.	Evaporation, in inches, at Cushman.	Temperature, in degrees Fahrenheit, at Cushman.		Wind, in miles per hour, at Tacoma.	Relative humidity, percentage, at Tacoma.	Precipitation, in inches, at Cushman.
		Water.	Air.			
January	0.05	39	40	10.6	85.0	20.53
February	0.25	39	43	8.9	74.5	20.18
March	1.47	45	46	6.9	67.0	2.42
April	1.76	50	47	5.9	70.5	4.80
May	4.02	62	62	7.2	58.5	1.98
June	4.75	67	65	7.4	55.5	1.97
July	5.50	63	67	6.0	53.5	0.53
August	3.92	65	53	7.4	59.0	0.42
September	3.01	58	48	5.6	62.5	1.73
October	1.17	48	46	8.8	70.5	20.30
November	0.31	40	42	7.5	81.0	17.28
December	0.06	38	35	8.7	89.0	9.59
Totals and means.	26.27	51.1	49.5	7.58	68.5	101.63

* Supt. of Design and Constr., Bull Run Storage Project, Water Bureau, City of Portland, Portland, Ore.

From Table 2, it is apparent that a single year's record will probably be within a range of 12.5% of the yearly mean. Such a record, therefore, has great value in estimating the evaporation from a proposed reservoir.

The Cushman Storage Reservoir, with a capacity of 450 000 acre-ft., has a maximum surface area of 4 000 acres and a minimum area at full drawdown of 2 000 acres, with a probable yearly mean area of 3 650 acres. In hydraulic studies of the probable yield of the reservoir for power purposes a mean annual evaporation of 36 in. was assumed to be liberal for the site. This is equal to a continuous flow of 15 cu. ft. per sec. and is almost 2% of the annual mean run-off of 800 cu. ft. per sec.

The year's record (Table 14) taken at Cushman for a check on this assumption gives a floating-pan evaporation of 26 in. and indicates that the 36 in. assumed was high. It is probable that the evaporation from the reservoir is not the same as from a floating pan. It is hoped that the proper factor will be found to apply to pans in determining the actual evaporation from a reservoir.

C. S. Jarvis,* M. Am. Soc. C. E. (by letter).—Although the data summarized by the author had been widely known in fragmentary form during past years, it has been enhanced in value by his orderly assemblage and clear presentation. Of special importance is the comparison between rates of evaporation within the same locality as affected by exposure to desert winds or to air currents tempered by contact with dense vegetation like grain or forage crops, the rate of transpiration of which may approach one-half, or more, of the evaporation rate for free water surfaces.

It is probable that shallow pondage, such as depths of 2 or 3 in., may promote greater evaporation than that recorded for either floating or land pans, due largely to increased temperature. It is not unusual, in the irrigation districts, to find alfalfa and other field crops "scalded out" by intense sunshine on the stagnant pools remaining for a few hours after each watering on dense clay soil, if the surface is slightly uneven or the drainage is imperfect. This danger is greater immediately after a crop of hay has been removed, leaving only the bare stubble. The preferable practice is to apply the water a few days before cutting, thus taking advantage of the protective shade and, at the same time, insuring a vigorous new growth without delay.

Where health seekers' colonies have been established in the arid regions, it has been demonstrated that the best results accrue from living above the canals and cultivated fields, because the humidity exhaled from the growing crops may have a depressing effect on the patient and prevent the healing of respiratory tissues.

In contrast with the cultivated vegetation, the native shrubs and plants of the desert have structures and covering which make each leaf, stalk, and root act as a reservoir. It is truly remarkable what sturdy growth is maintained under a few inches of annual rainfall, and how eagerly the pores open for incoming moisture only.

Investigations carried on by the U. S. Department of Agriculture and various College Experiment Stations have shown that selected crops may be

* Care. U. S. Bureau of Public Roads, Washington, D. C.

matured by strictly conserving the moisture which normally falls even in semi-arid regions. One of the most successful expedients is to plant and to fallow during alternate seasons, thus utilizing two years' rainfall for maturing one crop. Where the total precipitation is 12 in. or less per year, and the normal evaporation rate from a free water surface or from wet gound exceeds this total in one summer month, it is evident that the dry farmer or the settler on an irrigation project must be vigilant and able to reduce or nearly eliminate the opportunity for evaporation. Otherwise, the transpiration process would cease and the plant leaves would wilt. A week's delay beyond the proper time for cultivating and renewing the mulch or dust blanket may transform fair prospects to partial failure of crops.

In view of the enormous rates of evaporation reported for the desert regions of the Southwest, usually from 60 to 100 in. per year, the reservoir losses together with those sustained in transit to the farm unit may readily constitute more than one-half the total storage.

One of the many recent developments in the Salt River Valley of Arizona is the installation of pumping units to draw on the underground reservoir. The plane of saturation had been gradually rising since the improvement of the irrigation system by the Reclamation Service, and had water-logged extensive areas, flooding basements, and reducing the productivity of large tracts. The wells driven at strategic points and equipped with pumps have lowered the water-table, reclaimed the areas in need of drainage, added nearly 100 sec-ft. of dependable flow for the canals near the place of intended use, and eliminated the fear of further water-logging. Positive control of the underground water has transformed it from a menace to a potential resource, more valuable than surface storage because it is not subjected to such heavy tolls.

The practical irrigationist on a reclamation project soon realizes that the important consideration is not the quantity of water delivered on his land, but the portion retained within range of root systems. Where a surface mulch is carefully maintained, the crop may be matured in three months or more with the same quantity of soil moisture as might have been evaporated from the uncultivated ground in a few days of midsummer.

If properly interpreted, rates of evaporation from free water surfaces or from wet ground are indices to the attainable limits of transpiration and the consequent growth for ordinary crops. The great problem is to maintain an ample reserve of moisture to insure against wilting, and, at the same time, to avoid saturation and compaction of the soil such as results from flooding. A far better expedient is sub-irrigation wherever the humus content or the porosity of the subsoil makes it practicable. On various projects where drainage systems have been installed, it has proved advantageous to regulate the underground water surface in midsummer by the intermittent use of flash-boards in the silt chambers or in the open ditches. This process introduces the moisture exactly where it is needed, promotes capillarity to the rootlets but not to the surface, and thus maintains transpiration at a maximum rate and evaporation at a minimum. The same principle is utilized wherever shrubs or trees are supplied through sub-surface ducts or distributing pipes, and also

where deep furrows are prepared for the reception of irrigation water, and covered soon thereafter with a dust mulch.

With the construction of surface reservoirs and embankments to control floods and conserve the surplus flow, it is advisable to make investigations, tests, and basic provisions for the utilization of underground storage and also the underflow of streams during periods of drought. This, in turn, would provide additional capacity for dissipating flood crests, if adequate inlets are available.

On a limited scale, storm sewer outlets are satisfactorily functioning in coarse glacial drift or in the underlying sand and gravel strata of river valleys. This method of disposal not only saves the cost of deep trenching and large conduits, but conserves the water in a reservoir effectually roofed by the valley soil, and, therefore, immune from the usual losses. The City of Phœnix, Ariz., has utilized this method to great advantage.

Probably the underground waters would be more extensively used and depended on if there were some competent authority to prescribe regulations. The usual procedure thus far observed is to allow installations and pumping without restriction as long as the supply meets every demand, and without reference to depletion of reserve. However, after the water-plane has been lowered until the shallow wells are dry, the controversy naturally develops into litigation with its attendant delays, expense, and uncertainties. As a result, many meritorious projects have been abandoned with financial loss to those investors least able to afford such reverses; and the hazards of financing similar enterprises have multiplied.

If the underground water resources were administered by the same authority and with the same regard for the public welfare as now govern the use of surface streams, untold benefits would follow. This is especially true in regions where the evaporation and transpiration rates are excessive and where the chief problem of agriculture is to maintain the most favorable moisture content of the soil for promoting plant growth and fruitage, as on the typical successful reclamation project.

Reuben B. Sleight,* Assoc. M. Am. Soc. C. E. (by letter).—This paper should be welcomed by all members of the profession interested in irrigation and hydraulics. The writer's contribution will be confined mainly to the summaries of part of the data collected by himself in a two-year period during which practically his entire time was spent on evaporation research.

In the summer of 1915 he went to Denver, Colo., as Engineer for the U. S. Department of Agriculture to build and operate an Irrigation Field Laboratory. This project was conceived by, and the work was done under the direction of, Samuel Fortier, M. Am. Soc. C. E., then Chief of Irrigation Investigations, Office of Public Road and Rural Engineering. The major investigations at the Laboratory for the two years of its operation before the World War were evaporation studies. The report† of the first year's observations did not give to that eminent authority in irrigation engineering, Dr. Fortier,

* Asst. to Secy., U. S. Dept. of Commerce, Washington, D. C.

† "Evaporation from the Surfaces of Water and River-Bed Materials," by R. B. Sleight, *Journal of Agricultural Research*, Vol. X, No. 5, July 30, 1917.

due credit for the conception of the evaporation research project and guidance of the work throughout the period of operation of the Laboratory.

One of the problems set, and the one in partial answer to which the figures presented in this discussion were obtained, was the determination of ratios between evaporation amounts from the surfaces of water held in containers of different sizes, shapes, and positions or setting.

The Laboratory,* its floating pans, etc., are shown in Figs. 6, 7, and 8. It was located within the city limits of Denver, but on the open prairie south of the residential districts. Methods, accuracy of work, and full description of all water-surface evaporation investigations have been presented elsewhere.†

The Laboratory was completed in time to make the first evaporation measurements during the third week of November, 1915. These measurements were carried through until September 30, 1917, and summaries of data obtained from them are given herein. These summaries combine the data published in 1917 with one additional year's observations. Many evaporation tanks were used. In only two cases are the records continuous for the entire period, and as will be seen from Table 15 only two or three months' observations were possible from some of the types of tanks used.

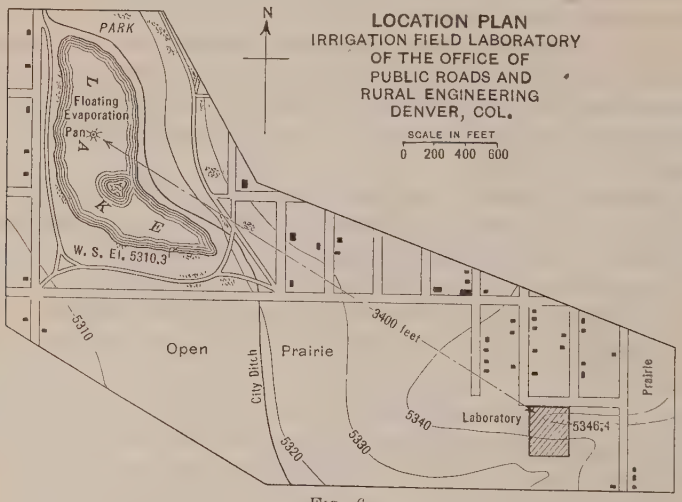

FIG. 6.

The first item listed in Table 15 is Ground Tank No. 7, 12 ft. in diameter, 3 ft. deep, set 2.75 ft. deep, and with a water depth as near 2.75 as it could be kept; this form has been selected as a base upon which to compute ratios of evaporation from tanks of different sizes and shapes. The evaporation from Tank No. 7—85.47 in. during the period, November 15, 1915, to September 30, 1917, or whatever other amount was evaporated from Tank No. 7 during the period of operation of the tank with which a comparison is made—is taken as 100 per cent.

* See, also, *Engineering News*, Vol. 76, No. 23, p. 1080.

† "Evaporation from the Surfaces of Water and River-Bed Materials," by R. B. Sleight, *Journal of Agricultural Research*, Vol. X, No. 5, July 30, 1917.

FIG. 7.—TANK ARRANGEMENT AT DENVER, COLO., LABORATORY, 1916, LOOKING WEST.

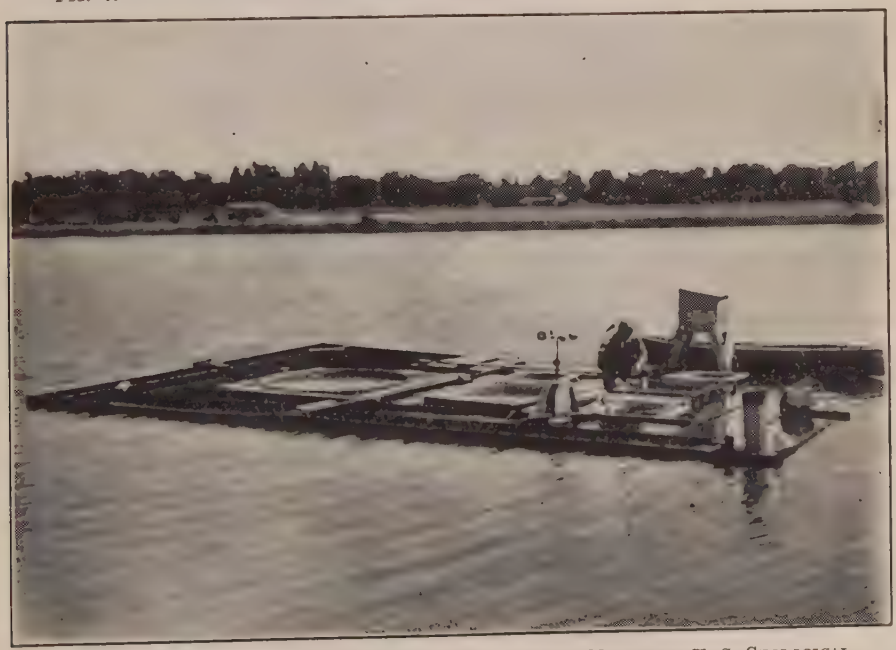

FIG. 8.—FLOATING PANS AT WASHINGTON PARK LAKE. MAN IS AT U. S. GEOLOGICAL TYPE NO. 9.

Tank descriptions, periods of observation, amounts evaporated, and the ratios between the amount evaporated from Tank No. 7 and that from other tanks are given in Table 15.

No attempt will be made here to explain the variations and ratios found between evaporation depths from tanks of different types. Some of them may be expressed quite readily by formulas, but it is believed that for practical use the curves of Fig. 9 will serve as well or better. It cannot be said that all the ratios are exactly comparable as it was not possible to operate each tank continuously, or all for equal lengths of time. However, the ratios between Ground Tank No. 7 and the others give factors for comparison that are believed to be the best bases for study.

It will be seen that, arranged according to type of container, there are five classifications or series of comparisons:

(*A*).—Cylindrical ground tanks, 3 ft. deep; set 2.75 ft. deep; water, 2.75 ft.; with diameters from 1 ft. to 12 ft. Tanks Nos. 1, 2, 4, 5, 6, and 7.

(*B*).—Ground tanks with square horizontal cross-section, 3 ft. deep; set 2.75 ft.; water depth, 2.75 ft., with water surface areas of 4 sq. ft. and 9 sq. ft. Tanks Nos. 3 and 15. For comparison between square and circular exposed water surfaces there are Tanks Nos. 3 and 4, each with an area of 9 sq. ft.

(*C*).—Cylindrical ground surface tanks of the U. S. Weather Bureau type for Class A Stations; 10 in. deep; setting on wood foundations above ground. The standard 4-ft. tank is compared with tanks 2 ft. in diameter and 6 ft. in diameter. Tanks Nos. 8, 69, and 70 give the relations.

(*D*).—Ground tanks cylindrical in form, having different depths. A series with diameters of 2 ft. and depths from 0.5 ft. to 6 ft. is furnished by Tanks Nos. 18, 19, 21, 22, and 2. Another series with diameters of 6 ft. and depths from 1 ft. to 3 ft., is given by Tanks Nos. 17, 5, and 16.

(*E*).—Floating cylindrical pans, 1 ft. deep, with diameters of 0.83 ft. to 6 ft., and a floating pan, 3 ft. by 3 ft. square and 1.5 ft. deep. Tanks Nos. 9, 92, 71, 72 and 73.

The results from each series are shown in the curves of Fig. 9.

Series (A).—Ground Tanks.—Curve *A* (Fig. 9) presents the data for Series (*A*) tanks. This set of results is perhaps one of the most significant and valuable obtained at the Laboratory. The curve, making only a small angle with the horizontal between the points for the 12-ft. and 9-ft. tanks, takes a turn upward and rises rapidly as the exposed water surface area becomes less than 9 sq. ft. The depth of evaporation from a ground tank 1 ft. in diameter was found to be nearly 60% greater than that from the 12-ft. tank.

It is likely that a majority of the tanks installed in the past for the purpose of obtaining indices of evaporation from large open water surfaces have been from 2 to 6 ft. in diameter, probably the greater number of these between 3 and 4 ft. The depth evaporated from the ground tank, 3.39 ft. in diameter and 9 sq. ft. in area, was more than 20% greater than that from the 12-ft. tank.

Curve *A* (Fig. 9), clearly indicates that the engineer should carefully investigate the method used to obtain whatever data on evaporation he may have occasion to use. Unfortunately, some of the tables presented in the past lack sufficient identification to justify their use.

TABLE 15.—SUMMARY OF EVAPORATION MEASUREMENTS, DENVER, COLO., IRRIGATION FIELD LABORATORY.

Tank No.	Description.	Period of observations.	Evaporation During Period.		
			For given tank, in inches.	For Tank No. 7, in inches.	Percentage of that from Tank No. 7.
		TANKS ON LAND AT LABORATORY			
7	12 ft. in diameter; 3 ft. deep; set in ground, 2.75 ft.; water depth, 2.75 ft.	Nov. 15, 1915–Sept. 30, 1917.	85.47	85.47	100
1	1 ft. in diameter, otherwise same as Tank No. 7.	Nov. 15, 1915–Sept. 30, 1917, except Dec. 4, 1916, to May 7, 1917.	126.59	79.68	158.9
2	2 ft. in diameter, otherwise same as Tank No. 7.	Nov. 15, 1915–Nov. 13, 1916.	63.12	49.16	128.4
4	3.39 ft. in diameter, otherwise same as Tank No. 7.	Nov. 15, 1915–Sept. 17, 1917, except Nov. 21, 1916, to Mar. 11, 1917.	96.74	80.51	120.2
5	6 ft. in diameter, otherwise same as Tank No. 7.	Nov. 15, 1915–Sept. 30, 1917.	92.87	85.47	108.9
6	9 ft. in diameter, otherwise same as Tank No. 7.	Nov. 15, 1915–Nov. 13, 1916.	49.63	49.16	100.9
15	2 by 2 ft. square by 3 ft. deep; set in ground, 2.75 ft.; water depth, 2.75 ft.	Apr. 17–Nov. 13, 1916, and from May 29–Sept. 24, 1917.	89.49	68.18	131.3
3	3 by 3 ft. square, otherwise same as Tank No. 15.	Nov. 15, 1915–Nov. 13, 1916, and May 8–Sept. 24, 1917.	98.63	78.33	125.9
8	4 ft. in diameter; 10 in. deep; setting on wood foundation; Weather Bureau Class A type.	Nov. 15–Dec. 13, 1915; Mar. 6–Nov. 13, 1916; and Apr. 11–Sept. 30, 1917.	116.09	81.69	142.1
69	2 ft. in diameter, otherwise same as Tank No. 8.	Apr. 11–Sept. 30, 1917.	54.91	32.53	168.8
70	6 ft. in diameter, otherwise same as Tank No. 8.	Apr. 11–July 30, 1917.	26.66	20.05	132.8
22	2 ft. in diameter; 0.5 ft. deep; set in ground, 0.25 ft.; and with water 0.25 ft. deep.	June 5–Oct. 9, 1916, and May 28–July 30, 1917.	55.62	43.47	127.9
21	2 ft. in diameter; 1.0 ft. deep; set in ground, 0.75 ft.; and with water, 0.75 ft. deep.	June 5–Oct. 9, 1916 and May 28–Sept. 24, 1917.	70.16	55.24	126.8
20	2 ft. in diameter; 1.5 ft. deep; set in ground, 1.25 ft.; and with water, 1.25 ft. deep.	June 5–Oct. 9, 1916, and May 28–Sept. 24, 1917.	70.69	55.24	127.9
19	2 ft. in diameter; 2 ft. deep; set in ground, 1.75 ft.; and with water, 1.75 ft. deep.	June 5–Oct. 9, 1916, and May 28, 1917.	55.30	43.47	127.9
18	2 ft. in diameter; 6 ft. deep; set in ground, 5.75 ft.; and with water, 5.75 ft. deep.	June 5–Oct. 9, 1916 and May 28–Sept. 24, 1917.	70.39	55.24	127.4
17	6 ft. in diameter; 1 ft. deep; set in ground, 0.75 ft.; and with water, 0.75 ft. deep.	June 5–Oct. 9, 1916.	30.48	28.49	106.9
16	6 ft. in diameter; 2 ft. deep; set in ground 1.75 ft.; and with water 1.75 ft. deep.	June 5–Oct. 9, 1916.	30.36	28.49	106.5

TABLE 15.—(*Continued*).

Tank No.	Description.	Period of observations.	Evaporation During Period.		
			For given tank, in inches.	For Tank No. 7, in inches.	Percentage of that from Tank No. 7.
Tanks Floating in Lake at Washington Park; Water Level in Tank Kept as Near as Possible to That in Lake					
9	3 by 3 ft. square; 1.5 ft. deep, with two supporting cylinders, U. S. Geological Survey type.....	Apr. 17–Nov. 6, 1916 and thirteen periods, ranging from 3 days to 1 week, from May 10–Sept. 17, 1917	72.04	64.06	112.5
92	0.83 ft. in diameter; 0.83 ft. deep.....	July 31–Sept. 17, 1917.....	13.42	10.27	130.7
71	2 ft. in diameter; 1 ft. deep.	May 10–19, May 23–28, June 1–4, June 7–12, July 31–Sept. 17, 1917.....	16.04	14.29	112.2
72	4 ft. in diameter; 1 ft. deep.	July 31–Sept. 17, 1917.....	10.63	10.27	103.5
73	6 ft. in diameter; 1 ft. deep.	May 10–19, June 1–4, June 7–12, Aug. 14–27, 1917.....	6.39	6.57	97.1

Series (B).—Ground Tanks.—Only two square tanks, Nos. 3 and 15, were used for these observations. On a percentage basis, with the cylindrical Tank No. 7 as 100%, results are shown as Curve *B* (Fig. 9). The general slope of the line for the two sizes—2.0 and 3.0 ft. square—is about the same as for the cylindrical ground tanks. With equal exposed areas the evaporation is slightly greater from the square surface. Its ratio of perimeter to area is greater and there is a greater wetted area of metal. It follows, therefore, that evaporation is taking place from a slightly greater surface when a tank pre-

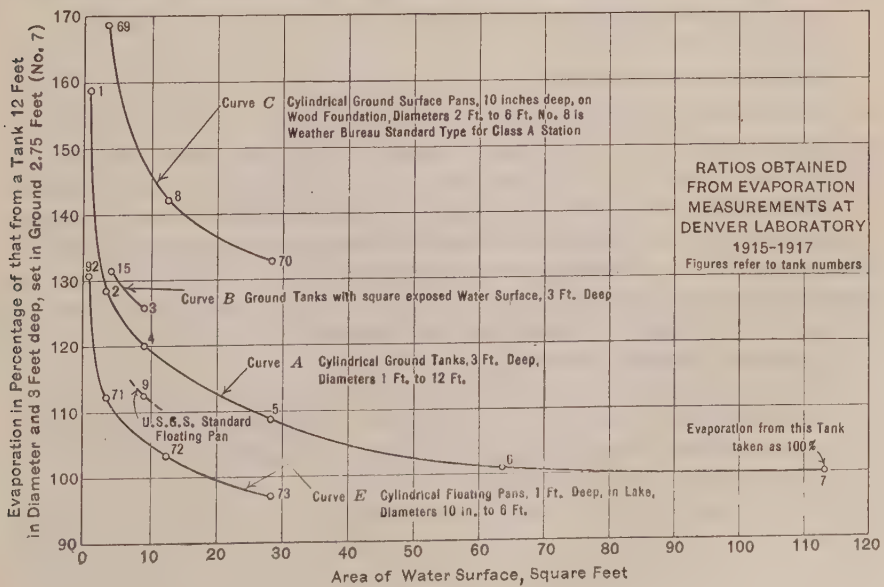
Evaporation in Percentage of that from a Tank 12 Feet in Diameter and 3 Feet deep, set in Ground 2.75 Feet (No. 7)
Area of Water Surface, Square Feet
Curve C Cylindrical Ground Surface Pans, 10 inches deep, on Wood Foundation, Diameters 2 Ft. to 6 Ft. No. 8 is Weather Bureau Standard Type for Class A Station
Curve B Ground Tanks with square exposed Water Surface, 3 Ft. Deep
Curve A Cylindrical Ground Tanks, 3 Ft. Deep, Diameters 1 Ft. to 12 Ft.
Curve E Cylindrical Floating Pans, 1 Ft. Deep, in Lake, Diameters 10 in. to 6 Ft.
U.S.G.S. Standard Floating Pan
RATIOS OBTAINED FROM EVAPORATION MEASUREMENTS AT DENVER LABORATORY 1915-1917
Figures refer to tank numbers
Evaporation from this Tank taken as 100%

FIG. 9.

senting a square water surface is used, than with a cylindrical one of equal cross-sectional area.

From Table 15 the relation between Tanks No. 3 (square surface, 3 ft. by 3 ft.) and No. 4 (circular, 3.39 ft. in diameter) based on Tank No. 7 is: 125.9 ÷ 120.2 = 1.047. There is no cylindrical tank of the surface area of Tank No. 15, but Curve *B* (Fig. 9), shows that the relation would be practically the same.

Thus, within the limits of experimental work done at the Denver Laboratory, evaporation from a small ground tank with square exposed area was slightly less than 5% greater than that from a ground tank with a circular exposed area of the same size, both tanks being 3.0 ft. deep and having settings and water depths of 2.75 ft.

Series (C).—Ground Surface Pans.—This series compares the U. S. Weather Bureau type for Class A stations (No. 8) with others of the same design, but with different diameters. Curve *C* (Fig. 9) shows graphically the evaporations. Taking Tank No. 8 as 100%, the ratios are as given in Table 16.

Curve *C* (Fig. 9) has about the same form as that for the ground tanks.

Series (D).—Ground Tanks.—Series (*D*) can be split into two sub-series, one, comprising Tanks Nos. 5, 16, and 17, with tanks, 6 ft. in diameter, and the other with 2-ft. tanks. From Table 15 it is seen that the ratios, based on Tank No. 7 as 100%, are as given in Table 17.

TABLE 16.—Comparison of Evaporation from Ground Surface Pans.

Tank No.	Diameter, in feet.	Evaporation, in percentage of Tank No. 8.
69	2.0	108.8
8	4.0	100.0
70	6.0	98.5

The results given in Table 17 do not indicate any material variation in evaporation amounts. One questionable point arises from the data for Tanks Nos. 5 and 2. In each case the tank was used for a longer period than the others of the series and shows a slightly greater relative amount of evaporation. It is believed that under conditions similar to those at the Denver Laboratory slight differences in evaporation would be recorded over long periods of observation if tanks of different depths are used. However, it is probably not advisable to install any less than 2.0 ft. deep, if the results are to be extended to lake or reservoir computations.

Series (E).—Floating Pans.—Storms interrupted the observations on several floating pans; the results possibly are not trustworthy and, therefore, have been omitted. There is some doubt about the results from Tank No. 73—a pan, 6.0 ft. in diameter. It was operated successfully for only a short time. One pan of the U. S. Geological Survey type was operated for two seasons; the others, for part of the second season only. Curve *E* (Fig. 9) presents the results graphically.

Although there was no exactly corresponding size, the evaporation from the floating square pan appears from the curve to be about 5% greater than that from a cylindrical one of the same exposed area. This is based on a size of 9 sq. ft. and is practically the same as the relation found between evaporation from ground tanks with square and circular water surfaces. It will be noted that Curve *E* (Fig. 9), follows the same general form as Curve *A*, with a possible tendency to flatten out more quickly as the area increases.

TABLE 17.—Comparison of Evaporation from Land Pans.

Tank No.	Diameter, in feet.	Depth of setting and water depth, in feet.	Evaporation, in percentage of that from Tank No. 7.
17	6.0	0.75	106.9
18	6.0	1.75	106.5
5	6.0	2.75	108.9
22	2.0	0.25	127.9
21	2.0	0.75	126.8
20	2.0	1 25	127.9
19	2.0	1.75	127.9
2	2.0	2.75	128.4
18	2.0	5.75	127.4

The ratios of evaporation from ground tanks at the laboratory divided by that from floating pans with the same exposed water area both in size and shape, are as follows:

Diameter of exposed water surface.	Area, in square feet.		Ratio.
2.0 ft.	4.0	128.4	÷ 112.2 = 1.144
3.39 ft.	9.0	120.2	÷ 106* = 1.134
4.0 in.	12.57	116.5*	÷ 103.5 = 1.125
6.0 ft.	28.27	108.9	÷ 97.1 = 1.121
3.0 by 3.0 ft. square	9.0	125.9	÷ 112.5 = 1.119

In the case of the cylindrical tanks, the ratio decreases as the area increases. This must necessarily be so; the condition of a large open water area is being approached.

As to convenience and reliability of observations, a large floating pan for practical use is not believed advisable. The 6.0-ft. size as used was not satisfactory. The use of the honeycomb arrangement described in the paper would perhaps have eliminated part of the trouble. The U. S. Geological Survey standard was found to be more satisfactory than any other tested.

Comparison of All Observations on a Common Basis.—For comparative purposes, Table 18, showing the results previously discussed, except the depth variation series, has been compiled.

Discussions of evaporation may be found in handbooks, in texts on hydraulics, and in technical papers. Some present formulas, others give tables of evaporation by districts or localities. Not all of them describe the method of evaporation measurement, or the size and kind of container used as a gauge. One glance at Table 18 shows the great necessity for a complete knowledge

* Interpolated.

of all conditions when attempting to make practical use of any figures on evaporation. There is a great chance of being led astray.

The few floating pans used show a variation in depth of water evaporated of nearly 35%, depending on the size. Similarly, the tanks on land show a 60% range. Then come the other differences that apparently depend on the shape and type of tank. It is true that the ratios to be used for the extension of any tank measurements to a large open water surface are still to some extent doubtful; on the other hand, if the bases for the tank figures are not fully understood with all their attendant conditions a guess may as well be made at the beginning.

TABLE 18.—Evaporation in Percentage of That from Ground Tank No. 7.

	Percentage Evaporation.				
	Circular Water Surface.			Square Water Surface.	
Exposed surface area, in square feet.	Floating pans, water about 0.75 ft. deep and at same level as lake.	Ground tanks, 3 ft. deep; 2.75 ft. water depth; 2.75 ft. setting.	Ground surface pans, Weather Bureau type, 10 in. deep.	Floating pan, U. S. Geological Survey type.	Ground tank, 3 ft. deep; 2.75 ft. water depth; 2.75 ft. setting.
113.09	.....	100	.....	.....	.....
63.62	.....	100.9	.....	.....	.....
28.27	97.1	108.9	132.8	.....	.....
12.56	103.5	117.5*	142.1	.....	.....
9.0	106.0*	120.2	145.0*	112.5	125.9
4.0	110.5*	126.0*	164.0*	.....	131.3
3.14	112.2	128.4	168.8	.....	.....
0.78	.....	158.9	.....	.....	.....
0.54	130.7	.....	.....	.....	.....

* Interpolated from curves, Fig. 9.

The writer fully agrees with Mr. Houk regarding the necessity of considering meteorological conditions. At the Denver Laboratory were kept records of air and water temperatures, relative humidity, wind movement and direction, and, of course, precipitation. In addition to the maximum and minimum and two other daily temperature readings by standard thermometers and two daily observations of relative humidity by wet and dry bulbs, continuous records were made by graphically recording thermometers and hygrometers. A graphic recorder was attached to one of the anemometers and a Hoff evaporimeter to one of the evaporation tanks. U. S. Weather Bureau standards were maintained throughout.

The results of the first year's meteorological observations, with other details, have been published.* Records on the second year's observations made at the Denver Laboratory are on file at the office of the U. S. Bureau of Public Roads, Berkeley, Calif. The summary of the first year's observations is given in Table 19.

In only one case can ratios obtained at the Denver Laboratory be directly compared with those presented by Mr. Houk, that of the U. S. Weather Bureau

* *Journal of Agricultural Research*, Vol. X, No. 5, July 30, 1917.

pan for Class A stations and the 3 ft. square ground tank. The depth of the 3 ft. square tank at Denver was 3 ft., while the one at Jackson Lake was 14 in. However, Denver observations indicated that the depths of ground tanks probably had little effect on evaporation except when the tanks were shallow.

TABLE 19.—Meteorological Records at Denver, Colo., Laboratory, 1915–1916.

Week ended:	Mean air temperature, in degrees Fahrenheit.	Percentage of mean relative humidity.	*Total wind movement, in miles.	Evaporation from Tank No. 7, in inches.
1915:				
November 22	40.6	..	1 112	0.69
" 29	38.8	..	1 015	0.70
December 6	38.8	..	633	0.25
" 13	39.5	..	518	0.43
1916:				
March 6	39.3	..	853	0.67
" 13	46.3	..	1 278	0.73
" 20	49.5	..	731	1.00
" 27	46.4	..	1 022	0.63
April 3	42.6	..	807	0.77
" 10	36.4	..	657	0.49
" 17	48.3	..	897	0.87
" 24	48.7	..	785	1.04
May 1	46.4	..	870	1.09
" 8	53.9	..	860	1.19
" 15	50.5	60	1 189	1.65
" 22	48.2	71	819	1.01
" 29	58.1	48	1 190	2.06
June 5	61.7	56	634	1.71
" 12	60.9	62	737	1.61
" 19	65.0	54	613	1.48
" 26	67.0	45	884	2.21
July 3	73.6	47	774	2.18
" 10	73.6	58	941	2.18
" 17	69.8	66	500	1.58
" 24	72.8	61	590	1.83
" 31	72.7	69	642	1.76
August 7	68.8	78	532	1.40
" 14	68.9	71	634	1.43
" 21	67.2	60	755	1.70
" 28	62.8	61	620	1.47
September 4	65.5	44	622	1.57
" 11	66.9	51	882	1.82
" 18	53.2	65	571	0.98
" 25	58.5	58	555	1.03
October 2	50.5	56	588	1.15
" 9	51.0	64	849	1.11
" 16	44.7	70	824	0.63
" 23	39.6	73	712	0.24
" 30	42.8	67	555	0.33
November 6	51.2	48	700	0.90
" 13	27.7	70	820	0.38

* Anemometer 2 ft. above ground.

Over the entire two-year period of operation at Denver, the depth evaporated from the 3 ft. square ground tank was 88% of that from the U. S. Weather Bureau Class A ground-surface pan, while the Jackson Lake ratio is 72 per cent. For the summer periods the Denver figures are as given in Table 20.

There is a slight difference in pan and tank settings. The circular pan at Jackson Lake was set on the ground, the one at Denver on a wooden foundation as specified by the U. S. Weather Bureau; the square tank at Jackson Lake was set in the ground (depth of setting not given) with earth banked around the sides to the water level, while the one at Denver was set in the ground to the water level. It seems, however, that these circumstances would tend to make a greater difference between the evaporation depths recorded at Denver than between those at Jackson Lake.

The results of the comparison emphasize the necessity for complete descriptions of all methods and equipment in connection with records of evaporation measurements. They further indicate the value of meteorological data. The mean air temperature at Jackson Lake during the three months of 1924 in which the comparative records were made was 53°; during the 5-month summer period at Denver, it was 64° in 1916 and 63° in 1917. The relative humidity was 59% in 1916 and 65% in 1917, as determined by the hygrometer. The total wind movement for the period in 1916 was 14 600 miles as compared with 14 000 miles in 1917.

TABLE 20.—Summer Evaporation, Denver, Colo., Laboratory.

Period.	Evaporation from 3 ft. square ground tank, in inches.	Evaporation from Class A type pan, in inches.	Ratio, in percentage.
May 8–September 24, 1916	42.08	49.77	85
May 9–September 25, 1917	36.56	41.59	88

This one comparison of the Denver and Jackson Lake results raises the question whether the U. S. Weather Bureau pan for Class A stations is the best index for attempting an estimate of evaporation from large open water surfaces. It is easily installed, easily maintained, and the extent of its use is probably greater than that of any other. The ground tank more nearly approximates the reservoir. The temperature changes of the water in the ground surface pan follow air temperature changes more closely than those of water in the reservoir or the ground tank. The temperature records of the water in the Class A pan at Denver show a maximum day temperature greater than, a minimum night temperature lower than, and an average temperature lower than, was recorded for the water of any ground tank. The high day temperature at the time of low relative humidity probably caused the rapid evaporation. It is to be expected that in districts where cool nights and rather high midday air temperatures prevail in summer the ratio between depths of evaporation from a ground tank or a reservoir and from the Class A pan will not be the same as in districts where daily variations in air temperatures are not so great. This may be the explanation of the difference in the ratios at Denver and Jackson Lake.

At the end of one season of its use the writer had formed the opinion that he could not recommend the U. S. Weather Bureau Class A pan (No. 8) in

preference to a ground tank as an evaporation index. He believes that a ground tank with a square or circular cross-section not less than 3 ft. across and at least 2 ft. deep, with the water level about even with the ground level, should be used.

There are, however, many records from Class A stations. It is hoped that the discussion of Mr. Houk's paper will bring out reliable data indicating how these records can be used to the best advantage.

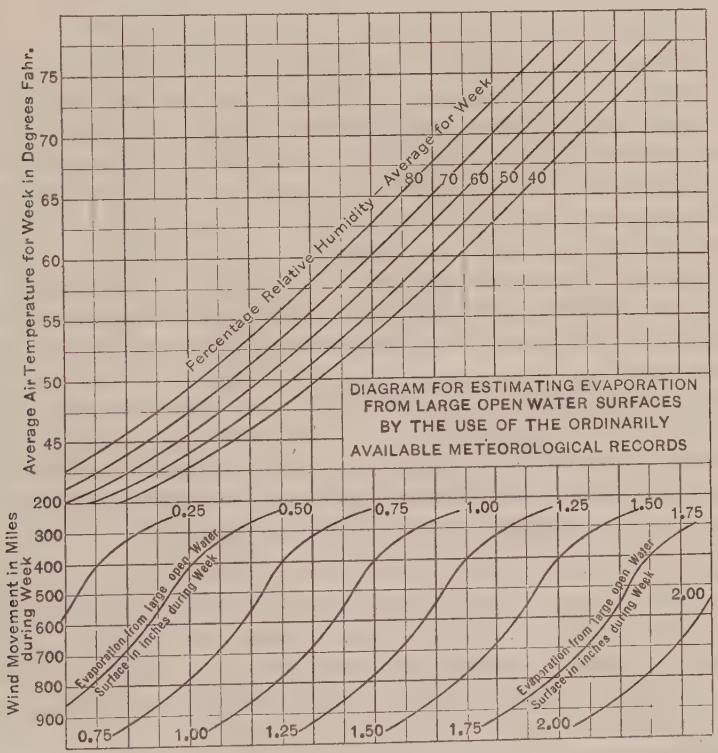

FIG. 10.

For a number of years the writer's Fig. 10 has been locked away awaiting the accumulation of data by which its value may be proven. It is presented as a means by which evaporation from large open water surfaces may be estimated from U. S. Weather Bureau meteorological data. It is based largely on the Denver research work, and meteorological conditions of the Denver region and involves the writer's conclusions regarding the extension of evaporation tank observations to large open water surfaces. It comprehends the main variables or influencing factors except barometric pressure. At an elevation lower than that of Denver, which is approximately 1 mile, under the same conditions as to wind movement, temperature, and relative humidity, the evaporation is less than, and at higher elevations it is greater than, at Denver.

As near as the writer can determine from the meager data available, three points on this curve are about as follows:

Elevation, in feet.	Evaporation in percentage of that at Denver under the same conditions of temperature, relative humidity, and wind movement.
127 (Yuma, Ariz.)	70
5345 (Denver, Colo.)	100
8180 (Nederland, Colo.)	120

The elements for Fig. 10 are to be found as follows: (1) Air temperature —from maximum and minimum, preferably two observations daily; (2) relative humidity—preferably from hygrograph records, approximated by the usual two observations per day; and (3) wind movement—from an anemometer 2 ft. above the water surface. For higher elevations, H ft. above the water or ground surface (up to 50 ft.), divide the wind movement by $(0.033\,H + 0.93)$ to get the movement at the 2-ft. elevation.

In using Fig. 10, enter the diagram with the air temperature; at the intersection of the horizontal temperature line with the relative humidity curve, drop vertically to the horizontal wind-movement line and read the evaporation from the curve.

Fig. 10 and the data as to the evaporations at various lengths should be checked further before applying them to the solution of a specific problem. The diagram is presented as a beginning of an analysis which the writer desires to develop further, and he earnestly hopes that the paper of Mr. Houk will bring out data of a character suitable for this development.

Lynn Perry,* M. Am. Soc. C. E. (by letter).—This paper, which presents much data in condensed form, should be of considerable value to the profession for this information is not readily obtained except from the files of the U. S. Reclamation Bureau. Although frequently discussed, reliable and accurate data on the subject of evaporation, rainfall, wind velocity, temperature, and humidity, are difficult to obtain for any given locality.

In this connection, the writer presents some data that were accumulated in connection with the studies leading to the design of the water-works at Christiansted, Virgin Islands. Rainfall has been observed at Christiansted, Kingshill, and Fredericksted, since 1852 and the records are in the files at the General Office. The temperatures on file at the office of the U. S. Weather Bureau cover a period of 17 years. The evaporation has been observed at Christiansted from 1919 to 1923. For this purpose, a land pan was constructed and placed in operation in April, 1919; with the exception of June and July, 1919, the records are continuous. No wind velocity or humidity records for Christiansted were available. The nearest available wind-velocity records are for St. John, Antigua, and Basseterre, St. Kitts. At St. John, the average wind velocity is 8 miles per hour (11-year average) and at Basseterre, it is

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9.5 miles. At San Juan, Porto Rico, observations over a long period average 10.6 miles per hour, which is probably higher than at Christiansted.

TABLE 21.

Station : Christiansted.
State : Virgin Islands.
Project : Municipal Water Supply.
Exposure : Ground.
Elevation : About 20.
Records : Precipitation since 1852.
Evaporation since 1919.
Wind velocity (see text) since 1909.
Maintained by : U. S. Navy Department.

MEAN RECORDS.

Month.	Evaporation, in inches.	Temperature, in degrees Fahrenheit.	Wind velocity (see text), in miles per hour.	Rainfall, in inches.
January	4.76	76.6	9.0	2.32
February	5.19	76.3	9.1	1.87
March	6.88	76.9	9.5	1.69
April	7.13	78.2	9.1	2.71
May	8.25	80.2	8.9	4.35
June	7.65	81.1	10.1	3.94
July	8.00	81.4	10.6	3.56
August	7.58	81.8	9.8	4.35
September	6.70	81.8	8.0	5.75
October	6.02	81.0	6.0	6.63
November	5.02	79.7	7.4	5.30
December	4.50	77.8	7.2	3.50
Totals and means	77.61	79.3	8.6	46.00

For the purpose of comparison the writer has used these data to construct Tables 21 and 22, and Fig. 11 to correspond with Tables 1 and 2 and Fig. 4 of Mr. Houk's paper.

TABLE 22.—RANGE IN ANNUAL EVAPORATION AT CHRISTIANSTED, VIRGIN ISLANDS.

Full year's record	5
Mean annual evaporation, in inches	77.61
Maximum Annual Evaporation :	
Year	1923
In inches	83.63
Percentage of mean	107.7
Total precipitation, in inches	32.04
Minimum Annual Evaporation :	
Year	1921
In inches	73.57
Percentage of mean	94.8
Total precipitation, in inches	25.34

A relatively few years ago no method was available for estimating run-off. As data became available, various observers suggested methods of calculating run-off based on their own observations, taking into account the information published by others. With carefully made measurements, such as those given

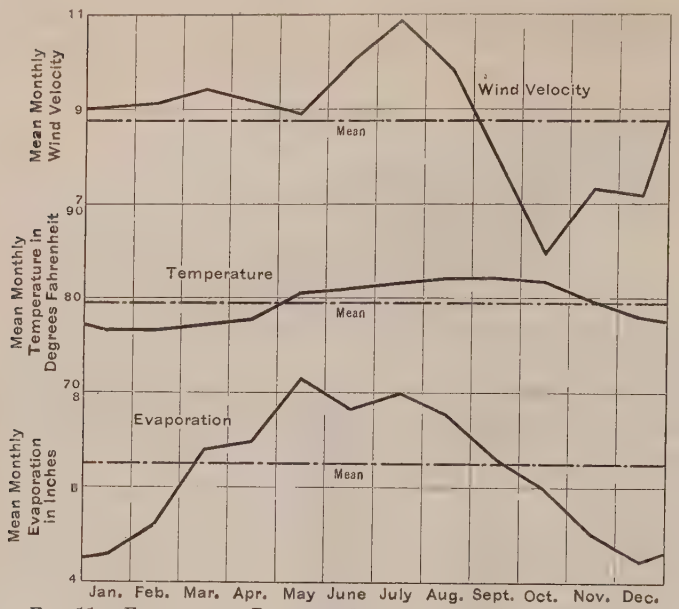

FIG. 11.—EVAPORATION DATA FOR CHRISTIANSTED, VIRGIN ISLANDS.

by the author, being made public at frequent intervals, it is possible that some method of estimating more accurately the probable evaporation at a given place will be suggested.

RALPH L. PARSHALL,* ASSOC. M. AM. SOC. C. E. (by letter).—The phenomenon of evaporation has engaged the attention of scientific investigators for a long time. Because of the relatively small quantity of water, as measured by depth, evaporated from an open water surface within a reasonably short time, the problem of accurately measuring the loss has involved some difficulty. Where sufficient time has been given between observations to allow for a measurable loss caused by evaporation the influencing factors have changed to such an extent that correlation becomes hard to effect and may not be possible.

The writer has given more or less attention to a study of evaporation from free water surfaces. In endeavoring to promote a greater degree of accuracy in measuring the depth loss and at the same time decreasing the interval between observations, he has devised special apparatus by use of which hourly readings disclose this loss and accumulate the necessary contributing meteorological data. The most accurate method of determining the loss is by weighing, but when the total weight of the mass of water involved is considerable the errors in weighing become so appreciable as to render uncertain the relation of this loss to other factors.

An optical evaporimeter (Fig. 12) was perfected to determine accurately the decrease in depth of the water surface, and at the same time to show a

* Irrig. Engr., U. S. Dept. of Agriculture, Fort Collins, Ohio.

measurable loss within a relatively short period. This apparatus includes a cylindrical brass float, 6 in. long and 2 in. in diameter. Reaching from the top of the float are two short horizontal, parallel arms that extend to a common bar. On the under side of this bar are two delicate pivot bearings supported by a rigid concrete pedestal placed close to the evaporation pan. Attached to the top of the pivot bar, at its middle point, is a small high-grade plane speculum mirror. At a distance of about 20 ft. from the mirror a high-powered telescope, with cross-hairs, is mounted on a concrete pedestal at a convenient height and focused on the image of a scale in the mirror. This scale is placed approximately 50 ft. from and about 20 ft. higher than the mirror. It is 6 ft. long and graduated accurately to 0.1 in. By adjusting the mirror and telescope in proper position, a clear and distinct image of the scale divisions may be observed. By careful calibration of this instrument, the constant or value of a scale division in difference in elevation of water surface was determined. In all observations the scale was read by estimation to one-tenth of the least scale division, or 0.01 in.

Calibration on this basis shows variations of from 3 to 5% from the mean constant. Different lengths of arms (float to pivot bar) were used, giving constants for the evaporimeter ranging from 0.000016 to 0.000060, that is, 0.01 in. on the scale representing, say, 0.000016-in., or 0.000060-in. difference in elevation of the water surface in the evaporation pan, depending on the length of arm used.

To determine the existing vapor pressure, an aspiration psychrometer (Fig. 12) was mounted so as to take air at a point 1 in. above the water surface in the evaporation pan. This instrument consisted of a thin-walled glass tube about 1⅛ in. in diameter and 24 in. long. Two thermometers with 14-in. stems, graduated to 0.1° Fahr. were fixed inside this tube. A single-ply linen sheath covered the bulb of one of these thermometers and was situated so that the bulb was immediately beneath a small aperture in the glass tube. A small pipette was used to moisten the wet bulb through this hole in the glass tube. When the psychrometer was in operation this aperture was stopped with a small cork. An aspirator connected to this instrument drew the air through the glass tube past the thermometers with a pressure of about 1 in. of water. The wet and dry bulb readings, taken through the wall of the glass tube, were recorded when the minimum temperature of the wet bulb occurred.

The air and water temperatures were taken using thermometers identical with those mounted in the psychrometer. The temperature of the air was observed at a point 1 in. above the water surface, and the temperature of the water surface at a point directly below the bulb of the air thermometer.

In taking the evaporation records a definite order was followed: First, the air and water temperatures were taken; second, the psychrometer readings; third, the barometric pressure; and, fourth, the optical evaporimeter reading, which included the time to 0.1 min. An hour later the optical evaporimeter was again read and the time noted, and the other readings were repeated in the foregoing order. The mean values of water and vapor pressures, temperatures, etc., were determined and assumed to be effective for the period between

the initial and final reading of the evaporimeter. The evaporimeter reading, as well as the time, was taken subsequent to the temperature and psychrometer readings as the initial observations for the beginning of the next test. This double reading of the evaporimeter scale, with the time interval necessary to observe the other data, gave a reliable check on scale readings as well as evidence as to the action of the float.

Two complete sets of apparatus were maintained, one within the laboratory (Fig. 12) and the other outside under fully exposed conditions (Fig. 13). Within the laboratory the observations were taken for still air conditions, and by means of a special air duct (Figs. 14 and 15), the effect of wind on the rate of evaporation was observed. The study under wind conditions covered velocities ranging from about 1.5 to 12 miles per hour.

Fig. 16 shows a series of observations taken in the laboratory under still air conditions. One of the striking facts observed in studying the evaporation loss, under still air, is the maximum rate occurring during the early hours of the morning. It is to be noted that the rate of evaporation bears a singular relation to the temperatures of the air and water. During the morning and early afternoon of September 15 the temperatures are rising while the rate of evaporation shows a marked reduction. This is apparently true for the following day. As these temperatures rise their difference increases, and as they fall the difference decreases. The relation, difference in vapor pressure, or vapor pressure of the water surface minus the vapor pressure of the dew point, according to Dalton, shows a direct variation with the rate of evaporation. The reciprocal of the relative humidity, $R\,H$, also varies directly with the evaporation, but its variation is through a more limited range than the difference in vapor pressure. Near midnight on September 15, a decided increase appears in the observed rate of evaporation, followed by a decrease. It would seem logical to suspect that an error in reading occurred at this point, as a mean value would agree quite nicely with the general trend of the loss during this interval. To account for this apparent error in the evaporimeter reading, the original notes for the point observed at 11:24 P. M., September 15, show a scale difference of 0.24 in. for a period of 61 min. To move this point down to the mean curve, the difference should have been about 0.20 in. This error of 0.04 in. in the scale reading amounts to 0.0001 in. in the elevation of the water surface for an interval of 61 min., or 0.0024 in. for a period of 24 hours.

The formula used to compute the rate of evaporation per 24 hours is based on a great number of observations, and inasmuch as the study of this subject has not been fully developed, certain minor alterations or combinations may later permit closer agreement with the observed rate of loss. The computed loss shown in Fig. 16 is based on the formula:

$$E = 0.08\,[3 - (T_a - T_s)]^{2/3}\,(E_s - E_d)$$

in which,

E = evaporation, in inches per 24 hours.

$(T_a - T_s)$ = temperature of air minus temperature of water, in degrees Fahrenheit.

$(E_s - E_d)$ = vapor pressure at water surface minus vapor pressure of dew point, in inches of mercury.

FIG. 12.—EVAPORATION EQUIPMENT FOR STILL AIR CONDITIONS, SHOWING ARRANGEMENT OF PSYCHROMETER, OPTICAL EVAPORIMETER, AND AIR AND WATER THERMOMETERS.

FIG. 13.—ARRANGEMENT OF APPARATUS FOR OBSERVATIONS UNDER FULLY EXPOSED CONDITIONS. ASPIRATOR FOR PSYCHROMETER IS SHOWN IN FOREGROUND.

FIG. 14.—EVAPORATION EQUIPMENT FOR CONTROLLED WIND. PART OF AIR DUCT IS SHOWN RAISED TO PERMIT MAKING OF TEMPERATURE OBSERVATIONS.

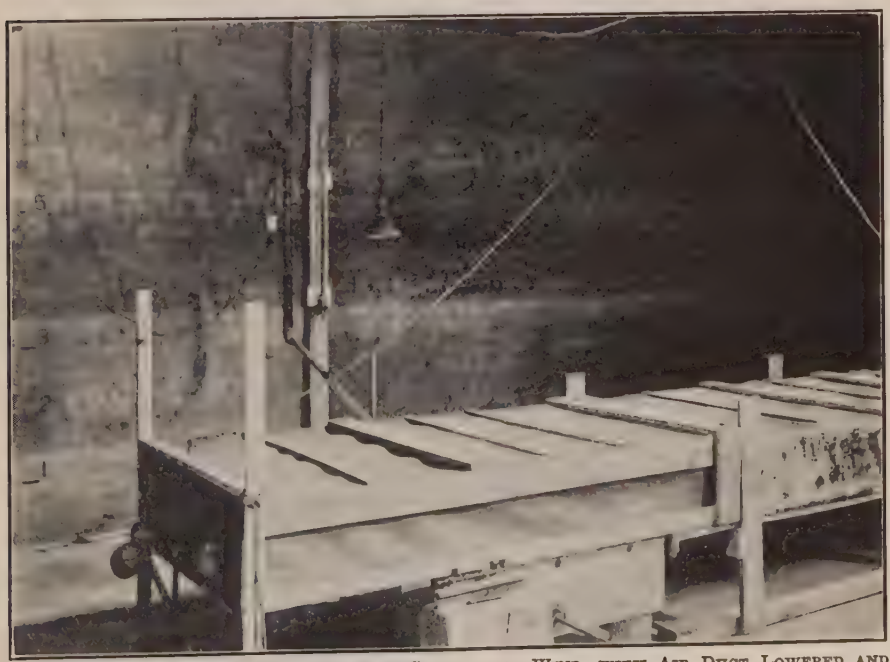

FIG. 15.—EVAPORATION EQUIPMENT FOR CONTROLLED WIND, WITH AIR DUCT LOWERED AND ANEMOMETER IN PLACE.

In Fig. 17 is shown a series of observations taken outside the laboratory under fully exposed conditions. The arrangement of the apparatus is shown in Fig. 15. Under these conditions the rate of evaporation appears to be at variance with the general trend as observed within the laboratory. For the period 6:00 P. M., October 12, to 6:00 A. M., October 13, the wind, as recorded by a standard Robinson anemometer set near the evaporation pan, indicated

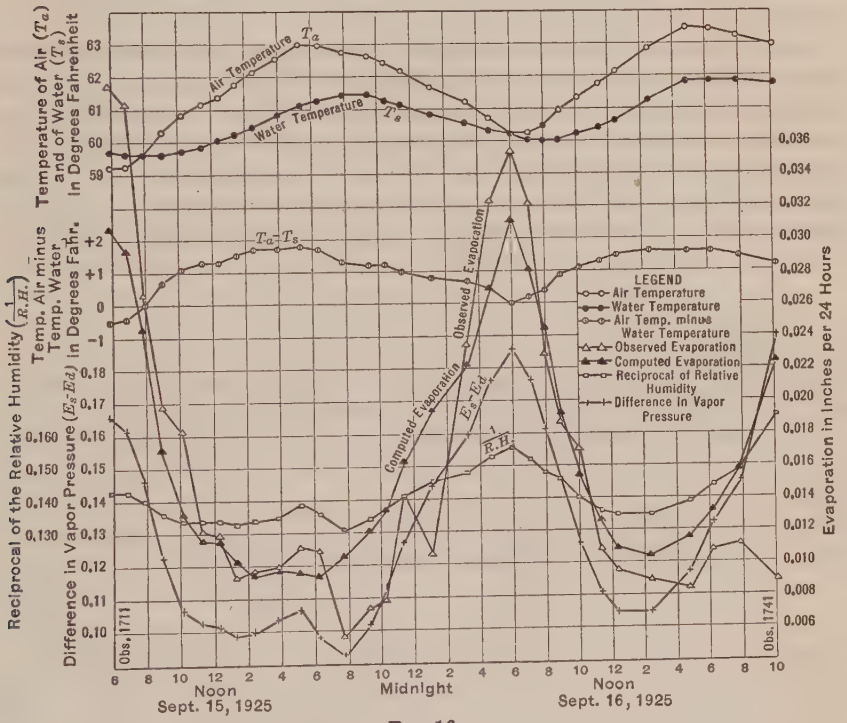

FIG. 16.

a low velocity. During this period the temperatures of air and water were decreasing, as was also the rate of evaporation, which is contrary to the findings inside the laboratory under still air conditions; however, in both cases the rate of loss is directly proportional to the difference in vapor pressures. During the day of October 13, there was considerable wind; also for this period the rate of evaporation was greatly increased. About 10:00 A. M. the rate of wind decreased but the rate of evaporation still increased, following the constantly rising difference in vapor pressure. As would be expected, the maximum rate of evaporation occurred at the time of maximum temperatures. The formula applied to compute the evaporation loss in this case differs somewhat from that developed to apply under still air conditions in that the factor for wind is introduced. This formula is:

$$E = (0.44 + 0.118\ W)\ (E_s - E_d)$$

in which,

E = evaporation, in inches per 24 hours.
W = wind velocity, in miles per hour.
$(E_s - E_d)$ = vapor pressure at water surface minus vapor pressure of dew point, in inches of mercury.

In applying this formula, with the wind factor considered, to an extended series of evaporation records of mean monthly values, promising agreement is shown.

In the matter of comparing the evaporation losses of the two stations at Yuma, Ariz., as mentioned by the author, it is believed that this difference may be explained in part by the fact that one station is situated on a desert mesa while the other is located in an irrigated alfalfa field. Because of these different exposures, it is thought that the vapor pressures are entirely different, and, further, that the difference in temperatures of water and air at the two stations may be considerable. The fact that the wind for the Citrus Station

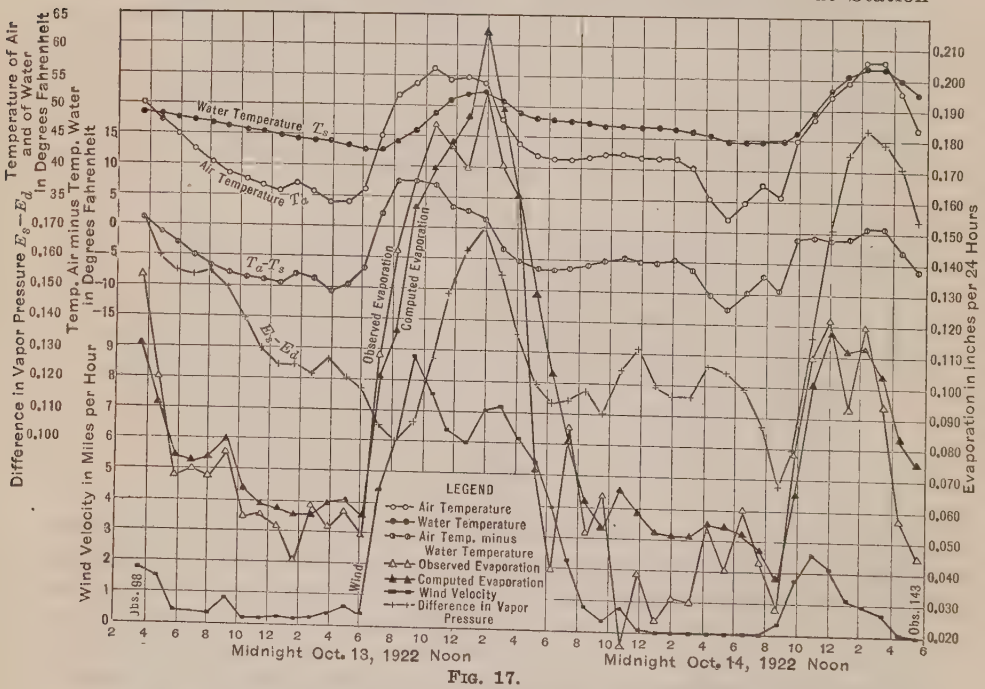

FIG. 17.

is greater than at the Evaporation Station would also cause a greater loss at the former place. A lower temperature at the pan located in the alfalfa field would be expected because the transpiration effect of the green vegetation modifies the air temperatures.

S. T. HARDING,* M. AM. SOC. C. E. (by letter).—Much uncertainty exists regarding the coefficients to be applied to the observations of evaporation from

* Prof. of Irrig., Univ. of California, Berkeley, Calif.

pans or tanks in order to deduce the loss from large water surfaces. The variations in different months of the coefficient for the same pan as shown in the paper indicate that other factors besides the size and condition of exposure of the pan are involved. There is need for careful investigations of these matters. To meet such need it is essential that any investigations be carefully planned and carried out and that they extend over a sufficiently long period to furnish a more complete checking of results than is possible with available data. Many of the present data have been secured for the particular needs of individual projects and the experiments are planned for that purpose. More extended study of the principles involved, based on observations of the effect of variations in the different individual factors, is needed.

Opportunities to observe evaporation from large water surfaces directly are seldom found. Unknown elements, such as seepage into or out of a reservoir, may be a material proportion of the evaporation loss and sometimes exceed such losses. Surface inflow may occur at scattered points and be difficult of measurement. It is not usual to have periods without inflow or outflow of sufficient length to enable the evaporation alone to be observed.

There are certain types of lakes in California in which opportunity is afforded to observe evaporation directly. These are the basins receiving the surplus flow of streams in which (1) the inflow is limited to a part of the year; (2) the outflow, if any, may be measured; and (3) soil and ground-water conditions are such that seepage inflow and outflow into the lake do not occur. Among such lakes are Tulare Lake, Buena Vista Lake, and Lake Elsinore.

Tulare Lake.—Gauge height records on Tulare Lake were begun in 1906 by the U. S. Geological Survey. Pumping from the lake began in 1916. During the intervening period the fluctuations of the lake at times of no surface inflow show the evaporation loss.

Tulare Lake is a shallow depression having an area at maximum stages of about 200 000 acres. It receives the surplus flow of the Kern, Tule, and Kaweah Rivers and part of the surplus flow of Kings River. The lake bed consists of fine, impervious material. In periods of small surface inflow the lake becomes entirely dry, with no indications of ground-water seepage into it. Until recently wells in the lake bed flowed, showing the ground-water gradient to be above the bed. There is no indication of absorption by the clay subsoil of the lake.

The records of fluctuation at periods when the flow in the contributing streams was too small to result in inflow to the lake, from 1906 to 1916, have been assembled, together with the mean monthly evaporation, in Table 23.*

The nearest station for which climatic records are available is Hanford, about 20 miles northeast of the lake. The rainfall at Hanford is probably somewhat higher than the average rainfall on the lake area. The temperatures may be generally similar.

Buena Vista Lake.—Buena Vista Lake is a broad shallow basin which is now used as a reservoir, a levee having been built along its eastern side.

* See, also, *Bulletin 9*, California State Dept. of Eng., entitled "Water Resources of Kern River and Adjacent Streams and Their Utilization."

Part of the flow of Kern River is stored in the lake for irrigation use. Some observations in 1920 of the fluctuation of the lake at times of no inflow and of only limited outflow have been secured (Table 24).*

TABLE 23.—Evaporation Data for Tulare Lake.

Month.	Mean evaporation, in inches.	Mean precipitation at Hanford, Calif., in inches.	Mean temperature at Hanford, Calif., in degrees Fahrenheit.
January	1.4	1.98	45.7
February	1.6	1.51	50.2
March	3.0	1.70	54.0
April	3.6	0.57	59.4
May	6.0	0.48	66.1
June	8.4	0.09	74.6
July	9.6	0.00	79.8
August	7.2	0.01	78.0
September	7.2	0.32	72.0
October	3.6	0.26	61.9
November	2.4	0.65	52.2
December	1.2	1.07	45.5
Year	55.2	8.64	61.6

The lake was only a few feet in depth during these months. Ground-water conditions are similar to those in Tulare Lake.

Lake Elsinore.—Lake Elsinore receives the surplus run-off of the San Jacinto River. In periods of excessive run-off the lake may overflow; its outlet channel is tributary to the Santa Ana River. The area westerly from the lake rises steeply, the valley being narrow on that side. The remainder of the surrounding area is flat with some low hills. The lake has gone dry in periods of deficient rainfall.

TABLE 24.—Evaporation from Buena Vista Lake.*

Month.	Total lowering of lake during month, in inches.	Lowering of lake due to outflow, in inches.	Indicated lowering due to evaporation, in inches.
July	7.3	...	7.3
August	8.4	1.4	7.0
September	8.8	1.7	7.1
October	5.4	0.8	4.6
November	2.2	0.2	2.0

* No rainfall during period.

The lake bed appears to be impervious. It is stated that no inflow from adjacent ground-water was apparent when the lake was dry. There is no indication of seepage from the lake, adjacent wells with water below lake level showing no indication of mixture with the salty water of the lake. The lake has been at relatively high stages since 1916. It has an area of about 6 000 acres and a depth of about 40 ft. at the present overflow level. There is no use for the lake water due to its high salt content.

* *Bulletin 9*, California State Dept. of Eng.

Since December, 1915, records of elevation have been secured by the U. S. Geological Survey. These records to September, 1922, were used by the writer in a report on the San Jacinto River Hydrographic Investigation made for the Division of Water Rights, California State Department of Public Works, as the basis of an estimate of the evaporation loss. A direct record of inflow and outflow was also available. These occurred only during a part of the period covered. Data on rainfall at Elsinore were also at hand.

The inflow, outflow, rainfall, and fluctuations were used, with the known area and capacity curves of the lake, to estimate the evaporation by months (Table 25). These are considered to include all elements of supply and draft. The records for different months were generally consistent; in some cases variations in the reading of the lake elevation, due probably to rough water at the time of reading, gave inconsistent results between adjacent months.

Observations with Pans.—Observations of evaporation were made under the writer's direction in 1920 on Buena Vista Lake, a natural depression now used as a reservoir.*

TABLE 25.—Estimated Mean Evaporation from Lake Elsinore.

Month.	Mean evaporation, in inches.	Mean precipitation, in inches.	Mean temperature, in degrees Fahrenheit.
January	1.8	3.43	50.0
February	0.6	2.36	52.4
March	3.0	2.72	55.3
April	4.8	0.51	60.2
May	5.4	0.39	64.7
June	6.0	0.02	72.1
July	7.2	0.10	78.1
August	7.2	0.15	77.2
September	6.6	0.20	73.2
October	5.4	0.49	65.2
November	3.6	0.95	56.9
December	2.4	2.07	51.5
Year	54.0	13.39	63.0

In 1920, Buena Vista Lake received little inflow and remained at depths of only a few feet. Its area was about 10 000 acres.

Both a floating and land pan were installed. These were of the type used by the U. S. Geological Survey, 3 ft. square and 2 ft. deep. The land pan was placed about 300 ft. from the edge of the lake, and the floating pan from 100 to 600 ft. from the shore, being moved with the fluctuations of the lake. Owing to the flat slopes of the lake bed the floating pan was at times set in an excavated area in order that it should be submerged to less than 6 in. from the top. The shallowness of the surrounding water may have introduced temperature factors not applicable in deeper areas.

The usual form of raft was placed around the floating pan to prevent wave action. Some washing into the pan occurred in June and July; corrections were made by comparison with the land pan. An unexpected source of error occurred in February and March due to the use of the pan for bathing purposes

* For detail results, see *Bulletin 9*, California State Dept. of Eng.

by pelicans. A repetition of this was prevented by placing chicken-wire screens above the pan. The results of these observations are given in Table 26.

These results indicate that for the particular conditions the evaporation from the floating pan exceeded that from the land pan and that the ratio varied during the season. These observations probably present an illustration of the extent to which local factors may affect the results rather than a basis for what would be expected under more normal conditions.

TABLE 26.—Evaporation from Pans, Buena Vista Lake.

Month.	Land pan, in inches.	Floating pan, in inches.	Ratio of evaporation, floating pan to land pan.
January	1.82	1.82	1.00
February	2.45	2.50*	1.30*
March	2.83	2.89*	1.88*
April	5.99	6.78	1.13
May	9.05	10.58	1.17
June	10.00	11.72	1.17
July	10.14	12.61	1.24
August	9.62	11.27	1.17
September	7.21	8.46	1.16
October	4.88	5.42	1.11
November	2.29	2.44	1.06
December	1.56	1.50	0.96
Total	67.84	77.99	1.15

* Errors in floating-pan record; figures given are estimated.

The direct observations of the lowering of the lake from July to November give the coefficients for the floating and land pans shown in Table 27.

TABLE 27.—Coefficients to Be Applied to Pans to Give Correct Evaporation.

Month.	Floating pan.	Land pan.
July	0.58	0.73
August	0.62	0.73
September	0.84	0.98
October	0.84	0.93
November	0.80	0.85

These coefficients may be affected by local conditions. The variations in the coefficients for different months, however, illustrate the uncertainties involved in applying pan records to the evaporation from large water surfaces.

Charles H. Lee,* M. Am. Soc. C. E. (by letter).—The author has performed a valuable service to the profession in collecting and compiling the numerous records of evaporation presented in his paper. These records cover a wide geographic range, and represent varied conditions of exposure and meteorology. The data have been assembled in convenient form for reference. Such efforts will advance the present deficient knowledge of this subject.

* Cons. Hydr. Engr., San Francisco, Calif.

The paper does not call for detailed comment. Before entering on a general discussion, however, the writer desires to point out that apparently there is no statement regarding precipitation, and particularly as to whether in all cases the reported depths of evaporation are gross or net. For comparative purposes it is desirable that the evaporation from rainfall caught by the pan be included in the monthly totals as well as that from water added by the observer.

A precipitation record is essential in connection with evaporation observations if the record is to have any but local use. The next most important auxiliary record is the temperature of the water surface. This can easily be obtained by means of a maximum and minimum recording thermometer floating on a small wooden raft, with the bulb just submerged. Although not definitely stated in the paper, it is assumed that the temperatures reported are for the air and not for the water surface. It is believed that if a comparative record of water surface temperatures at the two Yuma stations were available, it would be found that the record at the Yuma Evaporation Station would be much less than that at the Yuma Citrus Station, and would thus show a third important factor not referred to by the author. Temperatures at low levels in an alfalfa field are usually considerably less than on the adjacent bare ground.

A complete meteorological record should also include air temperature, wind movement, and relative humidity, as stated by the author.

Controlling Factors.—The process of evaporation consists essentially of the gradual "bombardment" into the adjacent atmosphere of the rapidly moving molecules which compose a liquid. The effect of various factors on this process may be illustrated as follows: If the temperature of the liquid at and near its surface is increased, the rate of molecular vibration is correspondingly increased, and more molecules break from the liquid surface in a given time; if the degree of saturation of the atmosphere immediately adjacent to the surface is decreased, either by a lowered relative humidity of the surrounding atmosphere, or by an increased rate of wind movement, the vapor pressure on the surface is diminished and it becomes easier for molecules to break from the surface, and hence more of them leave in a given time (variations in atmospheric pressure have a similar result); finally, if the liquid is heavily charged with mineral substances in solution the motion of the liquid molecules is retarded and fewer escape in a given time. If the specific gravity of the liquid advances to such a point that crystallization occurs and a film or crust forms over the surface, a marked decrease in the rate of loss occurs.

In brief, the important factors controlling evaporation are the relative temperatures of the liquid surface and of the adjacent atmosphere, the vapor and barometric pressure immediately above the liquid surface, and the specific gravity of the liquid. For ordinary conditions, temperature and vapor pressure are the most important factors, although in special cases the specific gravity of the water must be considered. Evaporation from capillary films, such as form on snow and ice, or from moist soil or crystal surfaces, involve changed conditions with respect to the form and continued existence of the

liquid body and hence introduce modifications in the rate. Otherwise, this process corresponds with that of evaporation from a free water surface.

Evaporation from Water Surface.—The author presents data in Tables 2 and 3 giving the range in annual and monthly evaporation at selected stations. For those who are interested, similar variations for 3-day periods during nearly 2½ years near Independence, Calif., have been published.* The data are assembled in diagrammatic form (Plate X), the maximum and minimum daily air temperature, relative humidity at 5:00 A. M. and 5:00 P. M., and average daily wind movement being shown in addition to evaporation. The diagram was prepared by the writer from submerged floating-pan evaporation observations made under his direction on Owens River, California, and meteorological observations by the U. S. Weather Bureau, 4 miles distant at Independence, Calif. The data show clearly the predominating influence that temperature exercises on the evaporation rate throughout the annual cycle, both directly, and indirectly through its effect on relative humidity. Short-period rates of evaporation show the temporary effect of high wind or abnormal humidity conditions, but these represent only minor irregularities on the curve.

The quantitative relation between the changes in the temperature of the water surface and the rate of evaporation has been carefully investigated by R. B. Sleight, Assoc. M. Am. Soc. C. E., at Denver, Colo.† All other factors being similar, he found that for each degree Fahrenheit within the natural range of large water bodies there occurred a change in the rate of evaporation averaging 6 per cent. This result agrees closely with the writer's experience at Independence, and elsewhere.

The effect of atmospheric pressure on evaporation at any one station is probably negligible. At Denver, Mr. Sleight could not detect any variation in evaporation for a range of 1 in. on the barometer. The maximum elevation at which reservoirs are constructed seldom exceeds 10 000 ft. As the normal barometric pressure at sea level is 30 in. and at an elevation of 10 000 ft., 21 in., the effect of atmospheric pressure is relatively small even when comparing evaporation rates from water at different altitudes. The observed differences of rates at different altitudes, it is believed, are due almost entirely to temperature differences.

The effect of specific gravity on the rate of evaporation has received little attention from investigators in the past as there has been no general demand for information. For the solution of problems arising in the writer's practice, he found it necessary, a number of years ago, to undertake an experimental study of the subject. The details and results of this study are presented herewith. The procedure adopted was as follows: Two samples of water, one distilled and the other a brine with a specific gravity of 1.11, taken from Owens Lake, California—a mineralized lake typical of the Great Basin—were evaporated under exactly similar conditions except as to specific gravity. The samples were placed in circular, flat-bottomed pans, 4½ in. deep and 12 in. in

* *Water Supply Paper 294*, U. S. Geological Survey, Plate X.

† "Evaporation from the Surfaces of Water and River-Bed Materials," *Journal of Agricultural Research*, Vol. X, 1917, pp. 229–230.

diameter, and filled to the same depth. The pans were left open at the top, immersed for 4 in. of their depth in an electrically heated air bath, and placed in a room free from drafts. A constant temperature was maintained in the air bath by means of a thermostat. The successive depths of water in the pans were calculated from the observed weight and specific gravity of the samples and the known dimensions of the pans. The results of the test are shown on Fig. 18. It was found that the rate of evaporation from Owens Lake brine decreased as the specific gravity increased, and was 27% less than distilled water when crusting commenced at a specific gravity of 1.275 (Reading 13, Fig. 18). When the crust began to form it was broken up at short intervals. The rate as then observed was 75% less than distilled water for a specific gravity of 1.32 (Reading 17, Fig. 18) and continued to decrease with increasing densities.

As a practical check on Fig. 18 there was available for the period, January 1, 1908, to December 31, 1914, detailed data of the inflow into Owens Lake and the fluctuations of the lake level. The lake basin has no surface outlet, and its bed is completely lined with clay deposits that preclude the existence of a subsurface outlet. The average area of the exposed water surface during this period was 58.173 acres and the specific gravity of the water, 1.11. The average annual depth of evaporation from the lake surface during the 7-year period of observation, as determined from the hydrographic data, was 60.8 in. (Table 28). The observed rate from fresh water at Owens River, near Independence, for the three years, 1908 to 1911, as observed in a floating submerged pan, was 67 in. annually.* Reducing this by 10% as indicated by Fig. 18, there results 60.3 in. for the annual loss from Owens Lake as compared with 60.8 in. as determined from the hydrographic data.

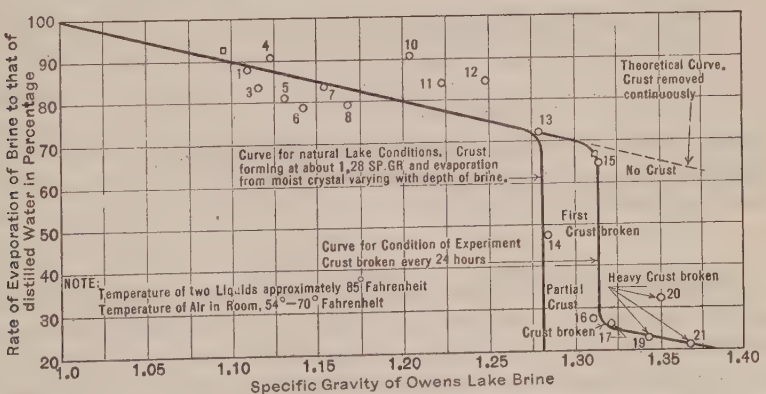
Rate of Evaporation of Brine to that of distilled Water in Percentage
Specific Gravity of Owens Lake Brine
Theoretical Curve. Crust removed continuously
Curve for natural Lake Conditions. Crust forming at about 1.28 SP.GR and evaporation from moist crystal varying with depth of brine.
No Crust
First Crust broken
Curve for Condition of Experiment Crust broken every 24 hours
NOTE: Temperature of two Liquids approximately 85° Fahrenheit Temperature of Air in Room, 54°–70° Fahrenheit
Heavy Crust broken
Partial Crust
Crust broken

FIG. 18.—COMPARISON OF EVAPORATION FROM OWENS LAKE BRINE AND DISTILLED WATER.

It is believed that Fig. 18 is generally applicable to bodies of highly mineralized water. It should be noted, however, that crusting will probably begin at slightly differing specific gravities, depending on the chemical composition of the salts in solution.

* *Transactions*, Am. Soc. C. E., Vol. LXXVIII (1915), p. 177.

Evaporation from Capillary Films.—The practical application of hydrology to water supply problems in Western America involves a knowledge of evaporation not only from free water surfaces, but also from capillary films surrounding soil grains, crystals of precipitated alkaline salts, and ice crystals, including snow. In addition, there should be included the evaporation from moist soil grains as modified by the transpiration losses from vegetation, the surface crusting due to the accumulation of salts precipitated from evaporating soil moisture, and the presence of clay in a highly colloidal state resulting from concentrated alkali solutions.

Less is known of the process of evaporation from capillary films than from free water surfaces. For example, it is probable that the modifying effect of concentrated alkaline salt solutions on clayey soils is unknown, except by a few soil physicists, and yet in the Great Basin region of Western America as well as in similar inland basins throughout the world, there are large areas of moist soils of this type to which some rate of evaporation must be applied by the hydrologist in estimating the ground-water supply. In this discussion it will be impossible to go into the details of these subjects. It is desired, however, to outline the problems in the hope that systematic investigation will be stimulated and ultimately lead to useful results.

TABLE 28.—Depth of Annual Evaporation from Owens Lake, California, Based on Measured Inflow and Lake Fluctuation.

(Period of Time Covered, January 1, 1906, to December 31, 1914.)

Elevation of lake surface, January 1, 1906, in feet	3 565.32
Elevation of lake surface, December 31, 1914, in feet	3 576.60
Rise in lake level, in feet	11.28
Area of lake surface, January 1, 1906, in acres	44 800.00
Area of lake surface, December 31, 1914, in acres	61 500.00
Water accumulated in lake, January 1, 1906, to December 31, 1914, in acre-feet	599 699.00
Inflow from all sources, except rainfall, January 1, 1906, to December 31, 1914, in acre-feet	3 120 125.00
Water lost by evaporation, January 1, 1906, to December 31, 1914, in acre-feet	2 520 426.00
Average area exposed water surface, January 1, 1906, to December 31, 1914, in acres	58 173.00
Total depth evaporation, net, January 1, 1906, to December 31, 1914, in feet	43.3
Annual depth evaporation, net, January 1, 1906, to December 31, 1914, in feet	4.81
Annual precipitation on surface of lake (Keeler), in feet	0.26
Annual depth of evaporation, gross, in feet	5.07
Annual depth of evaporation, gross, in inches	60.8

In general, evaporation from capillary films surrounding soil or crystal grains is controlled by the same factors as that from free water surfaces. Evaporation may take place from moist grains either at the surface of the granular body in contact with the atmosphere, or in the upper zone of the body if it is sufficiently open for atmospheric circulation. In the latter case it is termed interior evaporation. One complicating factor involved is the degree to which the capillary film is replenished with water as losses occur. If the source of replenishment is precipitation, surface flooding, or irrigation, the film has no permanent and constant source of supply, and passes through successive stages of fatness and leanness and may even disappear entirely. If, on the other hand, the source of supply is a permanent ground-water table

within capillary distance the film is replenished constantly by capillarity and suffers little variation, except as the water rises or falls. In the event that the water-table drops far enough below the surface to exceed the capillary limit the upper portions of the film might be withdrawn into the material so far that evaporation in measurable quantities would cease. A condition of leanness or complete desiccation of the film results in the diminution or complete cessation of evaporation. The availability of moisture controls directly the rate and total amount of evaporation from soil or crystal bodies; it has been termed the "evaporation opportunity."

Until recent years observations of evaporation losses from moist soil have been confined to conditions of widely varying evaporation opportunity, either for the purpose of determining in humid regions the proportion of rainfall returned to the atmosphere from a water-shed or, in arid regions, the atmospheric losses from water applied in irrigation. These observations are well known and need no discussion, as they constitute special cases. The drainage and ground-water problems of Western America during the past ten or fifteen years have led to observations of soil evaporation from areas of shallow ground-water where the evaporation opportunity is fairly constant. It is information of this kind which is especially valuable to the Engineering Profession, but not accessible.

For moist river-bed materials, ranging from fine sand to fine gravel, without vegetation, the investigations of Mr. Sleight at Denver give fairly complete and reliable data.* He used water-jacketed tanks, 2 ft. in diameter and 4 ft. deep, maintaining permanent water-tables at different depths and determining losses by weighing the tanks. The greatest depth to water-table was 24 in. The observations ran for three months.

For finer materials including sandy clay and clay soils having a greatest depth to water-table of 5 ft. and supporting a growth of salt grass, the writer's experiments in Owens Valley, California, may be noted.† Tanks, 7.5 ft. in diameter and 10 ft. deep, were used, set permanently in the ground, water being introduced in measured quantities into covered tanks feeding into the soil tanks at the bottom. These experiments were continued for two years.

Both sets of experiments indicated that the rate of evaporation was greatest for a saturated soil surface and diminished as the water-table lowered, becoming zero when the water-table fell below the capillary limit. They also indicated the possibility of expressing soil evaporation as a percentage of that from a free water surface, such percentage varying for different materials, and, for each material, for different depths to the water-table and different seasons of the year. The writer's conclusion, based on his experience in Owens Valley, modified by subsequent work in other localities and a comprehensive analysis of the work of other observers both in America and Europe, is that the evaporation loss from a saturated moist sand or soil is less than that from a free water surface. The ratio for loam soil and fine sand has been found to vary from 0.83 to 0.97 by different investigators; in ordinary

* "Evaporation from the Surfaces of Water and River-Bed Materials," *Journal of Agricultural Research*, Vol. X, 1917, No. 5, pp. 242–259.

† *Water Supply Paper 294*, U. S. Geological Survey, C. H. Lee, 1912 (preliminary results); *Transactions*, Am. Soc. C. E., Vol. LXXVIII (1915), pp. 181–196 (final results).

practice the writer uses 0.90 (Table 29). Mr. Sleight's work indicates that the loss from saturated coarse sands and fine gravel was less than from fine sand and soil. The ratio decreases for moisture content at the surface less than the optimum and becomes zero when the water-table drops to the capillary limit. The exact relation between the evaporation from the soil and the depth to the water-table has not been thoroughly determined, but the indications are that it is a direct proportion.

The growth of vegetation on the soil introduces a modifying factor of greatest importance. Although it may reduce the direct soil loss by shading, etc., it greatly multiplies the evaporating surface in foliage and often increases the loss as compared with that from bare soil. Transpiration depends on the plant species, light, soil moisture available to roots, soil texture, mineralization of soil moisture, temperature, wind movement, etc. The writer's experience and studies indicate that for fresh-water vegetation which thrives with a high water-table, or in swamps, the rate of loss during the active growing season may exceed the rate from a free water surface, but that when considered for the complete annual cycle the rate is less. For ordinary uses, therefore, the ratio of maximum loss by combined evaporation and transpiration lies between 0.90 and 1.00. For salt grass in Owens Valley the writer found it to be 0.95.

There have been numerous attempts to determine transpiration and evaporation-transpiration losses during the past seventy-five years both in Europe and America. European forestry experiments have been extensive, and in America the field of agriculture has been most active. Both in Europe and America the Engineering Profession has been vitally interested from the standpoint of water supply, but little fundamental work has been done. It is to be hoped that the recent impetus that Western irrigation and ground-water problems have given to the subject will produce results of practical value. The present interest among irrigation engineers in the determination of the so-called "consumptive use" of water in irrigation, is a most promising field of investigation, and with the existence of extensive areas of irrigated land, large-scale measurements covering projects and whole valleys are being found to produce important results. In the field of ground-water hydrology this method of measurement is also coming into general use for determining losses from both irrigated and natural vegetation. Two large-scale determinations of the latter made by the writer have given results as follows:

1.—Independence region of Owens Valley, California: 54 sq. miles of natural meadow land and moist soil varying from luxuriant fresh-water swamp growth to salt-grass pasture and bare crusted alkali flats:

Average annual evaporation from free water surface..5.58 ft.
Annual evaporation-transpiration (observed loss)....2.26 ft.*

2.—San Luis Rey Valley, San Diego County, California: 6 640 acres of moist bottom-land covered with willow groves, meadow grass, etc., and with bare moist sand along the river channel:

Average annual evaporation from free water surface..5.16 ft.
Annual evaporation-transpiration (observed loss)....2.19 ft.†

* *Transactions*, Am. Soc. C. E., Vol. LXXVIII (1915), p. 204.
† *Water Supply Paper 446*, U. S. Geological Survey, 1919, p. 105.

TABLE 29.—SUMMARY OF EXPERIMENTAL DATA EVAPORATION FROM SATURATED BARE SOIL.

No.	Observer.	Location.	Experimental Equipment.		Period of observation.	Depth of Evaporation, in Inches.		Ratio, evaporation from saturated soil to water.	Reference.
			Water evaporation.	Soil evaporation.		Water.	Soil.		
1	Ebermayer	Aschaffenburg, Bavaria	Lamont evaporometer	Tanks artificially fed at bottom	Two seasons: April to September	.....	.....	0.97	*Bulletin No. 7*, U. S. Dept. of Agriculture, Div. of Forestry.
2	Slichter	Deerfield, Kans.	Tank in soil	Tanks artificially fed at bottom	August 6 to September 8, 1905	12.11	10.0*	0.83	*Water Supply Paper 153*, U.S. Geological Survey, p. 44.
3	C. H. Lee	Owens Valley, California	"	Tanks artificially fed at bottom	September, 1909, and July, 1910	7.75 (Average)	6.5*	0.84	*Water Supply Paper 294*, U.S. Geological Survey, Plate XV and Table 51.
4	Sleight	Denver, Colo.	"	Tanks artificially fed at bottom	July 31 to October 16, 1916	18.93	*	0.91	*Journal of Agricultural Research*, Vol. X, No. 5, pp. 243-259.
5	Fortier	Davis, Calif.	"	Tanks artificially fed at bottom	August 2 to October 3	21.59	18.32	0.85	"Use of Water in Irrigation," Fortier. (Revised Edition).
6	Fortier	.....	"	Tanks artificially fed at bottom	.....	1.88	1.75	0.93	"Use of Water in Irrigation," Fortier (First Edition), p. 131.
	Average	.....	.....	.....	.....	.....	.....	0.89	

* Value taken from extrapolated portion of curve.

The writer is not aware of any thorough study of the effect of surface crusting resulting from precipitation of alkali salts, but his experience leads to the belief that material reduction in the rate of evaporation results from such crusting. It is a subject that needs investigation. Of a similar nature is the effect of an excessive amount of deflocculated colloids. The writer has observed soil in desert playas so moist that a footprint in the mud would fill with brine within a few minutes, but from which little if any evaporation was taking place, even under the great heat of summer. There is practically no data available for water losses from such materials.

Evaporation from Moist Crystal Deposits.—There are several saline and alkaline deposits in the Southwest that are free from mud, the crystal body being exposed directly to the atmosphere. Brine occupies the voids in the crystal, the upper surface of the brine during the summer period being at a depth of 1 in. to 1 ft., or more, below the surface, and flooding the surface in winter. The rate of evaporation from the brine that is drawn up around the surface crystals by capillarity is a matter about which little data exist, but one which is of sufficient importance in ground-water studies to need investigation. The writer has conducted some experimental work on this subject, but the results, beyond indicating that the rate of evaporation is small, are unsatisfactory.

Evaporation from Ice and Snow.—This phenomenon is important in hydraulic studies in connection with the determination of stream run-off. It is possible that loss takes place directly from ice crystal surfaces at low temperatures, although the greatest loss occurs when the temperature is sufficiently high to permit of melting. The source of water available for evaporation is thus not drawn up from below by capillarity, but is furnished directly by the crystal surface. Some experimental work has been done in recent years, particularly in the water-sheds of the Wasatch Mountains east of Salt Lake City, Utah, the results of which have been published by the U. S. Weather Bureau.* Further study of this subject is needed.

Methods of Measurement.—Although there is general recognition of the individual factors that control evaporation and the relative importance of these factors is becoming understood, yet, judged by the diversity in methods of observation and the frequent failure of observers to state the observational conditions, there is not full appreciation of the practical bearing of these factors. As a result there has been a lack of standardization in methods for measurement, the absence of which has resulted in much inaccuracy and confusion. It is believed that much of the difficulty could be obviated if comparable experimental conditions were secured in making the observations. It is common experience to find pans, established for the purpose of determining evaporation from a reservoir surface, placed so that the temperature of the water in them has no relation to that of the surface of a large body of water. Pans are sometimes completely surrounded by air; they are often placed on the ground; and, in some cases, they are set on a float anchored on the surface of a body of water. In these "unnatural" positions they are

* "Some Field Experiments on Evaporation from Snow Surfaces," by F. S. Baker, *Monthly Weather Review*, July, 1917, p. 363.

exposed on all sides to the air and direct sunlight and are subject to abnormal increase in temperature by the heat absorption and radiation effects of the metal pan.

The most disconcerting feature of these "unnatural" exposures is that no general relation can be established that will enable results to be converted into usable data. For example, the depth of evaporation from water in a land pan set on the surface of the ground or on a platform may vary from 10% less to 70% greater than that from an adjacent lake or reservoir surface, depending on the local combination of meteorological factors and the heat-absorptive character of the container and its surroundings. The writer has personally observed this range of variation in different localities by running parallel series of observations using the Class A land pan adopted by the U. S. Weather Bureau and standard submerged floating pans. The controlling factor is the relative speed with which the temperature of small and large bodies of water changes. The contents of the exposed pan heat rapidly in the day and cool just as rapidly at night. The large body, on the other hand, holds to a fairly constant temperature day and night. In a region of warm nights the temperature of the pan holds up at night and the average for the day exceeds the average for the large body. In a region of cold nights the reverse occurs. The latter condition is well illustrated by observations made under the writer's supervision at Lake Almanor, California, where the nights are characteristically cold (Fig. 19). On this diagram the pan designated "Sutter Butte Pan No. 1", was a Class A Weather Bureau pan while the lake pan was of the submerged floating type. As a result, in the first case the rate of evaporation from the exposed pan exceeds that from the large body of water, and in the second case it is less. The writer's experience has led to the conclusion that in order to use data obtained from the Class A land pan it is necessary to determine a correction factor, which will differ for every locality or meteorological condition.

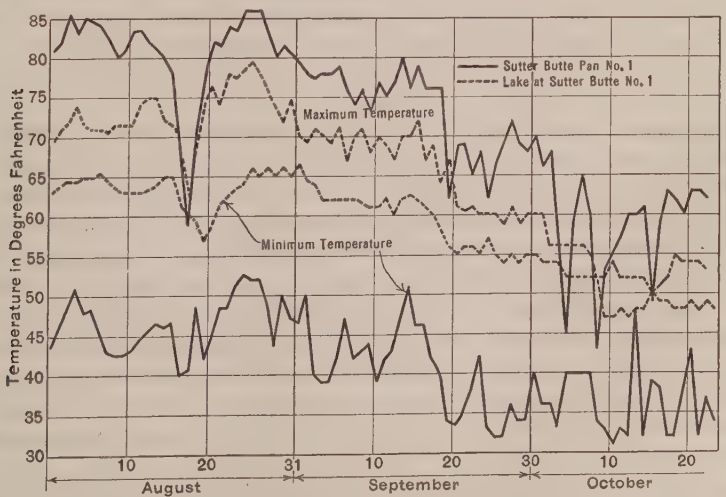

FIG. 19.—DAILY MAXIMUM AND MINIMUM TEMPERATURES OF SUTTER BUTTE PAN NO. 1 AND ADJACENT WATER SURFACE, LAKE ALMANOR, AUGUST TO OCTOBER, 1924.

Land pans, placed below the surface of the ground 2 or 3 ft., or still better, floating pans submerged in a large body of water, approach much closer to the actual temperature conditions of a lake or reservoir than any type of pan the sides of which are exposed. Experience has shown that the temperature of water in submerged floating pans that are made of light-colored metal and kept clean, does not ordinarily vary more than 1° or 2° Fahr., from that of the surrounding water. This is illustrated by Fig. 20 presenting the results of maximum and minimum thermometer readings for Lake Almanor, at Prattville, Calif. Thermometers were fastened to floats with the bulbs just submerged and the observations represented surface temperatures.

In the matter of vapor pressure there is an even greater departure from natural conditions. Often, attention is not given to insuring the same degree of free air movement. Pans are placed near the vertical faces or objects that break the wind, or are closely surrounded by protecting fences. The writer has even observed land pans about which the grass has been permitted to grow, so as almost to prevent any air movement across the water surface.

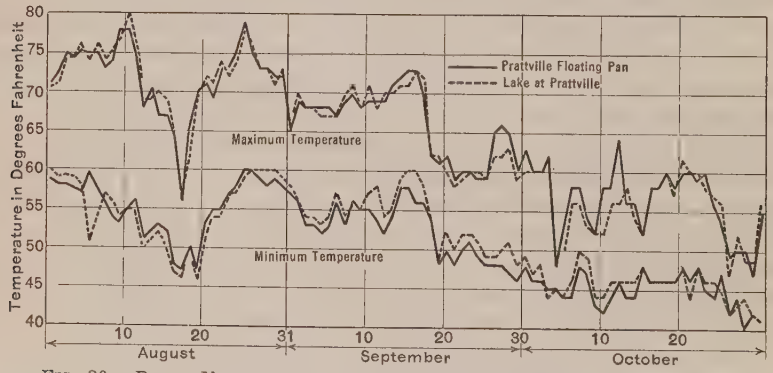

FIG. 20.—DAILY MAXIMUM AND MINIMUM TEMPERATURES OF PRATTVILLE FLOATING PAN AND ADJACENT WATER SURFACE, LAKE ALMANOR, CALIFORNIA, AUGUST TO OCTOBER, 1924.

Then, again, the size of the pan has an important influence, which often is not given consideration. The ease with which the saturated vapor from a water surface escapes into the atmosphere, is affected by the size of the surface. A lake or reservoir, considered as a whole, has not as great an opportunity for dissipation of its vapor as a pan, as escape is practically limited to the vertical direction. The perimeter of a small pan, however, is relatively large, compared to its area, as it varies directly with the diameter, while the area varies as the square of the diameter. The vapor dissipating horizontally from a small pan thus bears an appreciable ratio to the total, whereas from a large body of water it is practically negligible. The ratios of area and perimeter also have a bearing on marginal losses through wave action and capillary movement.

Another important relation, often lost sight of, is that of the ratio of rates of wind movement to the distance across a body of water. A wind mov-

ing at the rate of 10 miles per hour, for example, would cross a 3-ft. pan in 0.2 sec., whereas to cross a lake 1 mile in diameter would require 6 min. These examples are sufficient to show that marked differences in the rate of evaporation may occur from water surfaces of different sizes. In so far as the vapor pressure is involved, a submerged floating pan, if fully exposed to the wind, would seem to give results most comparable to the large body of water, as it is covered by the same vapor blanket and therefore its size does not introduce the element of horizontal vapor dissipation.

The specific gravity of the water is sometimes overlooked in making comparisons of evaporation rates. Well-placed pans are sometimes not emptied at regular intervals and the water concentrates until its density considerably exceeds that of fresh water. An error of considerable magnitude thus may be introduced. A similar error would result from the use of a correctly observed evaporation rate from fresh water in determining losses from mineralized bodies of water, such as Salton Sea, or Great Salt Lake, and *vice versa.*

A review of these examples, it is believed, will fully bear out the writer in the statement that much of the confusion and uncertainty which exists regarding evaporation rates is due to a failure to consider the fundamental factors in choosing observational methods. Another cause is the absence of full auxiliary information or description of the equipment. In 50% of the records reported by the author, for example, the description of the exposure is incomplete. If the pan is floating there is often no statement as to whether it is on a raft or submerged. If a land pan, it is a question whether it is set on the surface or into the ground.

The first step toward progress, either in determining the laws which govern evaporation or in obtaining data for practical use, is the general adoption of a standard method of measurement that will yield reasonably dependable and consistent results. In practical application evaporation data, whether for water surface, moist soil, or crystal bodies, are desired for relatively large areas. Investigations have shown that the standard U. S. Geological Survey floating pan submerged in water gives results that closely approximate the actual loss from a large body of water and that for this pan successive observations for short periods depart less from the mean than for any other measuring device in use. These considerations, and the fact that the floating submerged type of pan seems to reproduce most nearly the natural conditions with regard to factors controlling evaporation, weigh heavily in favor of the adoption of this method of measurement as a standard for water surfaces.

In certain localities where strong winds are prevalent great difficulty has been experienced in overcoming splash. The land pan or tank set into the ground as nearly to full depth as possible, and near the margin of the water body, is believed to be the best alternative method for use under such conditions. Neither the depth nor diameter of the tank should be less than 3 ft. Usually it is possible to obtain satisfactory observations from floating pans for occasional short periods even in windy districts. By establishing both land and floating pans the continuous record obtained from the land pan can be used as a basis for determining what the floating pan evaporation

would have been by use of the factors obtained when records of both are at hand. These factors should be available for different times of the year as they vary considerably.

To illustrate the relations of evaporation for a pair of land and floating pans Fig. 21 is presented. The observations were made under the writer's direction at Lake Almanor, near Prattville, Calif. The land pan was located within 100 ft. of the water's edge in moist soil, and was set in the ground so that the top rim was almost flush with the surface. The differences in depth of evaporation as observed in the two pans correspond closely with the observed temperature differences. During August and September the loss from the land pan was about 15% greater than from the floating pan. In October the two were practically the same. The writer's experience has been that for the full annual cycle there is little difference between the land pan set in the ground and the submerged floating pan, but that during the summer the rate of loss from the former is greater than and in winter is less than the latter. It is necessary, therefore, to obtain factors at intervals throughout the annual cycle in order to get complete records.

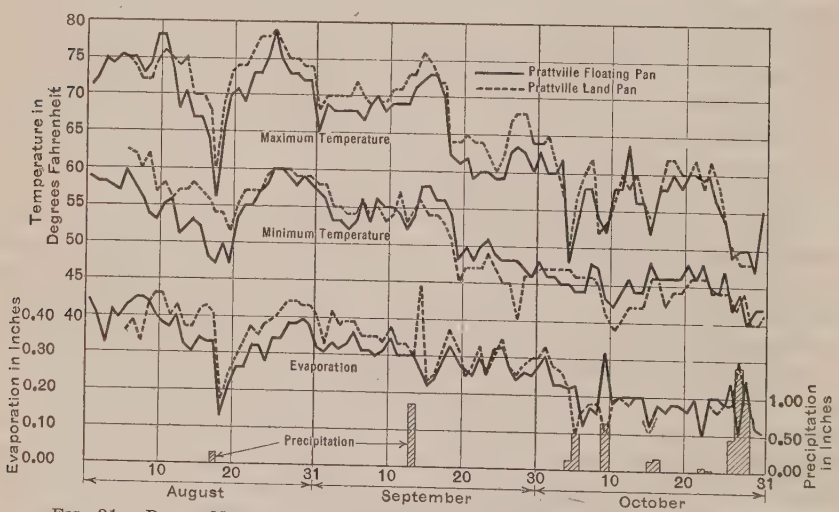

Fig. 21.—Daily Maximum and Minimum Temperatures of Water Surface, Precipitation, and Evaporation from Prattville Floating and Land Pans, August to October, 1924.

The measurement of evaporation from soil and the evaporation-transpiration loss from cultivated crops and natural vegetation is more in need of standardization even than that of evaporation from water surfaces. To be of use to engineers such data should be obtained by uniform methods and be expressed in terms of a generally accepted standard. Evaporation from a large free water surface as measured by the submerged floating-pan method is probably the best basis for reference. If a full annual temperature cycle is covered by the data it is believed that either the land or floating pan would be satisfactory for measuring water evaporation. In view of the wider

availability of floating pan records it is believed that this form of apparatus is preferable, although for shorter periods the land tank may be more desirable as temperature conditions are more comparable.

For moist soil and close growing vegetation, such as grass, the loss may be determined direct by means of soil tanks. For other types of low growing vegetation, such as shrubs and trees, the potometer is used. In the case of perennial plants and trees the dry leaves are weighed, while annuals are cut at the root crown, and then stalk, leaves, and seed are weighed. The ratio of the weight of dry matter produced to water absorbed is then obtained. A great many similar data have been observed by soil physicists and agricultural and forestry investigators. Such data give relative values for various types of vegetation but must be converted to obtain equivalent depth of water over the land area covered by the growth. For this purpose the weight of dry material produced by a unit of land area must be known. In the case of agricultural field crops the acre yield should be sufficient. For trees, shrubs, and natural vegetation, typical areas must be selected and the leaves stripped, dried, and weighed. It is probable that transpiration from the same variety of plant differs according to the locality. Certain investigators state that the water requirement varies proportionally to the evaporation from a free water surface. If this be true, the water requirement determined in one locality would require modification for use in another region where evaporation from the free water surface is different. The writer has followed this method in several instances when computing evaporation losses from river valleys containing natural deciduous trees of different types, using the results of European forestry experiments for the water requirement, and has arrived at reasonable results.

The use of tanks or potometers in determining plant losses is open to serious difficulties, however, and may produce results from which grossly erroneous conclusions could be drawn. The direct measurement in the field, either in plots, projects, or whole valleys, is more dependable, if physical conditions are favorable for measurement, and is coming more and more into use.

Conclusion.—In conclusion it is desired to emphasize two points, first, that the study of evaporation should not be confined to free water surfaces, but should be extended to capillary films surrounding soil grains, crystals of alkali salts and ice, and in connection with soils should include the process of transpiration; and, second, that there is greatly needed a study and an authoritative adoption of standard methods of measuring the various types of evaporation.

C. M. O'NEIL,* ESQ. (by letter).—The author has brought together in a concise and interesting paper much valuable data on the loss by evaporation from pans installed on U. S. Reclamation Projects.

It is to be hoped that in the not too distant future records of these losses obtained by the use of standard equipment and standard methods of installation may be available from many more points on the American Continent.

* Asst. Canal Supt., Dept. of Natural Resources, Canadian Pacific Ry. Co., Brooks, Alberta, Canada.

Not only the Engineering Profession but the public is beginning to feel the need of a better knowledge of losses from evaporation. A better general knowledge of the application of collected data to large bodies of water is also desirable. However, so many factors influence the rate of evaporation that any relevant formula can only tend toward increased accuracy in estimating or computing this loss.

The rate of evaporation varies locally from day to day and from year to year, particularly in latitudes subject to ice conditions during the winter season. It may also vary widely in different locations depending on climatic, topographic, and surface conditions as to cultivation, vegetation, etc. It also apparently varies considerably with the type of pan used and the method of installing and operating it.

During the years 1919 to 1924, inclusive, the writer conducted investigations, hoping to obtain data that would enable him to segregate seepage from evaporation losses occurring at the Lake Newell Reservoir on the Eastern Section of the Canadian Pacific Railway Company's Irrigation Block in Southern Alberta. Mr. B. A. Miles, the Hydrographer, continued the records of the land pans during 1925. The investigation was carried on in conjunction with the routine hydrometric surveys of the project under the supervision of Augustus Griffin, M. Am. Soc. C. E., Superintendent of Operation and Maintenance.

The Dominion Reclamation Service became interested in this investigation and supplied most of the equipment used. Since 1920 its engineers have made independent, periodic measurements of the discharges of inflowing and outflowing canals. The records were kindly made available to the Railway Company and added materially to the value of its data.

In the writer's opinion floating pans present a fairly common problem to all who attempt to secure records from them. Waves, water fowl, water rats, and other adverse agencies destroy records. Constant vigilance is required if reliable information is to be obtained. For this reason they are not likely to be widely used. If there is any practical means of protecting a floating pan without impairing the record the writer would appreciate details of it.

The Lake Newell Reservoir occupies a natural depression in an area of uncultivated, rolling prairie land. Its approximate location is 100 miles east of Calgary, Alberta. At full supply level it covers an area of 16 430 acres, is 8 miles in length, and averages $3\frac{1}{3}$ miles in width. Water is conveyed to it through two large canals and it, in turn, supplies two canals capable of diverting an aggregate of 1 280 cu. ft. per sec. Daily discharges of the canals are obtained from the continuous records of Steven's water-stage recorders. Its range of stage above the full supply level of the outflowing canals is from Elevation 2 471 to Elevation 2 485 and the available storage is 189 000 acre-ft. at full supply level. The maximum water depth is about 60 ft., the lower portion of the storage not being available. Corresponding to fluctuations in stage the drainage area of virgin prairie contiguous to the water surface varies roughly from 3 000 to 9 000 acres. Little is known of the nature of the

subsoil of this particular area. The surface soil of adjacent areas may be described as varying from a sandy to a silty loam, to 2 ft. in depth. During the summer months a dense growth of aquatic plants flourishes along the margin and in bays in water less than 12 ft. deep. Water was first admitted to the basin in the summer of 1914 and the storage was gradually built up. There were 12 000 acres under water in the fall of 1918.

The first complete season's record of evaporation from pans was obtained in 1919. Since then two land pans have been continually in operation during the open-water period of each year. A floating pan was operated from 1919 to 1924, inclusive.

One pan was set on the north shore of Lake Newell approximately 40 ft. back from the foreshore at a height above the lake surface varying from 5 to 10 ft., according to the lake elevation. As this point was 5.5 miles south of the project headquarters at Brooks it was seldom that observations could be taken more frequently than once a week until 1923 when an observer became available. This resulted at times in losses of as much as 2 in. before refilling.

As a check on this pan and to admit of a closer study, a similar land pan was installed 40 ft. from the shore of a reservoir covering about 40 acres in the immediate vicinity of Brooks. Here the water surface is about 35 ft. below the full supply level at Lake Newell and is 500 ft. wide in front of the land pan. The exposure and topographical features of both locations corresponded closely in 1919, but since then the area around Brooks has come under cultivation and irrigation with thousands of trees making a vigorous growth. As the prevalent winds are from the northwest and southwest and the parkland lies to the northeast across the reservoir from the pans, it is doubtful whether the trees have had any appreciable influence on the records.

On this latter body, centrally located on 6 ft. of water, a floating pan, 18 in. deep and 3 ft. in diameter, was suspended in the water from a raft at a point approximately 300 ft. from the land pan.

Both land pans were made from No. 22 gauge, galvanized metal, 4.0 ft. in diameter and 20 in. deep (Canadian Government standards). Wells for the pans were cut in the soil 20 in. deep and 4.0 ft. in diameter. A wooden post 4 in. square was driven into the center of the bottom leaving slightly more than 2 in. projecting, on which the bottoms of the pans had a bearing to prevent buckling. Two wooden breathers with openings 2 in. square were embedded vertically on opposite sides of the pans to provide for the free circulation of air through the 2-in. space beneath the pans. This arrangement left from $1\frac{1}{2}$ to 2 in. of the pans projecting above the ground surface. Water in the pans was set at ground level, or about 2 in. below the rim. This proved ample to accommodate the heaviest average rainfall. Rain gauges were placed 18 in. above the ground and 3 ft. from the pans. Both were protected by a 2-in., wire-mesh fence, 12 ft. square and 5 ft. high. An anemometer was placed near the pan at Brooks with the cups 2 ft. above ground.

The floating pan (No. 22 gauge, galvanized metal) was floated in much the same manner as those used on U. S. Reclamation projects except that no

honeycomb baffles were placed within. The water level was maintained as far as possible 4 in. below the rim or slightly below the surface of the surrounding water in the reservoir. The 4 in. were deemed necessary to prevent sloshing during light winds. As records from this floating pan, 3 ft. square, and from land pans, 4 ft. in diameter, were not considered properly comparable, a pan identical with the land pans was floated from a separate raft a few hundred feet from the square pan during the summer of 1923. The circular pan was immersed somewhat deeper than the square pan to equalize the reflecting areas of metal of each pan above the water, both inside and outside. The season's record showed that the amount evaporated from the smaller pan was 1% greater than that from the larger. The square floating pan was then discontinued.

Gauge readings were made once or twice daily for the land pan at Brooks Reservoir. Accurate readings of the gauges in the floating pans could only be made during calm weather when the pans were refilled. The water in the pans was changed often enough to prevent mossy formations but light silt adhering to the sides or deposited on the bottom was not removed as it was thought that this inside coating approached reservoir bed conditions. Little difficulty was experienced in the operation of land pans but floating-pan records were occasionally destroyed or rendered doubtful. All doubtful records were discarded and no attempt was made to interpolate the daily evaporation for periods of lost or doubtful records extending over two days. Since 1922 the average fluctuation of the water level in the land pan at Lake Newell has not exceeded ¼ in.

It is noteworthy that the records have consistently shown the average rate of evaporation from a floating pan to be greater than that from a land pan, while records collected on the U. S. Reclamation projects show the reverse ratio. This variation is possibly due chiefly to the difference in dimensions and to the method of installing the land pans.

In the aggregate these records have yielded the following results:

1.—The depth of water lost from a land pan, 20 in. deep and 4 ft. in diameter, set in soil is 80% of the loss from a similar pan suspended from a wooden raft.

2.—The loss from a floating pan, 18 in. deep and 3 ft. square, is 1% greater than from a floating pan 20 in. deep and 4 ft. in diameter.

3.—The loss from a land pan installed on the windward side of the 40-acre reservoir near cropped and irrigated areas varied from 93 to 101% (averaged 95%) of the loss from a similar pan, similarly installed, on the leeward side of a 12 000-acre reservoir 5.5 miles south in an uncultivated area of rolling prairie.

Mr. Houk gives a brief summary of the more important features of the investigations of the U. S. Reclamation Bureau to date. The following relations are extracted from this summary:

"4.—The evaporation from a land pan, 14 in. deep and 3 ft. square, set in the ground with earth banked around the sides, was found to be only 72% of the evaporation from a Class A land pan, 10 in. deep and 4 ft. in diameter, set on the ground with no earth around the sides."

* * * * * * * *

"6.—Evaporation from a floating Class A pan mounted on a raft was found to be 67% of the evaporation from a similar pan on land."

Assuming that this Class A floating pan and the Canadian Reclamation Service standard circular floating pan used on the Brooks Reservoir may be considered as forming a common basis for comparison, the following relation between the two records may be established:

(*a*).—The U. S. Reclamation Bureau pan, 14 in. deep and 3 ft. square, set in soil and banked gives a loss by evaporation of 108% of that from the Class A floating pan.

(*b*).—The Class A land pan set on land gives a loss of 149% of that from the Class A floating pan.

(*c*).—The Canadian Reclamation Service standard land pan as installed at Brooks gives a loss of 80% of that from the similar floating pan.

By deduction it is found that the amount of evaporation from the Canadian standard pan is 74% of the loss from the U. S. Reclamation Bureau square land pan and 54% of the loss from the Class A land pan.

It will be seen that the loss by evaporation from the Canadian Reclamation Service standard land pan is much less than from the U. S. Reclamation Bureau land pans when compared with the loss from floating pans.

It is quite probable that the distance below the rim of the pan at which the water surface within was maintained had a measurable influence on the rate of evaporation. However, considering the conditions under which it was necessary to operate it was not deemed advisable to maintain the water level any higher.

It has not been possible to obtain data from which to make comparisons of relative humidity and wind movement at the Lake Newell and Brooks Reservoirs. No doubt the larger size of the body of water at Lake Newell is conducive to a greater wind movement. It may be due to the fact that the land pan at Lake Newell was on the leeward side of the reservoir that the average rate of evaporation there has been found to exceed that of the land pan at Brooks, which was on the windward side, by 5 per cent.

TABLE 30.—Total Losses as Recorded for Winter Seasons.

Season.	Change in stage: depth, in feet.	Precipitation: depth, in feet.	Total water lost: depth, in feet.
1918–19	0.04 decrease	0.16	0.20
1919–20	0.07 increase	0.14	0.07
1920–21	0.05 increase	0.13	0.08
1921–22	0.07 decrease	0.09	0.16
1922–23	0.04 increase	0.08	0.04
1923–24	0.07 decrease	0.03	0.10
1924–25	0.01 increase	0.13	0.12

The average depth of water lost from the Lake Newell land pan for the 7-year period is 108% of the total loss of water from the reservoir due to all causes. A study of Table 35 will show that the seepage loss, or loss other than that measured by the evaporation pan, when taken by the season is only in evidence for seasons when storage has been increased and an area not

previously saturated was flooded. The figures given for run-off under balancing factors to satisfy the excessive evaporation loss are not compatible as they are in every case greatly in excess of the probable maximum run-off from the tributary drainage basin. It will further be observed that the land-pan record should be reduced by as much as 30% in certain seasons to correspond with the actual loss from the reservoir. That the correction varies widely for different seasons will also be observed. The problem that still presents itself is: What corrections can safely be applied to the evaporation measured in land pans that will reveal with some measure of accuracy the quantity of water lost from this reservoir to adjacent areas by seepage?

TABLE 31.—Relation Between the Rates of Evaporation as Measured from a Land Pan and a Floating Pan at Brooks Reservoir, Alberta, Canada.*

Month.	Ratio: Floating Pan / Land Pan.						Averages.	
	1919 May 9–September 30.	1920 June 1–October 5.	1921 May 15–October 31.	1922 June 1–October 17.	1923 July 1–October 18.	1924 June 1–October 7.	Floating pan / Land pan	Land pan / Floating pan
May	1.13	....	1.19	....	....	....	1.16	0.86
June	1.18	0.96	1.34	1.29	....	1.14	1.18	0.85
July	1.23	1.13	1.18	....	1.30	1.18	1.20	0.83
August	1.51	1.11	1.28	....	1.40	1.29	1.32	0.76
September	1.47	1.10	1.45	1.49	1.39	1.40	1.38	0.72
October	....	1.10	1.42	1.42	1.49	1.19	1.32	0.76
.....	1.30	1.08	1.31	1.40	1.39	1.24	1.28	0.80

* Floating pan used from 1919 to 1922 was 3 ft. square and 18 in. deep; that used from 1923 to 1925 was 4 ft. in diameter and 20 in. deep.

TABLE 32.—Difference Between Evaporation as Observed from Land Pans Set 40 Feet from the Margins of Lake Newell and Brooks Reservoirs.*

Year.	Lake Newell.		Brooks.		
	Period of record.	Evaporation, in feet.	Period of record.	Evaporation, in feet.	Brooks / Lake Newell
1919	May 20–October 23.....	2.573	May 9–October 23	2.549	....
1920	May 26–October 16.....	2.439	May 26–October 16	2.200	0.90
1921	May 16–October 31.....	2.640	May 16–October 31	2.477	0.94
1922	May 16–October 11.....	2.033	May 16–October 3	1.773	....
1923	May 9–October 31.....	2.229	May 5–October 18	2.030	....
1924	May 1–October 7.....	2.299	May 1–October 7	2.229	0.97
1925	May 4–September 30..	2.007	May 4–September 30 .	2.031	1.01

* Brooks land pan is located on the windward side of the reservoir near young park land and area under cultivation and irrigation. Lake Newell land pan is on the leeward side of the reservoir in virgin prairie.

Assuming that the seepage loss is negligible the corrections are indicated in Table 35; but what weight should be given to the various factors causing the seasonal differences? The writer is not able to make any suggestions that would be of value in this direction.

TABLE 33.—MONTHLY AVERAGES OF RECORDS FOR LAKE NEWELL RESERVOIR.*

Month.	Years of record.	Lake Newell Reservoir: Total Evaporation, in Inches.				Wind, in miles per hour.	Average temperature, in degrees Fahrenheit.	Average precipitation, in inches.
		Mean monthly.	Maximum.	Minimum.	Range.			
January	7	Ice conditions				2.3	12.83	0.47
February	7	" "				2.2	13.63	0.36
March	7	" "				2.9	22.33	0.64
April	7	" "				3.4	40.84	1.16
May	7	4.13†	....	....	....	2.9	52.23	1.21
June	7	6.14	7.13	5.25	1.88	2.1	61.31	1.56
July	7	7.04	8.78	5.42	3.36	2.0	67.97	1.73
August	7	5.69	7.61	4.14	3.47	2.0	64.82	1.58
September	7	3.98	5.10	2.05	3.05	1.9	54.21	1.27
October	6	2.12‡	....	....	....	2.1	41.72	0.62
November	7	Ice conditions				1.5	25.15	0.42
December	7	" "				1.8	14.35	0.50
Yearly average		29.11	(Open-water period)			2.26	39.28	11.52

* Temperatures and precipitation are complete for 7-year period, 1919 to 1925, inclusive. Anemometer records cover years 1923, 1924, and 1925.

† Average for last 18 days.

‡ Average for first 20 days.

TABLE 34.—EVAPORATION RECORDED AT LAKE NEWELL RESERVOIR FROM A LAND PAN.

(Depths are given in inches.)

Month.	1919 May 19 to October 23.	1920 May 26 to October 16.	1921 May 16 to October 31.	1922 May 16 to October 11.	1923 May 9 to October 31.	1924 May 1 to October 7.	1925 May 4 to September 30.	Average.	Remarks.
May	2.94	1.06	2.75	2.71	3.94	5.53	5.33	4.13	Average of last 18 days.
June	7.13	6.65	6.46	5.88	5.25	5.65	5.60	6.14	.....
July	8.56	7.00	8.78	5.42	5.77	7.68	6.11	7.04	.....
August	5.68	7.61	7.35	5.46	4.14	4.58	4.99	5.69	.....
September	4.94	5.10	3.49	4.14	4.43	3.72	2.05	3.98	.....
October	1.63	1.86	2.80	0.78	2.82	0.42	....	2.12	Average of first 20 days.
Total	30.88	29.27	31.63	24.40	26.35	27.59	24.08	29.10	.....

That loss by seepage from this particular reservoir is indeed small is fairly well indicated by records taken during the winter seasons when it was under complete ice cover and the canals were not in operation (Table 30). Winter precipitation is generally in the form of snow. The drifting over the

TABLE 35.—Synoptic Chart Showing Relation of Total Loss of Water from Lake Newell Reservoir to the Loss by Evaporation Based on the Record from a Land Pan 40 Feet from its Shore Line.*

Year.	Period of record.	Change in surface elevation.	Change in quantity stored, in acre-feet.	Delivered to reservoir canals and precipitation, in acre-feet.	Withdrawn by canals, in acre-feet.	Total loss from reservoir, in acre-feet.	Evaporation Loss.		Ratio of land pan evaporation to total reservoir loss, percentage.	Balancing factors + run-off etc. — seepage, in acre-feet.
							Depth from land pan, in feet.	Equivalent from reservoir, in acre-feet.		
1919	June 2 September 30	2 474.42 74.32	— 1 198	37 257	18 631	19 824	2.192	25 871	130	+ 6 047
1920	May 26 October 16	74.95 74.28	— 8 027	52 822	36 863	23 986	2.439	28 850	120	+ 6 699 May 25–July 31. — 2 229 August 1–September 30. + 394 October 1–16.
1921	May 16 October 31	74.24 75.78	+ 18 764	111 616	59 962	32 890	2.640	301 78	92	+ 4 473 May 16–August 31. — 7 185 September 1–October 31.
1922	May 16 October 11	75.86 73.52	— 28 179	58 118	65 728	20 569	2.033	24 181	118	+ 4 198 May 16–August 31. — 586 September 1–October 11.
1923	May 9 October 18	73.43 75.98	+ 31 053	119 424	64 383	23 988	2.229	25 401	106	+ 5 777 May 9–June 30. — 5 003 July 1–September 30. + 639 October 1–18.
1924	April 23 September 30	75 60 79.73	+ 55 038	173 702	82 622	36 042	2.359	31 056	86	+ 2 684 April 23–30. — 914 May. + 1 120 June and July. — 7 876 August and September.
1925	May 4 September 30	79.52 76.69	— 38 272	69 302	81 436	26 138	2.007	27 338	104	+ 4 276 May 4–July 31. — 3 076 August 1–September 30.
.....	Average	.....	.....	.....	.....	.....	.....	.....	108	.....

* Balancing factors have been copied from detailed annual reports. Yearly mean surface area of reservoir may be found by dividing depth of evaporation into the equivalent loss from the reservoir in acre-feet.

reservoir surface and from adjacent areas doubtless introduces an error. To what extent evaporation from the surface of ice and snow may account for the losses here given is problematical. Condensation from the atmosphere may also be an influencing factor.

In Table 30 the total losses in every case cover the months of December, January, and February, and while they may appear small there is no reason to believe that subterranean water reaches and augments the reservoir storage.

If the records from the floating pan used on this project are comparable with the records obtained from the Class A floating pan used by the U. S. Reclamation Bureau it appears that the evaporation loss measured in the Canadian type of land pan approaches actual reservoir loss more closely than does the loss from the land pans described in the paper.

In this discussion and in Tables 31 to 35, in an abridged form, an effort has made to point out some of the important factors that affect the records of evaporation from pans. The variable relation that these records may have to the loss from a large reservoir is shown by Table 35.

J. B. Lippincott,* M. Am. Soc. C. E. (by letter).—The author has presented a valuable record of evaporation as measured by standard methods on the various projects of the U. S. Reclamation Service in arid America. The determination of evaporation losses in the dryer sections of the country is of importance because cycles of drought as great as seven years in succession are recorded, which call for large storage capacity to hold over from years of flood. For instance, the Henshaw Reservoir in San Diego County, California, is built to a capacity of 164 000 acre-ft., and the net safe yield therefrom is taken at only 24 000 acre-ft. With evaporation ranging from 4 to 5 ft. and hold-overs for such long periods, the loss of impounded water by evaporation is likely to be from 30 to 50% of the total. There is much difference of opinion on the part of competent engineers as to what ratio exists between the loss from pans and from large water surfaces. This is difficult to determine, not only because of the variation in the evaporation from year to year, but in the results obtained from the relatively small shifting of the location in which the pan records are kept. Variations in such records of 10 and 20% within 3 or 4 miles have been noted at practically the same elevation. It is difficult to determine the evaporation from large reservoir surfaces because of the fluctuations in inflow and withdrawals. The information which has been compiled is not consistent, although not necessarily erroneous. It is important, however, for the water supply engineer to form a judgment as intelligent as possible as to this relation. The paper† by Edwin Duryea and H. L. Haehl, Members, Am. Soc. C. E., brought out much discussion on this subject and widely different opinions were expressed. Since that time the writer has been gathering data of evaporation from lake or reservoir surfaces. In California there is little summer rain, and it is possible to find places where the run-offs into the lakes during the summer months

* Cons. Hydr. Engr., Los Angeles, Calif.

† "A Study of the Depth of Annual Evaporation from Lake Conchos, Mexico," *Transactions*, Am. Soc. C. E., Vol. LXXX (1916), p. 1829.

are negligible and where the outflows are either nothing or may be measured with accuracy. It was the writer's intention to present these data in a paper to the Society. However, as the author's collection of pan evaporation records suggests a further discussion of this question, these notes are submitted.

Evaporation Losses from Large Water Surfaces.—In the paper by Messrs. Duryea and Haehl, previously mentioned, conclusions are drawn to the effect that the rate of evaporation from large water surfaces was substantially lower than that from a floating standard evaporation pan. It was stated that the evaporation loss from the water surface of Lake Conchos was certainly less than 67.5% (and probably as low as 62%) of that from an evaporation pan 3 ft. square, floating on the lake surface. Messrs. Duryea and Haehl state that the rate of evaporation from a small pan is greater than that from a large one, that is, the evaporation from a floating pan 2.0 ft. square is 108% of that from a pan 3.0 ft. square. Their conclusions have been accepted by many engineers, and two-thirds of the recorded rate of evaporation from a standard 3.0-ft. floating pan has frequently been used in estimating evaporation losses from reservoirs.

Relative to the subject, R. B. Sleight, Assoc. M. Am. Soc. C. E., has made the following statement:*

"A careful consideration of all factors mentioned, and the curve (Fig. 4) which is approaching the horizontal for the large areas, leads the writer to present the following conclusions:

"(*a*) Evaporation figures from tanks 2 feet or greater in depth, preferably circular, set in the ground so that but a narrow metal rim not over 3 inches wide projects above the ground and in which water is kept approximately at the ground level, are most applicable for extension to large open-water surfaces.

"(*b*) Data on such tanks may be quite safely extended to large open-water surfaces under exactly the same conditions of wind, air temperature, and relative humidity by multiplying the evaporation depth from *a:*

2-foot tank by 0.77	9-foot tank by 0.98
4-foot tank by 0.84	12-foot tank by 0.99
6-foot tank by 0.90	

"Factors for depths from other tanks used have been found:

3 by 3 by 3 feet, sitting 2.75 feet in ground (Fort Collins type)	0.80
U. S. Geological Survey floating standard 3 by 3 by 1.5 feet	0.91
U. S. Weather Bureau for class A station 4 ft. in diameter, 10 inches deep, above ground	0.66"

Comparative Evaporation from Tulare Lake and at Kingsburg, Calif.—Tulare Lake, in the San Joaquin Valley, in Central California, is a basin into which a number of rivers discharge when in flood. During the late summer and fall of ordinary and low years, little, if any, water now finds its way into the lake because of the irrigation diversions on tributary streams. Some pumping from the lake was begun in the fall of 1915, but since 1916 these pumping operations have been substantially increased for the irrigation

* "Surfaces of Water and River-Bed Materials," *Journal of Agricultural Research*, Vol. 10, July 30, 1917, p. 237.

of marginal lands. There were periods, however, prior to 1916, when the only fluctuation in the lake surface was caused by evaporation. The lake is without outlet, except when it is abnormally high, and it has not overflowed since 1878. Evaporation records from Tulare Lake are given in Table 36.

Thirty miles to the northeast of the lake, at Kingsburg, in the San Joaquin Valley, evaporation records were kept by the State Engineer of California, under conditions of elevation, air, temperature, and general exposure very similar to those at Tulare Lake.

The measurements of evaporation at Kingsburg were made during the period from November, 1881, to October, 1885, inclusive, (*a*) from a pan floating in Kings River; and (*b*) from a "pan in air".

The results by monthly averages for the period are given in Table 38, together with the evaporation figures as computed for Tulare Lake by the writer. The data for the Kingsburg Station were originally published by the California State Engineer, William Hammond Hall, M. Am. Soc. C. E., in 1886 under the title, "Physical Data and Statistics of California."

TABLE 36.—Evaporation, in Feet, from Tulare Lake.

Month.	1906.	1907.	1908.	1910.	1911.	1912.	1914.	1915.	Mean.
July	.....	.....	0.857	.....	.....	0.607	.....	0.748	0.737
August	0.845	0.690	.....	0.575	0.515	0.714	0.663	1.109	0.730
September	0.700	0.575	.....	0.489	0.607	0.374	.....	0.608	0.559
October	0.500	.....	.....	0.326	0.272	0.405	.....	0.816	0.464
November	0.310	.....	.....	0.120	0.219	0.151	.....	.....	0.200
December	.....	.....	.....	.....	0.146	0.335	.....	.....	.....

The floating pan near Kingsburg was 2 ft. square and 1 ft. deep. The pans used in making the observations in open ground at Kingsburg were 3 ft. square and 15 in. deep. The water surface was maintained about 5 in. below the rim of the pan. When floated in the river, the pans were protected from the splash of the waves. The contrivance used in Kings River, at Kingsburg, for this purpose was a large wooden frame, into which the pan was fitted so as to float horizontally, but was not protected from the sun or movement of the air. The water level in the pan was kept at uniform height by making daily additions, by measurement, into it during the summer months. Daily records of the temperatures of the air and the water were kept.

After the first three months, the "pan in air" referred to in Table 38 was set into the ground with earth banked around its sides to the elevation of the water in it, with its full surface exposed to the sun and wind. During the first three months, or from November, 1881, to January, 1882, inclusive, this pan was on a railroad bridge, exposed on all sides to sun and air.

Kings River flows from the western slope of the high Sierra Nevada Range, which culminates in a crest having an elevation of from 12 000 to 14 000 ft. The river is fed mostly from melting snows. The distance from the crest to Kingsburg is approximately 70 miles. The elevation of Kingsburg is 297 ft. The river waters at Kingsburg, therefore, are cold, especially during the period of melting snow. After July 1 the river is at low stage and the water

is relatively warmer. The low temperature of the river water would reduce its rate of evaporation as compared with warmer water. The water in the "pan in air" would be warm.

Tulare Lake receives its water from the Kings, Kaweah, Tule, and Kern Rivers. All these rivers increase in flow in the spring and early summer, due to the melting snows, culminating in discharge about June 1 and by July 1 reaching low stages which usually are maintained as such until winter. The low summer flow of these rivers is diverted for irrigation purposes before reaching the lake, and it is only during flood periods that the streams discharge into it. No direct measurements of these discharges into the lake prior to 1915 are available, so that a determination of the evaporation from the lake surface has to be restricted to periods during which there is no inflow into it, and also when the level of the lake is below its outlet. Hydrographs of these rivers indicate low-flow periods to extend from July to December. They coincide with the months of little or no rainfall in this region, as seen in the records of precipitation at Hanford, Calif. (Table 37), the point nearest the lake for which rainfall records are available. These records have been used in connection with the computation of evaporation on the lake. Any variation in the rainfall on the lake from that at Hanford, will have little effect on the level of the lake. The seepage into the lake from surrounding lands is negligible. If either seepage water or stream water entered the lake during the periods for which comparisons are being made between lake and pan evaporation, it would decrease the indicated evaporation from the lake surface, and if adjustment were made on such an hypothesis, it would emphasize the general conclusions presented in Table 38.

TABLE 37.—Rainfall, in Inches, at Hanford, California.

Month.	1906.	1907.	1908.	1909.	1910.	1911.	1912.	1913.	1914.	1915.
January	1.59	3.88	1.86	4.69	2.40	3.79	0.52	1.26	4.36	2.69
February	1.92	0.65	2.84	0.49	0	1.46	0.02	1.55	1.25	3.44
March	4.05	2.39	0	2.31	1.66	2.94	3.24	0.34	0.37	0.30
April	0.62	0.29	1.05	0	0.24	0.53	1.52	0.78	0.11	1.37
May	2.06	0	0.50	0	0.05	0	0.27	0.76	0.00	0.91
June	0.02	0	0	0.42	0	0	0	0.06	1.06	0.00
July	0	0	0	0	0	0	0	0.08	0.00	0.00
August	0	0	0	0	0	0	0	0	0	0
September	0	0	0.95	0	1.51	0.02	0	0.03	0	0
October	0	0.64	0	0.36	0.31	0.02	0	0	0	0
November	0.12	0	0.72	0.95	0.24	0.23	0.61	1.88	0.02	0.00
December	3.43	1.58	0.31	1.95	0.28	0.55	0.21	1.35	2.50	1.71
Annual	13.81	9.43	8.23	11.17	6.69	9.54	6.39	8.09	9.67	10.42

The mean annual rainfall in the neighborhood of the lake is about 10 in., nearly all of which occurs during the winter months. There are occasional storms in September, October, and November. The rise of the lake in November, 1912, over the October levels, suggests an October rain.

Tulare Lake is broad and shallow and during the period of this study, it ranged from an area of 103 000 to 198 000 acres. Its depth varied from 0 to 12 ft. The bottom of the lake is 180 ft. above sea level. The temperatures

of the San Joaquin Valley are high in summer and practically the same at Tulare Lake as those at Kingsburg where observations on the pans were made. The bed and shores of the lake are clay. Very little water would be lost by seepage into the soil or would return from the soil when the lake levels fell.

Systematic records of the stage of the lake are available since 1906, when the U. S. Geological Survey established a gauging station thereon. Measurements have been made at weekly and monthly intervals to date, but since 1916, the stage of the lake has been affected by withdrawals, by pumping of water for irrigation, and the computation of the evaporation has been carried from 1906 to 1916 only.

The gauge was usually read to 0.01 ft. and the observer was instructed not to take readings during wind storms which might cause abnormal levels in parts of the lake. Although the observations for evaporation were not taken at the same time from both the pans at Kingsburg and the surface of Tulare Lake, they were of such duration that it is believed they will throw light on the relation between the actual records of evaporation from the lake surface and the rate of evaporation from a floating pan in this locality.

Table 38 shows the evaporation from Tulare Lake as compared with that at Kingsburg, as measured in the pans.

TABLE 38.—Evaporation from Tulare Lake Compared with Evaporation at Kingsburg, Calif.

Month.	At Tulare Lake.		At Kingsburg.		
	In feet per month.	Percentage of floating pan evaporation at Kingsburg.	Pan floating in river, in feet per month.	Pan in air, in feet per month.	Percentage of floating pan evaporation.
July	0.737	117	0.628	0.904	144
August	0.730	101	0.721	0.861	119
September	0.559	103	0.541	0.615	114
October	0.464	189	0.339	0.289	85
November	0.200	114	0.176	0.174	99
	2.690	112	2.405	2.843	118

From Table 38 it will be noted that the rate of evaporation observed from the surface of the lake was 112% of that from the floating pan in the river and 95% of that from the "pan in air" at Kingsburg. The evaporation from the "pan in air" was 118% of that from the pan in the river at Kingsburg. The lake water probably was warmer than the river water, which would increase the rate of evaporation from the lake.

S. T. Harding and Harry Barnes, Members, Am. Soc. C. E., have made a study* of the evaporation losses in Kern County, California, with particular reference to their relation to Buena Vista Lake which much resembles Tulare Lake in position and exposure. For 1920 the evaporation pans were of the

* "Water Resources of Kern River and Adjacent Streams and Their Utilization," *Bulletin 9*, California State Dept. of Eng., 1920.

type used by the U. S. Geological Survey, 3 ft. square and 2 ft. deep. The floating pan was surrounded by a raft to baffle wave action. The water surface was maintained 3 in. from the top of the pan. Owing to the flat slopes of the bed of the lake and the shallowness of the water during a part of the season, the pan was set in an excavated hole so that the water at the outside did not get more than 6 in. below the top of the pan at any time. The floating pan was covered by a chicken wire screen so as to exclude water fowl. The evaporation from the land pan for the 12 months of 1920 was 67.84 in. The rate of evaporation from the floating pan is given as 115% of that of the land pan. During the months, July to November, inclusive, there was no inflow to the lake. The outflow was small, but was measured. The comparison of evaporation from the floating pan with the actual lowering of the water surface of Buena Vista Lake in 1920 was as given in Table 39.

TABLE 39.—Comparison of Evaporation from Floating Pan and from Surface of Buena Vista Lake.

Month.	Indicated lowering of Buena Vista Lake, due to evaporation, in feet.	Measured evaporation from floating pan, in feet.	Indicated coefficient for floating pan.
July	0.61	1.04	0.58
August	0.58	0.94	0.62
September	0.59	0.70	0.84
October	0.38	0.45	0.84
November	0.16	0.20	0.80
5 months	2.32	3.33	0.74

It has been estimated* that the evaporation for the floating pan for the full year was 77.99 in., that the estimated actual evaporation from the whole lake was 54.7 in., and that the estimated coefficient for the year was 0.70. The coefficient, however, for the months of actual observation was 74 per cent.

Messrs. Harding and Barnes also made a comparison between the evaporation from the surface of Tulare Lake and the pan observations made in previous years by the State Engineer at Kingsburg, referred to previously. It is stated† that the estimated evaporation measured from the pan at Kingsburg Bridge is 4.4 ft. per year and that from the lake itself the evaporation loss was 4.6 ft. It is concluded, as follows: "For this portion of the San Joaquin Valley the total evaporation for shallow lakes appears to be about 4½ ft. per year."

Comparative Evaporation at Lake Hodges.—Lake Hodges is in San Diego County, California, 10 miles from the coast. The high water surface is 315 ft. above sea level, and the maximum depth of the water at the dam is 115 ft. The sides of the reservoir are largely steep and rocky.

This study for Lake Hodges is based on the results obtained from a standard pan, 3 ft. square and 18 in. deep. The water in the pan was kept within 4 in. of the top, and the pan was buried flush in the ground on a slope with

* *Bulletin 9*, Dept. of Eng., State of California, 1920, p. 63.
† *Loc. cit.*, Table 25.

a southern exposure to the sun the whole day. The pan was protected by a screen of 2-in. chicken wire. This pan was at the west end of Lake Hodges.

There was a prevailing wind movement from the west over the pan from the spillway cut of the dam. This prevailing wind traversed a 12-mile stretch of uncultivated country, before it reached the lake surface. The lake is 7 miles long, about 2 000 ft. wide, and lies in the general direction of these prevailing westerly winds. The evaporation from a standard pan in the ground near the east end of the lake for 1922 was only 88% of that from the pan at the dam. The prevailing winds probably have absorbed some moisture from the lake before reaching the eastern pan and hence cause less evaporation.

There was a little rainfall during months of the observation period, but no resulting run-off as shown by the daily gauge readings at the reservoir. This rainfall was considered in the lake as well as in the pan evaporation record. There were no withdrawals from the lake during the period of comparison.

The lake evaporation was 96% of that from the pan at the dam at the west end of the lake. This is considered approximately the true relationship existing between the evaporation from the lake and the pan. The Hodges Reservoir, which was first filled in 1919, has few marginal swamps. The observer at Lake Hodges was an experienced engineer.

Comparative Evaporation at Cuyamaca Reservoir.—Cuyamaca Reservoir is in San Diego County, California, 40 miles from the sea, near the crest of the Coast Range, at an elevation of 4 600 ft.

These pan records were obtained from a standard pan 3 ft. square, 18 in. deep, the water being kept within 4 in. of the rim. For November, 1916, and January, 1919, the pan was floating in the water in the deep section of the lake near the dam, and was protected from wave motion by log booms. In November, 1921, and June, 1922, the pan was buried flush with the ground on the hillside north of the lake, fully exposed to the sunshine and wind movement, except for a 2-in. mesh screen.

The estimated evaporation from the lake based on the observations for these four months was 115% of that from the pans, which is 17% higher than the corresponding figures at Lake Hodges. This may be due to the rapid summer plant growth on the large marginal swamp area at Cuyamaca Lake. The evaporation figures from the lake for June, 1922, were 2 in. more than those from the pan. The lake has a mean depth of only 12 ft. and hence becomes warm during the summer months. A screen covering the pan may have somewhat reduced the evaporation therefrom. The wind movement over the Cuyamaca pan was practically the same as that over the lake. During the summer months the water temperature at Cuyamaca Lake was 73° Fahr. on the surface and 70° Fahr. at the bottom, as compared with 71° Fahr. on the surface at Lake Hodges and 55° Fahr. on the bottom. With due allowances for the plant growth and the screen over the pan, the evaporation from the surface of Cuyamaca Lake would probably be much the same as that from the pan.

The evaporation from a standard pan set in the ground at Cuyamaca Lake and observed from 1914 to 1917, inclusive, was 69.46 in. per year. From a floating pan, from 1914 to 1921, inclusive, it was 66.88 in. per year. The evaporation from the lake surface as expanded from fragmentary records, taken during the dry season of little run-off during two years* was at the rate of 57.6 in. per year. Based on these data, the lake surface evaporation was 86% of that of the floating pan. Table 40 shows the relative evaporation for Lake Hodges and Lake Cuyamaca.

TABLE 40.—Relative Evaporation, Lake Surface and Standard Pan, at Lake Hodges and Cuyamaca, for Periods of No Reservoir Draft and Little or No Rainfall.*

Lake Hodges, Area, 1 317 Acres.

(Pan located about 35 ft. above lake surface and near spillway opening, where wind movement is greatest. Two-inch mesh screen over pan.)

	Lake evaporation, in feet.	Pan evaporation, in feet.
June, 1919	0.62	0.60 (pan in ground)
July, 1919	0.70	0.68 (" " ")
September, 1919	0.50	0.57 (" " ")
June, 1920	0.57	0.61 (" " ")
October, 1920	0.44	0.45 (" " ")
October, 1921	0.37	0.42 (" " ")
Totals	3.20	3.33

Lake evaporation = 96% of pan.

Cuyamaca Lake, Area, 978 Acres.

	Lake evaporation, in inches.	Pan evaporation, in inches.
November, 1916	3.88	3.82 (pan in water)
January, 1919	2.50	2.07 (" " ")
November, 1921	3.25	3.21 (pan in ground)
June, 1922	9.25	7.28 (" " ")
Totals	18.88 (1.573 ft.)	16.38 (1.365 ft.)

Lake evaporation = 115% of pan.

* Comp. by F. E. Green, Hydrographer.

Evaporation from Upper Otay and Lower Otay Reservoirs Compared.—H. A. Whitney, M. Am. Soc. C. E., has given a statement† as to the relation between evaporation of the water surface of the Upper Otay Reservoir in San Diego County, California, to evaporation observed from floating pans 3 ft. square, at the Lower Otay Reservoir. The Upper Otay Reservoir is 13 miles

* *Transactions*, Am. Soc. C. E., Vol. LXXX (1916), p. 2052.

† *Loc. cit.*, p. 1988.

southeast of San Diego, Calif., and about 550 ft. above sea level, and is approximately 1 mile long and ½ mile wide. The drainage area is small with steep rocky sides. The reservoir has an area of 160 acres and an average depth of 20.7 ft. Accessions to it and withdrawals from it are comparatively infrequent. Mr. Whitney shows that during the three months, October to December, 1915, inclusive, the evaporation from the water surface of this reservoir was 87% of that from the floating pans.

In the paper by Messrs. Duryea and Haehl, previously mentioned, the probable mean annual evaporation from the Upper Otay Reservoir is given as 62.3 in., which is greater than the pan records at Sweetwater only 6 miles distant and with much the same exposure, the record of which is given subsequently.

Mr. Arthur J. Ellis and Charles H. Lee, M. Am. Soc. C. E., have made a study* of the comparative evaporation from a pan floating in the La Mesa Reservoir in San Diego County, California, with that of the Otay Reservoir which is situated 11 miles to the southeast and which was neither drawn upon nor received accessions during the period January, 1913, to December, 1915. The rainfall was observed at the Lower Otay Dam about 2½ miles to the west of the Upper Otay. After making corrections and expanding the record Mr. Lee concludes that the evaporation from the Otay Reservoir was 94.2% of that shown by the pan floating in the La Mesa Reservoir.

Evaporation from Sweetwater Reservoir.—The evaporation† from the Sweetwater Reservoir for the period, 1889 to 1894, inclusive, was measured for 4 years in a standard pan, averaging 58.71 in. and, subsequently, for 2 years with a Pische evaporimeter which showed a yearly loss of only 45.77 in.

Mr. John F. Couvert, Engineer for the Sweetwater System, computed the gross evaporation of the Sweetwater Reservoir from 1910 to 1914, inclusive.‡ Based on his meter readings, W. S. Post, Assoc. M. Am. Soc. C. E., found that the evaporation losses from the water surface amounted to 56.98 in., indicating that from this series of readings the loss from the reservoir surface was 97% of the pan record. The Sweetwater Reservoir is 8 miles east of San Diego, at an elevation of about 250 ft.

Lake Elsinore Evaporation.—Lake Elsinore, which is shallow and large, is situated 26 miles from the coast in Riverside County, Southern California, in an inland valley where summer temperatures are relatively high. The bottom of the lake is 1 218 ft. above sea level, and the lake overflows at Elevation 1261 when its area is 5 500 acres. The water is quite saline except during overflow. The floods of the San Jacinto River discharge into the lake, and it overflows at high stages through an outlet at the Town of Elsinore. On rare occasions the lake is entirely dry, but it was high during 1916, and overflowed.

Evaporation records have been kept from a 3-ft. pan floating in water and the gauge height has been taken on the lake itself. This work was done

* *Water Supply Paper 436*, U. S. Geological Survey.
† *Water Supply Paper 81*, U. S. Geological Survey, p. 344.
‡ *Transactions*, Am. Soc. C. E., Vol. LXXX (1916), p. 1914.

for the San Diego County Water Company, under the direction of its Engineer, Mr. W. S. Post, from May to September, 1916.

The lake is surrounded by quite extensive areas of flat land which possibly may drain into it as the water levels recede. Several hundred acres are irrigated by pumping from these marginal areas, the water being spread on the surface thereof. In the opinion of the engineers of the California State Division of Water Rights these ground-waters do not originate from the lake. The result of comparative evaporation measurements between the lake surface and the evaporation pan for 5 months during the summer of 1916 shows that the loss from the surface of the lake is 84% of that from the pan. (See Table 41.) During this period the inflow and outflow from the lake were known and the corrections made therefor as given in the table.

TABLE 41.—Lake Elsinore Evaporation.*

Date.	Evaporation from lake surface, in inches.	Evaporation from pan in water, in inches.
May, 1916	4.92	7.75
June, "	5.40	7.35
July, "	8.52	8.42
August, "	6.14	8.56
September, "	7.83	6.90
	32.81	38.98

Lake evaporation = 84% of pan.

* As computed by W. S. Post, Assoc. M. Am. Soc. C. E.

Professor S. T. Harding has written a report* in which he has presented the estimated gross evaporation of Lake Elsinore. The inflow of the San Jacinto River into the lake and the outflow of the lake were measured, together with the local rainfall, and, by deduction, the record of evaporation from the lake surface as shown in Table 42 is determined. Professor Harding states, as follows:

"The resulting figures cannot be considered as being more than a general estimate of the probable run-off. * * * All of the local run-off is considered as reaching the local ground water, direct inflow to the lake being small and infrequent."

Again, it is stated:

"The available records are sufficient to enable the evaporation from Lake Elsinore since 1916 to be computed. This has been done in Table 17. The fluctuation of the lake for each month is caused by evaporation, direct rainfall, and depth of inflow or outflow. The records of inflow and outflow, with the area and capacity of the lake, enable the depth of inflow or outflow to be computed. Fluctuations not accounted for by other items have been considered to represent evaporation."

Professor Harding estimates the evaporation from the surface of Elsinore Lake for the months, May to September, inclusive, to average 2.7 ft. for a

* "San Jacinto River Hydrographic Investigations, 1922," Div. of Water Rights, State of California, Table 17.

TABLE 42.—Estimated Gross Evaporation of Lake Elsinore.

(Based on fluctuations of lake, direct rainfall, and depth of inflow and outflow.)

Year.	Evaporation Depth, in Feet.											
	October.	November.	December.	January.	February.	March.	April.	May.	June.	July.	August.	September.	Year.
1915-16	....	....	....	2.34*	1.06*	0.30*	0.92*	0.50	0.50	0.70	0.50	0.60	....
1916-17	0.26	0.36	—0.3	0.08	0.18	0.23	0.33	0.59	0.30	0.28	0.40	0.70	3.68
1917-18	0.50	0.51	0.40	0.22	0.10	0.31	0.42	0.42	0.50	0.71	0.52	0.42	5.03
1918-19	0.57	0.26	0.28	0.12	0.01	0.14	0.42	0.63	0.40	0.40	0.70	0.54	4.47
1919-20	0.44	0.26	0.09	0.27	—0.17	—0.37*	0.47	1.21*	0.60	0.60	0.70	0.50	4.52
1920-21	0.48	0.21	0.14	0.18	0.06	0.23	0.40	0.29	0.50	0.61	0.81	0.71	4.62
1921-22	0.40	0.21	1.19*	—0.22*	0.23	0.69*	—0.16*	0.56	0.50	0.71	0.50	0.50	5.05
Mean except excluded months..	0.44	0.30	0.18	0.17	0.07	0.23	0.41	0.49	0.47	0.57	0.59	0.57	4.49
Adjusted estimated mean....	0.45	0.30	0.20	0.15	0.05	0.25	0.40	0.45	0.50	0.60	0.60	0.55	4.50

Note.—The large indicated evaporation in months of large inflow, such as January, 1916, to December, 1921, may be due to ground storage in lands newly submerged which would be lost by evaporation later after the lowering of the lake. The mean annual rainfall at Elsinore = 14.0 in.

* Months for which records are uncertain and which were omitted in computing the means.

TABLE 43.—Comparative Evaporation Between Floating Evaporation Pan and Large Water Surfaces in California.

Location.	Duration of Record.		Size of pan, in feet.	Evaporation, in Feet.		Ratio: Lake to pan.	Authority.	Remarks.
	Pan.	Lake.		Pan.	Lake.			
Tulare Lake.....	4 years	5 summers	2 (square)	2.405	2.690	112	State and U. S. Geological Survey.	Pan floating in river which was cooler than lake. Record for 5 summer months.
Tulare Lake.....	.....	.....	2 (square)	4.4	4.5	102	State Engineer.	Estimated for 12 months.
Buena Vista Lake.	5 months	5 months	3 (square) 2 (deep)	3.33	2.32	74	State Engineer.	.....
Lake Hodges.....	6 "	6 "	3 (square) 1½ (deep)	3.33	3.20	96	F. E. Green.	Pan in ground.
Cuyamaca Lake...	4 "	4 "	3 (square) 1½ (deep)	1.365	1.573	115	F. E. Green.	Pan, 2 months in water and 2 months in air.
Cuyamaca Lake...	4 years	Fragmentary	3 (square) 1½ (deep)	5.58	4.80	86	Duryea and Haehl.	Comparison with floating pan.
Upper Otay Lake.	3 months	3 months	3 (square)	1.05	0.917	87	Whitney.	Pan evaporation measured at Lower Otay Reservoir.
Upper Otay Lake.	4 years	Yearly mean	3 (square)	4.89	5.19	106	Duryea and Haehl.	Pan evaporation records at Sweetwater used in ratio. Lake evaporation at Upper Otay estimated by Duryea and Haehl.
Upper Otay Lake.	3 "	3 years	" "	5.51	5.19	94.2	Ellis and Lee.	Based on floating-pan record at La Mesa Reservoir.
Sweetwater Reservoir	4 "	5 "	" "	4.89	4.75	97	Couvert.	Lake evaporation estimated by Couvert, adjusted by Post.
Lake Elsinore.....	5 months	5 months	" "	3.25	2.73	84	Post.	.....
Lake Elsinore.....	.....	6 years	" "	3.25	2.7	83	Post and Harding.	Lake evaporation losses estimated by Harding for 6 years and averages compared with pan records taken by Post for May to September, 1916, inclusive.
Owens Lake.....	.....	.....	.....	.....	.....	99	Lee.	Based on record of pan floating in river near lake probably in cooler water.

period of six years. (See Table 42.) Mr. W. S. Post, during one season, found the loss from a floating pan, as given in Table 41, for these months to be 3.25 ft. On this basis of comparison the lake surface loss was 83% of that of the floating pan.

Mr. Lee has referred to certain evaporation observations that he has made.* A pan was floated on the surface of Owens River 20 miles north of Owens Lake. The water of this river during high stages comes from the melting snows of the adjacent high Sierras and during the late summer largely consists of return water from irrigation. Owens Lake is broad, shallow, and saline, and has no outlet. The wind movement in this region is high. The inflow was measured and the fluctuations in the surface of the lake were observed. Mr. Lee finds the evaporation from the pan and from the lake to agree within 1 per cent. As in the case of Kings River, however, the water of the river probably was much cooler than that of the lake, which may cause the rate of evaporation from the pan floating in the river to be less rapid as compared to the lake surface evaporation.

Conclusions.—From the data available, the writer believes that the loss from a reservoir surface is not quite as great as that from a floating pan under similar conditions of exposure, the difference depending on the general character of the reservoir or lake, probably amounting to 90 to 95% of that from a standard floating pan similarly exposed. This is a subject that requires much further information and study. When it is remembered how uncertain it is to compute the net safe yield from any given water-shed through a group of prospective dry years, it would appear that conservatism would lead to the acceptance of floating-pan evaporation records without deduction in determining the evaporation losses from lake water surfaces.

E. F. Chandler,† M. Am. Soc. C. E. (by letter).—The author deserves thanks for having summarized in such convenient and accessible form the results of so extensive a series of evaporation records from different localities. This illustrates that at any one station the evaporation has a definitely determinable average value, and that its total is nearly uniform at any one point, varying much less from year to year than the annual precipitation. This indeed might reasonably be expected, for, in most parts of the United States, rain actually falls during a very small portion of the year, only a few per cent. of the total. In most of the country a humidity of 100% is unusual, except when rain is falling, so that there is more or less rapid loss by evaporation nearly all the remaining time. Even if in very wet or very dry years the total hours of rainfall are doubled or halved, the resulting change in hours of the evaporation period will be an insignificant percentage.

Of course the evaporation at any station varies from hour to hour, day to day, and month to month, with temperature, wind movement, and humidity, but these factors are not so enormously different at the same point in annual averages or totals from year to year, and the total evaporation is fairly uniform.

* *Water Supply Paper 446*, U. S. Geological Survey, p. 103.

† Prof. Civ. Eng., and Dean Coll. of Eng., Univ. of North Dakota, University, N. Dak.

The observed amounts are affected by the form and arrangement of the tank and its exposure, so that care must be given to the conditions if comparable records are to be obtained from different stations. The paper draws attention to the diversity of results between different stations even in near-by adjoining regions, so that figures must not be heedlessly transferred from one region to another and considered applicable without adjustment or modification. These divergences are caused not only by differences in form and exposure of tank, but primarily by differences in climatologic factors, and especially by altitude which necessarily has a great effect on evaporation.

It may be of interest, therefore, to submit a summary (Table 44) of another record of this same kind which was maintained through many years under the writer's supervision for the U. S. Geological Survey. This record was continued through every open season from April, 1905, to June, 1920, at the University of North Dakota, in Latitude, 47° 55′ N. and Longitude, 97° 04′ W. The gauge was a floating metal tank, 3 ft. square and 18 in. deep, held in a raft so as to be protected from splashing. It was placed in a narrow pool, several acres in area, and 4 to 8 ft. deep, formed by a low dam on a small stream running through the level prairie of the University Campus. The water level is 820 ft. above sea level, and is about 10 ft. below the prairie level at a distance of 5, 10, or 20 rods from the stream. The water is at approximately the same level and the same temperature within the tank and outside it. The exposure to wind is about the same as on any ordinary small pond or pool, although less than on a large lake; but, on the other hand, the lower layer of air has not become humidified as it would be by passing over a large lake. This record, therefore, is presumably a reasonably accurate basis for predicting reservoir or lake evaporation in this region.

TABLE 44.—Summary of Records of Evaporation Gauge at University, North Dakota, 1905 to 1920.

Month.	Mean temperature, in degrees Fahrenheit.	Mean monthly evaporation, in inches.	Maximum evaporation recorded, in inches.	Minimum evaporation recorded, in inches.
January	3.6	....	....	....
February	6.5	....	....	....
March	22.4	....	....	....
April	42.0	2.69	....	....
May	53.4	4.35	5.77	3.48
June	62.9	4.94	7.09	3.22
July	67.4	5.69	7.01	4.83
August	64.9	4.94	7.03	3.94
September	56.2	3.63	4.24	2.32
October	43.5	1.98	3.32	1.29
November	26.1	0.68	....	....
December	10.6	....	....	....

The tank is refilled daily to a standard gauge mark, the loss by evaporation being accurately measured. Standard rain gauges of the U. S. Weather Bureau are within a few rods and, at the close of any day of rain, the difference between the recorded rainfall and the surplus to be removed from the tank to lower the water to the gauge mark is considered as the evaporation.

The recorded final results are, therefore, the gross total evaporation, and are slightly more than one and one-half times the total normal annual precipitation of 20 in.

No authentic continuous records of evaporation were secured during the frozen season. The record was begun usually after the ice cleared from the pool, during the first half of April, and continued until permanent ice closure sometime in November. April and November records for most of the years are therefore, in part mere estimates, but only for that part of the month having the smallest evaporation. There were mid-season interruptions for a few days or weeks in one or two years also, but for most of the months the figures given in Table 44 are the averages for fifteen years.

The temperature given is the average of the standard maximum and standard minimum readings of each day at the adjacent station of the U. S. Weather Bureau, and from these the mean temperature for the entire year of 12 months is 38.3° Fahr. The total evaporation for the 9-month period, April 1 to November 30 (as shown by these records, covering the whole time except the first and latest weeks of cold years presumably having small evaporation, so that the estimates for the omitted time will be fairly close), is 28.90 in. Winter records at other evaporation gauge stations would justify an estimate of slightly less than 3 in. for the four months not recorded, making an annual total evaporation of almost, but not quite, 32 in.

Ivan E. Houk,* M. Am. Soc. C. E. (by letter).—The purpose of presenting this paper was to make the data available for general use and to elicit discussion of the subject, hoping that through such discussion additional data of similar nature would be presented. The writer feels that the purpose has been well fulfilled. Many excellent discussions and much valuable data have been forthcoming.

Both Mr. Kleinschmidt and Mr. Lee ask about corrections for precipitation. Rain gauges were maintained at all stations, and all records in Table 1 were corrected for precipitation; that is, they represent actual, total evaporation (observed evaporation plus precipitation). The records would not be true evaporation records if they were not so corrected. The correction for precipitation is simply one of the incidental matters to be attended to in measuring and recording evaporation data.

Mr. Kleinschmidt asks about the methods of handling pans, frequency of adding water, etc. Naturally, the methods of handling the pans varied somewhat from project to project. Observers at Class A stations, as a general rule, followed the instructions for the installation and operation of such stations issued by the U. S. Weather Bureau.† These instructions provide that the pan shall be filled with water to within 2 in. of the top, readings made daily at about 7:00 a. m., local standard time, and additional water added, at the time of a regular observation, when the water surface has dropped 1 in. Thus, the depths of water in the pan would vary from 7 to 8 in., and the frequency of adding water would vary throughout the year, according to the rate of evaporation.

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* Engr., U. S. Bureau of Reclamation, Denver, Colo.

† *Circular L*, Instrument Div., U. S. Weather Bureau.

At the U. S. Department of Agriculture stations, maintained on the Experiment Farms by the Irrigation Officials of the Bureau of Public Roads, where the 24-in. by 6-ft. circular pans are used, the water surface levels are kept from 2 to 4 in. below the rims, so that the depths in the pans vary from 20 to 22 in. Apparently, the intention is to keep the water surface at the same elevation as the ground surface, since the pans are set in the ground to the same depths. At the other stations water surface levels are kept from 1½ in. to 6 in. below the rims. Usually it is necessary to maintain lower surfaces at the floating-pan stations than at the land-pan stations, to avoid losing water by splashing during windy weather. For instance, at the Nelson Reservoir on the Milk River Project of Montana, No. 29 in Table 1, the engineer in charge first tried submerging the pan to within about 2 in. of the rim, and keeping the water within the pan at the same elevation as the water outside. However, he soon found that greater free-board was necessary. Consequently, he changed the depth of submergence, and the depth of water inside, to 6 in., leaving a free-board of 4 in. This helped materially in reducing sloshing troubles, but did not entirely eliminate them.

The majority of the stations are provided with stilling wells and accurate hook-gauges. At such places the depths of evaporation are measured by direct readings of changes in the surface level. However, in a few cases the pans are provided with needle-points, set at fixed levels, and the evaporation losses determined by adding measured quantities of water until the water surfaces rise to the fixed levels.

Mr. Lee mentions the lack of necessary descriptive data for some of the stations in Table 1. At the time Table 1 was prepared all the available information was included. Letters had been written to the Project Superintendents, asking for complete installation details and other pertinent data. However, in some cases the information could not be furnished owing to the stations having been discontinued, the observers having moved away, the records having been misplaced, or some other legitimate reason. Since that time some of the missing data have been located and consequently the following additional notes can be presented.

At the Spanish Fork Station on the Strawberry Valley Project of Utah (No. 8 in Table 1), the evaporation pan was 17 in. deep by 3 ft. square, and was set in the ground 15 in.

At the Cold Springs floating-pan station on the Umatilla Project of Oregon, for the years 1909 to 1913 (No. 21 in Table 1), a square pan was located in the center of a raft and was partly submerged. The raft was placed inside a hollow square made of floating timbers, to which it was tied loosely at the corners; and the whole arrangement was anchored.

At the Nelson Reservoir floating-pan station on the Milk River Project of Montana (No. 29 of Table 1), the depth of submergence, and depth of water inside, was 6 in., as noted previously. At this station it was found advantageous to mount the pan on a square raft with a triangular protuberance, and to hold the raft by a cable about 200 ft. long, attached to a single anchor, thus allowing the raft to swing around keeping its nose continually against the wind.

FIG. 22.—FLOATING EVAPORATION PAN AT AVALON RESERVOIR, CARLSBAD PROJECT, RECORD 1, TABLE 1.

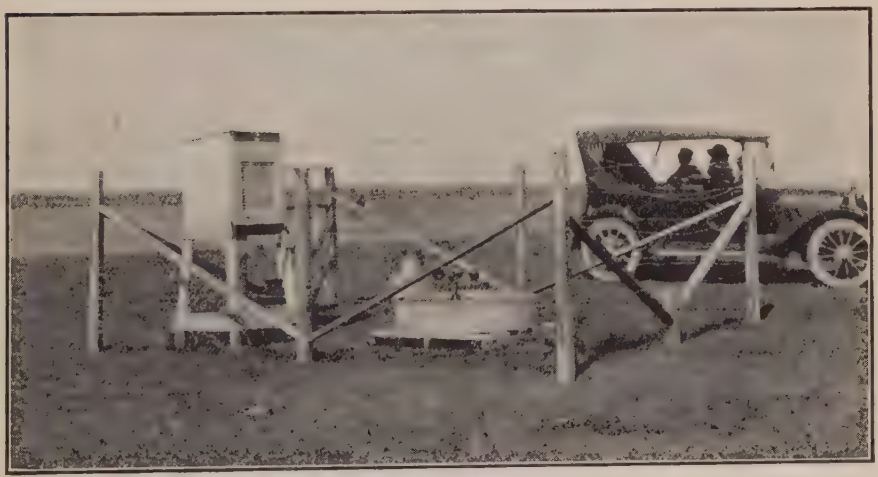

FIG. 23.—LAND EVAPORATION PAN AT NELSON RESERVOIR, MILK RIVER PROJECT, RECORD 29, TABLE 1.

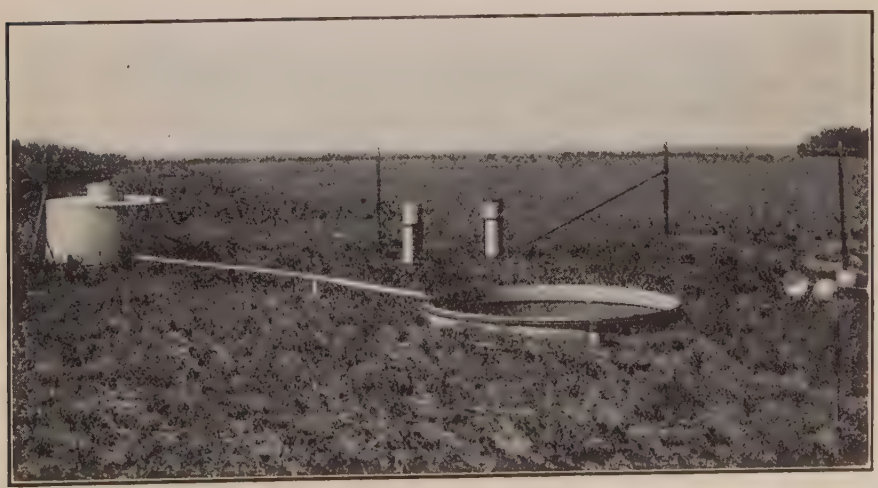

FIG. 24.—GROUND EVAPORATION PAN AT NEWELL EXPERIMENT FARM, BELLE FOURCHE

No information is at hand regarding the setting of the pan at the Hermiston Experiment Farm Station on the Umatilla Project of Oregon. However, since this station is maintained by the Bureau of Public Roads, U. S. Department of Agriculture, and since the pan is its standard 24-in. by 6-ft. circular type, it is quite likely that it is set in the ground 21 in., in accordance with the usual custom.

Figs. 22, 23, and 24 show the submergence, mounting, and exposure conditions at some of the stations. It was not considered necessary to give complete mounting details for all the Class A installations, inasmuch as the specified mounting for such stations is clearly shown in *Circular L* of the Instrument Division, U. S. Weather Bureau, previously mentioned.

Table 14, presented by Mr. Torpen, gives valuable evaporation and related meteorological data for Lake Cushman, near Tacoma, Wash. One of the unusual features of the table is the column of water temperatures. It is interesting to note that during the late summer and fall months the water temperatures reached values considerably higher than the air temperatures—as much as 12° higher in August and 10° higher in September. Such data are desirable, but unfortunately are seldom taken except in laboratory work. Mr. Lee gives some interesting and valuable water temperature comparisons for floating and land-pan installations at Lake Almanor, California (Figs. 19, 20, and 21), but does not include air temperatures. His assumptions regarding the temperature data in Table 1, are correct. The data are all air temperatures, based on maximum and minimum readings of instruments housed in standard U. S. Weather Bureau thermometer shelters. Occasional water temperature readings have been taken at some of the stations, but the records were not comprehensive enough to warrant their inclusion in the table; although it may be interesting to note, in connection with Mr. Lee's Fig. 20, that observations at the Lake Tahoe floating-pan station, Newlands Project, showed water temperatures in the pan from 1° to 4° colder than the river water early in the morning, and from 1° to 4° warmer in the evening.

Fig. 25 shows temperatures of the water surface in Marston Lake, about ten miles from Denver, Colo., and mean air temperatures at the U. S. Weather Bureau Station in Denver, for the period from April to September, inclusive, 1924. Marston Lake is one of Denver's impounding reservoirs for its municipal water supply. It has a capacity of about 19 800 acre-ft., when full, an area of about 650 acres, and an average depth of about 30 ft. The water temperatures were secured through the courtesy of D. D. Gross, Chief Engineer of the Denver Board of Water Commissioners. Data on the exact times of observation are not at hand, but the temperatures were probably measured during the warmer part of the day. It will be noticed from Fig. 25 that the water surface temperatures follow the air temperatures very closely. Individual readings vary from about 10° below the air temperatures to about 10° above, while the monthly mean water temperatures do not differ from the mean air temperatures by more than 1 or 2 degrees.

Fig. 26 compares air and water temperatures at the Los Griegos evaporation station near Albuquerque, N. Mex., for the period from September 14 to

October 23, 1926. This station was established in September, 1926, for the purpose of measuring evaporation from soil and water surfaces in the Middle Rio Grande Valley, the data being needed for studies of water supply available for irrigation. The station is equipped with a Class A evaporation pan, a ground pan for measuring water surface evaporation, five soil evaporation tanks, maximum and minimum thermometers, a rain gauge, and an anemometer. The ground pan is 4 ft. in diameter, 2 ft. deep, and is set in the ground with the rim projecting about 2 in. The depth to ground-water outside the pans varied from 1.1 to 1.7 ft. during the months of September and October. Observations were made at about 5:00 P. M. daily; consequently, the observed water temperatures approximate the maximum daily values rather than the mean daily values.[3] Temperatures of the water in the ground pan were slightly higher than the ground-water temperatures, but lower than the Class A pan temperatures, as would be expected. Class A pan temperatures were nearly as high as the maximum air temperatures.

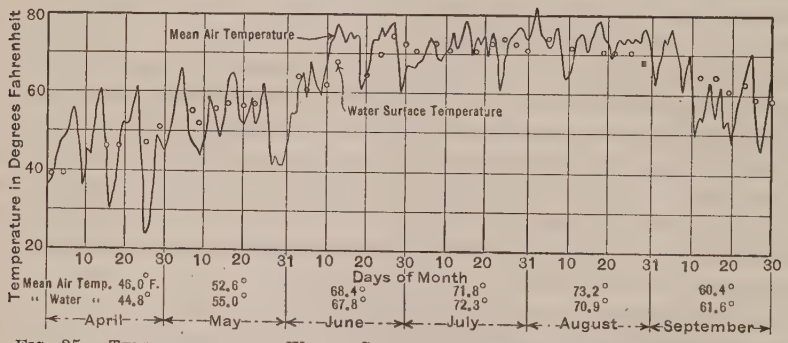

FIG. 25.—TEMPERATURES OF WATER SURFACE IN MARSTON LAKE, NEAR DENVER, COLO., AND MEAN AIR TEMPERATURES AT DENVER, APRIL TO SEPTEMBER, INCLUSIVE, 1924.

The necessity for observing water temperatures in connection with evaporation measurements depends on the type of station and the purpose for which it is maintained. At Class A land-pan stations, as well as at land-pan stations where shallow pans are set in the ground, it is doubtful whether the mean temperature of the water in the pan varies appreciably from the mean air temperature, at least as far as monthly averages are concerned. Consequently, it is doubtful whether water temperature measurements at such stations are necessary, although, of course, the data would furnish interesting information. It is desirable to observe water temperatures periodically at all floating-pan stations, timing the observations in such a manner as to obtain mean daily values. However, the measurement of maximum and minimum air temperatures should not be dispensed with at floating-pan stations; both air and water temperatures should be observed. If the floating or land-pan station is installed primarily as an experimental plant, for the purpose of investigating evaporation laws, it is especially important to obtain accurate data on both air and water temperatures, as well as on all other affecting meteorological phenomena.

Mr. Lee furnishes a very interesting discussion of the effect of specific gravity on the rate of evaporation. His Fig. 18 shows conclusively that the presence of alkaline salts in appreciable proportions results in a marked decrease in evaporation. The writer does not recall ever having seen this phase of evaporation mentioned, although it is a matter which doubtless should be considered in studying evaporation from alkali swamp lands such as exist in many parts of Western United States. Mr. Lee also includes a very interesting treatment of evaporation from capillary films, evaporation from moist crystal deposits, consumptive use of irrigation water, plant transpiration, and soil evaporation accompanying growth of vegetation. These matters are really phases of evaporation, using this term in its broadest sense; just as the evaporation from water surfaces is one phase of the general subject of evaporation. The U. S. Bureau of Reclamation has been collecting data on some of these matters for several years, particularly on the consumptive use of irrigation water; but the information has never been correlated in one report, in shape suitable for publication, and consequently cannot be included in this discussion.

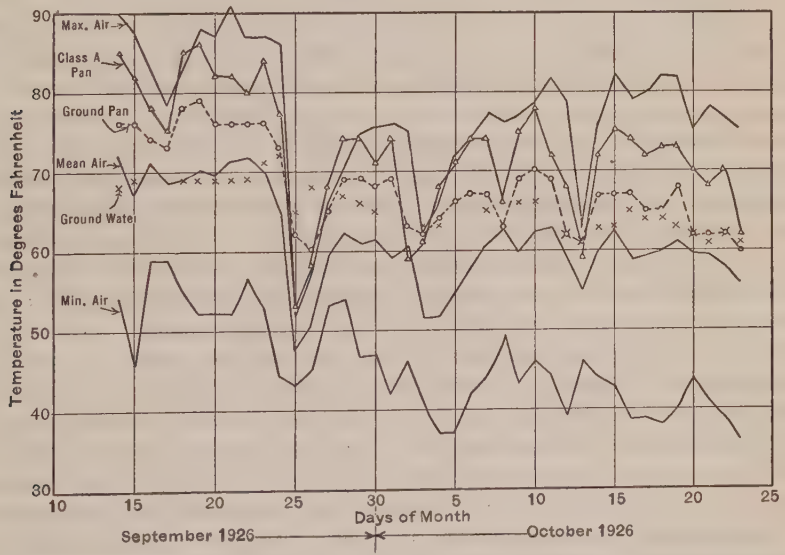

FIG. 26.—COMPARISON OF AIR AND WATER TEMPERATURES AT LOS GRIEGOS EVAPORATION STATION, NEAR ALBUQUERQUE, N. MEX.

It may be interesting to note that the difference between rainfall and run-off in some of the agricultural valleys of Eastern United States can be taken as an indication of the consumptive use of water. For instance, in the Miami Valley above Dayton, Ohio, an area including approximately 2 500 sq. miles, practically all of which is farm land producing paying crops each year, the average difference between annual rainfall and run-off for the 25-year period from 1894 to 1918, amounted to 25.20 in., or 2.10 ft.* The

* "Rainfall and Run-Off in the Miami Valley," by Ivan E. Houk, M. Am. Soc. C. E., Miami Conservancy Dist. Technical Repts., Pt. VIII, Dayton, Ohio, 1921.

twenty-five individual values varied from 1.54 ft. in 1913 to 3.17 ft. in 1896. However, individual values are subject to some uncertainty due to possible variations in ground-water storage. These figures include soil and snow evaporation during the winter months, which may amount to 3 or 4 in.

Tables 21 and 22, and Fig. 11, presented by Mr. Perry, give evaporation and related data for a ground pan maintained by the U. S. Navy Department at Christiansted, on the Virgin Islands. The data given in Table 21 are similar to those in Table 14, except that water temperatures and relative humidities are not included. The data in Table 22 show that the values of annual evaporation for the five years of record varied from 5.2% less, to 7.7% more, than the mean. Table 44, presented by Mr. Chandler, gives evaporation and temperature data for a floating-pan station maintained by the U. S. Geological survey at University, N. Dak., during the years 1905 to 1920. Differences between the maximum and minimum records for individual months, as shown by the records for different years, vary from 1.92 in. for September to 3.87 in. for June, values corresponding to 53 and 78% of the mean records for the respective months.

Mr. Jarvis mentions the matter of evaporation from shallow ponds, 2 or 3 in. deep, such as often occurs in irrigated fields following an irrigation, and suggests that under those conditions rates of evaporation may exceed the rates measured at either land or floating pans. The writer believes this to be true. However, he does not recall ever hearing of any evaporation measurements made under an environment approximating those conditions, although such experiments would be valuable and might easily be made at most experiment stations. It is to be hoped that practical investigations of this nature will be made some time, and that they will consider the effect of plant shade and wind protection in reducing the evaporation rate, as well as the maximum evaporation rates experienced in open ponds.

Both Mr. Follansbee and Mr. Meeker discuss the matter of converting pan-evaporation records into equivalent reservoir evaporation. They contribute tables of evaporation data, converted to reservoir equivalents on the basis of the coefficients determined by Sleight, at Denver, in 1916,* including in their tables records at some stations in Western United States not contained in Table 1. The writer has no criticisms to make regarding Mr. Sleight's excellent experimental work. However, in view of the fact that his coefficients have never been adequately confirmed by independent observations in other localities it seemed better to present the original records in Table 1, rather than converted values, so that engineers using the data can apply whatever coefficients they consider most applicable. Incidently, it might be mentioned that the U. S. Bureau of Reclamation is co-operating with the U. S. Bureau of Public Roads in measuring the actual evaporation at the East Park Reservoir on the Orland Reclamation Project of Northern California, and that floating and land-pan records are being obtained at the same time, so that the ratios of reservoir evaporation to pan evaporation can be determined. Although conclusive results are not available at this time, preliminary measure-

* *Journal of Agricultural Research*, Vol. X, No. 5, July 30, 1917.

ments seem to indicate that the ratio of reservoir evaporation to Class A pan evaporation at that particular reservoir is less than 0.66. The geological conditions are such that the quantity of water lost by seepage is believed to be negligible.

TABLE 45.—Comparison of Evaporation Coefficients Based on Different Lengths of Record at the U. S. Bureau of Public Roads Experiment Station, in Denver, Colo.

Tank No.	Description.	Evaporation in Percentage of Evaporation from Ground Tank 12 Feet in Diameter by 3 Feet Deep.		Differences.
		Records for 1915–1916.*	Records for 1915–1917.	
1.....	Ground tank, 1 ft. in diameter by 3 ft. deep.....	154.9	158.9	4.0
2.....	Ground tank, 2 ft. in diameter by 3 ft. deep.....	128.7	128.4	0.3
4.....	Ground tank, 3.31 ft. in diameter by 3 ft. deep.....	120.9	120.2	0.7
5.....	Ground tank, 6 ft. in diameter by 3 ft. deep.....	109.6	108.9	0.7
8.....	Land pan, Class A, 4 ft. in diameter by 10 in. deep ...	150.0	142.1	7.9
9.....	Floating pan, 3 ft. square by 1.5 ft. deep.....	108.9	112.5	3.6

* From *Journal of Agricultural Research*, Vol. X, No. 5, July 30, 1917.

Mr. Sleight furnishes a very interesting account of the experimental evaporation work carried on by the U. S. Bureau of Public Roads, at Denver, during the period from November, 1915, to September, 1917, including tabulated summaries of data for the entire period. It appears that the coefficients based on the complete records differ slightly from those based on the 1916 records.* In order to bring out this point Mr. Sleight has furnished the writer with the data in Table 45 and asked him to present it in his closing discussion. It will be noticed that the difference is greatest in the case of the U. S. Weather Bureau Class A pan; that it is appreciable in the case of Tanks Nos. 1 and 9 (the ground tank 1 ft. in diameter by 3 ft. deep and the floating pan 3 ft. square by 1½ ft. deep); and that it is insignificant in the case of the ground tanks, Nos. 2, 4, and 5 (tanks, 3 ft. deep, having diameters of 2, 3.31, and 6 ft., respectively). The data in Table 45 indicate the desirability of continuing evaporation experiments through a number of years rather than discontinuing them after records for one year or two years are obtained.

Regarding the ratios of ground-tank evaporation to Class A pan evaporation at Jackson Lake and Denver, discussed by Mr. Sleight, it is likely that the ground-pan evaporation at Jackson Lake was relatively lower than that at Denver due to a higher ground-water level. C. C. Elder, Assoc. M. Am. Soc. C. E., who installed the Jackson Lake ground pan and took the records during the years 1922 and 1923, states that the pan was in a swampy location about 6 ft. from a small drain channel, in which the water was

* "Evaporation from the Surfaces of Water and River-Bed Materials," by R. B. Sleight, Assoc. M. Am. Soc. C. E., *Journal of Agricultural Research*, Vol. X, No. 5, July 30, 1917.

always rather cold, probably never warmer than 50° Fahr. The pan was set in the ground to half its depth, and earth banked around the outside as high as the rim. It was filled with the comparatively cold drainage water. Since the ground-water was very close to the surface, sometimes at the surface, it must have had an appreciable cooling effect on the water within the pan. Somewhat similar conditions exist at the Los Griegos Evaporation Station, near Albuquerque, previously mentioned. Ratios of ground-pan evaporation to Class A pan evaporation at Los Griegos for the months of September, October, and November, 1926, were 79.0, 75.3, and 73.2%, respectively, averaging 75.8 per cent. During these months the depth to ground-water varied from about 1 ft. to 2 ft., being higher than the bottom of the pan the greater part of the time.

Mr. O'Neil furnishes a very interesting account of his evaporation investigations at the Lake Newell and Brooks Reservoirs, Alberta, Canada, including valuable tables of evaporation data, ratios of floating to land-pan evaporation, reservoir losses, etc. The ratios of floating to land-pan evaporation, given in Table 31, were determined at Brooks during the six years from 1919 to 1924. While their average value is 1.28, it is interesting to point out that the individual values vary from 1.08 to 1.40. This again indicates the desirability of continuing evaporation experiments through a number of years. Inasmuch as the land pan at Brooks was set in the ground it is not surprising that the differences between the land-pan and floating-pan evaporation should be less than the differences found at locations where Class A land-pan installations have been used. However, it is rather surprising that the floating-pan evaporation should be consistently higher than the land-pan evaporation.

The writer is inclined to believe that the actual winter losses, given in Table 30, may be fully accounted for by evaporation. They vary from 0.04 to 0.20 ft., or from 0.48 to 2.40 in., averaging 1.32 in. Such a total loss corresponds to a mean daily loss of 0.0147 in. for the 90-day period from December 1 to February 28, an amount which is not at all inconsistent with available data on evaporation from snow and ice surfaces. By referring to Table 4 it will be seen that the ice evaporation measurements on the Milk River Project in 1922 and 1923, showed average daily rates of 0.013, 0.010, and 0.015 in., respectively, for the months of December, January, and February. The average temperatures on the Milk River Project during January and February were lower than those for corresponding months at Lake Newell, given in Table 33. It might also be mentioned that observations in the Miami Valley of Southwestern Ohio, during the severe winter of 1917 and 1918, showed an average rate of snow evaporation of 0.023 in. per day for the period from December 3, 1917, to February 11, 1918.* During this period the average temperature was 19° Fahr., the mean wind velocity, 11.5 miles per hour, and the mean relative humidity, 83 per cent. R. E. Horton, M. Am. Soc. C. E., obtained an average rate of snow evaporation of 0.028 in. per

* "Rainfall and Run-Off in the Miami Valley" by Ivan E. Houk, M. Am. Soc. C. E., Miami Conservancy Dist. Technical Repts., Pt. VIII, Dayton, Ohio, 1921, p. 65.

day for the nine days from December 26, 1913, to January 4, 1914, at Albany, N. Y.,* when the mean maximum temperature was 26.6° Fahr.

Table 46 contains an abstract of snow and ice evaporation measurements made on the Milk River Project, of Northeastern Montana, since January, 1924. It is a continuation of Table 4. Measurements used in compiling Table 46 were made at Malta instead of Saco, the apparatus having been installed at Malta in November, 1923. Since that time evaporation readings have been taken through the summer as well as through the winter.

TABLE 46.—Ice and Snow Evaporation on the Milk River Project, Montana, During Winter Months, from 1924 to 1926.*

Period.	Number of days observed.	Total evaporation, in inches.	Average rate, in inches per day.	Equivalent, in inches per month.	Mean temperature, in degrees Fahrenheit.	Surface conditions.
1924:						
Jan. 1-31.	31	0.56	0.018	0.56	4	Snow and ice.
Feb. 14-26.	13	0.36	0.028	0.78	13	Ice.
Mar. 1-31.	31	1.43	0.046	1.43	26	Mostly ice, water around edges at times.
Dec. 15-31.	17	0.37	0.028	0.67	—8	Solid ice with 2 to 3 in. of snow on top.
1925:						
Jan. 1-17.	17	0.17	0.010	0.31	7	Solid ice with 1 to 3 in. of snow on top.
Jan. 18-23.	6	0.32	0.053	1.65	33	Ice with 1 to 3 in. of water around edge.
Jan. 24-29.	6	0.11	0.018	0.57	13	Solid ice, bare.
Feb. 11-28.	18	0.61	0.034	0.95	14	Solid ice with 1 to 2 in. of snow on top at times.
Mar. 9-17.	9	0.29	0.032	1.00	12	Solid ice with 2 to 4 in. of snow on top at times.
Mar. 18-22.	5	0.21	0.042	1.30	31	Ice with ½ in. to 2 in. of water around edge.
Nov. 22-30.	9	0.67	0.074	2.23	25	Ice with 1 to 2 in. of water around edge at times
1926:						
Jan. 1- 9.	9	0.84	0.093	2.89	20	Solid ice with 1 to 4 in. of snow on top.
Jan. 10-17.	8	0.72	0.090	2.79	30	Ice with ½ in. of water around edge.
Jan. 18-31.	14	0.52	0.037	1.15	19	Bare ice.
Feb. 13-21.	9	0.15	0.017	0.47	25	Ice.
Mar. 7-14.	8	0.31	0.039	1.20	34	Ice with 1 to 2 in. of water around edge.

* See Table 4 for earlier data.

G. E. Stratton, M. Am. Soc. C. E., Superintendent of the Milk River Project at the time the measurements were made, advises that in the winter of 1923 and 1924, and since about March 1, 1926, a semi-circular sheet of flume metal was placed around the small pan so as to protect its sides from the direct action of the sun during the greater part of the day. The water surface was kept about 8 in. below the rim during the winter months, so as to reduce freezing and bursting troubles as well as to provide space to catch the snowfall. During the summer months the water surface was kept about 2 in. below the rim.

The data in Table 46 show conclusively that evaporation continues at appreciable rates during the winter months, even when the temperatures are below zero. Daily rates vary from about 0.010 to about 0.093 in., for the periods included. While the values are somewhat erratic, as a general rule the higher rates correspond to the higher temperatures.

* "Evaporation from Snow and Errors of Rain Gage When Used to Catch Snowfall," by R. E. Horton, M. Am. Soc. C. E., *Monthly Weather Review*, February, 1914, p. 99.

Table 47 compares the evaporation measured in the small pan at Malta, with that measured in a Class A pan mounted near-by. The interesting point brought out by Table 47 is the effect of shading the sides of the small pan. During the months of August and September, 1924 and 1925, when the sides of the small pan were not shaded, the ratios of Class A pan evaporation to small pan evaporation, varied from 53.3 to 64.5%, averaging 59.3 per cent. During the summer of 1926, when the small pan was shaded, the corresponding ratios varied from 79.4 to 92.5%, averaging 85.3 per cent.

TABLE 47.—Ratios of Class A Pan Evaporation to Small Pan Evaporation at Malta, Montana, 1924 to 1926.

Month.	Year.	Evaporation, in Inches.		Ratio: Class A pan to small pan, in percentage.	Average ratios, in percentage.	Remarks.
		Class A pan.	Small pan.			
August	1924	6.85	10.62	64.5	....	Sides of small pan not shaded.
September	1924	5.61	9.18	61.1	....	
August	1925	8.70	14.94	58.2	....	
September	1925	4.43	8.31	53.3	59.3	
May	1926	7.54	8.48	88.9	....	Sides of small pan shaded by sheet of metal flume.
June	1926	8.36	10.52	79.4	....	
July	1926	10.21	12.71	80.3	....	
August	1926	7.84	8.47	92.5	85.3	

Professor Harding and Mr. Lippincott discuss the matter of actual reservoir evaporation and its relation to pan records, submitting data secured on California lakes as a basis for their discussions. Professor Harding submits data for Tulare Lake, Buena Vista Lake, and Lake Elsinore; while Mr. Lippincott submits data for these three and several additional reservoirs, summarizing his observations in Table 43. In some recent correspondence with Professor Harding the writer was advised that the evaporation data in Tables 23, 25, and 26 have been corrected for precipitation, that no appreciable quantity of aquatic growth was present in any of the three lakes, and that the land pan at Buena Vista Lake was set in the ground to within 3 in. of the rim, the water surface in the pan being kept 3 in. below the top. The slight differences in evaporation data for Tulare Lake, as given in Tables 23 and 36, are probably due to slightly different methods in working up the basic data. For instance, one man may have used some records which the other had discarded.

Mr. Parshall contributes an interesting account of the comprehensive evaporation measurements carried on by the Bureau of Public Roads, U. S. Department of Agriculture, at its laboratory in Fort Collins, Colo. Apparently, the measurements were made with unusual care and accuracy. The ingeniously devised optical evaporimeter is especially worthy of mention.

A rather hasty study of Figs. 16 and 17 seems to disclose an unusual confirmation of the Dalton theory. In both cases the observed evaporation is seen to vary directly with the quantity, $(E_s - E_d)$, which represents the difference between the vapor pressure corresponding to the temperature of the

water surface and the actual vapor pressure in the air immediately above. Mr. Parshall calls attention to this in his statement that,

"During this period the temperatures of air and water were decreasing, as was also the rate of evaporation, which is contrary to the findings inside the laboratory under still air conditions; however, in both cases the rate of loss is directly proportional to the difference in vapor pressures."

Mr. Parshall gives two formulas for computing evaporation; one for still air conditions, such as existed in the laboratory when the observations platted in Fig. 16 were taken, the other for windy conditions, such as existed outside when the measurements recorded in Fig. 17 were made. It is understood that both equations are tentative since he states that "inasmuch as the study of this subject has not been fully developed, certain minor alterations or combinations may later permit closer agreement with the observed rate of loss." In studying the formulas one notices that in both cases the computed evaporation varies with the quantity, $(E_s - E_d)$, that in the second case a wind factor is introduced, and that in the first case a function of the quantity, $(T_a - T_s)$, the temperature of the air 1 in. above the water surface minus the temperature of the water surface, is introduced.

There is a certain theoretical basis for the variation of evaporation with the quantity, $(E_s - E_d)$, aside from the observational basis shown by Figs. 16 and 17. The introduction of a wind factor of some kind to take care of the wind effect is logical. However, the writer would be inclined to question the introduction of the function of $(T_a - T_s)$, even for purely local laboratory conditions. It would seem that the effect of the difference in temperature of the air and water could be taken care of by some function of the difference in vapor pressures, since the latter quantity is dependent on the air and water temperatures and the relative humidity. Moreover, it would hardly seem probable that the law of evaporation inside the laboratory differs from that outside. It would seem more logical to have one formula for both conditions, and to obtain the evaporation rate for still air by the simple expedient of making the wind factor equal zero. If this is done in the second formula the equation becomes $E = 0.44\ (E_s - E_d)$, while if T_a is assumed equal to T_s the first formula becomes $E = 0.166\ (E_s - E_d)$. Values of evaporation computed by the latter equation would be only about 38% as large as those computed by the former, for the same meteorological conditions. However, the matter is only of academic interest to engineers, inasmuch as their problems do not necessitate the calculation of evaporation within a laboratory. They are concerned solely with the second formula, the equation for outdoor evaporation.

In the second formula the wind correction factor, $(0.44 + 0.118\ W)$, is a straight-line function of the wind velocity. In other words, its value increases directly with the wind velocity as long as that velocity increases, thus indicating an increase in the rate of evaporation with an increase in wind velocity regardless of the amount of the wind. While such a correction may be satisfactory for the range in wind velocity of from 0 to 12 miles per hour experienced at Fort Collins, the writer would not expect it to hold for indefi-

nite increases in velocity. As far as a general evaporation formula is concerned he would be inclined to favor some wind correction term along the lines recommended by Mr. R. E. Horton.*

Practising engineers must depend on experimental laboratory work, of such nature as that carried on at Fort Collins, to determine the physical laws governing evaporation phenomena. What engineers need most at the present time, as regards evaporation, is a general formula in which they can insert mean values of temperature, wind velocity, and relative humidity, observed at U. S. Weather Bureau stations, and calculate reliable values of reservoir evaporation for exposed areas of different sizes.

* *Engineering News-Record*, April 26, 1917.

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Paper No. 1601

STRESSES IN HELICALLY REINFORCED CONCRETE COLUMNS*

By A. W. Zesiger,† M. Am. Soc. C. E., and E. J. Affeldt,‡ Assoc. M. Am. Soc. C. E.

With Discussion by Messrs. Missac Thompson, C. L. Eddy, E. H. Harder, J. H. Griffith, John Tucker, Jr., Jacob Feld, and A. W. Zesiger and E. J. Affeldt.

Synopsis

The object of this paper is to analyze the effect of helical reinforcement on the concrete core of a helically reinforced concrete column, and to develop formulas for the stresses which obtain in the steel and concrete with varying percentages of reinforcement.

As the available tests on helically reinforced concrete columns are comparatively limited in number and have been made under such widely different conditions (as regards values of $\frac{l}{r}$, quality of materials, age of concrete, etc.) as to render their co-ordination extremely difficult, no claim is made for the absolute accuracy of the empirical constants hereinafter given. This paper is presented more with the idea of indicating the lines along which further research would be highly beneficial than as the final word on the subject. As a result of careful study of the available data, the writers believe, however, that the method of analysis proposed is substantially correct and that the following conclusions are justified:

(1) Plain concrete columns, of lengths up to at least $80\frac{l}{r}$, fail in shear rather than in compression.

* Published in January, 1926, *Proceedings*. The word, helical, is used instead of the commonly employed term, spiral, because a spiral lies in one plane only and is, therefore, not the proper word to use in describing the system of reinforcement which will be discussed in this paper.

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(2) Before a plain concrete column can fail in shear, the imposed load must be sufficient to overcome the shearing resistance of the concrete and the frictional resistance of the material along the impending plane of rupture.

(3) The frictional resistance of the material is a more important factor than is commonly assumed.

(4) Helical reinforcement is effective in resisting a shearing failure of the concrete core and, if sufficient reinforcement is used, it is possible to preclude a shearing failure of the concrete.

(5) Helical reinforcement has little effect in preventing crushing of the concrete core of the column.

(6) Three types of failure may be observed in helically reinforced concrete columns, tested to destruction, the type of failure depending on the amount of helical reinforcement:

a.—When only a small amount of reinforcement is used (up to about 1%) failure of the column is due to the shearing resistance of the concrete and the frictional resistance of the material, to incipient motion along the impending plane of rupture, being exceeded.

b.—With larger percentages of reinforcement (from about 1% to about 1.6%) failure of the column is no longer due to shear, but to crushing of the concrete.

c.—With reinforcement in excess of about 1.6% the column will not fail, even after the concrete has crushed, until the reinforcement is stressed considerably beyond its elastic limit and possibly, in some cases, almost to its ultimate strength.

(7) Even if the concrete shell outside the reinforcement, in a helically reinforced column, cracks and spalls off under a load only little greater than is necessary to cause failure in an unreinforced column, of the same dimensions and quality of concrete, the reinforcement tends to neutralize the effects of local imperfections in the concrete, thus increasing the reliability of the column. Furthermore, unlike the plain concrete column which usually fails suddenly, warning of impending failure is given in the reinforced column by the cracking and spalling of the outer shell of concrete. It is permissible, therefore, to allow a lower factor of safety and correspondingly increased working stresses in helically reinforced columns.

(8) A rational analysis of the stresses in the steel and concrete of a helically reinforced concrete column is possible, contrary to the opinion of many engineers, and the resulting equations are very simple.

(9) There is need of further research work on helically reinforced concrete columns in order to evaluate more accurately the constants in the rational formulas.

Although this paper is primarily concerned with the analysis of helically reinforced concrete columns, an analysis of plain concrete columns is essential to an understanding of reinforced columns. It seems necessary, therefore, to consider, first, the unreinforced column with the object of ascertaining the relations existing therein, between stress and imposed load, and then to

determine wherein these relations are modified by the introduction of reinforcement.

Authorities seem agreed that, for values of $\frac{l}{r}$ up to about 80, concrete columns may be assumed to act as short struts in which the effect of lateral flexure need not be considered. Furthermore, columns having a ratio of $\frac{l}{r}$ in excess of 80 are seldom used in practice. It will be assumed, therefore, in the following discussion, that the columns are sufficiently short to preclude the necessity of considering lateral flexure.

I.—Plain Concrete Columns

Any theoretical analysis of concrete columns is of value only to the extent that it explains the phenomena observed in tests, and it is essential to the development of a rational theory that the manner in which the column actually fails under load be determined. Upon this point there is a reported disagreement in tests, and it has been commonly accepted that plain concrete columns may fail either in shear or by a gradual crushing of the material—the type of failure depending on the quality of the concrete.

In the writers' opinion, there is a greater disagreement in the reported results than in actual tests. The majority of available tests of plain concrete columns disclose an unmistakable shearing failure, and it is probable that, in a large proportion of the columns where failure has been ascribed to crushing of the concrete, a more careful diagnosis would have shown that the column failed in shear, a contention which is substantiated in many instances by photographs of the test specimens taken after failure. Moreover, a large proportion of the so-called crushing failures have occurred in columns composed of concretes too lean for actual construction work, and are not a fair criterion. It seems reasonable, therefore, to assume that the average unreinforced concrete column, constructed of a grade of concrete ordinarily used in good practice, will fail in shear, a crushing failure being an exception which is due to some pronounced defect in materials or workmanship.

The resistance which an unreinforced concrete column offers to a shearing failure consists of: (1) the shearing strength of the concrete; and (2) the frictional resistance of the material along the impending plane of rupture. If failure of the column was resisted solely by the shearing strength of the material the plane of rupture would make an angle of 45° with the horizontal, as the shearing component of the imposed load is a maximum along a plane having this inclination. Tests, however, show conclusively that the actual plane of rupture is inclined considerably more than 45° with the horizontal and the difference in inclination can only be ascribed to the frictional resistance of the material.

Until recently, writers, even Rankine, have accepted the erroneous theory of Coulomb wherein friction is neglected and the plane of rupture is assumed to make an angle of 45° with the horizontal, despite the fact that Navier has given the true analysis of granular substances in compression. Although

engineers now admit the existence of friction in concrete, for some inexplicable reason they usually fail to give it proper consideration, as is evidenced by the frequent discussions regarding the cube *versus* the cylinder as the proper criterion of the compressive strength of a concrete column. The difference between the results obtained with cubical and cylindrical test specimens (having a height of 2 to 3 diameters) is fundamentally due to the effect of friction, a conclusion that may be inferred from a careful study of current literature, pertinent to the subject, which has not been clearly and concisely stated. If the true significance of frictional resistance in the concrete was properly appreciated the fallacy of considering the cube as a criterion of the compressive strength of a column would be so self-evident as to be almost axiomatic and there would be no need of any discussion.

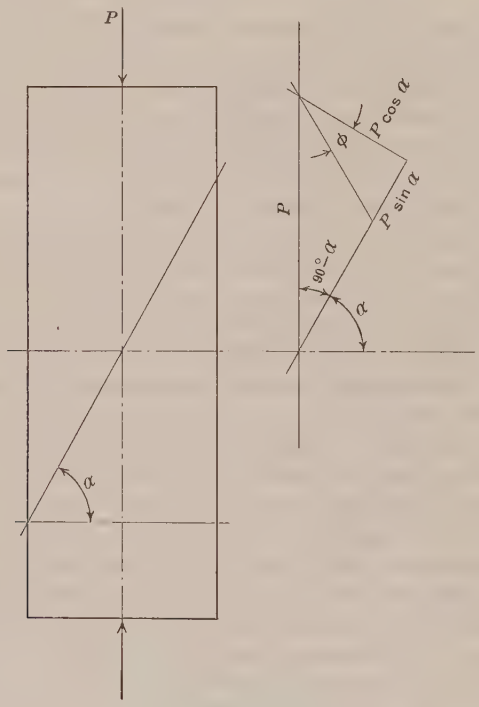

FIG. 1.

Tests show conclusively that the compressive strength developed by a cube exceeds that of a cylinder (having a height of 2 to 3 diameters), and that the results of cylinder tests agree reasonably close with those on actual columns of the same concrete, which is just what theory leads one to expect. Manifestly, the steepest plane of rupture, which can intersect the entire cross-section of a cube, has an inclination of 45° with the horizontal. Inasmuch as the plane of rupture concomitant with a shearing failure has a much steeper inclination, it is obviously impossible for a cube to fail in pure shear, and it is constrained, therefore, to fail in combined shear and crushing. However, in a cylinder having a height of 2 to 3 diameters the entire cross-section can be

intersected by a plane of rupture, having the inclination requisite to a true shearing failure, and the cylinder, therefore, will fail in shear alone. In other words, the difference in the mode of failure of cubes and cylinders and the discrepancy in compressive strength consequent thereon is due to the additional inclination of the plane of rupture, concomitant with a true shearing failure, which is introduced by the frictional resistance of the concrete.

Knowing the manner in which a plain concrete column fails, under imposed load, and the factors which tend to resist this failure, it becomes a simple matter to deduce formulas for the stress in the concrete and the inclination of the plane of rupture.

Referring to Fig. 1, let,

P = the imposed load, in pounds.

A = the cross-sectional area of the column, in square inches.

$f_c' = \frac{P}{A}$ = the unit compressive stress in the concrete, in pounds per square inch.

f_z = the unit shearing stress in the concrete, in pounds per square inch.

α = the angle which the plane of rupture makes with a normal to the axis of the column.

ϕ = the friction* angle of the concrete.

The component of P, which tends to cause sliding along the plane of rupture, is $P \sin \alpha$. This tendency to slide is resisted by $f_z A \sec \alpha + P \cos \alpha \tan \phi$, in which, $\tan \phi$ equals the coefficient of friction. Hence, for equilibrium,

$$P \sin \alpha = f_z A \sec \alpha + P \cos \alpha \tan \phi \quad \ldots\ldots (1)$$

or,

$$f_z = \frac{P}{A} (\sin \alpha \cos \alpha - \cos^2 \alpha \tan \phi) \quad \ldots\ldots (2)$$

Rupture of the column will manifestly occur along that plane on which f_z is a maximum ; consequently, at failure, $\frac{d f_z}{d \alpha} = 0$, whence,

$$\alpha = 45° + \frac{1}{2} \phi \quad \ldots\ldots (3)$$

From Equation (3) it is seen that the angle of rupture is 45° plus one-half the friction angle.

Substituting $\left(45° + \frac{1}{2} \phi\right)$ for α and $-\cot (90° + \phi)$ for $\tan \phi$ in Equation (2) :

$$f_z = \frac{1}{2} \frac{P}{A} \cot \left(45° + \frac{1}{2} \phi\right) \quad \ldots\ldots (4)$$

and,

$$f_c' = \frac{P}{A} = 2 f_z \tan \left(45° + \frac{1}{2} \phi\right) \quad \ldots\ldots (5)$$

* The term, friction angle, is used for lack of a better one. Strictly speaking, ϕ is an angle such that its tangent is equal to the coefficient of the friction in the concrete.

When the eccentricity of the imposed load is considerable, due either to the load not being applied along the axis of the column or to the non-homogeneity of the concrete, the maximum compressive stress on the extreme fiber may greatly exceed the unit compressive stress which, if uniformly distributed, would cause the column to shear across its entire section. Failure of the column would then be due to the progressive crushing or shearing of the extreme fibers. However, a shearing failure of the extreme fiber can not take place until a section in the shape of a wedge is sheared out, and this can not occur until the compressive stress on the extreme fiber becomes great enough to overcome the shearing strength of the concrete and the frictional resistance on the two faces of the wedge.

The angle of rupture, which is concomitant with the foregoing type of failure, may be determined as follows:

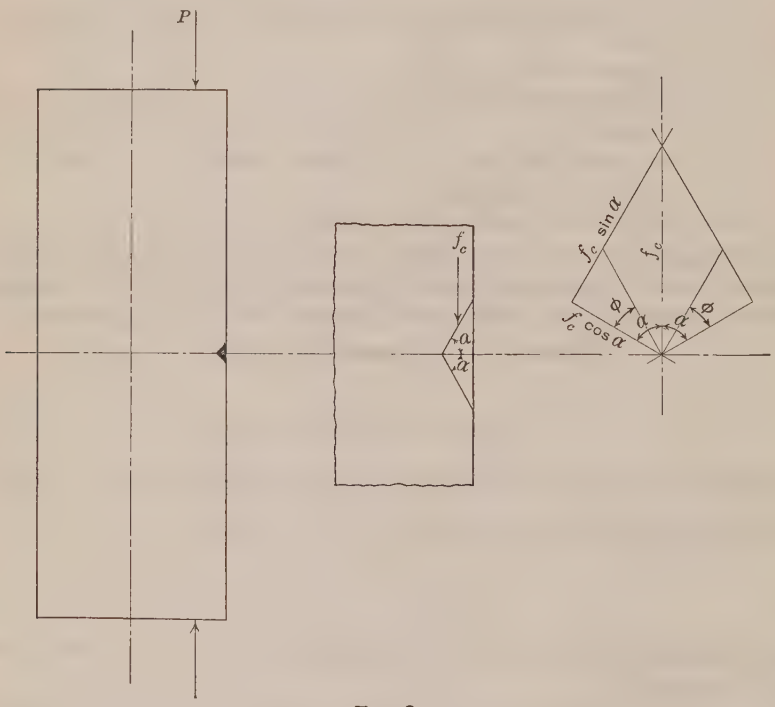

FIG. 2.

Referring to Fig. 2, let,

f_c = the maximum unit compressive stress on the extreme fiber, in pounds per square inch.

f_s = the unit shearing stress in the concrete, in pounds per square inch.

α = the angle which the two planes of rupture make with a normal to the axis of the column.

ϕ = the friction angle of the concrete.

The component of f_c, which tends to cause sliding along the two planes of rupture, is $f_c \sin \alpha$. This tendency to slide is resisted by $f_z \sec \alpha + 2 f_c \cos \alpha \tan \phi$. Hence, for equilibrium,

$$f_c \sin \alpha = f_z \sec \alpha + 2 f_c \cos \alpha \tan \phi \quad \ldots\ldots(6)$$

or,

$$f_z = f_c (\sin \alpha \cos \alpha - 2 \cos^2 \alpha \tan \phi) \quad \ldots\ldots(7)$$

As rupture will occur along the planes on which f_z is a maximum, it follows, at rupture, that $\frac{d f_z}{d \alpha} = 0$; whence,

$$\cot 2 \alpha = -2 \tan \phi = 2 \cot (90^\circ + \phi) \quad \ldots\ldots(8)$$

Substituting $\cot 2 \alpha$ for $-2 \tan \phi$ in Equation (7),

$$f_z = \frac{1}{2} f_c \cot \alpha \quad \ldots\ldots(9)$$

or,

$$f_c = 2 f_z \tan \alpha \quad \ldots\ldots(10)$$

The relation between the unit compressive stress which will cause a shearing failure across the entire section of the column and that which will shear out a wedge in the extreme fiber may be determined, for any given value of ϕ, as follows: Dividing Equation (10) by Equation (5) and solving for f_c:

$$f_c = f_c' \frac{\tan \alpha}{\tan \left(45^\circ + \frac{1}{2} \phi\right)}$$

If $\phi = 45^\circ$ (a commonly used value for concrete), $\alpha = 76^\circ 43'$ and $f_c = 1.75 f_c'$. In other words, the unit compressive stress which will cause failure by shearing out a wedge in the extreme fiber must be at least 1.75 times the unit compressive stress which will cause failure by shearing the entire section of the column.

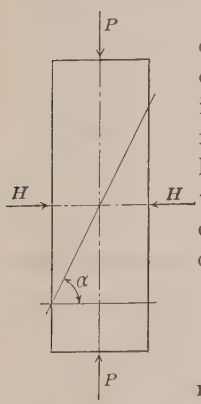

FIG. 3.

Having considered the plain concrete column, the analysis of the helically reinforced column may now be undertaken in order to ascertain wherein the relations between stress and imposed load which obtain in the unreinforced column are modified by the lateral restraint which is introduced by helical reinforcement. Helically reinforced concrete columns without longitudinal steel will be considered first, and the effect of longitudinal reinforcement will be discussed subsequently.

II.—Helically Reinforced Concrete Columns

Tests indicate that helically reinforced concrete columns may fail in any one of three ways, the type of failure depending on the quantity of reinforcement. With steel up to about 1%, the column will fail in shear the same as a plain concrete column. With steel in excess of 1% and up to about 1.6%, the lateral restraint of the reinforcement is sufficient to preclude a shearing failure and the column fails

through crushing of the concrete. With reinforcement in excess of about 1.6%, the column will not fail, even after the concrete has crushed, until the helical reinforcement has been stressed considerably beyond its elastic limit, causing undue elongation or rupture and thereby permitting the crushed concrete to slide.

The analysis of the helically reinforced concrete column, without longitudinal steel, is as follows:

Referring to Fig. 3, let,

P = the imposed load, in pounds.
H = the horizontal thrust, in pounds, supplied by the helical reinforcement.
v = the unit vertical pressure in the concrete, due to P.
h = the unit horizontal pressure in the concrete, due to H.
v_t = the unit tangential component of v along a plane making an angle, α, with a plane normal to the axis of the column.
v_n = the unit normal component of v on the plane mentioned.
h_t = the unit tangential component of h along the plane mentioned.
h_n = the unit normal component of h on the plane mentioned.
f_z = the unit shearing stress in the concrete along the plane mentioned.
α = the angle which the plane of rupture (for a shearing failure) makes with a normal to the axis of the column.
ϕ = the friction angle of the concrete.
D = the diameter of the helix, in inches.
a = the cross-sectional area, in square inches, of the steel composing the helix, per unit of column length.
f_s = the unit stress in the helical reinforcement.
e_v = the unit vertical deformation, in inches, of the column.
e_h = the unit horizontal deformation, in inches, of the column.
$\frac{1}{m}$ = Poisson's ratio.
E_s = the modulus of elasticity of steel.
E_c = the modulus of elasticity of concrete.
$n = \frac{E_s}{E_c}$
p' = the ratio of helical reinforcement.

The following equations are fundamental and, of course, require no explanation:

$$v_t = v \sin \alpha \cos \alpha$$
$$v_n = v \cos^2 \alpha$$
$$h_t = h \sin \alpha \cos \alpha$$
$$h_n = h \sin^2 \alpha$$

The tendency to slide on the plane of rupture is $v_t - h_t$. The resistance to sliding is $f_z + v_n \tan \phi + h_n \tan \phi$. Hence, for equilibrium,

$$v_t - h_t = f_z + (v_n + h_n) \tan \phi \quad \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

or,

$$f_z = v_t - h_t - (v_n + h_n) \tan \phi$$
$$= (v - h) \sin \alpha \cos \alpha - v \cos^2 \alpha \tan \phi - h \sin^2 \alpha \tan \phi \ldots\ldots (12)$$

Failure of the column, through shearing, will occur along that plane on which f_z is a maximum; therefore, at failure, $\frac{d f_z}{d \alpha} = 0$; whence,

$$\alpha = 45° + \frac{1}{2} \phi \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

From Equation (13) it is seen that, for a shearing failure, the inclination of the plane of rupture in a helically reinforced column, like that of a plain concrete column, is 45° plus one-half the friction angle.

In the analysis of the plain concrete column it was found, as shown by Equation (4), that the unit shearing stress is:

$$f_z = \frac{1}{2} \frac{P}{A} \times \cot \left(45° + \frac{1}{2} \phi\right)$$

The unit shearing stress in a helically reinforced column is:

$$f_z = (v - h) \sin \alpha \cos \alpha - v \cos^2 \alpha \tan \phi - h \sin^2 \alpha \tan \phi$$

as shown in Equation (12). For equal unit shearing stresses in the two types of columns these expressions become equal; whence,

$$\frac{1}{2} \frac{P}{A} \cot \left(45° + \frac{1}{2} \phi\right) = (v - h) \sin \alpha \cos \alpha - v \cos^2 \alpha \tan \phi$$
$$- h \sin^2 \alpha \tan \phi \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

In this manner, the unit shearing stresses may be eliminated and a relation established between the concomitant unit compressive stresses. The unit compressive stress that will cause a shearing failure in a plain concrete column, may readily be determined from cylinder tests of the same grade of concrete and the concomitant value of f_z calculated by Equation (4). At failure, the shearing stress in the concrete becomes equal to its shearing strength, which is constant for any given concrete whether used in plain or reinforced columns. Consequently, having determined the unit compressive stress concomitant with shearing failure in a plain concrete column, it becomes a simple matter to determine the corresponding unit compressive stress in a helically reinforced column, by the relation between $\frac{P}{A}$ and v expressed in Equation (14). In a similar manner, the unit compressive stress, concomitant with any given shearing stress in the helically reinforced column, may be expressed in terms of the unit compressive stress, which will produce the same unit shearing stress in a plain concrete column; or, letting f_c' equal the unit compressive stress which will produce any given unit shearing stress in a plain concrete column:

$$\frac{1}{2} f_c' \cot \left(45° + \frac{1}{2} \phi\right) = (v - h) \sin \alpha \cos \alpha - v \cos^2 \alpha \tan \phi$$
$$- h \sin^2 \alpha \tan \phi \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

Before Equation (15) can be solved for v, it is necessary to express h in terms of v. This may be accomplished by means of the relations which exist between the vertical and lateral deformations, and between the lateral deformation and the stress in the helix. It is shown in treatises on structural mechanics that,

$$e_v = \frac{v}{E_c} - \frac{2\,h}{m\,E_c} = \frac{1}{E_c}\left(v - \frac{2}{m}\,h\right) \quad \ldots\ldots(16)$$

and,

$$e_h = -\frac{h}{E_c} + \frac{h+v}{m\,E_c} = \frac{1}{E_c}\left(\frac{h+v}{m} - h\right) \quad \ldots\ldots(17)$$

In terms of the stress which it causes in the helix:

$$h = \frac{1}{D}\,(2\,a\,e_h\,E_s) = \frac{1}{2}\,p'\,e_h\,E_s \quad \ldots\ldots(18)$$

or,

$$e_h = \frac{2\,h}{p'\,E_s} \quad \ldots\ldots(19)$$

Whence, from Equation (17):

$$h = \frac{v\,n\,p'}{2\,m + (m-1)\,n\,p'} \quad \ldots\ldots(20)$$

Substituting the value of h from Equation (20) and the value of α from Equation (13), in Equation (15), and solving for v:

$$v = \frac{f_c'}{1 - \dfrac{n\,p'\tan^2\left(45^\circ + \dfrac{1}{2}\,\phi\right)}{2\,m + (m-1)\,n\,p'}} \quad \ldots\ldots(21)$$

When f_c' represents the unit vertical compressive stress required to produce a shearing failure in a plain concrete column, the corresponding stress, for a helically reinforced column, is represented by v in Equation (21). From Equation (21), it can readily be seen that v exceeds f_c' thus indicating the value of helical reinforcement in resisting a shearing failure. As the percentage of reinforcement increases, the denominator of the right-hand member of Equation (21) decreases, with a corresponding increase in the value of v. If the reinforcement is sufficient, v will become equal to the crushing strength of the concrete, in which case the column will no longer fail in shear, but by crushing of the concrete.

If v_c equals the compressive stress in plain concrete and e_v equals the corresponding deformation, Equation (16) becomes:

$$e_v\,E_c = v_c = v - \frac{2}{m}\,h = v\left\{1 - \frac{2\,n\,p'}{m\,[2\,m + (m-1)\,n\,p']}\right\} \quad \ldots\ldots(22)$$

Whence,

$$v = \frac{v_c}{1 - \dfrac{2\,n\,p'}{m\,[2\,m + (m-1)\,n\,p']}} \quad \ldots\ldots(23)$$

When the helical reinforcement is sufficient to preclude a shearing failure of the column, the vertical unit compressive stress, required to cause failure of the column by crushing, may be determined by Equation (23), when v_c represents the crushing strength of the same grade of plain concrete.

If the material within the helix is a granular mass, such as crushed concrete, the resistance to tangential movement or sliding is represented by Equation (12), in which f_z is now the resistance to sliding along the assumed plane of rupture. As failure will occur along that plane on which f_z is a minimum, $\frac{df_z}{d\alpha} = 0$; whence,

$$\alpha = 45° + \frac{1}{2}\,\phi \qquad (24)$$

When the resistance to sliding is overcome, Equation (12) becomes equal to zero ; whence, if $\alpha = 45° + \frac{1}{2}\,\phi$:

$$v = h \tan^2 \left(45° + \frac{1}{2}\,\phi\right) \qquad (25)$$

In terms of the stress in the helix, however:

$$h = \frac{1}{2}\, f_s\, p' \qquad (26)$$

Whence,

$$v = \frac{1}{2}\, f_s\, p' \tan^2 \left(45° + \frac{1}{2}\,\phi\right) \qquad (27)$$

As long as the value of v, determined from Equation (23), is less than the value of v, determined from Equation (27), when f_s represents the stress in the steel as it reaches its yield point, failure of the column will not occur even if the concrete within the helix is crushed by the imposed load.

If, in Equation (21), $\tan^2 \alpha$ is substituted for $\tan^2 \left(45° + \frac{1}{2}\,\phi\right)$, where α is determined from Equation (8), v will be the compressive stress necessary to shear out a wedge under eccentric load.

When longitudinal steel is used in conjunction with helical reinforcement, Equation (21) becomes:

$$v = \frac{f_c' \,[1 + (n' - 1)\, p]}{1 - \dfrac{n\, p' \tan^2 \left(45° + \dfrac{1}{2}\,\phi\right)}{2\, m + (m - 1)\, n\, p'}} \qquad (28)$$

The theoretically correct equation is:

$$v = \frac{f_c' \,[1 + (n' - 1)\, p]}{1 - \dfrac{n\, p' \tan^2 \left(45° + \dfrac{1}{2}\,\phi\right)}{2\, m + (m - 1)\, n\, p'}} \times \left[1 - \frac{2\, n\, p'}{m\,[2\, m + (m - 1)\, n\, p']}\right] \qquad (29)$$

The difference, however, in the results obtained with Equation (29) and with Equation (28) is small, and Equation (28) is much simpler and sufficiently accurate for all practical purposes.

Also, when longitudinal steel is used in conjunction with helical reinforcement, Equation (23) becomes

$$v = \frac{v_c\,[1 + (n' - 1)\,p]}{1 - \dfrac{2\,n\,p'}{m\,[2\,m + (m - 1)\,n\,p']}} \qquad (30)$$

in which, p equals the ratio of longitudinal steel.

Due to slipping, or some unknown cause, the longitudinal steel does not develop as much stress as is indicated by n at failure and, therefore, a lower value, n', must be used.

The theoretically correct equation is:

$$v = \frac{v_c}{1 - \dfrac{2\,n\,p'}{m\,[2\,m + (m - 1)\,n\,p']}} + v_c\,n'\,p \qquad (31)$$

but Equation (30) is used in order that the values of v, as determined from Equation (28) and Equation (30), respectively, may be equal for a certain value of p', regardless of the value of p. Tests (see graph of tests on helically reinforced concrete columns (Fig. 4) by Professor Wrentmore hereinafter discussed) indicate that curves, having as their equations, Equation (28) and Equation (30), respectively, intersect when p' equals about 0.01. In other words, when p' equals about 0.01, a helically reinforced concrete column will not fail in shear before the concrete core has crushed.

From actual tests,* it appears that when $f_c' = 1\,490$, $n = 44$ and $m = 2.5$; and when $f_c' = 2\,833$, $n = 21$ and $m = 3$. In other words, n decreases as f_c' increases and m increases as f_c' increases, but there is not sufficient information available to give the relation between n and f_c', and m and f_c'. Until these relations are determined it seems advisable to use values for the aforesaid factors which obtain in the average concrete. Assuming as probable values, $n = 27$, $m = 2.5$, $n' = 20$, and $\phi = 50°$, substituting these values in Equation (28) and Equation (30), respectively, and allowing a factor of safety of about 4†, the following working equations are obtained:

$$v = \frac{0.25\,f_c'\,(1 + 19\,p)}{1 - \dfrac{40\,p'}{1 + 8\,p'}} \qquad (28a)$$

and,

$$v = \frac{0.383\,f_c'\,(1 + 19\,p)}{1 - \dfrac{4\,p'}{1 + 8\,p'}} \qquad (30a)$$

A careful study of the test data, hereinafter given, indicates that n lies somewhere between 21 and 44, but nearer 21 than 44; and m lies somewhere

* See graph of tests (Fig. 4) conducted by Professor Wrentmore. These tests are reported in *Transactions*, Am. Soc. C. E., Vol. LXXVIII (1915), p. 97.

† The factor of safety is about 4 in Equation (28a), and about 4.5 in Equation (30a), a greater value being used in Equation (30a) because of the difficulty of determining the correct value of v_c.

between 2.5 and 3, but nearer 2.5. For the sake of convenience, m is assumed as 2.5, which is somewhat low, and to counteract this, the value of n is assumed as 27, which is undoubtedly less than the probable value. As previously stated, there appears to be a tendency for the longitudinal steel to slip, the amount of slip not remaining constant, and the value of n' is, therefore, indefinite. A study of tests, however, indicates that n' lies somewhere between 15 and 27. A value of 20 is chosen because, first, 20 is a convenient figure, and, second, its use in Equations (28a) and (30a) gives results which closely agree with the most modern practice. As regards the value of ϕ, tests indicate that, for percentages of reinforcement up to a percentage sufficient to preclude a shearing failure of the concrete, 50° is a reasonable value. For higher percentages of reinforcement, ϕ appears to be about 45 degrees. The only reason the writers can advance at present for the apparent decrease in the value of ϕ, for higher percentages of reinforcement, is that before the bond of the cement is broken the coarser particles of the aggregate dovetail into each other and are stronger in shear than the bonding cement. After the bond is broken, these particles have a tendency to roll upon each other, thereby diminishing the frictional resistance.

Although the values of n, m, n', and ϕ just given are only approximate, they are about as accurate as the available data will permit and, when substituted in Equations (28) and (30), the resulting values of v agree with actual tests as closely as may be expected when the great variation of these tests is taken into consideration. It should be clearly understood that the values of the aforesaid constants, which are used in obtaining the working Equations (28a) and (30a), are those that obtain at failure and the desired factor of safety is secured by using the proper coefficient of f_c'. Inasmuch as the values of n, m, and n' vary considerably as the stress in the concrete changes and are materially different at failure of the column than under working loads, the fallacy of attempting to secure a given factor of safety in a working formula by using therein the values of the aforesaid constants, which presumably obtain under working loads, should be self-evident. This point can not be stressed too strongly in view of the inexplicable tendency of many engineers to ignore it.

As helically reinforced concrete columns without longitudinal reinforcement are seldom if ever used, no attempt has been made to derive working equations for Equations (21) and (23).* As Equation (27) obtains only when the concrete core is crushed and is, therefore, only of academic interest and useless from the standpoint of design, a working equation of Equation (27) would be of no value.

A brief summary of various tests on concrete columns (principally helically reinforced concrete columns) is given in Table 1, wherein, for the sake of convenience in reference, the various columns are designated as Nos. (1) to (20), inclusive. Although the lengths of the test columns differed considerably and have not been noted in the table, it should be understood that no column had a ratio of $\frac{l}{r}$ exceeding the limits prescribed for short columns. For a few

* Equations (28a) and (30a) become working formulas for Equations (21) and (23), respectively, when $p = 0$.

columns, the tests indicated that failure was due to flexure, but this was due, no doubt, to some local defect. It is understood that the limiting value of $\frac{l}{r}$ decreases as p' increases and theoretically becomes zero when p' is sufficiently large to preclude a shearing failure. Of course, even when $p' = 0.01$ (the limit of shearing failure), the column curve (for a crushing failure in the extreme fiber) is almost horizontal for a considerable value of $\frac{l}{r}$ and for practical purposes the limiting value of $\frac{l}{r}$ would still be considerably greater than zero. As an aid to the understanding of the data presented in Table 1, the following explanatory notes may be of value:

The data given under Columns (1), (2), (3), (4), (5), (6), (7), (8), (9), (18), and (19), was furnished by the person, or persons, conducting the tests.

The ultimate strength of the column, as given under Column (10), was determined by dividing the maximum load by the area of the column cross-section. In other words, it would represent the ultimate strength of the column if the latter were perfectly homogeneous and each square inch of its cross-section were equally stressed.

The average cylinder test, given under Column (11), is the ultimate strength of the test specimen as determined by taking the average of all cylinder tests of a particular mixture of concrete in any given series of tests. It is, no doubt, unnecessary to state that, in all probability, concrete of given proportions in any one set of experiments will not be exactly the same as that of identical proportions in some other set of experiments.

The stress in the concrete, given under Column (12), was often rather difficult to determine, even approximately. Where longitudinal reinforcement was used, it was necessary first to determine (or assume) the stress in the longitudinal steel. Having determined (or assumed) this stress, it was then multiplied by the cross-sectional area of steel in order to obtain the total load carried by the longitudinal reinforcement. This load was then subtracted from the total load on the column and the remainder divided by the net cross-sectional area of the concrete in order to obtain the unit stress in the concrete.

The stress in the longitudinal steel, given under Column (13), was likewise rather difficult to determine. In certain series of tests (such as those by Withey), in which the amount of longitudinal steel varied while other conditions remained constant, the results were plotted on cross-section paper and a straight line passed through them in order to determine the increment of stress taken by the steel. In other cases, where the deformation was so great as to exceed considerably the elastic limit of the steel, some constant value a little in excess of the elastic limit, which best seemed to fit conditions, was assumed as the stress in the steel. In still other cases, where the elastic limit of the steel was not exceeded, the stress in the steel was determined by multiplying the observed unit deformation by the modulus of elasticity of the steel. In some instances, where the deformation was not given, the stress in the steel could not be determined.

The values of f_s, given under Column (14), were obtained from Equation (27) by using 45° as the value of ϕ and the results given under Column (12) as the values of v. The tests conducted by Messrs. Wrentmore, Brodie and Carey, and noted in Table 1, indicate that 45° is the most probable value of ϕ in concrete that is already ruptured.

The values of $n f_c$, given under Column (15), were computed by the formula, $f = f_c [1 + (n - 1) p]$, in which, f is given under Column (10), f_c is given under Column (11), and n is the unknown quantity. When lateral reinforcement is used, the formula just mentioned does not obtain and $n f_c$ can not be computed.

The values of n at failure, shown under Column (16), were obtained by extrapolation from the observed values of the tests whenever such data were given. No doubt these values are generally less than the real ones as the deformations become proportionately much greater at failure.

Having given the values of n, the values of m, given under Column (17), were then computed by Equation (21) wherein all the other quantities were assumed to be known. As may readily be seen, some values are less than those obtained by the assumption of constant volumes. However, the volume of concrete under compression always decreases; in such cases the computed values of m are, therefore, too small and either n, ϕ, or the strength of the concrete, are greater than the assumed values.

A study of Table 1 will readily indicate the difficulty of attempting to coordinate the majority of reported tests. Although there is considerable diversity in the mixture of concrete used and the age of the test specimens, there are only a few series of tests in which, with a constant mixture of concrete and age of test specimen, there is a sufficient variation in the percentage of helical reinforcement to indicate clearly what value of p' is the limit between Equation (21) and Equation (23), and what value of p' is the limit between Equation (23) and Equation (27). Inasmuch as v_c can only be determined when the exact intersection of curves, having as their equations, Equation (23) and Equation (27), respectively, is known, it can readily be seen how uncertain the value of v_c may be. The manner in which the helix influences the strength of the concrete may readily be seen, but the exact degree of influence is unknown as the exact values of m, n, ϕ, and v_c, to be used for the various concretes, are not known. Among the most instructive of the tests thus far reported are those which were conducted by Professor Wrentmore,* a graph of which is shown in Fig. 4.

In Fig. 4, the unit vertical pressures, sustained by the test columns at failure, were plotted as ordinates and the corresponding percentages of helical reinforcement as abscissas and curves, determined by Equations (21), (23), and (27), were then passed through the points thus obtained. For helical reinforcement up to about 1%, curves having as their equation, Equation (21), which is predicated upon a shearing failure of the column, seem to agree fairly well with the plotted tests. For reinforcement in excess of 1% and

* See Table 1, also "Report of a Series of Tests on Concrete Columns Reinforced with a Spiral of Steel", by C. G. Wrentmore, M. Am. Soc. C. E., and Messrs. Hugh Brodie and C. O. Carey, *Transactions*, Am. Soc. C. E., Vol. LXXVIII (1915), p. 97.

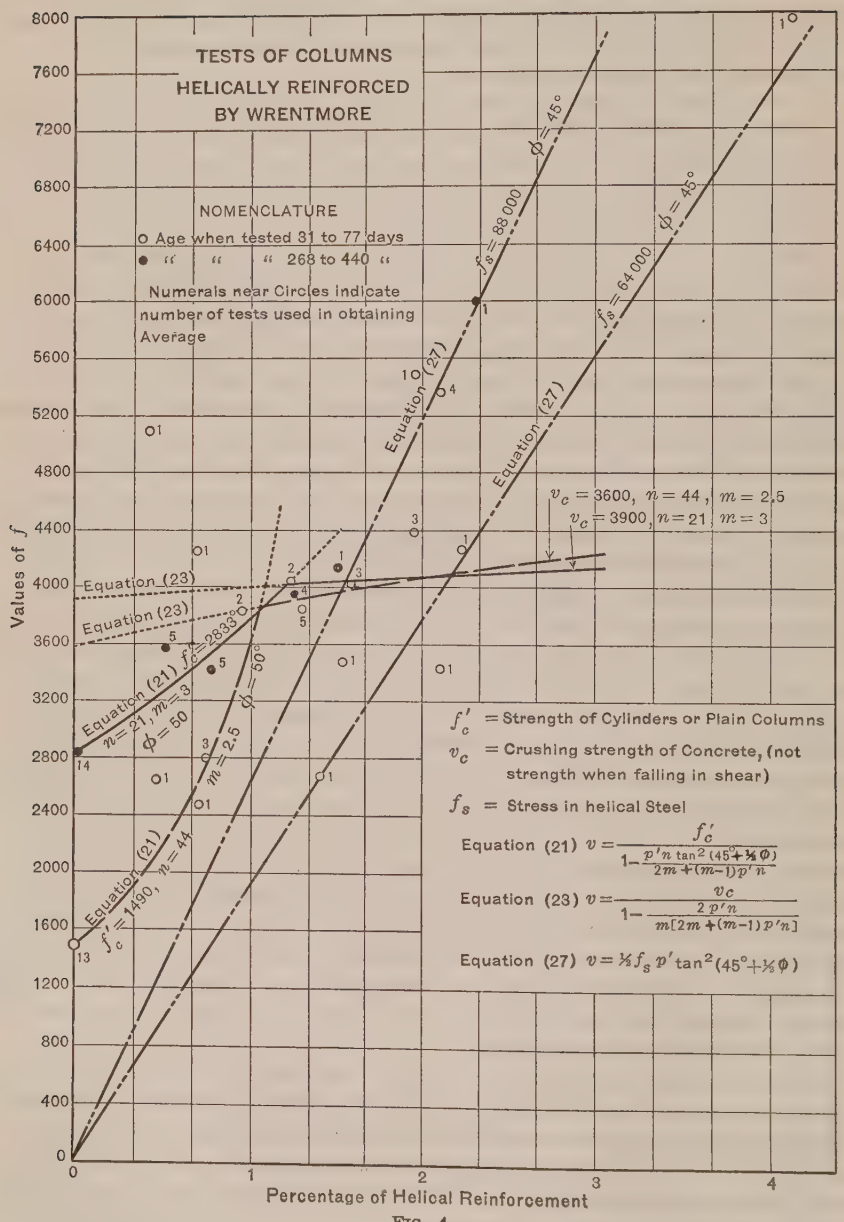
TESTS OF COLUMNS
HELICALLY REINFORCED
BY WRENTMORE
NOMENCLATURE
o Age when tested 31 to 77 days
● " " " 268 to 440 "
Numerals near Circles indicate number of tests used in obtaining Average
Values of f
Percentage of Helical Reinforcement
Equation (27)
Equation (23)
Equation (21)
$f_s = 88\,000$
$f_s = 64\,000$
$\phi = 45°$
$\phi = 50°$
$f'_c = 2833$
$f'_c = 1490$, $n = 44$
$n = 21$, $m = 3$
$\phi = 50$
$m = 2.5$
$v_c = 3600$, $n = 44$, $m = 2.5$
$v_c = 3900$, $n = 21$, $m = 3$
f'_c = Strength of Cylinders or Plain Columns
v_c = Crushing strength of Concrete, (not strength when failing in shear)
f_s = Stress in helical Steel
Equation (21) $v = \dfrac{f'_c}{1 - \dfrac{p'n \tan^2(45° + \frac{1}{2}\phi)}{2m + (m-1)p'n}}$
Equation (23) $v = \dfrac{v_c}{1 - \dfrac{2p'n}{m[2m + (m-1)p'n]}}$
Equation (27) $v = \frac{1}{2} f_s p' \tan^2(45° + \frac{1}{2}\phi)$

FIG. 4.

less than 2%, curves having as their equation, Equation (23), which is predicated upon a crushing failure of the concrete, seem to apply. For reinforcement in excess of from 1.6 to 2%, curves having as their equation, Equation (27), which is predicated upon the stress in the helical reinforcement, appear to be more nearly correct. On the basis of these tests, f_c', for 1 : 2 : 4 concrete, appears to lie somewhere between 1 490 and 2 833, n seems to lie somewhere between 21 and 44, m lies somewhere between 2.5 and 3, and ϕ is about 50° for the lower percentages of reinforcement and about 45° for percentages in excess of 1.6 per cent. The values of f_c', n, and m, seem to depend, to a great extent, on the age of the concrete.

Although the available test data, pertaining to helically reinforced columns, is inadequate to evaluate accurately the various constants used in the foregoing equations, the writers feel that a careful study of the data, thus far available, will justify the following conclusions:

(1) Plain concrete columns, of lengths up to at least 80 $\frac{l}{r}$, tend to fail in shear, barring some pronounced defect in materials or workmanship.

(2) The resistance to failure offered by a plain concrete column consists of the shearing resistance of the concrete and the frictional resistance of the material along the impending plane of rupture.

(3) The frictional resistance of the concrete is an important factor in the strength of the column.

(4) Helical reinforcement is very effective in resisting a shearing failure of the concrete core of the column. If sufficient helical reinforcement is used, it is possible to preclude a shearing failure of the column.

(5) Helical reinforcement is not very effective in preventing crushing of the concrete core.

(6) Helical reinforcement tends to increase the reliability of a concrete column by neutralizing the effects of local imperfections. Furthermore, with helically reinforced columns, a warning of impending failure is given, at 70 to 90% of the ultimate strength, by the cracking or spalling of the outer shell. It is permissible, therefore, to allow a lower factor of safety and correspondingly increased working stresses in helically reinforced columns.

(7) Three types of failure occur in helically reinforced columns, depending on the amount of reinforcement:

a.—With reinforcement up to about 1%, the column tends to fail in shear.

b.—With reinforcement from about 1% to about 1.6%, failure is due to the crushing of the concrete core of the column.

c.—With reinforcement in excess of about 1.6%, failure of the column will not occur even after the concrete is crushed, until the reinforcement is stressed considerably beyond its elastic limit and possibly, in some cases, up to its ultimate strength. Tests have been reported wherein the helix had ruptured under loads which, theoretically, could not stress the steel to anywhere near its ultimate strength. Such

failures were undoubtedly due to a pronounced local defect in materials or workmanship which resulted in a local overstressing of the helix.

(8) The pitch of the helix materially affects the strength of the column, especially with higher percentages of reinforcement, and should be as small as practicable; the smaller the pitch of the helix the less will be the tendency for the particles of crushed concrete to flow between the coils of the helix.

(9) A rational analysis of the stresses in the steel and concrete of a helically reinforced concrete column is possible.

(10) Longitudinal reinforcement, consisting of rods or bars, should not be used unless it is well tied together at frequent intervals with lateral bands. It should preferably not be used except in conjunction with helical reinforcement.

(11) There is urgent need of additional research work on helically reinforced concrete columns, especially with reinforcement ranging from 0.75 to 2 per cent.

TABLE 1.—SUMMARY OF VARIOUS TESTS ON CONCRETE COLUMNS, PRINCIPALLY HELICALLY REINFORCED CONCRETE COLUMNS.

Source.	Column designation.	Reinforcement. Kind.	Reinforcement. Vertical percentage.	Reinforcement. Lateral percentage.	Ratio of mix.	Age.	Area, in square inches.	Maximum load, in pounds.	Ultimate strength, in pounds per square inch.	Average cylinder test.	Stress in concrete.	Stress in longitudinal steel.	f_s (Equation (27)).	$n f_c$ *	$n.\|$	$m.$	Cylinders. Age.	Cylinders. Ultimate strength, in pounds per square inch.	Remarks.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
Bulletin 300, University of Wisconsin, by Withey→	A 1	None	0	0	1:2:4	65 days	144.0	294 000	2 040	.....	.....	.....	.....	.....	12.3	.....	63 days	2 427	.....
	A 2	"	0	0	1:2:4	63 "	143.0	303 000	2 110	.....	.....	.....	.....	.....	15.4	.....	63 "	2 395	.....
	A 3	"	0	0	1:2:4	64 "	142.0	292 000	2 055	.....	.....	.....	.....	.....	14.5	.....	62 "	2 240	.....
	A 4	"	0	0	1:2:4	65 "	141.0	294 000	2 080	.....	.....	.....	.....	.....	14.3	.....	62 "	2 360	.....
		Average	0	0	1:2:4	64¼ "	142.5	295 750	2 071	2 277†	.....	.....	.....	.....	14.1	.....	62½ "	2 363	.....
	B 1, B 4	Four, 2 by 2 in. by 3/16 in. latticed angles	4.5	0	1:2:4	64 "	64.0	237 000	3 700	.....	.....	.....	.....	.....	51.8	.....	62 "	2 270	.....
			4.5	0	1:2:4	63 "	64.0	242 000	3 780	.....	.....	.....	.....	.....	78.0	.....	62 "	2 140	.....
		Average	4.5	0	1:2:4	63½ "	64.0	239 500	3 740	2 277†	.....	.....	.....	34 900	64.9	.....	62 "	2 255	.....
	B 2, B 3	Four, 2 by 2 in. by 3/16 in. latticed angles	2.02	0	1:2:4	62 "	142.5	303 000	2 125	.....	.....	.....	.....	.....	.....	.....	62 "	2 640	Ultimate strength of column lower than cylinder tests.
			2.01	0	1:2:4	61 "	143.0	288 000	2 015	.....	.....	.....	.....	.....	.....	.....	61 "	2 205	
		Average	2.015	0	1:2:4	61½ "	142.75	.....	2 065	2 277†	.....	.....	.....	.....	.....	.....	61½ "	2 423	.....
	C 1, C 2, C 3, C 4	¼ in. high carbon steel, 10 in. in diameter; 1 in. pitch	0	2.00	1:2:4	62 "	78.54	366 000	4 660	.....	.....	.....	.....	.....	.....	.....	63 "	2 280	.....
			0	2.00	1:2:4	62 "	78.54	345 000	4 390	.....	.....	.....	.....	.....	.....	.....	62 "	2 190	.....
			0	2.00	1:2:4	64 "	78.54	288 000	3 650	.....	.....	.....	.....	.....	.....	.....	63 "	2 185	.....
			0	2.00	1:2:4	65 "	78.54	268 000	3 410	.....	.....	.....	.....	.....	.....	.....	62 "	2 150	.....
		Average	0	2.00	1:2:4	63¼ "	78.54	.....	4 030	2 277†	4 030	.....	39 100	.....	.....	.....	62½ "	2 201	.....
	D 1, D 2, D 3, D 4	Nine, 5/8 in. round mild steel rods and ¼ in. high-carbon steel spiral, 10 in. in diameter; 1 in. pitch	2.35	2.00	1:2:4	58 "	78.54	351 000	4 470	.....	.....	.....	.....	.....	.....	.....	56 "	2 150	.....
			2.35	2.00	1:2:4	60 "	78.54	330 000	4 200	.....	.....	.....	.....	.....	.....	.....	56 "	2 130	.....
			2.35	2.00	1:2:4	61 "	78.54	391 000	4 970	.....	.....	.....	.....	.....	.....	.....	56 "	2 380	.....
			2.35	2.00	1:2:4	62 "	78.54	421 000	5 360	.....	.....	.....	.....	.....	.....	.....	56 "	2 335	.....
		Average	2.35	2.00	1:2:4	60¼ "	78.54	373 250	4 750	2 277†	4 030	20 130	69 300	.....	.....	.....	56 "	2 259	Equation (27)

TABLE 1.—(*Continued.*)

Source.	Column designation.	Reinforcement. Kind.	Reinforcement. Vertical percentage.	Reinforcement. Lateral percentage.	Ratio of mix.	Age.	Area, in square inches.	Maximum load, in pounds.	Ultimate strength, in pounds per square inch.	Average cylinder test.	Stress in concrete.	Stress in longitudinal steel.	f_s (Equation (27)).	$n\,f_c$.*	n.‖	m.	Cylinders. Age.	Cylinders. Ultimate strength, in pounds per square inch.	Remarks.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
	E 1	Nine, ⅝ in. round	2.35	0.165	1 : 2 : 4	55 days	78.54	273 000	3 475	.....	.....	.....	.....	.....	.....	.....	55 days	2 430	.....
	E 2	mild steel rods,	2.35	0.165	1 : 2 : 4	57 "	78.51	273 000	3 475	.....	.....	.....	.....	.....	.....	.....	55 "	2 430	.....
	E 3	with ¼ in. ties 1 ft. c. to c.	2.35	0.165	1 : 2 : 4	57 "	78.54	318 006	4 050	.....	.....	.....	.....	.....	.....	.....	54 "	2 040	.....
		Average	2.35	0.165	1 : 2 : 4	56⅓ "	78.54	288 000	3 667	2 277†	.....	.....	.....	.....	.....	.....	54⅞ "	2 300	.....
	H 1	No. 7 wire spiral,	0	0.50	1 : 2 : 3.5	57 "	78.54	183 000	2 330	.....	.....	.....	.....	.....	37	.....	57 "	2 038	.....
	H 2	2 in. pitch.....	0	0.50	1 : 2 : 3.5	57 "	78.54	168 000	2 140	.....	.....	.....	.....	.....	39	.....	57 "	1 460	.....
		Average	0	0.50	1 : 2 : 3.5	57 "	78.54	175 500	2 235	1 881	2 235	.....	.....	.....	38	3.535	57 "	1 749	.....
	G 1	Eight, ½ in. rods	2.00	0.50	1 : 2 : 3.5	49 "	78.54	261 000	3 320	.....	.....	.....	.....	.....	35	.....	49 "	2 095	.....
	G 2	and No. 7 wire spiral, 2 in. pitch	2.00	0.50	1 : 2 : 3.5	58 "	78.51	258 000	3 280	.....	.....	.....	.....	.....	37	.....	57 "	1 430	.....
		Average	2.00	0.50	1 : 2 : 3.5	53½ "	78.54	259 500	3 300	1 881	2 374	49 000	.....	.....	36	3.092	53 "	1 763	.....
	I 1	Eight, 1 1/16 in. round rods and	3.78	0.50	1 : 2 : 3.5	57 "	78.54	330 000	4 240	.....	.....	.....	.....	.....	36	.....	59 "	2 235	.....
	I 2	No. 7 wire spiral, 2 in. pitch.....	3.78	0.50	1 : 2 : 3.5	57 "	78.54	321 000	4 080	.....	.....	.....	.....	.....	32	.....	59 "	2 115	.....
		Average	3.78	0.50	1 : 2 : 3.5	57 "	78.54	325 500	4 160	1 881	2 378	49 000	.....	.....	34	2.916	.....	2 175	.....
	J 1	Eight, ⅞ in. round rods and No. 7	6.11	0.50	1 : 2 : 3.5	58 "	78.54	408 000	5 190	.....	.....	.....	.....	.....	27	.....	59½ "	2 210	.....
	J 2	wire spiral, 2 in. pitch.....	6.11	0.50	1 : 2 : 3.5	58 "	78.54	397 500	5 050	.....	.....	.....	.....	.....	54	.....	59 "	1 995	.....
		Average.....	6.11	0.50	1 : 2 : 3.5	58 "	78.54	402 750	5 120	1 881	2 273	49 000	.....	.....	40.5	4.130	59¼ "	2 102	.....
	L 1	No. 7 wire spiral,	0	1.00	1 : 2 : 3.5	57 "	78.54	219 000	2 680	.....	.....	.....	.....	.....	65	.....	58 "	1 780	.....
	L 2	1 in. pitch.....	0	1.00	1 : 2 : 3.5	58 "	78.54	204 000	2 600	.....	.....	.....	.....	.....	65	.....	58 "	1 770	.....
		Average.....	0	1.00	1 : 2 : 3.5	57½ "	78.54	211 500	2 640	1 881	2 640	.....	.....	.....	65	.....	58 "	1 775	Equation (23) or Equation (27).

TABLE 1.—(*Continued.*)

Source.	Column designation.	Reinforcement. Kind.	Vertical percentage.	Lateral percentage.	Ratio of mix.	Age.	Area, in square inches.	Maximum load, in pounds.	Ultimate strength, in pounds per square inch.	Average cylinder test.	Stress in concrete.	Stress in longitudinal steel.	f_s (Equation (27)).	$n f_c$.*	n.‖	m.	Cylinders. Age.	Cylinders. Ultimate strength, in pounds per square inch.	Remarks.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
Bulletin 466, University of Wisconsin, by Withey	$K1$	Eight, ½ in. rods and No. 7 wire spiral, 1 in. pitch	2.00	1.00	1 : 2 : 3.5	57 days	78.54	318 000	4 050	.....	.....	.....	.....	.....	68	.....	58 days	2 100	.....
	$K2$		2.00	1.00	1 : 2 : 3.5	57 "	78.54	295 500	3 760	.....	.....	.....	.....	.....	68	.....	58 "	1 895	.....
		Average	2.00	1.00	1 : 2 : 3.5	57 "	78.54	306 750	3 905	1 881	3 517	23 100	.....	.....	68	4.20	58 "	1 998	Equation (23)
	$N1$	Eight, 11/16 in. rods and No. 7 wire spiral, 1 in. pitch	3.78	1.00	1 : 2 : 3.5	57 "	78.54	318 000	4 050	.....	.....	.....	.....	.....	46	.....	57 "	1 885	.....
	$N2$		3.78	1.00	1 : 2 : 3.5	58 "	78.54	340 500	4 340	.....	.....	.....	.....	.....	63	.....	57 "	1 720	.....
		Average	3.78	1.00	1 : 2 : 3.5	57½ "	78.54	329 250	4 195	1 881	3 452	23 100	.....	.....	54	3.59	57 "	1 802	Equation (23)
	$M1$	Eight, ⅞ in. rods and No. 7 wire spiral, 1 in. pitch	6.11	1.00	1 : 2 : 3.5	57 "	78.54	376 500	4 790	.....	.....	.....	.....	.....	65	.....	57 "	1 665	.....
	$M2$		6.11	1.00	1 : 2 : 3.5	58 "	78.54	360 000	4 580	.....	.....	.....	.....	.....	67	.....	57 "	1 710	.....
		Average	6.11	1.00	1 : 2 : 3.5	57½ "	78.54	368 250	4 685	1 881	3 493	23 100	.....	.....	66	4.16	57 "	1 688	Equation (23)
	$P1$	Eight, 1 in. rods and No. 7 wire spiral, 1 in. pitch	8.00	1.00	1 : 2 : 4	57 "	78.54	531 300	6 760	.....	.....	.....	.....	.....	44	.....	57 "	2 385	.....
	$P2$		8.00	1.00	1 : 2 : 4	60 "	78.54	556 500	7 090	.....	.....	.....	.....	.....	35	.....	57 "	2 360	.....
		Average	8.00	1.00	1 : 2 : 4	58½ "	78.54	542 750	6 925	2 380	3 500	46 270	.....	.....	39.5	4.10	57 "	2 372	Equation (23)
	$O1$	Eight, ⅞ in. rods and ¼ in. spiral, 1 in. pitch	6.11	1.96	1 : 2 : 4	57 "	78.54	511 500	6 510	.....	.....	.....	.....	.....	.....	.....	57 "	2 270	.....
	$O2$		6.11	1.96	1 : 2 : 4	57 "	78.54	522 000	6 650	.....	.....	.....	.....	.....	.....	.....	57 "	2 690	.....
		Average	6.11	1.96	1 : 2 : 4	57 "	78.54	516 750	6 580	2 380	5 278	26 600	92 600	.....	.....	4.16	57 "	2 480	Equation (23)
	$R1$	Eight, 1 in. rods and ¼ in. spiral, 1 in. pitch	8.00	1.96	1 : 2 : 4	53 "	78.54	570 000	7 250	.....	.....	.....	.....	.....	.....	.....	53 "	2 320	.....
	$R2$		8.00	1.96	1 : 2 : 4	53 "	78.54	525 000	6 680	.....	.....	.....	.....	.....	.....	.....	54½ "	2 475	.....
		Average	8.00	1.96	1 : 2 : 4	53 "	78.54	547 500	6 965	2 380	5 268	26 600	92 200	.....	.....	.....	53¾ "	2 398	Equation (27)
	$Q1$	Eight, 1⅛ in. rods and ¼ in. spiral, 1 in. pitch	10.12	1.96	1 : 2 : 4	53 "	78.54	486 000	6 190	.....	.....	.....	.....	.....	.....	.....	53 "	2 275	.....
	$Q2$		10.12	1.96	1 : 2 : 4	53 "	78.54	627 000	7 990	.....	.....	.....	.....	.....	.....	.....	54½ "	2 325	.....
		Average	10.12	1.96	1 : 2 : 4	53 "	78.54	556 500	7 090	2 380	4 894	26 600	85 700	.....	.....	.....	53¾ "	2 300	Equation (27)

TABLE 1.—(*Continued.*)

Source.	Column designation.	Reinforcement. Kind.	Reinforcement. Vertical percentage.	Reinforcement. Lateral percentage.	Ratio of mix.	Age.	Area, in square inches.	Maximum load, in pounds.	Ultimate strength, in pounds per square inch.	Average cylinder test.	Stress in concrete.	Stress in longitudinal steel.	f_s (Equation (2')).	$n.f_c$.*	n.‖	m.	Cylinders. Age.	Cylinders. Ultimate strength, in pounds per square inch.	Remarks.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
	W 1	None	0	0	1:2 :4	52 days	86.6	231 000	2 660	.....	.....	.....	.....	.....	10.35	.....	49 days	2 605	.....
	W 2	"	0	0	1:2 :4	52 "	86.6	231 000	2 660	.....	.....	.....	.....	.....	9.81	.....	49 "	2 395	.....
	W 3	"	0	0	1:2 :4	51 "	86.6	214 500	2 480	.....	.....	.....	.....	.....	9.36	.....	49 "	2 050	.....
		Average	0	0	1:2 :4	51⅔ "	86.6	.....	2 600	2 380	.....	.....	.....	.....	9.84	.....	49 "	2 350	.....
	Z 1	Eight, ⅞ in. rods and No. 7 high carbon steel wire, 1 in. pitch.	5.83	1.00	1:3 :6	48 "	82.6	476 000	5 760	.....	.....	.....	.....	.....	63.6	.....	45 "	2 020	.....
	Z 2		5.83	1.00	1:3 :6	49 "	82.6	411 000	4 980	.....	.....	.....	.....	.....	62.7	.....	45 "	1 520	.....
		Average	5.83	1.00	1:3 :6	48½ "	82.6	443 510	5 370	1 770	4 920	50 000 (Assumed)	.....	.....	63.1	2.22	45 "	1 770	.....
	X 1	No. 7 high carbon steel wire, 1 in. pitch.	0	1.00	1:1¾ : 3¼	56 "	82.6	523 000	6 330	.....	.....	.....	.....	.....	11.45	.....	55 "	4 730	.....
	X 2		0	0.99	1:1¾ : 3¼	57 "	84.6	560 000	5 900	.....	.....	.....	.....	.....	10.32	.....	55 "	4 665	.....
		Average	0	1.00	1:1¾ : 3¼	56½ "	83.6	.....	6 165	4 433	6 165	.....	.....	.....	10.9	1.46	55 "	4 698	.....
	Y 1	Eight, ⅞ in. rods and No. 7 high carbon steel wire, 1 in. pitch.	5.69	0.99	1:1¾ : 3¼	60 "	84.6	617 500	7 300	.....	.....	.....	.....	.....	20.0	.....	62½ "	4 220	.....
	Y 2		5.83	1.00	1:1¾ : 3¼	62 "	82.6	617 500	7 480	.....	.....	.....	.....	.....	19.5	.....	62½ "	4 115	.....
		Average	5.76	1.00	1:1¾ : 3¼	61 "	83.6	617 500	7 390	4 433	6 165	27 300	.....	.....	19.75	2.50	62½ "	4 168	.....
	S 1	No. 7 high carbon steel wire, 1 in. pitch.	0	1.00	1:1 :2	57 "	78.54	468 000	5 950	.....	.....	.....	.....	.....	14.15	.....	61 "	4 065	.....
	S 2		0	1.00	1:1 :2	57 "	78.54	453 000	5 760	.....	.....	.....	.....	.....	7.05	.....	61 "	4 080	.....
		Average	0	1.00	1:1 :2	57 "	78.54	.....	5 855	4 288	5 855	.....	.....	.....	10.6	1.47	61 "	4 072	.....
	T 1	Eight, ⅞ in. rods and No. 7 high carbon steel wire, 1 in. pitch.	6.11	1.00	1:1 :2	57 "	78.54	588 000	7 480	.....	.....	.....	.....	.....	8.86	.....	65 "	4 715	.....
	T 2		6.11	1.00	1:1 :2	56 "	78.54	558 000	7 100	.....	.....	.....	.....	.....	7.20	.....	65 "	4 080	.....
		Average	6.11	1.00	1:1 :2	56½ "	78.54	573 000	7 240	4 288	5 855	29 500	.....	.....	8.03	1.046	65 "	4 503	.....
	V 1	No. 7 high carbon steel wire.....	0	1.00	1:1.29	56 "	78.54	453 000	5 760	.....	.....	.....	.....	.....	8.31	.....	55 "	4 830	.....
	V 2		0	1.00	1:1.29	57 "	78.54	387 000	4 920	.....	.....	.....	.....	.....	7.37	.....	55 "	4 925	.....
		Average.....	0	1.00	1:1.29	56½ "	78.54	.....	5 340	4 708	5 340	.....	.....	.....	7.84	2.43	55 "	4 865	.....

TABLE 1.—(*Continued.*)

Source.	Column designation.	Reinforcement. Kind.	Vertical percentage.	Lateral percentage.	Ratio of mix.	Age.	Area, in square inches.	Maximum load, in pounds.	Ultimate strength, in pounds per square inch.	Average cylinder test.	Stress in concrete.	Stress in longitudinal steel.	f_s (Equation (27)).	nf_c*.	n.	m.	Cylinders. Age.	Cylinders. Ultimate strength, in pounds per square inch.	Remarks.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
	U 1	Eight, 7/8 in. rods and No. 7 high carbon steel wire, 1 in. pitch.	6.11	1.00	1 : 1.29	56 days	78.54	633 000	8 050	.....	.....	.....	.....	.....	.....	.....	55 days.	4 900	.....
	U 2		6.11	1.00	1 : 1.29	57 "	78.54	648 000	8 250	.....	.....	.....	.....	.....	.....	.....	55 "	4 205	.....
		Average.....	6.11	1.00	1 : 1.29	56½ "	78.54	640 500	8 150	4 708	5 430	51 500	.....	.....	8.46	2.61	55 "	4 552	.....
	AA 1	Eight, ½ in. rods and No. 7 high carbon steel wire, 1 in. pitch.	1.86	0.99	1 : 1.33	57 "	84.6	617 500	7 300	.....	.....	35 000	.....	.....	4.8	.....	59 "	6 135	Column did not fail.
	AA 2		1.86	0.99	1 : 1.33	57 "	84.6	617 500	7 300	.....	.....	35 800	.....	.....	4.9	.....	59 "	7 430	
		Average.....	1.86	0.99	1 : 1.33	57 "	84.6	617 500	7 300	6 783	6 783	35 400	.....	.....	4.85	.....	59 "	6 783	.....
	AC 1	Eight, 7/8 in. rods and No. 7 high carbon steel wire, 1 in. pitch.	5.83	1.00	1 : 1⅓	53 "	82.6	617 000	7 480	.....	.....	35 200	.....	.....	4.71	.....	52 "	6 075	Column did not fail.
	AC 2		5.56	0.97	1 : 1⅓	54 "	86.6	617 500	7 140	.....	.....	35 440	.....	.....	4.97	.....	52 "	5 835	
		Average.....	5.70	0.985	1 : 1⅓	53½ "	84.6	617 250	7 310	5 953	.....	35 320	.....	.....	4.84	.....	52 "	5 955	.....
	111	None.....	0	0	1 : 2 : 4	66 "	113.2	240 000	2 120	.....	.....	.....	.....	.....	20.52	.....	.....	.....	.....
	112	"	0	0	1 : 2 : 4	62 "	113.2	281 000	2 480	.....	.....	.....	.....	.....	13.10	.....	.....	.....	.....
		Average.....	0	0	1 : 2 : 4	64 "	113.2	.....	2 300	2 300	.....	.....	.....	.....	16.8	.....	.....	.....	.....
	101	None.....	0	0	1 : 2 : 4	58 "	113.2	132 000	1 165	.....	.....	.....	.....	.....	21.15	.....	.....	.....	.....
	102	"	0	0	1 : 2 : 4	69 "	109	218 000	2 000	....	.....	.....	.....	.....	16.38	.....	.....	.....	.....
	103	"	0	0	1 : 2 : 4	65 "	113.2	250 000	2 210	.....	.....	.....	.....	.....	16.01	.....	.....	.....	.....
	104	"	0	0	1 : 2 : 4	64 "	113.2	180 000	1 590	.....	.....	.....	.....	.....	16.05	.....	.....	.....	.....
	105	"	0	0	1 : 2 : 4	62 "	113.2	220 000	1 945	.....	.....	.....	.....	.....	20.86	.....	.....	.....	.....
	108	"	0	0	1 : 2 : 4	72 "	113.2	165 000	1 460	....	.....	.....	.....	.....	13.9	.....	.....	.....	.....
	109	"	0	0	1 : 2 : 4	64 "	113.2	205 000	1 810	.....	.....	.....	.....	.....	17.6	.....	.....	.....	.....
		Average.....	0	0	1 : 2 : 4	65 "	113.2	.....	1 740	1 740	.....	.....	.....	.....	17.4	.....	.....	.....	.....

TABLE 1.—(*Continued.*)

Source.	Column designation.	Reinforcement. Kind.	Vertical percentage.	Lateral percentage.	Ratio of mix.	Age.	Area, in square inches.	Maximum load, in pounds.	Ultimate strength, in pounds per square inch.	Average cylinder test.	Stress in concrete.	Stress in longitudinal steel.	f_s (Equation (27)).	nf_c*.	n.	m.	Cylinders. Age.	Cylinders. Ultimate strength, in pounds per square inch.	Remarks.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
—*Bulletin No. 20*, University of Illinois	116	None	0	0	1 : 3 : 6	61 days	113.2	108 000	955						17.62				
	117	"	0	0	1 : 3 : 6	62 "	113.2	126 000	1 110						18.80				
		Average	0	0	1 : 3 : 6	61½ "	113.2	.....	1 032	1 032					18.21				
	121	None	0	0	1 : 4 : 8	63 "	113.2	65 000	575						30.25				
	122	"	0	0	1 : 4 : 8	63 "	113.2	65 000	575						38.85				
		Average	0	0	1 : 4 : 8	63 "	113.2	.....	575	575					39.55				
	110	None	0	0	1 : 2 : 4	203 "	113.2	218 000	1 925						18.07				
	128	"	0	0	1 : 2 : 4	194 "	113.2	209 000	1 845						18.85				
	129	"	0	0	1 : 2 : 4	181 "	113.2	200 000	1 770						15.57				
	163	"	0	0	1 : 2 : 4	187 "	113.2	303 000	2 680						18.40				
	164	"	0	0	1 : 2 : 4	187 "	113.2	245 000	2 160						16.92				
	168	"	0	0	1 : 2 : 4	201 "	113.2	200 000	1 770						22.62				
		Average	0	0	1 : 2 : 4	192 "	113.2	.....	2 025	2 025					18.4				
	21	None	0	0	1 : 2 : 3¾	12 mos.	144	381 500	2 650						20.25				
	22	"	0	0	1 : 2 : 3¾	16 "	140.4	389 000	2 770						17.05				
		Average	0	0	1 : 2 : 3¾	14 "	142.2	.....	2 710	2 710					18.65				
	131	No. 16 bands (mild steel) 2 in. c. to c.	0	1.08	1 : 2 : 4	60 days	113.2	270 000	2 384						4.87				
	132		0	1.08	1 : 2 : 4	57 "	113.2	243 000	2 150						6.75				
	133		0	1.05	1 : 2 : 4	59 "	113.2	247 000	2 182						6.91				
		Average	0	1.07	1 : 2 : 4	58⅔ "	113.2	253 333	2 239	1 740	2 876				6.18	1.117			
	136	No. 12 bands (mild steel) 2 in. c. to c.	0	2.08	1 : 2 : 4	63 "	113.2	323 000	2 860						6.73				
	137		0	2.07	1 : 2 : 4	69 "	113.2	300 000	2 660										
	138		0	2.12	1 : 2 : 4	60 "	113.2	351 000	3 110										
		Average	0	2.09	1 : 2 : 4	64 "	113.2	.....	2 876	1 740	2 876		47 300		10.86	2.05			Equation (27)

TABLE 1.—(*Continued.*)

Source.	Column designation.	Reinforcement. Kind.	Vertical percentage.	Lateral percentage.	Ratio of mix.	Age.	Area, in square inches.	Maximum load, in pounds.	Ultimate strength, in pounds per square inch.	Average cylinder test.	Stress in concrete.	Stress in longitudinal steel.	f_s (Equation (27)).	nf_c*.	n ‖.	m.	Cylinders. Age.	Cylinders. Ultimate strength, in pounds per square inch.	Remarks.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
	146	No. 8 bands (mild steel), 2 in. c. to c.	0	3.22	1 : 2 : 4	60 days	113.2	339 000	3 000	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
	147		0	3.20	1 : 2 : 4	66 "	113.2	420 000	3 715	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
	148		0	3.20	1 : 2 : 4	63 "	113.2	327 000	2 890	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
		Average	0	3.207	1 : 2 : 4	63 "	113.2	.....	3 202	1 740	3 202	.....	34 250	.....	9.61	2.345	.....	.....	Equation (27).
	141	No. 12 bands (mild steel), 4 in. c. to c.	0	1.02	1 : 2 : 4	59 "	113.2	256 000	2 275	.....	.....	.....	.....	.....	7.90	.....	.....	.....	.....
	142		0	1.02	1 : 2 : 4	66 "	113.2	246 000	2 178	.....	.....	.....	.....	.....	7.99	.....	.....	.....	.....
		Average	0	1.02	1 : 2 : 4	62½ "	113.2	251 000	2 227	1 740	2 227	.....	.....	.....	7.95	1.887	.....	.....	.....
	143	No. 12 bands (mild steel), 3 in. c. to c.	0	1.39	1 : 2 : 4	60 "	113.2	309 000	2 735	1 740	2 735	.....	.....	.....	8.48	1.207	.....	.....	.....
	152	No. 16 bands (mild steel), 2 in. c. to c.	0	1.35	1 : 4 : 8	70 "	.....	86 000	1 345	.....	.....	.....	.....	.....	21.12	.....	.....	.....	.....
	153		0	1.41	1 : 4 : 8	56 "	.....	80 000	1 260	.....	.....	.....	.....	.....	23.93	.....	.....	.....	.....
		Average of 152 and 153	0	1.38	1 : 4 : 8	63 "	.....	83 000	1 303	575	1 304	.....	32 400	.....	22.5	1.95	.....	.....	Equation (27).
	151	No. 16 bands (mild steel), 2 in. c. to c.	0	1.47	1 : 2 : 4	65 "	.....	137 000	2 140	1 740	2 140	.....	.....	.....	5.69	1.66	.....	.....	Equation (23) or Equation (27).
	156	No. 12 bands (mild steel), 2 in. c. to c.	0	2.76	1 : 2 : 4	67 "	.....	190 000	2 970	1 740	2 970	.....	37 300	.....	6.93	1.665	.....	.....	Equation (27).
	157		0	2.94	1 : 1½ : 3	67 "	.....	228 000	3 561	.....	.....	.....	.....	.....	6.51	.....	.....	.....	.....
	158		0	2.80	1 : 1½ : 3	81 "	.....	234 000	3 685	.....	.....	.....	.....	.....	5.37	.....	.....	.....	.....
		Average of 157 and 158	0	2.87	1 : 1½ : 3	74 "	.....	231 000	3 623	2 300	3 623	.....	43 300	.....	5.94	1.708	.....	.....	Equation (27).
	171	No. 7 wire spiral (high carbon steel)	0	0.85	1 : 2 : 4	56 "	.....	283 000	2 503	.....	.....	.....	.....	.....	7.76	.....	.....	.....	.....
	172		0	0.85	1 : 2 : 4	63 "	.....	283 000	2 506	.....	.....	.....	.....	.....	9.84	.....	.....	.....	.....
		Average	0	0.85	1 : 2 : 4	59½ "	.....	.....	2 505	1 740	2 505	.....	.....	.....	8.80	0.926	.....	.....	.....

TABLE 1.—(*Continued.*)

Source.	Column designation.	Reinforcement. Kind.	Vertical percentage.	Lateral percentage.	Ratio of mix.	Age.	Area, in square inches.	Maximum load, in pounds.	Ultimate strength, in pounds per square inch.	Average cylinder test.	Stress in concrete.	Stress in longitudinal steel.	f_s (Equation (27)).	nf_c*.	n‖.	m.	Cylinders. Age.	Cylinders. Ultimate strength, in pounds per square inch.	Remarks.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
	176	No. 7 wire spiral (high carbon steel)	0	0.84	1:2:4	60 days	.....	235 000	2 080	.....	.....	.....	.....	.....	11.15	.....	.....	.....	.....
	177		0	0.85	1:2:4	64 "	.....	249 000	2 203	.....	.....	.....	.....	.....	7.90	.....	.....	.....	.....
	178		0	0.84	1:2:4	61 "	.....	251 000	2 220	.....	.....	.....	.....	.....	10.85	.....	.....	.....	.....
		Average	0	0.843	1:2:4	61⅔ "	.....	.....	2 168	1 740	2 168	.....	.....	.....	9.97	1.583	.....	.....	.....
	181	¼ in. spiral, 1 in. pitch (high carbon steel)	0	1.73	1:2:4	57 "	.....	307 000	2 718	.....	.....	.....	.....	.....	13.66	.....	.....	.....	.....
	182		0	1.67	1:2:4	60 "	.....	429 000	3 800	.....	.....	.....	.....	.....	9.16	.....	.....	.....	.....
	183		0	1.68	1:2:4	63 "	.....	428 000	3 793	.....	.....	.....	.....	.....	11.62	.....	.....	.....	.....
		Average	0	1.69	1:2:4	60 "	.....	.....	3 437	1 740	3 437	.....	69 800	.....	11.48	1.44	.....	.....	Equation (23) or Equation (27).
	186	¼ in. spiral, 1 in. pitch (mild steel)	0	1.64	1:2:4	58 "	.....	234 000	2 068	.....	.....	.....	.....	.....	16.12	.....	.....	.....	.....
	187		0	1.71	1:2:4	62 "	.....	385 000	3 404	.....	.....	.....	.....	.....	8.69	.....	.....	.....	.....
		Average	0	1.675	1:2:4	60 "	.....	.....	2 736	1 740	2 736	.....	56 000	.....	12.40	2.05	.....	.....	Equation (23) or Equation (27).
	1 684	None	0	0	1:2:0	6 mos. 0 days	158.0	395 000	2 500	3 358	.....	.....	.....	.....	11.1	4.69§	.....	.....	.....
	1 680	"	0	0	1:3:0	5 mos. 28 days	156.37	322 500	2 062	2 177	.....	.....	.....	.....	23.4	9.00§	.....	.....	.....
	1 679	"	0	0	1:4:0	5 mos. 21 days	156.0	244 000	1 564	1 408	.....	.....	.....	.....	22.55	8.21§	.....	.....	.....
	1 681	"	0	0	1:3:0	5 mos. 27 days	157.13	423 000	2 692	2 177	.....	.....	.....	.....	15.9	8.85§	.....	.....	.....
	1 685	"	0	0	1:4:0	6 mos. 9 days	157.75	232 100	1 471	1 408	.....	.....	.....	.....	26.5	5.79§	.....	.....	.....
	1 686	"	0	0	1:5:0	6 mos. 9 days	157.0	163 000	1 038	1 072	.....	.....	.....	.....	24.2	6.23§	.....	.....	.....

TABLE 1.—(*Continued.*)

Source.	Column designation.	Reinforcement. Kind.	Vertical percentage.	Lateral percentage.	Ratio of mix.	Age.	Area, in square inches.	Maximum load, in pounds.	Ultimate strength, in pounds per square inch.	Average cylinder test.	Stress in concrete.	Stress in longitudinal steel.	f_c (Equation (27)).	$n f_c$.*	n.‖	m.	Cylinders. Age.	Cylinders. Ultimate strength, in pounds per square inch.	Remarks.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
and Other Materials, Watertown Arsenal, 1905	1 673	Eight, ¾ in. square twisted bars	2.845	0	1:2:0	5 mos. 10 days	158.13	664 146	4 200	3 358	2 551	60 500	.....	31 870	23.7	....	.....	.....	.....
	1 690		2.845	0	1:3:0	7 mos. 21 days	157.0	603 000	3 841	2 177	2 156	61 400	.....	60 650	23.2	8.79§	.....	.....	.....
	1 691		2.83	0	1:4:0	7 mos. 25 days	157.25	531 100	3 377	1 408	1 911	53 800	.....	71 150	28.16	7.11§	.....	.....	.....
	1 688		2.826	0	1:5:0	6 mos. 17 days	157.50	443 000	2 813	1 072	1 387	56 100	.....	63 100	40.45	....	.....	.....	.....
	1 689	Thirteen, ¾ in. square twisted bars	4.63	0	1:5:0	7 mos. 15 days	157.75	616 000	3 905	1 072	1 269	58 200	.....	62 400	45.90	....	.....	.....	.....
	1 656	None	0	0	1:1:2	5 mos. 7 days	159.14	620 645	3 900	3 580	.....	.....	.....	.....	10.6	....	.....	.....	.....
	1 683	"	0	0	1:2:4	6 mos. 5 days	161.38	321 200	1 990	1 827	.....	.....	.....	.....	12.3	....	.....	.....	.....
	1 650	"	0	0	1:1:2	5 mos. 7 days	160.02	244 000	1 525	3 580	.....	.....	.....	.....	10.8	....	.....	.....	.....
	1 651	"	0	0	1:2:4	5 mos. 6 days	160.64	242 000	1 506	1 827	.....	.....	.....	.....	19.2	....	.....	.....	.....
	1 667	"	0	0	1:2:5	3 mos. 8 days	133.6	146 960	1 100	1 100	.....	.....	.....	.....	15.7	....	.....	.....	.....
	1 649	"	0	0	1:3:6	5 mos. 5 days	159.26	111 482	700	1 038	.....	.....	.....	.....	57.8	....	.....	.....	.....
	1 655	"	0	0	1:3:6	5 mos. 6 days	159.39	169 000	1 060	1 088	.....	.....	.....	.....	52.2	....	.....	.....	.....
	1 652	Eight, ¾ in. square twisted bars	2.817	0	1:2:4	5 mos. 5 days	159.77	484 000	3 029	1 827	1 276	63 500	.....	44 000	49.7	....	.....	.....	.....
	1 653	Four, ¾ in. square twisted bars	1.416	0	1:3:6	5 mos. 5 days	158.76	306 700	1 923	1 033	1 014	65 700	.....	64 000	65.1	....	.....	.....	.....
	1 654	Eight, ¾ in. square twisted bars	2.83	0	1:3:6	5 mos. 6 days	159.01	493 000	3 100	1 033	1 035	74 000	.....	73 900	71.6	....	.....	.....	.....
	1 687	None	0	0	1:5:0	6 mos. 18 days	158.76	171 800	1 080	1 072	.....	.....	.....	.....	24.3	....	.....	.....	.....

TABLE 1.—(*Continued.*)

Source.	Column designation.	Reinforcement. Kind.	Reinforcement. Vertical percentage.	Reinforcement. Lateral percentage.	Ratio of mix.	Age.	Area, in square inches.	Maximum load, in pounds.	Ultimate strength, in pounds per square inch.	Average cylinder test.	Stress in concrete.	Stress in longitudinal steel.	f_s (Equation (27)).	$n f_c$*	n.	m.	Cylinders. Age.	Cylinders. Ultimate strength, in pounds per square inch.	Remarks.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	19)	(20)
Report of Tests Metals	1 718	Steel wire cage, 2 by 2 in. mesh, 0.104 in. in diameter.....	0.135	0.1416	1 : 2 : 4	5 mos. 8 days	113.1	263 538	2 330	1 827	2 279	36 500	.....	.....	16.8	0.46	.....	.....	.....
	1 722	Steel wire cage. 3 by 3 in. mesh, 0.104 in. in diameter.....	0.0978	0.0945	1 : 2 : 4	5 mos. 13 days	113.1	242 000	2 140	1 827	2 110	32 200	.....	.....	15.3	0.411	.....	.....	.....
	1 724	Steel wire cage. 3 by 3 in. circular wires, 0.145 in. in diameter and vertical wires, 0.104 in. in diameter.....	0.0934	0.1833	1 : 2 : 4	5 mos. 13 days	113.1	318 000	2 810	1 827	2 767	47 140	.....	.....	17.0	0.354	.....	.....	.....
	1 720	Steel wire cage, 2 by 2 in. mesh, 0.104 in. in diameter.....	0.1426	0.1416	1 : 3 : 6	5 mos. 8 days	113.1	159 111	1 406	1 033	1 277	91 000	.....	.....	72.2	2.023	.....	.....	.....
	1 719		0.1426	0.1416	1 : 3 : 6	5 mos. 8 days	113.1	168 000	1 480	1 033	1 368	82 600	.....	.....	60.3	1.428	.....	.....	.....
	1 721		0.1426	0.1416	1 : 3 : 6	5 mos. 8 days	113.1	183 500	1 623	1 033	1 513	78 160	.....	.....	51.6	0.885	.....	.....	.....
	1 723	Steel wire cage, 3 by 3 in. mesh, 0.104 in. in diameter.....	0.0978	0.0945	1 : 3 : 6	5 mos. 12 days	113.1	197 340	1 745	1 033	1 707	39 000	.....	.....	22.9	0.173	.....	.....	.....
	1 725	Steel wire cage, 3 by 3 in. mesh 0.145 in. in diameter, circular and 0.104 in. in diameter vertical..	0.0978	0.1833	1 : 3 : 6	5 mos. 11 days	113.1	206 000	1 821	1 033	1 736	98 200	.....	.....	45.6	0.795	.....	.....	.....

TABLE 1.—(*Continued.*)

Source.	Column designation.	Reinforcement. Kind.	Reinforcement. Vertical percentage.	Reinforcement. Lateral percentage.	Ratio of mix.	Age.	Area, in square inches.	Maximum load, in pounds.	Ultimate strength, in pounds per square inch.	Average cylinder test.	Stress in concrete.	Stress in longitudinal steel.	f_s (Equation (27)).	$n f_c$.*	n.‖	m.	Cylinders. Age.	Cylinders. Ultimate strength, in pounds per square inch.	Remarks.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
and Other Materials, Watertown Arsenal, 1906	1734	None	0	0	1:1:0	6 mos. 11 days	84.95	367 000	4 320	4 292	.....	.....	.....	.....	10.88	.....	.....	.....	.....
	1735	Twenty-five hoops 1.5 by 0.12 in. and four angles, 1 by 1 by ⅛ in.	1.158	1.8	1:1:0	6 mos. 12 days	78.54	508 000	6 470	4 292	5 840	60 000‡	.....	.....	14.23	2.35	.....	.....	Equation (23).
	1730		1.158	1.8	1:1:2	5 mos. 0 days	78.54	466 000	5 930	3 580	5 300	60 000‡	.....	.....	21.95	4.0	.....	.....	" "
	1736	None	0	0	1:4:0	6 mos. 8 days	84.62	88 851	1 050	1 408	.....	.....	.....	.....	30.9	.....	.....	.....	.....
	1737	Same as No. 1735	1.158	1.8	1:4:0	6 mos. 8 days	78.54	235 000	2 992	1 408	2 323	60 000‡	44 050	.....	80.0	.....	.....	.....	Equation (27).
	1733	" " " "	1.158	1.8	1:4:8	6 mos. 11 days	78.54	255 000	3 246	1 408	2 580	60 000‡	48 900	.....	96.9	.....	.....	.....	" "
	1731	None	0	0	1:2:4	5 mos. 16 days	84.95	120 000	1 413	1 827	.....	.....	.....	.....	12.15	.....	.....	.....	.....
	1740	Thirteen hoops 1.5 by 0.12 in.; spaced 8 in. c. to c	0	0.9	1:2:4	6 mos. 6 days	78.54	188 900	2 405	1 827	2 405	.....	91 800	.....	49.0	5.96	.....	.....	Equation (27).
	1741	Twenty-five hoops 1.5 by 0.12 in.; spaced 4 in. c. to c	0	1.8	1:2:4	6 mos. 5 days	78.54	290 000	3 692	1 827	3 692	.....	68 800	.....	43.77	4.6	.....	.....	" "
	1739	Forty-seven hoops 1.5 by 0.12 in.; spaced 2 in. c. to c	0	3.6	1:2:4	6 mos. 6 days	78.54	445 000	5 665	1 827	5 665	.....	54 050	.....	.....	.....	.....	.....	" "

TABLE 1.—(*Continued*).

Source.	Column designation.	Reinforcement. Kind.	Reinforcement. Vertical percentage.	Reinforcement. Lateral percentage.	Ratio of mix.	Age.	Area, in square inches.	Maximum load, in pounds.	Ultimate strength, in pounds per square inch.	Average cylinder test.	Stress in concrete.	Stress in longitudinal steel.	f_s (Equation (27)).	$n f_c$.*	n.‖	m.	Cylinders. Age.	Cylinders. Ultimate strength, in pounds per square inch.	Remarks.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
Report of Tests of Metals	1738	Thirteen hoops 1.5 by 0.12 in.; and 4 angles, 1 by 1 by ⅛ in.....	1.158	0.90	1:2:4	6 mos. 7 days	78.54	256 300	3 262	1 827	2 597	60 000‡	.....	.....	40.7	4.0	.....	.....	.....
	1692	Twenty-five hoops 1.5 by 0.12 in. and four angles, 1 by 1 by ⅛ in.....	1.158	1.80	1:2:4	15 days	78.54	260 400	3 315	1 827	2 650	60 000‡	50 500	.....	.....	.....	.....	.....	Equation (27).
	1726		1.158	1.80	1:2:4	5 mos. 12 days	78.54	349 000	4 443	1 827	3 795	60 000‡	72 300	.....	59.9	.....	.....	.....	" "
	1728		1.158	1.80	1:2:4	5 mos. 12 days	78.54	370 000	4 710	1 827	4 060	60 000‡	77 400	.....	43.1	.....	.....	.....	" "
	1746	Forty-eight hoops 1.04 by 0.06 in. and four angles, 1 by 1 by ⅛ in...	1.158	1.248	1:2:4	5 mos. 12 days	78.54	324 000	4 125	1 827	3 470	60 000‡	.....	.....	34.6	3.01	.....	.....	.....
	1744		1.158	1.248	1:2:4	6 mos. 0 days	78.54	335 200	4 267	1 827	3 615	60 000‡	.....	.....	30.92	2.62	.....	.....	.....
	1732	None.....	0	0	1:3:6	5 mos. 11 days	84.62	63 465	750	1 033	.....	.....	.....	.....	22.1	.....	.....	.....	.....
	1742	Twenty-five hoops 1.5 by 0.12 in.; spaced 4 in. c. to c.....	0	1.80	1:3:6	6 mos. 4 days	78.54	162 300	2 066	1 033	2 066	.....	39 400	.....	80.0	.....	.....	.....	Equation (27).
	1743		0	1.80	1:3:6	6 mos. 4 days	78.54	96 000	1 222	1 033	1 222	....	23 300	.....	108.6	.....	.....	.....	" "
	1727	Twenty-five hoops 1.5 by 0.12 in. and 4 angles, 1 by 1 by ⅛ in.....	1.158	1.80	1:3:6	5 mos. 11 days	78.54	325 000	4 139	1 033	3 495	.....	66 600	.....	48.1	.....	.....	.....	" "
	1729		1.158	1.80	1:3:6	5 mos. 3 days	78.54	299 000	3 808	1 033	3 150	.....	60 000	.....	58.5	.....	.....	.....	" "
	1 747	48 hoops, 1.06 by 0.06 in. and 4 angles, 1 by 1 by ⅛ in.....	1.158	1.272	1:3:6	5 mos. 11 days	78.54	212 000	2 700	1 033	2 028	60 000‡	54 700	.....	91.35	.....	.....	.....	" "
	1 745		1.158	1.272	1:3:6	5 mos. 13 days	78.54	259 000	3 298	1 033	2 632	60 000‡	71 100	.....	66.9	.....	.....	.....	" "

TABLE 1.—(*Continued.*)

Source.	Column designation.	Reinforcement. Kind.	Reinforcement. Vertical percentage.	Reinforcement. Lateral percentage.	Ratio of mix.	Age.	Area, in square inches.	Maximum load, in pounds.	Ultimate strength, in pounds per square inch.	Average cylinder test.	Stress in concrete.	Stress in longitudinal steel.	f_s (Equation (27)).	$n\,f_c$.*	n.‖	m.	Cylinders. Age.	Cylinders. Ultimate strength, in pounds per square inch.	Remarks.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
and Other Materials, Watertown Arsenal, 1907.→	1 809	None	0	0	1 : 0 : 1	8 mos. 19 days	144.6	514 000	3 555	4 292					15.02				
	1 799	"	0	0	1 : 0 : 2	7 mos. 26 days	144.48	408 000	2 824	3 580					16.82				
	1 794	"	0	0	1 : 0 : 3	7 mos. 23 days	144.96	281 000	1 938	2 177					19.57				
	1 798	"	0	0	1 : 0 : 4	7 mos. 25 days	146.4	213 000	1 427	1 408					38.52				
	1 797	"	0	0	1 : 0 : 5	7 mos. 24 days	144.48	161 400	1 117	1 072					55.5				
	1 793	"	0	0	1 : 0 : 6	7 mos. 16 days	145.08	121 000	834	892					36.4				
	1 796	"	0	0	1 : 0 : 7	7 mos. 17 days	144.72	108 540	750	800					29.85				
	1 792	"	0	0	1 : 0 : 8	7 mos. 14 days	144.84	101 100	697	780					99.0				
	1 812	"	0	0	1 : 0 : 2	9 mos. 0 days	155.38	800 000	5 149	3 580									Column did not fail.
	1 788	"	0	0	1 : 0 : 3	8 mos. 5 days	154.38	389 000	2 520	2 177					19.47				
	1 791	"	0	0	1 : 0 : 4	8 mos. 4 days	155.0	170 500	1 100	1 408					51.6				
	1 790	"	0	0	1 : 0 : 1	8 mos. 11 days	155.75	643 000	4 128	4 292					14.37				
	1 813	"	0	0	1 : 0 : 1	9 mos. 9 days	155.87	792 000	5 081	4 292					9.97				
	1 810	"	0	0	1 : 0 : 2	9 mos. 8 days	154.88	431 000	2 783	3 358					15.45				

TABLE 1.—(*Continued.*)

Source.	Column designation.	Reinforcement. Kind.	Vertical percentage.	Lateral percentage.	Ratio of mix.	Age.	Area, in square inches.	Maximum load, in pounds.	Ultimate strength, in pounds per square inch.	Average cylinder test.	Stress in concrete.	Stress in longitudinal steel.	f_s (Equation (27)).	nf_c.*	n.‖	m.	Cylinders. Age.	Cylinders. Ultimate strength, in pounds per square inch.	Remarks.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
Report of Tests of Metals	1 811	None	0	0	1 : 0 : 2	9 mos. 11 days	155.75	528 000	3 390	3 358	.....	.....	.....	.....	10.56	.....	.....	.....	.....
	1 789	"	0	0	1 : 0 : 3	8 mos. 8 days	156.12	307 500	1 970	2 177	.....	.....	.....	.....	24.79	.....	.....	.....	.....
	1 802	"	0	0	1 : 0 : 2	7 mos. 19 days	156.52	549 900	3 753	3 358	.....	.....	.....	.....	7.12	.....	.....	.....	.....
	1 801	"	0	0	1 : 0 : 4	7 mos. 14 days	145.44	275 300	1 893	1 408	.....	.....	.....	.....	27.05	.....	.....	.....	.....
	1 795	"	0	0	1 : 0 : 6	7 mos. 12 days	144.96	137 712	950	892	.....	.....	.....	.....	55.6	.....	.....	.....	.....
	1 800	"	0	0	1 : 0 : 8	7 mos. 12 days	144.96	131 100	904	780	.....	.....	.....	.....	38.7	.....	.....	.....	.....
	1 808	"	0	0	1 : 0 : 1	7 mos. 19 days	146.05	639 000	4 375	4 292	.....	.....	.....	.....	11.28	.....	.....	.....	.....
	1 807	"	0	0	1 : 0 : 2	7 mos. 20 days	145.08	409 000	2 819	3 358	.....	.....	.....	.....	14.22	.....	.....	.....	.....
	1 806	"	0	0	1 : 0 : 3	7 mos. 4 days	144.60	272 000	1 881	2 177	.....	.....	.....	.....	15.84	.....	.....	.....	.....
	1 805	"	0	0	1 : 1 : 1	7 mos. 10 days	145.92	514 000	3 522	3 580	.....	.....	.....	.....	14.78	.....	.....	.....	.....
	1 804	"	0	0	1 : 2 : 2	7 mos. 8 days	144.84	192 000	1 326	2 000	.....	.....	.....	.....	17.10	.....	.....	.....	.....
	1 803	"	0	0	1 : 3 : 3	7 mos. 5 days	145.08	121 000	834	834	.....	.....	.....	.....	29.10	.....	.....	.....	.....
	1 773	"	0	0	1 : 0 : 0	10 mos. 25 days	84.46	591 220	7 000	.....	.....	.....	.....	.....	10.0	.....	.....	.....	Column did not fail.
	1 774	"	0	0	1 : 1.5 : 3	10 mos. 23 days	84.46	302 000	3 576	3 580	.....	.....	.....	.....	9.92	.....	.....	.....	.....

TABLE 1.—(*Continued.*)

Source.	Column designation.	Reinforcement.			Ratio of mix.	Age.	Area, in square inches.	Maximum load, in pounds.	Ultimate strength, in pounds per square inch.	Average cylinder test.	Stress in concrete.	Stress in longitudinal steel.	f_s (Equation (27)).	nf_c.*	n.‖	m.	Cylinders.		Remarks.
		Kind.	Vertical percentage.	Lateral percentage.													Age.	Ultimate strength, in pounds per square inch.	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
	1 775	None	0	0	1 : 2 : 4	10 mos. 22 days	112.53	270 072	2 400	1 827	.....	.....	.....	.....	12.92	.....	.....	.....	.....
	1 750	"	1.158	1.248	1 : 1 : 0	12 mos. 5 days	78.54	520 000	6 621	4 292	5 995	60 000	.....	.....	15.19	.....	.....	.....	.....
	1 748	48 hoops 1.04 by 0.06 in., 4 angles	1.158	1.248	1 : 1 : 2	12 mos. 3 days	78.54	479 000	6 099	3 580	5 470	60 000	.....	.....	16.05	.....	.....	.....	.....
	1 751	1 by 1 by ⅛ in...	1.158	1.248	1 : 4 : 0	12 mos. 4 days	78.54	233 000	2 967	1 408	2 298	60 000	63 100	.....	67.6	.....	.....	.....	Equation (27).
	1 749		1.158	1.248	1 : 4 : 8	12 mos. 4 days	78.54	215 000	2 737	.....	2 066	60 000	.....	.....	64.7	.....	.....	.....	No. 1:4:8 plain concrete test.
	1 757	4 angles 1 by 1 by ⅛ in. held by two	1.077	0	1 : 2 : 4	11 mos. 24 days	78.54	133 800	1 704	1 827	1 376	29 630	.....	.....	21.54	.....	.....	.....	Not as strong as plain concrete column.
	1 758	hoops 1.04 by 0.06 in.....	1.077	0	1 : 2 : 4	11 mos. 20 days	78.54	140 000	1 783	1 827	1 287	44 100	.....	.....	34.25	.....	.....	.....	
	1 756	4 angles 1 by 1 by ⅛ in., and 5 hoops 1.04 by 0.06 in....	1.075	0	1 : 2 : 4	11 mos. 25 days	78.54	135 392	1 700	1 827	1 269	37 300	.....	.....	31.9	.....	.....	.....	
	1 754	24 hoops 1.04 by 0.06 in., spaced 4 in. c. to c.....	0	0.624	1 : 2 : 4	11 mos. 23 days	78.54	186 000	2 367	1 827	2 368	.....	.....	.....	48.1	4.45	.....	.....	.....
	1 752	48 hoops 1.04 by 0.06 in., spaced	0	1.248	1 : 2 : 4	12 mos. 0 days	78.54	262 000	3 335	1 827	3 335	.....	.....	.....	26.7	2.5	.....	.....	.....
	1 753	2 in. c. to c.....	0	1.248	1 : 3 : 6	11 mos. 24 days	78.54	232 500	2 957	1 033	2 970	.....	.....	.....	66.5	3.7	.....	.....	.....
	1 765	25 hoops 1.5 by 0.12 in., 4 angles, 1 by 1 by ⅛ in....	1.158	1.80	1 : 3 : 6	14 mos. 0 days	78.54	288 000	3 667	1 033	3 012	60 000	89 200	.....	93.0	.....	.....	.....	Equation (27).

TABLE 1.—(*Continued.*)

Source.	Column designation.	Reinforcement. Kind.	Vertical percentage.	Lateral percentage.	Ratio of mix.	Age.	Area, in square inches.	Maximum load, in pounds.	Ultimate strength, in pounds per square inch.	Average cylinder test.	Stress in concrete.	Stress in longitudinal steel.	f_s (Equation (27)).	nf_c.*	n ‖.	m.	Cylinders. Age.	Cylinders. Ultimate strength, in pounds per square inch.	Remarks.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
Watertown Arsenal, 1906	1 755	25 hoops 1.04 by 0.06 in., spaced, 4 in. c. to c.	0	0.624	1 : 3 : 6	12 mos. 0 days	78.54	71 600	912	1 033	912	.....	50 150	.....	213.0	.....	.....	.....	Equation (27).
	1 784	Four, ¾ in. square twisted bars.	1.444	0	1 : 2 : 4	17 mos. 21 days	155.99	358 777	2 300	1 827	1 765	38 850	.....	34 500	22.0	.....	.....	.....	.....
	1 785	Eight, ¾ in. square twisted bars.	2.896	0	1 : 2 : 4	17 mos. 22 days	155.37	574 869	3 700	1 827	2 130	56 400	.....	59 600	26.46	.....	.....	.....	.....
	1 776	¼ in. helix, pitch 2 in., and 4 rods 0.62 in. in diameter.	3.306	1.57	1 : 2 : 0	9 mos. 20 days	36.6	172 000	4 702	3 358	2 786	57 500	60 800	.....	20.63	.....	.....	.....	Equation (27).
	1 777	0.3 in. helix, pitch 1½ in., 4 rods, 0.62 in. in diameter.	2.11	2.20	1 : 2 : 2	10 mos. 0 days	57.4	435 000	7 580	2 000	6 780	37 920	105 800	.....	5.58	0.677	.....	.....	Strength of plain concrete assumed.
	1 778	5/16 in. helix, pitch 1 in. 6 rods, 0.62 in. in diameter.	2.15	2.962	1 : 2 : 2	10 mos. 14 days	84.1	850 000	10 450	2 000	6 430	60 000‡	74 000	.....	9.80	.....	.....	.....	Equation (27).
	1 715	Four Truscan bars each 0.76 in. in diameter. Eleven ⅝ in. wire hoops.	1.163	0	1 : 2 : 4	5 mos. 10 days	155.5	367 500	2 363	1 827	1 650	62 900	.....	39 200	38.1	.....	.....	.....	.....
	1 705	None.	0	0	1 : 3 : 6	5 mos. 5 days	113.47	164 000	1 446	1 033	1 446	.....	.....	.....	21.1	.....	.....	.....	.....
	1 706	"	0	0	1 : 3 : 6	5 mos. 8 days	113.47	143 000	1 260	1 033	1 260	.....	.....	.....	17.2	.....	.....	.....	.....
	1 707	"	0	0	1 : 3 : 6	5 mos. 0 days	113.10	78 900	698	1 033	698	.....	.....	.....	30.8	.....	.....	.....	Cinder concrete.

TABLE 1.—(*Continued.*)

Source.	Column designation.	Reinforcement.			Ratio of mix.	Age.	Area, in square inches.	Maximum load, in pounds.	Ultimate strength, in pounds per square inch.	Average cylinder test.	Stress in concrete.	Stress in longitudinal steel.	f_s (Equation (27))	$n f_c$.*	n.‖	m.	Cylinders.		Remarks.
		Kind.	Vertical percentage.	Lateral percentage.													Age.	Ultimate strength, in pounds per square inch.	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
Am. Soc. C. E., Vol. LXXVIII (1915)	36	None	0	0	1 : 2 : 4	42 days	12.76	14 120	1 108										
	37	"	0	0	1 : 2 : 4	41 "	12.88	12 260	950										
	39	"	0	0	1 : 2 : 4	40 "	12.88	14 560	1 130										
	41	"	0	0	1 : 2 : 4	40 "	12.88	11 640	905										
	113	"	0	0	1 : 2 : 4	35 "	12.79	17 200	1 350										
		Average	0	0	1 : 2 : 4	40 "			1 089										
	38	None	0	0	1 : 2 : 4	63 "	12.69	22 660	1 790										
	42	"	0	0	1 : 2 : 4	58 "	12.76	21 980	1 720										
	43	"	0	0	1 : 2 : 4	73 "	13.01	16 600	1 275										
	44	"	0	0	1 : 2 : 4	74 "	12.88	17 840	1 385										
	49	"	0	0	1 : 2 : 4	71 "	12.69	29 890	2 345										
	58	"	0	0	1 : 2 : 4	61 "	12.76	22 420	1 760										
	61	"	0	0	1 : 2 : 4	77 "	12.82	29 450	2 300										
		Average	0	0	1 : 2 : 4	68 "			1 796										
	45	None	0	0	1 : 2 : 4	268 "	12.76	34 980	2 740										
	46	"	0	0	1 : 2 : 4	296 "	12.76	37 700	2 955										
	47	"	0	0	1 : 2 : 4	288 "	12.88	38 000	2 955										
	50*a*	"	0	0	1 : 2 : 4	340 "	12.76	37 000	2 900										
	53	"	0	0	1 : 2 : 4	295 "	12.69	39 570	3 120										
	67	"	0	0	1 : 2 : 4	323 "	12.69	31 300	2 470										
	76	"	0	0	1 : 2 : 4	376 "	12.76	28 000	2 200										
		Average	0	0	1 : 2 : 4	313 "			2 763										
	48	None	0	0	1 : 2 : 4	414 "	12.82	45 800	3 570										
	51	"	0	0	1 : 2 : 4	434 "	12.82	46 300	3 600										
	52	"	0	0	1 : 2 : 4	440 "	12.82	35 000	2 725										
	54	"	0	0	1 : 2 : 4	441 "	12.88	40 700	3 160										
	62	"	0	0	1 : 2 : 4	434 "	12.76	36 350	2 850										
	79	"	0	0	1 : 2 : 4	416 "	12.95	28 060	2 170										
	80	"	0	0	1 : 2 : 4	417 "	12.88	29 000	2 250										
		Average	0	0	1 : 2 : 4	428 "			2 903										

TABLE 1.—(*Continued.*)

Source.	Column designation.	Reinforcement. Kind.	Reinforcement. Vertical percentage.	Reinforcement. Lateral percentage.	Ratio of mix.	Age.	Area, in square inches.	Maximum load, in pounds.	Ultimate strength, in pounds per square inch.	Average cylinder test.	Stress in concrete.	Stress in longitudinal steel.	f_s (Equation (27))	$n f_c$.*	n.‖	m.	Cylinders. Age.	Cylinders. Ultimate strength, in pounds per square inch.	Remarks.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
Talbot and Slater, "Tests of Reinforced Concrete Columns," Bulletin No. 1, University of Illinois, 1904.	110	0.062 in. wire helix	0	0.47	1:2:4	35 days	.....	23 700	2 650	1 490	.....	.....	.....	.....	.....	.....	.....	.....	.....
	71	¾ in. pitch.....	0	0.45	1:2:4	317 "	.....	274 500	2 773	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
	74		0	0.44	1:2:4	370 "	.....	47 800	4 632	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
	75	0.062 in. wire helix,	0	0.54	1:2:4	300 "	.....	40 500	4 091	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
	78	0.62 in. pitch....	0	0.54	1:2:4	281 "	.....	31 300	3 160	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
	82		0	0.52	1:2:4	413 "	.....	33 700	3 265	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
		Average of 71-82,....	0	0.50	1:2:4	336 "	.....	.....	3 584	2 833	.....	.....	.....	.....	.....	.....	.....	.....	.....
	66	0.079 in. wire helix	0	0.72	1:2:4	74 "	.....	29 000	2 810	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
		¾ in. pitch.....																	
	101	0.062 in. wire helix,	0	0.71	1:2:4	29 "	.....	22 000	2 460	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
		½ in. pitch.....																	
	107	0.079 in. wire helix	0	0.78	1:2:4	35 "	.....	28 000	3 130	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
		¾ in. pitch.....																	
		Average.....	0	0.74	1:2:4	46 "	.....	.....	2 800	1 490	.....	.....	.....	.....	.....	.....	.....	.....	.....
	60	0.079 in. wire helix, ¾ in. pitch.....	0	0.72	1:2:4	311 "	.....	38 200	3 702	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
	64		0	0.67	1:2:4	435 "	.....	41 500	3 520	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
	69		0	0.72	1:2:4	323 "	.....	31 350	3 040	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
	72		0	0.72	1:2:4	388 "	.....	33 500	3 246	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
	81	0.079 in. wire helix, 0.62 in. pitch....	0	0.86	1:2:4	270 "	.....	36 250	3 560	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
		Average.....	0	0.74	1:2:4	345 "	.....	.....	3 406	2 833	.....	.....	.....	.....	.....	.....	.....	.....	.....
	59	0.015 in. wire helix, ¾ in. pitch.....	0	1.31	1:2:4	67 "	.....	40 250	4 184	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
	98	0.079 in. wire helix. ½ in. pitch.....	0	1.16	1:2:4	31 "	.....	31 500	3 520	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
	104	0.105 in. wire helix, ¾ in. pitch.....	0	1.35	1:2:4	35 "	.....	24 500	2 740	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
	105		0	1.35	1:2:4	49 "	.....	40 000	4 470	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
	106		0	1.35	1:2:4	44 "	.....	34 800	3 890	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
		Average.....	0	1.30	1:2:4	45 "	.....	.....	3 761	1 490	.....	.....	.....	.....	.....	.....	.....	.....	.....
	63	0.105 in. wire helix, ¾ in. pitch.....	0	1.24	1:2:4	395 "	.....	37 500	3 520	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
	65		0	1.26	1:2:4	316 "	.....	42 160	4 085	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
	68		0	1.24	1:2:4	321 "	.....	36 570	3 420	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
	73		0	1.26	1:2:4	428 "	.....	48 600	4 710	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
		Average.....	0	1.25	1:2:4	365 "	.....	.....	3 934	2 833	.....	.....	.....	.....	.....	.....	.....	.....	.....

TABLE 1.—(*Continued.*)

Source.	Column designation.	Reinforcement. Kind.	Reinforcement. Vertical percentage.	Reinforcement. Lateral percentage.	Ratio of mix.	Age.	Area, in square inches.	Maximum load, in pounds.	Ultimate strength, in pounds per square inch.	Average cylinder test.	Stress in concrete.	Stress in longitudinal steel.	f_s (Equation (27)).	nf_c.*	n.‖	m.	Cylinders. Age.	Cylinders. Ultimate strength, in pounds per square inch.	Remarks.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
Series of Tests on Concrete Columns Reinforced with a Spiral	56	0.079 in. wire helix,	0	1.68	1:2:4	58 days	.....	40 590	4 219	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
	57	⅛ in. pitch.....	0	1.65	1:2:4	59 "	.....	39 000	3 939	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
	94	0.062 in. wire helix,	0	1.41	1:2:4	35 "	.....	35 270	3 940	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
		¼ in. pitch.....																	
		Average.....	0	1.58	1:2:4	51 "	.....	.....	4 033	1 490	.....	.....	.....	.....	.....	.....	.....	.....	.....
	77	0.105 in. wire helix,	0	1.51	1:2:4	277 "	.....	42 700	4 138	2 833	.....	.....	.....	.....	.....	.....	.....	.....	.....
		0.62 in. pitch....																	
	89	0.079 in. wire helix,	0	2.31	1:2:4	35 "	.....	49 900	5 580	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
		¼ in. pitch																	
	96	0.105 in. wire helix,	0	2.02	1:2:4	35 "	.....	42 700	4 770	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
	97	½ in. pitch.....	0	2.02	1:2:4	56 "	.....	51 580	5 760	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
		Average of 89, 97, 96.	0	2.11	1:2:4	42 "	.....	.....	5 370	1 490	.....	.....	.....	.....	.....	.....	.....	.....	.....
	70	0.079 in. wire helix,	0	2.32	1:2:4	393 "	.....	53 690	6 000	2 833	.....	.....	.....	.....	.....	.....	.....	.....	.....
		¼ in. pitch.....																	
	87	0.105 in. wire helix,	0	4.10	1:2:4	35 "	.....	71 800	7 965	1 490	.....	.....	.....	.....	.....	.....	.....	.....	.....
		¼ in. pitch.....																	
	10	0.105 in. wire helix,	0	1.87	1:2:4	28 "	.....	30 770	3 020	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
		½ in. pitch.....																	
	16	0.079 in. wire helix,	0	1.87	1:2:4	28 "	.....	45 550	4 740	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
	17	¼ in. pitch.....	0	1.87	1:2:4	28 "	.....	51 100	5 310	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
		Average 10, 16, 17....	0	1.87	1:2:4	28 "	.....	.....	4 390	1 490	.....	.....	.....	.....	.....	.....	.....	.....	.....
	11	0.104 in. wire helix	0	1.25	1:2:4	28 "	.....	30 100	3 130	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
		¾ in. pitch.....																	
	18	0.079 in. wire helix,	0	1.26	1:2:4	28 "	.....	47 670	4 950	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
		⅛ in. pitch.....																	
		Average 11 and 18...	0	1.255	1:2:4	28 "	.....	.....	4 040	1 490	.....	.....	.....	.....	.....	.....	.....	.....	.....
	12	0.134 in. wire helix,	0	2.09	1:2:4	28 "	.....	32 300	3 360	1 490	.....	.....	.....	.....	.....	.....	.....	.....	.....
		¾ in. pitch.....																	
	13	0.165 in. wire helix,	0	2.30	1:2:4	28 "	.....	42 100	4 370	1 490	.....	.....	.....	.....	.....	.....	.....	.....	.....
		1 in. pitch.....																	
	14	0.165 in. wire helix,	0	1.54	1:2:4	28 "	.....	32 500	3 380	1 490	.....	.....	.....	.....	.....	.....	.....	.....	.....
		1½ in. pitch.....																	

TABLE 1.—(*Continued.*)

Source.	Column designation.	Reinforcement. Kind.	Vertical percentage.	Lateral percentage.	Ratio of mix.	Age.	Area, in square inches.	Maximum load, in pounds.	Ultimate strength, in pounds per square inch	Average cylinder test.	Stress in concrete.	Stress in longitudinal steel.	f_s (Equation (27)).	nf_c.*	n.‖	m.	Cylinders. Age.	Cylinders. Ultimate strength, in pounds per square inch.	Remarks.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
"Report on a	15	0.079 in. wire helix, ¼ in. pitch	0	1.95	1:2:4	28 days	.....	52 800	5 490	1 490	.....	.....	.....	.....	.....	.....	.....	.....	.....
	19	0.079 in. wire helix,	0	0.97	1:2:4	28 "	.....	39 600	4 120	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
	20	½ in. pitch.....	0	0.96	1:2:4	28 "	.....	34 200	3 560	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
		Average 19 and 20...	0	0.965	1:2:4	28 "	.....	.....	3 840	1 490	.....	.....	.....	.....	.....	.....	.....	.....	.....
	21	0.079 in. wire helix, ½ in. pitch.....	0	0.41	1:2:4	28 "	.....	49 000	5 090	1 490	.....	.....	.....	.....	.....	.....	.....	.....	.....
	22	0.079 in. wire helix, 0.70 in. pitch	0	0.69	1:2:4	28 "	.....	40 950	4 250	1 490	.....	.....	.....	.....	.....	.....	.....	.....	.....
	92	0.062 in. wire helix, ¼ in. pitch.....	0	1.41	1:2:4	34 "	.....	24 000	2 680	1 490	.....	.....	.....	.....	.....	.....	.....	.....	.....
	101	0.062 in. wire helix, ½ in. pitch.....	0	0.71	1:2:4	29 "	.....	22 000	2 460	1 490	.....	.....	.....	.....	.....	.....	.....	.....	.....
↓	113	None	0	0	1:2:4	35 "	.....	17 200	1 350	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....

* The value in Column (15) was obtained from the formula, $f = f_c [1 + (n - 1) p]$, in which, n was the unknown quantity, f_c being the value taken from the average cylinder tests or plain columns of the same mix. † This value is the average of all the cylinder tests of 1 : 2 : 4 concrete. ‡ The value was assumed, the longitudinal steel having been stressed considerably beyond the elastic limit. § Observed from extensometer readings some time before failure. ‖ The values shown in the columns for n (16) are taken from the observed values of the tests at failure by extrapolation whenever such data were given. No doubt these values are generally less than the real values, since the deformation becomes proportionately much greater at failure. Having given the value of n, the values in the column for m (17) were then calculated from Equation (21) wherein all the other values were assumed to be known. As may be seen some are less than those obtained under the assumption of constant volume. However, the volume of concrete under compression always decreases. In such cases these calculated values are therefore too small. In which case either n, ϕ, or the strength of the concrete was greater than the assumed value.

Equation (21), $v = \dfrac{f_c'}{1 - \dfrac{n\,p'\tan^2(45° + \frac{1}{2}\phi)}{2\,m + (m-1)\,np'}}$; Equation (23), $v = \dfrac{v_c}{1 - \dfrac{2\,n\,p'}{m[2\,m + (m-1)\,n\,p']}}$; Equation (27), $v = \dfrac{1}{2} f_s\, p' \tan^2\left(45° + \dfrac{1}{2}\phi\right)$.

DISCUSSION

MISSAC THOMPSON,* ASSOC. M. AM. SOC. C. E. (by letter).—This paper with its tabulation of experimental results, is certainly of interest to all engineers concerned with the design of concrete columns. It would have been far more interesting from a designer's point of view, if some data were included as to measurements of the actual shortening of columns under working loads with and without helical reinforcement. There has never been any real question in regard to the effectiveness of helical reinforcement as far as the ultimate strength of the column is concerned under the action of direct load, but rather as to what extent, if any, the helical reinforcement strengthens the column when the unit pressure on the concrete is 60% or less of its crushing strength.

Generally speaking, the total direct load coming on a column will seldom be greater than the amount a properly designed column can carry. A factor of safety is chiefly required to provide against (1) poor concrete; and (2) flexure due to unbalanced loading, rigidity, unhomogeneity, etc.

It is obvious that the latent strength of helically reinforced concrete columns will serve to good advantage when the concrete is poor, but that against flexure it will be of little use. It is a matter of judgment for a designer to decide when to rely on helical reinforcement. In one and two-story building columns the effect of flexure is important and, therefore, helical reinforcement is out of place. In a multiple-story building, however, the effect of poor concrete is more important, hence helical reinforcement serves a useful purpose.

In a reinforced concrete structure the amount of flexure to which a column is subjected, is indeterminate. The writer's observations on the behavior of concrete columns makes him skeptical about results based on the rigid frame theory. The construction joints materially change the results. The actual flexure on interior as well as exterior columns is invariably much less than that given by the rigid frame theory. Moreover, shrinkage, unequal settlement, temperature change, etc., will develop flexure in columns that could not be computed by any reasonable theory. In columns the reserve strength against flexure as well as against direct load is the only basis for proper design within the limits of the same factor of safety as is used for other parts of the structure.

It is to be noted that even if the pressure on the cross-section of the column is not uniform, as long as there is no reversal of stress on any part of the column, the resulting lateral elongation (and, consequently, the stress in the spirals and the resulting resistance to shearing failure) will be the same as if the pressure was uniformly distributed over the cross-section. Therefore, in that case, helical reinforcement will be useful to a certain extent in providing against flexure.

In the case of direct load and flexure such as will not cause any reversal of stress in any part of the column, the authors' reasoning regarding the benefit of helical reinforcement still holds, except that v of Equation (21) must be reduced by the ratio of average to maximum stress.

* Engr. and Archt., Brooklyn, N. Y.

The authors have disregarded the fact that, as the column shortens, each spiral coil lengthens by an amount:

$$\frac{e_v}{\text{Number of coils per unit of height}}$$

It is evident that, for columns 20 in. or more in diameter, this omission is of no consequence, but for smaller columns it affects the results considerably.

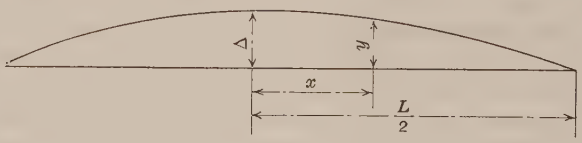

FIG. 5.

General Theory.—A general review from a theoretical standpoint of a homogeneous column under axial load is of interest. Consider a homogeneous column with hinged ends loaded axially:

Let A = cross-section of the column;
P = total load on the column;
I = least moment of inertia of the column section;
r = radius of gyration corresponding to I;
L = length of the column;
E = modulus of elasticity;
C = perpendicular distance from the extreme fiber to the neutral axis on which I is taken;
Δ = maximum deflection of the column under axial load;
R = radius of curvature of the bent column at any point, X, Y;
M = moment at any point of the column;
U = extreme fiber stress;
K = factor of safety.

Referring to Fig. 5, the column when bent, is subject to a positive moment,

$$M = P\,y \qquad (32)$$

Assuming y to be positive upward,

$$\frac{E\,I}{P\,y} = \frac{-\left[1 + \left(\frac{d\,y}{d\,x}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2\,y}{d\,x^2}} \qquad (33)$$

Assume the curve to be so flat that $\frac{d\,s}{d\,x} = I$, Equation (33) will reduce to:

$$-\frac{P\,y}{E\,I} = \frac{d^2\,y}{d\,x^2} \qquad (34)$$

or,

$$-\frac{P\,y}{E\,I}\,d\,y = \frac{d^2\,y}{d\,x^2} \cdot \frac{d\,y}{d\,x}\,d\,x$$

Integrating,

$$-\frac{P\,y^2}{E\,I} = \left(\frac{d\,y}{d\,x}\right)^2 + \text{constant} \quad \ldots\ldots (35)$$

When $x = 0, y = \Delta, \frac{d\,y}{d\,x} = 0$, and, therefore,

$$-\frac{P\,y^2}{E\,I} = \left(\frac{d\,y}{d\,x}\right)^2 - \frac{P\,\Delta^2}{E\,I}, \text{ or } \left(\frac{d\,y}{d\,x}\right)^2 = \frac{P}{E\,I}(\Delta^2 - y^2) \quad \ldots\ldots (36)$$

$$\frac{d\,y}{\sqrt{\Delta^2 - y^2}} = d\,x\sqrt{\frac{P}{E\,I}}$$

Integrating,

$$\sin^{-1}\frac{y}{\Delta} = x\sqrt{\frac{P}{E\,I}} + \text{constant} \quad \ldots\ldots (37)$$

Since, when $X = 0, Y = \Delta$ (constant) $= \sin^{-1} 1$:

$$\sin^{-1}\frac{y}{\Delta} = x\sqrt{\frac{P}{E\,I}} + \sin^{-1} 1 \quad \ldots\ldots (38)$$

Equation (38) represents the relation between x and y in transcendental form.

The maximum stress in the column,

$$U = \frac{P}{A} + \frac{P\,\Delta}{S} \quad \ldots\ldots (39)$$

in which, $S = \frac{A\,r^2}{c}$

Giving P in terms of the maximum deflection and maximum stress in Equation (38):

$$\sin^{-1}\frac{y}{\Delta} = x\sqrt{\frac{u\,A\,r^2}{(r^2 + \Delta\,c)\,E\,I}} + \frac{\pi}{2} \quad \ldots\ldots (40)$$

When $x = \frac{L}{2}, y = \Delta$, and from Equation (40):

$$\Delta = \frac{u\,L^2 - \pi^2\,E\,r^2}{\pi^2\,E\,c} \quad \ldots\ldots (41)$$

From Equation (41) it is evident that, when $U = \frac{\pi^2\,E\,r^2}{L^2}$, the maximum deflection at the center of the column $= 0$, and that when $u > \frac{\pi^2\,E\,r^2}{L^2}$, there is actual deflection at the center; for values of $u < \frac{\pi^2\,E\,r^2}{L^2}$, the deflection is negative and therefore imaginary.

Substituting the value of Δ from Equation (41) in Equation (39),

$$\frac{P}{A} = \pi^2\,E\left(\frac{r}{L}\right)^2 \quad \ldots\ldots (42)$$

Summary of Results.—

1.—Axially loaded homogeneous columns do not bend as long as $\frac{P}{A} < \pi^2\,E\left(\frac{r}{L}\right)^2$. For this condition, therefore, the average stress, $\frac{P}{A}$, equals the maximum stress, U.

2.—When the average stress, $\frac{P}{A} = \pi^2 E \left(\frac{r}{L}\right)^2$, the column begins to buckle and the maximum stress increases and continues to increase with the deflection without any change in the average stress. In other words, once a column buckles it will continue to buckle without increase in the load; therefore, it is not a stable structure. The proper conclusion is that for stability $\frac{P}{A}$ must be less than $\frac{\pi^2 E}{K}\left(\frac{r}{L}\right)^2$, in which, K is a proper factor of safety.

Taking 4 as the proper value for K, 2 000 000 as the proper value for E, and 500 as the proper value for $\frac{P}{A}$, the ratio of the length to the diameter of a round column must not exceed 25, and the ratio of the length to the side of a square column must not exceed 29. Within these ratios of length to diameter or side, the crushing value of concrete or its corresponding shearing resistance is the criterion for the strength of a column.

C. L. Eddy,* Esq. (by letter).—In their second conclusion of the Synopsis the authors state:

"(2) Before a plain concrete column can fail in shear, the imposed load must be sufficient to overcome the shearing resistance of the concrete and the frictional resistance of the material [to incipient motion] along the impending plane of rupture."

The writer fails to grasp the authors' conception as expressed in this statement. It would seem that there could be no frictional resistance until shear has occurred; that incipient motion assumes a plane of separation and, therefore, assumes that shear has occurred. It is clear that concrete is not a perfectly homogeneous material and that when it is subjected to stress, adjustment occurs between the different particles, producing permanent set, but it is also true that after this set has occurred there is a relation between stress and strain, and the writer's conception of the meaning of the elastic deformation of concrete is that the constituent parts of concrete are deforming according to some law.

If, therefore, along the plane of shear the stress in the concrete is approaching a maximum, deformations are occurring in the particles constituting the concrete but no slipping has occurred and cannot occur until failure takes place. This conception, therefore, is the basis for a statement that no friction can be developed until shear has occurred. Such a conclusion would preclude the acceptance of the authors' statement, which is the basis of their demonstration.

E. H. Harder,† Assoc. M. Am. Soc. C. E. (by letter).—The authors are to be commended for their paper on an old subject but one which will probably never be free from discussion. From a practical point of view, concrete is an exceptional building material, but from a purely philosophical one, it is

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disappointing, to say the least. Therefore, any attempt to develop exact theoretical formulas is bound to be futile. At no stage of its existence is concrete an absolutely fixed material. Shrinkage goes on for years; the value of E varies with the intensity of the stress, and with the age and quality of the concrete; consequently, n varies in the same manner, while m likewise varies with the intensity of stress and perhaps, also, with the age and quality.

As to the newness of the authors' method of theoretical analysis, twenty-two years ago Rudolf Saliger,* an Austrian, developed similar formulas but in different dress, and his formula is the basis of all design formulas used in Europe to-day. Countless experiments have been made in the interim to determine the values of the constants of his formula and, judging from the State specifications of the various continental countries in Europe, all investigators are not in absolute agreement.

Since this paper treats of the theoretical stress formulas for concrete columns with helical reinforcement, it will be proper to point out the difference between the authors' investigation and that of Saliger.

Equations (16) and (17) are the theoretical expressions for the longitudinal and radial deformations of a concrete cylinder subjected to a longitudinal compression and a lateral (in this case, radial) compression due to external pressures. Saliger simplifies Equation (17) by dropping the term, $\frac{h}{m\,E_c}$. Not only does this omission simplify the final result, but it is justified because the theoretical expressions of Equations (16) and (17) presuppose a simultaneous action of the longitudinal and lateral or radial forces. Actually, the radial forces do not come into play until the longitudinal compression has caused the concrete to expand radially and thereby engage the helical reinforcement. If the spiral had been wound under tension on the concrete cylinder, conditions would be different. It will be sufficiently accurate, therefore, to write:

$$e_h = \frac{v}{m\,E_c} - \frac{h}{E_{c_1}}$$

for Equation (17).

Thus, the first half of the right member signifies the radial expansion of a plain concrete column due to the longitudinal force, while the second half signifies the inward deformation due to the unit surface stresses, h. For convenience, the former will be written, e_h', and the latter, e_h''.

Thus, if A_s' is the cross-sectional area of the helical reinforcement for a height, g, then the tension in such a ring of reinforcement will be,

$$H = f_s\,A_s' = \frac{d\,g\,h}{2}$$

in which, d is the diameter of the concrete core and h is the radial unit horizontal pressure caused by H. Hence,

$$\frac{h}{f_s} = \frac{2\,A_s'}{d\,g}$$

* *Zeitschrift für Architektur u. Ingenieurwesen*, 1904, Heft 5; 1905, Heft 1; *Oesterreichische Wochenschrift für den öffentlichen Baudienst*, 1904, Heft 25; *Deutsche Bauzeitung (Mitteilungen)*, 1907, No. 16; *Rundschau für Technik u. Wirtschaft*, 1908, Heft 1–2; "Der Eisenbeton," 1920, Alfred Kroener, Stuttgart, Germany.

Furthermore, let $\frac{A_s' \pi d}{g}$ be the volume of steel in the helical reinforcement per linear inch of column and $A_c = \frac{d^2 \pi}{4}$ be the core area. Then the ratio of the helical reinforcement to the area of the core is $p' = \frac{4 A_s'}{d g}$. From this it is evident that $\frac{h}{f_s} = \frac{p'}{2}$.

Omitting all hyper-refinement alluded to in discussing Equation (17), $h = E_c e_h''$ and $f_s = E_s e_h$. It is easy to demonstrate the latter if one will remember that e_h is the horizontal deformation per inch of column diameter.

Then,

$$\frac{h}{f_s} = \frac{E_c e_h''}{E_s e_h} = \frac{p'}{2}$$

or,

$$\frac{e_h''}{e_h} = \frac{n p'}{2}$$

but e_h is equal to the residual lateral deformation, $e_h' - e_h''$; consequently,

$$e_h'' = e_h' - e_h$$

Substitute,

$$\frac{e_h' - e_h}{e_h} = \frac{n p'}{2}$$

or,

$$e_h = \frac{e_h'}{1 + \frac{n p'}{2}}$$

but $f_s = E_s e_h$ and $e_h' = \frac{v}{m E_c}$ and hence on substitution,

$$f_s = \frac{n v}{m \left(1 + \frac{n p'}{2}\right)}$$

or,

$$v = \frac{m}{n} f_s + \frac{m}{2} p' f_s$$

If there is little helical reinforcement, then p' approaches zero and the column will fail at a unit stress, f_c', which is the required stress to produce a shearing failure in a plain concrete column; consequently,

$$f_c' = \frac{m}{n} f_s$$

and,

$$v = f_c' + \frac{m}{2} p' f_s$$

By making use of the equivalent for $\frac{m}{n} f_s$, the second term of the equation for v becomes $\frac{m\,n}{2} p'\, f_c$ and the final formula is:

$$v = f_c' \left(1 + \frac{m\,n}{2} p'\right)$$

If longitudinal reinforcement is added and the ratio of its area to the core area is p, the total load carried by such reinforcement is $np\, f_c'$. For the combined longitudinal and helical reinforcement,

$$v = f_c' \left(1 + n\,p + \frac{m\,n}{2} p'\right)$$

If n is taken at its usual value of 15 and if an average value of $m = 6$ is used,

$$v = f_c\,(1 + 15p + 45p')$$

This is the design formula used in Germany. The restrictions imposed on the designer are that the longitudinal reinforcement must be at least one-third of the helical and may vary from $p = 0.008$ to 0.030. The pitch of the spiral must be less than one-fifth of the core diameter and not more than 3 in., while f_c is limited to 500 lb. per sq. in. Also,

$$A_{\text{core}} + 15\,A_s + 45\,A_s' = 2 \times (\text{total concrete area})$$

It is interesting to compare this formula with American formulas as specified by several building codes and others.

In the Chicago ordinance,

$$v = f_c\,(1 + 14p + 37.5p' + 525pp')$$

in the New York ordinance,

$$v = f_c\,(1 + 14p + 30p')$$

and in the American Concrete Institute specifications,

$$v = f_c\,(1 + 14p + 60p')$$

In these formulas, p' may vary from a minimum of 0.005 to a maximum of 0.015, while the percentage of vertical steel cannot be less than p' but may be equal to 0.005.

It should be noted that in the American formulas, the coefficient of p is 14, whereas in the German formula it is 15. This difference is due to the custom, in the United States, of subtracting the area of the steel from the area of the concrete core. This practice is theoretically correct, but it is surely a hair-splitting matter when there is such a divergent opinion regarding the coefficient of p'.

If one were interested in the history of theoretical derivations, the statement that, until recently, writers have neglected friction in concrete, will need some amplification. Saliger also considered it in his analysis of the helically reinforced column. His equation may be obtained directly from the authors' Equation (15). Let the left side of this equation be called f_z,

the shearing stress on a plane making an angle $\left(45° + \frac{\phi}{2}\right)$ with the horizontal (Equation 5); then solving for v,

$$v = \frac{f_z}{\sin \alpha \cos \alpha - \cos^2 \alpha \tan \phi} + \frac{\sin \alpha \cos \alpha + \sin^2 \alpha \tan \phi}{\sin \alpha \cos \alpha - \cos^2 \alpha \tan \phi} h$$

Let ρ be equal to tan ϕ, the coefficient of friction; then by trigonometrical manipulation, the foregoing equation may be simplified and written:

$$v = \frac{2 \operatorname{cosec} 2\alpha}{1 - \rho \cot \alpha} f_z + \frac{1 + \rho \tan \alpha}{1 - \rho \cot \alpha} h$$

This is the equation which Saliger published in 1904.

According to Equation (2),

$$f_z = \frac{P}{A} (\sin \alpha \cos \alpha - \cos^2 \alpha \tan \phi) = f_c' \times \frac{1}{2} \sin 2\alpha (1 - \rho \cot \phi)$$

$$= f_c' \times \frac{1 - \rho \cot \alpha}{2 \operatorname{cosec} 2\alpha}$$

therefore, it is clear that the first member of Saliger's equation is equal to f_c'.

The coefficient of h in the second member may be transformed into $\tan^2 \left(45 + \frac{\phi}{2}\right)$. Then,

$$v = f_c' + \tan^2 \left(45 + \frac{\phi}{2}\right) h$$

but,

$$h = \frac{f_s p'}{2} = \frac{n p' v}{2 m \left(1 + \frac{n p'}{2}\right)} = \frac{n p' v}{m (2 + n p)}$$

or,

$$v = \frac{f_c'}{1 - \tan^2 \left(45 + \frac{\phi}{2}\right) \frac{n p'}{m (2 + n p')}}$$

which agrees with Equation (21) if the previously mentioned omission is made in Equation (17).

The equation,

$$v = f_c' + \tan^2 \left(45 + \frac{\phi}{2}\right) \frac{f_s p'}{2}$$

agrees in form with

$$v = \frac{m}{n} f_s + \frac{m}{2} p' f_s$$

hence it may be concluded that,

$$\tan^2 \left(45 + \frac{\phi}{2}\right) = m$$

Also, from Equation (5),

$$f_c' = 2 f_z \times \tan \left(45° + \frac{\phi}{2}\right)$$

consequently,

$$m = \left(\frac{f_c'}{2\,f_z}\right)^2$$

If the values of f_c' and f_z at the point of rupture can be established, a ready means of obtaining the value of m is provided. As the authors have correctly stated, there is a difference between what one may call the cube strength and the cylinder or prism strength of concrete.

Bach gives* the following interesting tests using prisms of varying heights.

Ratio: Height to breadth.	Ratio: Prism to cube strength.
0.5	1.41
1	1.02
2	0.95
3.7	0.87
8	0.86
12	0.84

Thus, taking for the value of f_c', the average, 0.85, of the cube strength and for f_z, 0.20 of the cube strength, $m = 4.5$ and $\left(45° + \frac{\phi}{2}\right) = 65$ degrees.

J. H. GRIFFITH,† M. AM. SOC. C. E. (by letter).—The authors deal with the question of the causes of failure in concrete, more particularly in plain and reinforced concrete columns. Is failure in concrete to be attributed primarily to shearing, tensile, or compressive strain? What are the influences of internal friction, helical reinforcing, vertical rodding, etc? A perusal of the data obtained in past years shows quite conclusively that most failures in plain concrete have obtained under mongrel or ambiguous conditions of strain. The data must accordingly be regarded as possessing apparent rather than rigorous scientific import. The refinements of strict experimental technique usual in scientific testing have been too often neglected for the quite laudable aim of facilitating common sense working conditions. Thus, the real cause of failure has been destined to remain a moot point among engineers.

As a simple example of ambiguous straining the case of the test with the "shear-tool" so commonly used for concrete, timber, and steel, may be cited. The apparent pure shearing failure is, of course, in reality a combination of shearing, compression, and tensile strains of indeterminate amounts. Another type of experiment in the same category is the so-called punching test. Its unique position may be compared to that of the "yellow primrose by the shore" once quoted so aptly by Mr. George B. Grant in "A Treatise on Gear Wheels". It gives the punching strength and nothing more. For, again, the effect of shearing proper is completely masked by a combination of stresses the independent effects of which it is entirely impossible either to calculate or to measure.

* *Zeitschrift für Betonbau*, 1915, pp. 57–58.

† Prof. of Experimental Eng., Iowa State Coll., Ames, Iowa.

Indeterminate data in the case of cylinder crushing are numerous, the ambiguities arising from the presence of end "cappings" of variable thicknesses and degrees of plasticity, unknown axial eccentricities, and the consequent introduction of terminal stress couples, indeterminate radial frictional restraints from the platens of testing machines, sluggish universal bearings, etc. Accordingly, the relatively wide variations found in platting the data must be "camouflaged" by averaging the data in groups on the graph. The "stream of tendency" is thus narrowed to give the semblance of a statistical "law". Critical observers, however, who have made a comparative study in this field in the last twenty-five years, are aware that these laws are in reality too often mere zones of observations. In view of the many interesting points raised by the authors it would seem desirable that the perennial doubt which is made to encompass this whole subject of concrete test data should be dispelled by the occasional definite attainment of an objective. Why does the material fail and how?

The scientific method for obtaining a law, which is quite generally the practice among investigators, is to hold constant all variables but the one under consideration, to note the variation of the function with respect to this variable, and then to proceed with the others in detail in like manner. Mongrel conditions of testing should be abolished. As an instance, it is a well-known fact of rational dynamics that pure shearing stress under unequivocal conditions of loading may be obtained by twisting a circular shaft of the material. Special cylindrical moulds with slight enlargements of the ends to provide for local effects, and arranged to clamp together in halves, may be used to this end as easily as in the case of the compression tests. The few twisting strains conducted in this manner by Mörsch give definite helicoidal fractures, such as those obtained for cast iron. Little is known of stress-strain data for concrete in tension, and yet they are as easily determined as those for cement briquettes, steel, or earth specimens. As regards compression data, it was the writer's privilege to witness a number of the column tests of Wrentmore, Brodie, and Carey, cited by the authors. These authorities used cast-iron "cups" filled with sand and caulked with waste to "secure a full even bearing on the sand." The restrained sand bearing is still used with good results for tests of sewer pipe under conditions simulating those existing in Nature.

Perhaps the most important testing device for ascertaining determinate straining in compression tests of concrete is the hydraulic bearing designed by Schuyler* in 1913, which still remains practically unnoticed by professional committees. It possesses the signal merit of being able to test a cylinder under absolutely pure compression with the elimination of the unknown restraints, end couples, etc., which, in a large measure, cause the variations from the means. A few hundred precise data of this kind will easily outweigh in value the many thousand of the so-called "practical" or "rough-and-ready" results obtained under ambiguous conditions of straining, but from which the definite information is relatively small. Having precise data under pure conditions of straining it will be an easy matter

* *Proceedings*, Am. Soc. for Testing Materials, 1913, p. 1004, Fig. 14.

of course to proceed to definite determinations of failure under combined strains. For example, it is not a difficult problem to test in compression a concrete cylinder already exposed to a twisting couple. Hydraulic bearings with suitable ball races will suffice for applying the compressions. The methods of combining stresses given in dynamics are relatively much simpler than those given in engineering technology.

The pure compression test of Schuyler deserves more than passing comment. In the first place, the so-called Coulomb type of shear failure cited for so many years in textbooks disappears altogether, as do those of the Navier and Bouton type. The specimen fails in pure tension* in vertical planes through the axis usually 120° apart—for concrete having a tensile strength of only about one-tenth and one-half, respectively, of the compression and shearing values, it must fail in tension unless prevented from doing so by the methods adopted by investigators. Strictly speaking, one may usually call the compression strength of many materials infinity for most practical purposes as it is conditioned always, as in the case of concrete, by extraneous failures and not by compression proper. The apparent failure of timber in compression, for example, depends on the strength of the tracheid fibers; again, in a recent paper one author speaks of concrete sustaining 45 000 lb. per sq. in. under stresses from all sides by hydrostatic pressure, and it is doubtless capable of sustaining much higher pressures than this when suitably supported in the tests.

It has been known for many years, since the days of the scientists, Sedgwick, Lyell, and Sorby, that strain induces æolotropy of structure in tangible matter. The fact has already been commented on by Professor A. E. H. Love, the English elastician. The forces set up are akin to crystallization. The essential facts have been presented in an illuminating manner by the Irish scientist, John Tyndall, in a lecture before the Royal Institution of Great Britain, on June 6, 1856. Tyndall† remarks in connection with the discussion of a variety of other most interesting phenomena:

"There is no such thing in nature as a body of perfectly homogeneous structure. I break this clay which seems so uniform, and find that the fracture presents to my eyes innumerable surfaces along which it has given away, and it has yielded along those surfaces because in them the cohesion of the mass is less than elsewhere. I break this marble, and even this wax, and observe the same result; look at the mud at the bottom of a dried pond; look

* The writer has attributed these fractures on planes through the axis to failures in cohesion following the authority of John Tyndall as discussed later. That this is a gratuitous assumption, however, may be urged by a discerning mathematician; for, in dynamics, any stress distribution acting on a mechanical system must satisfy the conditions of rigid body equilibrium in accordance with the provisions of Newton's or d'Alembert's equations (see Love's "Theory of Elasticity," Second Edition, p. 166). Now, as the cylindrical face of the concrete specimen in a testing machine is free from stress, it is natural for the critical observer to raise the question (as did the writer) regarding the anomaly as to what balances the internal pull on the cylinder, assuming the stress distribution to be all tension and not a mutually balanced internal system?

In dynamics the radial and circumferential stresses on the sectors into which the cylinder breaks are rigorously zero. The writer can only state in the light of his own knowledge and experience that he believes this to be a legitimate field for the application of the hypothesis of "true stresses" of the late Mansfield Merriman, M. Am. Soc. C. E., and Prof. A. E. H. Love considers failure in tangible matter still to be a moot point. (See "Theory of Elasticity," Second Edition, p. 117). The hypotheses are: (*a*) That failure occurs under maximum stress; (*b*) under maximum shear; (*c*) under the difference of maximum and minimum principal stresses; and (*d*) under maximum strain.

† Fragments of Science, Chapter XII, "On Crystalline and Slaty Cleavage," pp. 241–243.

at some of the ungraveled walks in Kensington Gardens on drying after rain —they are cracked and split, and other circumstances being equal, they crack and split where the cohesion is a minimum. Take then a mass of partially consolidated mud. Such a mass is divided and subdivided by interior surfaces along which the cohesion is comparatively small. Penetrate the mass in idea, and you will see it is composed of numberless irregular polyhedra bounded by surfaces of weak cohesion. Imagine such a mass subjected to pressure—it yields and spreads out in the direction of least resistance; the little polyhedra become converted into laminæ, separated from each other by surfaces of weak cohesion, and the infallible result will be a tendency to cleave at right angles to the line of pressure."

He remarks further:

"Expanding our field of view, we find the selfsame law * * * extending into the domain of the pastry cook and iron founder; nay, a wheel cannot roll over the half-dried mud of our streets without revealing to us more or less of the features of this law."

The phenomena cited by Tyndall have been observed in a number of instances in research at Iowa State College. In the case of compression tests of virgin earth cylinders it has been possible to obtain a complete vertical network of cohesive failures—of which the Schuyler type of fracture may be regarded an elementary case, as perfect as those found in Iowa in depressions of the soil after water has evaporated and similar to those cited by Tyndall. In fact, if it were not known that these networks of fractures were produced by compression strains, they easily could be mistaken for the phenomena experienced in shrinkage of soils during the heat of summer. The writer has been informed by Dr. O. R. Sweeney, Professor of Chemical Engineering, who has studied the same class of phenomena, that this typically prismatic structure is observable in the product of the coke oven. On the other hand, in nature, the geologists, N. S. Shaler and J. B. Woodworth, have given an admirable illustration of the prismatic arrangement encountered in coke strata overlying the carboniferous beds of the Richmond, Va., Basin, north of Gayton on the James River.*

In the case of hollow building tile with cells running vertically the usual angular or shearing failures occur when the tests are conducted with plaster of Paris caps on the specimens. However, when the ends are ground smooth with a surfacing machine to realize partly the freedom from radical constraint of a hydraulic bearing, values as high as 12 000 lb. per sq. in. have been obtained by the writer. The specimens break simultaneously at the cell fillets into vertical "slats" without those "warnings" of early partial fractures which are so common in the tests with the plaster of Paris caps. The compression failures on soft iron and steel cited by Tetmajer clearly show the presence of vertical cracks. Rock cylinders tested by impact with the Page drop test give the same type of fractures as found by Schuyler for concrete. In the Michaelis "ball test", the balls of cement dropped on a slab from a given height to determine the relative consistency as a result of partial hydration of the colloidal particles show well-defined vertical cracks when they lack the requisite plasticity. In the case of brick pier tests, on the other

* 19th Annual Report, U. S. Geological Survey, 1899, pp. 498–499, Plate XIV.

hand, the influence of frictional constraint is in evidence as far as the quarter-points of the specimen. The exterior walls are "spalled off" by the wedging action of the pyramids in the same manner as the extraneous matter of a leaden bullet or a snowball slides off the cone of material which adheres to a wall or target. The cone must be caused by radial constraint at the platens. When there is no cone, there has been radial slippage on the platens; the material is free to expand circumferentially $2\ \pi$ times the radial strain until the Schuyler fracture occurs, a shear or compression fracture not being caused on account of the much greater resistances against these types of strain, except under conditions of platen supports.

In the case of helical reinforcement when present, as when absent, the question of flexure being neglected, there are two general types of matter to consider—the molar or continuous matter predicated in textbooks, and the actual granular material of the concrete. In the former case the construction of the helix regarded as a grating or series of rings—unless the winding cable effect is introduced to complicate vastly the analysis—will cause a radial constraint of h, using the authors' notation. This stress, h, may be obtained from the authors' rigorous dynamical Equation (16):

$$e_v = \frac{v}{E_c} - \frac{2\ h}{m\ E_c} = \frac{1}{E_c}\left(v - \frac{2}{m}\ h\right)$$

in which, e_v = the uniform axial strain, v = the axial pressure, E_c = the modulus of the concrete, h = the lateral pressure, and m = the reciprocal of Poisson's ratio. It would be expected from theoretical considerations that such a column, if absolutely concentrically loaded, would fail in tension as in the previous cases after the yield point of the reinforcement had been passed, for the resistance to expansion offered by the platens tends to dissipate itself with the distance from the ends. As it is the rule, however, that most square-ended columns have inherent eccentricities, it will be safer perhaps to assume in practice that the failure is in shear, in lieu of any definite analytical data as to the strain measurements necessary in making an analytical decision. By resolution the relation between the stresses, h and v, is found to be:

$$h = \frac{\tan\ \theta}{\tan\ (\theta \pm \phi)}\ v$$

in which, θ is the angle between the plane of the stress, h, and the inclined interface on which slippage takes place, and ϕ is the obliquity of the resultant stress on this plane or angle of friction. The extrema, or maximum and minimum values of h, are found by platting or by taking the derivative to be, respectively, multiples of $\theta_1 = \frac{\pi}{4} \pm \frac{\phi}{2}$ and $\theta_2 = \frac{3}{4}\ \pi \pm \frac{\phi}{2}$, which is in agreement with the authors' Equation (24) and Rankine's theory of earth pressure. The analysis from this point evidently will be similar to that of the authors or equivalent to it. They state in this connection that "when f_s represents the stress in the steel as it reaches its yield point, failure of the columns will not occur even if the concrete within the helix is crushed by the imposed load." Therefore, it is natural again to ask, are not "tension" failures of the Tyndall or Schuyler type already present even before the shear failures

occur? Indeed the records of tests show that frictional dilatancy alone is enormous. A porous sheet-steel, No. 16 gauge, container, 12 in. in diameter by 6 ft. long, filled merely with crushed rock, without cement, will carry 125 000 lb. A tracing cloth container, 4 in. in diameter by 1 ft. 2 in., filled with pebbles, will carry about 2 cu. ft. of cast iron. The helix acts like the tracing cloth so that a column, even if it has become more or less fractured by tension or cleavage, has a considerable inherent compressive strength until the yield point of the helix is reached.

It is natural to raise the question of the uni-planar angular fractures of the type cited by Bouton*—why do these sometimes occur instead of the pyramidal or conical fractures? The methods of elementary analysis ordinarily used cannot discriminate here; for it is tacitly assumed that either type of failure may occur in the analysis. The interpretation of homogeneous stress is thus framed *ad hoc* to suit the case. The writer prefers to think of the Bouton failure as sporadic—a mere accidental failure, as one might say. To deal with such freak phenomena it is necessary to part with the conception of the molar matter theory and think of the concrete in its true status as inherently granular. It starts to fracture locally on some "plane of limited stability" in the language of Osborne Reynolds; and when a crack starts in nature it seems to possess almost human volition and persistence. A slight strain developed in matter causes an "infinity" of stress at the "point of divellence" or re-entrant angle of the crack. A crack in a plate girder, for instance, will tend to seek out the holes and re-entrant points within and on the edge of the plates. The writer has noted on concrete roadways on numerous occasions that a crack in a concrete roadbed passes into the adjacent earth beds beyond. Sir John Herschel† has made the apropos remark that,

"A crack once produced has a tendency to run for this plain reason, that at its momentary limit at the point at which it has just arrived, the divellent force on the molecules there situated is counteracted only by half of the cohesive force which acted when there was no crack, *viz.*, the cohesion of the uncracked force alone."

In the same manner it is entirely logical to suppose that the crack latent or incipient at a cleavage surface or plane of limited stability between the matrix and pebble is gradually developed under the strain by the tendency of the crack at the point of divellence to propagate itself by continuance of minute tears until the structure fails *en masse*.

In conclusion, the writer wishes to emphasize the need of distinguishing between the several types of fracture which have been discussed. He believes the phenomena of failure may be given other interpretations than those now current with technologists.

John Tucker, Jr.,‡ Esq. (by letter).—The attack on the problem by a "rational analysis" is to be highly commended. Empirical solutions of engineering problems are only to be tolerated until they can be placed on a

* Johnson's "Materials," Fifth Edition, p. 118, Fig. 12.

† *Proceedings*, Royal Soc., Vol. XII, p. 678, quoted from Tyndall.

‡ Washington, D. C.

rational theoretical basis of fundamental physical concepts and laws. Consistency demands a "rational analysis" to determine safe working stresses or factors of safety. The authors, arbitrarily selecting a factor of safety, do not utilize the rational method which has already been developed.* Their Equation (23) gives promise of value as an aid in the analysis of the spirally reinforced column, but before the equation can be utilized, the constants, m and E_c, must be known, beyond the ultimate strain of concrete, and can not be "assumed * * * probable values." The other end equations, (21) and (27), are not valid, nor is the conclusion correct that the stress necessary to cause failure in the extreme column fiber is greater than the mean ultimate stress.

The authors have cited no proof, experimental or otherwise, that spiral† reinforcement "tends to neutralize the effects of local imperfections in the concrete." Although the spiral "tends to neutralize the effects of local imperfections" yet "tests indicated that failure was due to flexure, but this was due, no doubt, to some local defect."

The attempted proof that "the unit compressive stress which will cause failure by shearing out a wedge in the extreme fiber [of a column] must be at least 1.75 times the unit compressive stress which will cause failure by shearing the entire section of the column," is erroneous. No hypothesis was offered limiting the size of the wedge, and, therefore, the analysis would be valid for a wedge the thickness of which was equal to the column diameter, with results contradictory to those obtained from Fig. 1. This in itself is proof of error. Moreover, provided the proof presented were valid, it would have to be shown that the extreme column fiber could not fail in a manner requiring less stress than that required to shear out the wedge; for example, by the same type of failure that characterizes the failure of the column as a whole, a rupture in a plane or planes inclined to the column axis.

The analysis of the forces acting near the column surface may be simplified by considering one-half the small wedge shown in Fig. 2. The plane, $a\,b$ (Fig. 6), perpendicular to the paper is tangent to the column surface along an element; $c\,b$ is perpendicular to the paper and to $a\,b$. The shear along $c\,b$ is zero, since it is perpendicular to a free boundary surface, $a\,b$, along which no shear exists. By the same reasoning, the shear on the plane parallel to the paper is zero. The summation of the horizontal forces gives (identically with the authors' analysis of Fig. 1):

$$f_z = f_c\,(\sin\alpha\cos\alpha - \cos^2\alpha\tan\phi).$$

The compressive stress required to cause failure by shearing out a small wedge is the same as that required to cause a shearing rupture of the extreme fiber in one plane and is the same as the compressive stress necessary to cause failure in the interior of the column.

The Synopsis contains the conclusion "(1) Plain concrete columns * * * fail in shear rather than in compression," and the authors also state that

* *Transactions*, Am. Soc. C. E., Vol. LXXXVI (1923), p. 1127.

† When such a precise and scholarly authority as A. E. H. Love uses the term "spiral", it may be accepted as correct, see "Mathematical Theory of Elasticity," Second Edition, p. 396.

"the cylinder * * * will fail in shear alone." Pure shear cannot be produced in a material by compression. It is true that concrete fails by separation and sliding of the new surfaces along a plane, or surface, such as that of a cone, the common form observed in concrete test cylinders. This plane of failure is not a plane of pure shear, but has compression forces acting across it as well as the shearing stresses. The authors' intent in using "failure in compression", evidently a form of failure in distinction to "shear failure," is obscure. "Crushing," the other method of failure mentioned, may be the result of several causes.

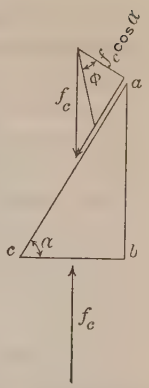

FIG. 6.

Compression failures may be classified as (*a*) plastic, exhibited in lead, short mild steel specimens, etc., a long specimen of the material failing by buckling (as a structural steel column); and (*b*) sliding or a shearing of the material, as in concrete, cast iron, terra cotta, etc. The so-called "splitting" failure is a secondary result of the action of wedges of the material originally produced by a shear failure, splitting the material between the wedge and the other surface producing the compression. Crushing may be a secondary effect of the shear failure in materials of low strength, the material being unable to remain uninjured beyond the point of original shearing, when the two surfaces are further compressed. The phenomena may also be due to a weakness or defect in the material throughout a volume of considerable magnitude. Thus, if a stratum of weaker material be included between two stronger strata, increase in stress will cause the "shearing" failure of the weaker material. If the stratum is not sufficiently thick to permit the two parts of the material to slide past one another, or if they are kept in the same axis, continued compression will break down the vertices of the shear cones originally produced, resulting in a series of additional "shearing" failures, until the weaker material is broken into small pieces, or "crushed." The phenomenon of "crushing" occurs in "lean" concrete, the failure being due to the wedge action of the large aggregate on the much weaker part of the concrete. No material is of absolutely uniform strength throughout. As the stress is increased, the concrete will rupture over small areas where the material is weakest, the areas of rupture increasing in number and magnitude with increasing stress. The greater the variation in strength of the material, the more pronounced and scattered will this network of ruptured areas be, and if sufficiently so, the result may possibly be the so-called crushing failure.

Great care must be exercised in the application of the friction concept. To one familiar with the irregular form of broken specimens of concrete, and also in comparison with the selection by the authors of values of "about 45°" and "about 50°", a value for α of 76° 43′ is absurdly accurate. Sound proof of the unquestionable experimental determination of ϕ, with the measure of the dependability of the result expressed by means of the probable error, is necessary before acceptance. It must be demonstrated that the friction angle is constant, as assumed in differentiating Equation (12), and that it is the same for plain concrete and for identical concrete laterally restrained by reinforcement. The constancy of ϕ is at present purely hypothetical.

Provided that the characteristics of ϕ be known, the analysis in the paper assumes a perfectly homogeneous material of uniform strength throughout, a condition that concrete does not even loosely approximate. The failure of concrete occurs along an irregular surface, the angle between the axis and the plane tangent to the surface varying greatly from point to point. The analysis given in the paper, therefore, cannot hold, for the "shearing" strength of the concrete, which has been considered equal in magnitude to f_z, is a variable throughout the concrete.

The second conclusion, that "the imposed load must be sufficient to overcome the shearing resistance of the concrete and the frictional resistance of the material along the impending plane of rupture", is not a conclusion, nor a unique and sole possible interpretation of pertinent test phenomena, but is a hypothesis on which to build the analysis of column action. "The frictional resistance of the material is a more important factor than is commonly assumed"—in fact, it is an absolutely indispensable factor if the friction concept is used as the working hypothesis. If the hypothesis is not used, the frictional resistance is not used and is of no importance whatever. One weakness of the frictional hypothesis is in that the equations developed upon it are single-valued, and valid only at the point of failure of the material, whereas the ordinary stress-strain relations are valid for all stresses up to and including the maximum or ultimate.

The conclusion that "helical reinforcement is effective in resisting a shearing failure of the concrete core and, if sufficient reinforcement is used, it is possible to preclude a shearing failure of the concrete," is an apparent contradiction to the authors' statement, to the effect that for more than 1.6% reinforcement "the column will not fail, even after the concrete has crushed, until the helical reinforcement has been stressed considerably beyond its elastic limit," for any "crushing" of the concrete is a compound shearing. The columns, *T* and *U*, with 1% spiral, are experimental evidence that spiral reinforcement in itself, for one example at least, is impotent in preventing shear failure of the concrete, the column failure (maximum load) occurring at the approximate ultimate stress and strain of the plain concrete. (The theoretical column strength is to be based on the strength of the concrete in the test cylinders plus the strength added by the rods at 36 500 lb. per sq. in.)*

That point at which the integral state of the concrete is destroyed, either by a shear rupture, "crushing", or whatever method, marks the limiting strength up to which the column serves a useful purpose. This strength, purported to be given by Equation (27), is, as stated, of "academic interest only." If, however, as claimed by the authors, concrete reinforced with more than 1.6% spiral, crushes before the maximum column strength is attained, then it is imperative that the strength at which crushing occurs be known. This latter strength, supposedly given by Equation (21), plotted in Fig. 4 indicates more than a proportionate increase in column strength with increase in reinforcement. Contrary to this, the majority of column tests indicate that the maximum strength (which would be greater than the "crushing" strength)

* *Transactions*, Am. Soc. C. E., Vol. LXXXVI (1923), p. 1099, or by methods used by the authors.

is proportional to the percentage of spiral, while the McKibben and Merrill tests indicate only a small increase in column strength with an increase of reinforcement beyond 0.5 per cent.

The statement of the mechanics of failure of a column with greater than 1.6% spiral reinforcement, is not clear: First, the concrete crushes, after which the "failure" occurs, when the reinforcement is stressed to some point "beyond its elastic limit and possibly, in some cases, almost to its ultimate strength." In exactly what phenomena the "failure" consists, is obscure. If failure has been used to connote the maximum column load sustained, the statement is then correct; the maximum load is sustained, however, at or shortly after the core concrete has begun its complete disintegration by "shear" or "crushing."

Experimental data fail to demonstrate that more than 1.6% of spiral reinforcement precludes a shearing failure. From the original source of the authors' contention, it appears that "the tests indicate, but can not be said to prove conclusively that * * * above 1.58% the ultimate strength is simply the amount which can be carried by the granular core supported by the spiral, with no cohesion of its own." This conclusion is mainly based on a unique interpretation of the plotted column strength test results. These points are represented by a most irregular empirical curve, whereas a straight line would do equally well, if not better. The test specimens, miniature columns with core diameters of about 3½ in., are poor means of obtaining information compared with the several columns 20 in. in diameter of the American Concrete Institute tests,* and the tests of columns 14 in. in diameter by McKibben and Merrill,† neither of which have been included in the test data of Table 1. Talbot states‡ that:

"The maximum load in some cases may not be reached until the hooping has been stressed to its elastic limit, and in other cases this load may be reached without developing the elastic limit of the hooping, especially when large lateral deflections of the column and general distortion from the general shape occur."

In the group thus referred to are five columns with more than 1.58% spiral reinforcement. This is a contradiction of the authors' statements. Neither Professors Withey and McKibben nor Mr. Merrill have remarked on any special phenomena occurring in columns with more than 1.58% spiral reinforcement.

The strain history of the laterally reinforced column has not been given. This is essential in the analysis of the column, for maximum column strength is attained at a large strain, many times the ultimate strain of unreinforced concrete (approximately 0.0015 in. per in.). Experimental proof has been cited§ that:

1.—For a 0.7% spiral reinforced concrete, strained to at least 0.006 in. per lin. in., the core concrete, except at the point in the column in which the failure

* *Proceedings*, Am. Concrete Inst., February, 1916.
† *Loc. cit.*, 1916, p. 200.
‡ *Bulletin No. 20*, Eng. Experiment Station, Univ. of Illinois.
§ *Transactions*, Am. Soc. C. E., Vol. LXXXVI (1923), p. 1227.

occurred, was practically uninjured, and required severe blows from a heavy maul to break it.

2.—A spiral reinforced column, approximately of 80 kg. per sq. cm. unreinforced strength, was stressed to 666.7 kg. per sq. cm. The spiral was then stripped for a length equal to three diameters, and broken under a load of 61.2 kg. per square cm.

3.—Four columns (*T*-1, *T*-2, *U*-1, *U*-2), with 1% spiral and 6.11% rods, failed to have their strength appreciably increased by the spiral reinforcement, and the ultimate strain was not appreciably different from that of the unreinforced concrete.

Therefore, since concrete that has its strength increased by reinforcement may be strained unbroken and integral far beyond the unreinforced ultimate strain, and since the ultimate column strength occurs at the ultimate strain of the unreinforced concrete, in the one case where the concrete strength is not appreciably increased by the spiral, it is apparent that the maximum strength of the column is developed when the core fails by a shear rupture, or crumbling. A column reinforced with more than 1.58% spiral will have an ultimate strain much beyond the ultimate strain of the unreinforced concrete, and the core will remain sound (that is, unbroken and integral) much beyond the corresponding strain of the unreinforced concrete. It is absurd to consider that the addition of more spiral reinforcement will cause the concrete to fail by shear or crushing at a smaller strain; that would be contrary to the conclusions to be drawn from Equation (12), for f_z decreases with an increase in h.

Equation (23) gives with close approximation* the relation between the stresses in plain and reinforced concrete for equal strains, (e_v). This, of course, becomes meaningless for stresses in plain concrete beyond the ultimate stress. If, however, $E_c\ e_v$ is substituted for v_c in the equation, then for any value of e_v (the longitudinal strain in the spirally reinforced column), and E_c (the modulus of elasticity at that strain), v will be the compressive stress in the column. The value of e_v increases far beyond the ultimate strain of plain concrete, and it has been shown† that the product, $e_v\ E_c$, increases greatly beyond its value at the ultimate strain of plain concrete. The authors' interpretation of the equation as giving the "crushing" strength of spirally reinforced concrete, in terms of the ultimate strength of plain concrete, is plainly in error, the interpretation connoting the "crushing" of the concrete at the ultimate strain of the unreinforced material, a phenomenon that holds only for concrete of high ultimate strength. In the form given in the paper, the equation is valid for one value of the variables only, and indicates the stress in a reinforced column which is compressed to the ultimate strain of the unreinforced concrete. No information of the ultimate strength of the reinforced concrete can be obtained from the equation.

Equation (27) does not give the relation between spiral stress and longitudinal column stress for a column with a crushed core. Moreover, because

* E_c is not constant.

† *Transactions*, Am. Soc. C. E., Vol. LXXXVI (1923), p. 1103, *et seq.*

of the inconsistency of the several operations in developing, the equation is devoid of physical significance. A maximum and a minimum value cannot occur at the same point, as the authors seem to believe.

The desired equation may be developed thus: For the plane in the crushed mass in which equilibrium obtains, the forces tending to cause sliding are equal to the resisting forces of friction. This is expressed by Equation (12) when f_z is placed equal to zero, for the equation expresses, by f_z, the difference between the tangential components and the friction force. The equation reduces to:

$$(v - h) \tan \alpha - v \mu - \mu h \tan^2 \alpha = 0$$

in which,

$$\mu = \tan \phi$$

Solving the equation as a quadratic:

$$\alpha = \tan^{-1} \left(\frac{v - h \pm \sqrt{(v - h)^2 - 4 h \mu^2}}{2 \mu h} \right)$$

In this case, α is not independent of the ratio of vertical to horizontal column stresses.

Equation (21) is apparently correct, provided ϕ has the same value for reinforced concrete as for unreinforced concrete. The equation of the curve well represents the 268 to 440-day observations. The straight line representative of these points indicates an increase in strength of 0.56 p, a close agreement with the value which has been empirically determined* as 0.573 p' from an entirely different group of columns. The points representative of the 31 to 77-day tests are so scattered that no law can be determined from them. The strength of concrete varies so considerably within this period and the curing conditions moreover in this particular case are so variable that consistency between the test results can not be expected.

In Table 1 are given values of m, the reciprocal of Poisson's ratio. As the value of m cannot be less than 2 without contradiction of the law of the conservation of energy,† the many values of less than 2 which are given in the table indicate the lack of reliance to be placed in this constant as experimentally determined.

Equation (21) is a function of m, and also of n, another constant difficult of determination for stresses near the ultimate of spiral reinforced concrete. E_c varies greatly for strains beyond the ultimate strain of plain concrete, decreasing with increasing strain, and at an increasing rapidity. It is not known that n and m are constant at the ultimate strength of reinforced concrete, independent of the percentage of spiral reinforcement, hence, for one-strength concrete, the curve representing Equation (21) in Fig. 4, for $f_c = 2\,833$, would probably have m and n variable with a variation in p', instead of constant values of $n = 21$ and $m = 3$, throughout, and would result in an altogether different form of curve.

* *Transactions*, Am. Soc. C. E., Vol. LXXXVI (1923), p. 1096.

† *Loc. cit.*, p. 1108.

JACOB FELD,* ASSOC. M. AM. SOC. C. E. (by letter).—The authors are perfectly justified in their contention that the addition of helical reinforcement to a concrete column should not give it the name of "spiral reinforced column". Neither the suggested nor the usual term is short enough to meet with general approval and the writer would be glad to see the development of a simple nomenclature in this as well as in other branches of reinforced concrete investigation. The expression "reinforced concrete" is also far from logical or truly descriptive.

The separation of the types of failure in concrete columns as given by the authors is quite logical, and may be described generally, using the same division as in the paper, as follows:

(*a*).—Some columns fail in accordance with the usual method of compression failure of solids (or granular materials) that are low in tensile strength, and, therefore, fail easily in diagonal shear. The equations for this type of failure are identical with those defining the limiting resistances of granular solids to displacement along a plane within the solid. In this theory, it must be assumed that the unit shearing stress, f_z, in the paper, is uniform over the entire area along which failure occurs. This assumption cannot be justified by experimental data on shear failure in concrete. The "friction angle" of the paper is the value now universally called the angle of internal friction of a solid body.

(*b*).—When sufficient steel is placed as a jacket around a granular body, failure by compressive load is a separation of the granular constituents of the core, that is, crushing. Separation of the internal structure necessarily requires a certain elastic yield of the steel helix. It is not possible, therefore, to admit the previous conclusion that helical reinforcement has little effect in preventing crushing of the concrete core.

(*c*).—Sufficient steel section can be placed around the concrete so that the elastic stretch under increasing loads is not sufficient to allow the internal structure to separate, or crush, and thus failure cannot occur, until, at some point, the steel helix is stressed beyond its elastic limit.

The authors fail to define the value, $\frac{l}{r}$; evidently, r is a radius of gyration, which expression is not easily defined in reinforced concrete theory. It is far simpler, and involves no greater (if as large) errors if the ratio of length to minimum width or diameter is used.

It is interesting to compare the equations for maximum stresses in axially and eccentrically loaded columns. One is surprised to find that these equations give the same possibility of failure for a given load placed axially or with an eccentricity of one-eighth the diameter. In addition, these results do not agree with the theoretical solution† of L. J. Mensch, M. Am. Soc. C. E., which was also based on a large collection of experimental data. The graphical solution of Mr. Mensch's formula requires that an eccentricity of one-eighth reduce the ultimate load to 70% of the ultimate axial load.

* Cons. Engr., New York, N. Y.

† *Transactions*, Am. Soc. C. E., Vol. LXXXVI (1923), p. 1195, Fig. 18.

Based on the authors' analysis, it follows that the percentage of helical reinforcement should be limited between 1% and 1.6% for the economical use of the steel. The analysis, however, is not complete in disregarding such factors as the relative contraction of concrete core, steel helix, and concrete shell, during the period of setting. Taking such factors into consideration, it seems not unlikely that the usual building code restriction of from 1% to 2% is not only theoretically justifiable, but also marks the most economical use of the reinforcement.

A. W. Zesiger,* M. Am. Soc. C. E. and E. J. Affeldt,† Assoc. M. Am. Soc. C. E. (by letter).—The desirability of data regarding the deformations of columns, with and without helical reinforcement, under working loads, is mentioned in Mr. Thompson's discussion. Information of this nature for cylinders and plain concrete columns under compression is fairly abundant. In such data the modulus of elasticity is given, and there is really much better information regarding the behavior of concrete under loads up to those that will produce a shearing failure of plain concrete than for loads in excess of these. The writers, however, are more concerned with the action that takes place after the shearing strength of plain concrete has been reached, in order to ascertain wherein helical reinforcement exerts such an influence on the ultimate strength of a column thus reinforced. As stated in the paper, the constants used in a working formula should always be predicated upon the properties of the material at failure. In this manner only is it possible to determine any given factor of safety, since, as is well known, the values of n, etc., do not by any means vary in direct ratio to the load.

The effect produced in the diameter of the helix by the shortening of the column under load, and its consequent lateral expansion, is far less than that assumed by Mr. Thompson. It is really so small that it may be considered negligible, in all cases, as the following extreme example will show. In addition to the notation previously used:

Let L = the length of one unstressed coil of the helix.
s = the pitch of the unstressed helix.
f = the compressive stress in the concrete.
s_1 = the pitch of the helix when stressed.
D_1 = the diameter of the helix when the pitch becomes equal to s_1.
D_2 = the diameter of the concrete core due to expansion.

Then,

$$D = \frac{1}{\pi}\sqrt{L^2 - s^2}$$

$$s_1 = s\left(1 - \frac{f}{E_c}\right)$$

$$D_1 = \frac{1}{\pi}\sqrt{L^2 - s^2\left(1 - \frac{f}{E_c}\right)^2}$$

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† Asst. Engr., The Grasselli Chemical Co., Cleveland, Ohio.

$$D_2 = D\left(1 + \frac{f}{m\,E_c}\right)$$

$$\frac{D_1}{D} = \frac{1}{\pi}\sqrt{\frac{L^2 - s^2\left(1 - \frac{f}{E_c}\right)^2}{(L^2 - s^2)}}$$

$$\frac{D_2}{D} = 1 + \frac{f}{m\,E_c}$$

Assume that, $D = 6$ in.; $s = 2$ in.; $n = 27$; $E_c = \frac{30\,000\,000}{27} = 1\,110\,000$; $m = 2.5$; and $f = 3\,704$ lb. per sq. in. Then, $\frac{D_1}{D} = 1.0000316$; $\frac{D_2}{D} = 1.001145$; and $\frac{D_2 - D}{D_1 - D} = \frac{0.001145}{0.0000316} = 36.2$, or the increase in diameter of the helix due to shortening of the column is less than one-thirty-fifth of that due to lateral expansion of the concrete for this extreme case; for columns of larger diameters it is proportionately less.

The column formula deduced by Mr. Thompson is quite ingenious, but is predicated on the same defective assumption as that of Euler, namely, that an ideal column will deflect without any force causing it to do so, whereas an ideal column with the load axially applied should not deflect. His demonstration would seem to indicate, however, that flexure does not cause failure of a concrete column for values of $\frac{l}{r}$ less than 100, and the writers, therefore, would appear to be conservative in suggesting a ratio of $\frac{l}{r}$ not greater than 80 as the safe limit.

Referring to Professor Eddy's discussion the writers are uncertain whether he takes exception to their original use of the word "incipient" or implies that there is no friction between the particles of a concrete column until rupture has occurred. In using the word "incipient" the writers had in mind a relation between the particles of a column such that motion was on the very verge of commencement, but had not yet started. This is probably an incorrect interpretation and the word has, therefore, been omitted.

If, however, Professor Eddy means to imply that friction cannot exist between the particles of a column until rupture has occurred, and that friction and shear, therefore, cannot occur simultaneously, the writers are unable to agree with him and feel confident that the consensus of engineering opinion will not support his view. If the latter interpretation of his discussion is correct it would be interesting to know how Professor Eddy reconciles his view with the fact that concrete columns do not shear along a plane having an inclination of 45° with the horizontal, since the shearing stresses would be a maximum and the shearing resistance a minimum along such a plane.

The writers feel indebted to Mr. Harder for his interest in their paper and his comparison of their treatment of the subject with that of Rudolf

Saliger. The practicability of theoretical reasoning, when applied to a material such as concrete, is, of course, a matter of opinion and Mr. Harder has a perfect right to his views. A study of literature pertinent to the subject of concrete construction would seem to indicate, however, that by far the majority of the design formulas now in use are based on theoretical reasoning.

Like Saliger, the writers originally made the same error in Equation (17) and omitted the term, $\frac{h}{m\,E_c}$, but later discovered their mistake. They cannot agree with Mr. Harder's contention that Equation (17) does not obtain unless the helix is wound on the concrete core under tension. It is undoubtedly true that the concrete core in setting has a slight tendency to shrink away from the helix, but this can be counterbalanced by assigning proper values to the constants. On the other hand, if the helix could be wound on the concrete core under tension, the lateral pressure against the concrete would thereby be increased unless the tension in the helix could be adjusted so as to compensate exactly for the shrinkage of the core, which is, of course, practically impossible, and Equation (17) would no longer apply. Furthermore, the writers cannot agree with Mr. Harder that the omission of the term, $\frac{h}{m\,E_c}$, will simplify in any way the final result as given in Equation (21) and especially in the working Equation (28*a*). About the only difference between Equation (21) and Mr. Harder's equation,

$$v = \frac{f_c'}{1 - \tan^2\left(45° + \frac{1}{2}\phi\right)\frac{n\,p'}{m\,(2 + n\,p')}}$$

is that the former is theoretically more nearly correct.

Mr. Harder's attempt to show that his formula,

$$v = f_c'\left(1 + \frac{m\,n}{2}\,p'\right)$$

is correctly deduced from the writers' Equation (21), is hardly successful. Although his demonstration is an ingenious example of algebraic manipulation, he seems to have lost sight of the basic facts on which the theoretical equation is predicted. When $p' = 0$, the equation, $f_c' = \frac{m}{n}\,f_s$, is correct, but for that value of p' only. In other words, f_c' is a constant for any given concrete, whereas f_s varies as p'. As p', and consequently the quantity of steel, increases, the lateral restraint of the column becomes greater and the vertical pressure necessary to produce a lateral deformation that will cause rupture of the concrete is proportionately increased, resulting in a greater longitudinal and lateral deformation and, consequently, in an increased

value of f_s. The equation, $f_c' = \frac{m}{n} f_s$, is, therefore, incorrect for any values of p' other than 0. It follows that the equation,

$$v = f_c' + \frac{m}{2} p' f_s$$

is likewise incorrect, and the equation,

$$v = f_c' \left(1 + n p + \frac{m n}{2} p'\right)$$

therefore, cannot be predicated on theory.

The coefficient of p' in the third term of the right-hand member of Mr. Harder's equation increases as m increases, whereas it should theoretically decrease. Consequently, the German design formula,

$$v = f_c (1 + 15 p + 45 p')$$

cannot be founded on theory, but must be purely empirical. Inasmuch as the equation,

$$v = f_c' + \frac{m}{2} p' f_s$$

is incorrect, Mr. Harder's conclusion that

$$m = \tan^2 \left(45° + \frac{1}{2} \phi\right)$$

must likewise be considered erroneous.

Referring to that part of Mr. Harder's discussion wherein he states that, "if one were interested in the history of theoretical derivations, the statement that, until recently, writers have neglected friction in the concrete, will need some amplification", the writers are well aware of the fact that friction in the concrete had been considered prior to the appearance of their paper. In fact, they explicitly stated that Navier has given the true analysis of granular substances in compression. The late J. B. Johnson, M. Am. Soc. C. E., and a few others, including Saliger, also have taken friction into account, but the great majority of engineers seem to have neglected it even if they may have been aware of its existence.

In using Bach's tests on prisms Mr. Harder seems to confuse the strength of cubes with what the writers term the crushing strength of concrete, as even cubes, when tested to destruction, will fail partly by shearing. Although the writers have never made or witnessed tests wherein the ratio of height to breadth was 0.5, it would appear to them that, in such a case, the ratio of shearing to crushing failure would be very small. A ratio of 0.85 to 1.41, although still somewhat too low, would represent a more nearly correct relation between shearing and crushing strength; that is,

$$v_c = \frac{1.41}{0.85} f_c' = 1.659 f_c'$$

From the foregoing it should be obvious that Mr. Harder's determination of m from Bach's tests is in error.

The writers were well aware of the complication of stresses in cylinders subjected to axial stress, as described by Professor Griffith. It was, however, and still is, their belief that the principal stresses causing failure in a short, helically reinforced concrete column are so-called shear and crushing. In using the expression "so-called" the writers have in mind that every failure, whatever its nature, or however it may be designated, is in reality some type of tension failure. When the cohesion or bond, by whatever term it may be designated, between the particles of a substance is ruptured, failure of the material takes place. Whether the particles are pulled apart, pushed apart, or twisted apart, the fact remains that they are separated. For the sake of convenience, and as a sort of mask for the prevailing ignorance concerning the phenomena attendant on failure of the material, it is customary to speak of shearing failures, tension failures, etc. It is the writers' belief, however, that the ultimate cause of all these types of failure is some kind of tension. In so-called crushing the principal factor tending to produce disintegration of the material seems to be an induced tension at right angles to the line of action of the principal stress that is, tension along the periphery of the material.

As Professor Griffith has so clearly stated, the majority of tests on concrete columns thus far reported are sadly lacking in the refinements essential to the attainment of precise data, and there is urgent need of further research along the proper scientific lines.

In his discussion Mr. Tucker states that certain equations are "not valid", that many of the conclusions are erroneous, etc. The writers frankly admit that they are unable to follow his criticisms. In view of the fact that Mr. Tucker so often gives an interpretation to their language and mathematics that is at utter variance with what is stated, they are constrained to believe that the majority of his criticisms are based on a misinterpretation of the paper. Had he offered any proof of his various assertions as to the non-validity of the writers' mathematics, etc., they would gladly have attempted to answer him.

In reply to the various points raised in Mr. Feld's discussion the writers wish to state that:

(*a*) It is true that their mathematics assumes a uniform shearing plane. They are aware, of course, of the fact that no granular mass will shear along a single plane. The plane assumed by them is merely the average of all planes.

(*b*) By the expression "little effect in preventing crushing", etc., the writers do not mean to imply that helical reinforcement has no effect. Its effectiveness in preventing a crushing failure is, however, so much less than in the prevention of a shearing failure that it does not seem economical to use steel for the former purpose. This is indicated by the graphical representation of Equation (23) in Fig. 4.

(*c*) There do not seem to be sufficient data available to indicate whether, if enough steel is used to preclude a shearing failure, crushing of the concrete will occur at or below the elastic limit of the steel.

Mr. Feld is correct in assuming that r is the radius of gyration of the column. The ratio of length to minimum width of column is undoubtedly a more simple quantity, but the writers are unable to understand wherein the use of such a relation would tend to minimize possible errors. Inasmuch as, for a constant width (or diameter) of column, r varies with the amount of reinforcement used and, therefore, takes it into account, whereas any relation between the length and minimum width (or diameter) of column ignores it, the use of the ratio of length to least radius of gyration would seem preferable.

Referring to that part of the writers' paper wherein it is demonstrated that the extreme fiber stress that will shear out a wedge must be about 1.75 times the average stress that will cause the column to shear across its entire cross-section, they did not state whether this extreme fiber stress equalled the crushing strength of the concrete. There are few data from which to determine the crushing strength of concrete. It would seem, however, that such strength is approximately equal to 1.75 times the stress that will cause shearing across the entire column cross-section. This, of course, applies only to plain concrete columns. Mr. Feld's statement that the authors' equations "give the same possibility of failure for a given load placed axially or with an eccentricity of one-eighth the diameter," is not correct even for plain concrete. The flexure that an eccentrically applied load will cause in a column of any appreciable length will produce an additional eccentricity resulting in an extreme fiber stress of more than 1.75 times the average. This action is even more noticeable in reinforced (especially helically reinforced) concrete columns in which the average stress approaches the crushing strength of the concrete. Even in the case of columns reinforced only with longitudinal rods tied together at intervals with bands or wire the steel resists a shearing failure to a greater or lesser extent and, therefore, increases the average stress in the concrete necessary to produce a shearing failure.

The writers were aware of the fact that shrinkage of the concrete in setting will affect the value of the helix, but believed that the constant selected for m in their working equation would tend to minimize this error.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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Paper No. 1602

PIPE TUNNEL UNDER GOWANUS CANAL, BROOKLYN, NEW YORK*

By L. S. Stiles,† M. Am. Soc. C. E.

With Discussion by Messrs. Robert Ridgway, J. C. Meem, John R. Slattery, Henry B. Seaman, T. Kennard Thomson, Frederick L. Cranford, Charles W. Staniford, John F. O'Rourke, and L. S. Stiles.

Synopsis

This paper describes the building of a tunnel under the Gowanus Canal, from the Citizens Works of The Brooklyn Union Gas Company, situated between Fifth and Huntington Streets, Brooklyn, N. Y. The purpose of the tunnel is to carry gas mains from this manufacturing plant and connect them to the distribution system, thus eliminating a lengthy and circuitous detour of pipe around the head of the canal.

The tunnel is 136 ft. from center to center of shafts, with an oval cross-section, 10 ft. 8 in. high and 16 ft. 1 in. wide. The tunnel and shafts are of steel plate, lined with reinforced concrete. The tunnel can accommodate two 30-in. and two 24-in. mains, with sufficient room for walkways on each side and ample space for repairs or replacement of the pipes.

The outstanding features in connection with this project, are as follows:

(1) The unique and inexpensive method used, which consisted of assembling the steel shell on ways, like a ship, launching it into a sheathed trench, and sinking it in position across the canal;
(2) The co-ordination of the many details of the work, obviating costly delays; and
(3) The perfect utility of the tunnel for the purpose for which it was designed.

* Presented at the meeting of March 3, 1926.

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The co-ordination of the work, insuring a minimum of delay, was doubly important by reason of the fact that all this construction had to be accomplished without interference with the extensive navigation in the canal. The Gowanus Canal is lined with large industrial enterprises, including a street railway power station, and their needs would brook no interference, however slight, in the navigation of the canal. The canal is closed to navigation from 5.00 P. M. to 7.00 A. M., and all day Sunday. Certain phases of the work had to be carefully co-ordinated with these periods.

Descriptive

The condition which prompted the construction of a pipe tunnel under the Gowanus Canal was the remarkable growth, within a few years, in the generating capacity of the Citizens Works of The Brooklyn Union Gas Company. One of the prime necessities for a gas plant is an abundantly adequate transmission system which will insure an easy "get-away" for the gas. The gas mains going through this tunnel provide a short cut eastward to the general transmission system of the Company at 12th Street and Second Avenue.

Prior to 1920, the capacity of this plant was 10 000 000 cu. ft. per day. An extensive construction program was launched about that time, and, in 1923, the capacity of the plant had been increased, by steps, to 29 000 000 cu. ft. daily, and additional transmission mains became absolutely necessary.

The Citizens Works is situated across the canal from some of its important mains (see Fig. 1). The original 24-in. main from the Citizens Works took a tortuous route in order to get around the head of the Gowanus Canal at Butler Street, and back to a 42-in. main that begins at the 12th Street Pumping Station, below Second Avenue. The length of the 24-in. main from the Citizens Works to the 42-in. main at Second Avenue and 12th Street is approximately 11 000 ft. Obviously, a much shorter line would be across the canal, and along Second Avenue to 12th Street, thereby eliminating approximately 8 000 ft. of transmission line.

As a matter of fact, the original route could not have been paralleled exactly with a second main as congested street conditions would have made it unreasonably costly; nor was it possible to go down Fourth or Fifth Avenues at a reasonable cost, because of the Brooklyn-Manhattan Transit Subway under Fourth Avenue and the elevated and trolley systems on Fifth Avenue. The route would necessarily have been to Sixth Avenue, and southward along Sixth Avenue, requiring an increased length of 2 200 ft. more, or about 13 000 ft. in all; and even this would have presented difficulties because of a large city water main running along part of the route in Sixth Avenue. In any case, the laying of a main along this route would have been complicated by the fact that it would cross the large Bond Street sewer, the Brooklyn-Manhattan Transit Subway line under Fourth Avenue, and the same Company's elevated structure along Fifth Avenue. A careful weighing of the relative cost of these two alternatives, both as to the initial cost of construction and as to the expense for loss of gas pressure which would have been entailed in the longer route, led

to the conclusion that the line under the canal was the more advantageous in every respect.

As early as 1912, the accumulation of data relative to this work was begun. Consideration was given to an aerial bridge, to support an overhead pipe across the canal, but it was apparent that the lofty rigging of the boats using the canal would make this prohibitive, and that it would not be a desirable form of construction, subject, as it would be, to storms, etc. This overhead line was never seriously considered, but it was desired to exhaust the subject in deciding what ultimately should be done.

Having decided on a tunnel under the canal, the next questions were the method of construction and the size and cross-section to be adopted. Consideration was given to the use of a shield, but such a method, with its attendant compressor plant and equipment, would have made the unit cost very high for a tunnel of such short length. It was decided, therefore, to use the open-cut method, assembling the steel tunnel on shipways, conveniently spaced, and launching it into a coffer-dam purposely located, dredging across the canal, sinking the tunnel in place below the bed of the canal, and back-filling.

One of the factors influencing the choice of this type was the volume of shipping on the canal and the shortness of time available for work in it, limited to the hours of 5:00 P. M. to 7:00 A. M., and all day on Sundays, during which periods the waterway was closed to navigation. The extent of the shipping in this canal may be comprehended by the following figures, taken from U. S. War Department data, on the tonnage and value of shipping handled along this waterway:

"Gowanus Creek, N. Y.

	Tons.	Value.
1895	266 299	(Not known)
1896	220 748	(" ")
1897	215 352	$2 631 752
1899	237 300	3 048 390
1902	237 300	3 048 390
1903	237 300	3 048 390
1915	658 000	354 000
1919	1 607 261	18 802 541
1920	1 550 732	21 248 128
1921	2 839 689	96 215 400"

As to the size and cross-section to be used, a number of shapes and sizes were considered, and the final cross-section selected was of an oval form with parallel sides and semi-circular ends. This shape gave a maximum area with a minimum height, thus reducing the extent of dredging, and lent itself admirably to the reception of the four mains required, with walkways on each side and ample room for inspection, caulking pipe joints, or replacing lengths of pipe.

Construction

The merest outline of the steps in the construction will visualize the work as a whole before its details and interesting phases are discussed.

Property was acquired on the canal just opposite the Citizens Works and shipways were erected for the assembling and launching of the structure,

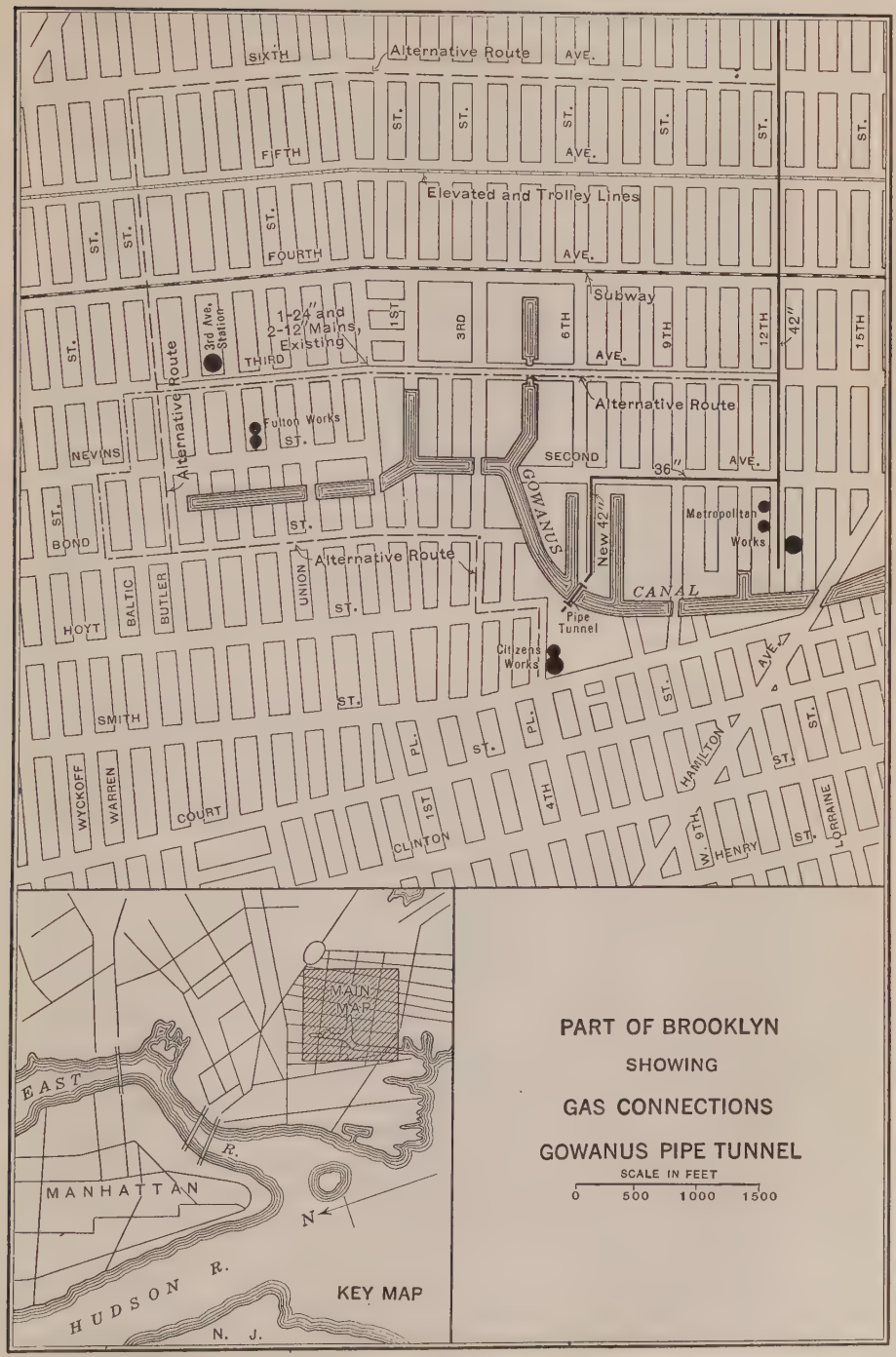
SIXTH
Alternative Route
AVE.
FIFTH
ST.
AVE.
Elevated and Trolley Lines
FOURTH
Subway
1-24" and 2-12" Mains, Existing
3rd Ave. Station
THIRD
1ST
3RD
6TH
9TH
12TH
42"
15TH
Alternative Route
Fulton Works
NEVINS
SECOND
36"
GOWANUS
New 42"
Metropolitan Works
CANAL
BOND
UNION
Alternative Route
HOYT
BALTIC
BUTLER
Pipe Tunnel
Citizens Works
SMITH
HAMILTON
WYCKOFF
WARREN
COURT
CLINTON
W. 9TH
HENRY
LORRAINE
MAIN MAP
EAST
R.
MANHATTAN
N
HUDSON R.
N. J.
KEY MAP
PART OF BROOKLYN
SHOWING
GAS CONNECTIONS
GOWANUS PIPE TUNNEL
SCALE IN FEET
0 500 1000 1500

FIG. 1.

which was delivered in sections. After launching into the coffer-dam previously built in the acquired property, the steelwork and concrete lining were partly completed. Meanwhile, the dredging for the shafts and the sheathed trench across the canal was completed. Finally, the steel tunnel, having been finished to a sufficient extent, was floated into place and sunk to its final position.

In Fig. 2 is shown a general view of the site in December, 1923. In the foreground the steel shell was erected and launched. This had formerly been a manure dump, and the clearing of the site for the coffer-dam and the shipways included a vast amount of fumigating for sanitary reasons. The total excavation for the shafts and tunnel section amounted to about 7 000 cu. yd.

The work on the acquired property east of the canal was comparatively simple, as there was practically an open field in which to begin, but on the Citizens Works (west) side the dredging for the shaft was complicated by its proximity to the congested gas plant. Inside the concrete retaining wall, there are stored approximately 20 000 tons of coal (Fig. 2), and it was necessary to take every precaution to insure against any settlement of this heavy and valuable stock. To the right, in the background, is shown a 5 000 000-cu. ft. gas-holder just previously erected.

The first shipments of steel for the tunnel came from the Bethlehem Shipbuilding Corporation on December 14, 1923, and the last shipment on January 7, 1924. On January 15, the work of assembling the shell was begun. The method of assembling the steel shell in sections on the launching cradle is shown in Fig. 3. The completed steel structure weighs about 264 tons.

In the meantime work had gone ahead on the Citizens Works side, dredging out the canal bottom, and driving the sheet-piling for the coffer-dam to receive the shaft of the tunnel. This work was complicated by the proximity of a future 1 000-cu. yd. overhead ash-pocket (since constructed) for which the structural steel alone amounted to more than 200 tons.

The next major operation was the preparation of the trench across the canal to receive the tunnel. This work was begun after the coffer-dam for the shaft on the Citizens Works side had been built. The bottom of the canal was dredged to a depth of about 3 ft. and steel master piles were driven on 10-ft. centers across the canal (Fig. 4). As finally driven, the tops of these master piles were 3 ft. below the bed of the canal. Their length was about 38 ft., so that they were actually driven to a depth of 41 ft. below the bottom. Along the outside of the master piles was the steel sheet-piling, driven down to the same elevation as the heads of the master piles. The sheet-piling was 20 ft. long, and thus penetrated to a depth of 23 ft. below the bottom of the canal.

The driving of the steel sheet-piling was one of the phases of the work that had to be done while the canal was closed to navigation. On Sunday, March 30, 1924, the master piles and the sheet-piles for the north side of the trench were driven across the canal, except for a gap of about 12 ft. on each side of the canal left open for navigation. This work was started at 6:30 A. M., Sunday, and finished at 3:30 A. M., Monday morning, March 31.

On the following Sunday a similar operation for the other (south) side of the trench was accomplished. Profiting by the previous experience, it was

FIG. 2.—CITIZENS WORKS, BROOKLYN UNION GAS COMPANY, VIEW LOOKING WEST ACROSS GOWANUS CANAL, SHOWING PILE-DRIVERS REMOVING OLD DOCK.

FIG. 3.—SECTION OF GAS CONDUCTOR BEING ASSEMBLED ON DOCK, CITIZENS WORKS, BROOKLYN UNION GAS COMPANY.

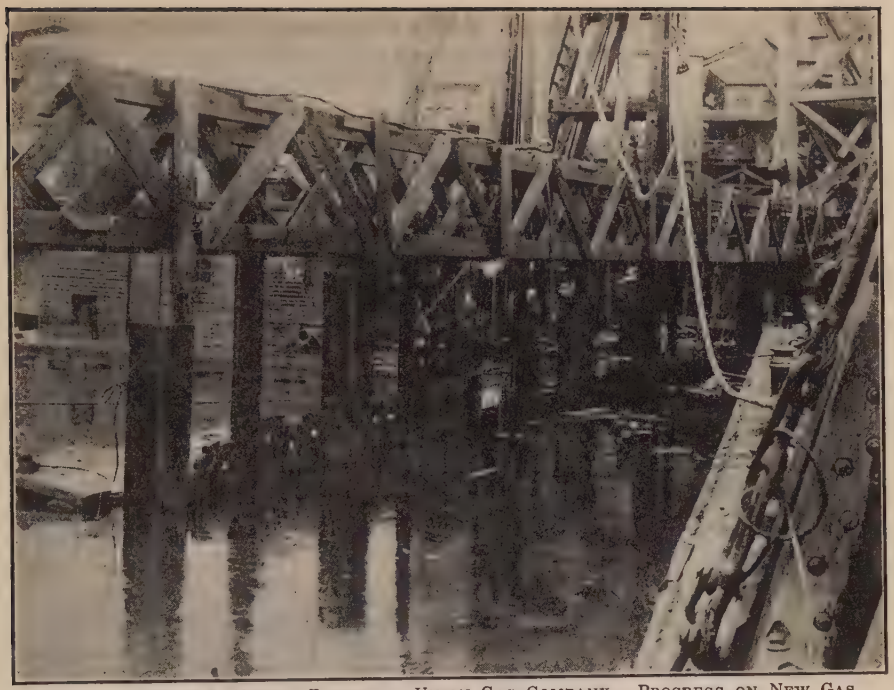

FIG. 4.—CITIZENS WORKS, BROOKLYN UNION GAS COMPANY. PROGRESS ON NEW GAS CONDUCTOR, APRIL 6, 1924.

FIG. 5.—VIEW SHOWING ASSEMBLING OF STEEL TUNNEL AND SHAFTS ALONGSIDE COFFER-DAM INTO WHICH IT WAS TO BE LAUNCHED, CITIZENS WORKS, BROOKLYN UNION GAS COMPANY.

found possible to drive the master piles and sheet-piling across the entire canal. The driving of the master piles is illustrated in Fig. 4. The wooden truss in position across the canal was used for centering those piles. Meanwhile, work on assembling the steel tunnel and shafts was progressing rapidly (Fig. 5). Incidentally, a total of 380 tons of piles was driven, of which all but about 55 tons for the coffer-dam was used in connection with the trench and shafts.

The dredging of the trench between the sheet-piling was then begun, and the sheeting temporarily braced. When the excavation was down to the final level, 22 ft. 6 in. below the bed of the canal, 24 by 24-in. concrete braces were constructed across the bottom of the trench on 10-ft. centers between corresponding master piles and the temporary braces at the top removed. The space between the braces was filled with gravel, to provide a secure and level foundation for the tunnel.

To meet the requirements of the War Department, it was ultimately necessary either to drive this sheeting down to 20 ft. 0 in. below mean low water, or to remove it entirely. The latter seemed the lesser evil, and all this piling eventually was pulled out, approximately 108 tons of sheet and master piles being recovered.

On April 19, everything was ready for launching the tunnel, the barrel and the two lower courses of the shafts having been assembled, riveted, and found absolutely tight. At 9:30 A. M., the structure was released from the ways and without the least difficulty floated into the coffer-dam prepared for it (Fig. 6).

While the tunnel was afloat in the coffer-dam, the reinforced concrete lining was placed in the barrel, and by Sunday, June 15, the concreting had progressed to a point where everything was ready to float the structure across the stream into position, and sink it. At this time the barrel was within a few inches of being awash. The great weight of the shafts, and, in counter relation, their greatly increased buoyancy per foot while sinking, made it necessary to brace the shaft shells preparatory to sinking. In order that the tunnel might sink on an even keel, the two ends of the barrel were bulkheaded, and when the structure had been towed to the correct position, the shafts were flooded and the tunnel submerged to its final position.

In Fig. 7 is shown the steel structure towed out to its position across the canal and gradually submerging to the bottom of the trench prepared for it. With the sinking of the structure to its final position, the critical stages of the work had been passed; what remained was comparatively simple. The shafts were assembled to their full height, the sheet-piling across the canal was drawn out, and the trench back-filled. The shafts were concreted, and the section of the barrel that had not been concreted before the sinking, was finished. The work on the reinforced concrete lining in the barrel of the tunnel is shown in Fig. 8.

While all this work had been going on, the major part of the main from the east tunnel shaft to 12th Street and Second Avenue had been laid, and on the Citizens Works side, the line from the Pumping Station to the Citizens Works shaft had also been brought up to a point where it was ready to be connected to the mains going through the tunnel. Two 30-in. lines were laid

through the tunnel, leaving ample room for two additional 24-in. lines in the future.

Although the tunnel had been built to stay perfectly dry as far as human skill could ensure this, provision was made for a possible flooding. Instead of American Gas Association standard pipe, American Water Works standard Class "D" pipe was selected, which is so heavy it will not float when the tunnel is filled with water. The slight additional expense entailed seemed to justify this extra precaution.

The mains in the tunnel were connected to the Company's system on November 13, 1924. The tunnel and shafts are absolutely dry. Both mains were made tight under 10 lb. of air pressure before subjecting them to gas pressure, and as the working pressure of the gas is only one-half of that, the mains are perfectly tight. Fig. 9 shows the two mains laid through the tunnel.

Structural Steel Design

The tunnel is a combination of a rectangle 5 ft. 5 in. wide and two semi-circular sections of 5 ft. 3½ in. radii, making the width of the tunnel 16 ft. 0 in. inside the steel shell, and the height 10 ft. 7 in. To take up the pressure on the straight sections a longitudinal bracing truss was placed through the center of the tunnel as shown clearly in Figs. 8 and 9. This truss was not intended to act as a beam.

A number of pipe tunnels were investigated in the preliminary research. Fig. 10 shows some of the comparative cross-sections. The final selection is really a compromise between the other two, combining the advantages of both, without their drawbacks. This permits two 30-in. pipes on one side, and two 24-in. pipes on the other. The pipes shown at the extreme ends of the cross-section represent replacement sections and emphasize the adequate space available in which to work. As a matter of fact, this plan was modified slightly. Instead of putting in two 30-in. pipes one on top of the other, they were placed side by side, and the two future 24-in. pipes will go over them; or, if it should be found necessary, two 30-in. mains can be added above, and the required clearances still maintained.

The shell is made up of ⅜-in. and ½-in. plates riveted to angles bent to the shape of the tunnel. The circumferential and tangential joints are covered with an exterior ½-in. butt-plate. All rivet holes were sub-punched and reamed, and all joints caulked.

The steel shell of the tunnel and shafts is lined with 12 in. of reinforced concrete. This lining is designed to support the entire load acting on the tunnel if, in the course of time, the outside steel should rust away. The longitudinal steel bracing through the center of the tunnel sets against the steel lining so that it will always function.

The plan and sections of the tunnel showing the general arrangement and the relation of the various appurtenances are indicated in Fig. 11.

Plotting Progress

The plotting of progress on plans and perspectives is an invaluable aid in co-ordinating the different phases of the work and in eliminating vexatious

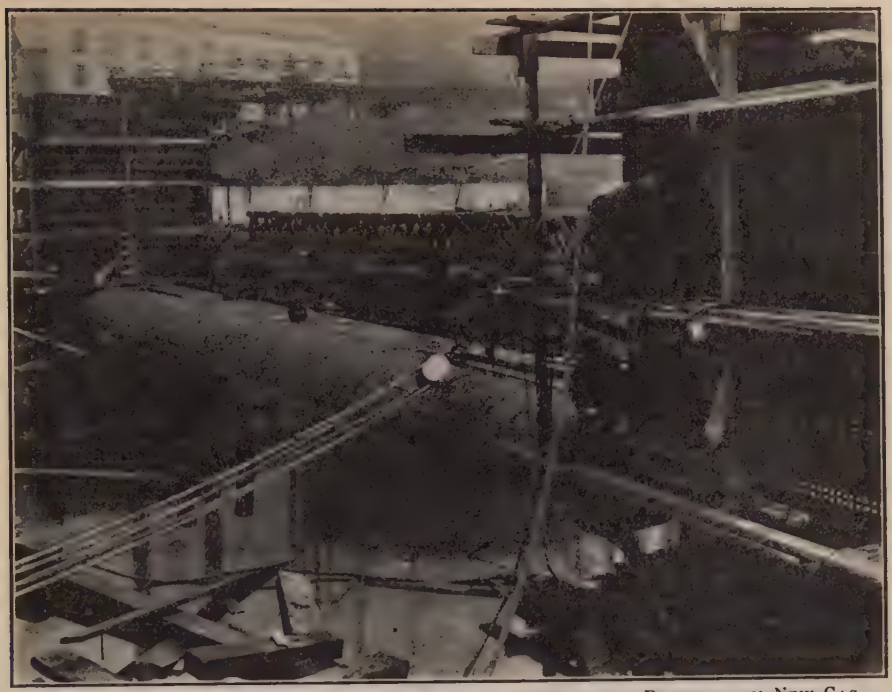

FIG. 6.—CITIZENS WORKS, BROOKLYN UNION GAS COMPANY. PROGRESS ON NEW GAS CONDUCTOR, JUNE 2, 1924.

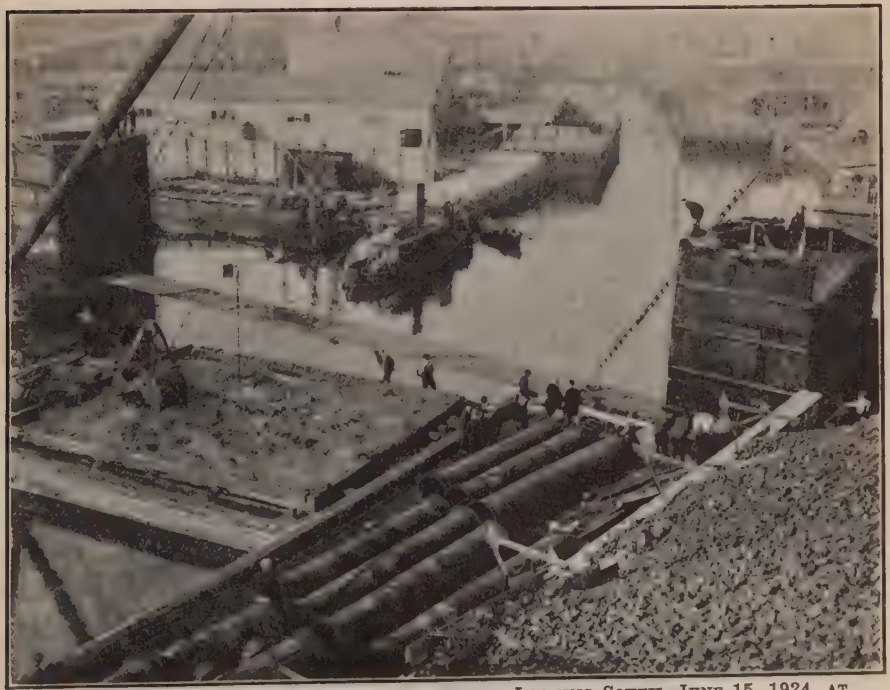

FIG. 7.—VIEW OF SINKING OF GAS CONDUCTOR, LOOKING SOUTH, JUNE 15, 1924, AT CITIZENS WORKS, BROOKLYN UNION GAS COMPANY.

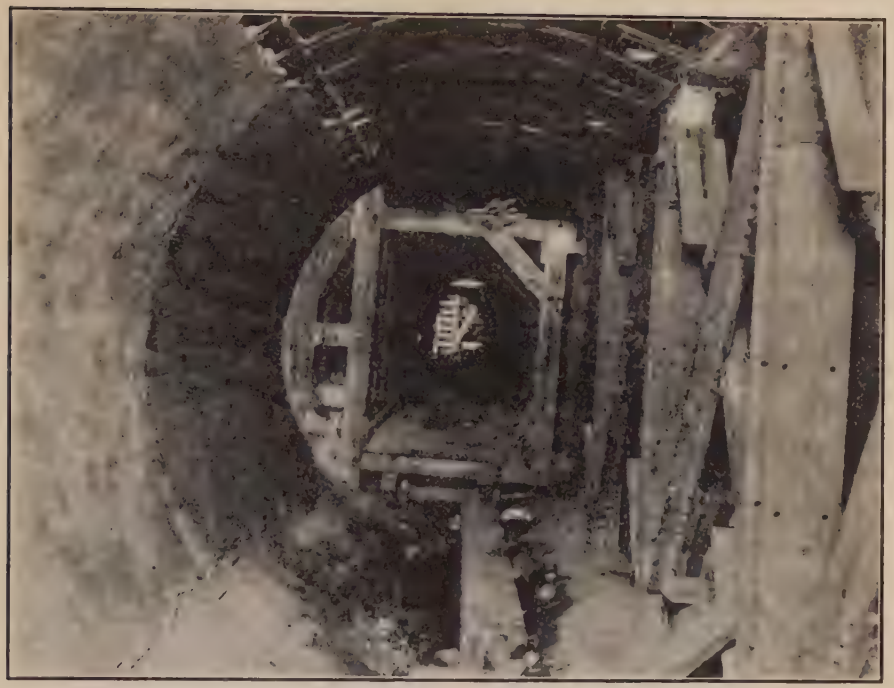

FIG. 8.—PROGRESS ON GAS CONDUCTOR, MAY 29, 1924, CITIZENS WORKS, BROOKLYN UNION GAS COMPANY.

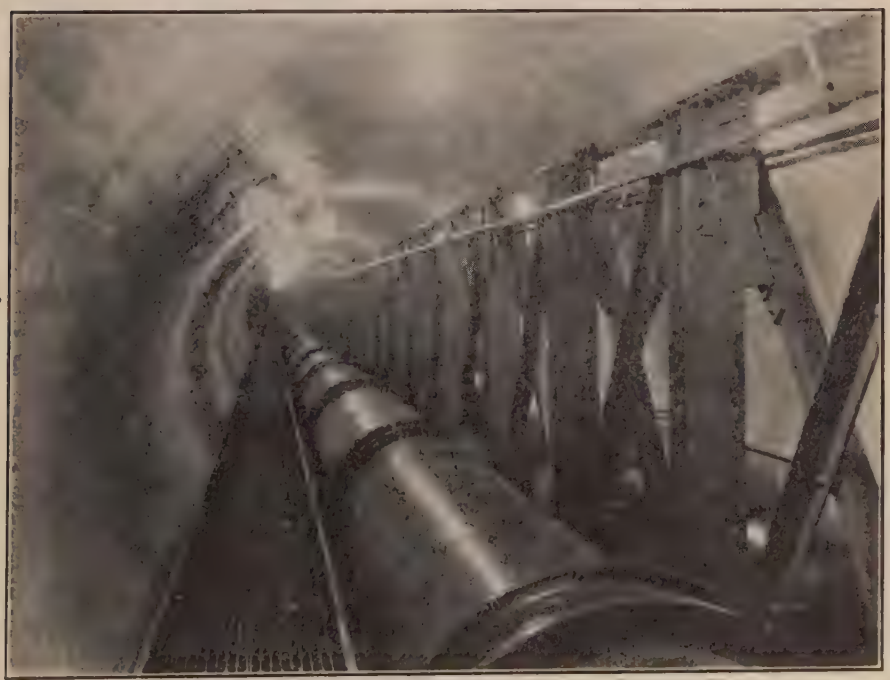

FIG. 9.—VIEW OF INTERIOR OF COMPLETED TUNNEL, CITIZENS WORKS, BROOKLYN UNION GAS COMPANY.

delays. It may appear elemental, but as a means of visualizing the work as a unit, it has no peer. It lessens the chance of halting the work for "lack of a tenpenny nail". There is a moment in almost every engineering project, when with the thousand loose ends that must dovetail not finished, or perhaps not begun, things look chaotic, and the engineer in charge, with all his planning and visualizing, trusts blindly to luck. At that moment the labor spent on keeping progress charts up to date more than repays its cost.

In looking over the actual work in its early stages the engineer is apt to be in the dilemma of the man who could not see the forest for the trees. A few plans and perspectives on which the progress of various items of the tunnel work were minutely and carefully plotted, will illustrate the methods used. During the progress of construction these charts hung on the walls of the field office, and a quick glance was all that was necessary to grasp the status of the work.

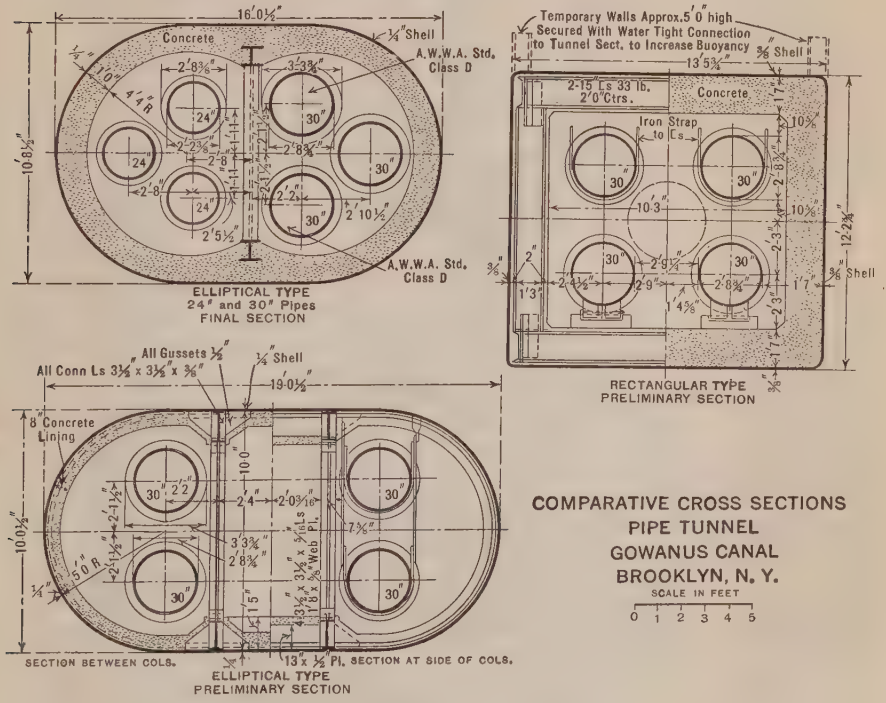

Fig. 10.

On a plan and longitudinal elevation (Fig. 12) was plotted a daily record of the dredging as it proceeded. Similar plans and longitudinal cross-sections were used for the delivery of the steel and for the progress of the riveting and of the concreting. On these plans a colored patch covering that part of the work which had been finished was filled in each day, so that there was available at all times a chronological record of the work and a comprehensive picture of what had been done, and what still remained to be done. Likewise Fig. 13

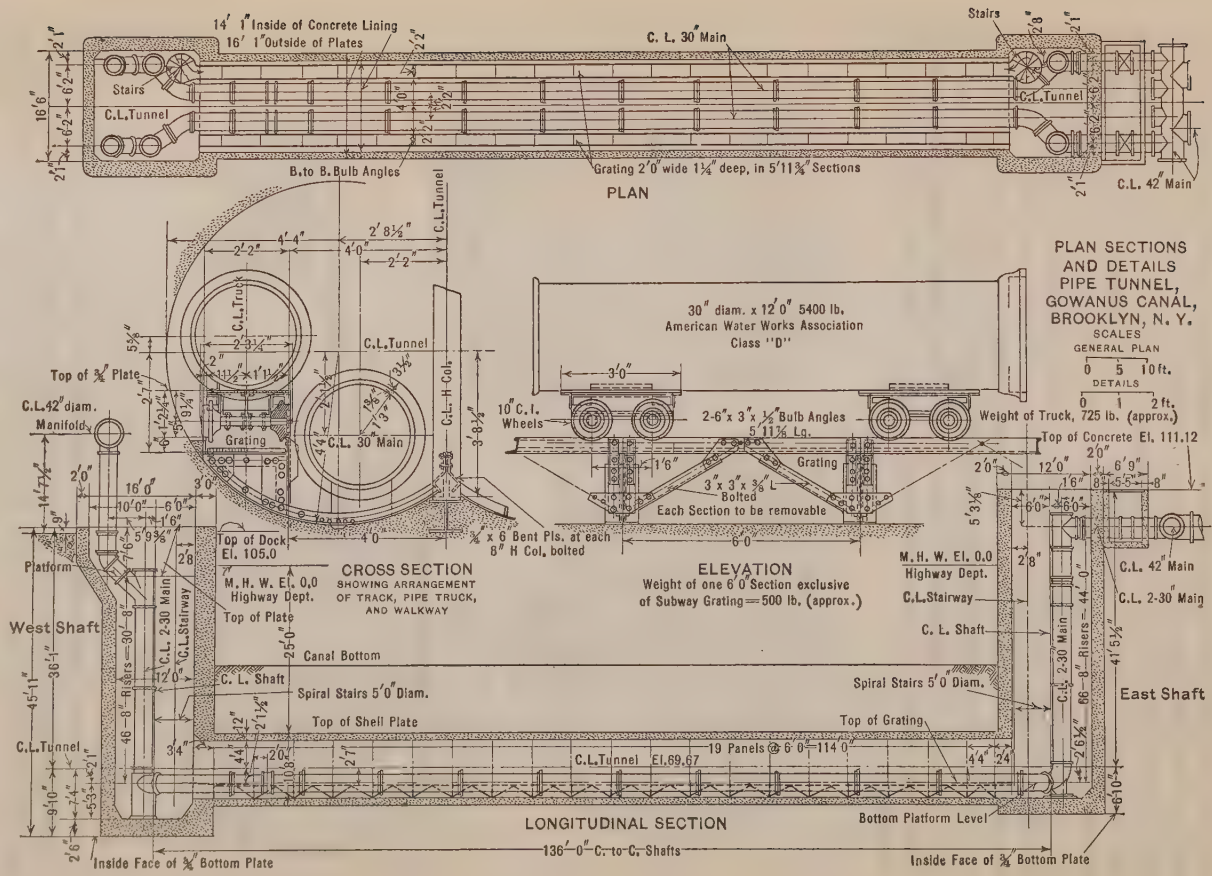
PLAN SECTIONS AND DETAILS PIPE TUNNEL, GOWANUS CANAL, BROOKLYN, N. Y.
SCALES
GENERAL PLAN
0 5 10 ft.
DETAILS
0 1 2 ft.
PLAN
14′ 1″ Inside of Concrete Lining
16′ 1″ Outside of Plates
C. L. 30″ Main
Stairs
C.L.Tunnel
Grating 2′0″ wide 1¼″ deep, in 5′11¾″ Sections
C.L. 42″ Main
B. to B. Bulb Angles
CROSS SECTION
SHOWING ARRANGEMENT OF TRACK, PIPE TRUCK, AND WALKWAY
C.L.Truck
Top of ¾″ Plate
C.L.42″diam. Manifold
Grating
C.L. 30″ Main
Tramway
Top of Dock El. 105.0
M.H.W. El. 0.0 Highway Dept.
Top of Plate
Platform
West Shaft
C.L. 2-30 Main
C.L.Stairway
Canal Bottom
C. L. Shaft
Spiral Stairs 5′0″ Diam.
Top of Shell Plate
C.L.Tunnel
¾″ x 6 Bent Pls. at each 8″ H Col. bolted
30″ diam. x 12′0″ 5400 lb. American Water Works Association Class "D"
10″ C.I. Wheels
2-6″x 3″x ½″ Bulb Angles 5′11¾ Lg.
Weight of Truck, 725 lb. (approx.)
3″x 3″x ⅜″ L Bolted Each Section to be removable
ELEVATION
Weight of one 6′0″ Section exclusive of Subway Grating = 500 lb. (approx.)
Top of Concrete El. 111.12
M.H.W. El. 0.0 Highway Dept.
C.L. Stairway
C. L. Shaft
C.L. 42″ Main
C.L. 2-30″ Main
Spiral Stairs 5′0″ Diam.
Top of Grating
East Shaft
C.L.Tunnel El.69.67
19 Panels @ 6′0″ = 114′0″
LONGITUDINAL SECTION
Bottom Platform Level
136′-0″ C. to C. Shafts
Inside Face of ¾″ Bottom Plate
Inside Face of ¾″ Bottom Plate

FIG. 11.

shows the percentage of riveting completed on the tunnel by days. It is self-explanatory.

General Details

Each side of the tunnel was provided with a walkway and rails for a pipe truck to facilitate the handling of pipe (Fig. 11). The installation of additional 24-in. mains above the present pipes will require this equipment. It will be noted that there is ample room for handling spare pipe, for removing sections of the present main, and for caulking or repairing the pipes. At the same time there is a minimum amount of waste space.

Provision was made for pumping out the "drip" resulting from the condensation of gas in the tunnel. The mains in the tunnel dip toward the Citizens Works side, which is deeper than the other, and contains the drip operated by a hand-pump from the platform level. No artificial ventilation was deemed necessary.

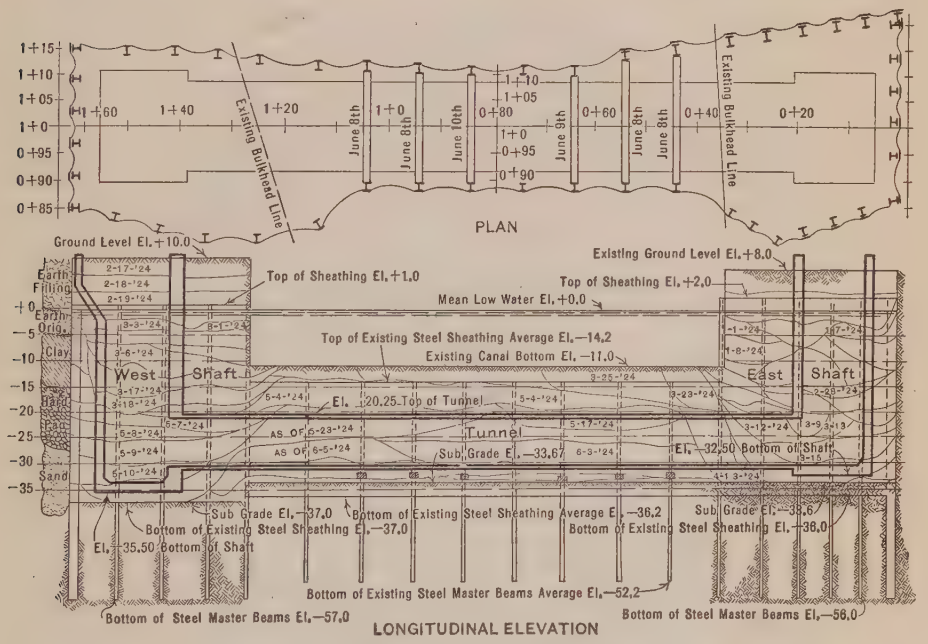

Fig. 12.—Progress Plan for Excavation and Dredging, Pipe Tunnel, Gowanus Canal, Brooklyn, N. Y.

The tunnel is lighted by electricity supplied from two separate and distinct services, one from the Citizens Works and the other from the Edison electric service of the 6th Street property of the Gas Company on the opposite side. At the entrance to the Citizens Works shaft, both feeders are connected to a three-pole double-throw switch, and a 4-circuit panel board was installed which is fed by either of the electric services.

The control of the lights throughout the tunnel and shafts is governed by four 3-way switches, two at the entrance to each shaft, fed by two circuits from the panel board. The lights are wired so that each set of 3-way switches controls every other light; thus, if a fuse should burn out on one circuit, the other circuit will continue to supply light. The same precaution has been taken with the Edison services; if one should fail the other is available, and either of them could be connected to the distribution panel boards of the double-throw switch. This 3-way switch makes it possible to turn the lights on or off at either entrance of both shafts. Galvanized hot-dipped conduit is used. The fixtures are of the vapor-proof type, consisting of an outer glass globe protected by a wire guard.

In each fixture the screw-shell parts are connected to the grounded neutral wire of the lighting circuit. By this method if, at any time, the screw shells should become loose and come in contact with the metal part of the fixture, since the fixture itself is grounded to the grounded neutral, no arc or short circuit would result.

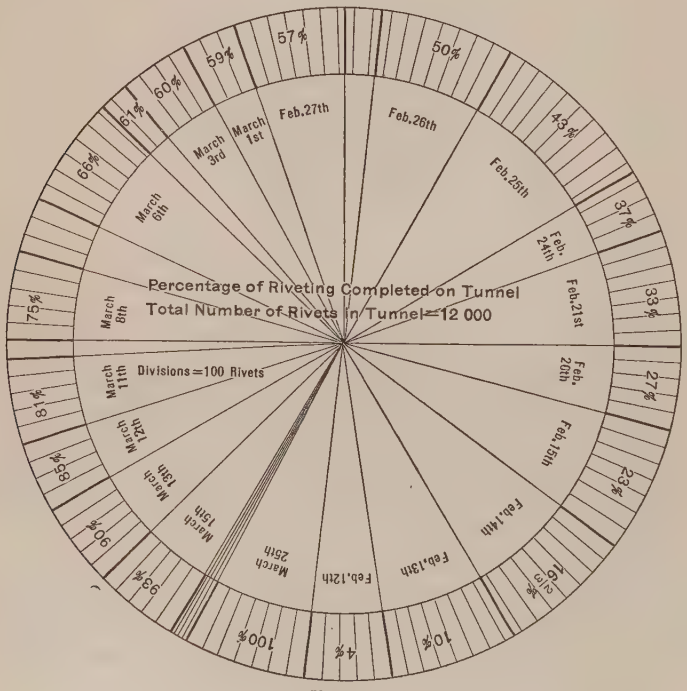

Fig. 13.

Conclusions

In the light of experience gained in the prosecution of the work, the following conclusions seem justified:

1.—That the method used in constructing this short tunnel was most economical, eliminating the use of an elaborate plant for driving the shield, and making it possible to use a cross-section best suited for the purpose.

2.—That the oval section is preferable to a circular section and ideal for this particular work, allowing a minimum vertical height, with sufficient clearance above and between the pipes, and affording maximum utility for the purpose required.

Acknowledgments

This work was executed under the direction of the writer by the Construction Department of The Brooklyn Union Gas Company. The writer acknowledges his indebtedness for the skillful aid of his assistant, Mr. W. M. Wepfer.

The steelwork was designed by Daniel E. Moran, M. Am. Soc. C. E., and the entire project was planned by Messrs. James C. Meem, M. Am. Soc. C. E., Chief Engineer of Frederick L. Cranford, Incorporated, Charles W. Staniford, M. Am. Soc. C. E., Mr. Moran, and the writer. Mr. Staniford was Consulting Engineer during construction. The Contractor was Frederick L. Cranford, Incorporated.

DISCUSSION

ROBERT RIDGWAY,* PAST-PRESIDENT, AM. SOC. C. E.—It was a privilege to witness the launching of this tube and, later, to see the completed work. Not a drop of leakage was observed, the tube being bone dry, which speaks well for a structure so deep under the water. Altogether it was a successful piece of work, both in design and construction, and the designers, the field engineers, and the contractors deserve great credit for the accomplishment. The design seems to be admirably adapted for the conditions that had to be met at Gowanus Canal.

Some day a rapid transit tunnel may be constructed under Gowanus Canal, in which case it will be well to consider whether this type of structure is the most feasible and economical one to adopt, notwithstanding the fact that such a tunnel would demand a much larger cross-section and, therefore, would require deeper and wider dredging. Certainly, the successful completion of the pipe tunnel brings it prominently to the attention of those who will have similar problems to face.

The speaker wonders whether a tube of similar design could be constructed under a wide stream like the East River where the traffic is very heavy and the tidal current strong. If the practicable length is limited, what would be the limit? Could the tube be built and sunk in sections, and the sections connected under the water? These are practical questions that would have to be answered if the method is to be considered for the crossing of large waterways.

All the East River and Hudson River transit tunnels have been constructed by the shield method under compressed air, cast-iron rings being generally used for the lining, with an inner lining of concrete. This method has been highly developed as a result of the work under New York waters, and its good features as well as its limitations are well known. It gives a water-tight structure which is believed to be permanent and its construction does not interfere with navigation. It is costly to build, however, and increasingly so, due principally to the rising schedule of pay for compressed air workers. Whether this cost will become so high as to make it necessary to adopt some other method is a question that is being given serious consideration by engineers concerned with rapid transit problems.

For the four-track Lexington Avenue rapid transit tunnel under the Harlem River—a waterway about 600 ft. wide between bulkhead lines—the open trench method was used, and the tubes were constructed and installed by the same general method as that used for the Michigan Central Railroad Tunnel under the Detroit River.† The Harlem River tubes had a total length of 1 080 ft. and were sunk in five sections, four of them each 220 ft. long, and the fifth, 200 ft. long. The steel in each 220-ft. section, together with the side lagging, which answered for the concrete form, weighed about 750 tons. The sections were built, as a ship is built, in a yard on the Bronx side of the Har-

* Chf. Engr., Board of Transportation, City of New York, New York, N. Y.

† "The Detroit River Tunnel," by W. S. Kinnear, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXIV (1911), p. 288.

lem River about a mile northwest of the site of the tunnel, launched, towed to the site, sunk in place, connected with the previous section, and then encased in concrete deposited by the tremie method, the concrete enclosure forming a structure 76 ft. wide and 24½ ft. high. The maximum depth of the bottom of the structure below mean high water was about 57 ft. Due to the admirable way in which the contractor, McMullen and Hoff Company, executed its work, there was no hitch in the operations and all the construction steps were co-ordinated and completed in an orderly manner with little or no complication. The leakage is negligible. The report of the operating company, the Interborough Rapid Transit Company, for the month of December, 1923, indicated an average leakage into the four tubes, including some leakage from the land approaches, of 1.27 gal. per min. for the preceding five years.

Inside the steel tubes of the Harlem River Tunnel is a 15-in. lining of reinforced concrete, designed to be strong enough to resist the external pressures in case of disintegration of the steel shell or the external concrete. The speaker assumes that the concrete lining of the Gowanus Canal tube was designed with the same thought in mind. This seems to be a precaution that should be taken, although even in the sewer-laden salt water of the rivers about New York, the steel probably will retain its strength for many years to come, as there is no circulating water in contact with it.

J. C. Meem,* M. Am. Soc. C. E.—In connection with the author's statement that the cost of this work was within the guaranteed estimate, the speaker considers that this was largely due to the hearty and generous co-operation of the author himself.

As to the strength of the tunnel, both longitudinally and laterally, it is not believed that any method of tunnel construction adapts itself so admirably to the greatest strength in either direction at relatively less cost. This is because a tunnel thus built may be designed for a cross-section adapted to the meeting points of the curves of strength and economy without reference to any other function, such as resistance to shield operation, excessive depth to gain cover, etc. The further fact that the section may be flattened to meet the minimum requirement of head-room, adds materially to the economical phases of construction and gradient.

Regarding the possibility of the rusting away of the steel shell, the speaker believes that steel buried in soil under water will never rust out; but rather that the initial rusting will be confined by the soil and by thus forming a rust-proof coating will prevent further deterioration. However, it is possible to provide at small additional cost a protective coating of concrete or mortar, which will prevent for all time the possibility of deterioration.

As to whether the tunnel can be built in great lengths, the answer is affirmative. Here the method provides temporary buoyant shafts at each end of sections about 200 to 400 ft. long. These sections are sunk by flooding the bulkheaded spaces under the buoyant shafts and as the water rises in these spaces and shafts the section sinks as described in the paper. Water-tight

* Chf. Engr. and Secy., Frederick L. Cranford-C. H. Locher, Inc., Brooklyn, N. Y.

diaphragms are placed at the ends of each section, and the end of the new section is brought by divers into full contact with the last submerged section and then bolted. A minimum number of light bearing piles, cut off to grade, enables the tunnel section to be brought to grade easily and with certainty. The temporary shafts are then removed and the tunnel is back-filled. Later, the diaphragms are burned out internally and the lining is completed. The only requirement is that the tunnel shall have sufficient internal or integral girder strength to overcome and submerge the remaining buoyancy of the unwatered tunnel section between bulkheads. The author's Fig. 7 shows how it was carried down on a perfectly even keel by its internal strength between bulkheads and by the proper manipulation of the valves flooding the shafts.

As to the adaptability of this method to a rock section, the speaker's answer would be that it is not well adapted to those trenches where rock forms a large percentage of the cross-section.

As to carrying this system into the approaches, the speaker believes that the experience had at the site described demonstrates conclusively that approaches may be carried by this method into actual city streets for considerable distances, if pipes and sub-surface structures are removed or by-passed.

It was demonstrated conclusively, not only in the case of the sheet-piling restraining the sides of the trench under the canal, but particularly in that of the west shaft, that the combined pressure of soil and water back of sheet-piling is not materially greater than that of the water itself, providing the soil is below the level of the lowest water at all times.

In the case of the canal piling, the master piles, 38 ft. long, were first driven 10 ft. apart, and the sheet-piling, 20 ft. long, was then driven between, the tops resting at an elevation 13 ft. below mean low water; 35-lb., straight web, Lackawanna piling was used. The master piles were 20-in., 65-lb. I-beams, with a standard sheet-pile riveted to the back flange. In the west shaft, the master piles were 58 ft. long, and the sheet-piling, 38 ft. long, driven to a penetration of 38 ft. below low water. The master piles were driven from 4 to 10 ft. apart according to conditions, and although this shaft was dredged to a full depth of 37 ft. below low water, no evidence of disturbing pressure was noted in the case of a single master pile. In spite of the inertia of the master piles and the fact that much cribbing and rip-rap, as well as hardpan, was encountered in driving, satisfactory results were obtained by using a No. 8 McKiernan-Terry steam hammer; in fact, it proved better able to do the work than a 3 000-lb. drop-hammer. Much difficulty was experienced, more especially on account of the restricted areas of operation, in dredging the hardpan, which appeared when dredged to be a clayey quicksand, but so dense in its natural bed as almost to exclude water. Even when jetted, it settled back into hardpan so quickly that the jet was not effective. The best results were obtained with a Hayward ¾-yd. multiple-power, clam-shell bucket, alternating occasionally with a Mead-Morrison, extra heavy, ¾-yd., orange-peel bucket.

All the canal piling was pulled out. The foreman of the dredge, Mr. Charles Hegstrom, was responsible for the rigging of an ingenious device for pulling this piling, by reversing the steam-hammer and driving the piling up.

To summarize: The completion of the tunnel as described shows conclusively that it is possible, under proper conditions, where it may be necessary to restrain the adjacent soil, to sheath and dredge a submerged trench without bracing; that it is possible to construct a tunnel of a strength and water-tightness not heretofore paralleled, and to submerge and set it easily to line and grade.

JOHN R. SLATTERY,* M. AM. SOC. C. E.—The constantly increasing wages that it is necessary to pay "sandhogs", and the constantly diminishing hours of labor under the various pressures that they work, make it of the utmost importance in the interest of economy to avoid the use of compressed air in building tunnels. The author's method supplies a satisfactory solution, it is believed, for under-river tunnels in many cases.

The Board of Transportation of the City of New York has now before it the problem of building tunnels under the East River at 53d Street, Fulton Street, and a little later, perhaps, at Rutgers Street. At 53d Street the tunnel will lie partly in rock; mostly in rock between Manhattan and Blackwell's Island. Between Blackwell's Island and the Borough of Queens, from what has been determined by boring thus far, there will be little rock.

There is a distinct limitation in the possibility of constructing a tunnel by this method where rock is encountered at such depths as are met in East River tunnels. The U. S. War Department requires a depth in the East River of 40 ft. over the roof of a tunnel.

The removal of rock under water can be economically carried out at the present time only by means of dipper dredges. The dipper dredge has yet to be built that will handle rock at the depth necessary to excavate a trench for a tunnel under the East River. It may be possible to design such a dredge, but it would be quite a step in advance of any dipper dredges that are now afloat. It is possible to remove rock at any depth with orange-peel buckets, but it is a heart-breaking operation. The best orange-peel bucket that has ever been built, will come up time after time with a piece of rock no bigger than a man's head. It is a serious question whether the cost of removing the rock by orange-peel buckets would not offset the other economies in the trench method of construction. In soft material it would be desirable to draw up specifications to permit of alternate bids being submitted for shield-driven tunnels and tubes laid in an open trench.

At Fulton Street there is another problem, that is, the current and the traffic in the river. The channel at that point is 1 200 ft. wide. It is intensively used and the current is swift. Dredges working in the East River, near the existing tunnels, are frequently damaged by towboats coming down the river, without apparent reason. The War Department would probably not permit the use of tunnel sections that would occupy more than one-third of the 1 200-ft. channel. That would reduce the possible length of sections to about

* Deputy Chf. Engr., Board of Transportation of the City of New York, New York, N. Y.

400 ft. The question is as to how safe the sinking of a 400-ft. section would be in the middle of that 1 200-ft. channel.

To carry out the plan safely, it would be almost essential to shut off traffic on the river while a section in the channel was being sunk. It has been stated that it required only 2½ hours to pull this section out of the coffer-dam into position and settle it. It might be possible to induce the War Department to shut off traffic for that length of time, but it is doubtful. Even if the War Department gave its consent, there would be some problem in actually stopping that traffic; towboat men are not easy to handle.

On Newtown Creek, where another tunnel is to be constructed, the boring sheets are not completed. Newtown Creek has probably the densest traffic of any stream in America, if not in the world. Nevertheless, this method could probably be applied there, for even if rock is encountered it will probably not be so deep but that it could be economically dredged.

There is one other problem in connection with this method that raises a question as to its economy; that is, whether approaches can be built without compressed air. If it is necessary to use a shield in the approaches, it is a question whether it would be economical to change the method for the river section. For any tunnels of such long lengths as those under the Hudson, it is believed that this method affords the best solution.

Henry B. Seaman,* M. Am. Soc. C. E.—This paper describing so clearly the novel method of construction, is extremely interesting and instructive from beginning to end. Yet there is one element which deserves particular commendation, that is, the driving of the sheet-piling to this depth, and the use of master piles. This seems to have been the key to the successful execution of the plan. Other matters, also, such as the adaptability of the design to the purpose intended, and its skillful prosecution, deserve the highest commendation.

It would seem that the method here adopted could be applied to work of indefinite magnitude, and may lead to a wholesome revision of tunnel methods.

T. Kennard Thomson,* M. Am. Soc. C. E.—More than thirty years ago a bright inventor tried to get the President of the Long Island Railroad, the late Austin Corbin, to appropriate $7 000 000 to build a tunnel under the East River. It was to have two iron shells, one inside of the other, with an annular space of 15 in. between them, which space was to be filled with oil (his great patent) to prevent rusting! As Mr. Meem has stated, iron has lasted for ages under water without rusting; but he should have added, "provided the water is pure". Steel does not stand rust as well as iron, and certainly the waters of New York Harbor are anything but pure. In at least one case in New York Harbor the impurities, combined with the heat from a power station, were sufficient to quickly destroy several tunnel linings, successively, of concrete, brick masonry, and stone masonry, whereas wood lining lasted much longer. Elsewhere in this harbor, wooden tunnels have been completely destroyed.

* Cons. Engr., New York, N. Y.

In the West some one estimated that 10% of the cost of a dam was to be saved by building it of steel, instead of gravity masonry construction, but shortly after the steel dam was completed, it collapsed. It was then found that an 8-lb. hammer which had been lying in the water for a few months was completely corroded due to the chemical impurities of the water.

The author, the consulting engineers, and the contractors are to be congratulated on their ingenious design, careful calculations, and successful work. The speaker's only criticism would be that a slight change of design would have saved a large sum, without reducing the factor of safety. Time could also have been saved.

About twenty-five years ago, the speaker took out patents for a trench tunnel. The proposition was: (1) To drive piles and cut them off, one row at a time, below the grade of the tunnel; and (2) to build the tunnel on floating pontoons, in lengths of from 500 to 1 000 ft. each, towing each section to the site and sinking it on the piles, the silt being dredged just before the sinking. One feature of these patents was a pneumatic caisson enclosing each joint, so that all the concrete would be placed in air instead of in running water. Sometimes heat and chemical impurities will destroy concrete before it has had time to set, even more quickly than steel will be destroyed.

A concrete tunnel with the roof and sides as well as the bottom 6 ft. thick, and with six compartments, each 25 ft. wide by 16 ft. high, will cost much less than the two 20-ft. roadways of the Holland (Vehicular) Tunnel and would be a little heavier than water, so that it would not rise and fall, like the present Hudson River Tunnels, inversely as the tides, hour after hour, year after year. The 6-ft. water main under Toronto Bay carried Toronto's water supply safely for years, and then, suddenly, a mile of it floated to the surface. No one can prove that the present tunnels will not last 1 000 years, nor that they will last 1 000 min. Then, if the bigger and safer tunnel costs much less why not adopt it?

Regarding the length of sections that could safely be used, it would be just as easy to handle sections 500 to 1 000 ft. long as a large ocean liner.

Where part of the excavation would be in rock there would be an air-chamber under the tunnel sections for completing the excavation under compressed air. An improvement over the device of sinking caissons over the joints would be to have the caissons attached to the tunnel sections before sinking so as to telescope those already sunk.

The reason given for rejecting this design for the Vehicular Tunnel was that with six compartments of 25 ft. each it would have too great a capacity for one location. To the speaker it seems absurd to expect one 20-ft. roadway in each direction to be at all ample to connect Long Island and Manhattan to the remainder of the United States, through New Jersey, even if such tunnels were placed at nearly every block. Even yet the peak of automobile congestion has not been reached.

Frederick L. Cranford,* M. Am. Soc. C. E.—The Gowanus Gas Tunnel is a perfect working model of what can be accomplished in the building of a sub-

* Pres., Frederick L. Cranford, Inc., Brooklyn, N. Y.

aqueous tunnel and demonstrates that a trench under water can be dug successfully and that the water pressure against the sides of the trench has great value in holding its banks even adjacent to important structures on the shore. The west shaft of the tunnel is right alongside a large sewer and concrete retaining wall holding several thousands of tons of coal and within about 40 ft. of important buildings of the Brooklyn Union Gas Company. There was no disturbance whatever to these structures, although this shaft was not braced in any way, except by the cantilever value of the master piles.

The operation of sinking such a tunnel, by bulkheading and sinking it either with a permanent or a temporary shaft fastened to the ends, provides a control that is absolute and perfect. The buoyancy of the tunnel having been neutralized the sinking was exactly regulated by permitting water to enter each shaft, through valves provided for that purpose; one man on the top of each shaft could control the speed of sinking and maintain the tunnel perfectly level by the regulation of the valves. Conversely, should there be any slip or hitch in the operation, by pumping the water out of the shaft the tunnel can be raised to the surface again.

The speed of sinking such a tunnel in navigable waters, is controlled by the size of the intake valves. This particular tunnel, with only 4-in. valves, sank in the beginning at the rate of 1 ft. in 6 min. and, later, 1 ft. in 4 min. There is no reason why this speed could not be greatly increased if it were desirable.

As regards the economics of such a structure, a four-track tunnel would be vastly more desirable proportionally than a single track. On large tunnels the economic gain very nearly multiplies as the size. Again, on small work where the necessary air plant and its operation is a large factor in relation to the value of the project, there would be an advantage in this method of construction.

Charles W. Staniford,* M. Am. Soc. C. E.—In common with the other engineers who have discussed this paper the speaker has often considered the possible adaptation of the method used in this pipe tunnel to deep river crossings and canal crossings adjacent to New York as well as to similar construction with the tunnel built in several sections.

It is certain that in shallow crossings this tunnel method will be widely recognized in the future. The difficulty of making land connections where the deep tunnel section proper ends may be obviated by continuing the tubes inshore up the approach grades, until a point is reached where the depth of portal cut may be assumed economically as an open braced excavation.

This again illustrates the advantage of the open trench method, in that the tunnel may be built very near the grade required by the navigation clearance. The elevations of the portal shafts, thus brought to a minimum depth, result in less grade to be overcome on the approaches, with a consequent lessening in the disturbance to property and streets, which is of prime importance.

* Cons. Engr., New York, N. Y.

The sinking of the structural tube of the Gowanus Tunnel was kept under perfect control by watering the shafts; after overcoming the air in the upper 18 in. near the roof, the tube sank symmetrically at each shaft, averaging about 2 in. per min. The sinking was halted about 1 ft. above the bottom of the trench for final alignment. There appears to be no reason why the operation should not be duplicated from shaft to shaft by proper bulkheading. On account of the perfect control over the sinking operation, connection may be made to adjoining sections by any of the various devices used at the St. Clair Tunnel, the Harlem River Tunnel, or by other mechanical means which may be improvised from the experiences gained with this particular undertaking.

In many river crossings, particularly in the approaches to the New York Metropolitan District, draw-bridges are becoming an impediment to the navigation and highway needs of even the present day. In these districts much of the marsh land is unused, but is rapidly being developed, each dock and terminal adding to the commerce on the streams, while the immense increase in the use of these highway crossings by motor trucks and pleasure cars renders the frequent opening of the draw-bridges a menace to both. Judging from present progress, the time is not far distant when these interferences will unquestionably prove a most serious drawback to the growing navigation needs; the new highways may not be able to function so as to permit of the intensive use estimated to meet their great cost to the States, and the railroads may fail to satisfy the demands of the quick suburban service required.

A structural steel section, built on land and sunk into a submerged trench built for it, offers at least one method that should henceforth receive the serious consideration of engineers as a means of eliminating some of these menaces from important navigable streams.

It was for this purpose that the speaker in collaboration with Victor Gelineau, M. Am. Soc. C. E., Director of the Board of Commerce and Navigation, State of New Jersey, has submitted to this Board a plan for proposed crossings of the Hackensack and Passaic Rivers to be built in an open trench, after the general plans used in the Gowanus Tunnel. These similar tunnels (Fig. 14) are designed to take the place of two proposed new draw-bridges, where the conditions even at the present time create a serious handicap to proper navigation and highway movement.

Some of the outstanding features developed at the Gowanus Tunnel are as follows:

First.—The adaptability and pliability of the steel interlocking sheet-piling in holding the sides of deep trenches, the piles being driven from 14 ft. to about 25 ft. below low water so that the top line of the submerged sheet-piling shall be in permanent position below the requirements of navigation.

At this tunnel not only the sheet-piles but the heavy 20-in. I-beams (the "master piles" which were placed 10 ft. apart for support against lateral earth pressure on the sides of the submerged trench) were driven well down by the aid of hammer, jet, and follower into a hard sand formation.

Second.—The minimum of open excavation under water required by this method. Obviously, the sheet-pile trench might be omitted, and the whole excavation made in open cut. This was considered but not adopted, not only on account of lack of control of the slopes in the river bottom itself, but on account of the danger of undermining the surrounding docks and structures.

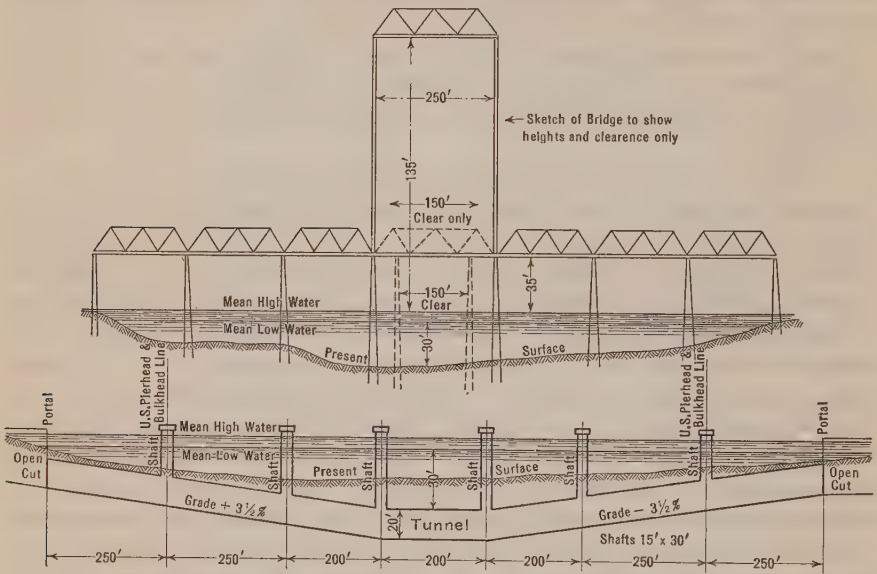

FIG. 14.—COMPARISON BETWEEN BRIDGE AND TUNNEL, HACKENSACK RIVER.

The excavation was further complicated by the remains on each side of the stream of old cribwork of logs and rock. In most of the outlying districts where tunnels may be required, and notably at the Hackensack and Passaic Crossings, the problem of excavation (Fig. 15) is much simpler than at the Gowanus Tunnel. At those sites there is a blanket of mud of moderate depth, underlaid with good sand. Therefore, the first under-water trench to create depth for navigation before the sheet-pile trench is started, could all be excavated most cheaply by pumping, using the material to be deposited on the adjacent marsh lands to make them usable and valuable.

Third.—The structural steel section built above ground under the most favorable circumstances. It was tested after launching and found to have suffered no distortion and to have no leaks.

Before being submerged in its trench, practically three-quarters of the reinforced concrete interior shell was placed. The tube then floated with an exposure of about 18 in., except for the shafts which were built high enough to give free-board above the level of high water when it had been sunk to place. All this work was accomplished while the tube was in the preparatory and temporary sheet-pile trench. On the other hand the Passaic and Hackensack Tunnels, or other similar tunnels, could be constructed in the open, longitudinally along the river banks, on side launching ways.

The tentative section for these tunnels (Fig. 15) approximates the oval Gowanus Section. The section for the four traffic lines shown could be modified to approach more nearly a rectangular form, similar to large scows and railroad car-floats. The longitudinal trusses between roadways offer opportunity for study in the development of longer sections between shafts before launching, even of a combination of one intermediate and two end shafts, which would still leave the length short as compared with a moderate sized ship. Lengthening the sections before launching would measurably decrease the cost, on account of the lesser number of under-water connections to be made at the joints.

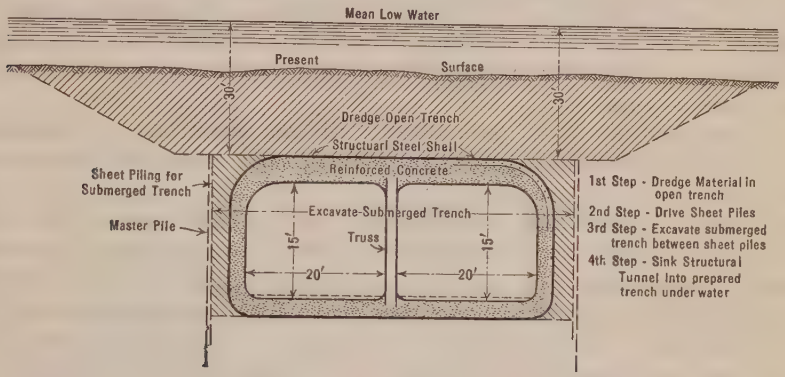

FIG. 15.—TYPICAL SECTION ILLUSTRATING METHOD OF CONSTRUCTION.

Fourth.—The side or lateral launching of this tube on a slope which caused comparatively rapid and free movement down the ways by gravity, the only control being tackle attached to anchors at intervals.

For future tunnels built on this principle it is easy to conceive of much longer units, by planning the side launching ways on a flatter slope, or on a slope whereby the tube would be pulled down the ways from the front, or pushed down by jacks in the rear.

Fifth.—The excellent general appearance that may be obtained with the simple steel shell and reinforced concrete lining. With ordinary rubbing and painting after the removal of the forms, the interior presents the appearance of a well-lighted, white enameled room. The spiral stairs at each shaft, easy walkways for inspection, ample room for handling pipe, and for work, in case of renewal or repair, together with absence of moisture, all indicate what may be expected from the same tunnel methods applied to vehicular or transit purposes.

JOHN F. O'ROURKE,* M. AM. SOC. C. E.—The design of this pipe tunnel and the manner in which it was constructed meet with the speaker's entire approval. What is said in regard to the earth pressures at the bottom of the excavation corresponds with a similar experience with the ground around the foundation piles under the piers of the Wisconsin River Bridge of the Chicago, Burlington, and Northern Railway Company, at Prairie du Chien, Wis.

* Cons. Engr. New York, N. Y.

It was necessary to saw off the piles about 10 ft. below the bed of the river. The sand surrounding the piles within the area of the foundations was removed with a sand pump. The sides of the excavation stood practically vertical and remained undisturbed while the piles were being sawed off and the floating caissons containing the concrete bottom of the piers were sunk to place, showing that no lateral pressure existed in the ground.

Referring to Mr. Thomson's statement in regard to the "floating" of the Hudson River tunnels, it was the speaker's experience that the ground there was also free from lateral pressures. This was shown by the fact that the clay of which the bed of the Hudson is composed, took almost a year to close in finally around the tunnels, with the air pressure in the tunnels during construction usually about equal to that due to the water above the river bottom, which balanced the water pressure and left the clay free to move if there was any lateral pressure.

The so-called floating of the tunnels is nothing more than the compressive action of the water of the river on the mass of clay resting on the rock about 200 ft. below the bottom of the tunnels. This was measured by a steel rod extending through the tunnel bottom and fastened in the rock, which showed that the tunnels moved up and down about $\frac{3}{16}$ in. between high and low water. This movement was continuously recorded by means of a clock mechanism with a pointer which gave a continuous curve, considerably magnified, for each 24 hours. The bed of the river was shown to be so elastic and sensitive to the varying pressures of the water that it was possible to determine the actual rise and fall of the tide each day from the movement of the tunnel. This interesting fact shows how stable the undisturbed clay in the bed of the Hudson River really is and its reliability for carrying loads without settlement when they are properly limited to the ordinary practice for that kind of ground.

L. S. Stiles,* M. Am. Soc. C. E. (by letter).—Answering Mr. Ridgway's question, the reinforced concrete lining is designed to support the entire load acting on the tunnel.

Mr. Thomson suggests a slight change of design. All things are possible, and there is no one thing that can be done perfectly. A third person often sees how the originators could have improved a particular conception. In the present instance, however, all types of cross-sections, and, in fact, all the elements entering into this tunnel were carefully weighed. As Mr. Meem points out, the design approximated the meeting points of the curves of strength and economy, and having developed a cross-section answering this requirement and the purpose for which the tunnel was built with a minimum of waste space, it was not considered good engineering to eliminate further any of the factors.

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INSTITUTED 1852

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Paper No. 1603

STRESSES IN THICK ARCHES OF DAMS*

By B. F. Jakobsen,† M. Am. Soc. C. E.

With Discussion by Messrs. William Cain, Elmo G. Harris, Lars R. Jorgensen, Fredrik Vogt, George Paaswell, W. A. Perkins, Fred A. Noetzli, and B. F. Jakobsen.

Synopsis

The stresses in arches considered as parts of an arch dam are determined on the assumptions that Poisson's ratio is zero and that the vertical downward stresses produced by the weight of the concrete and of the water acting on the up-stream face are negligible; it is further assumed that the modulus of elasticity is constant and that normal planes remain plane, so that deformations due to shear do not influence the distribution of the normal stresses due to bending. Making these the usual assumptions the writer finds formulas for stresses, which formulas are identical with those developed by William Cain, M. Am. Soc. C. E. This serves to check the method of development used in this paper.

Poisson's ratio is then assumed as finite and the structure is considered as an arch rather than as a straight beam, that is, the actual position of the neutral axis is found instead of, as usual, assuming it to coincide with the gravity axis, and the stress distribution is determined.

The introduction of Poisson's ratio and of the correct location of the neutral axis does not seem to influence the final results materially, except for short and thick arches. The influence of the vertical stress is found to be considerable for high dams.

The influence of temperature variations, swelling, and shrinkage is investigated; the formulas presented make it possible to assume different conditions

* Published in February, 1926, *Proceedings*.
† Cons. Engr., Los Angeles, Calif.

for the up-stream and for the down-stream faces. This assumption is probably nearer the truth than that of a uniform change of volume for the entire section of an arch, as the up-stream face, when loaded, is water-soaked, whereas the down-stream face is not, or perhaps to a lesser degree, depending on the thickness of the arch. Also, the temperature of the up-stream face is likely to differ from that of the down-stream face. The influence of the part of the water pressure that is carried by the dam considered as a cantilever and the effect of pressure grouting are also investigated.

The stress determination made by the use of Lamé's formulas for thick hollow cylinders and of Airy's stress function is on the assumptions that the modulus of elasticity and Poisson's ratio remain constant, and does not involve any of the other usual assumptions previously mentioned. Judging from the thick and rather short arch investigated, the usual assumptions are fairly accurate for ordinary arches in arch dams, but the more nearly correct formulas here developed show a somewhat more advantageous stress distribution than previous formulas indicated.

It is shown that radial sections cannot remain plane and that the correct stresses cannot be found unless the deformations of the abutments are taken into account.

The influence of the variation of the modulus of elasticity on the stress distribution is discussed, and it is shown that this variation tends to make the stress distribution more uniform than if the modulus of elasticity were to remain constant. Tables and diagrams are given to facilitate computations.

The theory of the secondary arch is discussed and convenient approximate formulas are derived. Approximate formulas are also given for an arch with a variable thickness.

It is shown that the swelling of the up-stream face due to water-soaking produces an up-stream displacement of the arches and a down-stream deflection of the cantilever; the deflection of a gravity dam due to the deformations of the foundation is also investigated. These factors are shown to be of relatively great magnitude. The usual methods of dividing the load between the cantilevers and the arches, which neglect those factors, cannot lead to trustworthy results.

I.—Introduction

The Pacoima Dam, a constant-angle arch dam approximately 400 ft. high, is now under construction by the Los Angeles County Flood Control District.* Its estimated cost is $1 750 000 and the saving, compared to a gravity dam, is in excess of $1 000 000. Besides, the constant angle arch dam has a much larger factor of safety than the gravity type.

In investigating the Pacoima Dam with the Cain formulas in the usual manner, that is, neglecting the influence of the vertical stresses due to the weight of the concrete, tension is found in the thicker arches. In order to

* A license arrangement has been entered into with the patentee, The Constant Angle Arch Company, San Francisco, Calif., to furnish the design and occasional inspection during construction.

transmit this tension about $60 000 worth of reinforcing steel in place would be necessary. The question, therefore, arose as to whether this tension actually existed or whether it was to be attributed to inaccuracies in the theory used in determining the stresses. It will be appreciated that the proper determination of this question is of considerable economic importance, especially as the constant angle type offers opportunities for rational design, that the constant radius type cannot offer, and as it is likely to be increasingly used.

In discussing Professor Cain's methods the writer has shown* that these reduce to the formulas for straight beams when the radius approaches infinity and the central angle approaches zero. In deriving the ordinary bending formulas it is assumed (see Fig. 1), that $\sigma_y = 0$, and that Poisson's ratio, $\frac{1}{m} = 0$. In reality, σ_y produces an elongation in the direction of σ_x which will modify the stress distribution. When the thickness of the beam is small compared to its length, as is generally the case, the influence of σ_y is quite negligible; but when the thickness is large compared to the length, the influence of σ_y becomes greater, and for a very short beam σ_y may be greater than σ_x. This is one reason why the Cain formulas cannot be expected to hold for very short arches and still be fairly accurate for the ordinary arches of arch dams. Another difficulty encountered in attempting to determine the stress distribution in very short arches is due to the restraining action of the abutments, which may influence this distribution throughout the entire arch. This difficulty is well known from the uncertainty in the determination of the stresses at the foundation of a gravity dam, where, unfortunately, they are greatest.

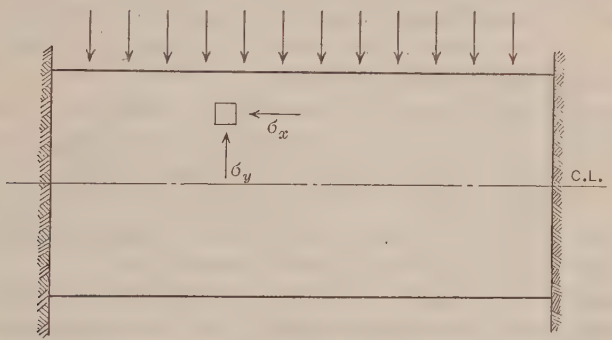

FIG. 1.

In an arch dam arch action and cantilever action exist simultaneously. To insure this double action, contraction joints should be provided at suitable intervals and these joints should be pressure-grouted after the concrete has set and cooled, that is, after most of the shrinkage has taken place and the temperature of the whole structure is near its minimum.† This provision also tends to eliminate internal stresses, as the contraction joints permit adjust-

* "The Circular Arch Under Normal Loads," *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 254, and Professor Cain's closing discussion, p. 272.

† "Improving Arch Action in Arch Dams," by Lars R. Jorgensen, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–20), p. 314.

ments of the various parts of the dam. Furthermore, the pressure grouting increases the vertical downward stresses in the up-stream face, which is an advantage.

That some skill is necessary in order to insure a continuous structure is illustrated by the Mulholland Dam, completed in December, 1924, by the City Water Department of Los Angeles, Calif. The dam is about 200 ft. high and is a so-called gravity arch, in which all or a large part of the load is carried by the cantilever, provided no uplift exists. In July, 1925, while the air temperature was high, the dam had developed a number of cracks, two of which were large and could be traced on the down-stream face down to the foundation and on the up-stream face down to the water surface, which was about 30 ft. below the crest. There was not much leakage, probably because the concrete inside the dam had not had time to cool. It is hardly safe to assume that this structure can act as an arch dam, which requires a continuous structure.

The contraction of concrete due to setting may amount to as much as 0.0002 and the thermal effect should be added to this. If a modulus of elasticity of 2 000 000 be assumed, the shrinkage of 0.0002 is equivalent to a tensile stress of 400 lb. per sq. in., or if Poisson's ratio is equal to 0.125, a vertical compression of 3 200 lb. per sq. in. would be necessary to compensate for a lateral shrinkage of 0.0002. In the thicker part of a dam no doubt the shrinkage is much less than 0.0002, and the water, being under considerable pressure in contact with the up-stream face, may also help to prevent the concrete from drying out. When the up-stream face is continually wet, a swelling of about 0.0001 may be expected.* Little is known regarding the volumetric changes of mass concrete, the individual designer usually making his own assumptions. The temperature variations inside a great mass of concrete† are known to be small after the chemical heat has been dissipated.

In designing an arch dam, it is convenient to determine the stresses due to the water pressure in two parts on the assumption that the arches and cantilevers divide the load and that no volumetric changes (due to temperature, swelling, and shrinkage) occur. Then some assumptions must be made regarding temperature variations, swelling, and shrinkage, depending on how the dam is built and whether or not pressure grouting is to be used.

It is planned to install in the Pacoima Dam some resistor type of cartridges with temperature coils, as developed by the U. S. Bureau of Standards. This will furnish information regarding the deformations due to stresses and the volumetric changes due to temperature variations, shrinkage in setting, effect of water-soaking, etc., will determine how effective the pressure grouting is, and will suggest the best method of procedure for accomplishing this grouting. Proper pressure grouting is essential to insure co-operation between the cantilevers and the arches and to prevent tension from developing in the arches.

* "Shrinkage and Time Effects in Reinforced Concrete," by F. R. McMillan, M. Am. Soc. C. E., Univ. of Minnesota, *Studies in Engineering No. 3*; see, also, the writer's discussion in *Transactions*, Am. Soc. C. E., Vol. LXXXIV (1921), p. 103, and especially Fig. 36 on p. 104, reproduced from Professor McMillan's paper.

† "Temperature Changes in Mass Concrete," by Charles H. Paul, M. Am. Soc. C. E., and the late A. B. Mayhew, Assoc. M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXIX (1915), p. 1225.

The purpose of this paper is to study the stresses in thick arches, including the influence of Poisson's ratio and of the vertical compression due to the weight of the concrete and the water acting vertically downward on the upstream face.

II.—Derivation of Professor Cain's Formulas

Professor Cain makes two assumptions which, for short thick arches, are questionable, namely,

(1) That the rotation of the section of the arch brought about by the uniformly distributed normal stresses, that is, compression or tension, may be neglected; and

(2) That Poisson's ratio $\frac{1}{m} = 0$. The customary method of computing stresses in an arch, by the formula:

$$\sigma = \frac{P}{t} \pm 6 \frac{M}{t^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

is only approximate, especially when $\frac{t}{r}$ is large. In Equation (1), P is the thrust; M, the moment; and t, the thickness. For high arch dams, $\frac{t}{r}$ may exceed 0.3 considerably, whereas $\frac{t}{r} = 0.3$ is the largest value considered by Professor Cain. Equation (1) is not involved in the Cain formulas and Professor Cain is, of course, fully aware of the approximation included, but the writer believes that it is generally used by engineers, even when $\frac{t}{r}$ is so large as to involve a considerable error. That this error is on the side of safety does not help matters, as it is the tension in an arch that limits the possibility of its use in dam design.

As the inaccuracies of the assumptions made by Professor Cain are inappreciable for an arch in which $\frac{t}{2 r \phi_1}$ is fairly small (ϕ_1 is one-half the central angle, see Fig. 2); that is, for a long thin arch, the formulas to be developed, based on the more accurate assumptions, should check Professor Cain's formulas for fairly small values of $\frac{t}{2 r \phi_1}$. This is actually the case, both for the stresses and for the deformations.

Professor Cain's formulas may be derived in the following manner, which lends itself better to the present investigation.* The water pressure acting on the outside of a cylinder of mean radius, r_m, and radial thickness, t, external radius, r_e, and internal radius, r_i, causes a shortening of the mean radius in that the center line of the cylinder moves from $A\,B$ to $A'\,B'$ (Fig. 2). If, now,

* Only arches encastré will be considered in this paper.

since the dam is fixed at the abutments, the arch be slid back along $C\,A$ so that the point, A', comes to A, the point, B', goes to D ($A\,A' = B'\,D$). Note that the crown section has not undergone rotation, but is still parallel to the center line, $C\,B$.

If now there be applied at the crown section a force, X, sufficient to pull the point, D, back into the center line, $C\,B$ (D does not go to the point, B, however), and its point of application A ft. from the center, C, be so determined that the crown section does not undergo rotation, then the same stresses as Professor Cain obtains should be found by combining the stresses due to the water acting on the cylinder and those due to the force, X.

In the following discussion all deformations are considered as positive when they are elongations, that is, an increase of length, and all stresses are considered positive when they bring about a positive deformation, that is, tension is taken as positive. The force, X, in Fig. 2 is taken as positive when acting as tension as indicated and the water pressure acting on the up-stream face, p_a, is also taken as positive, so that the radial stress produced at the up-stream face by p_a is negative. Displacements in the direction of X are also taken as positive and likewise displacements of the crown section toward the center, C, or in a down-stream direction, are taken as positive.

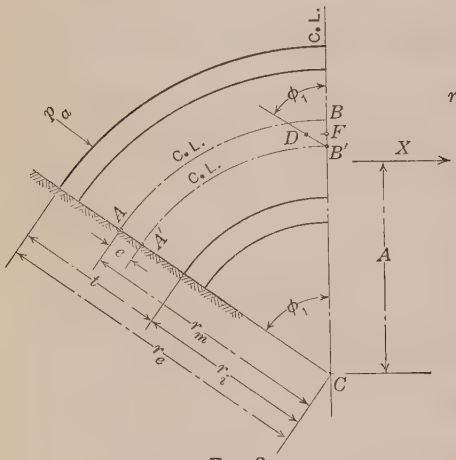

Fig. 2.

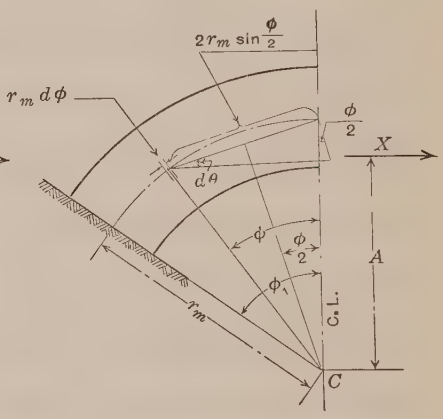

Fig. 3.

The displacement, $e = A\,A'$, in Fig. 2, computed from the ordinary cylinder formula, is,

$$e = \frac{p_a\,r_e}{E\,t}\,r_m = \frac{p_m\,r_m^{\,2}}{E\,t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

in which, p_m is the water pressure referred to the center line, so that

$$p_m = p_a\,\frac{r_e}{r_m} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

p_a and E both being in pounds per square foot, and t, r_e, r_m, and e being in feet. The force, X, must pull the crown section a distance, $e \sin \phi_1 = D\,F$,

in the direction of X. At the same time, X produces a movement at right angles to its own line of action; this will be considered later, when the deflection is to be determined.

In order to check Professor Cain's formulas, it is necessary to adhere to the assumptions made by him. That is, the ordinary cylinder formula must be used in determining the stresses and deformations in the cylinder due to water pressure, as that formula follows from the assumption that Poisson's ratio, $\frac{1}{m} = 0$, and the rotation of the crown section due to the uniformly distributed normal stresses must be neglected, that is, Professor Cain's Equation (6),*

$$\int_0^{\phi_1} M \, d\phi = 0 \quad \ldots\ldots (4)$$

must be used to determine A in Fig. 2.

The bending moment at any section of an arch due to $X = 1$, is,

$$M = A - r_m \cos \phi \quad \ldots\ldots (5)$$

M is taken as positive when it brings about an elongation in the extrados. From Equations (4) and (5),

$$A = r_m \frac{\sin \phi_1}{\phi_1} \quad \ldots\ldots (6)$$

Any force, X, acting a distance A ft. from the center, produces a movement of the crown section in the direction of X and in the direction of the center line, $B\,C$, of the arch, which center line is perpendicular to the direction of X; the crown section does not undergo rotation, but remains parallel to the center line, $B\,C$. Due to symmetry—a uniform pressure, p_a, is assumed—the shear is zero throughout the crown section.

III.—Movement of the Crown Section.

The movement of the crown section in the direction of X due to $X = 1$, is made up of three parts: (1) That due to bending moment; (2) that due to direct normal stress, in this case, tension; and (3) that due to shear. The moment is given by Equation (5). It brings about a rotation (Fig. 3), $d\theta = \Delta\, d\phi$ in a short piece of arch, $r_m\, d\phi$, situated at an angle, ϕ, from the center line, and hence,

$$d\theta = \Delta\, d\phi = \frac{M}{E\,I} r_m\, d\phi \quad \ldots\ldots (7)$$

in which, I is the moment of inertia. The displacement of the crown section in the direction of X due to the bending moment produced by $X = 1$ lb. acting on an arch slice 1 ft. thick, is then,

$$y_M = \int_0^{\phi_1} 2\, r_m \sin^2\left(\frac{\phi}{2}\right) d\theta \quad \ldots\ldots (8)$$

Introducing the value of M and $d\theta$ from Equations (5) and (7),

$$y_M = \int_0^{\phi_1} 2\, r_m \sin^2\left(\frac{\phi}{2}\right) \frac{A - r_m \cos \phi}{E\,I} r_m\, d\phi$$

* *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 237.

and integrating, since $\sin^2\left(\frac{\phi}{2}\right) = \frac{1}{2}(1 - \cos\phi)$,

$$y_M = \frac{r_m^2}{E\,I}\left[A\,(\phi_1 - \sin\phi_1) - r_m\left(\sin\phi_1 - \frac{\phi_1}{2} - \frac{1}{4}\sin 2\,\phi_1\right)\right] \ldots (9)$$

The unit direct normal stress (in this case, tension) is (Fig. 4) $\frac{\cos\phi}{t}$, and the displacement of the crown section in the direction of X due to the tension produced by $X = 1$, is,

$$y_N = \int_0^{\phi_1} \frac{\cos^2\phi}{E\,t}\,r_m\,d\,\phi = \frac{r_m}{2\,E\,t}\left[\phi_1 + \frac{1}{2}\sin 2\,\phi_1\right] \ldots\ldots (10)$$

The unit shear stress is $\frac{\sin\phi}{t}$ and the displacement of the crown section in the direction of X due to the shear stress produced by $X = 1$, is,

$$y_S = 2.88\int_0^{\phi_1} \frac{r_m}{E\,t}\sin^2\phi\,d\,\phi = 2.88\,\frac{r_m}{2\,E\,t}\left[\phi_1 - \frac{1}{2}\sin 2\,\phi_1\right] \ldots (11)$$

This is on the assumption that the shear is distributed across the section according to a parabola and that $\frac{E}{G} = 2.4$. It would be more correct to evaluate the ratio from the formula,

$$\frac{E}{G} = \frac{2\,(m+1)}{m} \ldots\ldots (12)$$

which, for $m = 8$, as will be used later, gives $\frac{E}{G} = 2.25$ instead of 2.4. The difference in the final result is small and moreover the assumption of parabolic distribution is not likely to be exact. Professor Cain has used the value

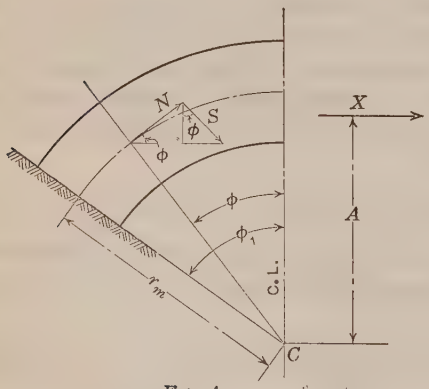

Fig. 4.

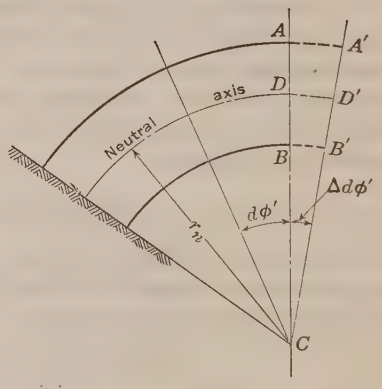

Fig. 5.

2.4, which will be used herein. The total displacement of the crown in the direction of X due to $X = 1$, is, therefore:

$$y = y_M + y_N + y_S$$

or,

$$y = \frac{r_m^2}{E\,I}\left[A\,(\phi_1 - \sin\phi_1) - r_m\left(\sin\phi_1 - \frac{\phi_1}{2} - \frac{1}{4}\sin 2\,\phi_1\right)\right]$$

$$+ \frac{r_m}{E\,t}\,[1.94\,\phi_1 - 0.47\sin 2\,\phi_1] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

IV.—Check on Cain's Thrusts and Moments

The crown section must be pulled over a distance of $e \sin \phi_1$ (see Fig. 2), in order to bring it back to the center line. As the force, $X = 1$, moves it a distance, y, the total force necessary is,

$$X = \frac{e \sin \phi_1}{y} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

The total compressive force on any radial section of the arch due to the water pressure is $p_a\,r_e = p_m\,r_m$ and, therefore, the resulting thrusts are:

At the crown,

At the abutments,

$$\left.\begin{aligned} P_0 &= -\,p_m\,r_m + X \\ P_1 &= -\,p_m\,r_m + X\cos\phi_1 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (15)$$

The moments are:

At the crown,

At the abutments,

$$\left.\begin{aligned} M_0 &= X\,(A - r_m) \\ M_1 &= X\,(A - r_m\cos\phi_1) \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (16)$$

Comparing Equations (15) and (16) with Professor Cain's equations, it will be seen that the writer's X is equivalent to Professor Cain's $(p\,r - P_0)$. In order to check this, substitute e from Equation (2) and y from Equation (13) in Equation (14). This gives Professor Cain's Equation (10),* with D_s taken from his closing discussion† instead of D, since shear is included. The two methods lead, therefore, to identical results, as they should. This affords a check on the writer's method, as Professor Cain's formulas have already been accepted as correct.

V.—Derivation of More Accurate Formulas

Professor Cain's Equation (6) (Equation (4) of this paper) neglects the rotation brought about by the uniformly distributed normal stresses produced by a force, X, acting at the crown section; he also assumes that the neutral axis coincides with the gravity axis, which is true for straight beams. The deformation brought about by a uniformly distributed normal stress is shown in Fig. 5, where Section ADB is brought to Section $A'D'B'$. The section remains radial, since the elongation of each fiber is proportional to its distance from the center, C, of the arch. The deformation, therefore, consists of an elongation, $D\,D'$, measured on the neutral axis, and a rotation, $\Delta\,d\,\phi'$.

In the preceding treatment, the stress on the radial section due to X was conceived as composed of: (1) A uniform tensile stress, with a resultant,

* *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 237.

† *Loc. cit.*, p. 269.

$X \cos \phi$, acting at the center of gravity of the section, a distance, $r_m = 0.5 (r_e + r_i)$, from the center, C; (2) a moment, $M = X (A - r_m \cos \phi)$; and, (3) a shear equal to $X \sin \phi$.

The uniform tensile stress produces rotation, as shown in Fig. 5. The most convenient manner of taking this rotation into account is to express the stresses produced by X' as: (a) A tensile stress with a resultant, $X' \cos \phi$, acting at the neutral axis a distance, r_n, from the center, C; (b) a moment, $M' = X' (A' - r_n \cos \phi)$; and, ($c$) a shear equal to $X' \sin \phi$.

The tensile stress is now no longer uniformly distributed, but the elongations of the various fibers are independent of r, the distance from the center, C (see Equation (25)), so that the rotation is produced by M' alone, and A' is determined from the fact that the rotation at the crown is zero, that is:

$$\int_0^{\phi_1} M' \, d\phi = X' \int_0^{\phi_1} (A' - r_n \cos \phi) \, d\phi = 0 \quad \ldots\ldots (17)$$

or,

$$A' = r_n \frac{\sin \phi_1}{\phi_1} = A \frac{r_n}{r_m} \quad \ldots\ldots (18)$$

This assumes that E and I are constants.

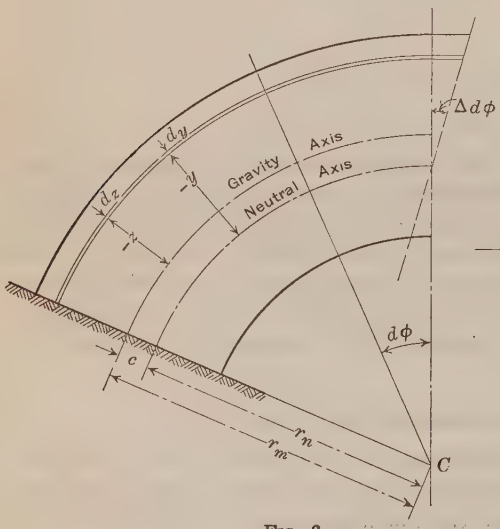

FIG. 6.

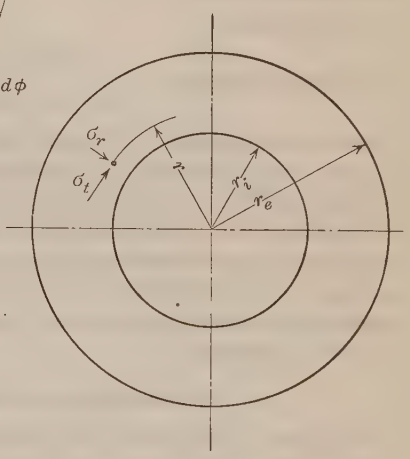

FIG. 7.

VI.—DETERMINATION OF THE FORCE, X'

In order to determine the displacement, y', the reasoning given in Article III is followed, but Equation (7) now becomes,

$$d\theta = \frac{M'}{E I'} r_n \, d\phi \quad \ldots\ldots (19)$$

in which*,

$$I' = r_n \, t \, c \quad \ldots\ldots (20)$$

* Bach's "Elasticität und Festigkeit", 9th Edition, p. 502; see, also, Ostenfeld's "Teknisk Elasticitetslaere" (Copenhagen), 3d Edition, p. 472, and Morley's "Strength of Materials".

is the moment of inertia with respect to the neutral axis, and (see Fig. 6),

$$r_n = r_m - c\ ; \qquad c = \frac{k}{1+k} r_m\ ; \qquad k = -\frac{1}{t}\int_{-\frac{t}{2}}^{\frac{t}{2}} \frac{z}{r_n - z}\, dz \ldots\ldots (21)$$

For a rectangular section,

$$k = -1 + \frac{r_m}{t} \log_e \left(\frac{r_e}{r_i}\right) \ldots\ldots\ldots\ldots\ldots\ldots (22)$$

in which, $\log_e$ is the natural logarithm.

The difference between I' and $I\left(=\frac{t^3}{12}\right)$ is always small. For $\frac{t}{r_m} = 0.3$ and $r_m = 100$, $k = 0.0076$, $c = 0.75427$, and $I' = 2\,245.74$, while $I = 2\,250$, or a difference of only 0.19 per cent. For $\frac{t}{r_m} = 0.6$ and $r_m = 100$, $k = 0.031717$, $c = 3.0742$, and $I' = 17\,878.16$, while $I = 18\,000$, or a difference of only about 0.7 per cent. Consequently, I may always be substituted for I'.

Then, from Equation (20):

$$c = \frac{I}{t\, r_n} = \frac{t^2}{12\, r_n} \ldots\ldots\ldots\ldots\ldots\ldots (23)$$

This is solved by first assuming $r_n = r_m$ and then substituting the value found for r_n and solving again.

The displacement, y'_M, of the crown section in the direction of X', produced by the bending moment, M', due to $X' = 1$, may now be obtained exactly as in Article III by substituting in Equations (5), (7), (8), and (9), M' for M, A' for A, and r_n for r_m. Then:

$$y'_M = \frac{{r_n}^2}{E\, I}\left[A'\,(\phi_1 - \sin\phi_1) - r_n\left(\sin\phi_1 - \frac{\phi_1}{2} - \frac{1}{4}\sin 2\,\phi_1\right)\right] \ldots (24)$$

The tensile stress is not now uniform across the section, but the elongation of each fiber is independent of r, so that the stress distribution is given by:

$$\left.\begin{aligned} &\frac{\sigma_t}{E}\, r = K \text{ (a constant)} \\ \text{and} \qquad &\int_{r_i}^{r_e} \sigma_t\, dr = X' \cos\phi \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots (25)$$

This gives,

$$K = \frac{X' \cos\phi}{E \log_e \left(\frac{r_e}{r_i}\right)}$$

and,

$$\sigma_t = \frac{K\,E}{r} = \frac{X' \cos\phi}{r \log_e \left(\frac{r_e}{r_i}\right)}$$

From Equations (21) and (22),

$$c = \frac{k}{1+k} r_m = \frac{-1 + \frac{r_m}{t}\log_e\left(\frac{r_e}{r_i}\right)}{\frac{r_m}{t}\log_e\left(\frac{r_e}{r_i}\right)} r_m = r_m - r_n$$

or,

$$\log_e \left(\frac{r_e}{r_i}\right) = \frac{t}{r_n}$$

Introducing this into the equation for σ_t, there results,

$$\sigma_t = \frac{X' \cos \phi}{r\ t}\ r_n \text{ lb. per sq. ft.} \quad \ldots\ldots\ldots\ldots\ldots\ldots (26)$$

This stress distribution amounts to a resultant, $X' \cos \phi$, acting in the neutral axis, as it should, since, taking moments about the center, C,

$$\int_{r_i}^{r_e} \sigma_t\, r\, d\, r = \int_{r_i}^{r_e} \frac{X' \cos \phi}{r\ t}\ r_n\, r\, d\, r = X'\, r_n \cos \phi$$

According to Equation (26) the stress at the neutral axis is $\frac{X' \cos \phi}{t}$, or exactly the same value as was used in Article III in deriving Equation (10). Therefore, y'_N, measured on the neutral axis, is,

$$y'_N = \frac{r_n}{2\ E\ t}\left(\phi_1 + \frac{1}{2}\ \sin 2\ \phi_1\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (27)$$

And for the shear,

$$y'_S = 2.88\ \frac{r_n}{2\ E\ t}\left(\phi_1 - \frac{1}{2}\ \sin 2\ \phi_1\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (28)*$$

The total displacement of the intersection of the neutral axis with the crown section, is, therefore,

$$y' = \frac{r_n^{\,2}}{E\ I}\left[A'\ (\phi_1 - \sin \phi_1) - r_n\left(\sin \phi_1 - \frac{\phi_1}{2} - \frac{1}{4}\ \sin 2\ \phi_1\right)\right] + \frac{r_n}{E\ t}\left[1.94\ \phi_1 - 0.47\ \sin 2\ \phi_1\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots (29)$$

For computations it is more convenient to introduce the value of A' from Equation (18), then,

$$y' = \frac{r_n^{\,3}}{E\ I}\left[\frac{\phi_1}{2} + \frac{1}{4}\ \sin 2\ \phi_1 - \frac{\sin 2\ \phi_1}{\phi_1}\right] + \frac{r_n}{E\ t}\left[1.94\ \phi_1 - 0.47\ \sin 2\ \phi_1\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots (29a)$$

It is evident that now the displacement, $e \sin \phi_1$, in Equation (14) must also be measured on r_n instead of on r_m, so that,

$$X' = \frac{e' \sin \phi_1}{y'} \quad \ldots\ldots\ldots\ldots\ldots\ldots (30)$$

*Professor Cain has called the writer's attention to the fact that r_m and not r_n should be used in this equation and also has pointed out that the error was most likely negligible. Using r_n simplifies the calculations (see Equation (56)). Moreover the coefficient, $2.88 = 1.2 \times 2.4$, is not exact. The value 2.4, has already been discussed in connection with Equation (12); the factor, 1.2, is derived on the assumption that the shear is distributed according to a parabola and is derived for a straight beam and not for an arch. Also, this coefficient, 1.2, is an average value found by the work formula and not an exact value.

The actual value of the coefficient, 2.88, as determined by the work formula for the arch considered in Article XII, for which, $\frac{t}{r_m} = 0.5$, is 2.63 for Poisson's ratio, $\frac{1}{m} = \frac{1}{8}$, and 2.63 r_m is in this case 2.687 r_n. Therefore, 2.88 r_n in Equation (28) is already too large and if $m = 8$, it is nearer the correct value, in this case, than 2.88 r_m.

in which, $e' = \frac{e\, r_n}{r_m}$, if Poisson's ratio is assumed to be zero. The correct value for e' will be given in Article VIII.

VII.—Stresses Due to the Force, X'

To determine the stresses produced by X' acting at the crown section, it is more convenient to locate the two equal but opposite auxiliary forces, X', at r_m, so that the resulting tensile stress is uniformly distributed and equal to $X' \frac{\cos \phi}{t}$. The moment must then also be referred to r_m and is $M = X' (A' - r_m \cos \phi)$. The stresses due to the bending moment alone are,

$$\sigma_t = \frac{M}{I} \frac{y}{1 - \frac{y}{r_n}} \quad \ldots\ldots (31)$$

y is measured from the neutral axis (see Fig. 6). For the extrados and intrados, respectively,

$$\left.\begin{aligned} y_e &= -\left(\frac{t}{2} + c\right) \\ y_i &= \frac{t}{2} - c \end{aligned}\right\} \quad \ldots\ldots (32)$$

The bending stresses at the extrados and intrados, respectively, are then,

$$\left.\begin{aligned} \sigma_{te} &= k_e \frac{M}{t^2}, \text{ in which, } k_e = 12 \frac{r_n}{r_e}\left(\frac{1}{2} + \frac{c}{t}\right) \\ \sigma_{ti} &= k_i \frac{M}{t^2}, \text{ in which, } k_i = 12 \frac{r_n}{r_i}\left(\frac{1}{2} - \frac{c}{t}\right) \end{aligned}\right\} \quad \ldots\ldots (33)$$

In Equation (1), $k_e = k_i = 6$. For large values of $\frac{t}{r_m}$, this may lead to a considerable error.

The stresses produced by X' on a radial section ϕ degrees from the crown section, are, therefore, for the extrados and intrados, respectively,

$$\left.\begin{aligned} \sigma_{te}' &= X' \left[k_e \frac{A' - r_m \cos \phi}{t^2} + \frac{\cos \phi}{t}\right] \text{ lb. per sq. ft.} \\ \sigma_{ti}' &= -X' \left[k_i \frac{A' - r_m \cos \phi}{t^2} - \frac{\cos \phi}{t}\right] \text{ lb. per sq. ft.} \end{aligned}\right\} \quad \ldots\ldots (34)$$

To these stresses must be added those produced by the water pressure, p_a, acting on the extrados. If the ordinary cylinder formula is used, this is,

$$\sigma_t'' = -\frac{p_a}{t} r_e \quad \ldots\ldots (35)$$

VIII.—Stresses Due to Water Pressure

The assumption that Poisson's ratio is zero is not correct for thick cylinders. The stresses and deformations for a thick hollow cylinder, are* (see Fig. 7):

* Bach's "Elasticität und Festigkeit," 9th Edition, Paragraph 58; see, also, Merriman's "Mechanics".

1.—The tangential stress,

$$\sigma_t'' = -\left(1 + \frac{r_i^2}{r^2}\right)\frac{r_e^2}{r_e^2 - r_i^2}\,p_a$$

or for the extrados and intrados, respectively,

$$\sigma_{te}'' = -\left(\frac{r_m}{t} + \frac{1}{4}\,\frac{t}{r_m}\right)p_a$$

and

$$\sigma_{ti}'' = -\left(1 + \frac{r_m}{t} + \frac{1}{4}\,\frac{t}{r_m}\right)p_a \qquad \text{.....(36)}$$

2.—The radial stress,

$$\sigma_r'' = -\left(1 - \frac{r_i^2}{r^2}\right)\frac{r_e^2}{r_e^2 - r_i^2}\,p_a \qquad \text{.....(37)}$$

(Here, p_a is that part of the water pressure which is supported by the arch. As will be seen, both the tangential and the radial stresses are independent of the axial stress, σ_x, as they should be for a cylinder. That is not true for an arch dam, for the reason that the vertical stress, σ_x, influences the deformation.)

3.—The tangential deformation, ϵ'. If σ_x is the vertical stress produced by the weight of the concrete above the arch under consideration and by the water acting vertically on the up-stream face, the tangential deformation per unit length a distance, r_n ft. from the center, is:

$$\varepsilon' = -\frac{1}{E}\left[\frac{m-1}{m} + \frac{m+1}{m}\,\frac{r_i^2}{r_n^2}\right]\frac{r_e^2}{r_e^2 - r_i^2}\,p_a - \frac{\sigma_x}{m\,E} \qquad \text{.....(38)}$$

For slide-rule computation this can be more conveniently written,

$$\varepsilon' = -\frac{1}{E}\left[\frac{m-1}{m} + \frac{m+1}{m}\left(\frac{1 + \dfrac{t}{2\,r_m}}{1 - \dfrac{c}{r_m}}\right)^2\right]\frac{\left(1 + \dfrac{t}{2\,r_m}\right)^2}{\dfrac{2\,t}{r_m}}\,p_a - \frac{\sigma_x}{m\,E} \qquad \text{.....(38a)}$$

The displacement, e', measured at the neutral axis, is therefore,

$$e' = -r_n\,\epsilon' \qquad \text{.....(39)}$$

If Poisson's ratio is assumed to be zero, and if the cylinder formula is used, the deformation is,

$$\varepsilon'' = -\frac{p_a\,r_e}{E\,t} \qquad \text{.....(38b)}$$

For $\dfrac{t}{r_m} = 0.5$, this is only 4.3% higher than the exact value, computed for $m = 8$; but sometimes a rather small change of X' may produce a considerable change in the resulting stresses.

When applying these formulas to an arch dam, it is tacitly assumed that the arches are free to deform radially but this is not true at the abutments. In

deriving Equations (36) to (38), inclusive, σ_x has been assumed as constant, that is, independent of r. This is, of course, not correct, but the average value of σ_x is practically the same whether the reservoir is empty or full. To take account of the variation of σ_x due to cantilever action would lead to great complications, and for the bottom part of the dam, the assumption that E and σ_x are constants, produces two errors, which are of opposite sign (see Article XVI).

It has further been assumed that the arch completely fills the site, so that if there were no concrete above the arch, both σ_x and ϵ' would be zero with the reservoir empty; or, in other words, it was assumed that no shrinkage would take place. This assumption may possibly be approximately correct for the very thick bottom part of a great arch dam, especially if σ_x is assumed as somewhat less than the average value for the vertical stress. Where the assumption is acceptable and p_a is the only water pressure acting on the extrados, Equations (38) and (39) give the displacement which X' must annul, and Equation (36) gives the tangential stress which must be added to Equation (34) to obtain the resulting stresses.

For the greater part of the dam, the shrinkage of the concrete must be accepted as a fact. To counteract this shrinkage, contraction joints should be provided at suitable intervals and after the dam has set and the chemical heat has been dissipated, pressure grouting should be effected.

With the reservoir empty, that is, for $p_a = 0$, there is a uniform tangential compression σ_{tg} in the arch and a variable vertical compression, σ_x, so that any element of the arch has undergone a tangential deformation,

$$\varepsilon_t = -\frac{\sigma_x}{m\,E} + \frac{\sigma_{tg}}{E} \qquad (40)$$

Both σ_x and σ_{tg} are compressions and, therefore, negative. Hence, when the water pressure, p_a, acts on the extrados, the deformation, ϵ', is computed from Equation (38), with $\sigma_x = 0$, and the tangential stress results from adding σ_{tg} to Equation (36) and then adding Equation (34) to obtain the resulting stress.

IX.—Accuracy of Assumptions

It has been assumed that E is constant and that the normal (radial) planes remain plane. Both these assumptions are at least approximately correct when the stresses are moderate, and both contribute an error on the side of safety. Professor Bach believes that radial planes will remain at least approximately plane also for such material where E is not constant,* and this will probably still be approximately true even when the abutment does not lie in one radial plane. For high stresses, E cannot be assumed as constant and the stress distribution will no doubt be materially changed. This will be discussed in detail in Article XVII.

The division of the water load between the cantilever and the arch cannot be made accurately and, moreover, it is not likely that p_a is constant for any one arch or even symmetrical with respect to the crown section. The influence

* "Elasticität und Festigkeit," 9th Edition, p. 289, last footnote.

of p_a decreases rapidly toward the abutment, and a small variation in symmetry should not affect the resulting stress greatly, so that in general the crown value of p_a may be used as a fair approximation, especially if care is taken, when determining the arch stresses, to select p_a somewhat larger than it actually is.

Some objection may be raised to the rather general application of Poisson's ratio. Tests made on hollow steel cylinders agree quite closely with Lamé's theory.* The writer does not know of any tests having been made by subjecting hollow concrete cylinders to external pressure, but Professor Bach reports some tests to destruction on hollow unreinforced concrete cylinders by subjecting them to internal pressure.† The internal diameter was 70 cm., and the thickness, 8, 12, and 18 cm. The tangential stress as computed by the Lamé formula was 15.7 kg. per sq. cm. and the tensile strength of the concrete was found to be 15.5 kg. per sq. cm. If the stress had been uniformly distributed throughout the largest cylinder (18 cm. thick) it should have withstood a pressure 30% higher. These tests show, therefore, an excellent agreement with the Lamé formula. It is likely, however, that cylinders tested under external loading would show some deviation, because E is not nearly so constant in compression as in tension.

If it were feasible properly to take account of the swelling of the up-stream face due to water-soaking, to the time effect (the yielding of the concrete with time in proportion to the stress), and to the restraining action of the foundation as distinguished from the abutment, a still more advantageous stress distribution probably would be found.

X.—Radial Deflection of Arch

The displacement, BF, of the point, B (Fig. 2) is $e\ (1 - \sin \phi_1)$ in the direction of the center line. This displacement is due to the water pressure on the up-stream face and to the fact that the arch has been slid back from A' to A. To this displacement must be added the displacement in the direction of the center line produced by the force, X', found previously.

The displacement caused by $X' = 1$ is most conveniently determined by referring the normal stresses and the moments to r_n as in Articles V and VI. The rotation produced is given by Equation (19) and the corresponding displacement of the crown section in the direction of the center line, BC (taken positive from B toward C), neglecting the change in thickness of the arch (see Fig. 3), is

$$\eta_M = \int_0^{\phi_1} 2 r_n \sin \frac{\phi}{2} (A' - r_n \cos \phi) \frac{r_n}{E I} \cos \frac{\phi}{2} d\phi$$

or, integrating and re-arranging,

$$\eta_M = \frac{r_n^2}{E I} \left[A' (1 - \cos \phi_1) - \frac{r_n}{4} (1 - \cos 2 \phi_1) \right] \ldots\ldots\ldots (41)$$

Since according to Equation (26) the tension at the neutral axis is $\frac{X' \cos \phi}{t}$

* See, for example, Fig. 333(*b*), p. 350, of Goodman's "Mechanics Applied to Engineering," 1912 Edition.

† "Elasticität und Festigkeit," 9th Edition, p. 579, footnote.

(see, also, Fig. 4), the displacements due to tension and shear produced by $X' = 1$, measured on the neutral axis, are,

$$\eta_N = -\int_0^{\phi_1} \frac{\cos\phi}{E\,t}\, r_n \sin\phi\, d\,\phi = -\frac{r_n}{4\,E\,t}(1 - \cos 2\,\phi_1) \ldots\ldots (42)$$

and,

$$\eta_S = 2.88\int_0^{\phi_1} \frac{\sin\phi}{E\,t}\, r_n \cos\phi\, d\,\phi = 2.88\,\frac{r_n}{4\,E\,t}(1 - \cos 2\,\phi_1) \ldots (43)$$

The deflection caused by $X' = 1$, is, therefore,

$$\eta = \eta_M + \eta_N + \eta_S$$

or,

$$\eta = \frac{r_n^{\,2}}{E\,I}\left[A'(1 - \cos\phi_1) - \frac{r_n}{4}(1 - \cos 2\,\phi_1)\right] + 0.47\,\frac{r_n}{E\,t}(1 - \cos 2\,\phi_1) \ldots\ldots (44)$$

The total deflection will be,

$$\eta_t = \eta\,X' - \epsilon'\,r_n(1 - \cos\phi_1) \ldots\ldots (45)$$

Here, ϵ' is, as before, determined from Equation (38). If the deflection for the up-stream face is wanted rather than for the neutral axis, the change of thickness of the arch, which may be found from Equation (37), must be considered. This will generally and perhaps always be negligible. If pressure grouting is undertaken, this may also affect the deflection slightly.

XI.—Temperature Variations, Swelling, and Shrinkage

Let u_e and u_i be the deformations per unit length at the extrados and intrados, respectively, caused by the temperature variation, swelling, and shrinkage. Let u_e and u_i be positive when they represent increase in length and assume that u varies from u_e to u_i in such a manner that radial vertical planes remain plane.

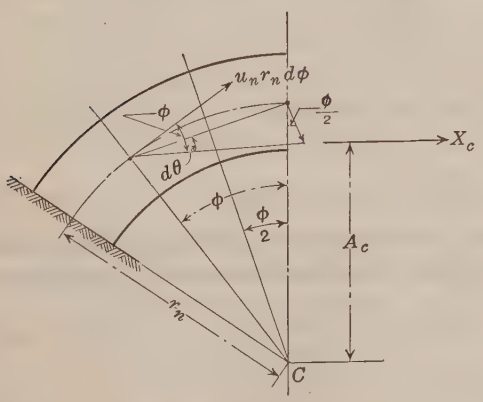

Fig. 8.

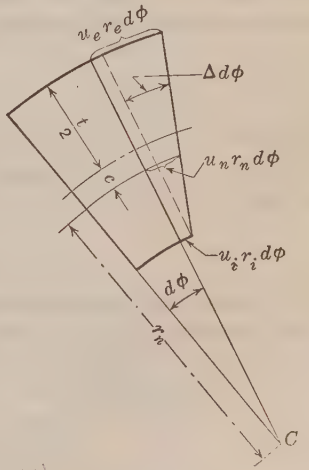

Fig. 9.

If in Fig. 8 the semi-arch is free to move at the crown, but is fixed at the abutment, the deformations, u, produce a rotation of the crown section and

also displacements parallel to and at right angles to X_c. The force, X_c, acting a distance, A_c, from the center, C, of the arch, and considered positive when in tension, is the force necessary to bring the crown section back into the center line, $C\,D$. In order to determine X_c and A_c it is necessary, therefore, to determine the rotation and the displacement in the direction of X_c of the crown section.

The unit deformation of the neutral axis (see Fig. 9) is determined by,

$$u_n\, r_n = u_i\, r_i + \frac{u_e\, r_e - u_i\, r_i}{t}\left(\frac{t}{2} - c\right)$$

which, for $c = 0$, gives,

$$u_n = \frac{u_e\, r_e + u_i\, r_i}{2\, r_n} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (46)$$

The rotation is,

$$d\,\Delta\,\phi = d\,\theta = \frac{u_e\, r_e - u_i\, r_i}{t}\, d\,\phi \qquad \ldots\ldots\ldots\ldots\ldots\ldots (47)$$

The rotation of the crown section is, therefore,

$$\theta = (u_e\, r_e - u_i\, r_i)\,\frac{\phi_1}{t} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (48)$$

The displacement of the crown section in the direction of X_c is (see Fig. 8),

$$y_c = \int_0^{\phi_1} 2\, r_n \sin^2\left(\frac{\phi}{2}\right) d\,\theta + \int_0^{\phi_1} u_n\, r_n \cos\phi\, d\,\phi$$

Integrating,

$$y_c = \frac{r_n}{t}\,(u_e\, r_e - u_i\, r_i)\,(\phi_1 - \sin\phi_1) + \frac{1}{2}\,(u_e\, r_e + u_i\, r_i)\sin\phi_1 \qquad \ldots\ldots\ldots\ldots (49)$$

The force, X_c, acting at a distance, A_c, from the center, C, must annul both the rotation, θ, and the displacement, y_c. From Equation (19),

$$-\theta = \int_0^{\phi_1} \frac{M_c}{E\,I}\, r_n\, d\,\phi$$

in which,

$$M_c = X_c\,(A_c - r_n\cos\phi)$$

so that,

$$\theta = -\frac{X_c\, r_n}{E\,I}\,(A_c\,\phi_1 - r_n\sin\phi_1) \qquad \ldots\ldots\ldots\ldots\ldots\ldots (50)$$

and θ is given by Equation (48). The displacement produced by X_c is obtained from Equation (29) by substituting A_c for A' and multiplying by X_c; it is:

$$y_c = -X_c\left\{\frac{r_n^2}{E\,I}\left[A_c\,(\phi_1 - \sin\phi_1) - r_n\left(\sin\phi_1 - \frac{\phi_1}{2} - \frac{1}{4}\sin 2\,\phi_1\right)\right] + \frac{r_n}{E\,t}\,[1.94\,\phi_1 - 0.47\sin 2\,\phi_1]\right\} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (51)$$

In addition, y_c is given by Equation (49). Solving Equations (50) and (51) gives X_c and A_c. The deflection of the crown section in an up-stream and

down-stream direction caused by the deformations, u, may be obtained from Equation (44) by substituting A_c for A and multiplying by X_c, giving,

$$\eta_c = X_c \left\{ \frac{r_n^2}{E\,I} \left[A_c\,(1 - \cos\phi_1) - \frac{r_n}{4}\,(1 - \cos 2\,\phi_1) \right] + 0.47\,\frac{r_n}{E\,t}\,(1 - \cos 2\,\phi_1) \right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (52)$$

The stresses produced by X_c acting a distance, A_c, from the center is found from Equation (34) by substituting X_c and A_c for X' and A'. The final resulting deformations and stresses are then found by simple addition.

The computations are greatly simplified, if θ in Equation (48) is zero, that is,

$$u_e\,r_e = u_i\,r_i = u_n\,r_n \quad \ldots\ldots\ldots\ldots\ldots\ldots (53)$$

Then, from Equation (50):

$$A_c = r_n\,\frac{\sin\phi_1}{\phi_1} = A' \quad \ldots\ldots\ldots\ldots\ldots\ldots (54)$$

In this case, u'_n, the deformation per unit length at the neutral axis, due to temperature variation, swelling, and shrinkage, may be added directly to ϵ' of Equation (38) and the force required, including temperature variation, etc., may be determined from:

$$X_c' = \frac{\varepsilon' + u_n'}{y'}\,r_n \sin\phi_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots (55)$$

in which, ϵ' is the deformation at the neutral axis per unit length due to the water pressure acting on the extrados. In case $\epsilon' + u'_n = 0$, $X'_c = 0$, and the only tangential stresses are those given by Equation (36); there are no bending stresses. This is also evident from an inspection of Fig. 2 and from the reasoning of Article II.

Obviously, good construction demands that the pressure grouting should take place while the concrete is at or near its annual minimum temperature. Also, near the foundation, where pressure grouting cannot be made effective, the concrete should be poured during the coldest part of the season, or preferably during that period when the air temperature is approximately equal to or somewhat below the annual mean. In this manner the temperature variations will lower the maximum stresses, since ϵ' and u'_n are of opposite sign.

XII.—Numerical Computations

In order to facilitate the use of the formulas here presented and to enable the computations to be made by slide-rule, some of the equations can be put in a form that facilitates the use of tables and curves. Let,

$$B = 12\,\frac{r_n^2}{t^2} \left[\frac{\phi_1}{2} + \frac{1}{4}\sin 2\,\phi_1 - \frac{\sin^2\phi_1}{\phi_1} \right] + 1.94\,\phi_1 - 0.47 \sin 2\,\phi_1 \quad (56)$$

so that, from Equation (29a),

$$y' = \frac{r_n}{E\,t}\,B \quad \ldots\ldots\ldots\ldots\ldots\ldots (57)$$

The value of B is given in Table 1 and is plotted in Fig. 10. The values of $100\,\frac{c}{r_m}$ (c when $r_m = 100$ ft.), k_e, and k_i are given in Table 2 and plotted in Figs. 11 and 12.

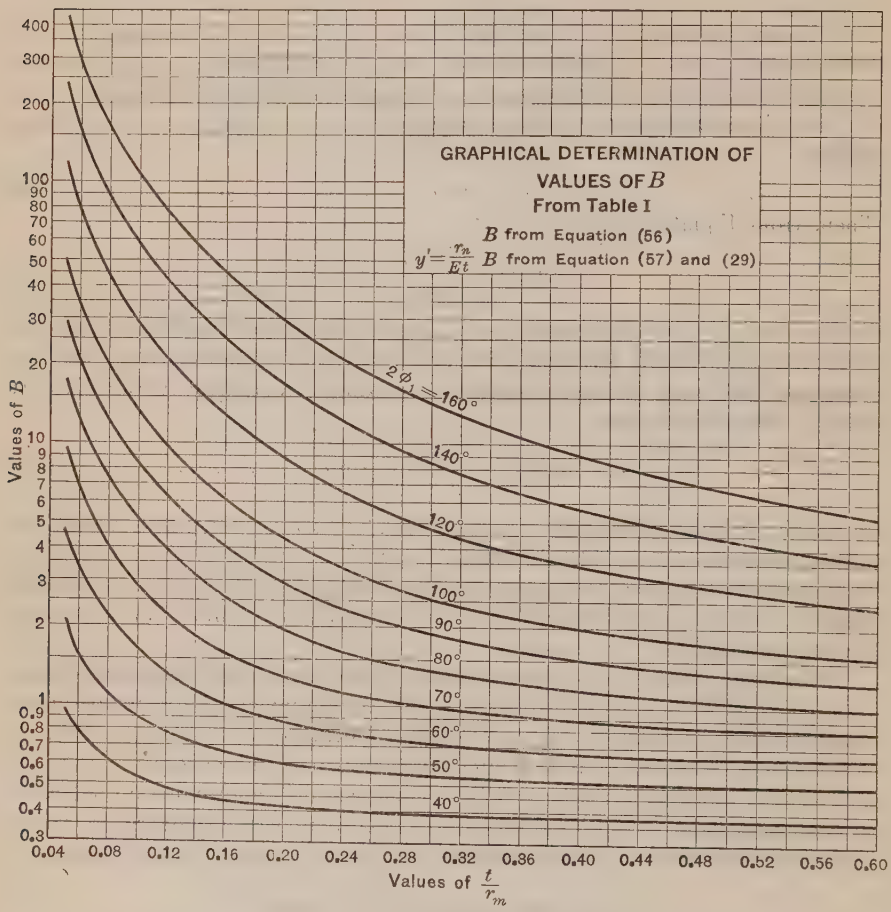

FIG. 10.

From Equations (30) and (57),

$$X' = -\frac{\varepsilon'\, r_n}{B\,\frac{r_n}{t}}\, E \sin \phi_1 = -\frac{\varepsilon'}{B}\, t\, E \sin \phi_1$$

or,

$$\frac{X'}{t} = -\frac{\varepsilon'}{B}\, E \sin \phi_1 \qquad \text{.....(58)}$$

TABLE 1.—VALUES OF B FROM EQUATION (56).

$\frac{t}{r_m}$	Values of $2\phi_1$.									
	40°	50°	60°	70°	80°	90°	100°	120°	140°	160°
0.05	0.9511	2.094	4.665	9.335	17.88	30.25	49.64	116.4	236.4	429.9
0.10	0.5188	0.8876	1.619	2.887	5.131	8.338	13.31	30.26	60.53	109.2
0.15	0.4388	0.6644	1.069	1.695	2.772	4.285	6.588	14.33	28.01	49.85
0.20	0.4108	0.5862	0.8605	1.277	1.885	2.866	4.235	8.748	16.61	29.07
0.25	0.3979	0.5501	0.7734	1.084	1.564	2.209	3.146	6.168	11.34	19.47
0.30	0.3907	0.5302	0.7219	0.9770	1.354	1.847	2.546	4.745	8.439	14.17
0.35	0.3866	0.5186	0.6899	0.9152	1.231	1.637	2.198	3.920	6.753	11.09
0.40	0.3838	0.5109	0.6704	0.8741	1.150	1.498	1.966	3.370	5.631	9.045
0.45	0.3820	0.5056	0.6579	0.8459	1.094	1.402	1.808	2.994	4.863	7.645
0.50	0.3806	0.5019	0.6486	0.8258	1.054	1.333	1.694	2.724	4.313	6.643
0.55	0.3796	0.4991	0.6414	0.8108	1.025	1.283	1.610	2.525	3.906	5.901
0.60	0.3788	0.4969	0.6359	0.7995	1.002	1.244	1.546	2.373	3.597	5.336

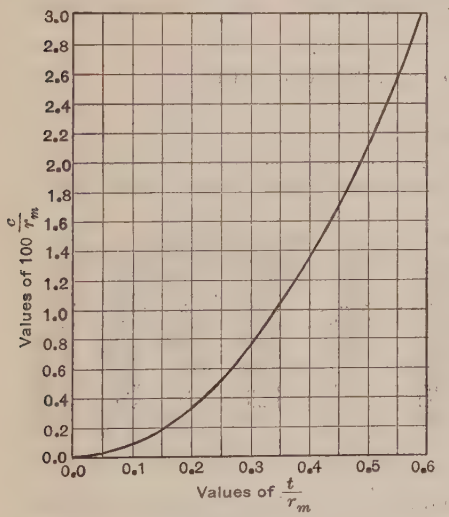

FIG. 11.

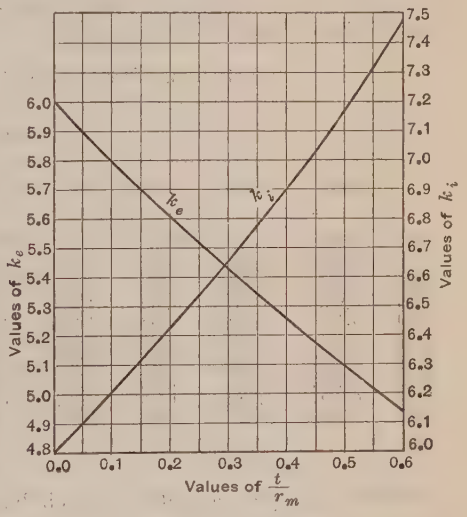

FIG. 12.

TABLE 2.—VALUES OF k_e, k_i, AND $100\frac{c}{r_m}$.

$\frac{t}{r_m}$ =	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60
k_e =	5.900	5.801	5.710	5.618	5.528	5.438	5.351	5.264	5.181	5.096	5.013	4.932
k_i =	6.105	6.210	6.312	6.422	6.535	6.654	6.775	6.902	7.030	7.167	7.310	7.457
$100\frac{c}{r_m}$ =	0.000	0.084	0.188	0.334	0.525	0.754	1.030	1.347	1.712	2.118	2.572	3.074

If this value and the value for A' are introduced into Equation (34) and the equation is re-arranged, there results,

$$\sigma_{te}' = -\frac{\varepsilon'}{B} E \sin\phi_1 \left\{ k_e \frac{r_m}{t} \left[\frac{\sin\phi_1}{\phi_1} - \cos\phi - \frac{c}{r_m} \frac{\sin\phi_1}{\phi_1} \right] + \cos\phi \right\}$$

$$\sigma_{ti}' = \frac{\varepsilon'}{B} E \sin\phi_1 \left\{ k_i \frac{r_m}{t} \left[\frac{\sin\phi_1}{\phi_1} - \cos\phi - \frac{c}{r_m} \frac{\sin\phi_1}{\phi_1} \right] - \cos\phi \right\} \quad (59)$$

At the crown, $\cos \phi = 1$, and, at the abutment, it is $\cos \phi_1$. The value of $\frac{\sin \phi_1}{\phi_1}$ can be determined quite closely on the slide-rule by first computing

$$1 - \frac{\sin \phi_1}{\phi_1} = \frac{\phi_1 - \sin \phi_1}{\phi_1}$$

The radial deflection at the crown due to $X' = 1$ is, from Equation (44),

$$\eta = \frac{r_n}{E\,t} D \qquad (60)$$

in which,

$$D = 3 \frac{r_n^2}{t^2} \left[2 \frac{2 \sin \phi_1 - \sin 2\phi_1}{\phi_1} - (1 - \cos 2\phi_1) \right] + 0.47 (1 - \cos 2\phi_1) \qquad (61)$$

The value for D is given in Table 3 and plotted in Fig. 13. The total deflection is then given by Equation (45).

TABLE 3.—VALUES OF D FROM EQUATION (61).

$\frac{t}{r_m}$	VALUES OF $2\phi_1$.									
	40°	50°	60°	70°	80°	90°	100°	120°	140°	160°
0.05	2.97	7.15	14.3	25.8	42.7	66.2	102.2	185.5	310.8	471
0.10	0.823	1.91	3.75	6.68	10.96	16.58	24.68	46.8	78.2	118.2
0.15	0.426	0.941	1.793	3.13	5.08	7.75	11.26	21.16	35.1	52.9
0.20	0.287	0.602	1.109	1.894	3.02	4.55	6.56	12.18	20.07	30.1
0.25	0.223	0.444	0.792	1.320	2.066	3.07	4.38	8.02	13.01	19.52
0.30	0.188	0.358	0.618	1.004	1.541	2.259	3.18	5.73	9.26	13.70
0.35	0.167	0.308	0.516	0.820	1.235	1.785	2.486	4.40	7.03	10.31
0.40	0.1534	0.274	0.449	0.697	1.032	1.470	2.02	3.51	5.54	8.06
0.45	0.1440	0.251	0.403	0.614	0.894	1.254	1.705	2.91	4.53	6.53
0.50	0.1373	0.235	0.370	0.554	0.794	1.100	1.478	2.48	3.80	5.42
0.55	0.1324	0.223	0.345	0.507	0.721	0.986	1.310	2.16	3.26	4.60
0.60	0.1286	0.213	0.327	0.476	0.665	0.899	1.183	1.911	2.85	3.98

As an example, consider an arch having $\frac{t}{r_m} = 0.5$, $r_m = 100$ ft., and $2\phi_1 = 90°$, and assume that the arch is 400 ft. below the maximum water surface and that it carries 40% of the water load, so that,

$$p_a = 0.4 \times 400 \times 0.434 = 69.5 \text{ lb. per sq. in.}$$

Assume, further, that $\sigma_x = -300$ lb. per sq. in., that $m = 8$, and that the influence of the cantilever load, p_c, and of the swelling, shrinkage, and temperature variation may be neglected. From Table 2, $c = 2.118$. Then, from Equation (38a), with E, in pounds per square inch,

$$\varepsilon' = -\frac{1}{E} \left[\frac{7}{8} + \frac{9}{8} \left(\frac{0.7500}{0.9788} \right)^2 \right] \times 1.25^2 \times 69.5 + \frac{300}{8E}$$

$$= \frac{1}{E} (-166.8 + 37.5) = -\frac{129.3}{E}$$

If $\sigma_x = 0$, $\varepsilon' = -\frac{166.8}{E}$, while Equation (38$b$) gives $\varepsilon'' = -\frac{173.9}{E}$, or only 4.26% greater than ε' for $\sigma_x = 0$.

From Equation (36) is obtained,

$$\sigma''_{te} = -147.7; \text{ and } \sigma''_{ti} = -217.2$$

From Tables 1 and 2, $B = 1.333$, $k_e = 5.096$, and $k_i = 7.167$, so that from Equation (59), the stress at the crown section is,

$$\sigma_{te}' = \frac{129.3}{1.333}\left\{5.096 \times 2\,[0.90032 - 1.0 - 0.0191] + 1\right\} \times 0.70711$$

$$= -14.4 \text{ lb. per sq. in. (compression).}$$

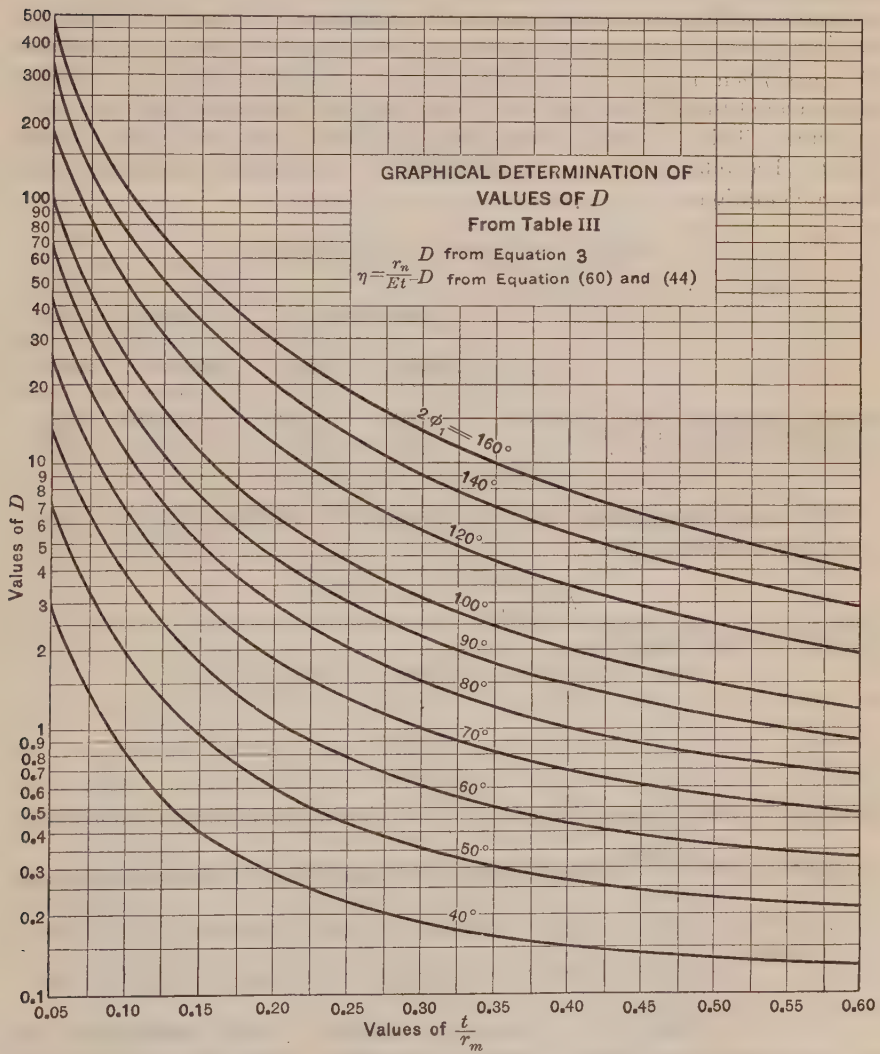

FIG. 13.

Likewise it is found that $\sigma'_{ti} = 185.3$ lb. per sq. in. (tension). For the stresses at the abutment, where $\cos\phi = \cos\phi_1 = 0.70711$, there is found $\sigma'_{te} = 170.2$

lb. per sq. in. (tension) and $\sigma'_{ti} = -122.9$ lb. per sq. in. (compression). The resulting tangential stresses are then obtained by adding to these the proper values of σ''_t found previously. The results are given in Table 4.

For $\sigma_x = -300$ lb. per sq. in. (compression), $X' = \frac{129.3}{1.333}\, 0.707 \times 50 =$ 3 430 lb.,* while for $\sigma_x = 0$, $X' = 4\,420$. Professor Cain's formulas assume $\sigma_x = 0$ and his value for D_s was computed as $0.05606\ \phi_1$, giving $p_r - P_o = 4\,565.8$ lb., or only about 3.3% greater than the writer's value for X'. The moments found at the crown and at the abutments by Professor Cain's formulas are 45 513 and 88 215 as against 52 500 and 77 100 by the writer's formulas when $\sigma_x = 0$; or Professor Cain's values for the moments at the crown and at the abutments are 86.7% and 114% of those found by the writer's formulas. This is a fairly close check considering that the arch is rather heavy. The influence of σ_x is considerable in this case, as will be seen from the computations.

TABLE 4.—Comparison of Stresses.

	Crown Section.		Abutment Section.	
	Extrados.	Intrados.	Extrados.	Intrados.
Jakobsen's formulas, $\sigma_x = 300$	− 162.1	− 31.9	+ 22.5	− 340.1
Cain's formulas and Equation (1)	− 191.7	+ 26.8	+ 102.5	− 320.9
Jakobsen's formulas, $\sigma_x = 0$	− 166.2	+ 21.3	+ 71.3	− 375.2

The radial deflection is for $\sigma_x = 0$, since $X' = 4\,420$ and $D = 1.100$, so that from Equation (60):

$$\eta = \frac{97.88}{50\,E}\, 1.10 = \frac{2.155}{E}.$$

and from Equation (45),

$$E\eta_t = 2.155 \times 4\,420 + 166.8 \times 97.88 \times 0.293$$

or,

$$\eta_t = \frac{14\,320}{E}$$

For $\sigma_x = -300$ lb. per sq. in., $\eta_t = \frac{12\,200}{E}$. Professor Cain's formula including shear† gives $\frac{15\,400}{E}$ as the deflection which agrees quite well with the value found by the writer's formula. Neglecting shear, Professor Cain's formula gives $\frac{10\,420}{E}$; for thick arches shear contributes considerably to the deflection.

That the influence of a temperature variation is quite considerable may be shown readily as follows: For $\sigma_x = -300$ lb. per sq. in. (compression),

* When computing stresses in pounds per square inch, it is more convenient to express X' in pounds acting on an arch slice $\frac{1}{12}$ in. thick, as has been done here, but the formulas assume X' acting upon an arch slice 1 ft. thick.

† *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 270.

$\varepsilon' = -\frac{129.3}{E}$, and if $\varepsilon' + u_n' = 0$, $X_c' = 0$. The increase in length for an increase of temperature of 1° Fahr. is approximately 6×10^{-6} ft. per ft. of length, and the temperature increase required to annul X'_c is, if $E = 2 \times 10^6$,

$$T = \frac{129.3}{2 \times 6} = 10.8° \text{ Fahr.}$$

XIII.—Influence of Cantilever Load and Pressure Grouting

That part, p_c, of the water pressure, which is carried by the cantilever and transmitted vertically by shear, also has an effect on the arch in that it produces a radial stress, σ_r, similar to σ_y in Fig. 1. This radial stress, σ_r, produces a tangential stress, which may be found as follows.

At the up-stream face,*

$$\sigma_r = \sigma_x \cot^2 \alpha + p_a (1 - \cot^2 \alpha)$$

in which, α is the angle which the up-stream face makes with the horizontal. If σ_x and α are both fairly small, σ_{re} is nearly equal to p_a. Assuming the down-stream face to be vertical, as is generally the case with constant-angle arch dams, except quite near the bottom, $\sigma_{ri} = 0$. For a multiple-arch buttress, the writer has shown† that σ_r decreases nearly uniformly; this variation may be assumed to hold at least approximately in this case also.

Professor Cain has shown that for a triangular section of a gravity dam with a nearly vertical up-stream face, σ_r is almost constant for the entire section.‡ In the case of an arch dam that acts also as a gravity dam, no doubt σ_r likewise varies with ϕ. The actual determination of σ_r is complicated, and since the purpose of the present discussion is to show that the influence of σ_r on the stress distribution is quite small, the assumption made seems permissible.

If the assumption that σ_r is constant be made, the result is nearly the same as if the temperature of the concrete were increased sufficiently to produce a unit elongation, $\frac{-\sigma_r}{m E}$, so that Equation (53) holds approximately; this case was discussed in Article XII.

It seems to the writer that no matter how σ_r varies, it cannot influence the stress distribution greatly except for very short and thick arches, which are not likely to be of any great practical importance.

Assuming, therefore, that $\sigma_{re} = -p_a$ and $\sigma_{ri} = 0$ and that σ_r varies linearly, the influence of p_c on the arch deflections and stresses may be found by adding $\frac{p_c}{m E}$ to u_e in Article XI; or if the influence of p_c alone is wanted, by putting $u_e = \frac{p_c}{m E}$ and $u_i = 0$ and determining the corresponding values of X_c, A_c, and the deflection in an up-stream direction.

* Equation (30) of the writer's paper "Stresses in Multiple-Arch-Dams," *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 298.

† *Loc. cit.*, p. 306.

‡ "Stresses in Masonry Dams," *Transactions*, Am. Soc. C. E., Vol. LXIV (1909), p. 208.

For the arch considered in Article XII,

$$p_c = 0.6 \times 400 \times 0.434 = 104.1 \text{ lb. per sq. in.}$$

so that,

$$u_e = \frac{104.1}{8\,E} = \frac{13.02}{E}$$

From Equation (48),

$$\theta = \frac{25.58}{E}$$

From Equation (49),

$$y_c = \frac{624.8}{E}$$

Equations (50) and (51) then give:

$$25.58 = -0.0094\,X_c\,[0.7854\,A_c - 69.2]$$

and,

$$624.8 = -X_c\,[0.072\,A_c - 4.242]$$

Solving,

$$A_c = 107.4 \text{ ft. and } X_c = -180.2 \text{ lb.}$$

The stresses produced at the abutment are, therefore, from Equation (34):

$$\sigma'_{te} = -16.0 \text{ lb. per sq. in (compression)}$$

and,

$$\sigma'_{ti} = +21.5 \text{ lb. per sq. in. (tension).}$$

These stresses tend to equalize those produced by the water pressure. The tension at the abutment is 22.5 lb. per sq. in., so that the resulting tension is only 6.5 lb. per sq. in. The value of $\frac{p_c}{m\,E}$ is 13.0 lb. per sq. in. For a shorter arch the influence of p_c will be greater.

In Article VIII the grouting pressure, σ_{tg}, was assumed to be uniformly distributed tangential compression. This is true where the contraction joints are fairly close together and all are grouted simultaneously. By suitable grouting methods, however, it is possible to produce in the unloaded arch bending moments that would tend to neutralize those due to the water pressure. At times, this may be an advantage well worth considering. For example, if all joints except the one at the crown section are grouted first, the grouting of the crown joint under pressure tends to put compression in the extrados of the abutment section.

It is customary to assume that at the bottom of an arch dam the cantilever carries the entire load and the arch carries nothing. This may not be true in a high dam, such as the Pacoima Dam, where the bottom arches support nearly 400 ft. of concrete and their thickness may prevent shrinkage to a considerable extent.

XIV.—Formulas for the Stresses Produced by X, when Poisson's Ratio Is Considered

In computing the stresses and displacements caused by a force, X (Fig. 2), Poisson's ratio was assumed to be zero, or, in other words, it was assumed that

normal stresses are produced in one direction only, namely, tangentially, and that the normal stresses in radial and axial directions are zero. The axial stress, σ_x, has been included with the deformation of the cylinder due to water pressure and, therefore, σ_x may properly be neglected when considering the stresses produced by X. It remains to determine how large an error is committed by assuming the radial stress, $\sigma'_r = 0$, since this is true only at the up-stream and down-stream faces. Here, σ'_r is the radial stress produced by X only, since account has already been taken of the radial stress produced by the water pressure and by p_c.

Föppl* gives Airy's stress function† for pure bending of a plane curved beam, Fig. 14, and for the loading indicated in Fig. 15. In deriving the stress function, it is assumed that $\sigma'_x = 0$, that is, that the arch is free to deform vertically as the plane stresses, σ'_t and σ'_r, may require, and that the up-stream and down-stream faces are vertical.

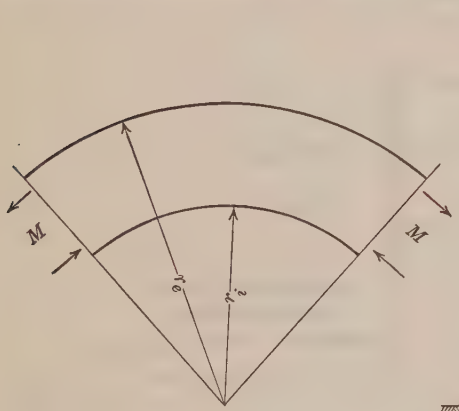

FIG. 14.

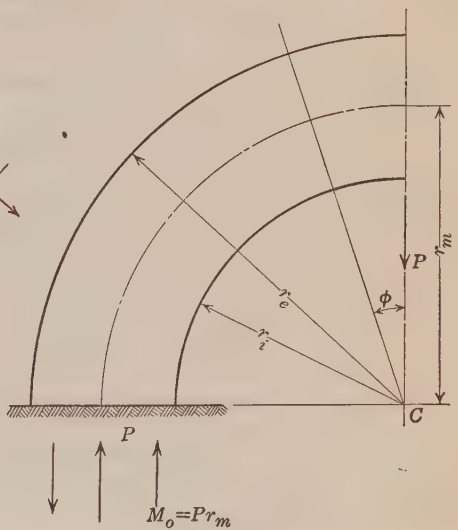

FIG. 15.

Fig. 16 shows the condition here involved, and Fig. 15 may evidently be converted into Fig. 16 by writing $90° - \phi$, in place of ϕ, changing P to $-X$ and, consequently, the moment, $M_o = P\ r_m$, to $M'_o = -X\ r_m$, and adding a moment, $M' = A\ X$. The stresses produced by the loading (Fig. 16), therefore, may be obtained from Föppl's solution by superimposing the conditions of Figs. 14 and 15, that is, by setting $M' = A\ X$, $P = -X$, and $90° - \phi$ for ϕ. The stress function is,

$$F = c_0 + c_1 \log_e r + c_2 r^2 + c_3 r^2 \log_e r + [\alpha r^3 + \beta r \log_e r + \gamma r + \delta r^{-1}] \cos \phi \quad \ldots\ldots (62)$$

* "Drang und Zwang" Bd. I, 2d Edition, p. 302.

† "Transmission of Pressure Through Solids and Soils and the Related Engineering Phenomena," by George Paaswell, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 1563. Mr. Paaswell refers to "A Mathematical Treatise on the Theory of Elasticity," by A. E. H. Love, and to the Appendix of "The Elasticity and Resistance of Materials," by W. H. Burr, M. Am. Soc. C. E.

in which, r is the distance from the center, C, to any point and r and ϕ are the only variables.

The function, F, must satisfy the fundamental differential equation,

$$\left(\frac{\delta^2}{\delta r^2} + \frac{1}{r^2}\frac{\delta^2}{\delta \phi^2} + \frac{1}{r}\frac{\delta}{\delta r}\right)\left(\frac{\delta^2 F}{\delta r^2} + \frac{1}{r^2}\frac{\delta^2 F}{\delta \phi^2} + \frac{1}{r}\frac{\delta F}{\delta r}\right) = 0 \quad \ldots\ldots (63)$$

The stresses are,

$$\left.\begin{aligned} \sigma_r' &= \frac{1}{r}\frac{\delta F}{\delta r} + \frac{1}{r^2}\frac{\delta^2 F}{\delta \phi^2}; \quad \sigma_t' = \frac{\delta^2 F}{\delta r^2} \\ \tau &= -\frac{\delta}{\delta r}\left(\frac{1}{r}\frac{\delta F}{\delta \phi}\right); \text{ and } \sigma_x' = 0 \end{aligned}\right\} \quad \ldots\ldots (64)$$

The axial, that is, vertical, deformation is then,

$$\frac{\delta \xi}{\delta x} = -\frac{1}{2\,G\,(m+1)}(\sigma_t' + \sigma_r') \quad \ldots\ldots (65)$$

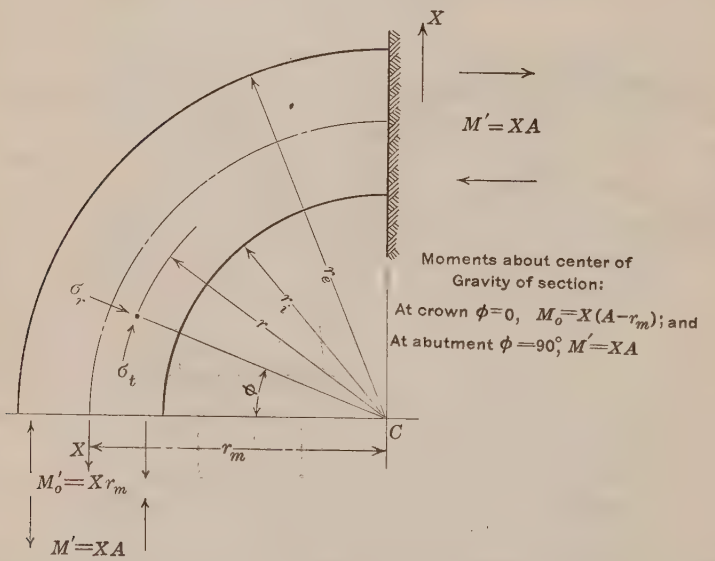

FIG. 16.

Since, in general, σ'_t and σ'_r are functions of both r and ϕ, the originally plane horizontal limiting surfaces of the arch do not remain plane under stress. However, Equations (62), (63), and (64) are equally correct if it is assumed that the originally plane horizontal limiting surfaces do remain plane and horizontal; but, in that case, it is required that,

$$\sigma_x' = \frac{1}{m}(\sigma_t' + \sigma_r') + \frac{m+1}{m}\,2\,G\,k \quad \ldots\ldots (66)$$

in which, $k = \dfrac{\delta \xi}{\delta x}$ is a constant representing the vertical displacement of the horizontal limiting planes. Only when $\sigma'_t + \sigma'_r$ is constant, can σ'_x in Equation (66) remain constant over the entire section.

XV.—Stress Equations

The equations for the stresses are obtained by differentiating Equation (62) in accordance with Equation (64) and determining the constants from the conditions at the surfaces and from the known loading, that is:

For $r = r_e$ and $r = r_i$,

$$\sigma_r' = 0 \text{ and } \tau = 0$$

for $\phi = 0$ (Fig. 16),

$$\tau = 0\,; \int_{r_i}^{r_e} \sigma'_t\, dr = -X\,; \int_{r_i}^{r_e} \sigma_t'\, r\, dr = -X(r_m - A) \qquad \ldots\ldots (67)$$

and, for $\phi = 90°$,

$$\int_{r_i}^{r_e} \tau\, dr = -X\,; \text{ and } \int_{r_i}^{r_e} \sigma_t'\, dr = 0$$

This gives for the stresses corresponding to the loading in Fig. 16,

$$\left.\begin{aligned}
\sigma'_r &= \frac{4AX}{N}\left[-r_i^2 \log_e\left(\frac{r}{r_i}\right) - r_e^2 \log_e\left(\frac{r_e}{r}\right) + \frac{r_e^2 r_i^2}{r^2}\log_e\left(\frac{r_e}{r_i}\right)\right] + 2Q\cos\phi\left[r - \frac{r_e^2 + r_i^2}{r} + \frac{r_e^2 r_i^2}{r^3}\right] \\
\sigma_t' &= \frac{4AX}{N}\left[r_e^2 - r_i^2 - r_i^2 \log_e\left(\frac{r}{r_i}\right) - r_e^2 \log_e\left(\frac{r_e}{r}\right) - \frac{r_e^2 r_i^2}{r^2}\log_e\left(\frac{r_e}{r_i}\right)\right] + 2Q\cos\phi\left[3r - \frac{r_e^2 + r_i^2}{r} - \frac{r_e^2 r_i^2}{r^3}\right] \\
\tau &= 2Q\sin\phi\left[r - \frac{r_e^2 + r_i^2}{r} + \frac{r_e^2 r_i^2}{r^3}\right]
\end{aligned}\right\} \qquad \ldots\ldots (68)$$

The constants, N and Q, are given by:

$$\left.\begin{aligned}
N &= (r_e^2 - r_i^2)^2 - 4 r_e^2 r_i^2 \left[\log_e\left(\frac{r_e}{r_i}\right)\right]^2 \\
Q &= -\frac{X}{2\left[r_i^2 - r_e^2 + (r_e^2 + r_i^2)\log_e\left(\frac{r_e}{r_i}\right)\right]}
\end{aligned}\right\} \qquad \ldots\ldots (69)$$

All the formulas previously developed for stresses and deformations produced by X were based on the usual assumptions that original plane normal sections remain plane and that Poisson's ratio is zero, so that X does not produce stresses in a radial direction. Equations (68) do not involve those two assumptions, but only that the modulus of elasticity and Poisson's ratio are constant throughout (which assumptions are also involved in the equations previously developed). Equations (68) are likely, therefore, to be the more nearly correct ones, but in order to make them complete the deformations should be computed for the stress distribution given in Equations (68). As computations indicate that the influence of Poisson's ratio is small and that the additional work of determining the deformations according to the stress distribution in Equations (68) may be quite considerable, it was not attempted.

The tangential stresses at the up-stream and down-stream faces are obtained by inserting r_e and r_i in Equations (68),

$$\left.\begin{aligned} \sigma_{te}' &= \frac{4\,A\,X}{N}\left[r_e^2 - r_i^2 - 2\,r_i^2 \log_e\left(\frac{r_e}{r_i}\right)\right] + 4\,Q\,\frac{r_e^2 - r_i^2}{r_e}\cos\phi \\ \sigma_{ti}' &= -\frac{4\,A\,X}{N}\left[2\,r_e^2 \log_e\left(\frac{r_e}{r_i}\right) - (r_e^2 - r_i^2)\right] - 4\,Q\,\frac{r_e^2 - r_i^2}{r_i}\cos\phi \end{aligned}\right\} \quad . . (70)$$

These equations replace Equations (34) and by adding Equations (36) to Equations (70), the resulting stresses are obtained, neglecting temperature effect, etc.

The stress distribution at the abutment of the arch considered in Article XII, computed from Equations (68) and (36) is given in Table 5, and plotted in Fig. 17. It will be noted that the tangential stresses at the extrados and intrados agree well with the values found previously (Table 4). Also, the assumption that the shear stress is distributed in accordance with a parabola is at least approximately correct.

The agreement between these stresses found by entirely different methods should give confidence in the correctness of the formulas presented. It has already been noted that for $\sigma_x = 0$, the writer's formulas agree well with those of Professor Cain. For shorter and thicker arches, the influence of Poisson's ratio, of σ_x, and of p_c, is likely to be more pronounced and for very short arches the Cain formulas are apt to be seriously in error but this latter possibility is likely to be of little practical importance.

TABLE 5.—Stress Distribution at Abutment.

r	σ_t	σ_r	τ
125	+ 20.5	— 69.5	0.0
120	+ 1.2	— 72.5	17.6
115	— 21.4	— 74.1	34.0
110	— 47.3	— 74.2	48.8
105	— 76.3	— 71.9	61.0
100	— 109.3	— 67.7	69.7
95	— 146.1	— 60.5	74.3
90	— 186.9	— 50.9	72.3
85	— 231.9	— 37.3	62.1
80	— 280.8	— 20.7	39.4
75	— 333.2	0.00	0.0

It should be noted that the stresses, σ_t and σ_r, are the principal stresses at the crown section, and likewise the stresses, σ_t, σ_r, and σ_x, at other sections on the faces, that is, in both cases where the shear stress, $\tau = 0$. Regarding the stresses which determine failure, the writer discussed these in his paper, "Stresses in Multiple-Arch Dams".* It may be of interest, however, to add that Föppl has subjected various materials, also concrete cubes (1 cm. to 3 cm.) to a hydrostatic oil pressure of about 45 000 lb. per sq. in. and reports† that by far the greater number of specimens showed no evidence whatever of having been submitted to stresses, while a very few had their corners broken. Likewise, several kinds of crystals were subjected to a heavy hydrostatic pressure.

* *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 308.

† *Mitteilungen* des Münchner Laboratoriums, 27 Heft; see, also, Föppl's "Drang und Zwang," Bd. I, 2d Edition, p. 41.

They were examined optically before and after the test, as such an examination would reveal even very minor structural changes; but no sign of any change in these crystals was found, although some of them had low compressive strengths when stressed in only one direction. These tests would indicate that for concrete the maximum stress in any one direction is not the deciding factor.

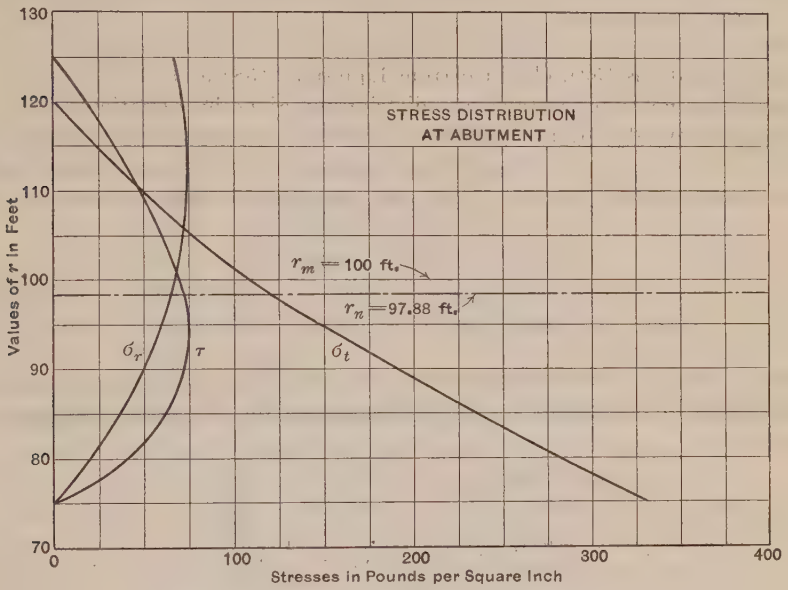

FIG. 17.

The writer would also call attention to a new theory advanced by Chidô Sunatani in his paper, "Laws of Failure of Solid Bodies Due to Stress".* This theory is an elaboration of Mohr's theory† and seems to the writer to be an improvement.

XVI.—FUNDAMENTAL EQUATIONS

As an example assume $\frac{t}{r_m} = 1.00$ and $2\,\phi_1 = 60$ degrees. Using the Lamé stresses in combination with Equations (59) and (70), respectively, the stresses given in Table 6 are found.

TABLE 6.

	ABUTMENTS.		CROWN.	
	Extrados.	Intrados.	Extrados.	Intrados.
Equations (36) and (59)	$-0.28\,p_a$	$-1.32\,p_a$	$-0.76\,p_a$	$+0.15\,p_a$
Equations (36) and (70)	$-0.33\,p_a$	$-0.78\,p_a$	$-0.86\,p_a$	$+0.81\,p_a$

* Technology Reports, Tôhoku Imperial Univ., Vol. III, No. 1, November, 1922, abstracted by George Paaswell, M. Am. Soc. C. E., in *Engineering News-Record*, Vol. 91, 1923, p. 379.

† "Abhandlungen aus dem Gebiete der Technischen Mechanik" (1906), 5th Abhandlung, p. 187.

If the central angle of this arch is decreased indefinitely, the stress at the intrados of the crown as computed from Equations (36) and (59) or from Equations (36) and (70), is, $+0.21\ p_a$ or $+\ 1.00\ p_a$, that is, there is still tension at the intrados of the crown. To the writer, this result seems improbable, since, when the central angle is sufficiently small, he would expect to find only compression.

The values used for X and A in Equation (70) are the values, X' and A', used in Equations (59), obtained from Equations (58) and (18); these values, therefore, do not apply to the stress distribution given by Equations (70).

To give an idea of the problems involved in determining X and A correctly, it is necessary to consider the fundamental differential equations for equilibrium and deformation. These are, assuming the vertical stress, $\sigma_x = 0$:

$$\left.\begin{aligned} \frac{\delta\ \sigma_r}{\delta\ r} + \frac{\sigma_r - \sigma_t}{r} + \frac{\delta\ \tau}{r\ \delta\ \phi} &= 0 \\ \frac{\delta\ \sigma_t}{r\ \delta\ \phi} + \frac{\delta\ \tau}{\delta\ r} + 2\ \frac{\tau}{r} &= 0 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (71)$$

When u_r and u_t are the tangential and radial displacements, or change of co-ordinates of any point, (r, ϕ), the equations for the elastic deformations are:

$$\left.\begin{aligned} E\ \frac{\delta\ u_r}{\delta\ r} &= \sigma_r - \frac{1}{m}\ \sigma_t \\ E\left(\frac{\delta\ u_t}{r\ \delta\ \phi} + \frac{u_r}{r}\right) &= \sigma_t - \frac{1}{m}\ \sigma_r \\ G\left(\frac{\delta\ u_r}{r\ \delta\ \phi} + \frac{\delta\ u_t}{\delta\ r} - \frac{u_t}{r}\right) &= \tau \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (72)$$

Referring to the equilibrium equations, Equations (71), it will be found that the Lamé formulas and Equations (68) satisfy Equations (71); but none of the other formulas does, nor does the formula developed by Professor Cain. In fact, no formula that disregards the radial stress, σ_r, can satisfy the equilibrium equations. This is clear also from the fact that the two tangential stresses which act on an element, $r\ d\phi\ d_r$, are not parallel and therefore there must be a component in the radial direction. At the crown the shear is zero and hence σ_r cannot be zero. For slender arches the error introduced by neglecting σ_r is unquestionably insignificant, while for short thick arches this error may be considerable.

Since Equations (68) satisfies the equilibrium equations, the displacements of each point might be found from Equations (72) by integration. This would involve considerable labor and is not likely to lead to simple relations. Moreover, the formulas found would not be correct, for the following reasons:

1.—Because the arch is assumed as free to deform radially and this is not true at the abutments.

2.—Because the stresses as given by Equations (68) do not maintain the radial sections plane, since the shear varies with r. All radial sections (except at the crown, where the shear is zero), are deformed into **S**-shaped surfaces.

This objection applies to all formulas, since the shear is zero at the up-stream and down-stream faces.*

3.—Because the rock abutment is also deformed and, for short thick arches especially, this deformation cannot offhand be assumed as negligible.

Therefore even when neglecting the effect of temperature, water-soaking of the up-stream face, and shrinkage, it is evident that correct formulas cannot be derived without taking the foundation deformation into account and this would greatly complicate the problem. A good approximation might be obtained, however, by the use of the formulas developed by Dr. Vogt (Article XXI).

XVII.—Stress Distribution at Failure

Lamé's formulas and the formulas developed in Article XV for the stresses caused by X assume that the modulus of elasticity, E, and Poisson's ratio, m, are constant. This is approximately true for stresses well beyond the ordinary allowable working stresses, up to, perhaps, one-third the ultimate compressive strength.†

A considerable variation of Poisson's ratio will not change the results materially, but a variation in E, such as takes place as the stresses approach their ultimate values, will influence the stress distribution materially.

Tests made on cast iron by Professor von Bach‡ throw some light on this subject, since for cast iron E also varies considerably with the stress. He experimented on machined cast iron and found its tensile strength to be 1 315 kg. per sq. cm. When testing this cast iron in bending, by supporting it at the ends and loading it in the center, Bach found that the bending moment at failure was 1.64 times greater than the value computed from the ordinary formula:

$$s_t = \frac{M}{I}\,\frac{t}{2}$$

in which, s_t is the tensile strength and t the thickness of the beam. The stress-strain curve for this cast iron is shown in Fig. 18. From the fact that the total compression must equal the total tension and from the known thickness of the test specimen, the neutral axis can be located by means of a planimeter. The centers of gravity of the two areas are then determined. If the distance between them is Y and the total compression area represents N kg., the moment at failure would be $N\,Y$, on the assumption that normal planes remain plane. Bach found the moment thus computed to agree with bending tests to within 3 per cent. The error in assuming E constant in this case results in underestimating the factor of safety in the ratio of 1.64 to 1.

* In the paper by Dr. Vogt, referred to in Article XXI, it is shown that a moment warps the surface of a rock foundation into an S-shaped curve.

† See, for example, Hool and Johnson, "Concrete Engineer's Handbook," 1st Edition (1918), Fig. 26, p. 250; also, "Modulus of Elasticity of Concrete," by Stanton Walker, *Bulletin 5*, Structural Materials Research Laboratory, Lewis Inst., Chicago, Ill., Fig. 3, p. 16.

‡ "Elasticität und Festigkeit," 9th Edition, p. 298.

A stress-strain curve for concrete in compression is shown in Fig. 20.* This shows that the deformations increase faster than the stresses and the more so the greener the concrete. Other experimenters find the same to be true, but the actual shape of the curve seems to depend on the age, ultimate strength, mix, water-cement ratio, etc. The variation of E will affect the stress distribution found by Lamé's formulas, decreasing the difference between the stress at the extrados and intrados.† It will also affect the distribution of the stresses due to X. A picture of what takes place may be formed by assuming that in Equation (29) for y', E associated with I becomes smaller, while E associated with t remains constant. Also, both k_e and k_i are decreased and the stress distribution in bending is as indicated in Fig. 19.

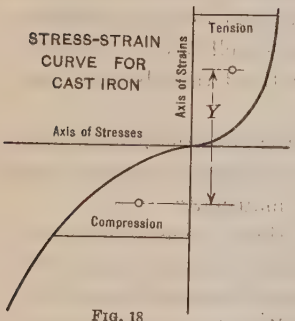

FIG. 18

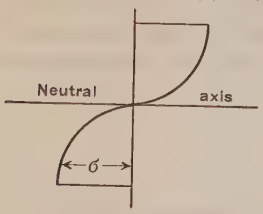

FIG. 19.

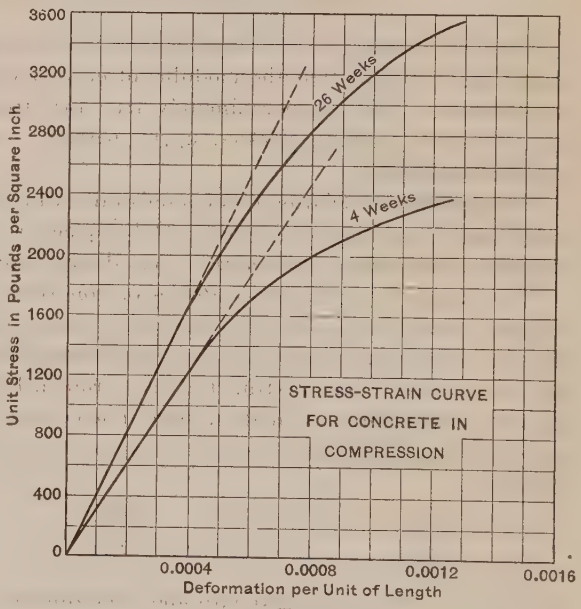

FIG. 20.

In order to obtain an idea of how these modifications will influence the stresses, assume arbitrarily, in the example given in Article XII, that the Lamé stresses from Equations (36) are increased 20 lb. per sq. in. for the extrados and decreased 20 lb. per sq. in. for the intrados. Assume, also, that E associated with I is reduced to one-third its former value, while E associated with t remains unaltered. Then, y' increases to $\frac{3.706}{E}$. Since ϵ' and ϵ'' are nearly alike, it may be assumed that ϵ' remains unchanged; X' is therefore decreased in proportion as y' is increased, or the new value of X' is 70.5% of its former value. Further, let it be arbitrarily assumed that both k_e and k_i

* Taylor and Thompson, "Concrete, Plain and Reinforced," 2d Edition (1909), Fig. 129, p. 403.

† Bach, "Elasticität und Festigkeit," 9th Edition, p. 579.

are reduced to 85% of their former values. Then, when $\sigma_x = -300.0$ lb. per sq. in., the stresses at the crown become:

$$\sigma_{te} = -167.7 + 1.5 = -166.2 \text{ lb. per sq. in. (compression)}$$
$$\sigma_{ti} = -197.2 + 118.2 = -79.0 \text{ lb. per sq. in.}$$

and at the abutments,

$$\sigma_{te} = -167.7 + 107.2 = -60.5 \text{ lb. per sq. in.}$$
$$\sigma_{ti} = -197.2 - 68.5 = -265.7 \text{ lb. per sq. in.}$$

If the stress were uniformly distributed, it would be:

$$\sigma \text{ (cylinder)} = -\frac{69.5 \times 125}{50} = -174.0 \text{ lb. per sq. in.}$$

Comparing the stresses here obtained with those given in Table 4, it will be seen that the maximum compression has been lowered from 340.1 to 265.7 lb. per sq. in.; but this is still about 50% higher than the stress computed from the cylinder formula. The tension which existed at the extrados of the abutments, as shown in Table 4, has been converted into compression. It seems obvious that the effect of having E decrease with the stress is to bring about a more uniform stress distribution than with E constant. A thick arch tested to destruction therefore should show a considerably higher factor of safety than is indicated by calculations based on E as a constant. The assumption that the cylinder formula holds good, however, errs on the side of danger; no conservative engineer can afford to make it.

XVIII.—Approximate Formulas

If,

$$\epsilon'' = -\frac{p_a r_e}{E t}$$

from Equation (38 *b*) is substituted for ϵ' from Equation (38), that is, if σ_x is assumed to be zero, there results from Equation (58):

$$X = \frac{r_e}{B} p_a \sin \phi_1 \quad \ldots\ldots (73)$$

The normal thrust at any section is then,

$$P = -p_a r_e + X \cos \phi \quad \ldots\ldots (74)$$

and the moment at any section is,

$$M = X r_m \left(\frac{\sin \phi_1}{\phi_1} - \cos \phi\right) \quad \ldots\ldots (75)$$

M is positive when it represents tension at the extrados. The stresses at the extrados and intrados of any section are, therefore, by Equation (1),

$$\left.\begin{matrix}\sigma_{te} \\ \sigma_{ti}\end{matrix}\right\} = \frac{P}{t} \pm 6 \frac{M}{t^2}$$

The total shear at any section is,

$$S = X \sin \phi \quad \ldots\ldots (76)$$

The resultant at any section is $R = \sqrt{P^2 + S^2}$, and if this makes an angle, ω, with the normal thrust, P,

$$\tan \omega = -\frac{S}{P} = \frac{\sin \phi_1 \sin \phi}{B - \sin \phi_1 \cos \phi} \quad \ldots\ldots (77)$$

These equations should agree closely with those of Professor Cain. If p_a is in pounds per square inch, X is the force applied to an arch slice $\frac{1}{12}$ in. thick.

The angle, ω, between the total thrust and the total resultant is a fictitious angle. The real angle between the resultant stress and the normal stress at any point varies from point to point of a radial section; it is zero at the up-stream and down-stream faces, since the shear is zero, and it reaches a maximum somewhere near the middle point of the section. At the abutment of the arch investigated in Article XV (see Table 5), for $r = 95$, $\tau = 74.3$ and $\sigma_t = -145.1$, so that $\tan \omega = 0.511$ and $\omega = 27$ degrees.

From Equation (45) (substituting ϵ'' for ϵ') and Equation (60) the crown deflection is,

$$\eta_t = \frac{r_e r_m}{E_t} p_a \left(\frac{D}{B} \sin \phi_1 + 1 - \cos \phi_1 \right) \quad (78)$$

Professor Cain's equation in connection with his Table 4* and Fig. 9† is more convenient. The results from both methods should agree closely.

XIX.—Secondary Arch

It frequently happens in the design of arch dams that some of the arches cannot be given a sufficiently large central angle for a proper ratio of $\frac{t}{r_m}$ and that, therefore, tension is found at the abutment, or perhaps both at the abutment and at the crown. The question then necessarily arises as to what the maximum compression in such an arch is, assuming that the concrete does not transmit any tension.

In this case a secondary arch may be conceived, which fulfills the following conditions:

(1) It must lie entirely within the original arch; and
(2) It must be free from tension in a tangential direction.

Such an arch, in general and perhaps always, will be free from tension in all directions, but to make sure of this the principal stresses must be investigated.

The theory of the secondary arch has been discussed by L. J. Mensch, M. Am. Soc. C. E., and by Professor Cain in the latter's paper "The Circular Arch under Normal Loads"‡ and also by Professor Résal in his paper, "Formes et Dimensions des Grands Barrages en Maçonnerie."§

This theory, since it affords the best and possibly the only means of investigating satisfactorily an arch that shows tension, will be explained in more detail. It must be borne in mind that the secondary arch is merely a possible solution and probably does not furnish the actual stress distribution. No doubt the actual stresses are smaller than those in the secondary arch, because parts of the arch that are assumed not to be effective actually do participate and bring about a lowering of the stresses; but as long as the theory furnishes a fair approximation and errs on the side of safety it is

* *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 264.
† *Loc. cit.*, p. 265.
‡ *Loc. cit.*, p. 249 and p. 272, foot-note.
§ *Annales des Ponts et Chaussées*, II, 1919.

useful in engineering, especially as a first approximation in getting a design started, when the influence of the weight of concrete above an arch, of the temperature, swelling, shrinkage, and cantilever action must be neglected in order to simplify the problem.

On the assumption that the vertical downward pressure, σ_x, is zero, the stress at the extrados of the abutment is, from Equations (36) and (59):

$$\sigma_{te(ab)} = -\left(\frac{r_m}{t} + \frac{1}{4}\frac{t}{r_m}\right) p_a$$

$$-\frac{\epsilon'}{B} E \sin \phi_1 \left\{ k_e \frac{r_m}{t} \left[\frac{\sin \phi_1}{\phi_1} \frac{r_n}{r_m} - \cos \phi_1 \right] + \cos \phi_1 \right\} \ldots\ldots (79)$$

The value of ϵ' is obtained from Equation (38) by putting $\sigma_x = 0$. If $\sigma_{te(ab)} = 0$ in Equation (79), there may be found for any value of $\frac{t}{r_m}$ a corresponding limiting value, $2\phi_0$, for the central angle, $2\phi_1$. For any larger value of the central angle, there is compression at the extrados of the abutment, and for a smaller value, tension at the abutment and perhaps also at the crown. Let the limiting ratio for $\frac{t}{r_m}$ be called x_0, then the corresponding values, as determined from Equation (79), are given in Table 7 and plotted in Fig. 21.

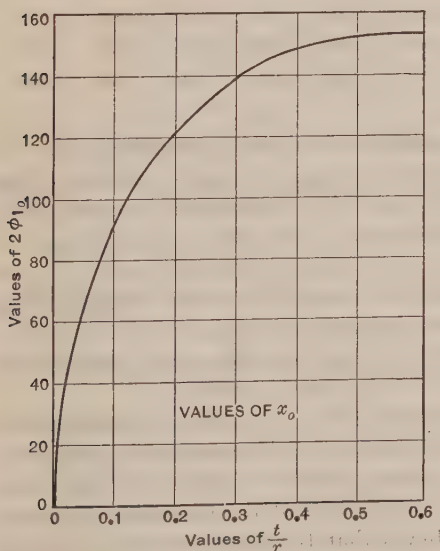

FIG. 21.

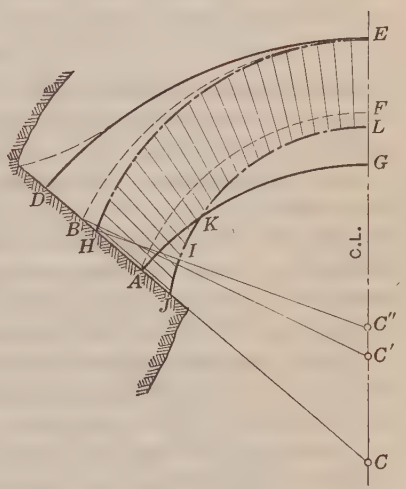

FIG. 22.

To illustrate, assume an arch with $\frac{t}{r_m} = 0.353$ and $2\phi_1 = 100°$, as shown to scale in Fig. 22. As these values locate a point in Fig. 21 which lies below the curve, this arch will develop tension at the abutments when $\sigma_x = 0$. By

drawing a few secondary arches, the secondary arch which has no stress at the extrados of the abutment can be quickly located by referring to Fig. 21. In the case under consideration, the corresponding values are $x_0 = \frac{16.5}{65.5} = 0.252$ and $2\phi_0 = 132$ degrees. The stress in this arch is zero at Point B; at Point A, where the maximum compression exists, it is nearly (as will be shown later):

$$\sigma_{max.} = -2\frac{r_e^2}{t}\frac{p_a}{r_m} = -\left(2 + \frac{2}{x_0} + \frac{x_0}{2}\right) p_a$$

or,

$$x_0 = \frac{5}{2\sqrt{\frac{\sigma_{max.}}{p_a} - 1}} \qquad \text{.....(80)*}$$

The maximum stress is nearly equal to $2\frac{r_e}{r_m}$ times the stress found by the cylinder formula, when this is applied to the secondary arch. In this case it is,

$$\sigma_{max.} = -10.05\, p_a\dagger$$

TABLE 7.—Corresponding Limiting Values of $x_0 = \frac{t}{r_m}$ and Central Angle.

x_0 =	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.50	0.60
$2\phi_0$ =	68.5°	92.5°	106°	121.5°	130°	138°	144°	148°	151°	152°

It may happen that this value is considered excessive by the designer. In the case of a new design, he can lower this maximum stress by adding to the intrados at the abutment, as indicated in Fig. 22; it is assumed, of course, that r_e cannot be decreased. This change increases x_0 and calls for a corresponding increase of the limiting central angle. By moving the center to C'', the new value for the central angle is 144° and from Fig. 21 the corresponding value of x_0 is 0.35 and the maximum compression becomes only $-7.89\, p_a$ instead of $-10.05\, p_a$. The up-stream radius of the secondary arch, scaled from the drawing, is 67 ft. and this gives $r_m = 57$ ft. and $t = 20.0$ ft.

In this case the arch carrying the load is $E\,H\,J\,K\,L$ and p_a is assumed to act on $E\,H$. If the original arch actually cracks at $D\,H$ permitting water pressure to act on $D\,H$, the resultant of p_a acting on $E\,H$ is equal to the resultant of p_a acting on $E\,D$ and $D\,H$. Due to the swelling of the up-stream face produced by water-soaking, the writer believes that the up-stream face actually will not crack and that the stresses found from Equation (80) are in excess of the actual stresses.

* r_e, r_m, t, and x_0 apply to the secondary arch.

† It is interesting to note that the stress computed by the cylinder formula applied to the original arch gives $-3.33\, p_a$, the error amounting to 300% on the side of danger. The cylinder formula, therefore, can hardly be considered as a useful approximation. This case is by no means extreme.

In case the swelling is not sufficient to relieve all tension, the secondary arch actually formed will be thicker at the abutment than at the crown, at least for thin and medium thick arches. For example, it might be such as *E B J K L* in Fig. 22, the stress at Point *B* being zero. The stresses in such an arch may be investigated as will be shown in Article XX, but this involves much more labor and would not result in lowering them materially.

It should be noted that it is unnecessary to continue the arch, *L K I*, in Fig. 22 beyond the point, *I*, and it is sufficient to draw *I J* perpendicular on the radius, *H I C″*. This is so because with the stress at Point *H* equal to zero, the angle that the resultant makes with the perpendicular on the radius, *H I C″*, is always small and the resultant acts one-third of the distance, *I H*, from *I;* if this resultant intersects the abutment between *H* and *J*, nothing is gained by continuing the curve beyond Point *I*. From Equation (77) the angle, ω, for values of $\frac{t}{r_m}$ of 0.1, 0.3, and 0.6 will be 3°, 6° 30′, and 12° 40′, respectively.

The maximum compression in a secondary arch occurs at the abutments and may be determined as follows, if ϵ'' is substituted for ϵ':

$$\left.\begin{aligned}
\sigma_{t\,e\,(a\,b)} &= -\left(\frac{1}{x_0}+\frac{1}{4}x_0\right)p_a+\frac{p_a r_e}{t B}\sin\phi_1 \\
&\quad \left\{\frac{k_e}{x_0}\left[\frac{\sin\phi_1}{\phi_1}\frac{r_n}{r_m}-\cos\phi_1\right]+\cos\phi_1\right. \\
\sigma_{t\,i\,(a\,b)} &= -\left(1+\frac{1}{x_0}+\frac{x_0}{4}\right)p_a-\frac{p_a r_e}{t B}\sin\phi_1 \\
&\quad \left\{\frac{k_i}{x_0}\left[\frac{\sin\phi_1}{\phi_1}\frac{r_n}{r_m}-\cos\phi_1\right]-\cos\phi_1\right.
\end{aligned}\right\} \quad \ldots\ldots(81)$$

For a secondary arch, subtracting $\sigma_{t\,e\,(a\,b)}$ (equal to zero) from $\sigma_{t\,i\,(a\,b)}$, since $k_e + k_i$ is nearly equal to 12, and assuming that $\frac{r_n}{r_m}=1$,

$$\sigma_{t\,i\,(\text{max.})} = -p_a-\frac{12}{B}\frac{r_e r_m}{t^2}\left[\frac{\sin^2\phi_1}{\phi_1}-\frac{1}{2}\sin 2\phi_1\right] \quad \ldots\ldots(82)$$

The values for $\sigma_{\text{max.}}$ and $\sigma_{t\,i\,(\text{max.})}$ are given in Table 8 ; it will be seen that Equation (80) is a good approximation for values of $\frac{t}{r_m}$ up to 0.6. The deflection of the secondary arch may be found from Equation (78).

TABLE 8.—Comparison of Stresses from Equations (80) and (82).

$\frac{t}{r_m}$ =	0.1	0.2	0.3	0.4	0.5	0.6
$\sigma_{\text{max.}}$ =	$-22.3p_a$	$-12.1p_a$	$-8.81p_a$	$-7.2p_a$	$-6.25p_a$	$-5.63p_a$
$\sigma_{t\,i\,(\text{max.})}$ =	$-20.6p_a$	$-11.1p_a$	$-8.91p_a$	$-7.13p_a$	$-6.34p_a$	$-5.89p_a$

The formulas given by Professor Résal are based on a parabolic arch.* They check Equations (80) and (82) for secondary arches fairly well, are more convenient to use, and will, therefore, be given here. Referring to Fig. 23, in

* See remarks on p. 576.

which the hatched portion is the parabolic secondary arch, the maximum compression, which occurs at the intrados of the abutment, is,

$$\sigma_{max.} = -1.41\, p_a \frac{L^2}{F^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (83)$$

The crown deflection is,

$$E\,\eta_a = 0.209 \frac{L^4}{F^3}\, p_a \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (84)$$

The radial thickness, t', of the secondary arch, and the values of F and L (Fig. 23) are:

$$\left.\begin{aligned} t' &= \frac{F}{4.75} \\ F &= t + r_i \cos \phi_i \\ L &= 2\, r_e \sin \phi_e \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (85)$$

It is generally better to draw the arch to scale and then measure L and F; care should be taken to draw the abutments on both sides as they actually occur for each arch to be investigated.

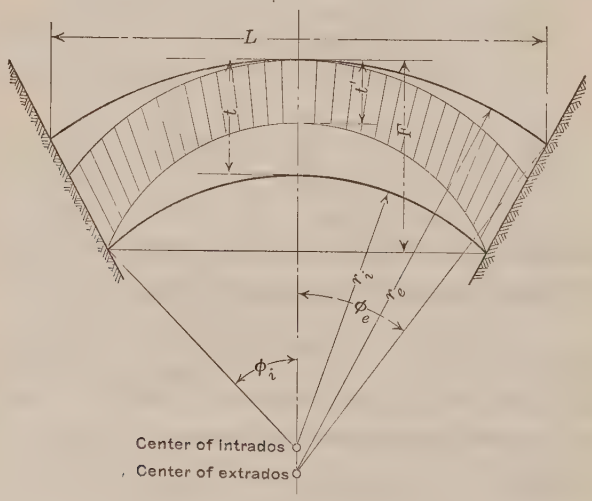

FIG. 23.

If the maximum stress is found to be higher than is thought advisable and if r_e cannot be made smaller without producing excessive overhang in the dam, F must be made larger. It can be determined from Equation (83), if the maximum stress is assumed. The corresponding thickness is $t' = \frac{F}{4.75}$ and a parabola must be constructed in order to find the shape of the down-stream face, although, in general, a circle is a sufficiently close approximation.

Referring to Fig. 24, the equation of a parabola is:

$$y^2 = 2\, p\, x$$

The value found for F locates a point on the abutment a distance, H, from the center line, where,

$$H = (t + r_i - F) \tan \phi_i$$

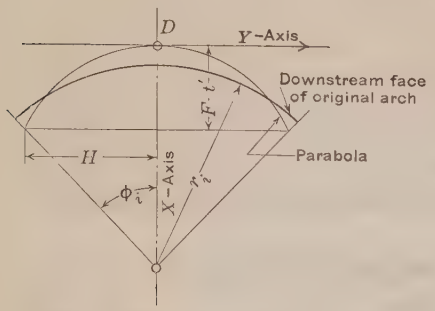

FIG. 24.

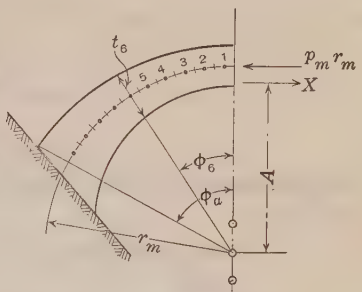

FIG. 25.

For this point, $y = H$ and $x = F - t'$; therefore,

$$2p = \frac{H^2}{F - t'}$$

The parabola should be continued only to Point I in Fig. 22, as explained previously. The arch investigated previously gave a maximum compression of 10.05 p_a with $\frac{t}{r_m} = 0.353$ and $2\phi_1 = 100°$; for the arch of Fig. 24, $\frac{L}{F} = 2.79$ and from Equation (83) the maximum compression is 10.95 p_a. This is a fairly close check. It is immaterial which of the two values is the more nearly correct. The difficulty in arch dam design arises from the fact that concrete is not reliable for transmitting tension and, therefore, a solution that results in tension is not satisfactory.

XX.—Arch of Variable Thickness

It is sometimes desirable to investigate an arch of variable thickness. This may be accomplished approximatively by dividing the semi-arch into a number of equal parts, say, ten, of length, $s = r_m \Delta \phi$ (see Fig. 25) and then by applying the formulas of Articles II, III, and IV. If the mean radius is approximately constant, the stress due to the water pressure alone will be,

$$\sigma'' = -\frac{p_m r_m}{t} \qquad (86)$$

in which, p_m is the water pressure, referred to the center line. The thrust, $p_m r_m$, acting at the middle points of the crown section and of the abutment section, holds the water pressure in equilibrium. These two reactions and the water pressure produce a displacement, $D F$, in Fig. 2 and the force, X, pulls the crown section back into the center line. Let the displacement, $D F = D$, then,

$$D = \sum \frac{p_m r^2_m}{E t} \cos \phi \Delta \phi \qquad (87)$$

In order to find A (by Equation (4)) since the moment of inertia, I, is not now a constant,

$$\sum \frac{M}{I} s' = 0$$

M is given by Equation (5), so that,

$$A = \frac{\sum \frac{r_m \cos \phi}{I}}{\sum \frac{1}{I}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (88)$$

The displacement of the crown section in the direction of X produced by $X = 1$, is,

$$\left.\begin{aligned} y = & \sum \frac{r_m \cos^2 \phi}{E\, t} \Delta \phi + 2.88 \sum \frac{r_m \sin^2 \phi}{E\, t} \Delta \phi \\ & + A\, s \sum \frac{r_m (1 - \cos \phi)}{E\, I} - s \sum \frac{r_m (1 - \cos \phi)}{E\, I} r_m \cos \phi \end{aligned}\right\} \ldots (89)$$

and,

$$X = \frac{D}{y} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (90)$$

The values for t, $r_m \cos \phi$, $r_m \sin \phi$, and $r_m (1 - \cos \phi)$ can be scaled from a drawing, a table for the values needed for each point can be calculated, and these values added. The method involves little labor; additional refinements can readily be introduced, but are likely to add materially to the labor involved.

The thrusts and moments are found by Equations (15) and (16) and the stresses from Equation (1). By comparing the formulas of Articles II, III, and IV with the more accurate formulas, it will be noted that Equation (86) over-estimates the stresses at the extrados and under-estimates them at the intrados. The fact that k_e is always less than 6 and k_i always more than 6, compensates, in a measure, for this inaccuracy. Hence, these formulas are a good approximation.

If r_m is not approximately constant, Equations (86) and (87) can not be applied and the resulting labor is considerably greater. If the deviation from a constant value occurs only near the abutment the stress produced on the abutment section by the water pressure and the assumed reaction, $p_m r_m$, acting at the crown section can be found easily by graphical construction and an allowance can be made for irregularities in r_m. The influence of σ_x and of temperature, swelling, and shrinkage, can also be included separately in the calculations, exactly as has been done previously.

XXI.—Effect of Cantilever Action

The cantilevers and the arches divide the load imposed by the water pressure. The determination of this distribution has been attempted by several writers, recently by Alfred Stucky* in connection with the constant angle

* "Etude sur les barrages arqués," Extrait du *Bulletin technique de la Suisse romande*, 1922, Lausanne, F. Rouge et Cie.

arch dam on the Jogne River. Stucky determines the cantilever action at five sections of the semi-arch and also investigates the effects of temperature. The labor involved is considerable. Professor Résal investigates the crown section only, but uses the secondary arches and makes sure that the bending moment at the base of the cantilever does not reduce the compression in the up-stream face below the hydrostatic pressure. F. A. Noetzli,* M. Am. Soc. C. E., also investigates only the crown section and although he finds tension in both arches and cantilevers, yet he determines the deflections assuming the concrete to transmit this tension.

In Mr. Noetzli's paper,† he made the usual assumptions, as follows:

1.—That the force acting to bend the dam was at right angles to the neutral axis;

2.—That the modulus of elasticity is constant for both compression and tension;

3.—That the modulus of elasticity is the same for compression as for tension;

4.—That the deformation due to shear is negligible;

5.—That the influence of lateral deformation (Poisson's ratio) is negligible;

6.—That the foundation is unyielding; and

7.—That the water-soaking of the up-stream face does not increase the volume of the concrete, that is, does not cause swelling.

The writer discussed these assumptions in detail and showed that they were so approximate for dams that they could not furnish a sound basis for the sweeping conclusions made by Mr. Noetzli, more especially since these conclusions were not supported by any facts whatever, but on the contrary appeared to be at variance with facts.

It seems to the writer that the cantilever cannot relieve its tension by a single crack as Mr. Noetzli suggests,‡ but that it must yield a little over a considerable vertical zone, so as to afford the arches an opportunity to carry the load, which actually they were doing if Mr. Noetzli's thesis was correct, as the dams had not failed nor shown any tendency in that direction. It is probable that a secondary cantilever would be formed in much the same manner as the secondary arches. This secondary cantilever can probably be determined by trial computations, but the labor might be considerable. At any rate a solution such as that offered by Mr. Noetzli, which showed tension much in excess of what could exist, is not satisfactory, since it is an impossible condition.

That the effect of swelling of the up-stream face (according to Mr. Noetzli there is no swelling) is not by any means negligible is illustrated by the Santa Anita Dam, a constant angle arch dam now being constructed by the Los Angeles County Flood Control District. This dam is 225 ft. high and of dimensions as given in Table 9. Assume the dam to have been closed or pressure grouted after the concrete was well dried; the shrinkage would prob-

* "Gravity and Arch Action in Curved Dams," *Transactions*, Am. Soc. C. E., Vol. LXXXIV (1921), p. 1.

† The writer called attention (*Transactions*, Am. Soc. C. E., Vol. LXXXIV (1921), p. 92) to the many inaccuracies in the assumptions made by Mr. Noetzli.

‡ *Transactions*, Am. Soc. C. E., Vol. LXXXIV (1921), p. 54.

ably amount to 0.0002. Water against the up-stream face would cause a swelling such that the volume would be 0.0001 larger than when poured, that is, a relative swelling of 0.0003. It seems fair to assume that a relative swelling of at least 0.0001 occurs; this is no doubt an under-estimate, since for a fairly thick dam the down-stream face probably does not participate in the swelling to any marked degree. A swelling of 0.0001 is equivalent to imposing a tension of 200 lb. per sq. in. on the up-stream face, on the assumption that $E = 2\,000\,000$ lb. per sq. in. The deflection caused by this swelling is therefore considerably greater than the cantilever could otherwise make without failing in tension. The swelling of the up-stream face, besides producing a down-stream deflection of the cantilever, also produces an up-stream deflection of the arches. This latter is not readily computed because the arches in the Santa Anita Dam are thicker at the abutments than at the crown (except near the top), and it might also be necessary to determine new secondary arches, which would be thicker than the secondary arches carrying the load when there is no swelling. The following computation, however, may furnish an indication of the influence of swelling. Consider the arch at Elevation 1280; from Equation (38 *b*), if $E = 2\,000\,000$,

$$\epsilon'' = -\frac{19.5 \quad 287.0}{E \; 18} = -\frac{1.55}{10\,000}$$

If a swelling equal to ϵ'' occurs uniformly throughout the arch (see Equations (53) and (55)), the arch would not deflect at all when the water load was imposed. At Elevation 1220, using an average value of 40 ft. for t in Equation (38 *b*), $\epsilon'' = -0.000163$. If the swelling amounts to as much as this, the arch would not deflect at all when taking its full load.

TABLE 9.—Data for Santa Anita Dam.

Elevation.	t_c, in feet.	t_a, in feet.	$2\,\phi_1$*, in degrees.	r_a, in feet.	v, in feet.	η_a, in feet.
1325	7.0	7.0	115	305	0.075	0.0000
1300	12.5	12.5	106	299	0.056	0.0287
1280	18.0	18.0	101	287	0.043	0.0503
1260	23.0	27.0	94	274	0.033	0.0695
1240	26.0	47.0	90	269	0.024	0.0816
1220	29.0	50.0	79	254	0.017	0.1015
1200	32.0	60.0	65	238	0.011	0.0886
1180	34.0	63.0	55	214	0.0066	0.0608
1160	38.5	60.0	47	190	0.0028	0.0292
1140	42.0	48.0	32	158	0.0010	0.00466
1120	45.0	46.5	30	135	0.0004	0.00199
1100	48.0	48.0	30	109	.....	.....

* Measured from the center for the up-stream radius to the junction of the extrados with the abutments.

The cantilever deflections were computed as follows, referring to Fig. 26, in which, θ denotes the rotation brought about at the points, 1, 2, 3, . . . n, by the swelling, t denotes the average thickness, s, the lengths of each section,

and v, the down-stream deflections at the points, 1, 2, 3, . . . n. For a swelling of 0.0001,

$$\theta_1 = \frac{0.0001\ s_1}{t_1}$$

$$\theta_2 = \theta_1 + \frac{0.0001\ s_2}{t_2}$$

$$\theta_n = \theta_{n-1} + \frac{0.0001\ s_n}{t_n}$$

and the cantilever deflections,

$$v_1 = 0.5\ s_1\ \theta_1$$

$$v_2 = v_1 + s_2\ \theta_1 + 0.5\ s_2\ (\theta_2 - \theta_1)$$

$$v_n = v_{n-1} + s_n\ \theta_{n-1} + 0.5\ s_n\ (\theta_n - \theta_{n-1})$$

These deflections are given in Table 9, also the deflections of the secondary arches, on the assumption that they carry the entire water load. The arch deflections were computed by Résal's method, Equation (84). The two deflection curves are shown to scale in Fig. 27, from which it is quite obvious that the deflection of the cantilever caused by the swelling is by no means negligible. It is considerably greater than the cantilever deflection obtained by computing only for bending as was done by Mr. Noetzli. The influence of this cantilever deflection due to swelling is likely to be comparatively greater at other parts of the dam.

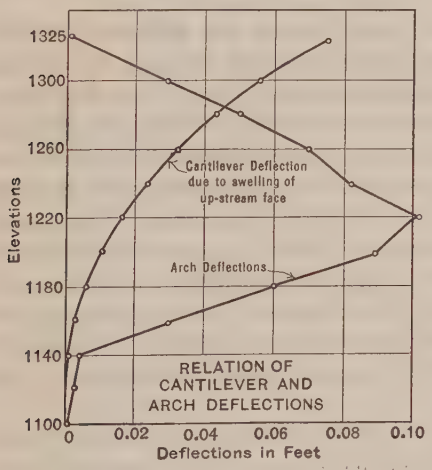

FIG. 26.

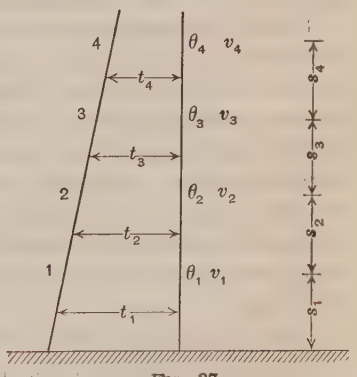

FIG. 27.

The writer* further disagreed with Mr. Noetzli and asserted that the deformation of the foundation might contribute materially to the deflection of the cantilever. In a thesis kindly furnished by a Norwegian engineer, Dr. Fredrik Vogt, the deformations of rock foundations are investigated.† Dr. Vogt considers a gravity dam having a base two-thirds of its height and

* *Transactions*, Am. Soc. C. E., Vol. LXXXIV (1921), p. 91.

† "Ueber die Berechnung der Fundament-deformation," *Det Norske Videnskapsakademi*, Oslo, Norway, Math.-Naturv Klasse 1925, No. 2.

a length five times the height. The usual triangular section is evidently considered, although the author does not say so specifically, but his figures for deflection of the cantilever checks the writer's calculations.* Poisson's ratio is assumed as 0.167. Dr. Vogt gives the deflection, in feet, at the crest of the cantilever, when sustaining the full water pressure, as

$$f_1 = 4.5\, w\, \frac{h^2}{E}$$

in which, $w = 62.5$, the weight of a cubic foot of water, h, the height of the dam, in feet, and E is in pounds per square foot. About three-fourths of this is due to moment and one-fourth to shear.

The crest deflection, in feet, caused by the deformation of the foundation alone, is, according to Dr. Vogt,

$$f_2 = 3.74\, w\, \frac{h^2}{E}$$

Thus, the crown deflection produced by the yielding of the foundation alone amounts to about 85% of the crest deflection due to the deformation of the cantilever alone.† Such a percentage can hardly be called negligible.

Dr. Vogt investigates the influence of a normal force, a moment, and shear separately, and has derived simple approximate formulas. The paper merits a study by any one seriously interested in dams or other massive structures.

The yielding of the foundation will also increase the deflection of the arches, but the writer believes to a much smaller extent, because the arches are fixed at the crown and not free to follow the deformations at the abutment.

It will be evident that the assumptions usually made, in finding the division of load between the cantilevers and the arches, are not sufficiently accurate to warrant any conclusions. The writer would like to see some fairly reliable formulas developed, which would take account of all the factors of the same order of importance. Until this can be done he prefers to neglect the influence of the cantilevers and make sure that the arches can support the full water pressure with a large margin of safety. He does not deny the influence of the cantilevers, but it is obviously without avail to select arbitrarily one of the many factors, and not the largest one at that, and neglect the influence of all the others.

That the standard assumptions are correct for standard conditions of ordinary structures is not denied, and moreover that question is not here involved; the question is whether they are acceptable for those conditions involved in designing dams.

It is not clear how neglecting the cantilever action can mean that the actual factor of safety of the dam is less than the factor calculated on the assumption that the cantilevers carry no part of the load. There may be calculated tensile stresses, but, as long as the arches can carry the full water load with a great

* *Transactions*, Am. Soc. C. E., Vol. LXXXIV (1921), Fig. 32, p. 101.

† Dr. Vogt assumes E to be nearly the same for rock and concrete.

margin, it is not evident how such a dam can possibly fail or become a dangerous structure.

The voice of experience is also entitled to be heard, and it indicates that even poorly designed and constructed single arch dams are relatively safe structures.* Theory supports this view. The fact that no single arch dam has ever failed, although several freak single arch dams exist, is also to be given some weight, especially in view of the fact that this type is the only one that has not produced a fair number of failures.†

ACKNOWLEDGMENT

This paper was submitted for criticism to Professor William Cain, to whom the writer's sincere thanks are due for discussing it and encouraging its submission to the profession.

* See A. J. Wiley, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–20) p. 609. The radius at the top varies between 58 and 80 ft. In cold weather, with reservoir empty, a crack of ⅜ in. appeared; this closed when the reservoir filled.

† "The Record of 100 Dam Failures," L. R. Jorgensen, M. Am. Soc. C. E., *Journal of Electricity*, March 15 and April 1, 1920.

DISCUSSION

WILLIAM CAIN,* M. AM. SOC. C. E. (by letter).—The author has effected a solution of the thick arch by a geometrical method that is of decided value. For Poisson's ratio, $\frac{1}{m} = 0$, the results agree with those derived by the "work method", as given subsequently. When σ_x, the vertical unit stress, is considered, and m is finite, the author's method leads to a solution which is nearly as simple as the preceding and is not included in the writer's formulas.

After the publication of his paper on "The Circular Arch Under Normal Loads,"† the writer began investigating the subject of thick arches and sent, in 1922, certain results to interested friends. Finally, when Mr. Jakobsen kindly sent the first draft of his paper in manuscript form, the writer's interest was aroused and he completed a thorough treatment of the thick arch by the work method as herein given. To make it convincing, every preliminary step was proved from elementary principles; then the work method was applied, giving X' and the deflection, which, moreover, agree with the author's values, for $m = \infty$. In this investigation, the most signal advance was made by referring moments and thrusts to the neutral axis. The writer was first led to this by the demonstration given later in connection with Fig. 33, although another briefer proof is first given. Using as a base the neutral axis, the general formula for L, the elastic work of the half-arch was expressed in as simple a form as in the writer's paper cited, so that the subsequent derivations were almost as simple as before. The analysis follows.

Tangential Stresses.—Let Fig. 28 represent a part of the arch 1 ft. thick perpendicular to the plane of the paper, bounded by two right sections making, with each other, the angle, α, with the center line of radius, r, passing through C and the neutral line of radius, r_n, passing through D. Let, $r - r_n$ = distance $O\,C$ — distance $O\,D$ = distance $D\,C = c$. Since the full water pressure is carried partly by the arch and partly by the corresponding cantilever, let p_e equal the unit water pressure carried by the arch and applied at its extrados. Suppose that the equilibrium polygon for the full arch fixed at the ends and subjected to the forces, p_e, passes through Point E, where the components (normal and along the section) of the resultant are P and S, respectively, the latter being the shear. Then, on applying at D two forces, P, equal and opposed and both normal to the section, the single force, P, at E is now replaced by the couple of moment, P times the distance, $D\,E$, the single force, P, at D on the neutral axis tending to compress the section. It will be shown later that this force, P, at D causes only a translation of the section to a parallel position and that the rotation of this section is caused entirely by the couple of moment, $M = P$ times the distance, $D\,E$. This would not be true if the two opposed forces P, were supposed to act at the center point, C; for then, it can be shown that rotation of the section is caused not only by the couple of moment, P times the distance, $C\,E$, but also, in part, by the force,

* Prof. Emeritus, Univ. of North Carolina, Chapel Hill, N. C.

† *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 233.

P, at C, which causes a uniform stress, $\frac{P}{t}$, on the section.* To find the rotation of the section, $\alpha' - \alpha = \Delta\alpha$, caused by the couple of moment, P times the distance, $D\,E$, regard the left section as fixed and the angle, α, of the voussoir as indefinitely small, which is equivalent to taking the limit of the result as α approaches zero.

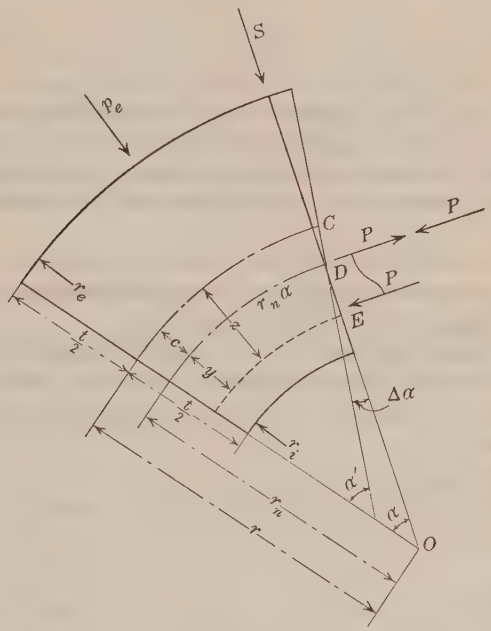

FIG. 28.

In Fig. 28, y, the distance of a fiber from the neutral axis, will be taken as positive or negative, according as the fiber lies nearer to or farther from O than the neutral axis. The length of the fiber is thus, $(r_n - y)\,\alpha$, and it is shortened by the rotation by an amount, $y\,\Delta\,\alpha$, so that its unit deformation or strain $= \frac{y\,\Delta\,\alpha}{(r_n - y)\,\alpha}$, and the unit stress, s, on this fiber $= E\,\frac{y\,\Delta\,\alpha}{(r_n - y)\,\alpha}$.

Regarding this fiber as having the area, $d\,y \times 1$, its stress will be, $E\,\frac{y\,d\,y\,\Delta\,\alpha}{(r_n - y)\,\alpha}$, and the sum of the moments of such stresses for the entire section must equal M.

Then,

$$M = \frac{E\,\Delta\,\alpha}{r_n\,\alpha}\int_{-\left(\frac{t}{2}+c\right)}^{\left(\frac{t}{2}-c\right)} \frac{y^2\,d\,y}{1 - \frac{y}{r_n}} = E\,\frac{\Delta\,\alpha}{r_n\,\alpha}\,I_n$$

* See demonstration in connection with Fig. 33.

in which,

$$I_n = \int_{-\left(\frac{t}{2}+c\right)}^{\left(\frac{t}{2}-c\right)} \frac{y^2\,dy}{1-\frac{y}{r_n}} = r_n \int_{-\left(\frac{t}{2}+c\right)}^{\left(\frac{t}{2}-c\right)} \frac{y^2\,dy}{r_n - y} \quad \ldots\ldots (91)$$

Then,

$$\Delta\,\alpha = \frac{M\,r_n\,\alpha}{E\,I_n} \quad \ldots\ldots (92)$$

and,

$$s = \frac{E\,y\,\Delta\,\alpha}{(r_n - y)\,\alpha} = \frac{E\,y}{(r_n - y)\,\alpha} \times \frac{M\,\alpha\,r_n}{E\,I_n} = \frac{M}{I_n}\,\frac{y\,r_n}{r_n - y} \quad \ldots\ldots (93)$$

As to the signs, M is regarded as positive when the rotation is clockwise, and, therefore, when E is nearer O than D and M ($= P$ times the distance, DE), is positive, s is compressive when y is positive and tensile when y is negative; the reverse occurs when M is negative or E is farther from the center, O, than D.

Again, from Equation (92), the rotation is clockwise or anti-clockwise according as M is positive or negative. Also, the rotation, $\Delta\,\alpha$, is evidently the rotation of the tangent to the neutral line at D.

To deduce simple formulas for r_n, c, and I_n, reckon z, the radial ordinate, from the medial line of the arch. Then, $y = z - c$, and $r_n = r - c$; therefore, $r_n - y = r - z$, and $dy = dz$. Since the sum of the bending stresses on the cross-section is 0,

$$\int_{-\left(\frac{t}{2}+c\right)}^{\left(\frac{t}{2}-c\right)} \frac{y\,dy}{r_n - y} = \int_{-\frac{t}{2}}^{\frac{t}{2}} \frac{z-c}{r-z}\,dz = 0, \text{ and } c = r - r_n$$

therefore,

$$\int_{-\frac{t}{2}}^{\frac{t}{2}} \frac{z - r + r_n}{r - z}\,dz = -\int_{-\frac{t}{2}}^{\frac{t}{2}} dz + r_n \int_{-\frac{t}{2}}^{\frac{t}{2}} \frac{dz}{r-z} = 0$$

Now,

$$\int_{-\frac{t}{2}}^{\frac{t}{2}} \frac{dz}{r-z} = -\log_e (r-z)\Big]_{-\frac{t}{2}}^{\frac{t}{2}} = \log_e \frac{r + \frac{t}{2}}{r - \frac{t}{2}} = \log_e \frac{r_e}{r_i}$$

therefore,

$$r_n \log_e \frac{r_e}{r_i} = t$$

or,

$$r_n = \frac{t}{\log_e \left(\frac{r_e}{r_i}\right)} \quad \ldots\ldots (94)^*$$

---

* Expanding,

$$\log_e \frac{r_e}{r_i} = \log_e \frac{r + \frac{t}{2}}{r - \frac{t}{2}},$$

$$\log_e \frac{r_e}{r_i} = \frac{t}{r}\left\{1 + \frac{1}{12}\left(\frac{t}{r}\right)^2 + \frac{1}{80}\left(\frac{t}{r}\right)^4 + \frac{1}{448}\left(\frac{t}{r}\right)^6 + \ldots\ldots\right\}$$

If this is substituted in Equation (94), it is seen that $r_n < r$, or the neutral line, is nearer 0 than the medial line of the arch, as was assumed.

$$c = r - r_n = r - \frac{t}{\log_e \left(\frac{r_e}{r_i}\right)} \quad \ldots\ldots\ldots\ldots\ldots\ldots (95)$$

$$I_n = r_n \int_{-\left(\frac{t}{2}+c\right)}^{\left(\frac{t}{2}-c\right)} \frac{y^2\, d\, y}{r_n - y} = r_n \int_{-\left(\frac{t}{2}+c\right)}^{\left(\frac{t}{2}-c\right)} \left(r_n \frac{y\, d\, y}{r_n - y} - y\, d\, y\right)$$

$$= 0 - r_n \int_{-\left(\frac{t}{2}+c\right)}^{\left(\frac{t}{2}-c\right)} y\, d\, y = - r_n \int_{-\frac{t}{2}}^{\frac{t}{2}} (z - c)\, d\, z$$

Now, since z is measured from the center of gravity of the section,

$$\int_{-\frac{t}{2}}^{\frac{t}{2}} z\, d\, z = 0$$

and,

$$I_n = c\, r_n\, t \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (96)$$

The numerical computation of $\log_e \frac{r_e}{r_i} = \log_e r_e - \log_e r_i$, is quickly effected when an adequate table of Napierian logarithms is at hand. Otherwise, $\log_e N = 2.30258509 \times \log_{10} N$. By use of Equations (95) and (96), letting $I = \frac{1}{12} t^3$ = moment of inertia of cross-section,

If $t = 30$ and $r = 100$, or $\frac{t}{r} = 0.3$, then $I_n = 0.99841\, I$

If $t = 50$ and $r = 100$, or $\frac{t}{r} = 0.5$, then $I_n = 0.99568\, I$

If $t = 70$ and $r = 100$, or $\frac{t}{r} = 0.7$, then $I_n = 0.99123\, I$

Thus, I_n differs from I by less than 1% for any case likely to occur in practice, and it will be replaced by I in the numerical computations.

Moments Taken About Points in Neutral Axis.—It will now be proved that the resultant, P, of the normal stresses on the cross-section that cause it to move to a parallel position, or that cause translation only, must pass through D.

Let,

σ = one of these unit stresses, acting at a distance, r', from 0.

α = the angle between the two sections of the voussoir, Fig. 29.

The length of a fiber at $I = I\, I' = r'\, \alpha$. Since the unit stress acting on it equals σ, it will shorten the amount, $I\, I'' = \frac{\sigma}{E}\, r'\, \alpha$. Since by assumption, the shortening is to be the same at any point of the section, $I\, I'' = A\, A'' = BB''$, therefore,

$$\sigma\, r' = \text{constant} = k \text{ (say)}$$

Then,

$$P = \int_{r_i}^{r_e} \sigma\, d\, r' = k \int_{r_i}^{r_e} \frac{d\, r'}{r'} = k \log_e \frac{r_e}{r_i}$$

or,

$$k = \frac{P}{\log_e \left(\frac{r_e}{r_i}\right)}$$

that is,

$$\sigma = \frac{k}{r'} = \frac{P}{r' \log_e \left(\frac{r_e}{r_i}\right)} = \frac{P}{t} \frac{r_n}{r'} \quad \ldots\ldots\ldots\ldots\ldots\ldots (97a)$$

Note that at D, $r' = r_n$, and $\sigma = \frac{P}{t}$.

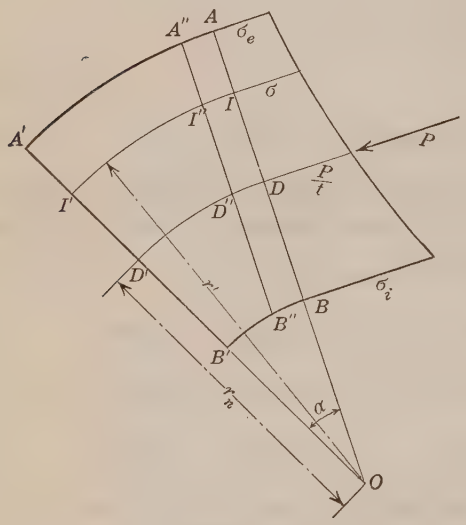

FIG. 29.

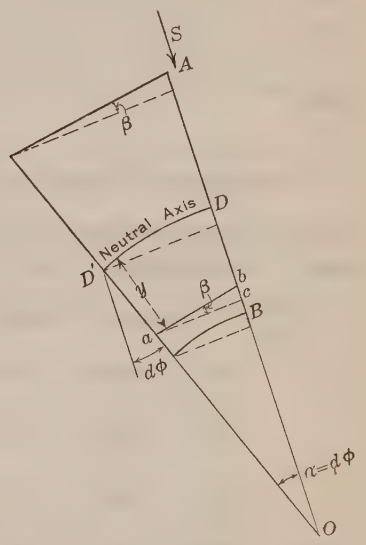

FIG. 30.

The unit stresses, σ_e and σ_i, at the extrados and intrados, respectively, are:

$$\sigma_e = \frac{P}{r_e \log_e \left(\frac{r_e}{r_i}\right)}; \quad \sigma_i = \frac{P}{r_i \log_e \left(\frac{r_e}{r_i}\right)} \quad \ldots\ldots\ldots\ldots\ldots\ldots (97b)$$

The position of the resultant, P, of these stresses can be most conveniently found by taking moments about O. From Equation (97a),

$$\int_{r_2}^{r_e} \sigma \, r' \, dr' = \int_{r_2}^{r_e} k \, dr' = k \, (r_e - r_i) = k \, t = P \, r_n$$

Thus, P passes through D.

It follows, since P, or the forces, σ, cause translation only, that the couple of moment, P times the distance, $D\,E$ (Fig. 28), causes all the rotation. This moment is always equal to the moment of all forces supposedly acting from the crown to the section about Point D (and not C). To include all the rotation of sections, therefore, the bending moments must be taken about points on the

neutral axis. For thin arches, such as those used on highways or railways, it makes no practical difference whether the moments, M, are taken about C or D, but for the thick arches of the lower parts of arched dams 100 to 400 ft. high, the difference is quite appreciable.

Elastic Work of Semi-Arch Due to P.—To find the elastic work of the voussoir of angle, $\alpha = d\,\phi$, note that the unit strain at y is, $\frac{\sigma\, r'\, d\,\phi}{E} = \frac{k\, d\,\phi}{E}$, and that the work of the fiber of cross-section, $d\,y \times 1$, for a uniformly increasing load, is,

$$\frac{1}{2}\,\sigma\,(d\,r' \times 1)\,\frac{k\,d\,\phi}{E} = \frac{d\,\phi}{2\,E}\,k^2\,\frac{d\,r'}{r'}$$

since

$$\sigma = \frac{k}{r'}$$

Thus, the work due to P acting at the point, D, of the voussoir is,

$$\frac{d\,\phi}{2\,E}\,k^2 \int_{r_2}^{r_e} \frac{d\,r'}{r'} = \frac{k^2\,d\,\phi}{2\,E}\log_e \frac{r_e}{r_i}$$

or, since,

$$k = \frac{P}{\log_e\left(\frac{r_e}{r_i}\right)}, \text{ and } \log_e\left(\frac{r_e}{r_i}\right) = \frac{t}{r_n}$$

the work is,

$$\frac{r_n}{2\,E\,t}\,P^2\,d\,\phi$$

The work for the semi-arch having a central angle, $2\,\phi_1$, is,

$$\frac{r_n}{2\,E\,t}\int_0^{\phi_1} P^2\,d\,\phi$$

This is the work due to translation only caused by the normal component, P.

The work done by the shearing component, S, can be found by aid of Fig. 30, in which, the central angle, $d\,\phi$, of the voussoir is infinitesimal, so that the limit, as $d\,\phi = 0$, is tacitly assumed in stating results. The shear is supposed to displace the fibers relative to the section through D' to the positions shown by the dotted lines (really curved), the angle of rotation being $\beta = \frac{S}{t\,F}$, $t \times 1$ being the area of the section and F the shearing modulus. The shear is not uniformly distributed over the cross-section, $A\,B$; but assume for the present that it is, and that the correction will be eventually applied. Thus, for the fiber, $a\,b$, distant y from the neutral axis, the unit shear acting at b will be assumed to be $\frac{S}{t}$.

The length, $a\,b$, of this fiber is $D'\,D - y\,\alpha = (r_n - y)\,d\,\phi$, and the displacement, $b\,c$, of its end is, $(r_n - y)\,d\,\phi\,\beta$. Hence, the work, for a gradually applied shear, on the fiber, $a\,b$, of cross-section, $d\,y \times 1$, is,

$$\frac{1}{2}\,d\,y\,\frac{S}{t}\,(r_n - y)\,d\,\phi\,\beta = \frac{1}{2}\,\frac{S^2}{F\,t^2}\,(r_n - y)\,d\,\phi\,d\,y$$

so that the work for the whole voussoir is,

$$\frac{d\,\phi}{2\,F\,t^2}\int_{-\left(\frac{t}{2}+c\right)}^{\left(\frac{t}{2}-c\right)} S^2\,(r_n-y)\,d\,y = \frac{d\,\phi}{2\,F\,t}\,S^2\,(r_n+c) = \frac{r\,d\,\phi\,S^2}{2\,F\,t}$$

If $F = \frac{E}{2.4}$, this becomes $\frac{2.4\,r}{2\,E\,t}\,S\;d\,\phi$, and six-fifths of this, $\frac{2.88\,r}{2\,F\,t}\,S^2\;d\;\phi$, corresponds to a parabolic distribution of the shear on $A\,B$.* Therefore, the elastic work due to shear for the half-arch $= \frac{2.88\,r}{2\,E\,t}\int_0^{\phi_1} S^2\,d\,\phi$.

The bending unit stress is given by Equation (93); the fiber at y (Fig. 28) has a length, $(r_n - y)\,\alpha$, and a cross-section, $d\,y \times 1$; hence the work of this fiber is,

$$\frac{1}{2}\,s\,d\,y\,\frac{(r_n-y)\,\alpha\,s}{E} = \frac{M^2}{2\,E\,{I_n}^2}\,\frac{{r_n}^2\,y^2}{(r_n-y)^2}\,(r_n-y)\,\alpha\,d\,y = \frac{{r_n}^2\,\alpha}{2\,E\,{I_n}^2}\,\frac{M^2\,y^2\,d\,y}{r_n-y}$$

By Equation (91),

$$\int_{-\left(\frac{t}{2}+c\right)}^{\left(\frac{t}{2}-c\right)} \frac{y^2\;d\;y}{r_n-y} = \frac{I_n}{r_n}$$

and, therefore, the elastic work of the voussoir of angle, $d\,\phi$, due to M (taken about D) is,

$$\frac{r_n}{2\,E\,I_n}\,M^2\;d\;\phi$$

Then, the work for the semi-arch of the central angle, $2\,\phi$, due to M is,

$$\frac{r_n}{2\,E\,I_n}\int_0^{\phi_1} M^2\;d\;\phi$$

Hence, the elastic work for the semi-arch due to M, P, and S is,

$$L = \frac{r_n}{2\,E\,I_n}\int_0^{\phi_1} M^2\,d\,\phi + \frac{r_n}{2\,E\,t}\int_0^{\phi_1} P^2\;d\;\phi + \frac{2.88\,r}{2\,E\,t}\int_0^{\phi_1} S^2\,d\,\phi \ldots (98)\dagger$$

Solution of the Arch.—Thrusts and Moments.—The arch to be considered is a horizontal circular arch of constant radial thickness, t, and a vertical height of 1 ft., fixed at the ends, and subjected on the extrados to a water pressure of p_e lb. per sq. ft. Generally, this is not the full water pressure at the level of the medial plane of the arch, but the part carried by the arch, the remainder being carried by the supposed cantilever.

In Fig. 31, representing the half-arch,

t = uniform radial thickness of arch, in feet;
r = radius of the center line, in feet;
r_n = radius of neutral line, in feet;
r_e = radius of extrados, in feet;
r_i = radius of intrados, in feet;
p_e = normal radial pressure, in pounds per square foot, on the extrados;
ϕ = angle with radius of crown for any point, D;
ϕ_1 = one-half the central angle = $A\,O\,C$.

* This is, of course, only approximate, and is only true when $r_n = r$.

† In this formula for L, only the work of the radial stress is omitted. Its influence is very small, as will appear subsequently.

The arc, $A\ D\ C$, is the neutral line, so that if s = the length of the arc, the distance, $C\ D = r_n\ \phi$, $d\ s = r_n\ d\ \phi$.

P_0 = thrust at the crown, acting at C, on the neutral line;
M_0 = moment at crown, taken positive when clockwise;
M, P, and S = respectively, the moment, normal component, and shear on the section through D $(r_n\ \phi)$;
$c = r - r_n$;
η = radial deflection at the crown; and
E = modulus of elasticity of the arch.

At the abutments, the components of the reaction parallel to and perpendicular to $O\ C$ are V and H and the reaction moment is M_1. For the semi-arch of Fig. 31, the reactions, V, H, M_1, at the left abutment, with P_0, M_0, at the crown, and the water load on the semi-arch, hold the latter in equilibrium, so that it can be treated as a free body.

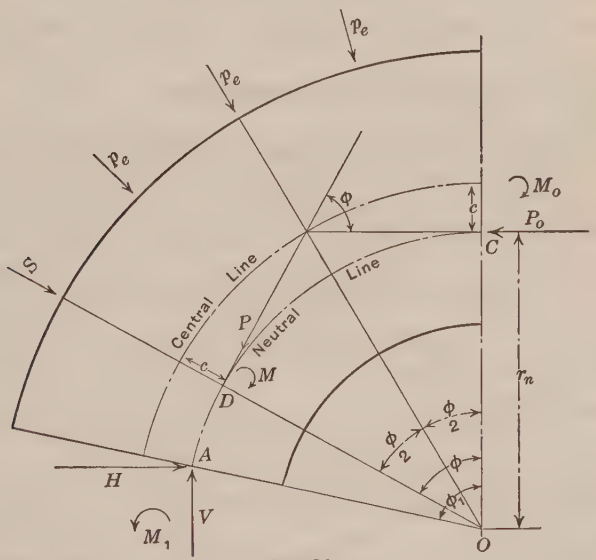

FIG. 31.

The analysis now proceeds exactly according to the method used by the writer in his paper on "The Circular Arch Under Normal Loads"*. The water pressure, however, is no longer referred to the center line of the arch ring, but to the extrados, and the cylinder formula is not used. Then, in the paper† cited, letting, $X = p_e r_e - P_0$,

$$\left.\begin{aligned} P &= p_e r_e - X \cos \phi \\ M &= M_0 + X r_n (1 - \cos \phi) \\ S &= X \sin \phi \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (99)$$

* *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), pp. 236–237.
† *Loc. cit.*, p. 236.

Taking partial derivations of these equations,

$$\frac{\delta M}{\delta M_0} = 1, \frac{\delta M}{\delta P_0} = -r_n (1 - \cos \phi); \frac{\delta P}{\delta M_0} = 0, \frac{\delta P}{\delta P_0} = \cos \phi;$$

$$\frac{\delta S}{\delta M_0} = 0; \frac{\delta S}{\delta P_0} = -\sin \phi$$

The shear at the crown being zero, it is necessary to determine only M_0, and P_0. The rotation of the crown section, being zero, $\frac{\delta L}{\delta M_0} = 0$; also, since the displacement of the crown section in the direction of P_0 is zero, $\frac{\delta L}{\delta P_0} = 0$.

By Equation (98), from $\frac{\delta L}{\delta M_0} = 0$,

$$\int_0^{\phi_1} M \frac{\delta M}{\delta M_0} d\phi = \int_0^{\phi_1} M d\phi = 0 \quad \ldots\ldots\ldots\ldots (100)$$

therefore,

$$\int_0^{\phi_1} [M_0 + X r_n (1 - \cos \phi)] d\phi = M_0 \phi_1 + X r_n (\phi_1 - \sin \phi_1) = 0 \ldots (101)$$

From, $\frac{\delta L}{\delta P_0} = 0$, L being given by Equation (98),

$$\frac{r_n}{I_n} \int_0^{\phi_1} M \frac{\delta M}{\delta P_0} d\phi + \frac{r_n}{t} \int_0^{\phi_1} P \frac{\delta P}{\delta P_0} d\phi + \frac{2.88\, r}{t} \int_0^{\phi_1} S \frac{\delta S}{\delta P_0} d\phi = 0$$

Multiplying by $\frac{I_n}{r_n}$ and putting $\frac{I_n}{t} = k^2$,

$$-r_n \int_0^{\phi_1} M (1 - \cos \phi) d\phi + k^2 \int_0^{\phi_1} P \cos \phi \, d\phi + \frac{2.88\, r}{r_n} k^2 \int_0^{\phi_1} S (-\sin \phi) d\phi = 0$$

By Equation (100),

$$\int_0^{\phi_1} M d\phi = 0$$

hence, the first term reduces to,

$$r_n \int_0^{\phi_1} M \cos \phi \, d\phi = r_n \int_0^{\phi_1} [M_0 + X r_n (1 - \cos \phi)] \cos \phi \, d\phi$$

$$= r_n \left[M_0 \sin \phi_1 + X r_n \left(\sin \phi_1 - \frac{\phi_1}{2} + \frac{1}{4} \sin 2\phi_1 \right) \right]$$

Likewise,

$$k^2 \int_0^{\phi_1} P \cos \phi \, d\phi = k^2 \int_0^{\phi_1} (p_e r_e - X \cos \phi) \cos \phi \, d\phi$$

$$= k^2 \left[p_e r_e \sin \phi_1 - X \left(\frac{\phi_1}{2} + \frac{1}{4} \sin 2\phi_1 \right) \right]$$

Finally, the term,

$$-2.88 \frac{r k^2}{r_n} \int_0^{\phi_1} S \sin \phi \, d\phi = -\frac{2.88}{2} \frac{r}{r_n} k^2 X \left(\phi_1 - \frac{1}{2} \sin 2\phi_1 \right)$$

Hence, on adding, the whole original equation reduces to,

$$r_n \left[M_0 \sin \phi_1 + X r_n \left(\sin \phi_1 - \frac{\phi_1}{2} + \frac{1}{4} \sin 2 \phi_1 \right) \right] + k^2 \left[p_e r_e \sin \phi_1 - X \left(\frac{\phi_1}{2} + \frac{1}{4} \sin 2 \phi_1 \right) \right] - \frac{2.88}{2} \frac{r}{r_n} k^2 X \left(\phi_1 - \frac{1}{2} \sin 2 \phi_1 \right) = 0 \quad \ldots\ldots(102)$$

To eliminate M_0, multiply Equation (102) by ϕ_1 and subtract it from ($r_n \sin \phi_1$) times Equation (101) and solve for X. Then,

$$X = p_e r_e - P_0 = \frac{p_e r_e}{D_n} 2 \sin \phi_1 \frac{k^2}{r_n^2} \quad \ldots\ldots(103)*$$

in which,

$$D_n = \left(\phi_1 + \frac{1}{2} \sin 2 \phi_1 \right) \left(1 + \frac{k^2}{r_n^2} \right) - \frac{1 - \cos 2 \phi_1}{\phi_1} + 2.88 \frac{r}{r_n} \frac{k^2}{r_n^2} \left(\phi_1 - \frac{1}{2} \sin 2 \phi_1 \right) \quad \ldots\ldots(104)$$

From Equation (101):

$$M_0 = - X r_n \left(1 - \frac{\sin \phi_1}{\phi_1} \right) \quad \ldots\ldots(105)$$

This moment at the crown is always negative or counterclockwise, giving tension at the intrados and compression at the extrados.

If this value of M_0 is substituted in Equation (99), the value of M at any point, (r_n, ϕ), is found, as follows:

$$M = + X r_n \left(\frac{\sin \phi_1}{\phi} - \cos \phi \right) \quad \ldots\ldots(106)$$

This moment for a point, D' (Fig. 32), is equal to $-X$ times the distance, $E' D'$, and for a point, D'', it equals $+X$ times the distance, $D'' E''$, if X is laid off as a force, acting to the right at E on $O C$, where $O E = r_n \frac{\sin \phi_1}{\phi_1}$.

The moment is zero at D_0, where the line of action of X crosses the neutral line; it is positive for points between D_0 and A and negative for points on the neutral axis between D_0 and C.

At the abutment, A, $\phi = \phi_1$ in Equation (106), and since the moment is positive or clockwise, it causes tension at the extrados of the abutment section and compression at the intrados.

---

* To compare with the author's Equation (30), the water pressure must be referred to the extrados so that in the formula for deflection of the full cylinder, $e = \frac{p r \cdot r}{E t}$, $p r$ must be replaced by $p_e r_e$. Then, as given by the author,

$$e' = e \frac{r_n}{r} = \frac{p_e r_e \cdot r_n}{E t}$$

which agrees with Equation (38b). Therefore, Equation (30), becomes,

$$X' = \frac{p_e r_e r_n \sin \phi_1}{E t \quad y'}$$

In Equation (28), r_n should be replaced by r, according to the usual theory. With this correction, the value of X' can be easily reduced to Equation (103). The numerical value of the term in which this error occurs is very small and the error is negligible. In fact, it is perhaps well to replace $\frac{r}{r_n}$ by unity in the last term of Equation (104). It is gratifying to have this complete check of the fundamental formula for X by a totally different method.

In Equations (103) and (104), $k^2 = \frac{I_n}{t}$. As already noted, I_n differs from I by less than 1% for most cases in practice. Assuming,

$$k^2 = \frac{I}{t} = \frac{1}{12} t^2$$

and, since by Equation (94),

$$r_n = \frac{t}{\log_e \left(\frac{r_e}{r_i}\right)}$$

therefore,

$$\frac{k^2}{r_n^2} = \frac{1}{12} \left(\log_e \frac{r_e}{r_i}\right)^2$$

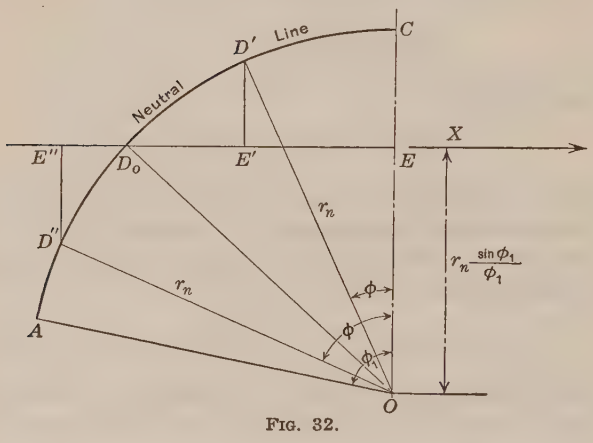

FIG. 32.

In numerical computations, it is well to use a seven-place logarithmic table for the first two terms of D_n, as their difference for small angles is minute.

Example.—The unit being the foot, let $t = 30$, $r = 100$, $2\,\phi_1 = 40$ degrees. Then,

$$r_e = 115; \quad r_i = 85; \quad \log_e \left(\frac{r_e}{r_i}\right) = \log_e 115 - \log_e 85 = 0.302281$$

$$r_n = \frac{30.000}{0.302281} = 99.2455; \quad c = 0.7545; \quad \frac{k^2}{r_n^2} = \frac{(0.3023)^2}{12} = 0.007615$$

$$\frac{r}{r_n} = 1.00760; \quad 2.88 \frac{r}{r_n} \frac{k^2}{r_n^2} = 0.022098$$

From Equations (103) and (104),

$$D_n = 0.6755655 - 0.6702333 + 0.0006115 = 0.0059437$$

$$X = \frac{115\, p_e \times 2 \times 0.007615 \sin 20°}{0.0059437} = 100.78\, p_e$$

$$P_0 = p_e r_e - X = (115 - 100.78)\, p_e = 14.22\, p_e$$

At the abutment,

$$P_1 = p_e r_e - X \cos \phi_1 = (115 - 100.78 \times \cos 20°)\, p_e = 20.29\, p_e.$$

From Equations (105) and (106),

$$M_0 = -100.78 \times 99.2455\ (1 - 0.979815)\, p_e = -202\, p_e,$$
$$M_1 = 100.78 \times 99.2455\ (0.979815 - 0.939693)\, p_e = 401.3\, p_e.$$

Unit Tangential Stresses.—The unit stresses at the extrados and the intrados, at the crown and the abutment, will now be computed. Tension will be taken as positive and compression as negative, except that p_e will always be treated as positive. This agrees with the author's conventions. If p_e is expressed in pounds per square foot, then the stresses will be in pounds per square foot, and it will be necessary to divide by 144 to reduce to pounds per square inch. This is equivalent to expressing p_e in pounds per square inch, and this will be supposed in the formulas, so that the total unit stresses given by the formulas will be in pounds per square inch.

The bending stresses are obtained from Equation (93) by putting $y = \frac{t}{2} - c$ for the intrados, $y = -\left(\frac{t}{2} + c\right)$ for the extrados, and assuming $I_n = I = \frac{1}{12} t^3$. They are represented by the first terms in the formulas, the signs being assigned by the conventions stated.

The stresses due to the component, P, normal to the section and acting where the neutral line crosses it, are given by Equation (97*a*), with the negative sign, since these stresses are all compressive.

Let s_e, s_i, equal the total unit stress on a fiber at the extrados and intrados, respectively.

At the crown,

$$\left.\begin{aligned} s_e &= \frac{M_0}{I} \frac{\frac{t}{2} + c}{r_e} r_n - \frac{P_0}{r_e \log_e \left(\frac{r_e}{r_i}\right)} \\ s_i &= -\frac{M_0}{I} \frac{\frac{t}{2} - c}{r_i} r_n - \frac{P_0}{r_i \log_e \left(\frac{r_e}{r_i}\right)} \end{aligned}\right\} \quad \text{(107)}$$

At the abutment,

$$\left.\begin{aligned} s_e &= \frac{M_1}{I} \frac{\frac{t}{2} + c}{r_e} r_n - \frac{P_1}{r_e \log_e \left(\frac{r_e}{r_i}\right)} \\ s_i &= -\frac{M_1}{I} \frac{\frac{t}{2} - c}{r_i} r_n - \frac{P_1}{r_i \log_e \left(\frac{r_e}{r_i}\right)} \end{aligned}\right\}$$

On substituting numerical values, always giving M_0 its negative sign:* At the crown,

$$s_e = (-1.221 - 0.409)\ p_e = -1.630\ p_e \text{ (compression)}$$
$$s_i = (+1.493 - 0.553)\ p_e = +0.940\ p_e \text{ (tension)}$$

At the abutment,

$$s_e = (+2.425 - 0.584)\ p_e = +1.841\ p_e \text{ (tension)}$$
$$s_i = (-2.967 - 0.789)\ p_e = -3.756\ p_e \text{ (compression)}$$

Since the author's value of X', as given by the Equation (30), with the negligible correction, can be reduced to the writer's Equation (103), the values of s_e and s_i, as computed by either method, should give identical results. Thus far, the check is complete.

Equations (107) can be put in another form. Thus, in Fig. 28, E being a point on the equilibrium polygon, suppose the two opposed forces, P, to be placed at C, the center of the section, in place of at D on the neutral axis. Then the single force, P, at C is the resultant of the uniform stress, $\frac{P}{t}$, acting on the section, and the couple gives the moment,

$M_c = P$ times the distance, $E\ C = P$ (distance, $E\ D +$ distance, $D\ C$)
$= M_D + P\,c$

in which, $M_D = M$ of Equations (107) equals the moment of all forces to the right of the section about D, a point on the neutral axis. The same formula holds when E is farther from O than D, the sign of M then being minus, but the sign of $P\,c$ is always plus.

The formulas for s_e and s_i, for any section the normal component of which is P and the moment about D, a point on the neutral axis, is M, become:

$$\left.\begin{aligned} s_e &= \frac{(M + P\,c)}{I}\,\frac{\frac{t}{2} + c}{r_e}\,r_n - \frac{P}{t} \\ s_i &= -\frac{(M + P\,c)}{I}\,\frac{\frac{t}{2} - c}{r_i}\,r_n - \frac{P}{t} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (108)$$

At the crown, $M = M_0$, $P = P_0$, and at the abutment, $M = M_1$, $P = P_1$. Thus, at the crown, $M = -202\ p_e$, $P_0 = 14.22\ p_e$, and $c = 0.7545$. Then, $(M + P\,c) = -191.28\ p_e$ and by Equations (108),

$$s_e = \left[-191.28 \times 0.0060426 - \frac{14.22}{30} \right] p_e = -1.630\ p_e$$

exactly as found previously; and, similarly, for other cases.

The effect of the uniform compression, $\frac{P}{t}$, represented by the ordinates of the rectangle, $eabf$, of Fig. 33, on the voussoir of the central angle, α, may now be examined.

* Here, $\frac{\frac{t}{2} + c}{Ir_e}\,r_n = 0.00604$; $\frac{\frac{t}{2} - c}{Ir_i}\,r_n = 0.00740$; $r_e \log_e \frac{r_e}{r_i} = 34.73$; $r_i \log_e \frac{r_e}{r_i} = 25.27$.

The forces, $\frac{P}{t}$, act on the face, $A\ B$, and rotate it to the position, $A'\ B'$, since any fiber, $I\ J$, the length of which is $r'\ \alpha$, is shortened by the distance, $I\ I' = \frac{P}{t}\ \frac{r'\ \alpha}{E}$, which varies with r' and, thus, $A'\ I'\ B'\ O$ is a straight line. The angle of rotation is, $\Delta\ \alpha = \frac{I\ I'}{r''} = \frac{P}{t}\ \frac{\alpha}{E}$.

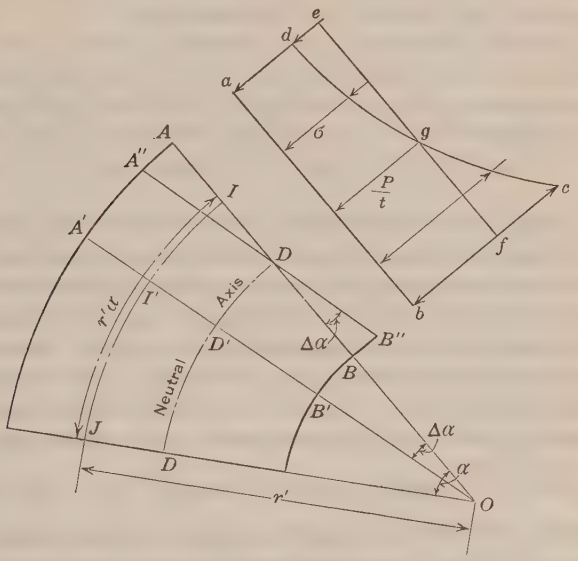

FIG. 33.

Now suppose the forces in the areas, *edg* and *gfc*, form a couple of moment, m, that causes $A\ B$ to rotate to $A''\ B''$ about D through the angle, $\Delta\ \alpha$; then the forces, σ, in the area, *gdabc*, cause the translation from $A''\ B''$ to $A'\ B'$ and are given by Equation (97*a*).

By Equation (92), $\Delta\ \alpha = \frac{m\ r_n\ \alpha}{E\ I_n}$, and this equals $\frac{P}{t}\ \frac{\alpha}{E}$. Therefore, $m = \frac{P\ I_n}{r_n t} = P\ c$, by Equation (96), or since the section, $A\ B$, turns counterclockwise, $m = -P\ c$. This moment combined with the moment, $M + P\ c$, of Equations (108), gives M, the moment about D, on the neutral axis, as the moment corresponding to the entire bending. Thus is proved this important principle in another way. Equations (108) are now transformed to Equations (107).

Suppose that the pressure grouting, which the author properly insists upon, to be properly applied at low temperatures to fill up any cracks, still the stresses as given by Equations (107) are not exactly realized. The vertical stress on a horizontal arch, due to the weight of the masonry and water vertically over it, has been ignored. The average vertical unit stress is designated σ_x by the

author, and this will affect results. Besides, the variation in E for large stresses, the flow of concrete under stress with time, the water-soaking of the upper face, and the probability that the water pressure on the arch (which equals the full water pressure minus that carried by the cantilever) is not constant from crown to abutment, tend to modify the results for all arches. For those horizontal arches near the foundation, its restraining influence will be so marked that exact computation of the stresses there seems rather a hopeless task. For the arches higher up, the results are more nearly correct, but even there, difficulties are met when the influence of σ_x is considered.

The Lamé Stresses.—Criticism.—To appreciate properly these difficulties, consider the author's first step in connection with Fig. 2, first assuming that $\sigma_x = 0$, where the semi-arch is supposed to be slid to the dotted position along the skewback a distance, $A\,A' = e$, under the influence of the water pressure acting on the arch as if it were a complete cylinder, free to move and not fixed at the ends.

In this first stage, the stresses induced are the Lamé stresses, both tangential and radial. In deducing these stresses it was assumed that radial sections remain plane under the stresses, so that the assumed sliding can be effected. However, it will conduce to simplicity if the Lamé tangential stresses are considered as decomposed into a uniform stress and a couple. Thus, if R equals the resultant of the Lamé stresses, acting a distance, l, from the center of the section, on adding the forces, $+R$ and $-R$, at this center, there results the single compressive force, R, at the center, and a couple which can be resolved into a uniform stress, $\frac{R}{t}$, and a couple of moment, Rl. The latter affects the bending only and will be necessarily included in the moment of X' about the point where the neutral axis cuts the section, since the writer has demonstrated that all bending corresponds to such moments.* The only stresses then to be considered in this sliding stage are the uniform stresses, $\frac{R}{t}$, for which radial sections remain radial after strain, so that the sliding can be effected. To find the value of R, write the Lamé unit stress in the form, $\frac{b}{x^2} + a$, in which, x = radius to the point on the section, and, $a = \frac{p_e\, r_e^{\,2}}{r_e^{\,2} - r_i^{\,2}}$, and $b = \frac{p_e\, r_e^{\,2}\, r_i^{\,2}}{r_e^{\,2} - r_i^{\,2}}$. Then, by a simple integration and reduction,

$$R = \int_{r_i}^{r_e} \left(\frac{b}{x^2} + a \right) d\,x = p_e\, r_e$$

With the corresponding value of the constant unit stress, $\left(-\frac{p_e\, r_e}{t} \right)$ on each section, the author's solution follows. The values of X' thus deduced can be reduced to the identical form, Equation (103), found by the "work method". The numerical solution of stresses is effected precisely as in the last example given.

* Hence, bending stresses due to this couple of moment, $R\,l$, should be neglected in finding the total tangential stresses.

The author writes for the moment about the center of any section, C, $M_c = X' \left(r_n \frac{\sin \phi_1}{\phi_1} - r \cos \phi \right)$, which involves an error that is only partly compensated by the use of the Lamé stresses. Thus, for the crown section $A\ B$ (Fig 34), the center of which is C and the neutral point, D, this becomes $M_c = -X'$ times the distance, $E\ C$, whereas, it has been proved that the true moment at C is,

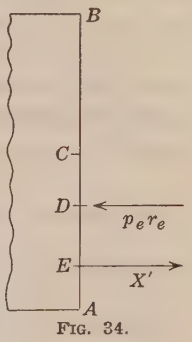

FIG. 34.

$M + Pc = -X'$ times the distance $E\ D + (p_e\ r_e - X')$ times the distance $D\ C$

$= -X'$ times the distance $E\ C + p_e\ r_e$ times the distance $D\ C$

which also follows from the forces shown in Fig. 34.

The moments are different and, although the introduction of the Lamé stresses tends to correct the error, there is nevertheless a difference in the total tangential stress at the extrados and intrados as computed by the author's and the writer's methods. Thus, for $t = 30$ ft., $r = 100$ ft., $2\phi_1 = 40°$, and $m = \infty$, at the crown, by the writer's Equations (108) it was found that $s_e = -1.63\ p_e$, and $s_i = +0.94\ p_e$; whereas by the author's formulas, $s_e = -1.73\ p_e$, and $s_i = +1.01\ p_e$.

For the same arch, only assuming $m = 8$, $\sigma_x = 4.32\ p_e$, the writer's Equations (108) gives, $s_e = -1.91\ p_e$ and $s_i = +0.04\ p_e$, which may be compared with the author's $s_e = -2.00\ p_e$ and $s_i = +0.12\ p_e$.

For the very thick arch, $t = 100$ ft., $r = 100$ ft., $m = \infty$, and $2\phi_1 = 40°$, the writer's Equations (107) gives at the crown, $s_e = -0.198\ p_e$, $s_i = -0.051\ p_e$ —both compression—whereas from the author's formulas, $s_e = -0.527\ p_e$ (compression) and $s_i = +0.408\ p_e$ (tension)!

The error by the author's formulas is too great. Part of it is due to the use of the Lamé stresses involving, as just shown, bending stresses which should be neglected, since the moment of X' takes care of all bending stresses. By the writer's method the Lamé stresses are not introduced at all and Equations (107) or Equations (108) will give accurately the tangential stresses. The examples solved will illustrate the procedure.

The writer regrets to have to criticize this excellent paper, but he is constrained to regard the author's formulas for tangential stress as approximate. Some idea of the amount of error can be gained from a consideration of the numerical results just given.

Influence of the Unit Circumferential Strain.—The true unit circumferential strain, ϵ', at the neutral axis for the full cylinder, under the water load, p_e lb. per sq. ft., is,

$$\varepsilon' = \frac{\sigma_t''}{E} - \frac{\sigma_r''}{E} - \frac{\sigma_x}{E}$$

in which, σ_t'', σ_r'', and σ_x, denote the tangential, radial, and vertical unit stresses, respectively, at the neutral axis. These stresses, being compressive,

are to be given the minus sign. By aid of Equations (36) and (37), the expressions for Equations (38) and (38*a*) follow immediately.

It will be observed that these formulas include the effect of the radial as well as of the vertical stress, omitted in the solution by the "work method", which thus corresponds to $m = \infty$, or Poisson's ratio equal to zero.

For $m = \infty$ and $\frac{t}{r} = 0.3$, Equation (38*a*) gives, $\epsilon' = -3.821 \frac{p_e}{E}$. Now, if in Equation (38*a*), after developing, the terms in $\frac{t^2}{r^2}$ and $\frac{t^4}{r^4}$ be omitted, and c be put equal to 0, the formula reduces to Equation (38*b*), or $\epsilon'' = -\frac{p_e r_e}{E t}$, which, for $\frac{t}{r} = 0.3$, gives, $\epsilon'' = 3.833 \frac{p_e}{E}$, or practically the same as ϵ'. Thus, when $m = \infty$, the relative thickness of the arch affects very little the strain, ϵ'. Note that this value of ϵ'' is exactly the same as that used previously in checking Equation (103).

The influence of m is more pronounced. Thus, when $\sigma_x = 0$, for $m = 8$, let ϵ' be derived from Equation (38*a*), and for $m = \infty$, let ϵ'' be computed from Equation (38 *b*), and let X' and X'' be the corresponding values of X. Then,

$$X' = -r_n \epsilon' \frac{\sin \phi_1}{y'}; \quad X'' = -r_n \epsilon'' \frac{\sin \phi_1}{y'}$$

or,

$$\frac{X''}{X'} = \frac{\epsilon''}{\epsilon'}$$

Thus, for,

$$\frac{t}{r} = 0, \frac{X''}{X'} = 1.000$$

$$\frac{t}{r} = 0.3, \frac{X''}{X'} = \frac{3.833}{3.747} = 1.023$$

$$\frac{t}{r} = 0.5, \frac{X''}{X'} = \frac{2.500}{2.399} = 1.042$$

Thus, X'' exceeds X' by only 2.3% for $\frac{t}{r} = 0.3$, and by 4.2% for $\frac{t}{r} = 0.5$. The values of X'' are exactly equal to those given by Equation (103), as derived by the "work method," and the excess of X'' over the exact value X', is almost entirely due to neglecting the elastic work of the radial stress in applying the work method. These numerical illustrations will give a good idea of the error involved which is generally negligible, so that Equation (103) can be regarded as a good practical formula for finding X when σ_x is ignored. The influence of σ_x is to lessen numerically ϵ' and thus to increase the percentages just found.

Influence of Vertical Load and of Poisson's Ratio.—Next, consider the case where σ_x is not zero and Poisson's ratio is introduced. The displacement for the free cylinder (Fig. 2), is $A A' = e' = -r_n \epsilon'$, in which, ϵ' is given by Equations (38) or (38*a*). The Lamé tangential stresses are as before, since their derivation, as given in textbooks, shows that they are independent of ϵ'; hence,

reasoning as before for the sliding stage, each section will sustain a uniform compression of $p_e r_e$ lb., the sum of the Lamé stresses. Thus,

$$e = A\,A' = -\,r_n\,\varepsilon';\ X' = \frac{e' \sin \phi_1}{y'}$$

If, now, bending stresses are computed from moments taken about points on the center line of the arch to get the total unit tangential stress, it is necessary, for any section, to add the uniform unit stress,

$$\frac{X' \cos \phi}{t} - \frac{p_e\,r_e}{t}$$

the first term being positive (tension) and the second, negative (compression).

Example.—The author, for an arch 400 ft. below the water surface, assumes that $\sigma_x = 300$ lb. per sq. in. and $p_e = 69.5$ lb. per sq. in. Then, $\sigma_x = 4.32\,p_e$. For the arch previously considered ($t = 30$ ft., $r = 100$ ft., $2\phi_1 = 40°$), assume that $m = 8$ and $\sigma_x = 4.32\,p_e$; and, for the present, let p_e, σ_x, and E be expressed in pounds per square foot. Then, Equation (38) gives,

$$E\,\epsilon' = -\,3.206\,p_e$$

and Equations (29a) and (30) give:

$$y' = \frac{r_n^3}{E\,k^2\,t}\,\frac{D_n}{2} = 0.0029718\,\frac{r_n^3}{E\,k^2\,t}$$

and,

$$X' = \frac{-\,r_n\,\varepsilon' \sin \phi_1}{y'} = \frac{3.206 \sin \phi_1}{0.0029718}\,\frac{k^2}{r_n^2}\,t\,p_e = 84.29\,p_e$$

As the writer's Equations (107) and (108) will be used in what follows, P_0 and P_1 will be given the plus sign. Therefore,

$$P_0 = p_e r_e - X' = (115 - 84.29)\,p_e = 30.71\,p_e$$

$$P_1 = p_e r_e - X' \cos \phi_1 = 35.97\,p_e$$

$$M_0 = -\,X'\,r_n\left(1 - \frac{\sin \phi_1}{\phi_1}\right) = -\,168.94\,p_e$$

$$M_1 = +\,X'\,r_n\left(\frac{\sin \phi_1}{\phi_1} - \cos \phi_1\right) = +\,335.81\,p_e$$

Utilizing certain numerical values of the last example, by Equations (107): At the crown,

$$s_e = 0.00604\,M_0 - \frac{P_0}{34.73} = -\,1.91\,p_e \text{ (compression)}$$

$$s_i = -\,0.0074\,M_0 - \frac{P_0}{25.27} = +\,0.04\,p_e \text{ (tension)}$$

At the springing line:

$$s_e = 0.00604\,M_1 - \frac{P_1}{34.73} = +\,0.99\,p_e \text{ (tension)}$$

$$s_i = -\,0.0074\,M_1 - \frac{P_1}{25.27} = -\,3.90\,p_e \text{ (compression)}$$

To check these values, use Equations (108), writing $c = 0.754$ ft.: At the crown,

$$M_0 + P_0 c = -145.8\ p_e,$$

At the springing line,

$$M_1 + P_1 c = +363.0\ p_e$$

the moments being taken about the center of the sections. By substituting in Equation (108), the same values as before are derived.

Summary of Numerical Results.—The results are in pounds per square foot. On dividing by 144 the stress will be given in pounds per square inch. This amounts to expressing p_e in pounds per square inch in the final results. A summary of the numerical results are given in Table 10.

Influence of Shear on Tangential Stresses.—The influence of shear is not well understood, and in dealing with it, certain inconsistencies appear. It is known that, ordinarily, shear changes a plane section to a warped surface, and the amount of the departure affects the computation of the tangential stress due to shear. However, at the crown, from considerations of symmetry, the section must remain plane; similarly at the abutment the section, being fixed by assumption, must remain plane. At these two sections the usual tangential stresses due to shear do not seem to be realized. Following the general lines of the analysis of J. P. J. Williams, M. Am. Soc. C. E.,* a rather cumbersome formula has been derived for the change in tangential stress for the arch due to shear. This may apply to intermediate sections, not too near the crown or the abutment, but it is not given because it cannot apply to the sections at the crown or abutment—the very ones to be investigated. The same reasoning applies to the center of a beam uniformly loaded, where the section must remain plane after the application of the load.

TABLE 10.—Comparison of Unit Stresses as Factors of p_e

Assumptions.	Crown Section.		Springing Line.	
	Extrados.	Intrados.	Extrados.	Intrados.
$\sigma_x = 0$; $m = \infty$; r put for r_n as for a thin arch*	−1.45	+1.56	+2.44	−3.71
σ_x not included; $m = \infty$	−1.63	+0.94	+1.84	−3.76
$\sigma_x = 4.32\ p_e$; $m = 8$	−1.91	+0.04	+0.99	−3.90

* The value of X was found by aid of the writer's Table 6 in "The Circular Arch Under Normal Loads," *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 266, shear being disregarded. The unit stresses are those given by Equations (107).

If the distribution of the shear stress along the sections at the crown and abutment is constant instead of varying by the usual parabolic law, the sections remain plane and there are no tangential stresses; but of course this would modify the distribution on other sections. The whole subject evidently demands much more study than has been given to it.

Radial Deflection at the Neutral Axis at the Crown.—Fixed Ends.—The formula for deflection is derived exactly as given in the writer's previous

* *Transactions*, Am. Soc. C. E., Vol. LXXV (1912), pp. 937–939.

paper,* using Equation (98) for L, the elastic work of deformation, and Equation (99) for the values of M, P, and S. Thus, the value of $\frac{E\,I_n}{r_n}\,\eta$ is derived in terms of X and then Equation (103) is used for the value of X, reducing to the common denominator, D_n, of Equation (104). As before, the factor, $(1 - \cos\,\phi_1)$, can be taken out and $\left(2 \sin \frac{\phi_1}{r_n^2}\right)$ inserted in the bracketed term; then by use of the formulas, $\sin 2\,\phi_1 = 2 \sin \phi_1 \cos \phi_1$ and $2 \sin^2 \phi_1 = 1 - \cos 2\,\phi_1$, a material reduction can be effected, giving,

$$\frac{E\,I_n}{r_n}\,\eta = \frac{p_e\,r_e\,k^2\,(1 - \cos\,\phi_1)}{D_n}\left[(\phi_1 - \sin\,\phi_1)\left(1 + \frac{k^2}{r_n^2}\right) + 2.88\,\frac{r}{r_n}\,\frac{k^2}{r_n^2}\,(\phi_1 + \sin\,\phi_1)\right]$$

On multiplying by $\frac{r_n}{E\,I_n} = \frac{r_n}{E\,t\,k^2}$,

$$\eta = \frac{p_e\,r_e\,r_n}{E\,t}\,\frac{(1 - \cos\,\phi_1)\left[(\phi_1 - \sin\,\phi_1)\left(1 + \frac{k^2}{r_n^2}\right) + 2.88\,\frac{r}{r_n}\,\frac{k^2}{r_n^2}\,(\phi_1 + \sin 2\,\phi_1)\right]}{D_n} \quad \ldots(109)$$

or,

$$\eta = c_n\,\frac{p_e\,r_e\,r_n}{E\,t}$$

(in which, c_n is the value of the last fraction in Equation (109)).

$$D_n = (\phi_1 + \frac{1}{2}\sin 2\,\phi_1)\left(1 + \frac{k^2}{r_n^2}\right) - \frac{1 - \cos 2\,\phi_1}{\phi_1} + 2.88\,\frac{r}{r_n}\,\frac{k^2}{r_n^2}\left(\phi_1 - \frac{1}{2}\sin 2\,\phi_1\right)$$

$$r_n = \frac{t}{\log_e\left(\frac{r_e}{r_i}\right)}$$

and,

$$\frac{k^2}{r_n^2} = \frac{t^2}{12\,r_n^2} = \frac{1}{12}\left(\log_e \frac{r}{r_i}\right)^2$$

For small values of ϕ_1, seven or eight-place logarithmic tables must be used for determining the first two terms of D_n.

For comparatively thin arches, on putting $r_n = r$, the formula reduces to that previously derived.† In the formula for η, both p_e and E must be expressed in the same unit, pound per square foot, or pound per square inch, in which case η is expressed in feet, since $\frac{p_e}{E}$ is simply a ratio.

The influence of shear seems to be very large for thick arches. Thus, for $t = 30$, $r = 100$, and $\phi_1 = 20°$, Equation (109) gives, with shear included,

* *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), pp. 240 and 270.

† *Loc. cit.*, p. 270.

$E\,\eta = 0.751\ p_e\,r_e$. Omitting the terms containing 2.88 as a factor (that is, the influence of shear) $E\,\eta = 0.266\ p_e\,r_e$. Therefore the deflection including shear is 2.82 times the deflection when the influence of shear is omitted.

The author's Equation (45) can be reduced to Equation (109), when $m = \infty$ and ε' is replaced by $-\frac{p_e\,r_e}{E\,t}$. When m is finite and σ_x becomes a factor, the author's formula gives a ready solution.

Thrusts and Moments Due to Temperature Changes.—Let e equal the expansion per foot for a rise of 1° Fahr.; and t_0 equal the number of degrees of change in temperature.

For a free cylinder, the span of the neutral line will be increased $(e\,t_0\ 2\ r_n \sin\phi_1)$ ft. Now regard the arch fixed at the right end and free to move at the left end, where H, the horizontal force, is just sufficient to bring this end back to its original position. Fig. 31 can be used to illustrate the conditions if $p_e = 0$, $V = 0$, and $P_0 = H$, so that the half-arch is in equilibrium under the forces, H, at A and C, the couples of moments, M_1 and M_0, at A and C, being just sufficient to prevent rotation of the sections. If L is the elastic work for the entire arch (twice the value given by Equation (98)), the horizontal displacement of the left end toward the right, is,

$$\frac{\delta\,L}{\delta\,H} = e\,t_0\ 2\ r_n \sin\phi_1$$

Also, since there is no rotation of the crown section, $\frac{\delta\,L}{\delta\,M_0} = 0$. Then,

$$M = M_0 - H\,r_n\,(1 - \cos\phi);\ P = H\cos\phi;\ S = -H\sin\phi;$$
$$\frac{\delta\,M}{\delta\,M_0} = 1;\ \frac{\delta\,M}{\delta\,H} = r_n\,(\cos\phi - 1);\ \frac{\delta\,P}{\delta\,M_0} = 0;$$
$$\frac{\delta\,P}{\delta\,H} = \cos\phi;\ \frac{\delta\,S}{\delta\,M_0} = 0;\ \frac{\delta\,S}{\delta\,H} = -\sin\phi.$$

Differentiating under the integral sign as before, there is derived from the condition, $\frac{\delta\,L}{\delta\,M_0} = 0$, the relation,

$$M_0 = H\,r_n\left(1 - \frac{\sin\phi_1}{\phi_1}\right)$$

and from the condition, $\frac{\delta\,L}{\delta\,H} = 2\ e\ t_0\ r_n \sin\phi_1$, after reduction, the relations,

$$H = \frac{2\sin\phi_1}{D_n}\left(E\,e\,t_0\frac{k^2}{r_n^{\,2}}\,t\right)$$

and,

$$D_n = \left(\phi_1 + \frac{1}{2}\sin 2\ \phi_1\right)\left(1 + \frac{k^2}{r_n^{\,2}}\right) - \frac{1 - \cos 2\ \phi_1}{\phi_1}$$
$$+ 2.88\,\frac{r}{r_n}\,\frac{k^2}{r_n^{\,2}}\left(\phi_1 - \frac{1}{2}\sin 2\ \phi_1\right)$$

These formulas for H and M_0 are for a free arch with a rise of temperature of t_0 degrees Fahrenheit above an assumed mean, the arch not being restrained by the cantilever action. The latter will be considered subsequently. Of

course, for a fall of t_0 degrees of temperature, t_0 is negative and the direction of H is reversed.

Radial Deflection at Crown Under a Rise of Temperature of t_0 Degrees.— For a free cylinder the span of the neutral line will be increased $2\,e\,t_0\,r_n \sin\phi$, and its rise, $h = r_n\,(1 - \cos\phi_1)$, will be increased $e\,t_0\,r_n\,(1 - \cos\phi_1) = h\,e\,t_0$. In this stage no work is done. Now, for this expanded arch, regard the right end as fixed and the left end as free and as subjected to the forces, H and M_1, just derived, which bring the section back to its original position. By the work method, the corresponding value of the crown deflection, η', is found.

As deflection toward the center is positive, the total radial deflection of the crown is,

$$\eta_t = \eta' - h\,e\,t_0$$

To find η', since $P = H\cos\phi$, and $S = -H\sin\phi$

$$M = M_0 - H\,r_n\,(1 - \cos\phi) = H\,r_n\left(\cos\phi - \frac{\sin\phi_1}{\phi_1}\right)$$

The solution proceeds now exactly as in the writer's previous paper* and leads to the formula,

$$\eta' = \frac{2\sin\phi_1}{D_n}\,h\,e\,t_0\left[\frac{1}{2}\,(1+\cos\phi_1)\left\{1 + \frac{k^2}{r_n^2}\left(1 - 2.88\,\frac{r}{r_n}\right)\right\} - \frac{\sin\phi_1}{\phi_1}\right]$$

Therefore, the full deflection reduces to,

$$\eta_t = \eta' - h\,e\,t_0 = -c_n\,(r_n\,e\,t_0)$$

in which, c_n is the coefficient of $\dfrac{p_e\,r_e\,r_n}{E\,t}$ in Equation (109).†

Hitherto, the horizontal arch fixed at the ends, under the change of temperature has been considered as free, or unrestrained by the dam acting as a

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* *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), pp. 240 and 270.

† This result can be obtained in another way. When a cylinder is subjected to the rise of temperature, the unit elongation along or parallel to its medial line is $e\,t_0$. Assume that the expansion is due to a tangential tension, $t.Eet_0$, on each cross-section and that the arch is brought back to its original position at the left abutment by the forces, H and M, acting there. The tangential component at (r_n, ϕ), the moment, and the shear are:

$$P_t = H\cos\phi - t\,E\,e\,t_0 = \frac{2\sin\phi_1}{D_n}\,E\,e\,t_0\,\frac{k^2\,t}{r_n^2}\cos\phi - t\,E\,e\,t_0$$

$$M_t = \frac{2\sin\phi_1}{D_n}\,E\,e\,t_0\,\frac{k^2 t}{r_n}\left(\cos\phi - \frac{\sin\phi_1}{\phi_1}\right)$$

and,

$$S_t = -\frac{2\sin\phi_1}{D_n}\,E\,e\,t_0\,\frac{k^2\,t}{r_n^2}\sin\phi$$

For an arch subjected to normal loads, p_e, it has been found that,

$$P = -\left(\frac{2\sin\phi_1}{D_n}\,p_e\,r_e\,\frac{k^2}{r_n^2}\cos\phi - p_e\,r_e\right)$$

$$M = -\frac{2\sin\phi_1}{D_n}\,p_e\,r_e\,\frac{k^2}{r_n}\left(\cos\phi - \frac{\sin\phi_1}{\phi_1}\right)$$

and,

$$S = \frac{2\sin\phi_1}{D_n}\,p_e\,r_e\,\frac{k^2}{r_n^2}\sin\phi$$

Observe that,

$$\frac{P_t}{P} = \frac{M_t}{M} = \frac{S_t}{S} = -\frac{t\,E\,e\,t_0}{p_e\,r_e}$$

hence, the crown deflections must be in the same ratio, or

$$\frac{\eta_t}{\eta} = -\frac{E\,t\,(e\,t_0)}{p_e\,r_e}$$

therefore,

$$\eta_t = -c_n\,\frac{p_e\,r_e\,r_n}{E\,t}\cdot\frac{E\,t\,(e\,t_0)}{p_e\,r_e} = -c_n\,(r_n\,e\,t_0)$$

cantilever. Necessarily, this constraint must be considered. When this is done, the problem offers difficulties. The writer has given* a solution after the hypothesis of B. A. Smith,† M. Am. Soc. C. E., in which the dam with the up-stream face vertical, under a rise of temperature of t_0 degrees, was assumed (1) to have a "cylinder expansion" on a smooth base, with a radial up-stream displacement of the neutral line equal to $(-r_n\ e\ t_0)$; and then (2) if p_e' is the unit normal force exerted at the extrados by the dam acting as a cantilever, assumed constant for any one horizontal arch, the corresponding down-stream displacement of the arch, η, is $c_n \frac{p_e' r_e r_n}{E\ t}$; hence, the resulting displacement of the crown section is,

$$r_n \left(-\ e\ t_0 + c_n \frac{p_e' r_e}{E\ t} \right)$$

The advantage of assuming a cylinder expansion in Assumption (1), is that it ensures a constant p_e' for any one arch. The disadvantage is that the forces, p_e', cause the same displacement all around, and thus the dam is not vertical at the abutments, as it should be. The solution cannot be regarded as exact.

Another approximate solution will now be given, in which the cylinder expansion is not used in the first stage. In Fig. 35, let $A\ D\ B$ represent the plan of the neutral line of a free arch fixed at the ends, which, under the temperature rise, takes the position, $A\ C\ B$. The up-stream displacement of the free arch, as just shown, is, $\eta_t = D\ C = -\ c_n\ (r_n\ e\ t)$, and the dam at the abutments remains vertical. However, in considering the action of the dam as a cantilever, its forces, p_e', can no longer be regarded as constant for the same arch. They probably decrease in going from the crown to the abutments. Experiment will have to determine this matter, not only for this case, but for the general solution of an arch under water pressure alone.

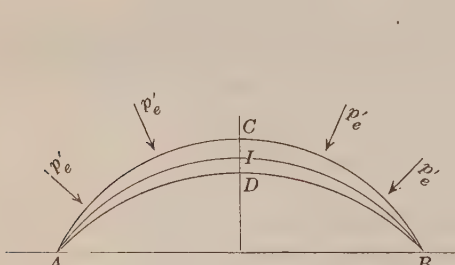


FIG. 35.

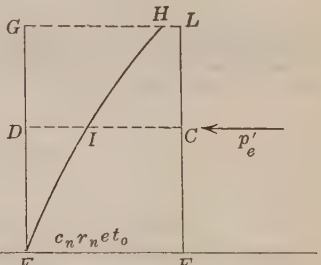


FIG. 36.

To effect a solution in the present state of knowledge, it is necessary to assume p_e' as constant; then,

$$\eta = I\ C = c_n \frac{p_e'\ r_e\ r_n}{E\ t}$$

Therefore,

$$D\ I = c_n\ r_n \left(-\ e\ t_0 + \frac{p_e'\ r_e}{e\ t} \right)$$

* *Transactions*, Am. Soc. C. E., Vol. LXXXIV (1921), pp. 80–81.

† "Arched Dams," *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–20), p. 2027.

in which, deflection up stream is negative, and down stream, positive. The value of p_e' for the level considered, being the pull or resistance of the cantilever to the expansion of the arch, is supposed to be known from the solution of the arch.

The movements indicated in this last solution are clearly shown in Fig. 36, representing a vertical radial section at the crown, in which, $F\,D$ represents the original position of a vertical line through D (Fig. 35), which is moved to $E\,L$ under the expansion of the free arch, a distance, $D\,C = F\,E = -\,c_n\,r_n\,e\,t_0$. The cantilever forces, p_e' acting on an arch at the level, C, force C back a distance, $I\,C = c_n\,\frac{p_e'\,r_e\,r_n}{E\,t}$, so that $D\,I$ is the actual deflection reckoned from $F\,G$. The elastic curve is $F\,I\,H$.

If Fig. 36 is revolved 180° about $F\,G$, it can represent the displacements due to a fall of temperature and shrinkage, e' per unit of length, the cantilever resistance now acting away from the center. The resulting deflection down stream is,

$$D\,I = c_n\,r_n\left[-\,e\,t_0 + e' - \frac{p_e'\,r_e}{E\,t}\right],$$

in which, t_0 is negative and $-\,e\,t_0$, positive.

As none of these formulas can be regarded as exact, it is hoped that experiment will come to the rescue and aid in producing a practically exact formula. To consider the case for combined water pressure, shrinkage, and temperature rise, suppose the solution of the arch dam gives, for an arch at a particular level,

p_e = normal pressure on the arch, in pounds per square foot, due to water; and

p_e' = normal pull of the cantilever or its resistance to the temperature expansion.

Then, adding the displacement down stream due to the water to the displacement due to the temperature rise, the total radial displacement at the crown will be:

$$D = c_n\,r_n\left[-\,e\,t_0 + e' + \frac{(p_e + p_e')\,r_e}{E\,t}\right]$$

This is down stream when positive and up stream when negative. For a fall of temperature, shrinkage, and water pressure, the formula applies on taking t_0 and p_e' as negative. The formulas, of course, are only valid where no cracks are formed, either from the shrinkage or from the exertion of too much tension.

If precise observation at the crown of an arch gives the total deflection, D, the solution of the last equation for $(p_e + p_e')\,r_e$ gives the total thrust. The Smith hypotheses (which may prove the better) lead to the formulas previously given by the writer.* The moment at the crown can be expressed in terms of the deflection, as follows:

* *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 277.

$$M_0 = -\,a\,\frac{E\,t^3}{r_n\,h}\,\eta$$

in which,

$$a = \frac{1}{6}\,\frac{\sin\phi_1\left(1 - \dfrac{\sin\phi_1}{\phi_1}\right)}{(\phi_1 - \sin\phi_1)\left(1 + \dfrac{k^2}{{r_n}^2}\right) + 2.88\,\dfrac{r}{r_n}\,\dfrac{k^2}{{r_n}^2}\,(\phi_1 + \sin 2\,\phi_1)}$$

It is possible for η to represent the deflection due to the water pressure alone, or to represent η_t, the deflection due to a change of temperature for a "free arch" and not for one the expansion (or contraction) of which is resisted by the arch acting as a cantilever. Experiments should be made to determine the coefficients in the formulas referring to temperature changes on the dam with the reservoir empty.

Then, if D_t = the observed crown deflection,

$$\eta_t = D_t - c_n\,r_n\,\frac{p'\,r_e}{E\,t}$$

As η_t varies directly with t_0, it can be found for any t_0 different from that of the observation.

Next, with the reservoir full, if possible, observe the deflection, η, due to water pressure only when there is no change of temperature. If this is not feasible, let D be the observed crown deflection for combined water pressure and t_0 degrees change of temperature; then the formula for D can be written as,

$$D = \eta_t + \frac{p_e + p_e'}{p_e}\,\eta$$

whence,

$$\eta = \frac{p_e}{p_e + p_e'}\,(D - \eta_t)$$

and, finally,

$$M_0 = -\,a\,\frac{E_t^3}{r_n\,h}\,(\eta_t + \eta)$$

The author's notes on the "Secondary Arch" (page 510) are particularly full and useful. The approximate solution of arches of varying section appears to be as brief as the subject permits. Castigliano's method, utilizing Simpson's formula, might be adapted to the case, but possibly the solution would be longer.

The writer is unable to agree with the author that as $2\,\phi_1 \doteq 0$, the intrados at the crown can ever suffer compression, since the arch then reduces to a straight beam, fixed at the ends and loaded uniformly, under which condition the center on the intrados side always experiences tension.

With regard to the conservation of plane sections of a beam, column, or arch, to stress-strain curves, and to the influence of shear in warping plane sections, George F. Swain, Past-President, Am. Soc. C. E., has thrown* a flood of light on a subject not well understood.

The author has written very convincingly of the serious effect of water-soaking in an arch dam. There can be no doubt of its influence, of the deformation of the foundation, and of the variation in E. He seems well aware that an exact solution of the arch dam is not to be looked for, so that all an engineer can do is to examine the various influences and combine them to effect a practical solution. In that direction Mr. Jakobsen has done much.

Elmo G. Harris,† M. Am. Soc. C. E. (by letter).—The writer has laboriously followed the several mathematical papers, and has read with interest —also some profit and satisfaction—the many discussions that have been published in recent years by the Society relating to arched dams. He has found the subject fascinating on account of the mixed mechanical principles involved in it, and of its importance economically. He has given this type of structure considerable study, together with other styles of reinforced concrete dams. These studies were published in May, 1925.‡ The following is in substance a part of that study modified and expanded.

Several of the mathematical analyses alluded to are based on the assumption that one of the major mechanical principles involved is cantilever action in any small vertical prism of the dam. The writer does not concede that such cantilever action is of sufficient importance to justify the intricate analyses, but believes that conclusions thus reached may be quite misleading and that, instead of cantilever action, vertical-beam action§ is a major mechanical principle active in carrying and distributing the loads on an arch dam. This conclusion results from the following considerations.

In Figs. 37, 38, and 39 are shown various types of arch dams. Fig. 37 gives three cross-sections; Fig. 38 represents a dam designed by the cylinder method, with no surplus material, whereas Fig. 39 shows a dam also designed as a cylinder, but with surplus material at the top. Referring to Fig. 37, let (a), (b), and (c) represent vertical cross-sections of the wall of a dam. The question as to whether or not the designer should consider cantilever action as a major factor will depend on whether or not the foundation, C, can be made so rigid that the vertical tangent to the face of the dam at B can be assumed as unmoved when the dam is loaded and all the stresses and strains are in action. It is the writer's opinion that it is not necessary for safety, and certainly not economical, to design the foundation with this intent. The designer will usually find no difficulty in obtaining a safe bearing stress under the foundation. The more exacting condition will be that of providing an effective cut-off wall below B D. In this part of the study it should be noted that any unequal bearing on the foundation will cause a movement at A inde-

* "Strength of Materials," pp. 234–242.

† Prof. of Civ. Eng., Missouri School of Mines, Rolla, Mo.

‡ "A Study of Reinforced Concrete Dams," Missouri School of Mines *Bulletin*, Vol. 8, No. 3.

§ Analysis of vertical-beam dams constitutes part of the study alluded to previously.

pendent of the elastic deflection due to stresses above B D. Evidently, the more the cantilever action with consequent varying intensity of stress on the foundation, the more will be this independent movement at A. It is conceivable that a foundation could be made safe in bearing and safe against leakage and left in the condition shown in Fig. 37 (c), yet there would be but little rigidity in such a design. The point is that an infinitesimal movement of the tangent at B would render vain any attempt at calculating the proportion of the load carried by the cantilever and by the arch. Doubtless some movement of the tangent at B will occur under loading in the majority of dams.

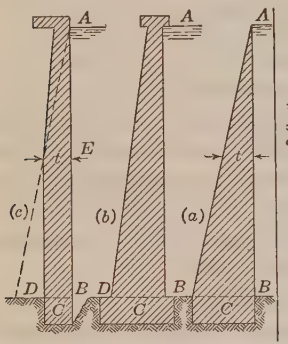


Fig. 37.

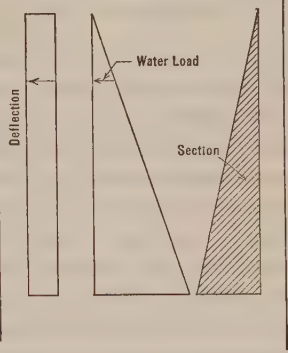


Fig. 38.

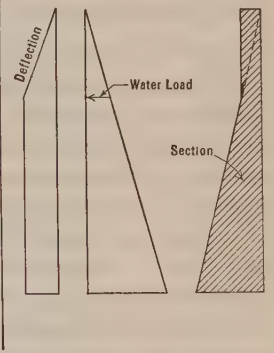


Fig. 39.

The cylinder formula would indicate a zero thickness or no stress at the water surface in an arched dam. If pure cantilever action occurs there must be a movement down stream at A. This down-stream movement would be resisted by the arch. Inevitably pure cantilever action cannot extend to the top of an arch dam.

At the top the vertical beam rests against a more or less elastic support. The more surplus materal there is at the top of the dam (as measured by the cylinder formula) the more rigid will be the support at the top end of the vertical beam and the farther down from the top will be the point of contraflexure of the beam considered as a cantilever.

Taking up now the question as to which is the more logical, the cantilever or the vertical beam, or which action is probably the more dominant in dams as built, or which should be developed by the design. Suppose Sections (a) and (c) (Fig. 37), be taken from straight walls. For the cylinder formula (in which T, the axial stress, is equal to the product of the radius, r, by the unit pressure, p), r would be infinite and, therefore, T would also be infinite. Evidently the cylinder formula fails to apply and in both cases simple beam action must carry all the load. In Section (a) the action would be purely that of a cantilever even if some movement of the tangent at B should occur. In Section (c), if the top girder be supported at intervals by piers or abutments and the beams, A B (making up the wall), simply rest against the girder at the top and against the cut-off wall at D, the reactions will be, at A, one-third the total pressure between A and B and, at D, two-thirds. As such dams would be built the vertical beams (curtain wall, A B) would be monolithic

with the girder at the top and with the cut-off wall or foundation, *C*, at the base. Whatever rigidity or fixedness of the beam may thus be obtained at the top and bottom will reduce the bending moment near the center, and therefore (theoretically) the beams could be made lighter the more nearly fixed they are at the ends.

Now assume that both dams, (*a*) and (*c*) (Fig. 37), are built on a circular arc in plan, thus becoming arched dams. It has been shown that Case (*a*) cannot then be under pure cantilever action in combination with arch action; but in Case (*c*) vertical beam action must continue although the load now carried by the beam will be reduced in amount and changed in relative intensity at all points due to the introduction of arch action.

That the beam must carry some of the load is evident by considering the fact that the arch stresses causing arch deflection are at right angles to the plane in which the arch is bent (plane of the paper in Fig. 40 and Fig. 42). The beam could not be held in this bent condition except by loads in the plane of the paper. Of course, if vertical beam action is to be utilized, the wall must be reinforced accordingly.

A possible fallacy exists as to the limit in the length of radius for an arched dam. Without exception all studies on the subject that the writer has followed fix, by the cylinder formula alone, the maximum radius that may be economically used. With a conservative permissible average unit stress this maximum radius is usually put at 300 or 400 ft. The fallacy is in failing to take account of vertical beam action which will be present in arched dams as usually proportioned, and also in failing to utilize this additional recourse available to the engineer if he will modify his design with the purpose of bringing into action and economically utilizing vertical beam action. To illustrate: In the writer's studies* it was shown that for straight vertical-beam dams the appropriate thickness at mid-height is $0.018\ h^{\frac{3}{2}}$ (h being the total height) and that that thickness may be made constant from mid-height down to the base. For a beam 50 ft. high this thickness would be 6.3 ft. By the cylinder formula, assuming about 50 000 lb. per sq. ft. as the average horizontal thrust in the arch, the radius that would require such a thickness at 25-ft. depth would be 200 ft. and for the same thickness at 50-ft. depth the radius would be 100 ft. Yet the section designed as a vertical beam would be safe with an infinite radius (straight wall). Evidently any degree of curvature will add strength instead of reducing it. A rational design of arched dams will not be accomplished until vertical beam action is recognized and utilized.

From an economic point of view the combination of vertical beam and of arch actions is not as promising as might seem from the foregoing, for, to carry the top reaction of the vertical beams, a horizontal arch rib must be placed along the top of the dam (replacing the top girder of the straight vertical-beam dam). For long radii this arch rib will assume large proportions.

The top reaction of the vertical beams in a combination vertical-beam and arch dam depends on several factors, among which are:

* Missouri School of Mines *Bulletin*, Vol. 8, No. 3, p. 1.

(*a*) The rigidity of the base or foundation (Fig. 37 (*c*)). The more rigid the base the less will be the top reaction.

(*b*) The radius of curvature—everything else holding constant, the flatter the arc the greater will be the top reaction.

(*c*) The lateral location of the beam between the crown and the abutment. It will probably be greatest for a vertical beam at the crown and will gradually reduce to zero at the abutment.

(*d*) The rigidity of the arch rib against deflection radially inward. This depends on the radius and the unit stress or area; the more rigid the rib, as compared to the wall below, the greater will be the top reaction.

From these considerations it is evident that a satisfactory determination of the reactions will be difficult and may be impossible.

Since the load on the top arch rib is not constant the cylinder formula cannot be applied to that rib. All the thrust going through the top arch rib is so much relief to the arch wall below.

One important matter that seems to be generally ignored (intentionally or otherwise) is the large portion of the water load on an arched dam that goes into the base or foundation without affecting the arch. This is due to fixedness of the base against horizontal movement inward or down stream. Consider a horizontal slice, say, 1 ft. vertically, adjacent to and above the base. If the cylinder formula applies to this slice it would be under compression in the direction of the axis of the arch; the arch would be shortened and would slide (deflect) inward a determinate distance. This motion is prevented because the slice is rigidly fixed at the base. It may then be fairly assumed that all the water pressure on the bottom foot of the arch goes into the base as pure punching shear. Likewise, the next horizontal slice above will be fixed to the slice below but some deflection will develop; and this same relation will increase as points are taken progressively up the face. This deflection will be the sum of the effects due to pure shear, vertical-beam action, and arch action. Evidently, the water pressure at any point (as at *P*, Fig. 40 and Fig. 42) will likewise be divided, Part *s* going down into the base without affecting the beam or arch, Part *b* going into the vertical beam, and the remainder, Part *a*, going into the arch.

If any one has difficulty in understanding the difference between the shear, *s*, and that in a simple vertical beam it can be made clear by imagining two adjacent horizontal slices of the arch (say, 1 ft. thick) as at *J*, Fig. 40, without cohesion or friction in the dividing plane, but having dowel pins set between the two slices. Evidently the lower slice is more rigidly held against arch deflection than the upper one. Thus will be developed a pure shear in the dowel pins. It is this shear that is represented by *s* in Figs. 40 and 42.

In Figs. 39 to 42, inclusive, the writer presents some suggestive diagrams illustrating what takes place in an arched dam, such as distribution of the water load, deflections, and shears. Of course the proportions in the diagrams may not be correct even for any one dam, and they would change with every change in the proportions of a dam, in the rigidity of the base, and with the place in the dam at which the section is taken.

For Fig. 40 the cylinder formula was used taking account of shear and of resistance to deflection by surplus material placed at the top of the dam; vertical beam action is ignored. In Fig. 41 on the other hand, vertical beam action alone is considered, with due recognition of the arch deflection in the rib at the top. In Fig. 42 the two previous effects are combined and the deflections in general are less, due to the divided load. The distance, $G\,K$, allows for the arch action from the horizontal compression of the trench wall below C. The reaction, M-N, will include all the areas, $D\,G\,B$, and the greater part of the area, $I\,G\,D$.

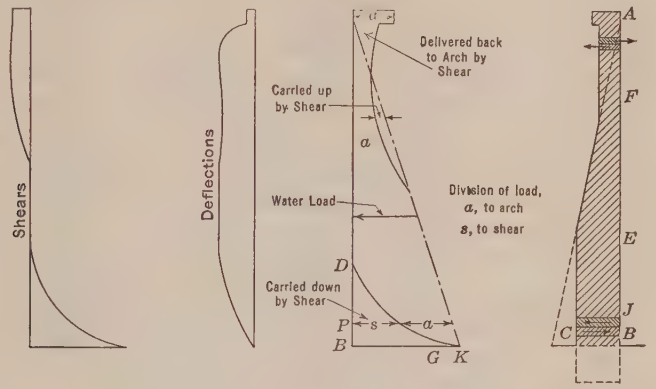


FIG. 40.

The point, D (Fig. 42), will not be at the same height in all the vertical sections. It will descend toward the base as the section approaches the abutments. The beam load, b, will become less as the section approaches the abutment. The reason for these variations is that the inward deflection of the arch becomes less as the abutment is approached and therefore the deflections causing beam and shear loads will be less.

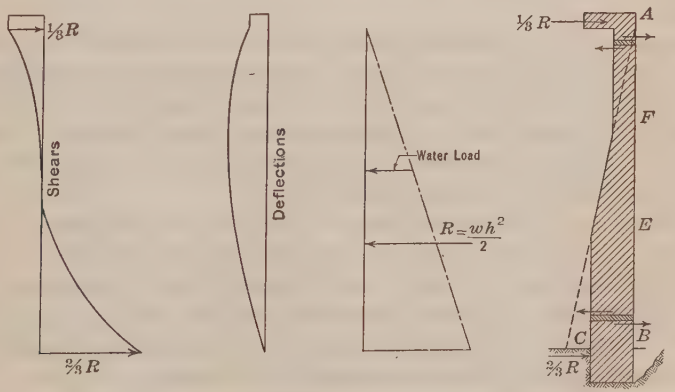


FIG. 41.

It is with some temerity that the writer shows that the cylinder formula applies correctly at only one point in a vertical section, the point, I (Fig. 42),

where the beam load line crosses the vertical. If this conclusion is correct, it should have some influence in designing the section of the dam at the crown, but not so much at the abutments. The sub-divisions of the load (s and b) will be relatively large at the crown on account of the relatively large arch deflections at that point. Near the abutments the vertical-beam load line, $G\ I\ M$, will lie close to the vertical, $A_1\ B_1$, and the point at which they cross will matter little.

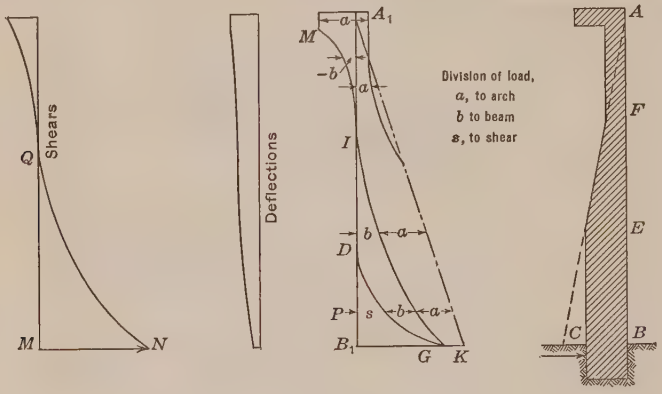


FIG. 42.

If the foregoing conclusions are correct it would indicate that the economic design of an arched dam would require that near the abutments the dam should be more massive than at the crown, both in the wall and in the top arched rib.

As a last comment about vertical beam action in a dam, an advantage worth noting is that the wet surface would be in compression which would tend to make the wall more impervious while cantilever action would have the reverse effect.

Referring to the experimental dam now under construction on Stevenson Creek, in California, it will be easy to determine whether the deflection is that of a vertical beam or a cantilever, by fixing several mirrors in the down-stream face, and noting the trace, on some distant fixed surface, of a reflected ray from a fixed light. More than this hint would be superfluous. The writer hopes that this subject will receive attention.

Thus far, the writer has not alluded to the effect of temperature changes, nor will he go further than to express the belief that temperature stresses can be practically eliminated by one or more flexible joints. These would usually need to go only part way down from the top. If it is practicable to put "hinged" joints in a concrete arched bridge surely it could be done with safety in an arched dam where there are no shocks or vibrations.

Although the numerical determination of the stresses in an arched dam seems hopeless, the writer wishes, in conclusion, to call attention to the comforting and reassuring fact that, excepting temperature stresses, each and every one of the mechanical agencies coming into play in an arched dam co-operate

in giving it strength, and starting with the cylinder formula the question is not, "Where is the danger?" but rather, "How far can the section be reduced below what would be required by the cylinder formula?"

Lars R. Jorgensen,* M. Am. Soc. C. E. (by letter).—The theory of stresses in arch dams is undergoing a gradual development as evidenced by Mr. Jakobsen's instructive paper and Professor Cain's extensive discussion.

When almost every phase of the subject is thus examined the resulting theory becomes somewhat elaborate for every-day use, and some short-cut method will be desirable and, in fact, necessary if arch dams are to enjoy the popularity they deserve. Such empirical formulas would have to be worked out from results obtained by testing arch dams to destruction, that is, from such tests on a large scale as are now under way by Engineering Foundation's Arch Dam Committee near Huntington Lake in Central California.

As the subject is too complicated for absolute scientific solution, empirical formulas based on actual test data will undoubtedly prove expedient for designing purposes. Several tests to destruction were made on small arch dams in connection with the construction of Lake Cushman Dam, a constant angle arch, 275 ft. high, built by the City of Tacoma, Wash. While these test dams (1 in. thick, 36 in. span, and 36 in. radius, first series, and 15 in. thick, with a 7-ft. up-stream radius) were too small to use as a foundation for empirical formulas based on results from crushing tests, it was rather interesting to find that the unit crushing strength of these arches was from 94 to 100% + of the unit crushing strength of cylinders made from the same material.† The crushing strength of the arches was obtained by using the simple arch formula (water pressure $\times$ up-stream radius = unit compression $\times$ thickness), and it was demonstrated (94 to 100% +) that at or near the point of failure the stresses were distributed evenly over the section. It is evident from this that the concrete must have "flowed" when the arch was highly loaded, to give the compressive stress an opportunity to average itself over the whole area just before the arch crushed.

At normal water load (reservoir full) the stress distribution is far from being uniform over the cross-section, but it is reassuring to know that before complete failure can take place the average crushing strength of the building material will be developed, corresponding to a uniform distribution of the compression over the section. With more data available from actual tests on large dams, it should be possible to develop fairly simple empirical formulas, which will give the amount of stress and the stress distribution in the building material for normal loads on the arch.

It is not safe to assume the cantilever as taking any load, since whether it does or does not depends on the degree of water-soaking and its distribution between the up-stream and the down-stream face of the dam body; and this cannot be calculated. It seems fair to assume, however, that the effect of the water-soaking (swelling) is distributed similarly to the effect of uplift, or

* Cons. Engr., Constant Angle Arch Dam Co., San Francisco, Calif.

† These data will be published by B. E. Torpen, M. Am. Soc. C. E., who was largely responsible for making the tests on these dams.

that it is a maximum at the up-stream face decreasing to zero at the down-stream face.

For the sort of concrete used for massive dams (1 : 3 : 6 approximately) the coefficient of swelling can be taken at 0.0003.* Then if a dam were 300 ft. high before the water came on its up-stream face, it would become $300 \times 0.0003 \times 12 = 1.08$ in. higher along the up-stream face with the reservoir full. As the down-stream face remains at its original length the dam would deflect in a down-stream direction without developing any secondary stresses to resist this movement. The amount of this deflection depends on the shape of the cross-section of the dam and may easily be as much as the deflection of the arch carrying the full water load, in which case the cantilever cannot relieve the arch of any load. This also explains why the cantilever never cracks along the up-stream face at or near the bottom, or at any other place, and why apparent tension in the arch along the up-stream side is not serious; tension simply does not exist in either place, due to the swelling of the wet concrete. Since it is doubtful whether the cantilever relieves the arch of any load, the arch should be calculated to support the entire load.

Dr. Fredrik Vogt† (by letter).—The writer has studied this valuable paper with the greatest interest. Its reference to his paper, "Ueber die Berechnung der Fundamentdeformation",‡ gives him the opportunity of offering some supplementary remarks on the influence that the yielding of the foundation has on these thick arches. For so complicated a body statically as an arch dam, it would be fiction to claim any exact stress calculations. The writer's calculations, therefore, do not pretend to be exact, but only to show approximately how the stresses are changed due to the yielding of the foundation. At first glance it might seem obvious to consider this yielding as negligible. Closer investigations, however, show that if a more or less correct calculation is wanted, it should by no means be neglected in the case of thick arches.

The writer agrees with Mr. Jakobsen's remarks relative to division of loads between the cantilevers and the arches. The following, therefore, refers to the case in which the arches have a known, uniformly distributed pressure, p_a (the author's Fig. 2). The writer expects to publish a paper on the influence of the yielding of foundation on the division of load between the cantilevers and the arches. Consequently, that part of the question is not discussed.

(*A*).—In Fig. 43 a cantilever is acted on by a force, P, per unit length (that is, load = $P\ b$). The influence of the yielding of the foundation on the deflection of this cantilever will be approximately equivalent to the replacement of the elastic foundation by a continuation of the cantilever with the length, h_1, down to an imaginary unyielding foundation. The calculations assume that the foundation is bounded by an infinite plane surface, and that the material of the foundation follows Hooke's law. To simplify matters, it is assumed that the modulus of elasticity, E, is the same for the foundation as for the cantilever, a condition which will be nearly fulfilled in the case of a dam.

* See Campbell and White's experiments, *Engineering News*, July 6, 1911.

† Civ. Engr., Trondhjem, Norway.

‡ *Det Norske Videnskapsakademi*, Oslo, 1925, No. 2.

If the difference is appreciable, the displacements of the foundation here given may be corrected by multiplying them by the ratio of E for the arch to E for the foundation. If the rock in the foundation is shattered, the yielding will be greater of course than is here given, since the tensile strength in this case has disappeared. A calculation on this assumption is obviously impossible. Further, the following calculations assume an arch without cracks or open contraction joints.

When the moment is of main importance for calculating the deflection, and when Poisson's ratio, $\frac{1}{m}$, is small:

$$h_1 = 0.45\ t \text{ (approximately)} \quad \ldots\ldots\ldots\ldots\ldots\ldots (110)$$

When the normal force or the shear is of importance, h_1 must be given a larger value.

If the deformation due to the normal compression is considerable as compared with the deflection due to the moment, the calculation of the deflection based on Equation (110) will be as given in Article (*D*).

Approximate formulas for the average displacement and rotation of the loaded part of the surface, Fig. 43, are* for values of b varying from a to 30 a:†

$$\left.\begin{aligned} u &= \frac{m^2-1}{m^2} \times \frac{N}{E\,a} \times \sqrt[3]{a^2\,b} \\ v &= \frac{m^2-1}{m^2} \times \frac{S}{E\,a} \times \sqrt[3]{a^2\,b} + \frac{(m-2)\,(m+1)}{m^2} \\ &\quad \times \frac{M}{E\,a} \times \frac{1}{1+1.1\frac{a}{b}} \\ \Delta\,\phi &= \frac{18}{\pi} \times \frac{m^2-1}{m^2} \times \frac{M}{E\,a^2} \times \frac{1}{1+0.25\frac{a}{b}} \\ &\quad + \frac{(m-2)\,(m+1)}{m^2} \times \frac{S}{E\,a} \times \frac{1}{1+1.1\frac{a}{b}} \end{aligned}\right\} \ldots (111)$$

in which,

$N = P.\ \sin\alpha =$ normal force per unit length,
$S = P.\ \cos\alpha =$ shear " " "
$M = P.\ h.\ \cos\alpha =$ moment " " "

Besides this average displacement, there will also be a curvature on the originally plane surface, due mainly to the normal force and the moment.‡ This curving may also have some influence on the distribution of stress near the abutments, as will be treated in Article (*C*).

* See *Det Norske Videnskapsakademi*, Oslo, 1925, No. 2, Equations 8, 17, 20, 23, and 27. The pressure, p, per square inch in these formulas is here expressed by N, S, and M, in the usual manner. The formulas are developed by means of the known equations from Boussinesq and Cerrutti.

† For values of b varying from a to $\frac{a}{30}$, the expression, $\sqrt[3]{a^2\,b}$, may be replaced by $\sqrt[3]{a\,b^2}$.

‡ See Figs. 4, 5, and 6, "Ueber die Berechnung der Fundamentdeformation," in *Det Norske Videnskapsakademi*, Oslo, 1925, No. 2.

In applying these formulas for the average displacement of the foundation in arch dam calculations, it must be remembered that the surface of the foundation is not plane in a sufficiently wide area from the abutment, and that the formulas therefore give only approximate displacements.

When each side of the arch dam is taken separately, for a dam with the ratio of b to a between 3 and 10 and Poisson's ratio, $\frac{1}{m}$, between $\frac{1}{6}$ and $\frac{1}{10}$, Equations (111) may be written:

$$\left.\begin{aligned} u &= \xi \times \frac{N}{E} \\ v &= \xi \times \frac{S}{E} + \eta \times \frac{M}{E\,a} \\ \Delta\,\phi &= \mu \times \frac{M}{E\,a^2} + \eta \times \frac{S}{E\,a} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (112)$$

in which,

$\xi = 1.40$ to 2.14
$\eta = 0.57$ to 0.79
$\mu = 5.14$ to 5.53

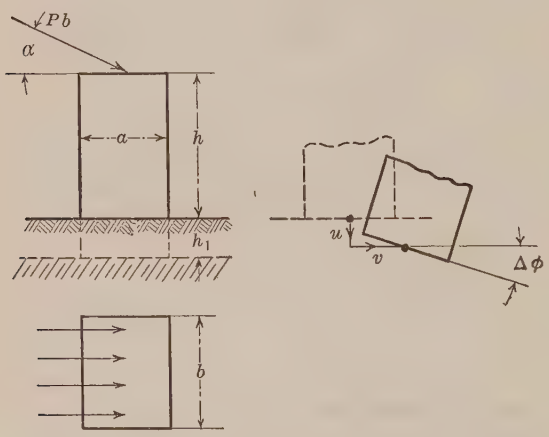


Fig. 43.

(B).—The yielding of the foundation due to the direct water pressure on the sides of the valley will be considered subsequently. In the meantime, the yielding of the foundation due to the reaction of the arch ring on the abutments only will be considered (Fig. 44). As in the paper, the force, X', per unit length is denoted by X'', and its distance from the center, C, of the arch by A''. The abutment is assumed to be vertical, making an angle, ψ_1, with the arch radius. For convenience, the water pressure is assumed to be uniformly distributed over the whole length, E_0-E_1, and consequently also over E_1-E_2. As the angle, ψ_1, usually will be small, the resulting error will be negligible.

At an arbitrary point (Fig. 44), the moment about the neutral axis is:

$$M' = X'' (A'' - r_n \cos \phi)$$

the normal pressure,

$$N' = R - X'' \cos \phi$$

in which,

$$R = p_a r_e$$

and the shear,

$$S' = X'' \sin \phi$$

} .....(113)

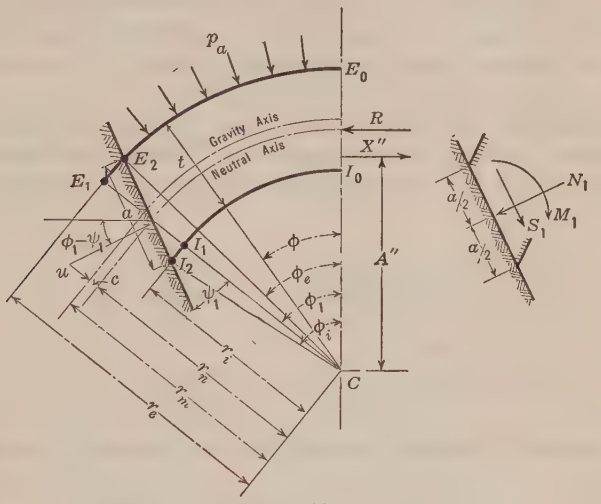


FIG. 44.

The moment, etc., at the abutment, referred to the center of gravity* and plane of the abutment, is:

$$M_1 = X'' \left[A'' - r_n \cos \phi_1 - c \frac{\cos (\phi_1 - \psi_1)}{\cos \psi_1} \right] + R\, c \text{ (approx.)}$$

$$N_1 = R \cos \psi_1 - X'' \cos (\phi_1 - \psi_1)$$

$$S_1 = R \sin \psi_1 + X'' \sin (\phi_1 - \psi_1)$$

} ..(114)

The angle, ψ_1, is usually relatively small, and, therefore,

$$a = \frac{t}{\cos \psi_1} \text{ (approx.)} \quad \ldots\ldots(115)$$

With the notation, $q = \frac{t}{2\, r_n}$, see the author's Equation (23):

$$\frac{c}{r_n} = \frac{q^2}{3}, \text{ and } \frac{c}{t} = \frac{q}{6} \text{ (approx.)}\dagger \quad \ldots\ldots(116)$$

* M_1 should be taken about the center of gravity and not about the neutral axis of the curved arch, so that it may be used in Equations (112). As the distribution of stress in curved beams is not linear, Equations (112) are only approximate.

† More accurately,

$$\frac{c}{r_n} = \frac{q^2}{3} \left[1 - \frac{q^2}{15} + \frac{2}{315} \frac{q^4}{} - + \cdots \right]$$

Further notations are as follows:

$$c_1 = \frac{1}{2}\left(\frac{\kappa E}{G} + 1\right) \text{ and } c_2 = \frac{1}{2}\left(\frac{\kappa E}{G} - 1\right)^*$$

From Article III of the paper, the values, $c_1 = 1.94$ and $c_2 = 0.94$, are obtained.

If temperature variations, swelling, shrinkage, or lateral elongations due to vertical pressure (σ_x), are acting on the free movable arch, Fig. 44, a tangential elongation, u_n, per unit length of the neutral axis, and a rotation, θ, of the crown section relative to the abutment section may be produced (see Article XI of the paper). A uniform change in temperature on the free movable arch will produce a geometrically similar arch, and therefore no rotation. The author's Equation (48) gives a value of θ not equal to zero, when $u_e = u_i$. This equation, therefore, cannot be correct; the error is caused by neglecting the radial elongations. On the other hand a rotation may be produced when u_e is not equal to u_i. When u varies linearly along the radius from u_i to u_e, the radial sections will remain approximately plane, and,

$$\theta = \frac{(u_e - u_i)\, r_m\, \phi_1}{t} = \Delta u\, \phi_1 \frac{1 + \frac{q^2}{3}}{2\, q} \quad \ldots\ldots\ldots\ldots (117)$$

in which, $\Delta u = u_e - u_i$, and

$$u_n = u_e \left(\frac{1}{2} - \frac{c}{t}\right) + u_i \left(\frac{1}{2} + \frac{c}{t}\right) \quad \ldots\ldots\ldots\ldots (118)$$

From symmetry, the crown section has no rotation, so that (see Equations (112)),

$$\mu \cdot \frac{M_1}{E\, a^2} + \eta \times \frac{S_1}{E\, a} + \int_0^{\phi_1} \frac{M'\, r_n\, d\, \phi}{E\, I'} + \theta = 0 \quad \ldots\ldots (119a)$$

or, re-arranged (see Equations (114)),

$$X''\, r_n \left\{ \sin \phi_1 + \mu \frac{q}{6} \cos \psi_1 \left[\cos \phi_1 \cos \psi_1 + \frac{q^2}{3} \cos (\phi_1 - \psi_1) \right] - \eta \frac{q^2}{3} \cos \psi_1 \sin (\phi_1 - \psi_1) \right\} - X''\, A'' \left\{ \phi_1 + \mu \frac{q}{6} \cos^2 \psi_1 \right\}$$

$$= R\, r_n \frac{q^2}{3} \cos \psi_1 \left\{ \mu \frac{q}{6} \cos \psi_1 + \eta \sin \psi_1 \right\} + \Delta u\, E\, r_n{}^2 \frac{q^2}{3} \left(1 + \frac{q^2}{3}\right) \phi_1 \quad (119b)$$

From the author's Equations (38) and (39), etc., the radial displacement in a cylinder may be given by,

$$e' = -r_n\, \varepsilon' = \sigma \frac{R\, r_n}{E\, t} - r_n\, u_n$$

in which,

$$\sigma = \left[\frac{m-1}{2_m} + \frac{m+1}{2_m} \left(\frac{r_i}{r_n}\right)^2 \right] \frac{r_e}{r_m} \quad \ldots\ldots\ldots\ldots (120)$$

* For uniform distribution of shear, $\kappa = 1$; for parabolic distribution in a rectangular section $\kappa = 1.2$. The author assumes $\frac{E}{G} = 2.4$.

The tangential displacement of the crown section must be zero, or, see Equations (112) and (114),*

$$\frac{\xi}{E}\left[S_1 \sin(\phi_1 - \psi_1) - N_1 \cos(\phi_1 - \psi_1)\right] + \eta \frac{M_1}{E\,a} \sin(\phi_1 - \psi_1)$$
$$+ \left[\mu \frac{M_1}{E\,a^2} + \eta \frac{S_1}{E\,a}\right]\left[r_n (1 - \cos\phi_1) - c \frac{\cos(\phi_1 - \psi_1)}{\cos\psi_1}\right] - e' \sin\phi_1$$
$$+ X'' y'' + \int_0^{\phi_1} \frac{\theta}{\phi_1} r_n (1 - \cos\phi)\, d\phi = 0 \ldots\ldots\ldots\ldots (121a)$$

in which, y'' is the quantity, y', in the author's Equation (29), when A' is replaced by A''. Re-arranged,

$$X'' r_n \Big\{ - 1.5\left[2 \sin\phi_1 - \phi_1 - \sin\phi_1 \cos\phi_1\right]$$
$$+ q^2\left[c_1 \phi_1 - c_2 \sin\phi_1 \cos\phi_1\right] + 2\,\xi\, q^3 - \eta\, q^2 \sin(\phi_1 - \psi_1)$$
$$\left[\cos\psi_1 (2\cos\phi_1 - 1) + \frac{2}{3} q^2 \cos(\phi_1 - \psi_1)\right] - \mu \frac{q}{2}\left[\cos\phi_1 \cos\psi_1\right.$$
$$\left. + \frac{q^2}{3}\cos(\phi_1 - \psi_1)\right]\left[\cos\psi_1 (1 - \cos\phi_1) - \frac{q^2}{3}\cos(\phi_1 - \psi_1)\right]\Big\}$$
$$+ X'' A'' \Big\{ 3\left[\phi_1 - \sin\phi_1\right] + \eta\, q^2 \cos\psi_1 \sin(\phi_1 - \psi_1)$$
$$+ \mu \frac{q}{2} \cos\psi_1 \left[\cos\psi_1 (1 - \cos\phi_1) - \frac{q^2}{3}\cos(\phi_1 - \psi_1)\right]\Big\}$$
$$= R\, r_n\, q^2 \Big\{ \sigma \sin\phi_1 + 2\,\xi\, q \cos\phi_1 - \eta\left[\sin\psi_1\left[\cos\psi_1 (1 - \cos\phi_1)\right.\right.$$
$$\left.\left. - \frac{q^2}{3}\cos(\phi_1 - \psi_1)\right] + \frac{q^2}{3}\cos\psi_1 \sin(\phi_1 - \psi_1)\right]$$
$$- \mu \frac{q}{6} \cos\psi_1 \left[\cos\psi_1 (1 - \cos\phi_1) - \frac{q^2}{3}\cos(\phi_1 - \psi_1)\right]\Big\}$$
$$- u_n\, E\, r_n^2\, 2\, q^3 \sin\phi_1 - \Delta\, u\, E\, r_n^2\, q^2 \left(1 + \frac{q^2}{3}\right)(\phi_1 - \sin\phi_1) \ldots . (121b)$$

From Equations (119) and (121), X'' and A'' may be found. The arch stresses due to X'' and $R = p_a\, r_e$ may be found from the author's Equations (34) and (36).

For instance, when $b = 4.5a$ and $m = 8$, see Equations (111) and (112),

$$\xi = 1.6,\ \eta = 0.7, \text{ and } \mu = 5.4$$

With these values, using the example given by the author in Article XII ($r_m = 100$ ft., $t = 50$ ft., $\phi_1 = 45°$, $p_a = 69.5$ lb. per sq. in., and $u_e = u_i = 0$ (that is, $\sigma_x = 0$), which gives $c = 2.118$ ft., $r_n = 97.882$ ft., and $q = 0.25541$), and with $\psi_1 = 0$,

$R = 8687.5$ lb. per arch slice $\frac{1}{12}$ in. thick;
$X'' = 0.47896\ R = 4\,161$ lb.; and
$A'' = 0.83918\ r_n = 82.140$ ft.

* The quantity, u, in Equations (112) is the displacement normal to the abutment surface, $E_2\, I_2$, in Fig. 44, and $-\mu \cos(\phi_1 - \psi_1)$ is the displacement in the direction of X'' resulting from the displacement, u.

Consequently, the yielding of the foundation causes a reduction in the force, X, and at the same time the force is acting considerably nearer the center, C, of the arch (Fig. 44). The moment due to X is therefore increased at the crown and reduced at the abutments. It follows that the tensile stress in the intrados is increased and that in the extrados is reduced.

Take now $\psi_1 = 20°$ instead of $0°$; then,

$$X'' = 0.48856\ R = 4\,244 \text{ lb.};$$

and,

$$A'' = 0.83956\ r_n = 82.178 \text{ ft.}$$

TABLE 11.—Comparison of Stresses According to Three Methods of Computation.

	X, in pounds.	A, in feet.	Tensions, in Pounds per Square Inch.			
			Crown Section.		Abutment Section.	
			Extrados.	Intrados.	Extrados.	Intrados.
With $\psi_1 = 0°$						
Cain	4 566	90.032	−191.7	+26.8	+102.5	−320.9
Jakobsen	4 420	88.125	−166.2	+21.3	+ 71.3	−375.2
Vogt	4 161	82.140	−216.0	+79.0	+ 8.1	−294.7
With $\psi_1 = 20°$						
Cain	4 566	90.032	−191.7	+26.8	+ 49.4	−421.5
Jakobsen	4 420	88.125	−166.2	+21.3	+ 28.8	−490.3
Vogt	4 244	82.178	−217.0	+84.5	− 29.8	−407.0

The numerical values of the stresses are given in Table 11, calculated from Mr. Jakobsen's Equations (34) and (36), for the abutment section with the angles, ϕ_e and ϕ_i (see Fig. 44). When ψ_1 is not equal to zero these formulas are certainly not exact even when the abutment section is a plane, since the abutment section and the radial sections cannot at the same time remain plane. The writer supposes, however, that the error is negligible, when ψ_1 is small. The corrections that follow from the curving of the abutments are treated in Article (*C*).

The coefficients, η and μ, are only slightly changed by a change in the ratio of b to a, and the values, $\eta = 0.7$ and $\mu = 5.4$, may be used as approximate values in most cases. In any case the calculations are only approximate, and therefore the coefficients are taken only to the first decimal. The coefficient, ξ, on the other hand, depends more on the ratio of b to a, that is, on the length of the loaded part of the surface. For instance, with $b = 6a$ instead of $4.5a$, η and μ are unchanged (to the nearest tenth), but ξ is increased from 1.6 to 1.8. The resulting additional stresses at the crown and abutment sections are $-1.1; +5.2; +3.0$; and -1.6 lb. per sq. in. The stresses are thus increased, but not to such an extent that it is of importance.

For comparison the calculations are recapitulated in Table 12 for the same value of $\frac{t}{r_m}$, but with $\phi_1 = 15°$ and using the values, $\xi = 1.6$, $\eta = 0.7$, and $\mu = 5.4$.

TABLE 12.—Comparison of Stresses According to Three Methods of Computation.

	X, in pounds.	A, in feet.	Tensions, in Pounds per Square Inch.			
			Crown Section.		Abutment Section.	
			Extrados.	Intrados.	Extrados.	Intrados.
With $\psi_1 = 0°$						
Cain	8 200.5	98.862	—32.1	+12.7	+29.3	— 60.0
Jakobsen	7 871.4	96.768	—42.1	+13.2	+ 7.3	— 69.1
Vogt	6 233.5	94.520	—92.7	+ 5.4	—53.6	— 59.7
With $\psi_1 = 15°$						
Cain	8 200.5	98.862	—32.1	+12.7	+ 5.3	—112.7
Jakobsen	7 871.4	96.768	—42.1	+13.2	—12.1	—128.7
Vogt	8 260.8	94.929	—67.9	+68.1	—36.3	— 80.8

With $t = 0.5\ r_m$ and $\phi_1 = 15°$, the span of the arch is about equal to its thickness, so that the dam is acting more like a plug or wedge than like an arch. As may be seen from Table 12, the tension in this case materially depends on the angle, ψ_1. By such a wedge the displacements mentioned in Article (*F*), also may be of importance, and the calculations may be somewhat uncertain.

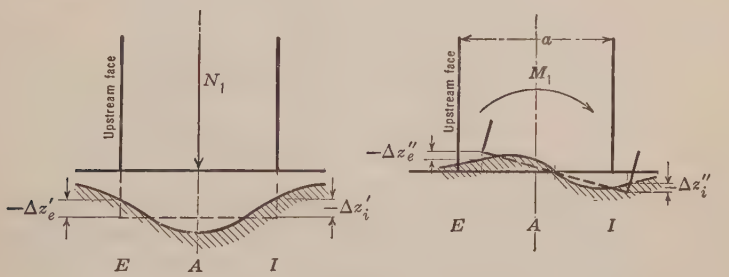


Fig. 45.

(*C*).—As already mentioned, the abutment section, besides having the mean displacements given by Equations (112), will also suffer a curvature, even if the distribution of the load is linear.* This curving is caused more by the normal force and the moment than by the shear. The deformation and the mean displacements of the loaded part of the surface are shown diagrammatically (dotted) in Fig. 45. As an average over the whole length

* See Figs. 4, 5, and 6, "Ueber die Berechnung der Fundamentdeformation," *Det Norske Videnskapsakademi*, Oslo, 1925, No. 2.

for the edges, E and I, the deformation due to the normal force, N_1, is about 8% less for $\xi = 1.6$ than the mean displacement, and for the gravity axis, A, it is about 4% greater.*

Consequently,

$$\Delta z_e' = \Delta z_i' = -0.08 \times 1.6 \times \frac{N_1}{E} = -0.128 \frac{N_1}{E} \text{ (approx.)}$$

When the moment is acting alone, the deformation for the edges† will be:

$$\Delta z_e'' = -\Delta z_i'' = \frac{1}{3} \Delta \phi \frac{a}{2} = \frac{1}{3} \times 5.4 \frac{M_1}{E a^2} \times \frac{a}{2} = 0.9 \frac{M_1}{E a} \text{ (approx.)}$$

Hence, for the numerical example, with $r_m = 100$ ft., $t = 50$ ft., $\phi_1 = 45°$, $\psi_1 = 0°$, and $p_a = 69.5$ lb. per sq. in.,

$$\Delta z_e = \Delta z_e' + \Delta z_e'' = -0.128 \frac{N_1}{E} + 0.9 \frac{M_1}{E a} = \frac{p_a r_e}{E} \times 0.009$$

$$\Delta z_i = \Delta z_i' + \Delta z_i'' = -0.128 \frac{N_1}{E} - 0.9 \frac{M_1}{E a} = -\frac{p_a r_e}{E} \times 0.265$$

The distribution of stress is then supposed to be approximately linear. The deformations, Δz_e and Δz_i, must be produced by additional stresses, which have their resultant and their moment equal to zero, since they have to change only the distribution of stress across the section. It follows from St. Venant's principle that the space through which these stresses act, in the direction of the arch axis, is small. Let Δz be produced by the stress, σ, on a length equal to t (for instance, 0.5 t in the arch and 0.5 t in the foundation); the corresponding additional stresses may be found from:

$$\frac{\sigma_e}{E} t = \Delta z_e = \frac{p_a r_e}{E} \times 0.009$$

$$\frac{\sigma_i}{E} t = \Delta z_i = -\frac{p_a r_e}{E} \times 0.265$$

or,

$$\sigma_e = p_a \frac{r_e}{t} \times 0.009 = +1.5 \text{ lb. per sq. in.}$$

$$\sigma_i = -p_a \frac{r_e}{t} \times 0.265 = -46.0 \text{ lb. per sq. in.}$$

The length on which the additional stresses act is chosen arbitrarily and is rather uncertain. The numerical values of these stresses then also become uncertain. The calculation is given here only to show that the local distribution of stress in the abutment section, especially at the intrados, will be materially influenced by such conditions, which are difficult to estimate.

At a distance from the abutments—at the crown, say—this correction, according to St. Venant's principle, will not appreciably influence the distribution of stress. The curving of the radial sections due to shear, mentioned in the author's Article XVI, will also cause a change in the distribution of stress. The corresponding additional stresses are, however, usually small.

* See Equation 5, "Ueber die Berechnung der Fundamentdeformation," *Det Norske Videnskapsakademi*, Oslo, 1925, No. 2.

† *Loc. cit.*, Equation 10.

(*D*).—If the temperature of the arch is changed and the temperature variation is linear along the radius, from Equations (117) and (118), for the free movable arch the values, u_n and θ, may be expressed by the temperature variation at the extrados, T_e, and at the intrados, T_i, and the coefficient of expansion, β. The corresponding values for X'' and A'' may be found from Equations (120) and (121).

A numerical example, for $r_m = 100$ ft., $t = 50$ ft., $\phi_1 = 45°$, $\psi_1 = 0°$, and $\beta E = 12$ lb. per sq. in., is given in Table 13. The values are calculated for $T_e = 0°$ Fahr. and 1° Fahr., and for $T_i = 0°$ Fahr. and 1° Fahr. The actual stresses can be found by multiplying these values by the actual temperature variations.

TABLE 13.—Numerical Example.—Stresses for Temperature Variations.

	X, in pounds.	A, in feet.	Tensions, in Pounds per Square Inch.			
			Crown Section.		Abutment Section.	
			Extrados.	Intrados.	Extrados.	Intrados.
Yielding of Foundation Taken into Account.						
$T_e = 1°$, $T_i = 0°$..	— 57.565	117.469	—3.201	+ 1.732	—6.301	+6.902
$T_e = 0°$, $T_i = 1°$..	—109.460	65.096	+5.599	—13.142	—0.295	—3.310
$T_e = T_i$ = 1°..	—167.025	83.146	+2.398	—11.410	—6.596	+3.592
Yielding of Foundation Neglected.						
$T_e = 1°$, $T_i = 0°$..	—145.630	105.663	—4.594	— 0.548	—12.435	+12.533
$T_e = 0°$, $T_i = 1°$..	—172.588	73.326	+5.932	—16.649	— 3.361	— 1.147
$T_e = T_i$ = 1°..	—318.218	88.125	+1.338	—17.197	—15.796	+11.386

For these stresses, too, it is seen from Table 13, that the yielding of the foundation is of importance, reducing the stresses to a marked degree at most points.

Corresponding values may be obtained of course for the equivalent shrinkage and swelling, and for the lateral elongations due to a vertical pressure, σ_x. For instance, a swelling of the up-stream face due to water-soaking produces a compression at the extrados and a tension at the intrados, but the maximum stresses at the abutment are for the given example only about one-half the stress found by neglecting the yielding of the foundation (see Table 13).

(*E*).—The radial deflection at the arch crown relative to the abutment due to water pressure alone will become:*

$$\Delta r_{0.1} = \int_0^{\phi_1} \frac{M' r_n^2 (\sin\phi - \sin\phi_1)\, d\phi}{E I'} - \int_0^{\phi_1} \frac{X'' r_n \sin\phi \cos\phi\, d\phi}{E t} + \frac{\kappa E}{G} \int_0^{\phi_1} \frac{X'' r_n \sin\phi \cos\phi\, d\phi}{E t} + \sigma \frac{R r_n}{E t} (1 - \cos\phi_1)$$

* The calculation is carried out for the displacement of the foundation relative to the arch crown to get an expression, which holds also when the abutment is turned through the angle, $\Delta\phi$.

$$= \frac{R\,r_n}{E\,t}\left\{\sigma\,(1-\cos\phi_1) + \frac{X''}{R}\,e_2\sin^2\phi_1 + \frac{X'}{R}\,\frac{3}{2\,q^2}\left[\sin^2\phi_1 - 2\,\frac{A''}{r_n}\,(\phi_1\sin\phi_1 + \cos\phi_1 - 1)\right]\right\} \quad \ldots(122)$$

In addition to this there is the displacement of the abutment in the downstream direction

$$\Delta r_{0.2} = \frac{\xi}{E}\,[N_1\sin(\phi_1-\psi_1) + S_1\cos(\phi_1-\psi_1)] + \eta\,\frac{M_1}{E\,a}\cos(\phi_1-\psi_1)$$

$$= \frac{R\,r_n}{E\,t}\left\{2\,q\,\xi\,\sin\phi_1 + \eta\cos(\phi_1-\psi_1)\left[\frac{q^2}{3}\cos\psi_1 + \frac{X''}{R}\left[\left(\frac{A''}{r_n}-\cos\phi_1\right)\cos\psi_1 - \frac{q^2}{3}\cos(\phi_1-\psi_1)\right]\right]\right\} \quad \ldots(123)$$

The total radial deflection for the arch crown is:

$$\Delta r_0 = \Delta r_{0.1} + \Delta r_{0.2} \quad \ldots(124)$$

With $t = 0.5\,r_m$, and $\phi_1 = 45°$:

For $\psi_1 = 0°$:

$$\Delta r_0 = \frac{R\,r_n}{E\,t}\,[1.1613 + 0.6164] = \frac{R\,r_n}{E\,t} \times 1.7777$$

For $\psi_1 = 20°$:

$$\Delta r_0 = \frac{R\,r_n}{E\,t}\,[1.1767 + 0.6234] = \frac{R\,r_n}{E\,t} \times 1.8001$$

It is apparent that this deflection does not, to any great extent, depend on the angle, ψ_1.

For unyielding foundations the expression may be put in the form:*

$$\Delta r_0 = \lambda_0\,\sigma\,\frac{R\,r_n}{E\,t} \quad \ldots(125)$$

in which,

$$\lambda_0 = \frac{(1-\cos\phi_1)\,[1.5\,(\phi_1 - \sin\phi_1) + q^2\,(c_1\,\phi_1 + c_2\sin\phi_1)]}{1.5\left(\phi_1 + \sin\phi_1\cos\phi_1 - 2\,\frac{\sin^2\phi_1}{\phi_1}\right) + q^2\,(c_1\,\phi_1 - c_2\,\sin\phi_1\cos\phi_1)}$$

In Fig. 46, λ_0 is shown as a function of q and ϕ_1, calculated from $c_1 = 1.9$ and $c_2 = 0.9$. These coefficients correspond to $\frac{\kappa\,E}{G} = 2.8$, for which Mr. Jakobsen uses 2.88 (see his Article III); the difference in the final result is small.†

For unyielding foundations in the numerical example previously given:

$$\Delta r_0 = \frac{R\,r_n}{E\,t} \times 0.876 \text{ when } \frac{\kappa\,E}{G} = 2.88$$

or,

$$\Delta r_0 = \frac{R\,r_n}{E\,t} \times 0.871 \text{ when } \frac{\kappa\,E}{G} = 2.80$$

which is less than one-half the correct value.

* See, also, "The Circular Arch Under Normal Loads," by William Cain, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 233.

† The writer had these curves drawn before Mr. Jakobsen's paper had appeared, hence the reason for using the coefficients, 1.9 and 0.9.

Suppose the arch to be continued to an imaginary unyielding foundation at a depth, $h_1 = \frac{\mu}{12} t = 0.45\ t$, below the actual abutment (see Fig. 43 and Equation (110)), one-half the corresponding central angle, ϕ_2 (see Fig. 47), is:

$$\phi_2 = \phi_1 + \frac{\mu}{12}\frac{t}{r_m} = \phi_1 + 0.45\ \frac{t}{r_m} \quad \ldots\ldots\ldots\ldots\ldots (126)$$

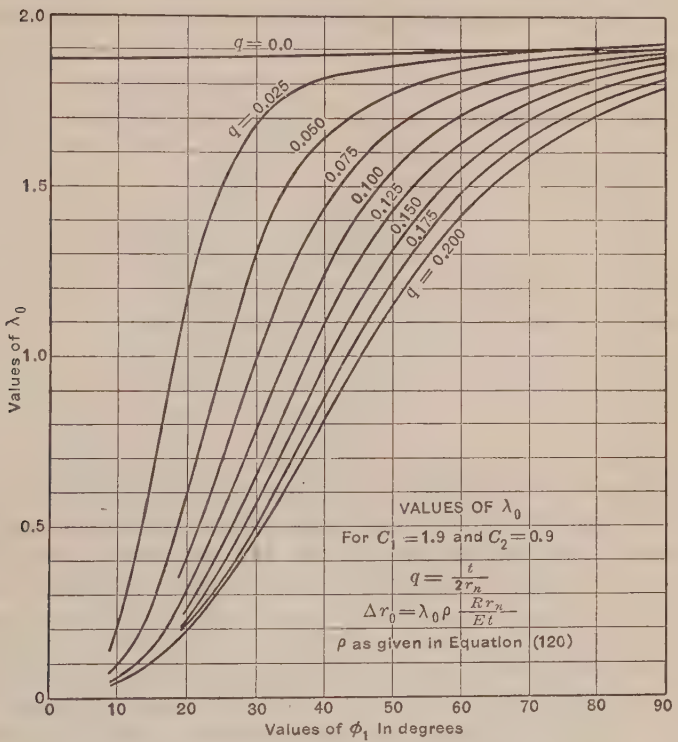


FIG. 46.

In such an arch the displacements due to the moment are approximately equal to the displacement in the actual arch; but the displacement due to the normal force and the shear in the foundation gives an additional displacement of the crown section. The total deflection, Δr_0, of the crown may be found approximately from the expression:

$$\Delta r_0 = \frac{R r_n}{E t}\left[\lambda_{0,\phi_2} \times \sigma + 2\ q\ \xi \sin \phi_1\right] \quad \ldots\ldots\ldots\ldots (127)$$

For the numerical example, with $t = 0.5\ r_m$ and $\phi_1 = 45°$, $\phi_2 = 57°\ 53.5'$, and from Equation (127):

$$\Delta r_0 = \frac{R r_n}{E t}\left[1.19 + 0.58\right] = \frac{R r_n}{E t} \times 1.77$$

which is practically correct.

The error for different values of ϕ_1 and q is:

$\phi_1 = 30°, q = 0.05$, error $= -7\%$
$\phi_1 = 30°, q = 0.125$, error $= +2$ to 3%
$\phi_1 = 45°, q = 0.255$, error $=$ 0 to -1.5%
$\phi_1 = 60°, q = 0.05$, error $= -2.5\%$
$\phi_1 = 60°, q = 0.125$, error $= -3\%$

Compared with all the other uncertainties, these errors are not great, and this method for calculating the deflection may be sufficiently accurate.

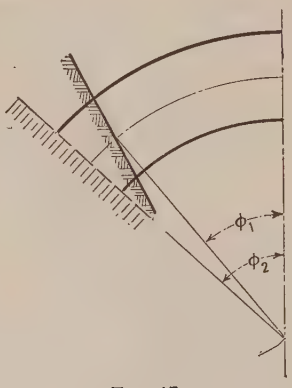


FIG. 47.

The calculation of Δr_0 is of vital importance for finding the division of load between the cantilevers and arches, and it is therefore desirable to get a simple approximate method like the one here given. For calculating the stresses, the writer would prefer the more accurate methods given in Article (*C*).

(*F*).—Since it has been made clear that even for a rock foundation the yielding must not be neglected in calculating the stresses or the deflections in dams, a study should be made as to what displacements the water pressure on the bottom and the sides of the valley produce at the abutments. Evidently it is not the absolute displacements of the foundation that are of interest for the stress calculations, but only the displacements of the different parts of the arch foundation relative to one another.*

From this it follows that only the pressure in the neighborhood of the dam counts, since a force acting at a great distance from the dam will give nearly the same displacements for all parts of the foundation. It is extremely difficult to make an accurate calculation, since the surface of the valley is not plane. A rough estimate† seems to show that, for a wide dam site, for instance, those with the length of dam about three times its height, or more,

* The following may serve as an illustration of the absolute displacements in large reservoirs: In a reservoir having an area of 1 by 2 miles and a depth of 150 ft. (that is, about 8 400 000 000 cu. ft.), the filling of the reservoir will give a mean lowering of the ground of about 3 in., due to the elastic deformations alone. This calculation assumes $E = 2.10^6$ lb. per sq. in., and follows from Equation (111). The measurements of such displacements may be used to determine the average value of E for solid rock.

† Based on Equations 4*a*, 4*b*, and 4*c* in "Ueber die Berechnung der Fundamentdeformation," *Det Norske Videnskapsakademi*, Oslo, 1925, No. 2.

the abutments will have practically no relative displacements due to the pressure. In narrow canyons, however, this pressure produces a displacement which acts in the same manner as a lowering of the temperature of the dam, for instance, in a dam having a span and height equal to 150 ft., perhaps about 3 to 5° Fahr. If the canyon is formed along a geologic line of weakness, as is often the case, the displacements will be increased to some extent, while the bottom will yield more, due to tensile stresses.

Compared with all the other uncertain factors, the writer believes that this effect in most cases will not be of importance for the stress calculations. It may be of importance, however, in exceptional cases, especially for high dams in very narrow canyons, and then it reduces the arch effect.

(*G*).—As a conclusion it may be said, that the yielding of the foundation in thick arches reduces the bending moment at the abutments, and therefore reduces the tensile stress in the extrados. The bending moment in the crown as well as the tensile stresses in the intrados are usually increased. The stresses due to change in temperature, shrinkage, and swelling are usually reduced. The radial deflection is increased. When the arches are thick, all these changes in stresses, etc., are so great that they by no means should be neglected. With thin arches, on the other hand, they are small,* because with q equal to zero all these changes are equal to zero. The "secondary arch", treated in the author's Article XIX, may then be drawn in accordance with the alterations in stresses.

The yielding of the foundation obviously makes the calculations more complicated, and it has been shown that small variations in the conditions, for instance in the angle, ψ_1, in short thick arches alter the stresses. Further, a number of corrections have been mentioned that are difficult to estimate accurately. In addition, there is the influence on the division of load between the cantilevers and arches, depending on the deflection. It should be realized therefore that there will always be considerable uncertainty in the calculations, even if there is complete and accurate knowledge, not only of the water pressure and the temperature, but also of the elastic properties of the rock and concrete, and the shrinkage and swelling of concrete. No calculation of this kind may claim to be more than a rough approximation.

George Paaswell,† M. Am. Soc. C. E. (by letter).—The writer is keenly interested in this analysis of the arch and presents herewith some minor notes of discussion introducing a novel analysis of the arch, with the hope that his method will lead to a more thorough analysis of the actual problem and the correlation of the results with the data gathered by experiments.

As regards temperature changes in large masses, the writer would like to call attention to some equations he has derived to show the variation in the range of temperature as the distance from the exposed face increases.‡ The periodic cycle of temperature changes may be given by an equation of the form:

$$u = A + B \cos \frac{2\pi}{T} t$$

* For instance, for the test dam in Stevenson Creek, California, the yielding of the foundation will hardly be of great importance.

† Engr., Corson Constr. Corporation, Brooklyn, N. Y.

‡ "Retaining Walls: Their Design and Construction," p. 151.

in which, u is the temperature, A and B, are constants determined from the range, T is the period of the season, and t is the time when u is measured. If x is the distance, in feet, of any point within the mass from the surface, the ratio of the range at this point to that at the surface is $e^{-kx} \cos kx$, in which, e is the base of natural logarithms and k is a coefficient such that,

$$k = \frac{1}{a}\sqrt{\frac{\pi}{T}}$$

The quantity, a^2, is the coefficient of thermal diffusivity, which, in the centimeter-gramme-second system for concrete is 0.0058. For a period of one year, with T measured in seconds, $k = 0.00413$; and for a period of one day, $k = 0.079$. From the experiments of Messrs. Paul and Mayhew quoted by the author, Table 14 has been compiled to show a comparison between the actual and the theoretical ranges.

TABLE 14.

Yearly Range.				Daily Range.			
x.	$e^{-kx} \cos kx$.	Theoretical.	Actual.	x.	$e^{-kx} \cos kx$.	Theoretical.	Actual.
0	1.00	75	75	0	1.00	50	50
1.0	0.87	65	..	0.25	0.45	22	..
2.0	0.76	57	..	0.50	0.11	5	..
3.5	0.57	43	32	1.00	0.07	3	2
5.0	0.42	31	..	1.5	0.02	1	..
10.0	0.09	7	12	2.0	0.01	1	1
20.0	0.04	3	0	2.5	0.00	0	..
....	....	..	..	3.0	....	..	..
....	....	..	..	3.5	....	..	0

It would appear that the formula, $e^{-kx} \cos kx$, is simple enough, and sufficiently accurate to warrant its use in the determination of temperature ranges.

To design an arch barrel by the usual formula is, of course, as approximate as to design a plate by the beam formula. The writer has attempted, therefore, to analyze the arch as a surface rather than as a ring and, in spite of the intricate mathematical analysis involved, hopes that at least qualitative results may be obtained. In a general study of flexure it is assumed as the basis of the common theory that the middle surface is unextended, that is, the neutral and the gravity surfaces coincide. The author has shown that this assumption is invalid for an arch of finite ratio of thickness to radius. However, the writer has predicated his analysis on the common theory. Take a typical arch barrel of average central radius, a, and thickness, $2h$, subjected to water pressure. For simplicity in computation the radius, a, is taken as unity and the height of the arch, L, as 2. The arch is assumed to be fixed at the bottom, and freely supported at the sides. Measure the co-ordinate, x, vertically downward from the crest of the dam and the angle, ϕ, in the horizontal plane from a central plane bisecting the arch sector. Let the total sector angle be $2\phi_0$. The flexure of the arch surface introduces a change in curvature vertically, and circumferentially, and also a twist of the normal to the arch

about itself. Let k be the change in curvature in the x direction; K, in the ϕ direction; and t, the twist; and let w measure the radial deflection at any point. The changes in curvature may be represented (see any text, such as Love, for a demonstration) by:

$$k = \frac{d^2 w}{d x^2}$$

$$K = \frac{1}{a^2}\left(\frac{d^2 w}{d \phi^2} + w\right)$$

$$t = \frac{1}{a}\frac{d^2 w}{d x\, d \phi}$$

The energy, $d W$, expended by this change in curvature may be expressed in terms of these coefficients by:

$$d W = \frac{C}{2}[(k + K)^2 - 2(1 - \sigma)(k K - t^2)]$$

in which, C is the flexural rigidity, and σ the Poisson ratio; so that

$$C = \frac{E I}{1 - \sigma^2}$$

If p is the intensity of water pressure at any point with the co-ordinates, x and ϕ, the theorem of virtual work states,

$$p w - d W = 0$$

The symbol, d, is here a coefficient of variation and may be applied to the usual differential coefficients previously given. Substituting the values of the coefficients, k, K, and t, and simplifying by applying some theorems in the calculus of variation, there results a general differential equation:

$$p = \frac{E I}{1 - \sigma^2}\left(\frac{d^4 w}{d x^4} + \frac{2}{a^2}\frac{d^4 w}{d x^2\, d \phi^2} + \frac{1}{a^4}\frac{d^4 w}{d \phi^4} + \frac{2}{a^2}\frac{d^2 w}{d x^2} + \frac{2}{a^4}\frac{d^2 w}{d \phi^2} + \frac{w}{a^4}\right)$$

This may be termed a general flexural equation. If the radius is made infinite, the general equation of the plate becomes:

$$p = \frac{E I}{1 - \sigma^2}\left(\frac{d^4 w}{d x^4} + 2\frac{d^4 w}{d x^2\, d y^2} + \frac{d^4 w}{d y^4}\right)$$

Again, if the plate is narrowed down to the beam,

$$p = E I \frac{d^4 w}{d x^4}$$

If the cylinder is taken of uniform thickness and of infinite length (the pipe problem, or tunnel lining), the equation is directly solvable and gives the stress system with due consideration for the continuity of the structure. If the length dimension is suppressed there results the usual equations for the arch ring. Unfortunately, there is no direct solution for the general equation given and a particular form of solution is assumed, satisfying the equation and the conditions at the edges and containing a constant which may be computed for specific values of the thickness and conditions of loading. Assume a solution in the form,

$$w = f(L - x) \sin \theta\, [\sinh (L - x) - j \cosh (L - x)]$$

in which, $\theta = \sqrt{2}\ (\phi_o - \phi)$ and j is a constant which has been evaluated for the condition that the cantilever moment must vanish at the crest. Let g represent the moment in the vertical direction, that is, the so-called cantilever moment, and let G represent the ordinary radial or arch moment:

$$g = C\ (k + \sigma\ K).$$
$$G = C\ (K + \sigma\ k)$$

The values of k and K are obtained from the expression for w and the moments read:

$$g = C f \sin\theta \left\{ [2 - (1 - \sigma)\ j\ y] \cosh y + [y\ (1 - \sigma) - 2\ j] \sinh y \right\}$$

$$G = C f \sin\theta \left\{ [y\ (1 + \sigma) - 2\ j] \sinh y + [2 - j\ y\ (1 + \sigma)] \cosh y \right\}$$

$$y = L - x.$$

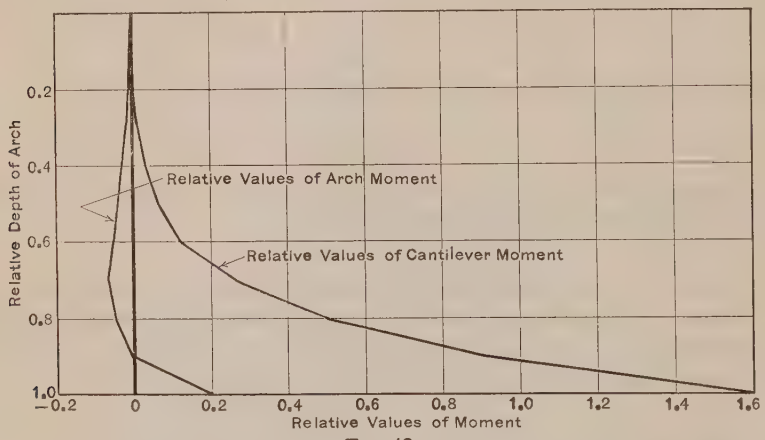


FIG. 48.

These enable a qualitative study to be made of the moment variations. In Fig. 48 is shown the variation of the cantilever and of the arch moments taken vertically along the crown line of the arch. At any other vertical plane the moments are the crown moments multiplied by the sin θ. This study shows that the cantilever moment acquires a maximum value at the foot far exceeding the maximum arch moment. It is noticed that the arch moment changes sign toward the bottom.

It may be that these results are based on assumptions too approximate for the actual conditions. The writer therefore has contented himself with merely presenting the outline of moment values with the hope that data may be found to give sufficient color to the study to justify its further extension.

W. A. PERKINS,* ASSOC. M. AM. SOC. C. E. (by letter).—This paper and the discussion on it by Professor Cain throw much new light on this complicated subject. The writer, however, feels that the problem is not completely solved for very thick arches which show heavy tension at the abutments and crown; nor does the proposed solution by the secondary arches give the correct answer.

* Associate Hydr. Engr., Div. of Eng., State Dept. of Public Works, Sacramento, Calif.

For thick arches in which analysis shows heavy tensile stresses, it is probable that cracks will develop at the junction of the extrados and the abutments. There is less likelihood of these cracks appearing at the crown, because conditions there are more favorable for withstanding tension, and the moments are little more than one-half those at the abutments. Assuming, however, that failure in tension occurs at both points, it is not probable that more than one crack would appear at either place. It is certain that no cracks will appear at the points of no moment, approximately half way between the crown and the abutments, and there are not likely to be any for some distance on either side of these points. All that part of the arch ring in which no cracks have occurred will be working to overcome water pressure. If this tension failure occurs, the assumptions on which the analysis of the primary arch was based no longer exist and, therefore, the formulas cannot strictly apply.

The principal actions that are taking place within a thick arch under stress are a shortening of what may be called the diagonal from the crown at the extrados to the abutment at the intrados, and a lengthening of the diagonal from the crown at the intrados to the abutment at the extrados. Some of the load that is assumed to be resisted by moments in the primary arch analysis is actually being carried by compression (corresponding to the stresses represented by P in Professor Cain's formula) and the remainder, resulting from the reduction of the moments, is being carried by shear.

When the assumption is made that the secondary arch is carrying the load, there is omitted from consideration the greater part of the concrete that lies along the diagonal from the extrados at the abutment to the intrados at the crown, and the work of tension, compression, and shear which is actually done by this neglected body of concrete is assumed to be done by the thinner, secondary arch. It is evident, then, that the true stresses and deflections must lie between those found by the analysis of the primary and of the secondary arches.

The writer recently attempted to arrive at a more correct solution of the problem by substituting a variable moment of inertia in Equation (5) and in the equation for deflection in Professor Cain's formulas.* This variable, I, is derived from the formula, $t = t_0 \div \cos a\,\phi$, in which, t_0 = the crown thickness; t = the thickness at any point the angle to which from the center line is ϕ; and a = a constant the value of which depends on the relative thickness of t_0 and t at the abutment.

The resulting formulas are somewhat longer than those of Professor Cain, but are not difficult to apply. The center line of the arch was assumed to be circular, and the thickening was assumed symmetrical about the center line. The writer does not believe, however, that this gives the best form of arch. The addition of concrete at the extrados of the abutment serves no useful purpose except to eliminate over-hang at this point. This over-hang, however, unless excessive, can do no harm.

A true analytical solution of the problem is perhaps impracticable because of the irregular variation in the value of the moment of inertia of the arch after cracking has occurred. A close approximation, however, may possibly be

* *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 236 and 240.

obtained by the graphical method suggested by Mr. Jakobsen. There still remains the question of the true deflection of the arch, by means of which the relation between arch and cantilever action is determined.

This relation is difficult to determine because of the many uncertain factors that enter, the principal ones of which are temperature changes, water-soaking of the up-stream face, and the variation in the relation that will be found to exist as successive vertical slices from the crown to the abutment are studied Uncertainty in the value of E does not have so much bearing because of the fact that E enters in the same way in both cantilever and arch action, and, therefore, in a sense, is cancelled.

The writer was recently called on to analyze the Pacoima Dam and another arch dam of constant radius, which will be called Dam No. 1. To study the stresses at the base of the cantilever, the division of loading, at the crown, was determined from the deflection curves, successive trials being made until a division was found which gave practically the same deflections for the arch and the cantilever. Owing to the difficulty of reconciling the deflection lines of the arch and the cantilever near the base, the effect of the deflection of the cantilever due to shear was investigated.

The deflections for Dam No. 1 are shown on Fig. 49. This arch has a constant up-stream radius of 175 ft., a thickness at the top of 5 ft., and at the bottom of 64.5 ft., a vertical up-stream face, and a concave face down stream. The arch deflections were computed by Cain's formula for the primary arches, and the deflection of the cantilever due to moments was found by the method of moment-area diagrams. The cantilever deflections due to shear were found for each 10-ft. block from the three formulas: $y_s = S\,l + G\,A$; $y_s = P\,l^2 + 2\,G\,A$; and, $y_s = w\,l^3 + 6\,G\,A$, in which, S = the total load above the section being studied, that is, the total shear or concentrated load at that point; P = the unit value of a uniformly distributed load against the section; and w = the unit weight of a liquid or triangular load. If the triangular load is upside down, the deflections are twice the value of those given by the last formula. Any load acting to produce shearing deflection in any section can be resolved, practically, into these three types of loading.

The deflection of each 10-ft. elevation about the 10-ft. elevation below was computed, and the summation taken, starting from the bottom. To simplify the work, S, P, and w were assumed to be equal to 1, and the deflection coefficients for each 10-ft. section were determined. After the probable division was found, the deflection coefficients were multiplied by the loads in the table from which Fig. 49 was drawn. The shear deflections were added to the scaled moment deflections of the cantilever to determine the total deflection of the latter. The mean area of the section considered was A, and G was assumed as 1 000 000 lb. per sq. in., or 0.4 of the value used for E. Because of the parabolic distribution of the shearing stresses, the true shear deflection was assumed to be 1.2 times that given by the formulas, which assume a uniform distribution of the stresses.

It will be noted that near the base, the shear deflections are much greater than the moment deflections, and they do not become equal until a height of

about 35 ft. above the base is reached. It will also be noted that the deflection line makes an abrupt angle with the tangent to the undeflected section. This fact probably explains why there are no cracks at the base of many dams which, by analysis without introducing this shear action, would seem to be cracked. The same general conditions were found to apply in the Pacoima Dam, although the ratio of cantilever load to arch load is much smaller than that in Dam No. 1.

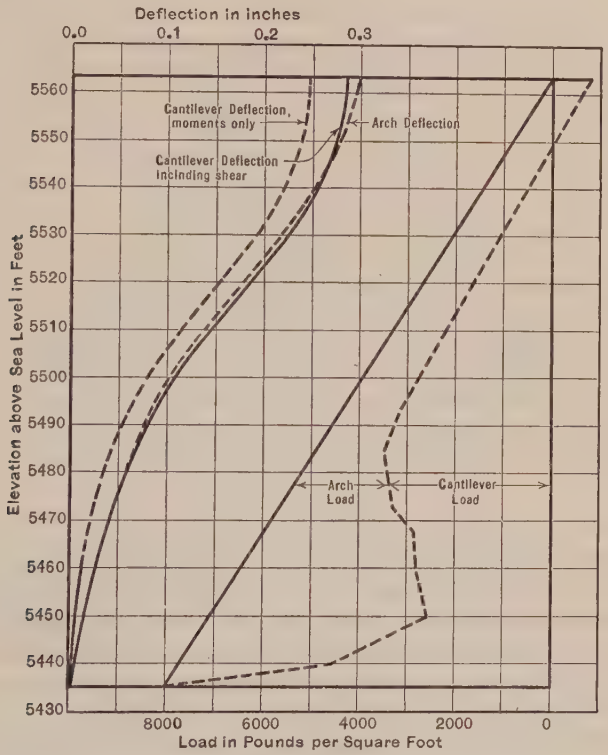


FIG. 49.—DAM NO. 1, DIVISION OF LOAD AND DEFLECTION CURVE, JUNE 10, 1926.

There seems to be some hesitation among engineers to accept the results obtained from the use of the Cain, Jakobsen, or Noetzli formulas which show such large moments with thick arches with small central angles. A simple illustration should clear up this doubt. Fig. 50 illustrates the action in two struts supporting a concentrated load that is analogous to the action in an arch, for as the struts are the proper supports for a concentrated load, so the arch of circular section is the proper support for a uniformly distributed load.

Angle ϕ in Fig. 50 corresponds to the central angle in the arch. For Struts S_1, ϕ_1 is large, θ_1 (its supplement) is small, and the load is carried almost entirely by column action. The angular movement of the apex of the struts about the abutment, caused by the compression in the struts—the latter corresponds to the rib-shortening in an arch—is very small, and the moments, which

are a function of this angular movement, are correspondingly small. In Struts S_2, with the small central angle, ϕ_2, it is evident that thrusts and moments are much larger for the same load, P, than they are in Struts S_1.

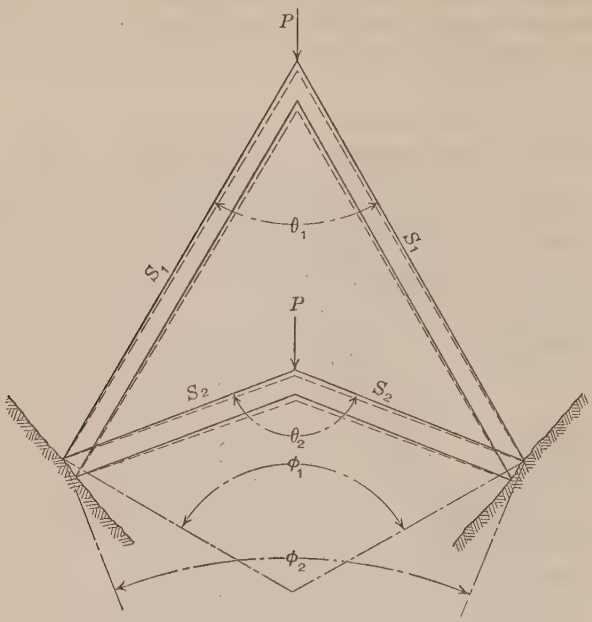


FIG. 50.

If the load, P, should be divided into several partial loads, to maintain equilibrium it would be necessary to increase the number of struts, so that the total number of sections in the partial polygon forming the span would always be one more than the number of loads. If the number of loads were increased without limit, they would approach a condition of uniform loading as their limit, and the polygon would approach a circle as its limit.

This illustration suggests a possible method of analysis of the arch, but the resulting formulas should not differ from those already published.

FRED A. NOETZLI,* M. AM. SOC. C. E. (by letter).—Mr. Jakobsen's paper and Professor Cain's discussion, which latter might well be considered a paper by itself, are of much interest to the writer, especially in view of the experiments in progress at present on the Stevenson Creek Test Dam of Engineering Foundation.

The writer is pleased that Professor Cain once more voices the opinion that it may prove feasible to establish a definite mathematical relation between the deflections and stresses in arch dams. Mr. Jakobsen, on the other hand, seems to differ with this opinion. In this connection it is of interest that the experiments on the Test Dam give promise of a definite answer to this question.

The analysis of the stresses in thick arches of dams is a rather complicated problem. In order to have a basis for deriving theoretically more nearly cor-

* Cons. Hydr. Engr., Los Angeles, Calif.

rect formulas than those available heretofore, notably those of Professor Cain,* the author introduces a number of assumptions not ordinarily made in the design of arch dams, such as the location of the neutral axis, Poisson's ratio, the influence of the vertical stress, etc. Except for benefitting by the correct location of the neutral axis, as was done also by Professor Cain, it is a question whether Mr. Jakobsen's new formulas will lead to a materially higher degree of accuracy in actual stress computations. There would appear to be too much uncertainty involved as to some of the assumptions that vitally affect the result of any stress analysis in arch dams, namely: (1) Modulus of elasticity of concrete; (2) temperature changes in dam, both uniform and variable, from up-stream to down-stream face; (3) shrinkage due to setting of concrete; (4) swelling due to moisture in concrete; (5) arching in inclined and vertical planes; (6) proportion of water pressure supported by horizontal arching and by vertical cantilever, beam, or other action; (7) variation from the uniformity of load on the horizontal arch elements between arch crowns and abutments; and (8) flow of concrete under a sustained load.

It is rather doubtful whether the author's theory of secondary arch action can be used in the analysis of stresses in arch dams for which the ordinary theory indicates the occurrence of excessive tension and open cracks in the concrete. Such cracks form restrained hinges which will tend to convert the hingeless (fixed) arches into three-hinged or multiple-hinged arches.† The rib-shortening and temperature stresses will thereby be reduced materially. This is one of the reasons why arch dams are relatively safe structures even after the formation of cracks.

Professor Résal's formulas, as used by the author, are based on a flat parabolic arch. This should be kept in mind when applying this theory.

In a discussion of the writer's paper, "Gravity and Arch Action in Curved Dams",‡ Mr. Jakobsen stated that:

"The theory of Professor Moersch [which is fundamentally the same theory as that expounded in this paper] is not applicable to arched concrete dams, * * * and that for this reason the Rankine [cylinder] formula is the most satisfactory available, * * *."

Now Mr. Jakobsen believes "the assumption that the cylinder formula holds good, however, errs on the side of danger; no conservative engineer can afford to make it."

Thus, within a few years, he apparently has changed from an ardent adherent of the cylinder formula to an advocate of a fairly complicated theory, and the writer wonders whether Mr. Jakobsen has not gone a little too far in this direction.

It is to be hoped that the experiments on the Test Dam will furnish reliable data and information on arch dams. These data will be helpful especially in two ways: First, for developing a theory of arch dam design, which although probably only approximate, will give reasonably accurate results; and, second, for establishing sound assumptions on which such a theory may be based.

* *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 233.
† *Loc. cit.*, p. 263.
‡ *Loc. cit.*, Vol. LXXXIV (1921), p. 108.

The author seems to have misquoted the writer relative to a statement on the probability of cracks in the vertical elements of some arch dams.*

With regard to the influence of cantilever action, swelling of the concrete due to water-soaking, deformations of the bed-rock, etc., which Mr. Jakobsen has discussed at some length in this and in previous papers on dams, the writer prefers to wait for the outcome of the experiments on the Test Dam and on other dams under investigation by the Committee on Arch Dam Investigation. It would be interesting, however, if Mr. Jakobsen would apply his ideas and theories to the interpretation of the deflections of some dams, as, for instance, in connection with actual measurements on the Salmon Creek Dam.†

B. F. Jakobsen,‡ M. Am. Soc. C. E. (by letter).—In Article XIX of the paper, Professor Résal's formulas and the maximum stress given by Equation (83) were discussed. The writer should have stated explicitly that this equation was derived for a loading parallel to the center line at the crown and not for radial loading.§ Therefore, the abutments in Fig. 23 should have been shown parallel, with L as the distance between them. Résal also assumes (see Fig. 24), that the length of the parabolic arc, ds, is approximately equal to its projection, dy, on the Y-axis, and this is true only for a flat arc. The secondary parabolic arch for a short thick arch is far from being flat and for this condition the Résal formulas become unavailable.

In designing the Pacoima Dam it became clear that the secondary arch plays a considerable rôle in determining the maximum stress. The subject, therefore, will be further discussed.

In order to avoid undue complications and generalities, it is expedient to consider a specific case. Consider, therefore, the arch at Elevation 1 700 in the Pacoima Dam, shown in Fig. 51 with the irregular abutments, as they exist, except for the stepping of the abutment. The secondary arch as determined from the writer's formulas and from Fig. 21 is also shown. The dimensions of this secondary arch are, $r_e = 60$ ft.; $t = 30$ ft.; $\frac{t}{r_m} = 0.667$; and $2\phi_0 = 152$ degrees. Résal's secondary arch, determined from Equation (85), is only 24 ft. thick.

There are two objections to the secondary arch. The first is that the actual load does not coincide with the assumed load. The actual load is normal to the primary arch, whereas the load is assumed to be normal to the secondary arch. Considerable calculation has convinced the writer that this objection is not of much importance, especially since the actual loading is not known, but is taken as equal to the water pressure, which is, of course, not exact.

The second objection is much more serious. In Fig. 51 the points, A and B, lie just inside the secondary arch, while the points, A' and B', lie just outside. If the secondary arch carries all the load, the distance between the

* See p. 517, and "Gravity and Arch Action in Curved Dams," *Transactions*, Am. Soc. C. E., Vol. LXXXIV (1921), p. 54.

† *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1920), p. 321.

‡ Cons. Engr., Los Angeles, Calif.

§ "Stabilité des Constructions," J. Résal, Librairie polytechnique, Paris, 1901, pp. 357 and 370.

points, A and B, is decreased when the load is applied, but the distance between the points, A' and B', remains the same, since there is no stress outside the secondary arch. This is not possible and the error involved is likely to increase as the ratio of thickness of original arch to that of secondary arch increases. For short thick arches the maximum stress found for the secondary arch is, therefore, likely to be much in excess of the actual maximum stress.

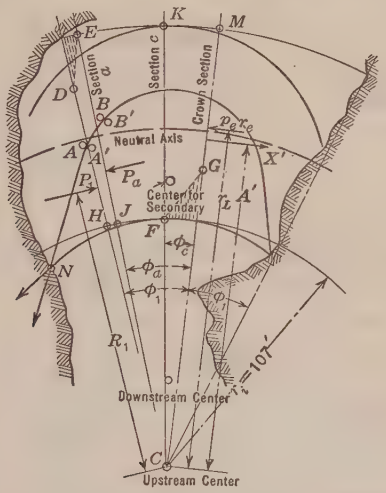


FIG. 51.—PACOIMA DAM AT ELEVATION 1700.

In addition, the direction of the maximum stress is not the same for the secondary arch as for the original arch and in order to judge intelligently of the sufficiency of the abutments, it is necessary to know something about the direction of that stress, which at Point N in Fig. 51 will be tangent either to the original or to the secondary arch. Since there is compression in the intrados near the abutment, it is not reasonable to assume, as is done for the secondary arch, that the concrete lying down stream from the secondary arch does not transmit stresses. In order to arrive at an understanding of what actually happens, the following calculations were made.

Neglecting the vertical stress due to the weight of the concrete and the water pressure on the up-stream face and, also, the effect of swelling, shrinkage, and temperature, as well as the deformation of the abutments, and assuming Poisson's ratio, $\frac{1}{m} = 8$, the resulting tangential stresses at the extrados and intrados at crown and at abutment can be determined. The constants for this arch are: $2\,\phi_1 = 41°$; $r_m = 149$ ft.; $t = 84$ ft.; $r_e = 191$ ft.; $r_i = 107$ ft.; $\frac{t}{r_m} = 0.564$; and, from Fig. 11, $100\,\frac{c}{r_m} = 2.7$, or $c = 4.025$ ft., so that $r_n = r_m - c = 144.975$ ft. From Equation (18), $A' = 141.904$ ft.; from Equation (38), $\varepsilon' = -2.17\,\frac{p_e}{E}$; and, as a check, from Equation (38b), $\varepsilon'' = -2.2273\,\frac{p_e}{E}$. The constant, B, is found by interpolation from Table 1, $B = 0.392$, and

Equation (58) gives $X' = 162.7\ p_e$. From Fig. 12, $k_e = 5.0$ and $k_i = 7.35$; p_e at Elevation 1 700 is due to 325 ft. of water on the assumption that the water stands level with the crest and that cantilever action is neglected, so that $p_e = 141.0$ lb. per sq. in. The stresses due to X' acting a distance, A', from the center of the arch may now be found from Equation (34); they are, compression being negative, as given in Table 15.

TABLE 15.

Section.	Extrados.	Intrados.
Crown section	$+1.12\ p_e$	$+3.14\ p_e$
Abutments	$+2.08\ p_e$	$+1.42\ p_e$

The Lamé stresses from Equation (36) are, for the extrados and intrados, respectively, $-1.92\ p_e$ and $-2.92\ p_e$. The resulting stresses are as given in Table 16.

TABLE 16.

Section.	Extrados.	Intrados.
Crown section	$-0.80\ p_e$	$+0.22\ p_e$
Abutments	$+0.16\ p_e$	$-1.50\ p_e$

By assumption the concrete cannot transmit tension and, consequently, the stress distribution found in Tables 15 and 16 cannot exist since it involves tension.

The tension in the extrados decreases from the abutment toward the crown, so that a point on the extrados close to the abutment can be found where the stress is zero; points closer to the crown section have compression. Likewise, there is a point on the intrados close to the crown section, where the stress is zero. Referring to Fig. 51, denote these sections as a and c; their location is determined from Equations (34) and (36):

$$\left.\begin{array}{ll} \text{For Section } a, & \\ & \sigma_{te}' + \sigma_{te}'' = 0 \\ \text{For Section } c, & \\ & \sigma_{ti}' + \sigma_{ti}'' = 0 \end{array}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (128)$$

The solution of Equations (128) gives the values of ϕ_a and ϕ_c (see Fig. 51).

The arch will yield in tension without resisting, approximately as indicated in Fig. 51 by the hatched areas. No tension exists in the arch between the two sections, a and c, and this part of the arch is therefore not affected.

Assume, now, that X' and A' remain unchanged and consider the part of the arch between Section a and the abutment. The shear forces acting radially in these two sections as well as the water pressure acting on the extrados, produce no moment about the center, C, of the arch. If P_a is the resultant

of all tangential stresses acting on Section a and R_a is the distance from P_a to the center, C, and if P_1 and R_1 apply to the abutment section, it is,

$$P_a R_a = P_1 R_1 = M_c = \text{constant} \quad \ldots\ldots\ldots\ldots\ldots\ldots (129)$$

Also, from Fig. 51, when P is the sum of all the tangential stresses on any section ϕ° from the crown section,

$$\left.\begin{aligned} P &= p_e r_e - X' \cos \phi \\ M_c &= p_e r_e r_L - X' A' \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (130)$$

in which, r_L is the distance from the center, C, to the point of application of the Lamé stresses. For any section the Lamé stresses must total $p_e r_e$ (see, also, Professor Cain's discussion) and the lever arm, r_L, is obtained from Equation (36),

$$r_L = \frac{\int_{r_i}^{r_e} \sigma_t'' \, r \, dr}{p_e r_e} = \frac{r_e}{2}\left(1 + \frac{r_i^2}{r_m r_n}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (131)$$

On the assumption that X' and A' remain unchanged, the stress distribution for Section a is known and from Equations (129), (130), and (131), P_1 and R_1 may be determined. This gives the sum of the tangential stresses, or the thrust at the abutment and the point of application of this thrust, but it does not give the stress distribution, nor the thickness, $t' = DH$, and, therefore, the maximum stress that occurs in Point H is not determined.

It should be noted that the calculations are carried out for the condition that the up-stream and down-stream faces are concentric, which is true for the arc through the points, F, J, and H; the down-stream face is not concentric to the up-stream face and is shown as the arc through the points, F and N. Also, the calculations assume that the arch is symmetrical so that the radial line, CHD, is the abutment. If symmetry cannot be assumed, the calculations become considerably more involved.

The stress distribution near the abutment might be determined by making use of the fundamental Equations (71) and (72), by assuming the stress distribution as given by Equations (36) and (68). This would certainly be very complicated and not quite correct unless the abutment deformations were also taken into account (see Dr. Vogt's discussion, especially Fig. 45). The writer does not feel equal to the task of determining the stress distribution by calculation, and the only other avenue open is to select one or more plausible assumptions and see how these affect the maximum stress that occurs in the point, H. Two assumptions will be investigated:

I.—That the tangential stress distribution at the abutment section, DH, is similar to that of Section a. The stress in the points, D and E, is zero.

II.—That the tangential stress distribution at the abutment section is linear.

Assumption I.—

Let,

$$\left.\begin{aligned} R_a &= r_i + k\,t \\ \text{and} \\ R_1 &= r_i + k\,t' \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (132)$$

Since by assumption the stress distribution is similar for both sections, the coefficient, k, is a constant. M_c and P_a may be obtained from Equations (130) and R_a from Equation (129) and then k follows from the first equation in Equations (132). Likewise, P_1 and R_1 are obtained from Equations (130) and (129) and, therefore, t' can be found from the last equation in Equations (132). Designate the stress in the points, J and H, of Fig. 51* by σ_J and σ_H, then,

$$\sigma_H = \sigma_J \frac{t}{t'} \frac{P_1}{P_a} \quad \ldots\ldots\ldots\

Assumption II.—Since now the stress distribution is linear,

$$\left.\begin{aligned} R_a &= r_i + \frac{t'}{3} \\ \text{and,} \qquad \sigma_H &= -\frac{2\,P_1}{t'} \end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (135)$$

R_1 was 124.9 ft., so that now, $t' = 53.7$ ft., as against 63.02 ft. found by Assumption I. P_1 was 38.60 p_e, so that now.

$$\sigma_H = -2\,\frac{38.6}{53.7} = -1.44\,p_e$$

For linear stress distribution both t' and the maximum stress are smaller than under Assumption I.

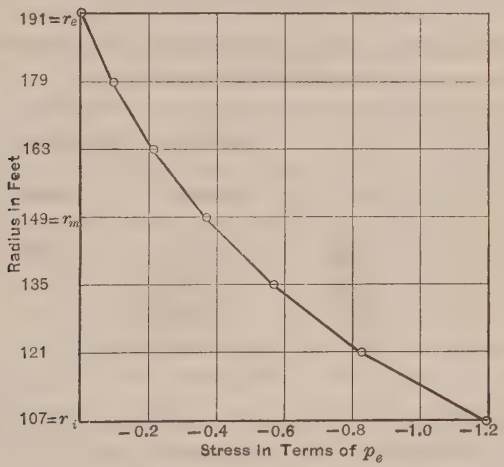


FIG. 52.—RESULTING TANGENTIAL STRESSES ON SECTION *a* OF FIG. 51.

In order to form an idea of the tangential stress distribution at Section *a*, this is shown in Fig. 52 and Table 17. The Lamé stresses are obtained from Equation (36) and the bending stresses from Equation (31), in which,

$$M = X'\,(A' - r_m \cos \phi_a) = 119.58\,p_e$$

The moment of inertia is $I = 49\,392$, so that $\frac{M}{I} = \frac{p_e}{413.05}$, and the tensile stress is,

$$\frac{X' \cos \phi_a}{t} = 1.835\,p_e$$

The calculations that were made previously for Section *a* near the abutment were repeated for Section *c* near the crown section. The results were found to be, $\cos \phi_c = 0.99193$, or $\phi_c = 7°\ 17'$; this is much larger than the angle, $\phi_1 - \phi_a$, because at the abutments the moment changes rapidly while at the crown it changes slowly (see, Professor Cain's discussion, Fig. 32). The

stress at Point K for Assumption I is found to be, $-0.68\ p_e$ and $P_c = 29.613\ p_e$; $R_c = 162.80$ ft.; and $k = 0.33571$, when Equations (132) is replaced by,

and,

$$\left.\begin{aligned} R_c &= r_e - k\,t \\ R_0 &= r_e - k\,t' \end{aligned}\right\} \qquad (132a)$$

At the crown-section, $P_0 = 28.30\ p_e$; $R_0 = 170.36$ ft.; $t' = 61.48$ ft.; and $\sigma_M = -0.887\ p_e$; this is the maximum stress at the crown section.

In comparison the secondary arch as determined from Fig. 21, the dimensions of which have been previously given, has a maximum stress as determined from Equation (80) of $5.35\ p_e = 738$ lb. per sq. in., since $p_e = 141$ lb. per sq. in. Under Assumption I the maximum stress was $1.69\ p_e$, so that the maximum stress of the secondary arch is 4.36 times greater. The writer stated specifically that the stress found in the secondary arch was not the true stress, but a safe stress to use, especially as a first approximation.

TABLE 17.—Tangential Stress Distribution on Section a in Fig. 51.
(Compression is negative.)

r, in feet.	Stresses in Terms of p_e.			
	Lamé.	Bending.	Tensile.	Total.
191	— 1.913	0.0846	+ 1.835	+ 0.0066
177	— 1.988	0.0631	"	— 0.0899
163	— 2.084	0.0388	"	— 0.2102
149	— 2.207	0.0094	"	— 0.3626
135	— 2.371	— 0.0260	"	— 0.5620
121	— 2.595	— 0.0696	"	— 0.8296
107	— 2.913	— 0.1248	"	— 1.203

The stress distribution thus found is predicated on the assumption that X' and A' remain unchanged when the hatched areas in Fig. 51 yield in tension without resisting. The writer believes that this assumption is approximately correct for the arch under consideration. He has made some calculations which seem to substantiate this belief, but they are of course only roughly approximate and can furnish only an indication; moreover, they involve additional assumptions and since they are tedious and of no particular significance, they will not be given here.

In the stress calculations previously given it was assumed that the line, $H\,D$, coincides with the abutment and this is not true (see Fig. 51). The deformation and consequent yielding of the abutments were also neglected. It is evident that this subject needs considerably more study and the assumption of symmetry, which so materially simplifies the calculations, may have to be discarded. At any rate it needs to be demonstrated that the assumption of symmetry does not lead to large errors. An examination of Fig. 51 shows plainly that no line of symmetry exists and this is also the case at most other elevations for the Pacoima Dam, and, no doubt, is generally so. Also, it will be found that when a line of symmetry is drawn for each elevation, these lines do not lie in one vertical plane.

It should be noted that the line, *E D*, of zero stress in Fig. 51, is determined from the assumption that *C H D* is the actual abutment. Equations (129) and (130) can not be applied to find the stress distribution on a section to the left of Section *C H D*, because such procedure involves the assumption that the central angle is larger than that assumed in determining *A'* and *X'* and both *A'* and *X'* depend on the central angle. It would possibly have been more nearly correct to have assumed the central angle to be 10 to 15° larger, but the writer believes this would have given slightly smaller maximum stresses, since, at least, in general the maximum stresses decrease when the central angle is increased. In reality, the line, *E D*, of zero stress in Fig. 51 lies farther to the left and probably entirely inside the rock abutment.

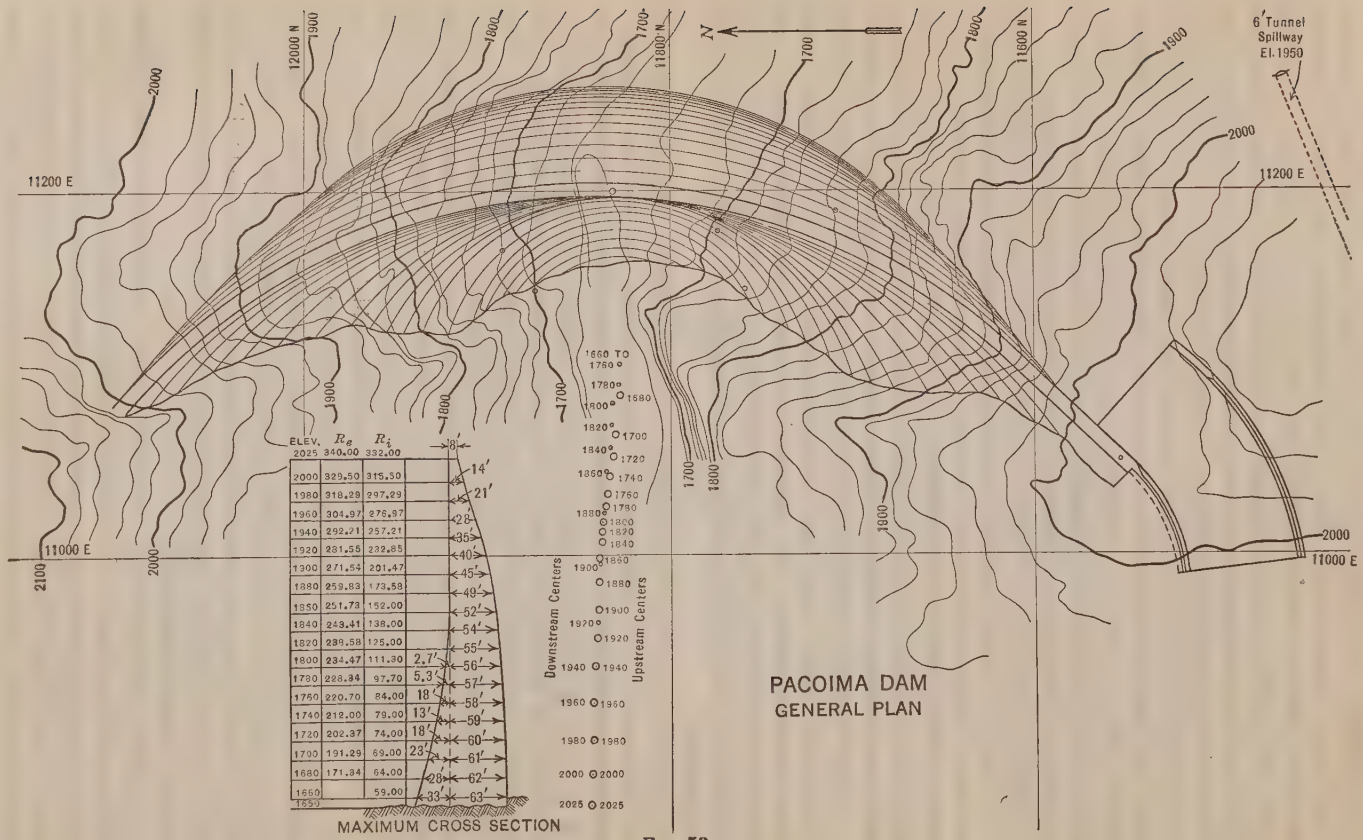
General Description.—Pacoima and Santa Anita Dams.—The subject matter of the paper is largely confined to mathematical theory; some engineers might even go so far as to say it had nothing to do with actual dam design. As a matter of fact, the actual construction is intimately bound up with the design and unless the dam is so constructed that the assumptions underlying the theory are actually fulfilled, the whole is unrelated and may be worthless.

The principles here developed have been used to advantage in both the Pacoima and Santa Anita Dams now being built for the Los Angeles County Flood Control District. To illustrate, therefore, the relation between theoretical investigations and practical construction a brief general idea of these two dams will be given, together with a description of some of the methods of construction.

In the Pacoima Dam* (Fig. 53) contraction joints are provided 50 ft. apart and are made in 5-ft. sections across the dam such that each 5-ft. section is offset tangentially 12 in. with reference to its two adjoining 5-ft. sections. In this manner at least one-half the section must be sheared across before a failure due to shear can occur. Each 5-ft. section is provided with a 2-in. split grout pipe rising vertically and these are connected horizontally every 50 ft., so that the contraction joint can be pressure-grouted later. An asphalt water-stop is provided near the up-stream face, and a V-shaped copper strip near the down-stream face seals the contraction joint. Pouring was begun on August 5, 1926, proceeding at an average rate of about 1 ft. in elevation per day. The engineer each day lays out a trapezoidal section, limited on each side by radial vertical planes and extending from the down-stream to the up-stream face, and this section is poured to completion that day. The concrete elevation at the down-stream face is kept somewhat higher than that at the up-stream face, and near this latter a raised portion about 12 in. high and 12 in. wide, running tangentially, is formed to act as a water-stop for the horizontal construction joints.

A rather complete physical testing laboratory, including a 200 000-lb. Riehle testing machine, has been provided at Pacoima and standard 6 by 12-in. cylinders are tested daily, the concrete being taken both at the dam and at the mixers. Screening operations and tests for purity are carried on and the mix has been carefully designed in accordance with the theory of Duff A. Abrams,

* This dam is the highest attempted to date.


6' Tunnel Spillway El. 1950
12000 N
11800 N
11600 N
11200 E
11000 E
N
1660 TO 1760
1780
1800
1820
1840
1860
1880
1900
1920
1940
1960
1980
2000
2025
Downstream Centers
Upstream Centers
ELEV. R_s R_i
2025 340.00 332.00
2000 329.50 315.50
1980 318.29 297.29
1960 304.97 276.97
1940 292.21 257.21
1920 281.55 232.85
1900 271.54 201.47
1880 259.83 173.58
1860 251.73 152.00
1840 243.41 138.00
1820 239.58 125.00
1800 234.47 111.30
1780 228.34 97.70
1760 220.70 84.00
1740 212.00 79.00
1720 202.37 74.00
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60'
61'
62'
63'
2.7'
5.3'
18'
13'
18'
23'
28'
33'
MAXIMUM CROSS SECTION
PACOIMA DAM
GENERAL PLAN

FIG. 53.

M. Am. Soc. C. E., the cement content being determined by the limiting value of the water-cement ratio.

The maximum water-cement ratio used is 1.00, but this has been reached only about six times and then only for short periods not exceeding 1 hour during the hottest part of a day; the average water-cement ratio has been about 0.9. The aggregates used are dry-crushed, unwashed rock, the maximum size of which is 3 in., and with 1 bbl. of cement per cu. yd. of concrete the average compressive strength to date (6 by 12-in. cylinders) is a little more than 2 650 lb. per sq. in. at 28 days. This agrees well with what should be expected for a water-cement ratio of 0.9.* The water-cement ratio given is by volume and exclusive of absorption by the rock aggregates (the sand is washed and wet), which amounts to about 1.12 cu. ft. of water per cu. yd. of concrete. The total quantity of water added is, therefore, on an average, $4 \times 0.9 + 1.12 = 4.72$ cu. ft. of water per cu. yd. of concrete.

An inundator is used both at Pacoima Dam and at the Big Santa Anita Dam (Fig. 55), the latter being, as far as the writer knows, the first dam to be constructed at which an inundator has been used. The inundators have been thoroughly satisfactory so far, and the writer would not want to go back to the old method of adding water according to the judgment of the man operating the mixer. The cement is stored at the mill until 28-day concrete compression tests have been made. In this way it is possible to vary the cement ratio as may be found necessary and the compressive strength of the concrete is known before the cement goes into the dam. The standard physical and chemical tests are made by testing laboratories in Los Angeles, Calif., and an inspector is kept constantly at the mill. The California Portland Cement Company furnishes all the cement for both dams from its mills located at Colton, Calif.

Views of these dams are shown herewith, Fig. 54, of the Pacoima Dam and Fig. 55, of the Big Santa Anita Dam. The latter will be finished by about February 1, 1927, while the Pacoima Dam containing about 210 000 cu. yd., will not be finished until about a year later.

Influence of Swelling.—Referring to the original arch (Fig. 51), assume swelling of the up-stream face due to water-soaking and that the lateral deformation due to the weight of the concrete and the downward water pressure on the up-stream face is sufficient to compensate for shrinkage and temperature effects. Then (see Article XI of the paper)), let $u_e = 0.0001$ and $u_i = 0$, so that from Equation (48), $\theta = \frac{0.813}{10\,000}$ and from Equation (49), $y_c = 0.003595$; from Equations (50) and (51) are found,

$$\theta = \frac{1.0503}{1\,000} \frac{X_c}{E} (A_c - 141.904)$$

and,

$$y_c = -\frac{X_c}{E} (0.00322551\ A_c + 0.2154425)$$

* "Design and Control of Concrete Mixtures," by Portland Cement Assoc., Curve *A*, Fig. 1, p. 4.

Inserting the values of θ and y_c just found and solving the two equations gives, $A_c = 157.47 \pm$ ft. and, if $E = 2\,000\,000 \times 144$ lb. per sq. ft., $X_c = -1\,431\,200$ lb. acting on an arch slice 1 ft. thick vertically. The stresses produced by X_c acting a distance, A_c, from the center are from Equation (34), compression being negative, as given in Table 18.

TABLE 18.

Section.	Stress at extrados, in pounds per square inch.	Stress at intrados, in pounds per square inch.
Crown section	−178.0	−30.6
Abutment	−235.5	+74.5

Adding these to the stresses previously found for the original arch (see Tables 15 and 16), and since $p_e = 141$ lb. per sq. in., the resulting stresses obtained including the effect of swelling, are as given in Table 19.

TABLE 19.

Section.	Stress at extrados, in pounds per square inch.	Stress at intrados, in pounds per square inch.	Remarks.
Crown section	−112.9	+ 31.1	Original arch.
	−178.0	− 30.6	Due to swelling.
	−290.9	+ 0.5	Resulting stress.
Abutment	+ 22.6	−211.8	Original arch.
	−235.5	+ 74.5	Due to swelling.
	−212.9	−137.3	Resulting stress.

There is now compression throughout, except a very small tension, 0.5 lb. per sq. in., at the intrados of the crown. The maximum compression in the original arch is less than the resulting stress. It is quite evident that swelling is not even approximately negligible, but the writer does not claim that the assumptions here made cover the facts. Very little is known about the effect of swelling and shrinkage in a great body of concrete, but that does not warrant the assumption that it is negligible.* The writer had hoped to install some resistor type of cartridges for measuring deformations in both the Pacoima and the Big Santa Anita Dams, but was unable to obtain the necessary funds. He hopes other engineers may have better luck, since the questions involved are of considerable importance. Also, the writer felt the cartridges would afford an indication of how successful the pressure grouting was and, therefore, a guide as to how this grouting should be carried out in order to secure maximum effectiveness. As a direct result of the theoretical investigations it was possible to eliminate about $60 000 worth of steel from the Pacoima Dam.

* *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 270.

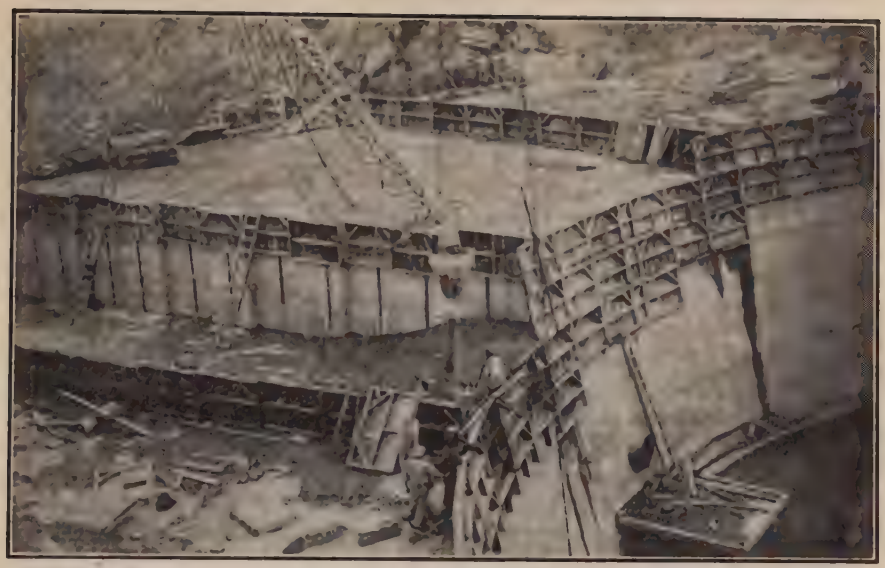

FIG. 54.—PACOIMA DAM, OCTOBER 6, 1926, SHOWING A CONTRACTION JOINT AND PART OF DOWN-STREAM FACE. BENT BROTHERS, LOS ANGELES, CALIF., IS THE CONTRACTOR.

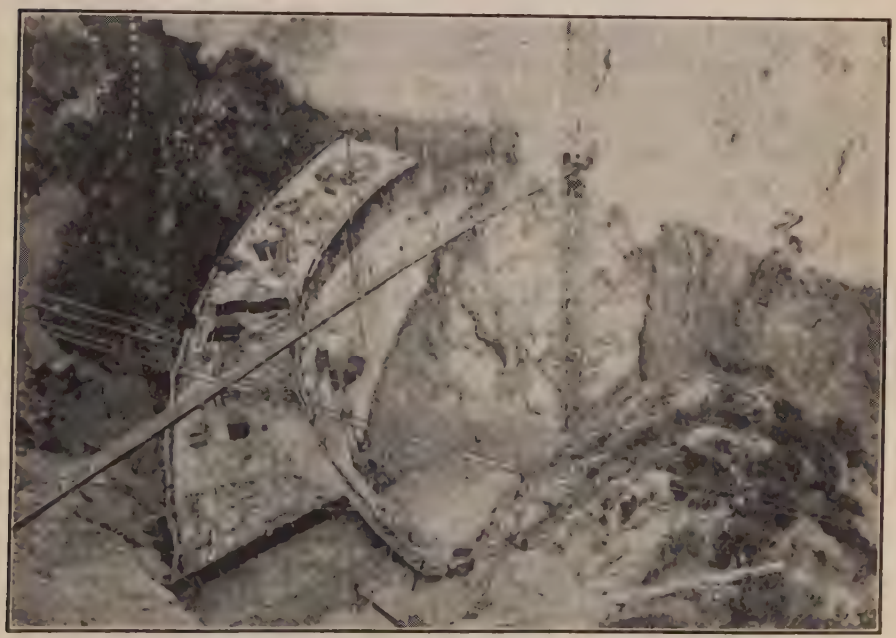

FIG. 55.—SANTA ANITA DAM, 150 FEET ABOVE BED-ROCK IN STREAM BED. MIXING PLANT TO THE LEFT NOT SHOWN. ROSS CONSTRUCTION COMPANY, LOS ANGELES, CALIF., IS THE CONTRACTOR.

Discussions.—Professor Cain's discussion constitutes, as is usual with him, a valuable contribution to a subject in which he has so largely pioneered, and the check afforded is, of course, gratifying. The writer prefers his own method of development, because it permits him to form a mental picture of the operation of the stresses and deformations, but Professor Cain is quite right when he insists on deriving the formulas by what he calls the "work method," also known as the principle of least work. This principle is so fundamental that the entire theory of structures may be derived from it without further assumptions, except Hooke's law (proportionality between stress and strain), which is a generalization of experimental results rather than a theoretical assumption. The ordinary bending theory is based on Mariotte's assumption that normal sections remain plane. This is strictly true only when E is constant and when shear is not involved. By the method of least work the stress distribution due to a moment may be found by variational calculus and without making Mariotte's assumption.* When using the principle of least work, however, care must be taken in deriving the formula for work, since any assumption made for this purpose naturally involves the final result.

An interesting illustration of how an assumption is sometimes unwittingly introduced into reasoning is given by Föppl.† It was desired to compute the deflection of a piece of bent tubing. The result obtained by applying the standard bending theory gave deflections which were only about one-fifth the measured deflections. On examination it was found that the implied assumption that the section of the tubing was not deformed, was the cause of the discrepancy. A theoretical investigation, which made use of the principle of least work as applied by Ritz‡ and took account of the fact that the section of the tube did not remain circular, led to a correct formula.

Professor Cain takes exception to the use of the Lamé formulas and the following is intended to meet his criticism. In order to determine the stresses in a cylinder subjected to external pressures, three different assumptions may be made:

1.—Neglect the radial and vertical stresses and deformations, that is, assume Poisson's ratio, $\frac{1}{m} = 0$, and assume that the curvature of the arch is so slight that the arch may be treated as a straight beam.

This assumption was made by Professor Cain in his paper, "The Circular Arch Under Normal Loads".§ It is not explicitly stated, but is indicated by his work formula, Equation (5), which neglects curvature, and by Equation (6), which neglects the rotation brought about by the normal stresses. Professor Cain at that time limited his investigation to a ratio of thickness of arch to mean radius, $\frac{t}{r} = 0.3$, and Assumption 1 may be considered a sufficiently close approximation for such arches.

* *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 335.

† "Drang und Zwang," Bd. I, Second Edition, p. 70.

‡ "Stresses in Multiple-Arch Dams," *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 325.

§ *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 233.

It may be well to insist here that the reasoning in Professor Cain's paper, "The Circular Arch Under Normal Loads", is impeccable and it is not now a question of whether or not Professor Cain's reasoning was correct, but whether the assumptions he made are sufficiently close to the truth to yield reliable results, that is, results that will accord fairly well with actual experience. In this connection it should also be emphasized that whether or not the fundamental assumptions made in deducing a formula are acceptable for a particular case occurring in actual practice, this is something the engineer himself must judge of and for which the originator of the formula can in no wise be held responsible. It is the ability to judge in this matter which distinguishes an engineer as a man of scientific training and attainments from the mere artisan, who applies formulas mechanically and without understanding.

2.—Neglect the radial and vertical stresses and deformations, as under Assumption 1, but take account of curvature.

Assumption 2 leads to Professor Cain's revised formulas, as given in his discussion of the writer's paper (see, the work formula, Equation (98)). In a footnote to this formula Professor Cain states that only the work of the radial stresses is omitted; this is not exact, since also the lateral deformations are neglected, that is, Poisson's ratio is assumed as zero. A comparison between this Equation (98) with Equation (5)* of Professor Cain's paper, "The Circular Arch Under Normal Loads", shows that the only change is from r, the mean radius, to r_n, the radius of the neutral axis. The writer's Fig. 11 shows that r_n and r_m are practically alike for small values of $\frac{t}{r_m}$.

3.—Radial as well as vertical stresses and deformations cannot be neglected, nor can curvature.

Assumption 3 leads to the Lamé stresses, used by the writer and to Equation (68) for the stresses, in which, as outlined in Article XVI of the paper, the force, X, should be determined from Equations (68), (71) and (72). Since this led to complications the writer compromised and used the Lamé stresses combined with the stresses obtained from Equation (68), in which, X' was derived as shown in his Article VI by neglecting the radial stresses and deformations due to X. The error committed is believed to be small, because stresses found from Equation (34) check quite closely those obtained from Equation (68), when the value of X is—as assumed to be—the same in both cases.

Assumption 1 implies that the force, $p_e\ r_e$, which holds the water pressure in equilibrium, acts in the middle point of the section a distance, r_m, from the center, C, of the arch. This produces a uniform compression, $-\frac{p_e\ r_e}{t}$. The resulting stress is found by adding to it the stress due to the force, X, acting at the crown section. Assumption 2, which Professor Cain accepts, implies that the force, $p_e\ r_e$, acts at the neutral axis a distance, r_n, from the center, C. This is equivalent to a force, $p_e\ r_e$, acting in the middle of the sec-

* *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 236, and p. 268, where the influence of shear is added.

tion and producing a uniform compression, plus a moment, $M = p_e r_e (r_m - r_n)$, producing compression at the intrados. The stress due to this moment is given by the writer's Equation (31) and also by Professor Cain's Equation (93). Combining this uniform stress with the bending stress leads to Professor Cain's Equation (97*a*), in which, $P = p_e r_e$ To these stresses must be added those produced by X or X', which are determined in the same manner by both Professor Cain and the writer.

The only difference between Professor Cain's results and those of the writer is due to the difference between Assumptions 2 and 3; Professor Cain calculates the stresses due to water pressure by Equation (97*a*), while the writer used the Lamé stresses. That there is not a great deal of difference, except for very thick arches,* is shown by Table 20, which was calculated for the short thick arch of Fig. 51. The stresses are shown in Fig. 56. The maximum differences occur at the faces and amount to only 11 and 12.4 per cent.† When the maximum stress (compression) is computed (page 577), the difference between the two values is quite small and either value is acceptable.

Professor Cain states:

"The only stresses then to be considered in this sliding stage [see Fig. 2] are the uniform stresses, $\frac{R}{t}$, for which plane [radial] sections remain plane after strain, so that the sliding can be effected."

TABLE 20.

r, in feet.	Stresses, in Terms of p_e.	
	Lamé, Equation (36).	Cain, Equation (97*a*).
191	— 1.913	— 1.725
177	— 1.900	— 1.863
163	— 2.084	— 2.022
149	— 2.207	— 2.212
135	— 2.371	— 2.440
121	— 2.595	— 2.725
107	— 2.913	— 3.160

The stress required to cause only translation of a section is not a uniform stress, but is given by Professor Cain's Equation (97*a*), using Assumption 2, as he shows himself. This was also shown by the writer in his Articles V and VI, in which the radial stresses and deformations were neglected. When, however, radial stresses and deformations are to be considered, the Lamé stresses are needed to conserve radial sections plane, as may be shown by applying Equation (72) to the Lamé stresses. Referring to Fig. 57, u_r and u_t are the

* For the very thick arch, $\frac{t}{r_m} = 1.00$, the difference is considerable, but is due solely to the difference between Assumptions 2 and 3, as may be readily checked. Since tension is involved in both cases, the question arises as to how great the difference would be between the maximum stresses in the two cases, when these are determined as shown (see page 577). The writer has not checked this, but believes the difference would not be material.

† It should be noted that these stresses are not the final stresses; the stresses produced by X must be added, see Table 17.

radial and tangential displacements of the point, P, positive in the directions indicated. Likewise, σ_r and σ_t are the radial and tangential stresses. The Lamé stresses are given Equations (36) and (37), and the shear is zero. If the Lamé stresses are introduced into the equilibrium, Equations (71), it will be found that these stresses satisfy Equations (71) and that Professor Cain's Equation (97*a*) does not, as already pointed out by the writer (page 506).

From the Lamé stresses and the first equation of Equations (72).

$$E \frac{\delta u_r}{\delta r} = -\left(1 - \frac{r_i^2}{r^2}\right) k + \frac{1}{m}\left(1 + \frac{r_i^2}{r^2}\right) k$$

in which,

$$k = \frac{r_e^2 p_e}{r_e^2 - r_i^2}$$

Integrating toward r, gives,

$$E u_r = -k\left(\frac{m+1}{m}\frac{r_i^2}{r} + \frac{m-1}{m} r\right) + f(\phi)$$

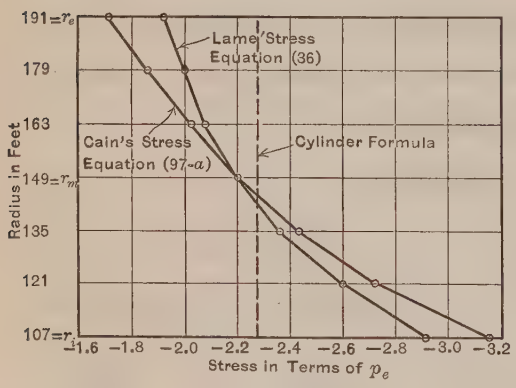

FIG. 56.

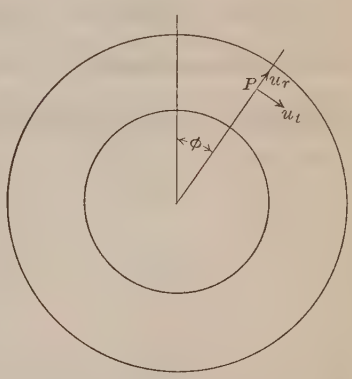

FIG. 57.

in which, $f(\phi)$ designates a function of ϕ, which may be constant or zero, but does not contain r, that is, is independent of r. From the second equation in Equations (72) and the Lamé stresses,

$$E\left(\frac{\delta u_t}{\delta \phi} + u_r\right) = -k\left(\frac{m+1}{m}\frac{r_i^2}{r} + \frac{m-1}{m} r\right)$$

Substituting the value of $E u_r$ found previously,

$$E \frac{\delta u_t}{\delta \phi} = f(\phi)$$

or,

$$E u_t = \int f(\phi)\, d\phi + F(r)$$

in which, $F(r)$ designates a function of r, which is independent of ϕ. From the third equation in Equations (72),

$$E \frac{\delta u_r}{\delta \phi} + r E \frac{\delta u_t}{\delta r} = u_t E$$

Introducing the various values found previously,

$$\frac{\delta f(\phi)}{\delta \phi} + r \frac{\delta F(r)}{\delta r} = \int f(\phi)\, d\phi + F(r)$$

This equation is satisfied when $f(\phi) = 0$ and $F(r) = 0$, so that, $E u_t = 0$, and,

$$E u_r = -k \left(\frac{m+1}{m} \frac{r_i^2}{r} + \frac{m-1}{m} r \right) \ldots\ldots\ldots\ldots\ldots\ldots (136)$$

That these two equations satisfy Equations (72) may be readily verified.* Equation (136) for u_r affords an opportunity to investigate whether and when the thickness, t, of the arch is decreased or increased due to water pressure. Inserting $r = r_e$ and $r = r_i$, and subtracting, gives,

$$E(u_{re} - u_{ri}) = -k \left(\frac{m+1}{m} \frac{r_i^2}{r_e} + \frac{m-1}{m} r_e - 2 r_i \right)$$

$$= k \left(\frac{2}{m} t - \frac{m+1}{m} \frac{t^2}{r_e} \right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (137)$$

When this is positive the thickness, t, of the arch is increased. For $m = 8$, for example, the following holds: If $\frac{t}{r_e} = \frac{2}{9}$, t remains constant; if $\frac{t}{r_e} > \frac{2}{9}$, t decreases (thick arches); and, if $\frac{t}{r_e} < \frac{2}{9}$, t increases (thin arches).

For the arch shown in Fig. 51, it is,

$$u_{re} - u_{ri} = -30.1 \frac{p_e}{E}$$

that is, t is decreased. If $m = \infty$, and the radial stress decreases linearly from p_e to zero, the contraction would amount to $42.0 \frac{p_e}{E}$. As a comparison, the displacement, $DF = e'$, in Fig. 2, is $r_n \varepsilon'$, and ε' was $-2.17 \frac{p_e}{E}$ (see page 577), so that $e' = -415.0 \frac{p_e}{E}$.

Professor Cain discusses the influence of shear on tangential stresses; Equation (68) takes this influence into account, on the assumption that the arch is free to deform at the abutment as this stress distribution requires. In order to find the deformations, the writer made an attempt to solve Equations (72) when the stresses were those given by Equation (68). This would give the warping of the originally plane radial sections, but would not be a solution after all, because the arch is not free to warp its radial section at the abutment, but must conform to the abutment deformations (See pages 554 and 506, and especially Fig. 45).

What is here involved is more correctly speaking the influence of the shear deformation (warping) on the tangential stresses. Judging from the fact that the arch computed in Article XII by Equation (34) gives practically

*The assumption, $u_t = 0$, is made in developing the Lamé formulas, see, for example, Bach-Baumann, "Elasticitat und Festigkeit," 9th Edition (1924), p. 571. The calculations just given show that this assumption is correct.

the same stresses as when the more nearly correct Equation (68) is used, the writer concludes that the influence of this shear distortion on the tangential stresses can not be great and Dr. Vogt comes to the same conclusion. It is also clear, as Professor Cain points out, that the crown section must remain plane and radial, while the abutment section cannot remain plane, since unevenly distributed shear exists in every section except at the crown.

Professor Cain's discussion is a very welcome addition to the writer's paper, and he is pleased to note that for all practical purposes agreement exists. The writer believes that a dam designed by either formula will result in a safe and economical structure. He would like to add that, in his opinion, neither Assumption 1 or Assumption 2 is exact since Poisson's ratio is not zero and, on the other hand, the arch is not free to deform laterally at the abutments, as assumed in the Lamé formulas.

In order to find the true stresses in the arch in Fig. 51 or in the part lying between the abutment and Section *a*, the fundamental equations of Article XVI, as already stated, would have to be applied. It is quite likely that this would be very complicated and that Professor Cain's Equation (98) would lend itself much more readily and with quite sufficient accuracy to such a stress determination. The writer would like to suggest that Professor Cain make such an attempt since this would be of great value to the engineer in determining the actual maximum compression on the assumption that tension cannot be transmitted.

Professor Harris brings up nothing new. That cantilever action ought to be considered is generally agreed, but how this can be done is the question, and Professor Harris contributes nothing to its solution. The writer is reminded of a remark by Lord Kelvin to the effect that "a phenomenon is well known, when it is known in figures", and there are no figures in Professor Harris' discussion. The writer tried some years ago to take account of the cantilever by using the method of least work as applied by Ritz,* but he came to the conclusion that the calculations, even if possible, were so extremely complicated as to be of little practical use, and that in order to render them fairly simple so many approximations would have to be made in the assumptions, that he had no confidence in the results. At that time he also read over the discussion of plates given by Föppl† which is quite involved despite the fact that he does not consider temperature influences, swelling, and shrinkage, but only plates in which $\frac{t}{r_m}$ may be assumed to be small. Add to that the fact that plates are generally symmetrical while dams are not and the resulting mathematical complications are obvious.

Professor Harris seems to consider only the cylinder formula and he concludes that no matter how close together the canyon walls are, a dam with a straight face carries all the load on the cantilever, while the truth is that it carries but very little load in that manner, since the load is carried by the

* "Stresses in Multiple-Arch Dams," *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 325.

† "Drang und Zwang," Bd. I, Second Edition, pp. 123 to 244.

horizontal secondary arches.* The cylinder formula is approximately correct for fairly thin long arches, but it is not a general formula and when used as such in reasoning it leads to absurdities. Referring to Fig. 2, if a uniform stress, σ, is applied to the semi-arch the displacement, $e = DF$, at the crown is,

$$e = -\int \frac{\sigma}{E} r \cos \phi \, d\phi = -\frac{\sigma}{E} r \sin \phi_1$$

and this is not zero except when $E = \infty$. This is not an assumption that can be safely made, but one that must be shown to lead to a fair approximation in each specific case. It is also evident that if a sufficient number of simple assumptions be made, it is quite easy to determine the distribution of load between the cantilever and the arches, but this would afford merely an exercise in elementary mathematics and no competent engineer could be induced to trust to the results of such formulas.

Mr. Jorgensen considers the formulas presented by Professor Cain and by the writer as somewhat elaborate for every-day use. These formulas are for use with thick arches, which are expensive. In the case of the Pacoima Dam, for example, the saving of the constant angle arch dam over a gravity dam is in excess of $1 000 000, and for a small fraction of this amount some very extensive theoretical investigations could be and should be made. Mr. Jorgensen has possibly confused the work of deriving these formulas with that of applying them and while the writer knows nothing about bridge design, he doubts whether a careful theoretical investigation of the Pacoima Dam would require more time or skill than is required for the proper design of a bridge costing approximately the same, that is, $2 000 000. The writer doubts very much whether it will ever be possible to obtain empirical formulas for dams of a sufficient range to be of much value. Tests on models, however, might be made valuable as a help to theoretical considerations.

Mr Jorgensen refers to some tests on models, made by B. E. Torpen, M. Am. Soc. C. E., in connection with the construction of the Lake Cushman Dam. The tests showed, as Mr. Jorgensen states, that the water pressure required to break one of these arches, or perhaps several, would give rise to a tangential stress (computed by the cylinder formula), about 100% higher than the compressive stress found for the concrete when tested in ordinary 6 by 12-in. compression cylinders. Mr. Jorgensen then states: "* * * and it was demonstrated * * * that at or near the point of failure the stresses were distributed evenly over the section." As a matter of fact, this was not demonstrated, since no determination was made of the stress distribution at or near breaking. Mr. Jorgensen evidently reasons that since the stresses found in arches by the formulas of Professor Cain or the writer are higher than the cylinder stresses (and that is especially true of the stresses in the secondary arches given by the writer), and since the water pressure required to break these arches corresponds to a cylinder stress of 100% in excess of the strength of the concrete in 6 by 12-in. cylinders, the cylinder formula applies,

* "Formes et dimensions des grands barrages en maçonnerie," Résal, *Annales des Ponts et Chaussées*, p. 26, and, on p. 29, where he states that when the length of a straight dam equals 2.5 times the height, the arch action is insignificant, but if the length does not exceed one-third the height, it is the arch that carries all the load.

at least more nearly than the others. However, 100% in excess is not a very satisfactory check and certainly such a divergence should not be taken as proof that the stresses were uniformly distributed.

The writer stated specifically in his paper, that the secondary arch stresses were not the true stresses, but were higher than the true stresses and, therefore, safe to use. He has already shown that the maximum stress in a thick arch is very likely less than the cylinder stress. The model arch did not break at the abutment, probably on account of lateral restraint, but the fact that the cylinder formula gives results that are 100% in excess of the compressive strength of the concrete, coupled with the fact that the arch did not break at the abutment, leads the writer to conclude that the cylinder formula is certainly incorrect.

The writer thoroughly agrees with Mr. Jorgensen that it is not safe to assume that any load is taken by the cantilever, for the reason that at present no method of obtaining the correct distribution is known.

Dr. Vogt's discussion is a valuable contribution to the subject in that he makes it possible to take the yielding of the abutments into account with but little more work than is required when it is neglected. The writer had prepared a discussion of the influence of the yielding of the abutments based on Dr. Vogt's thesis*, but Dr. Vogt's discussion covers the subject much more ably. His criticism of the writer's Equation (48) is just; the radial deformation was neglected. As already discussed in replying to Professor Cain's criticism, it is not correct to neglect radial deformation, and on the other hand near the abutments the radial deformation is no doubt restrained to some extent, even if it be argued that the rock near the abutment can not vary much in temperature from the concrete near the abutment. The writer prefers Dr. Vogt's Equation (117) to his own Equation (48), but the difference in the results obtained by these formulas is probably less than the probable error of the temperatures, etc., involved.

Dr. Vogt's Equations (119*b*) and (121*b*) take into account temperature effects, swelling, shrinkage, the lateral deformation due to the vertical stress, σ_x, and the deformation of the abutments. All things considered, these equations are remarkably simple. They may, however, be still further simplified by assuming the angle, $\psi_1 = 0$ (see Fig. 44), which assumption he believes is usually approximately correct and when not correct the influence of this angle, ψ_1, is very difficult to determine, as Dr. Vogt states; moreover this angle is not likely to be the same for both abutments (see, for example, Fig. 51). The writer also prefers to write the equations so that they are independent of the assumption that the modulus of elasticity for rock and concrete is the same, although this is probably nearly correct after the concrete is a year or two old. Johnson,† for example, gives for Branford (Conn.), granite, 8 333 300; for Milford (Mass.), granite, 6 663 000; and for Troy (N. H.), granite, 4 545 400; these moduli are for working loads for granite under compression and are taken from tests made at the Watertown

* "Ueber die Berechnung der Fundamentdeformation," *Det Norske Videnskapsakademi*, Oslo, 1925, No. 2.

† "The Materials of Construction," 4th Edition (1912), p. 645.

Arsenal. Stanton Walker, Assoc. M. Am. Soc. C. E., has shown* the rapid increase of the modulus with age, and that, for example, for a concrete with aggregates up to 2 in., the modulus at 7 days is about 2 500 000 lb. per sq. in., while at 1 year, it is 7 000 000 lb. The writer also prefers to omit Dr. Vogt's constant, q, as defined in Equation (116).

Let E_f equal the modulus of elasticity of the rock; and E that of concrete. Also, referring to Equation (112), let, $\xi' = \xi \frac{E}{E_f}$; $\eta' = \eta \frac{E}{E_f}$; and $\mu' = \mu \frac{E}{E_f}$. In case $E = E_f$, $\xi' = \xi$, etc. Then, with the angle, $\psi_1 = 0$ (see Fig. 44), Dr. Vogt's Equations (119b) and (121b) can be written more simply, as follows:

$$X'' \left\{ r_n \sin \phi_1 + \mu' c \frac{r_m}{t} \cos \phi_1 - \eta' c \sin \phi_1 \right\} - X'' A'' \left(\phi_1 + \mu' \frac{c}{t} \right) = p_e r_e \mu' \frac{c^2}{t} + \Delta u E c r_m \phi_1 \quad \ldots\ldots(138)$$

and,

$$X'' \left\{ -0.5 r_n \left(2 \sin \phi_1 - \phi_1 - \frac{1}{2} \sin 2\phi_1 \right) + c \left(c_1 \phi_1 - \frac{c_2}{2} \sin 2\phi_1 \right) + 12 \xi' \frac{c^2}{t} - \eta' c \sin \phi_1 \left(2 \frac{r_m}{r_n} \cos \phi_1 - 1 \right) + \mu' c \frac{r_m}{t} \left(\frac{r_m}{r_n} \cos \phi_1 - 1 \right) \cos \phi_1 \right\}$$

$$+ X'' A'' \left\{ (\phi_1 - \sin \phi_1) + \eta' \frac{c}{r_n} \sin \phi_1 - \mu' \frac{c}{t} \left(\frac{r_m}{r_n} \cos \phi_1 - 1 \right) \right\}$$

$$= p_e r_e c \left\{ \sigma \sin \phi_1 + 12 \xi' \frac{c}{t} \cos \phi_1 - \eta' \frac{c}{r_n} \sin \phi_1 + \mu' \frac{c}{t} \left(\frac{r_m}{r_n} \cos \phi_1 - 1 \right) \right\}$$

$$- 12 u_n c^2 \frac{r_n}{t} E \sin \phi_1 - \Delta u E c r_m (\phi_1 - \sin \phi_1) \quad \ldots\ldots(139)$$

Equations (138) and (139) have been carefully checked and the example given by Dr. Vogt has been calculated to make sure that no error was committed in re-writing them. There is undoubtedly a considerable uncertainty in determining the coefficients, ξ, η, and μ in Equations (112). The values used by Dr. Vogt† are nearly the mean values from Equations (112), and these may no doubt be used quite generally.

In using Equations (138) and (139) some care must be taken since u_n in accordance with Dr. Vogt's Equation (120) for e' includes the temperature effect, swelling, and shrinkage, besides the deformation due to the vertical stress, σ_x, which latter only is included in the writer's Equation (38) to which Dr. Vogt refers. Equations (117) and (118) should be used, but as stated, u_e and u_i contain also the deformation produced by σ_x, which is (see Equation (38)) $-\frac{\sigma_x}{m E}$. Since the stress, σ_x, does not produce rotation

* "Modulus of Elasticity of Concrete," *Bulletin 5*, Structural Materials Research Laboratory, Lewis Inst., Chicago, Ill., January, 1920, Edition, p. 41.

† See p. 563; the mean values from Equations (112) are $\xi = 1.77$; $\eta = 0.68$; and $\mu = 5.34$.

of radial sections, it is to be added to both u_e and u_i and, therefore, does not affect θ from Equation (117). The two constants, c_1 and c_2, in Equation (139) are given by Dr. Vogt in the equations immediately following Equation (116) and the values used in the paper are, $c_1 = 1.94$ and $c_2 = 0.94$; and σ is obtained from Equation (120).

Dr. Vogt assumes the resultant, $R = p_e r_e$, of the water pressure as acting in the neutral axis (see Fig. 44 and Equations (114) for the moment, M_1, referred to the center of gravity axis), so that R produces a moment equal to $Rc = R(r_m - r_n)$. This is, as Dr. Vogt states, only approximate, and the writer thinks it sufficiently correct even if R actually acts at a point farther from the center, C, of the arch (see Fig. 44), namely, at a distance, r_L, as given by Equation (131). This could be taken care of by substituting for $p_e r_e c$, in Equations (138) and (139), $p_e r_e (r_m - r_L)$. However, this is not correct either, since this moment would not give the Lamé distribution of stresses and, therefore, the correction is hardly worth while. Dr. Vogt, as well as the writer, determined the stresses produced by $p_e r_e = R$ in the arch, from Equation (36).

When in Equations (138) and (139), the constants, ξ', η', and μ', are put equal to zero and the proper value for u_n is introduced (that is, $u_n = 0$, if no temperature effects, swelling, and shrinkage and no lateral deformation due to σ_x are to be considered), the writer's Equations (18) and (29a) result, in which, X' is given by Equation (30) and $\sigma_x = 0$ in Equation (38). If the influence of σ_x is to be included in Equations (138) and (139), then $u_n = -\frac{\sigma_x}{m E}$; here, u_n is positive since σ_x, being a compression, is negative.

Equation (127), for the deflection of the arch when the yielding of the abutment is considered, is simple to use in connection with Fig. 46 and shows that the deflection in the example considered by Dr. Vogt is twice as great when the abutment yielding is considered as when it is assumed to be negligible, which it is not. Equation (127) is obtained by considering the arch to have a somewhat larger central angle than it actually has. This can not be used in connection with the stresses, since it would lead to a decrease in all stresses, while as a matter of fact the yielding of the abutment increases the tension at the intrados of the crown. (See page 560 and Table 12.) The writer hopes that Dr. Vogt will publish his proposed paper on the distribution of load between the arches and the cantilevers, when the yielding of the abutments of the arches and the foundations of the cantilevers are taken into account, as this would be a great step toward a rational solution of an important problem in dam design and, one which would likely lead to considerable savings in material. Dr. Vogt's excellent work has already prepared the way for this.

Dr. Vogt does well in emphasizing, as he does repeatedly in his discussion, that the results obtained from his calculations can only be roughly approximate; the same is true of most stress determinations in which anything but simple stresses are involved. This is due to the fact that the various assumptions which must be made in order to treat the subject mathematically and

without too great complications, are only approximate. In order not to be misunderstood, it should also be emphasized and insisted on that this is not an admission that the engineer who reasons by "hunches" and from "common sense", and without any specific knowledge of the subject involved, is as likely to arrive at the correct or approximately correct result, as he who starts from fundamental assumptions and develops his subject mathematically, which is the only way it can be developed. Common sense is quite requisite, but it does not supersede specific knowledge* and, moreover, the engineer with some training in mathematics has as valid a claim on common sense as any one.

Mr. Paaswell's discussion brings up some interesting points. The formula for determining the temperature variations given by Mr. Paaswell, and quoted from his work on retaining walls,† checks quite well with the results of tests, and ought to prove of value to engineers in the design of dams. Mr. Paaswell considers a thin section of an arch barrel treating it as a surface. This development the writer cannot follow as he is not acquainted with Love's treatise. From Fig. 48 the writer gathers that since the moment in the arch at the crest is zero the crown deflection is zero, and this is not in agreement with what has been generally found for arch dams. Some tests made on the deformation of a cylinder,‡ however, show no deformation at the crest, and, of course, Mr. Paaswell's calculations are not claimed by him to be directly applicable to an arch dam, as usually built. If an equation could be developed for an arch with variable central angle and variable thickness, this might afford a means of analyzing arch action and cantilever action combined. This might be of value even if it should become necessary to resort to some simplifications of the assumptions, such, for example, as that the neutral and gravity surfaces (axes) coincide, as was assumed by Mr. Paaswell. The writer sincerely hopes that Mr. Paaswell will return to this subject and make an attempt to apply his method of attack.

Mr. Perkins has been charged for a number of years with the investigation of designs of dams submitted to the State Engineer of California and in this capacity he has accumulated a considerable knowledge of the subject. The writer agrees with Mr. Perkins' discussion except perhaps that he places less confidence in the calculations of the division of load between cantilever and arches. Mr. Perkins' remarks concerning the secondary arches are true and have already been dealt with in a fashion, but there still remains to be proven that X' and A' remain approximately unchanged.

Mr. Perkins states that the "uncertainty in the value of E does not have so much bearing because of the fact that E enters in the same way in both cantilever and arch action, and, therefore, in a sense, is cancelled." This is not quite correct in large dams which require considerable time to construct, due to the fact that E increases with age, so that although E may be fairly

* See Kant's scathing remarks in his "Prolegomena," edited in English by Dr. Paul Carus, The Open Court Publishing Co. (1909), Introduction, p. 6: "To appeal to common sense, when insight and science fail, and no sooner—this is one of the subtle discoveries of modern times, by means of which the most superficial ranter can safely enter the lists with the most thorough thinker, and hold his own. But as long as a particle of insight remains, no one would think of having recourse to this subterfuge."

† "Retaining Walls: Their Design and Construction."

‡ "Experimental Deformation of a Cylindrical Arched Dam," by B. A. Smith, M. Am. Soc. C. E., *Proceedings*, Am. Soc. C. E., October, 1926, Papers and Discussions, p. 1596.

constant for any horizontal arch, it varies from elevation to elevation, and is not constant for the cantilever, but may vary quite considerably, just as does the strength.* Also, the wetting of the up-stream face may be of considerable importance, as well as the yielding of the abutments. A study such as Mr. Perkins' Fig. 49, made in order to determine the probable division of load between the arches and the cantilever, is always of interest, but the engineer should bear in mind the fact that important factors have been left out of consideration and that, therefore, the results should not be taken too seriously.

Mr. Noetzli in his discussion remarks "that the author has introduced a number of assumptions * * *", while the truth is that the writer reduced the number of assumptions and it is this circumstance which leads to the greater mathematical complexity. In deriving the stresses by the Lamé formulas and Equation (68) there is assumed only that E and m are constants. In deriving Equation (34) it was further assumed that radial sections remain plane and that Poisson's ratio is zero. Finally, in deriving the cylinder formula, it is assumed that the stress distribution is uniform over the cross-section. The assumptions which Mr. Noetzli lists, he will find in the writer's paper as well as in his discussion of Mr. Noetzli's paper, "Gravity and Arch Action in Curved Dams".† Regarding Mr. Noetzli's quotation from the writer's discussion of the paper on "Gravity and Arch Action in Curved Dams", the writer devoted about eighteen pages to this and is quite content to rest his case there. He wishes to add, however, that if he were to design an arched dam and had the choice between the cylinder formula and Mr. Noetzli's theory, he would choose the cylinder formula for the reasons given in the aforementioned discussion. He might add that, according to Mr. Noetzli's theory, the Kerckhoff Dam, designed by the writer‡ should long ago have failed as it should have high tension in the heel, but this dam is still functioning and earning dividends.

Regarding Mr. Noetzli's statement as to what Professor Cain thinks about deflections of arched dams, the writer would refer to Professor Cain's own statement (page 546). Rather than reply in detail to several of Mr. Noetzli's other statements, the writer would quote Henri Bergson, as given by Will Durant:§

"I believe that the time given to refutation in philosophy is usually time lost. Of the many attacks directed by the many thinkers against each other, what now remains? Nothing, or assuredly very little. That which counts and endures is the modicum of positive truth which each contributes. The true statement is of itself able to displace the erroneous idea, and becomes, without having taken the trouble of refuting any one, the best of refutations."

The writer believes Mr. Noetzli is somewhat too optimistic about what the tests on the Stevenson Creek Dam will bring forth. It is one thing to deter-

* *Bulletin 5*, Structural Materials Research Laboratory, Lewis Inst., Chicago, Ill., January, 1921, Fig. 22, p. 41, E varies logarithmically with the age, for example, from 2 500 000 at 7 days to 7 000 000 at 1 year.

† *Transactions*, Am. Soc. C. E., Vol. LXXXIV (1921), p. 92.

‡ *Loc. cit.*, p. 107.

§ "The Story of Philosophy," 1926, p. 503.

mine a few constants, as is done, for example, when the over-all efficiency of a power plant is determined, and it is quite another matter to derive a natural law, especially when this is to be done from a limited number of tests on a single structure. The writer speaks from experience, for in 1914 he made some tests on corona losses on a high-tension transmission line in the Peruvian Andes with a view to determining the law for these losses* and he, therefore, believes that he has a fair appreciation of the difficulties to be overcome. To that may be added that it is not directly obvious how tests on a relatively thin dam can furnish any information as to the stress distribution in thick dams. The upper part of the test dam is so thin and long that there seems to be danger of failure by buckling rather than failure in arch action. The writer has investigated the test dam, using the formula for buckling,†

$$p_c = \frac{E}{4}\left(\frac{t}{r}\right)^3$$

in which p_c is the critical water pressure, that is, the pressure at which buckling may be expected. He called this to the attention of one member of the Los Angeles Committee in charge of the tests several months before concrete pouring was started and he was told that the matter had been called to the attention of other members of this local committee, but apparently nothing was done to remedy it. If buckling should be found to have actually occurred the applicability of the test to actual dam design will be extremely limited.

In conclusion, the writer wishes to quote Professor Cain's closing remarks as they so exactly and so aptly express his own attitude toward the whole subject:

"The author has written very convincingly of the serious effect of water-soaking in an arch dam. There can be no doubt of its influence, of the deformation of the foundation, and of the variation in E. He seems well aware that an exact solution of the arch dam is not to be looked for, so that all an engineer can do is to examine the various influences and combine them to effect a practical solution."

The writer wishes to thank those who have contributed to the discussion of his paper and thereby added materially to its value.

* "Corona Tests at High Altitude," *Transactions*, Am. Inst. Elec. Engrs., Vol. XXXVII, Pt. I (1918), p. 91.

† "Drang und Zwang," by A. Föppl and L. Föppl, Bd. I, Second Edition, p. 94.

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Paper No. 1604

TREND OF CONSTRUCTION COST OF CERTAIN PUBLIC UTILITIES*

By William Breuer,† Jun. Am. Soc. C. E.

With Discussion by Messrs. Frank C. Wight, J. W. Hummer, and William Breuer.

Synopsis

In 1925 a valuation was made of one of the largest public utilities in the United States and in connection therewith a detailed study of the trend, year by year, from 1914 to date, of the construction cost of various public utility properties.

It was found that, since 1918, whether a street railway, a gas or an electric property, a power plant or a distribution system, a sub-station or a transmission line, for all these, regardless of location or magnitude, the composite trend was substantially the same. In no instance was an exception found.

An appraisal of a large electric light and power property was made about a year or two ago (1925), as of present-day prices; and for retirement purposes an investigation to determine approximately the original cost of any property in question was also made.

It is the practice in making such analyses to use either 1913 or 1914 price levels for a base. We are, however, in a new and normal period of post-war prices and because more accurate results are obtained by the use of present-day cost data, a 1924 base was used. Furthermore, 1913 was a year of depression, and in order to determine whether the results obtained were typical of utilities as a whole, other light, power, and street railway properties in various parts of the country were also trended.

The electric light and power property was, of course, first analyzed, and it was necessary to adhere to the Classification of Accounts prescribed by the

* Published in March, 1926, *Proceedings*.

† Engr., Day & Zimmermann, Inc., Philadelphia, Pa.

Public Service Commission. A separate trend was compiled for each of more than thirty accounts, and a composite trend derived by combining into a weighted average these thirty accounts. The trends for these accounts were not at all uniform which was also true of each item of each account.

In a similar manner a gas property was analyzed, and as its composite trend was substantially the same as that of the electric light and power property, it was thought that this was merely a coincidence. A street railway property was then studied, but its composite trend did not differ greatly from that of the gas or electric property. Other light and power and street railway utilities were also analyzed, but regardless of location or magnitude no exception was found.

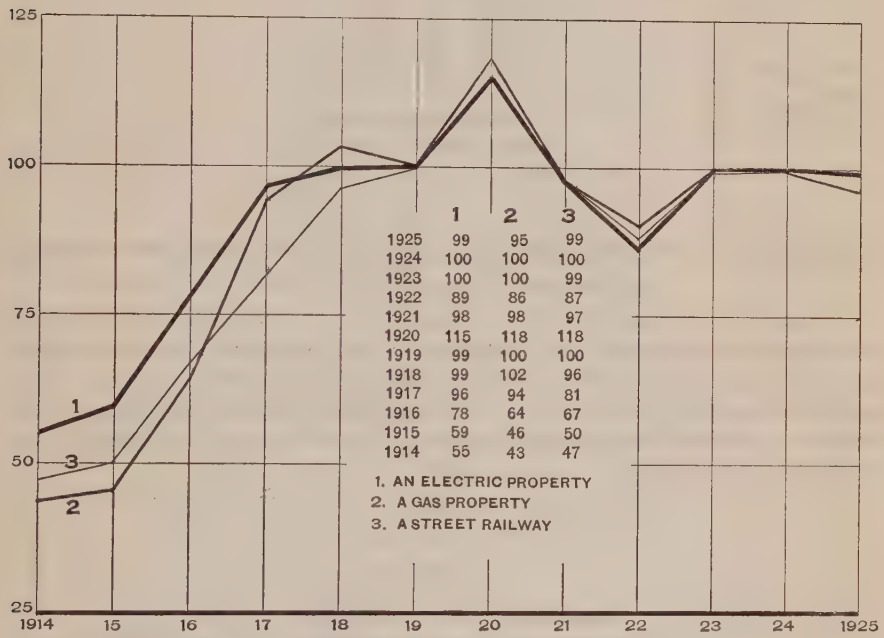

FIG. 1.—COMPOSITE PUBLIC UTILITY TRENDS.

A study was likewise made of the trend of steam power plants, the plants studied representing a wide range of conditions and many stages in the progress of the art of power plant design. Some were equipped with turbo-generators, others with engine-driven generators, with conversion equipment for street railway purposes, and even with ice plant machinery, and the installations ranged from capacities of 120 to more than 100 000 kw. A study was also made of electric distribution systems and of transmission lines, but again in no instance was an exception found, in that all power plant trends were about the same and all distribution trends the same.

Several months were necessary to make the analyses and at no time were references made to other sources of information. Of course, the research work was very exhaustive. The material trends used were those secured from manufacturers which were further checked by means of unit costs from purchase order records.

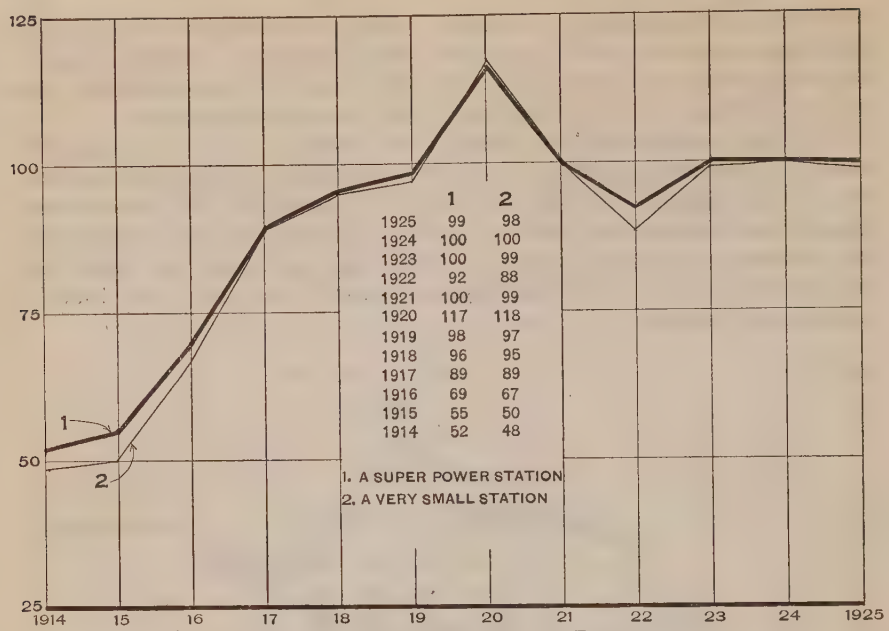

FIG. 2.—COMPOSITE POWER PLANT TRENDS.

So many of the data are also confidential and not for publication that it is rather difficult to include the fundamental facts from which the conclusions have been derived. There is, however, submitted a tabulation (Table 1) in which the method of procedure is shown in detail. Table 1 is typical of the analyses, and in order to show more clearly the results obtained, several charts have also been prepared.

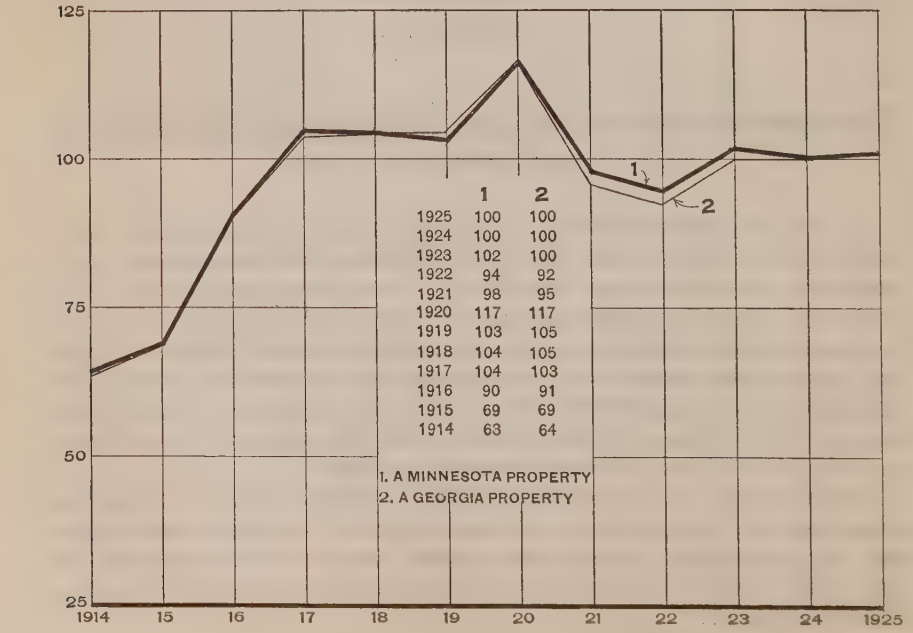

FIG. 3.—COMPOSITE DISTRIBUTION LINE TRENDS.

TABLE 1.—COMPOSITE TREND FOR A STEAM POWER PLANT.

Year.	Power plant building.	Yard improvements.	Boilers.	Boiler brick work.	Soot blowers.	Stokers.	Steel stack.	Breeching.	Economizers.	Induced fans.	Forced-draft fans.	Forced-draft air-ducts.	Feed-water heaters.	Pumps.	Tanks.	Piping, meters, gauges.	Coal-handling machinery.	Locomotive cranes.
	34.6%	3.1%	6.9%	1.3%	0.3%	2.7%	1.6%	1.1%	1.9%	0.5%	0.6%	0.3%	0.6%	0.3%	0.1%	8.0%	3.0%	0.4%
1925	33.9 98	3.0 98	6.9 100	1.2 96	0.3 100	2.7 100	1.6 97	1.1 97	1.9 100	0.5 100	0.6 100	0.3 97	0.6 100	0.3 100	0.1 97	8.0 100	3.2 105	0.4 96
1924	34.6 100	3.1 100	6.9 100	1.3 100	0.3 100	2.7 100	1.6 100	1.1 100	1.9 100	0.5 100	0.6 100	0.3 100	0.6 100	0.3 100	0.1 100	8.0 100	3.0 100	0.4 100
1923	34.6 100	3.1 100	6.9 100	1.2 93	0.3 100	2.7 100	1.9 116	1.3 116	1.9 100	0.5 100	0.6 100	0.3 116	0.6 96	0.3 99	0.1 116	8.0 100	2.9 95	0.4 93
1922	30.4 88	2.7 88	6.6 96	1.0 74	0.3 96	3.1 116	1.5 95	1.0 95	1.8 96	0.5 100	0.6 100	0.3 95	0.6 96	0.3 99	0.1 95	7.4 92	2.6 87	0.3 84
1921	32.5 94	2.9 94	6.9 100	1.0 79	0.3 100	2.7 98	1.7 106	1.2 106	1.9 100	0.5 105	0.6 105	0.3 106	0.6 95	0.4 124	0.1 106	8.1 101	3.3 111	0.4 104
1920	39.8 115	3.6 115	8.8 128	1.3 103	0.4 128	2.2 81	2.1 128	1.4 128	2.4 128	0.8 150	0.9 150	0.4 128	0.6 108	0.4 141	0.1 128	9.4 118	3.8 125	0.5 124
1919	31.1 90	2.8 90	6.9 100	0.8 63	0.3 100	2.2 81	1.9 121	1.3 121	1.9 100	0.6 120	0.7 120	0.4 121	0.6 105	0.4 128	0.1 121	8.6 107	3.3 111	0.4 111
1918	27.7 80	2.5 80	6.9 100	0.7 56	0.3 100	2.2 83	2.6 160	1.8 160	1.9 100	0.6 110	0.7 110	0.5 160	0.5 83	0.3 113	0.2 160	8.0 100	3.2 105	0.5 137
1917	24.6 71	2.2 71	6.4 93	0.7 53	0.3 93	1.8 68	2.8 174	1.9 174	1.8 93	0.4 83	0.5 83	0.5 174	0.4 66	0.2 81	0.2 174	7.2 90	3.0 100	0.5 122
1916	20.4 59	1.8 59	4.2 61	0.5 37	0.2 61	1.5 57	1.7 103	1.1 103	1.2 61	0.3 67	0.4 67	0.3 103	0.3 57	0.2 70	0.1 103	5.8 72	2.2 74	0.3 66
1915	17.0 49	1.5 49	3.0 44	0.4 30	0.1 44	1.5 57	1.1 69	0.8 69	0.8 44	0.3 55	0.3 55	0.2 69	0.3 57	0.2 63	0.1 69	4.4 55	2.1 69	0.2 51
1914	16.6 48	1.5 48	3.0 44	0.4 30	0.1 44	1.5 57	0.9 58	0.6 58	0.8 44	0.3 50	0.3 50	0.2 58	0.3 57	0.2 63	0.1 58	4.4 55	2.0 65	0.2 52

Year.	Locomotives.	Ash hoppers.	Condensers.	Turbo-generators.	Motor generators.	Reactance coils.	Air washers.	Air-washer ducts.	Brick bus compartments.	Disconnecting switches.	Oil circuit breakers.	Switchboard, including bus-bar.	Conduit cable wiring.	Power transformers.	Current transformers, also potential.	Insulators.	Lightning arresters.	Composite.
	0.3%	0.5%	4.5%	14.3%	0.1%	0.5%	0.2%	0.4%	1.1%	0.7%	0.7%	2.0%	2.7%	0.3%	0.3%	0.7%	0.2%	99.0%
1925	0.3	0.5 100	4.4 97	14.3 100	0.1 100	0.5 100	0.2 100	0.4 97	1.1 98	0.7 100	2.9 100	2.0 100	2.9 107	0.3 100	0.3 100	0.6 89	0.2 100	98.3 = 99 1925
1924	0.3 100	0.5 100	4.5 100	14.3 100	0.1 100	0.5 100	0.2 100	0.4 100	1.1 100	0.7 100	2.9 100	2.0 100	2.7 100	0.3 100	0.3 100	0.7 100	0.2 100	99.0 = = 100 1924
1923	0.3 104	0.5 100	4.5 100	14.3 100	0.1 98	0.5 96	0.2 100	0.5 116	1.1 100	0.6 92	2.7 92	1.8 92	2.8 103	0.3 96	0.3 96	0.7 100	0.2 96	99.0 = = 100 1923
1922	0.3 94	0.5 96	4.2 94	14.6 102	0.1 95	0.4 88	0.2 102	0.4 95	1.0 88	0.6 82	2.4 82	1.6 82	2.5 91	0.3 89	0.3 89	0.7 95	0.2 89	91.4 = 92 1922
1921	0.3 100	0.5 100	4.6 102	17.2 120	0.1 113	0.5 104	0.2 120	0.4 106	1.0 94	0 6 92	2.7 92	1.8 92	2.2 81	0.3 105	0.3 105	0.7 105	0.2 105	99.0 = = 100 1921
1920	0.3 119	0.6 128	5.8 129	17.2 120	0.1 123	0.6 114	0.2 120	0.5 128	1.3 115	0.6 91	2.6 91	1.8 91	3.3 121	0.3 114	0.3 114	0.9 131	0.2 114	115.5 = = = = 117 1920
1919	0.3 98	0.5 100	5.0 110	15.5 108	0.1 105	0.5 100	0.2 108	0.5 121	1.0 90	0.6 81	2.3 81	1.6 81	3.1 116	0.3 100	0.3 100	0.7 93	0.2 100	97.0 = = 98 1919
1918	0.3 95	0.5 100	5.4 121	15.5 108	0.1 107	0.5 100	0.2 108	0.6 160	0.9 80	0.6 80	2.3 80	1.6 80	4.1 151	0.3 101	0.3 101	0.6 85	0.2 101	95.1 = = 96 1918
1917	0.3 97	0.5 93	4.5 99	14.5 101	0.1 95	0.5 94	0.2 101	0.7 174	0.8 71	0.5 74	2.1 74	1.5 74	4.8 178	0.3 94	0.3 94	0.7 94	0.2 94	87.9 = = 89 1917
1916	0.2 66	0.3 61	3.5 77	11.6 81	0.1 76	0.4 75	0.2 81	0.4 103	0.6 59	0.4 58	1.7 58	1.2 58	4.5 166	0.2 75	0.2 75	0.4 52	0.2 75	68.6 = = = = 69 1916
1915	0.1 46	0.2 44	3.0 67	9.5 66	0.1 61	0.3 65	0.1 66	0.3 69	0.5 49	0.3 46	1.3 46	0.9 46	2.8 104	0.2 65	0.2 65	0.3 36	0.1 65	54.5 = = = = 55 1915
1914	0.1 43	0.2 44	2.7 59	8.5 99	0.1 62	0.3 61	0.1 59	0.2 58	0.5 48	0.3 43	1.2 43	0.9 43	2 2 81	0.2 61	0.2 61	0.3 42	0.1 61	51.5 = = = = 52 1914

On one of these (Fig. 1) composite trends typical of street railway, gas, and electric properties have been plotted, and the uniformity from 1919 to 1925, during the post-war years, is very notable. The component parts of these properties differ widely, the one with its boilers, stokers, and turbo-generators, the gas plant with its holders, trunk lines, and mains, and the trolley property with its track and rolling stock, but in each of these properties certain basic materials and also labor predominate sufficiently in their physical elements, to keep the trends approximately in line.

Between 1914 and 1919 deviations did develop, but these were due to abnormal war-time conditions which affected the demand for certain commodities to a much greater extent than others. Everything was then in confusion; prices in many cases were simply prohibitive, with no relation to either wages or material costs, and others were governed almost wholly by the eagerness to keep their shops going at a time when little or no business was being offered. The similarity of the last seven years, since the return to more staple business conditions, is in many ways remarkable.

A comparison is given on Fig. 2, for a super-power station costing $15 000 000 and a very small station costing only $50 000; and on Fig. 3, a comparison between the composite trends of a distribution system in Minnesota and another in Georgia is made. Figs. 4 and 5 are merely representative of the detail trends used.

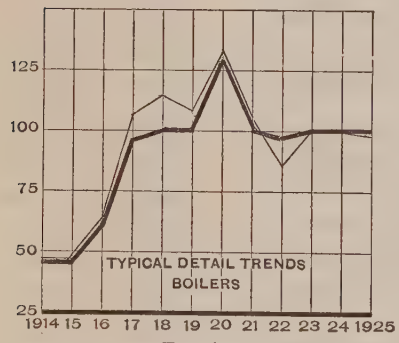

FIG. 4.

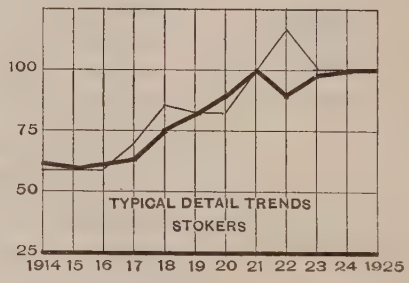

FIG. 5.

Table 1 is a composite of the super-power station shown on Fig. 2, and it will be noted that 99% of the equipment was trended. The year 1924 was used as a base of 100, and multipliers were compiled for each of the items, year by year, from price trend letters. The composite summary at the end of Table 1 is not predetermined, but is the aggregate of the weighted averages of each of the thirty-five major items.

That all these utilities, power plants, and distribution systems would trend so much alike, was not expected, but if similar research work of others were readily available, the writer is very confident that their experience would be but a substantiation of these findings.

DISCUSSION

FRANK C. WIGHT,* M. AM. SOC. C. E. (by letter).—In the last paragraph of his interesting paper, Mr. Breuer states:

"That all these utilities, power plants, and distribution systems would trend so much alike, was not expected, but if similar research of others were readily available, the writer is very confident that their experience would be but a substantiation of these findings."

This last stated opinion the writer shares. Construction costs are made up of a number of items, each of which varies individually and in the proportion it bears to the total. Nevertheless, except for unusual types, the total cost of construction seems to follow about the same curve over a period of years.

The writer is confirmed in this view by a study of the *Engineering News-Record* Construction Cost Index and its relation to the data Mr. Breuer presents. That Index is purely an index; that is, it is not the cost of any specific project or even of typical projects or structures. It is a simple mathematical derivation of the composite costs of four of the items which go to make up construction cost, each weighted arbitrarily according to the part it is judged to take in the total cost of an engineering work. The value of the Index, therefore, being based on judgment, must be proved by the accuracy with which it gauges actual cost and not by any individual opinion of the propriety of the relative weights of each item on the selection of the several items. The Index has been worked out for the years 1914 to 1926 and is based on an assumption of normal costs in 1913. The Index for any month of any year, therefore, indicates the cost of a construction project built at that time in its relation to the cost of the same project as built in 1913.

The Index is based on the prices of steel, cement, lumber, and common labor in 1913, each item being weighted as follows: Steel, 37.50%; cement, 7.14%; lumber, 17.10%; and common labor, 38 per cent. The weightings were determined by studying the annual production of the three prime materials and the number of common laborers (excluding farm labor) in the United States. The markets considered are structural steel in Pittsburgh, Pa.; cement without bags, f. o. b. Chicago, Ill.; 3 by 12-in. to 12 by 12-in. Southern pine to contractors in New York, N. Y.; and the average price of common labor in twenty cities listed in *Engineering News-Record* in the first issue of each month. The prices are those obtaining in the last week of each month, published in the first issue of each month. The quantity of these materials and the number of man-hours used in computing the Index were selected so that the total cost (quantities multiplied by prices and added together) for 1913 would be $100 for a "unit of construction".

On that basis, the Construction Cost Index for the years for which it has been computed is, as follows:

1913	100.00	1916	147.35
1914	88.56	1917	181.24
1915	92.58	1918	189.20

* Editor, *Engineering News-Record*, New York, N. Y.

1919	198.42	1923	214.07
1920	251.28	1924	215.36
1921	201.78	1925	206.68
1922	174.45		

A study of these figures will reveal that they form a curve remarkably similar to the curves of construction costs of actual properties presented in the paper. This will be evident from an inspection of the curves of Fig. 6, a series

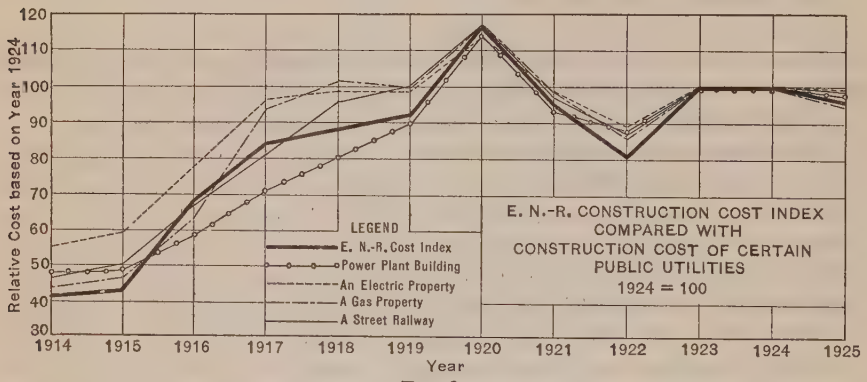

FIG. 6.

of graphs showing the trends of cost of (1) a power plant building; (2) an electric property; (3) a gas property; and (4) a street railway property, for the years 1914 to 1925, all plotted from Mr. Breuer's figures. Superposed on these graphs is the graph of the *Engineering News-Record* Construction Cost Index through the same years, with its base referred to the year 1924 instead of to 1913, because 1924 has been used as the base of the relative figures presented by Mr. Breuer.

It is submitted that this similarity of trend amounts almost to identity, indicating not only the truth of Mr. Breuer's observation that there is a similar trend in all construction costs, but also that the fundamental characteristics of cost trends are fairly portrayed in the basic elements which go to make up the *Engineering News-Record* Construction Cost Index.

J. W. HUMMER,* JUN. AM. SOC. C. E. (by letter).—A number of interesting comparisons are brought out in Mr. Breuer's paper. It seems rather a coincidence that the trends for an electric property, a gas property, and a street railway property should parallel from 1919 to 1925. One would expect them to be somewhat similar, but the maximum variation during the 7-year period, as shown in Fig. 1, is 4 per cent. The writer made a similar study of the data at his command, and the results were approximately the same. The trends for the three types of utilities from 1919 to date, closely paralleled, the maximum variation being 10 per cent. Also, the data for the period from 1914 to 1919 showed a much closer correlation between the three types than is shown in Fig. 1. The gas and street railway trends were almost coincident

* Asst., Appraisal Dept., The United Light & Power Co., Davenport, Iowa.

for that 6-year interval and the electric property trend, with the exception of two years, did not differ from the other two by more than 7 per cent.

The fact, as shown in Fig. 2, that the trends for a super-power station and a small power station extend so evenly over the 12-year period seems unusual. With the very decided change to large central stations, and the abandonment of small stations, one would expect that the economies, due to improved design, would cause the trend of the former to deviate more and more from that of the latter.

In determining the trend for the electric utility the writer compared the yearly cost factors for the individual items, as listed in Table 1, with data in his use. It was somewhat of a surprise to find that with a number of items variations of 20% and more existed. Because the yearly indexes were probably not secured from the same source and because the items under consideration were not of the same type or make, variations were expected to occur; but 20% and more seems altogether too high. However, when all the items were weighted and the yearly trends for the property as a whole determined, the writer's work compared very closely with figures given by Mr. Breuer.

William Breuer,* Jun. Am. Soc. C. E. (by letter).—The marked similarity, year by year since 1918, in the composite trends of construction costs for street railway, gas, and electric properties, is again confirmed by the year 1926. These statistics are frequently in use, and seemingly always, with more and better data at hand, one would expect discrepancies; but an impartial judgment discloses the fact that the conclusions derived in the previous investigations are sound, and only minor changes in detail trends are found. The year 1926 compares very favorably with 1925.

* Engr., Day & Zimmermann, Inc., Philadelphia, Pa.

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Paper No. 1605

VIRTUAL WORK: A RESTATEMENT*

By Hardy Cross,† M. Am. Soc. C. E.

With Discussion by Messrs. E. A. MacLean, Richard G. Doerfling, J. Charles Rathbun, and Hardy Cross.

Synopsis

The fundamental relations that determine the distortions produced in structures by internal strains may be stated in the mathematical language of the calculus and of geometry and in the engineering terminology of moment, shear, product of inertia, and moment of inertia. These theorems are, however, most readily correlated by use of the principle of virtual work. The paper restates this principle, emphasizing that the reactions to the imaginary external resistance are themselves purely imaginary and in no way necessarily related to the supports of the structure. It then illustrates in part the very broad application of the principle.

Most of the theorems developed are not new. Indeed, the writer is not sure that any of them are entirely new, nor is he here especially concerned with their lack of novelty. The paper may serve a useful purpose in bringing into logical sequence a group of geometrical principles which are essential to the study of statically indeterminate structures. If it has any such value, the omission of references to the work of many who have helped to develop or illuminate the principle of virtual work, may be pardoned.

Virtual Work—Usual Statement

Although essentially a principle of geometry, the principle of virtual work has proved the most powerful tool ever applied to the analysis of statically

* Published in February, 1926, *Proceedings*.
† Prof., Structural Eng., Univ. of Illinois, Urbana, Ill.

indeterminate structures. An examination of American textbooks leads to the belief that the broadest application of the principle has not been properly emphasized.

As usually presented, the principle may be stated somewhat as follows:* Assume any rigid body acted on by a force and reactions and let it be required to determine the deflection, Δ, in the direction, $a\,b$, of any point, a. Consider, first, the stretch—call it δ—of any differential fiber. Now considering a unit hypothetical resistance to motion at a to produce a stress, u, in the fiber, then the external work will be $1 \times \Delta$ and the internal work will be $u \times \delta$. Then,

$$\Delta = u\,\delta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Each fiber produces its independent deflection and, therefore,

$$\text{Total } \Delta = \Sigma\, u\,\delta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

If the internal hypothetical stress resists the distortion, the internal work will be positive and the external movement opposite in direction to the hypothetical load. The same result follows from the more convenient rule, which treats lengthening and tension as positive, and indicates a positive displacement in the direction of the hypothetical resistance when the algebraic summation, $\Sigma\, u\,\delta$, is positive.

Reference has been made to the stretch of a differential fiber—the argument would have applied equally well to the shearing distortion of a differential cube, to the rotational distortion of a differential cylinder, to internal angular displacement, or to any other internal distortion.

Also, as Professor Swain has pointed out,† it is possible to deal with external rotation at a instead of translation. The hypothetical unit resistance is then a unit moment. Then,

$$\text{Rotation at } a = \Sigma\, u\,\delta$$

in which, u is the stress in each small particle due to a unit moment at a and δ is the strain actually existing in the particle.

Virtual Work—Restatement

As the equations of virtual work just stated involve no work done by the reactions from the hypothetical unit resistance it must have been assumed that they do no work and hence that they do not move. This is commonly—although not always—stated. A more correct statement, however, is that these reactions do not have any motion which appears as a part of the described displacement, and hence the reactions to the hypothetical unit resistance are such as to fix in direction the line with reference to which the desired displacement is measured and to fix in position any assumed point on this line.

Commonly this line of reference is the line joining the supports of the structure, but it might equally well be either (1) the line joining any two points on the deflected structure; or (2) any tangent to it. In the former case the unit load or moment acts on a structure simply supported at the two

* For the first presentation of the principle in American literature, see a paper by George F. Swain, Past-President, Am. Soc. C. E., *Journal*, Franklin Inst., February, March, and April, 1883.

† *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–20), p. 622 *et seq.*

given points; in the latter, the structure is fixed, or cantilevered, at the point of tangency of the fixed tangent.

The general principle deduced, therefore, may be stated, as follows: To find with reference to any line of the distorted structure and with reference to any point on that line the displacement produced at any other point by given internal distortions, consider a unit resistance to such displacement applied externally at the given point, the structure being held stable by reactions fixing the point and the line of reference. Find the internal resistances produced by this imaginary external resistance. The sum of the products of the internal distortions multiplied by the imaginary internal resistances is the desired displacement.

The internal distortions may be due to any condition of stress and the strains may be either elastic or plastic, or they may be due to temperature or inaccurate workmanship. The external movement may be either of rotation or of translation. Rotation may be measured with reference to any line in the deflected structure. Translation may be measured in any direction with reference to any given line in the deflected structure and any given point on that line.

Virtual Work Applied to Trusses

Thus presented the broad statement of the relations of external movements to internal distortions leads directly to the usual theorems dealing with slopes and deflections. In trusses it takes the familiar form, $\sum \frac{s\,u\,l}{A\,E}$ (the properties of the member being s, the stress; l, the length; and A, the area; E being the modulus of elasticity), except that u should be defined with reference to the fixed or reaction points. Thus, it is possible to find the deflection of any panel point, A, with reference to a line through any other two panel points, B and C, by considering the hypothetical unit load applied at A, the truss being supported at B and C. Either B or C may be taken as fixed, the other point being, for hypothetical loads, on rollers. Similarly, the relative deflection of opposite corners of any quadrilateral of a truss may be found by taking $\Sigma\,\delta\,u$, in which, u is the stress in any bar due to a unit load at one corner in the direction of the other when the truss is supported at the other corner. This is a familiar problem in internal indetermination.

Virtual Work Applied to Beams

In the case of beams the displacements due to moment only are:

$$\Delta = \int M'\,d\,\phi \qquad (3)$$

in which, Δ is the external relative movement of any point, $d\,\phi$ is the rotation (produced by the bending moments) in any differential length, and M' is the imaginary moment over this differential length produced by a unit resistance to the external movement.

If the deflection at any point from a chord of a structure, due to moments only, is to be found, the values, M', will be identical with the ordinates to an

influence line for bending moment at this point on the chord. From this it follows that the deflections of any slightly curved line away from its chord equal the bending moments on the chord, treated as a beam, simply supported at its ends, due to the angle changes as loads. This is applicable in any case to a beam, a floor line, or a truss chord, if the angle changes can be computed. From it the moment-area theorems of Mohr and Greene may be shown to follow directly, or they may be taken as direct corollaries of the principle of virtual work, as will be shown.

The angle change in a differential length of the axis of a beam, if plane sections remain plane, is the strain of an outer fiber, divided by the distance of this fiber from the neutral axis. The strain of the outer fiber is $\frac{f}{E}\,d\,l$, in which, f is the intensity of stress in the outer fiber. Then, $d\,\phi = \frac{f}{E\,y}\,d\,l$, in which, y is the distance from outer fiber to neutral axis.

In all the theorems dealing with moment-areas, the term, $\frac{f}{y}$, may be substituted for $\frac{M}{I}$. Sometimes this is convenient. It also makes possible a clearer understanding of the deflection of reinforced concrete beams and, in general, gives a clearer view of the assumptions that are involved in the analysis of indeterminate structures.

If the beam formula applies, then,

$$f = \frac{M\,y}{I};\; d\,\phi = \frac{f}{E\,y}\,d\,l = \frac{M}{E\,I}\,d\,l$$

Hence,

$$\Delta = \int M'\,M\,\frac{d\,l}{E\,I} \qquad (4)$$

If applied to a part of a beam containing a frictionless hinge, this expression is evidently indeterminate, since at the hinge both M and I are zero. Moment-area theorems are, therefore, indeterminate for those parts of a beam in which a hinge occurs unless the change of angle at the hinge is computed independently.

The Reciprocal Theorem

If the expressions, $\sum \frac{s\,u\,l}{A\,E}$ for trusses and $\int M'\,M\,\frac{d\,l}{E\,I}$ for beams, be used to find absolute displacements due to loads on the structure, the interchangeability of the terms, s and M, which are due to the loads, with u and M', respectively, which are due to the hypothetical external unit resistances to displacement, indicates at once the general theorem of reciprocal displacements. If "displacement" is interpreted in a general sense as either linear or angular, and "load" in a general sense as either force or moment, then in any structure the displacement at A due to a load at B has the same value as the displacement at B due to a load at A, provided that both at A and at B the force and the displacement are of the same nature, linear displacement corresponding to force along its line of action, and rotation corresponding to moment.

This theorem is useful principally in interpreting as influence ordinates those displacements of the load line of a structure which would be produced by an imaginary unit internal distortion corresponding to the internal stress for which the influence line is desired.

Greene's Theorems Derived Directly by Virtual Work

If the relative rotation at one point on a beam referred to the tangent at another point is desired, consider the beam cantilevered from one point and loaded with a unit moment at the other point. Then, $M' =$ constant $= 1$, and,

$$\int \frac{M\,M'}{E\,I}\,d\,l = \frac{1}{E\,I} \times (\text{area under moment curve between the points of reference}) \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

If, however, the deflection of the second point with reference to a tangent from the first is wanted, consider the beam to be loaded and cantilevered from the first point, with a unit load at the second. Then M' equals the distance of each section from the second point and $\int \left(\frac{M}{E\,I}\,d\,l\right) M'$ is evidently $\frac{1}{E\,I}$ times the statical moment of the area of the moment curve about the second point.

Mohr's Theorems

In the case of a beam on fixed supports the change of slope at any point, A, relative to the line joining these supports may be found by applying a unit moment at the point, the beam being simply supported at the fixed points. The moment curve for M' will be found to be identical with an influence line for shear at A. Hence,

$$\int \frac{M\,M'}{E\,I}\,d\,l = \int \frac{M\,d\,l}{E\,I} \times (\text{shear at } A \text{ due to a unit load at each section})$$

$$= \int \left(\text{shears at } A \text{ due to each } \frac{M\,d\,l}{E\,I}\right)$$

$$= \text{shear at } A \text{ on a simple beam due to the } \frac{M}{E\,I}\text{-curve considered as a load.} \quad (6)$$

For the deflection at A, on a simply supported beam, $D\,E$, apply a unit load at A. The curve for M' is then identical with an influence line for bending moment at A. Hence, the deflection of A is the bending moment at A due to the $\frac{M}{E\,I}$-curve considered as a load.

The method herein indicated of establishing the identity of the curve for M' with the influence line for some easily evaluated function, such as shear or moment, offers a general method for stating theorems of slope and deflection in the language of structural engineering. By its use may be readily determined those analogies which form the foundation for what has come to be known as the method of conjugate beams.

Evidently, any line on the beam other than $D\,E$ may be considered as fixed. Thus, consider a beam bent as shown in Fig. 1. The deflection of Point A with reference to the line, $B\,C$—Δ_a in the diagram—is the bending moment at A produced by the $\frac{M}{E\,I}$-curve between B and C on a simply supported beam, $B\,C$. This is sometimes a convenient theorem.

Also, the slope of the tangent at A with reference to the line, $B\,C$, is the shear at A on the simply supported beam, $B\,C$, due to the $\frac{M}{E\,I}$-curve between B and C acting as a load.

Angle Changes in Trusses

Another interesting application of the principle of virtual work is in finding the change, $\Delta\,\alpha$, of any angle, α, of a triangle (Fig. 2) due to stresses in the three sides. Apply at A and C the elements of a unit couple, causing reactions such as to hold $A\,B$ fixed in direction. Tension and increase of angle are taken as positive. Then,

$$\Delta\,\alpha = \Sigma\,\delta\,u = \Sigma\,\frac{f\,l}{E}\,u$$

or,

$$E\,\Delta\,\alpha = \Sigma\,f\,l\,u \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

in which, for any bar, f is the fiber stress, l, the length, and E, the elastic modulus.

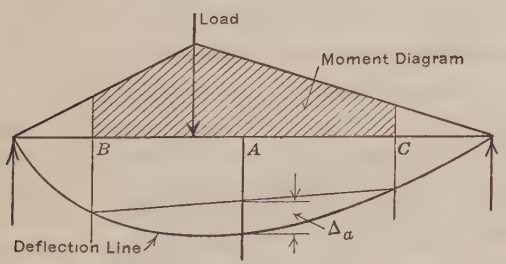

Fig. 1.

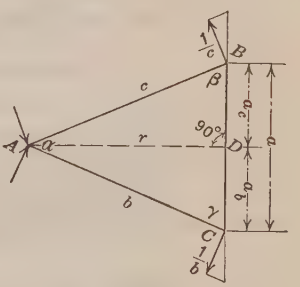

Fig. 2.

Resolving the forces at Joints B and C,

$$u_a = \frac{1}{r}\,; \quad u_b = -\frac{1}{b}\,\frac{a_b}{r}\,; \quad u_c = -\frac{1}{c}\,\frac{a_c}{r}$$

$$E\,\Delta\,\alpha = f_a\,a\,\frac{1}{r} - f_b\,b\,\frac{1}{b}\,\frac{a_b}{r} - f_c\,c\,\frac{1}{c}\,\frac{a_c}{r} = \frac{f_a\,a - (f_b\,a_b + f_c\,a_c)}{r}$$

$$= \left[f_a - \left(f_b\,\frac{a_b}{a} + f_c\,\frac{a_c}{a}\right)\right]\frac{a}{r} \quad \ldots\ldots\ldots\ldots\ldots (8)$$

in which, f_a, f_b, and f_c are fiber stresses and the other terms are as shown in Fig. 2. Equation (8) is probably the most convenient formula for the angle changes. It may be modified to:

$$E \Delta \alpha = f_a \frac{a_b}{r} + f_a \frac{a_c}{r} - f_b \frac{a_b}{r} - f_c \frac{a_c}{r}$$

$$= (f_a - f_b) \frac{a_b}{r} + (f_a - f_c) \frac{a_c}{r}$$

$$= (f_a - f_b) \cot \gamma + (f_a - f_c) \cot \beta \ldots\ldots\ldots\ldots (9)$$

Equation (9) is familiar to students of secondary stresses.

Evidently, the method here presented may be readily extended to the quadrilaterals which occur in subdivided trusses and K-trusses.

Slope Deflection

The application of this method to the special case of a beam or part of a beam loaded only with moments at its two ends gives directly either the end slopes in terms of the end moments or the latter in terms of the former. In the first case given the loads on the beam, the reactions (end shears) are desired; in the second, the magnitude of the loads is to be determined for given reactions. By taking moments about B (Fig. 3), there follows,

$$\theta_a = \frac{2}{3} M_a \frac{l}{2 E I} - \frac{1}{3} M_b \frac{l}{2 E I} = \frac{l}{6 E I} (2 M_a - M_b) \ldots\ldots (10)$$

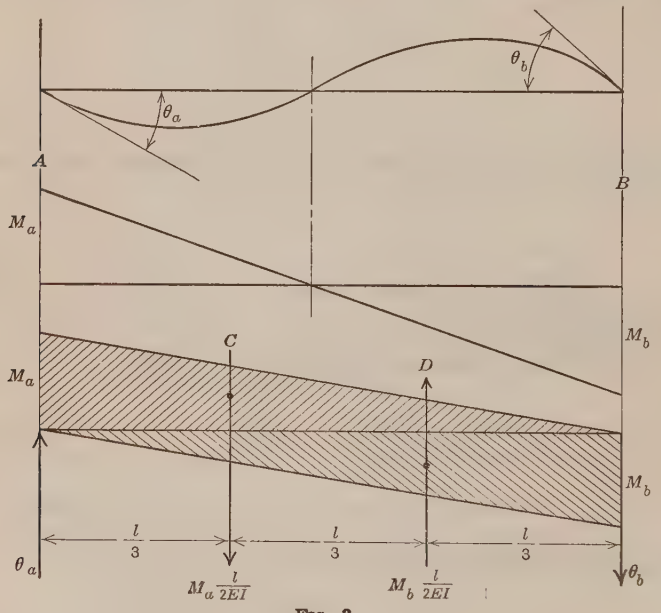

Fig. 3.

By taking moments about D,

$$\frac{M_a l}{2 E I} = 2 \theta_a + \theta_b$$

or,

$$M_a = \frac{2 E I}{l} (2 \theta_a + \theta_b) \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

Both forms of equations have long been used in evaluating secondary stresses. Equation (11), combined with the device of using as unknowns the rotations

of parts of the structure and evaluating them by simultaneous equations of statics, has had an elaborate development in the literature of the "slope-deflection method."

Moment Weights

The principle of virtual work furnishes directly a method of computing moment weights, or, as some writers call them, "elastic weights", for determining the deflected load line, or influence line, for trusses. Assume that it is desired to draw the deflected load line for a unit load as shown (Fig. 4), that is, an influence line for horizontal reaction. The angle change at a may be computed by applying a unit moment resisting this angle change. This is effected by applying loads as shown in Fig. 4 (b), acting at the points on the load line or floor as indicated by circles, and then computing $\Sigma\, \delta\, u$, in which, δ is the change in length of any bar due to the horizontal reaction, and u is the stress for the loading shown in Fig. 4 (b). These angle changes may then be treated as loads at the panel points and, when corrected for the deflection of the ends of the load line, the moment curve thus produced will have the shape of the influence line.

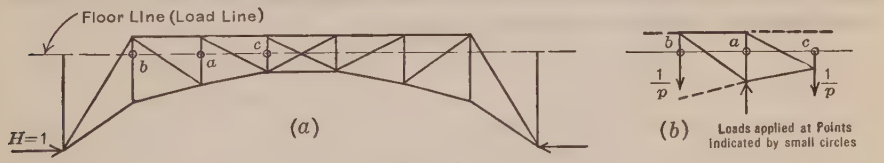

Fig. 4.

This method presents advantages in directness in some cases as where the floor-beams frame into the verticals between upper and lower panel points.

Conclusion

The illustrations given indicate the broad usefulness of the general principle of virtual work. The writer has thought it wise to limit them within the space which the specialized interest of the paper justifies. For the same reason, the paper is confined to the geometrical relations of distortions and does not include other relations which, in the theory of indeterminate structures, follow therefrom.*

In closing it may be well again to call attention to the purely geometrical nature of these theorems. They do not bear on the accuracy of the physical assumptions regarding the action of structures. Like all other geometrical relations, they are not open to dispute, nor are they proper subjects for experimental verification, and they are also subject to statement in an endless variety of forms or aspects of the relations involved. Familiarity with these relations is essential to facility in analyzing indeterminate structures, but neither ordinary nor extraordinary expertness in this field necessarily connotes either unusual or even common ability in designing structures or in understanding their action.

* For an interesting correlation of virtual work with the theorems of Castigliano, see "The Principle of Virtual Velocities and Its Application to the Theory of Elastic Structures", by Ernest Lamb, *Selected Engineering Papers No. 10*, Inst. C. E., December, 1923.

To apply these theorems to engineering structures, it is simply necessary either to show that for given values of the moments, shears, and thrusts, the strains can be predicted, or, if exact prediction is not possible, to determine by direct computation what error results from the inexactness. In analyzing indeterminate structures it is also to be noted that usually the relative and not the absolute values of these strains are in question.

Because of the definiteness of the moments, shears, and thrusts, strain measurements on statically determinate structures seem for this purpose more valuable and dependable than similar data derived from measurements on indeterminate frames. When it is established that, for any type of construction, these strains—or their relative values—can be definitely predicted, the whole theory of indeterminate stress analysis follows from the relations of the distortions as a matter of geometry. In order to be of real value in designing indeterminate structures, however, the load-strain relations must be those for conditions approaching failure and not merely those that exist at working stresses. An understanding of these facts will make clearer the limitations of the theory of elasticity as applied in much of the literature dealing with indeterminate structures of reinforced concrete, will make possible the application of more correct theory to such structures, and will give greater confidence in the results obtained by its use.

DISCUSSION

E. A. MacLean,* Esq. (by letter).—The writer believes that Professor Cross has done the Structural Engineering Profession a service by calling to the attention of its members the value of virtual work as a tool in the solution of indeterminate structures. He has shown that many special methods, slope deflection, elastic weights, etc., come directly and logically from virtual work. A careful study of the paper should convince the reader of the wide applicability of the method and the desirability of mastering it thoroughly.

To those already familiar with the principle of virtual work the applications and modifications are valuable and instructive; to others the paper will give little except a desire to know more about the subject. It is unfortunate that the engineer wishing to master this principle must study so much literature, which often contains a great deal of extraneous material, before the matter set forth by Professor Cross can be understood.

The writer feels that the paper could profitably be elaborated into a more understandable and usable form. Such a development of the theory of virtual work with its numerous applications should be of even greater service to the profession and a valuable addition to engineering literature.

Richard G. Doerfling,† M. Am. Soc. C. E. (by letter).—The term, "Restatement", in the title alone seemed promising, the writer understanding it first to mean a statement clearer than prevailing ones—one furnishing a working knowledge of the subject and making the study of its practical application more attractive. However, it can hardly be classed as such. Further, too little importance is attached to the theorem of reciprocal deformation, although it is considered by many to be the most fundamental as well as the most comprehensive theorem in the analysis of statically indeterminate structures.

In the practical application such factors as imaginary reactions or hypothetical stresses do not enter; it seems that their inclusion in the demonstration only confuses the mind.

Virtual Work.—Work as the product of force and the distance through which it acts is a familiar enough conception; but work as the product of a moment and the angle (in circular measure) through which the moment acts, is a conception less familiar but of equal importance. That both are really equivalent may be illustrated in a simple way by a displacement diagram like Fig. 5:

$$\text{Work of Force } F = F \times ra = Fra$$
$$\text{Work of Moment } Fr = Fr \times a = Fra$$

In considering a framed structure under the action of external forces it seems clear that all panel points, or points of application of the forces, may be regarded as small rigid bodies connected by elastic members, and that the motion of these rigid panel points and the deformation of the whole structure are due entirely to the change in length of the connecting members. In other

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† Cons. Engr., San Francisco, Calif.

words, while the deformation takes place, or during the transition from one state of equilibrium to the other, each panel point moves with all its forces in equilibrium. Hence the resultant of these forces at each panel point and also its work remains zero, or the resultant of all external forces at each panel point remains equal but opposite in direction to the resultant of all internal forces at the point; the internal work remains equal to the external work because their sum remains zero.

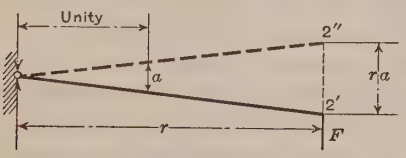

FIG. 5.

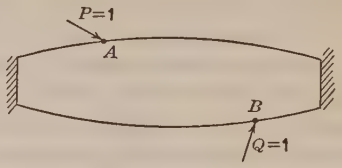

FIG. 6.

Now, let P be an external force or the resultant of a number of external forces at a panel point, and d be its motion in the direction of P. Let S be the resultant of the internal forces or stresses at the same panel point and Δl its motion in the direction of S. If the panel point moves with all forces in equilibrium then P, d, S, and Δl remain in line, and from the beginning of motion, at any instant during motion, until the end of motion, $P = S$, $d = \Delta l$, and $Pd = S \Delta l$. If the loads are brought on without shock all four values increase from zero to a maximum at the same rate, hence, $\frac{1}{2} \Sigma Pd = \frac{1}{2} \Sigma S \Delta l$.

Assume now two different systems of loading, and indicate all values due to the first system by a subscript, 1, and all values due to the second system by a subscript, 2. The two equations of work will be:

$$P_1 d_1 = S_1 \Delta l_1$$

and

$$P_2 d_2 = S_2 \Delta l_2$$

Since, however, the strain in any member is proportional to its stress and in line with it, the projection of the strain on any line must be proportional to the projection of the stress on that line. Hence,

$$\frac{S_1}{S_2} = \frac{\Delta l_1}{\Delta l_2} = \frac{d_1}{d_2} = \frac{P_1}{P_2}$$

or,

$$S_1 \Delta l_2 = P_1 d_2$$

This is a true equation—there is nothing hypothetical or imaginary about it; but it is not an equation of real work, because S_1 and P_1 are due to one system in equilibrium whereas Δl_2 and d_2 are due to another system in equilibrium. It might be called an equation of potential work (possible work), but is known as the equation of virtual work. It is the most important equation of the subject and of constant application in determining statically indeterminate quantities. Expressed for the whole structure it is written:

$$\Sigma P_1 d_2 = \Sigma S_1 \Delta l_2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

The Reciprocal Theorem.—Consider two forces, P', each equal to unity, acting on two panel points, *A-A*, of a statically determinate structure, changing the distance between the points, *A-A*. Designate the stresses due to this loading by S', the change in length of the members by $\Delta\, l'$, and the change in distance between two other points, *B-B*, by p'. Then, consider two other forces, P'', each equal to unity, acting on the panel points, *B-B;* designate the stresses due to this loading by S'', the change in length of the members by $\Delta\, l''$, and the change in distance between the panel points, *A-A*, by p''. Now, write one equation of virtual work using the loads, P', and the stresses, S', of the first loading, but the deformations, p'' and $\Delta\, l''$, of the second loading; and another equation of virtual work using the loads, P'', and the stresses, S'', of the second loading, but the deformations, p' and $\Delta\, l'$, of the first loading. The two equations are:

$$P'\, p'' = \Sigma\, S'\, \Delta\, l''$$

or,

$$p'' = \Sigma\, S'\, \frac{S''\, l}{E\, A} \left(\text{because } P' = 1 \text{ and } \Delta\, l'' = \frac{S''\, l}{E\, A}\right)$$

and,

$$P''\, p' = \Sigma\, S''\, \Delta\, l'$$

or,

$$p' = \Sigma\, S''\, \frac{S'\, l}{E\, A} \left(\text{because } P'' = 1 \text{ and } \Delta\, l' = \frac{S'\, l}{E\, A}\right)$$

showing that $p'' = p'$.

If *A-A* and *B-B* had been points of application of the unit loads, P' and P'', on a plate girder or solid arch, the two equations of virtual work would be:

$$p'' = \Sigma\, S'\, \frac{S''\, \Delta\, V}{E}$$

and

$$p' = \Sigma\, S''\, \frac{S'\, \Delta\, V}{E}$$

($\Delta\, V$ being the volume of a fiber between two sections considered), showing the same equality of p' and p'' as for framed structures. Hence, for any structure: Two unit loads acting on two points, *A-A*, will change the distance between two other points, *B-B*, the same amount as two unit loads acting on the points, *B-B*, will change the distance between the points, *A-A*.

Remembering now that the work of a moment equals the moment multiplied by the angle (in circular measure) through which the moment acts, and that this is equivalent to the work of a force as explained before by Fig. 5, the theorem may be expressed in a more general way, thus: The cause of elastic deformation of a structure may be a force, a pair of forces, a moment, a pair of moments, etc. If such a cause (equal to unity) is acting on one element of the structure, say, on a point, a straight line, a pair of straight lines, etc., it will displace a second element of the structure the same amount as a similar cause (equal to unity), acting on the second element, will displace the first.

Expressed more briefly: The displacement which $Q = 1$ must suffer while $P = 1$ is acting equals the displacement which $P = 1$ must suffer while $Q = 1$ is acting.

For particular cases the theorem leads to a number of statements of which the following three are the most important:

(1) The force, $P = 1$ (Fig. 6) will move the point, B, in the direction of Q the same amount as the force, $Q = 1$, will move the point, A, in the direction of P.

(2) The force, $P = 1$ (Fig. 7) will turn the line, CD (in circular measure) the same amount as the moment, $M = 1$, will move the point, A, in the direction of P.

(3) The two forces, $P = 1$ and $Q = 1$ (Fig. 8) will change the angle between the lines, CD and EF (in circular measure), the same amount as the two moments, $M = 1$ and $N = 1$, will change the distance between the points, AB.

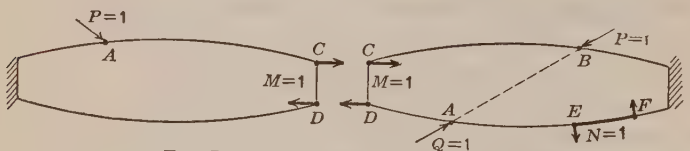

FIG. 7. FIG. 8.

Note that a moment equal to unity, or a unit moment, is represented by a couple, $\frac{1}{a}$, acting on a lever arm, a, because $\frac{1}{a} \times a = 1$.

The reciprocal theorem is fundamental in the analysis of statically indeterminate structures and leads directly to the theory of influence lines considered as deflection diagrams.

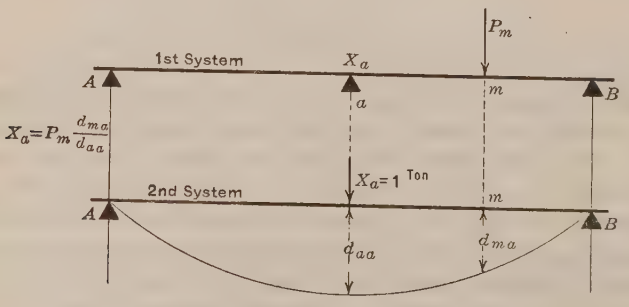

FIG. 9.

Let $A\ a\ B$ (Fig. 9) be a continuous beam on three supports with a load, P_m, at Point m; required the reaction, X_a. Let the statically determinate main system be the simple beam, AB, loaded with $X_a = 1$ at Point a. Construct the deflection line for this loading or compute the deflections, d_{aa} and d_{ma}. Then, according to the reciprocal theorem: The load, $P_m = 1$, will

cause a deflection at Point a equal to d_{ma}; hence the load, P_m, will cause a deflection at a equal to $P_m\ d_{ma}$. The force, $X_a = 1$, causes a deflection, d_{aa}, at a, hence a force, X_a, will cause a deflection, $X_a\ d_{aa}$. Thus,

$$X_a\ d_{aa} = P_m\ d_{ma} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

Again this is a true equation—not an equation of real work but of virtual work. X_a and P_m are elements of the first system loaded with P_m only, while d_{aa} and d_{ma} are elements of the second system loaded with $X_a = 1$ only.

Note that while Equation (12) gives a relation between external and internal forces or between loads and stresses, Equation (13) gives a relation between loads and reactions. Together they form the foundation of the subject.

J. Charles Rathbun,* M. Am. Soc. C. E. (by letter).—Professor Cross has pointed out the application of the theory of virtual work when the fibers of the structure are in tension or compression, and the manner of computing deflections in a given direction by assuming a hypothetical force acting in that direction. He also mentions the method of computing rotation based on tension and compression in the fibers.

If, in addition, shear in the fibers is considered, the torsional deformation (as in a shaft) will also apply, as well as the deformation due to simple shear. Thus, in the cantilever beam of Fig. 10, to obtain the full deformation in the direction of the force at the point, P, not only is the deformation due to the direct thrust in the direction of the X-axis and the moments about the Z and Y-axes to be taken, but also the shear in the direction of each of these axes and the torque about the X-axis. As a rule these shears may be neglected, while the torsion does not often occur in engineering design. However, when it does, it may become an important element in the deformation.

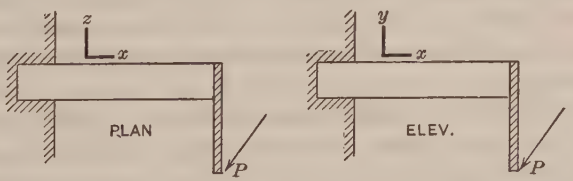

Fig. 10.

Each one of these six elements of the deflection can be obtained by a separate computation by virtual work.

Hardy Cross,† M. Am. Soc. C. E. (by letter).—The writer has read with interest the discussion by Mr. Doerfling, but is unable to accept his conceptions either of virtual work or of the laws of statics. Mr. Doerfling states that "during the transition from one state of equilibrium to the other, each panel point moves with all its forces in equilibrium", a view of the phenomenon of motion which is also stated algebraically. This, if correct, would be an important addition to the laws of physics.

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† Prof., Structural Eng., Univ. of Illinois, Urbana, Ill.

The equality assumed by Mr. Doerfling that $S_1 = P_1$, is simply an identity. The quantities, $\Delta\ l_2$ and d_2, are by definition the movement of a point in a given direction (the direction of P_2 and S_2). Hence, $S_1\ \Delta\ l_2 = P_1\ d_2$. Mr. Doerfling is correct in stating that this is not an equation of real work; the terms, "hypothetical equation" and "imaginary equation", are new to the writer and he is unable to interpret them.

Apparently, Mr. Doerfling is trying to deduce displacement relations from considerations of the true internal work of structures. Of course, such a method, if logically applied to elastic structures, leads to equations identical with those reached by virtual work; and it may even appear to lead to the same results if followed illogically, but with determination and purpose. Considerations of real work, however, are not used in the method of virtual work, a method which Mr. Doerfling has failed entirely to grasp, since he states that the inclusion of imaginary reactions or hypothetical stresses is unnecessary and confuses his mind.

The method of virtual work leads to the equation, $\Delta = \Sigma\ u\ \delta$, in which, Δ is the unknown external movement resulting from the internal distortions, δ. The meaning of the term, u, however, is not always made clear nor are the limits of the summation. Perhaps the most comprehensive statement is that the u-values are the imaginary internal resistances to distortion (internal stresses) produced by an imaginary unit load, coincident with Δ, in any imaginary framework or structure which will support this imaginary load from the points of reference. It is essential to understand that the reactions to the "dummy" load with which the principle operates are fixed in position and direction by the reference from which the displacements are measured.

Thus, in the truss shown in Fig. 11(a), the displacement of Point a, in the direction shown relative to the line, $b\ c$, and Point b, may be found from $\Sigma\ u\ \delta$, in which, u is the stress due to load unity on the truss shown in Fig. 11(b). However, any framework that is made up of any or all of the bars of the original truss and that would support a load at a from Points b and c, might as well have been chosen. A framework as shown in Fig. 11(c) might even be used, provided the value of δ were known for the false bar shown by dashes.

Also, in the beam shown in Fig. 12(a), in finding the displacement of a with reference to b and c, u may be taken as the moments along the chord bc, due to a unit load at a and then the integration, $\int u\ \phi$, may be performed; or u may be taken as the stress in the truss shown in Fig. 12(b), or in any such framework connecting a with b and c and $\Sigma\ u\ \delta$ may be applied to the imaginary bars.

The principle is entirely a geometrical tool derivable from the principle of the conservation of energy. It does not deal with, nor does it depend on, considerations of true internal work. It is not, therefore, dependent on the validity of Hooke's law, as Mr. Doerfling seems to think. That it is to be found almost exclusively in the literature of engineering results from the fact that only engineers find it especially convenient.

The phenomena considered in virtual work are entirely imaginary and the use of the coefficient, $\frac{1}{2}$, on both sides of the equation, $\frac{1}{2}\ \Delta = \frac{1}{2}\ \Sigma\, u\ \delta$, seems to involve a logical absurdity, although an unimportant one, since it is obviously possible to imagine the unit force remaining constant during the deformation and producing constant resistances in an inelastic structure.

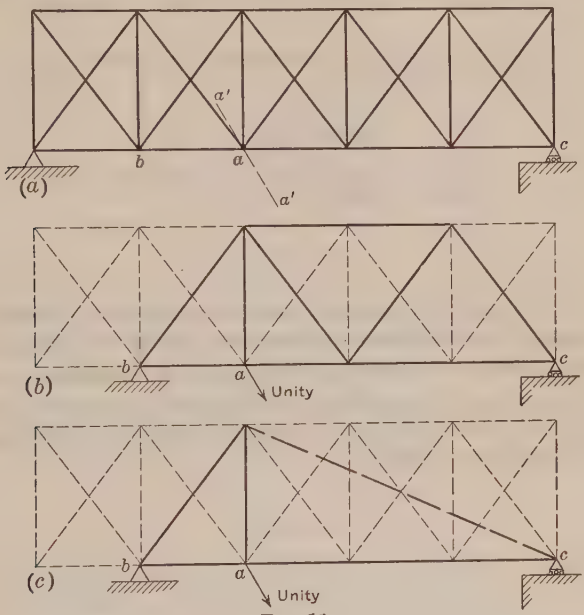

FIG. 11.

The interchangeability of s with u and of m with m' in the expressions, $\Delta = \Sigma\ \frac{s\ u\ l}{A\ E}$, or $\Delta = \int m\ m'\ \frac{d\ s}{E\ I}$, furnishes so simple and complete a proof of the reciprocal theorem that the writer is surprised that Mr. Doerfling considered it necessary to substitute a demonstration which is both tedious and incomplete. Mr. Doerfling begins his demonstration with a statically determinate structure and at some point not defined extends it to indeterminate structures. The limitation of stress-strain proportionality makes it inadvisable to consider the reciprocal theorem as a fundamental.

Illustration of the utility of the reciprocal theorem does not properly belong in this paper, as the writer specifically stated. The writer tried to condense the paper as much as possible; Professor MacLean thinks that he condensed it too much. Probably Professor MacLean is right, but the writer ventures the opinion that engineering papers in general are too extensive and that the literature of indeterminate structures especially has suffered from elaboration.

Professor Rathbun is correct in stating the broad applicability of the principle, although such application often presents mathematical difficulty.

In beams, the expression, $u\ \delta$, takes the form, $m\ \phi$, and it follows as a corollary that the deflections due to slightly bending any line away from its original position can be evaluated as bending moment on the chord due to the angle changes as loads. This may be readily proved by the geometry of small angle changes. From this, other identities—slopes as shears, tangent offsets as statical moments—follow directly.

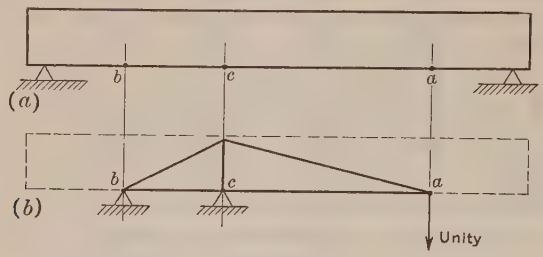

FIG. 12.

Theories of structural action appear only when an effort is made to evaluate δ and ϕ in terms of actual forces and bending moments. Thus, the relation of external displacements to internal distortions in structures is not subject to question; the values of these distortions in actual structures is subject to some question, but the significance of these distortions as indicating failure is subject to considerable dispute.

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Paper No. 1606

AEROPLANE TOPOGRAPHIC SURVEYS*

By George T. Bergen,† M. Am. Soc. C. E.

With Discussion by Messrs. Harold C. Fiske, Arthur P. Davis, Haywood R. Faison, Gerard H. Matthes, and George T. Bergen.

Synopsis

The object of this paper is to acquaint engineers with a method of making accurate contour maps directly from aerial photographs. The process discussed is the product of research and development since the World War. Some important elements of the problem are stated, followed by a brief exposition of the means adopted for a solution.

The control needed is confined to a few ground elevations for each photograph and a small number of distance measurements for each survey. The resulting maps are available in a fraction of the time necessary for surveys by old methods. Their accuracy often exceeds that of comparable ground surveys.

The method is applicable to a wide range of conditions. It offers marked advantages in making maps of large areas for engineering studies and preliminary locations, particularly in country of bold relief and considerable growth of timber or brush.

Introduction

Aerial photography is a relatively new resource of the surveyor and map maker. Extravagant claims have doubtless been made for it, as is not infrequent with new processes. On the other hand, its scope has been widened and its use adopted sufficiently to indicate that it has come to stay. Its limits

* Published in March, 1926, *Proceedings*.

† Pres., Bergen Eng. Service Corporation, Lakeland, Fla.

are by no means reached. "In future mapping the use of aircraft will play an important part."*

The speed of the aeroplane, the rapidity and detailed record of the camera, suggest to any engineer tremendous advantages for a process by which accurate topographic maps can be produced from aerial photographs, with relatively little instrumental ground measurement required for control and correction of the pictures. The photographs themselves cannot ordinarily be used as maps, because of the variation in scale. Particularly in country having any substantial relief, the horizontal scale of pictures varies to an extent not permissible for engineering purposes. In any event, a single photograph furnishes no elevations.

Although aerial mapping is new, the use of photography in surveying is not. Most engineers have some acquaintance with the theory, if not the practice, of the photo-theodolite. Records are available of the scientific use of photography in surveying as early as 1850. European army officers at that time made use of the camera somewhat after the present manner of using the transit. At the end of measured base lines, pictures were taken on vertical planes, and the angles between base lines and optical axes determined, thus locating objects by triangulation. Photographs were later made from balloons, some by cameras having several lenses and picture planes simultaneously exposed, so as to record more information and supply some check on the position of the viewpoint.

Developments of the World War revolutionized the use of aerial photography. The changes were so radical in their nature and the uses so extensive as to give the concept of something wholly new. André Carlier, Président de l'Association Française Aérienne, is authority for the statement that, during the last years of the war, 80% of the information about the enemy was due to aerial photography.†

An important war development was the mosaic of aeroplane photographs. A series of views were pieced together to form a battle map, constantly subject to replacement by a flow of new photographs showing everchanging details.

After the war, attention was soon directed toward the development of peaceful uses for this new resource. Numerous manufacturers engaged in the commercial production of photographic mosaics. Because of the rapidity with which they can be obtained and the vivid conception they give of a large area, they have many uses. A mosaic, however, is subject to severe scale variation, particularly where it covers ground of marked relief.

Various efforts have been made to bridge the gap between photographs and maps. One method is to use the picture for what it is worth as a step in mapping by old methods. Individual pictures, mosaics, or some derivatives of these have thus been used simply as plane-table sheets. Having before him the

* Report of September 30, 1919, to the President by Conference of Representatives of Federal Map-Making Organizations, including W. M. Black, Maj-Gen., U. S. A. (*Retired*), M. Am. Soc. C. E. (then Chief of Engineers, U. S. A.), William Bowie, M. Am. Soc. C. E., Chief, Division of Geodesy, U. S. Coast and Geodetic Survey, George Otis Smith, Director, U. S. Geological Survey, A. P. Davis, Past-President, Am. Soc. C. E. (then Director, U. S. Reclamation Service), Thomas H. MacDonald, Chief of Bureau of Public Roads, O. C. Merrill, M. Am. Soc. C. E., Chief Engineer, Forest Service, and eight others.

† "Tout le monde à présent a l'esprit des services qu'elle rendit pendant la guerre, et il n'est pas exagéré de dire que, dans les dernières années, 80% des renseignements sur l'ennemi étaient donnés par elle," "La Photographie Aérienne", *L'Illustration*, 13 décembre, 1924.

wealth of detail shown by the photographs, the topographer is aided and an economy is effected in ground surveys, particularly in making small-scale maps of flat country for general purposes. In such a case, the absence of relief and the reduction of scale attending the processes through which the map assumes final form, minimize the scale discrepancy and render possible considerable use of the photographic information. In such methods, all measurements of elevation and the work of locating contours is done on the ground in the old manner; aerial photography is merely an auxiliary.

The ideal method will reverse this relation, will make maps directly from the camera records, and dispense with ground measurements or reduce these to a minimum needed for control only. One effort toward this goal uses the three-point method of control; that is, if the distances between three points of equal elevation are known and their images clearly distinguishable on the photograph, it is possible by computation to approximate the picture plane. This method has serious objections. Many such triangles must be located on the ground, involving much field work. The computation is laborious, the result unsatisfactory. In practice it is possible to make an indefinite number of re-projected positions answer to the calculated size and position of the triangle. Often it is difficult or impossible to locate a suitable triangle with points of equal elevation. Resort may be had to points with measured differences of elevation, but the computation is then made more laborious. In any event, there remains the problem of locating contours.

Such considerations have led to efforts to evolve a practical method for producing directly from the data in the photographs an accurate contour map, without excessive work on the ground. Although the writer is not directly connected with the development of the particular process to be described, he has had unusual opportunity to become familiar with it, having had occasion to check certain work done by it and to study closely the operations and instruments it involves.

The Problem

It is not to be expected that an area of marked relief will ever be reproduced directly with the attributes of a good map. With proper equipment photographs can be made into records of high precision, but they follow certain laws inapplicable to maps. To convert the data recorded in pictures to the form of maps requires analysis and changes to meet these inherent differences as well as to overcome the effects of practical working obstacles. Distances are represented on maps unaffected by the point of view; on photographs, even a series of aerial views from a great altitude, points and lines follow the phenomenon of parallax, suffering changes in their apparent positions with changes of the point of view. A variation in the height of the aeroplane and camera changes the photographic scales. Efforts to hold aerial cameras truly vertical have failed, and the tilt causes further displacements of points in the photographs, varying between succeeding views and within a single picture.

In short, to convert the data of photographs to a uniform scale a variety of obstacles and limitations must be met, some by nature inherent in all photographs, some characteristic of photographic materials, some due to difficulties

met in practical operation, but the most troublesome are those involving mechanical obstacles.

Limitations Due to Nature of Photographs

Pictures are conic projections.* Their fundamental mathematical properties are simple. The light reflected from the ground photographed travels in straight lines to the camera lens, and thence to the photographic negative. A pencil of light rays reaching the lens from any point on the ground is brought back to convergence or focus in the corresponding image at the focal distance behind the lens. For all practical purposes, there is a cone with its vertex in the lens, one nappe cut by the negative, one by the ground. Each element of the cone represents the axis of a pencil of light rays from ground point to image point. If a plane be passed through the optical axis, the light rays it contains may be simply represented by straight lines crossing at the lens as in the diagrams.

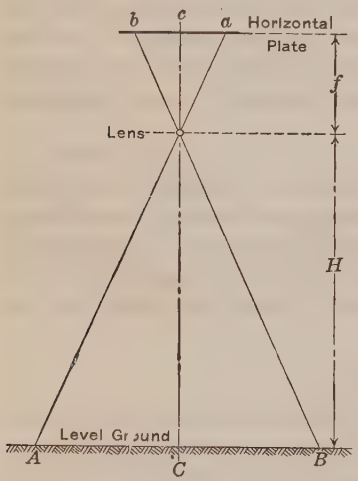

FIG. 1.—CONIC PROJECTION.

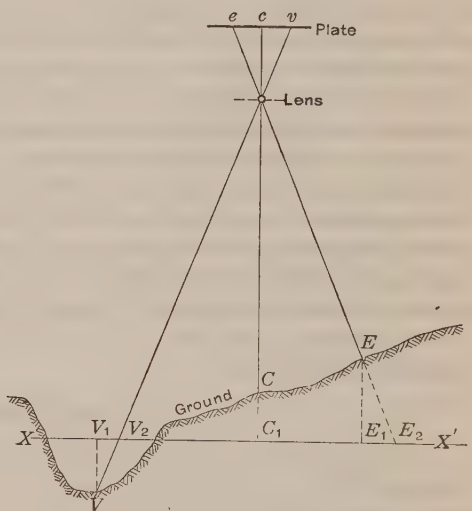

FIG. 2.—DISPLACEMENT DUE TO GROUND RELIEF.

Scale and Area of Single Photograph.—Fig. 1 represents the ideal case—a horizontal photographic plate and level ground. A lens with a focal length, f, projects light rays from the points, A and B, on the ground to the image points, a and b, on the photographic plate. From the similar triangles, $A\,B : a\,b :: H : f$, and the photographic scale, $S = \frac{A\,B}{a\,b} = \frac{H}{f}$; that is, the scale of the photograph is the ratio of the height of the lens to the focal length. If, as is customary, H is expressed in feet and f in inches, then $\frac{H}{f}$ is the scale

* The expression, "conic projection," herein signifies a projection by rays constituting a cone, a sense supported by natural meaning and certain usage, which should be distinguished from use of the same expression to refer to any projection to a conical surface.

in feet to the inch; if expressed in the reverse ratio, as is frequently done in the case of maps, such ratio or "principal fraction" is $\frac{f}{H}$.

If a and b mark the length or width of the photographic image, from the same proportion, $A\,B = a\,b \times \frac{H}{f}$, which expresses the simple fact that either dimension of the photographic image multiplied by the scale gives the corresponding ground dimension, thus limiting the area that can be shown in the photograph. Similarly, $a\,b = A\,B \times \frac{f}{H}$. By these simple relations it is possible to determine the area of ground represented in a given picture with H and f fixed.

Scale Variation Due to Ground Elevation.—Such ideal conditions seldom if ever exist. If the earth were level, contour maps would have no place. The scale of an aerial photograph becomes changeable as soon as there is ground relief. Because the photograph is a conic projection, a vertical dimension on the ground causes a horizontal displacement on the plate. Fig. 2 is a simple illustration of this. It represents a section formed by passing a plane through the optical axis and the ground, including a stream channel or valley at V. In an orthographic or map projection to a horizontal plane such as $X\,X'$, the points, V and E, would show at V_1 and E_1 with their true horizontal distance between, but the conic projection to e and v on the picture plane corresponds to the positions, V_2 and E_2, on the datum plane, $X\,X'$. That is, to scale, the point, E, is displaced away from the center of the picture by the distance, $E_1\,E_2$, and V is displaced toward the center by the distance, $V_1\,V_2$, in accordance with the differences in elevation. The projection of the center point, C, is the same regardless of its elevation. If the displacement, such as $E_1\,E_2$, be called d, the elevation, such as $E_1\,E$, be called h, and the angle, such as $E_1\,E\,E_2$, be called α, then $d = h \tan \alpha$. As tan α varies with the distance from the axis of the lens, the displacement varies with this distance and with the ground elevation. At the axis of the lens, α is zero, tan α is zero, and the displacement is zero, regardless of the elevation, as already noted. In other words, the error in the scale of a photograph due to a difference in the elevation of the ground varies with the slope of the ground, the height of the lens, and the distance of the point involved from the center of the picture. It follows that, due to these causes, there is inherent in the photograph a variation in scale as irregular as the ground relief and varying with the other factors named. "It is not practicable to make accurate use of aerial photographs by scaling."*

In Fig. 2 an arbitrary datum plane, X-X', was assumed. If the photographic scale of any other datum plane be used as a basis, the effect in general is to increase the numerical values for some points and decrease those for others. If the results for uniform slopes be considered and the horizontal plane through the foot-point of the lens be taken as the datum, the total change of scale is then divided between plus and minus amounts for the slopes above and below.

* War Department Training Regulations, Aerial Photographic Mapping, prepared under direction of Chief of Engineers, January 23, 1925, p. 9.

For areas having marked relief these errors are very substantial. Viewed from an altitude of 14 400 ft., over ground having a 3% slope, a point down grade from the foot-point of the lens and 4 500 ft. horizontally from it, is displaced in the photograph inward toward the center of the picture an amount equivalent to 72.5 ft. at the scale of the datum plane through the foot-point. To cite an extreme case, a point up grade on a 30% slope and 5 400 ft. horizontally from the axis of the lens is displaced outward an amount equivalent to 1 492.2 ft.

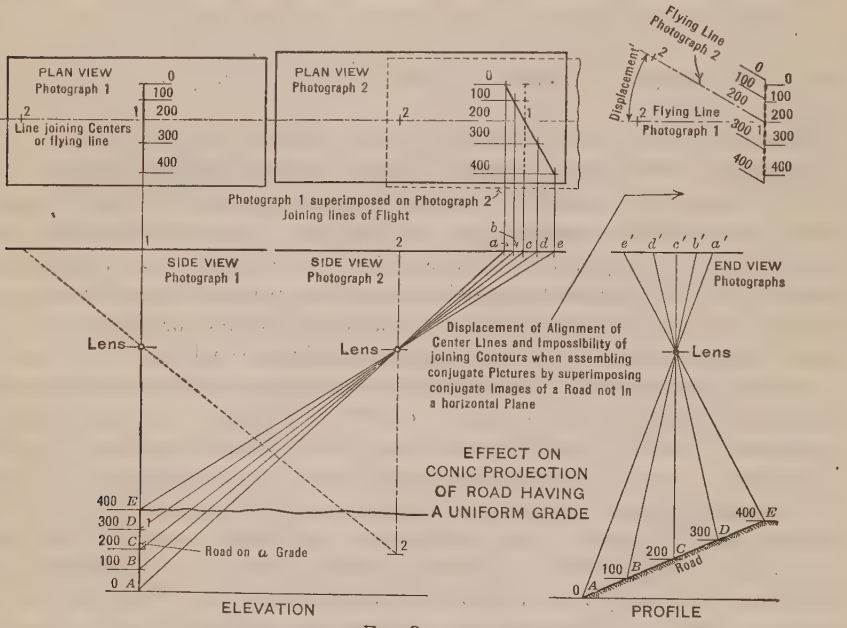

FIG. 3.

Such gross errors will exist in a single photograph of bold relief. In a mosaic the errors can be lessened by suitable trimming of overlapping prints, but errors due to relief exist throughout mosaics and maps made from them or from photographs without correction. In fact, this condition makes it impossible to obtain a mosaic with perfect junctions between pictures showing ground relief, because two pictures of the same ground or object, photographed from successive viewpoints, take different forms. Some curious effects of this sort result, as shown in Fig. 3. It exaggerates what happens in the case of a road of uniform grade, running normal to the line of flight. At the bottom of the diagram the road appears in elevation and profile; above are indicated two photographic plates in side view and in plan. The plates are assumed to be horizontal and on the one exposed directly above the road, it properly appears as normal to the course of the plane in its flight, but on Photograph 2, exposed to the right, the road appears at an angle to the flight. In the upper right-hand corner of Fig. 3 is illustrated the impossibility of correctly matching the photographs along the line of the road. The images along the flying

line are thrown out of position through the angle marked "displacement" and if the images of one point be made to coincide the other points along the road do not, although on both photographs the road appears as straight. If it is of uniform width on the ground its width on the plate will taper slightly.

Fig. 4 represents a more common case of a road of varying slope. This again may be straight in plan or in orthographic projection on a map, but, on plates exposed in the aeroplane before and after it passes above the road, it will appear as a curve; and the concavities or convexities of the two projections will face each other. At the right the two are superimposed, showing the impossibility of trimming prints so as to make them join along such a line.

Ground relief, then, causes the displacement of points in the photographs, and, for a given elevation, the displacement in a single picture differs for points of different distance from the axis of the lens. Similarly, for the same point on the ground a change in the position of the lens and the optical axis causes a change in the displacement of the point in the photographic images. Thus, in two photographs taken with the axis of the lens vertical but on opposite sides of the same point on the ground, it is displaced in opposite directions, that is, away from the axis of each position or the center of each picture. In other words, in overlapping, photographic images common to both suffer a change of relative position, or parallax.

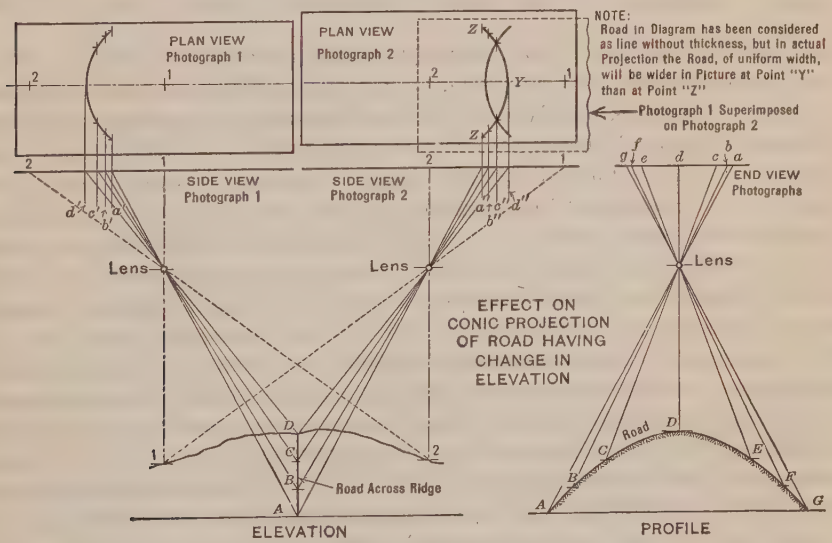

FIG. 4.

Parallax.—Parallax in general is an apparent change of position of objects due to a change in the point of view. Familiar examples of this are the relative shifting of objects seen through the window of a moving train and the different readings of time by persons looking at a clock from different angles. Similarly, in overlapping aerial photographs, points in relief are projected differently on a datum plane. Thus, Fig. 5 represents two plates exposed in the same horizontal plane above the points, C_1 and C_2, on the ground with

the lens at successive points, L_1 and L_2; on the first plate, the point, A, on the ground has its image at a_1, corresponding to A_1 in the datum plane through C_1, whereas on the second plate its image is a_2, corresponding to A_2 in the datum plane. The horizontal distance between C_1 and A is represented by $a_1\ c_1$ and $a_2\ c_1'$ on the two plates, corresponding to the lengths, $A_1\ C_1$ and $A_2\ C_1$, in the datum plane. In the photographic scale of the datum plane the difference in the length of the two images is: $A_1\ C_1 - A_2\ C_1 = A_1\ A_2$. The triangles, $A_1\ AA_2$ and $L_1\ AL_2$, are similar, and, $A_1\ A_2 : L_1\ L_2 :: h : H - h$.

Then,

$$A_1\ A_2 = L_1\ L_2\ \frac{h}{H - h}$$

if the distance between exposures, or the base, $L_1\ L_2$, be called B, the difference of distances, $A_1\ A_2$, be called D, and the corresponding difference in length of image, or parallax, be called p, then p represents D to the photographic scale of the datum plane, that is,

$$p = \frac{f}{H} \times D = \frac{f}{H} \times B\ \frac{h}{H - h} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

For any pair of plates exposed in a horizontal plane, the base, B, is constant, for any given datum, the lens height, H, is constant, the scale ratio, $\frac{f}{H}$, is constant, and the parallax, p, for any point is a function of its elevation. From the foregoing relation, Equation (1), the parallax may be computed for a given elevation; conversely, the elevation may be determined if the value of the parallax can be correctly measured on the photographs. The measurements may be made in conjunction with the stereoscopic treatment of overlapping photographs; if the pictures of the common area from two viewpoints are examined stereoscopically they are seen as one, with ground forms, buildings, and other objects in relief. In aerial stereoscopic photography, parallax may be defined as the difference in distance between two pairs of conjugate points (images) due to the difference in elevation of the two corresponding points (objects). For convenience the representation of parallax in Fig. 5 is in some respects a special case, but the relations deduced may be generalized. Thus, the point, A, is shown as above the datum plane, but might have been shown below, in which case, $A_2\ C_1$ would exceed $A_1\ C_1$, the difference, $A_1\ C_1 - A_2\ C_1$, would be negative, and for a minus elevation the value of the parallax would be negative, as it should be. For a point in the datum plane the elevation would be zero, and likewise the parallax, for the difference, $A_1\ C_1 - A_2\ C_1$, would be zero.

Again, for simplicity, Fig. 5 shows relations in a single plane; it represents a section through the two vertical positions of the axis of the lens. The effective movement of the aeroplane and camera between the two exposures has been in this plane; regardless of what lateral movements took place between L_1 and L_2, the mathematical result is the same as if the center of the lens had traveled along the line, $L_1\ L_2$, which, therefore, may be called the line of flight. It is represented in the diagrams by the line on Photograph 1 joining the

images of the points, C_1 and C_2, and by the conjugate image, $c_1' c_2$, on Photograph 2.

The parallax measurements between the two plates are parallel to such lines. In other words, although in Fig. 5 the point, A, and the lines to it are shown in the vertical plane through the line of flight, they may represent equally well projections on that plane of similar points and lines located elsewhere. Thus, if A is such a projection of any point, A', of equal elevation situated away from the vertical plane through the line of flight, $L_1 A$ is such a projection of $L_1 A'$. If the line, $L_1 A'$, pierces the datum plane at A_1', the projection of A_1' on the vertical plane through the line of flight must lie in the vertical plane, in the datum plane, and in the inclined plane of the triangle,

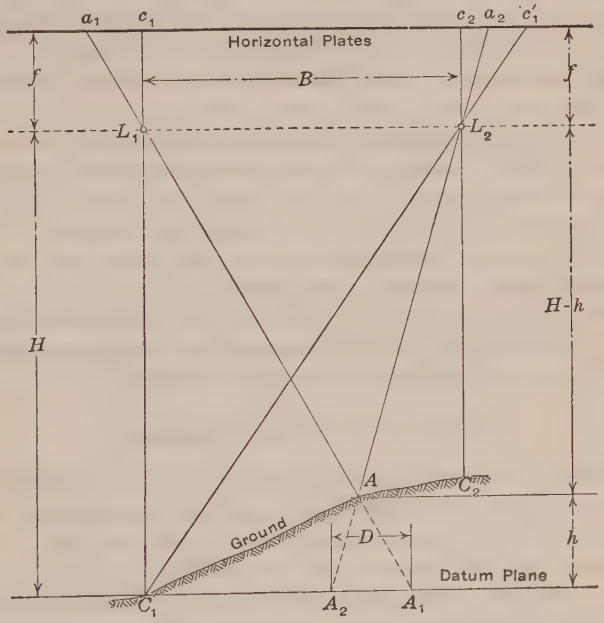

FIG. 5.—EXAMPLE OF PARALLAX.

$L_1 AA'$. The only point meeting this conditions, is A_1. Hence A_1 is the projection of A_1' on the vertical plane through the line of flight and, similarly, A_2 is such projection of A_2', the point where the line, $L_2 A'$, pierces the datum plane. Then, $A_1 A_2$ is the projection of $A_1' A_2'$ and is again for C, and for any point having the elevation of A, the measure of "the difference in distance between two pairs of conjugate image points due to the difference in elevation of the corresponding object points", provided these distances and difference are measured parallel to the line of flight. That is, parallax values as here defined are always to be measured parallel to the line of flight for a pair of photographic plates.

Limitations of Photographic Material

This is not the place for a discussion of all the minutiæ of the photographic art. Only a few points are mentioned that assume special importance in con-

nection with the use of photographic materials for producing engineering maps.

One requirement is that the base carrying the negative emulsion shall be flat or so nearly a perfect plane as not to cause errors in highly refined measurements. It is strange that in all that has been written regarding aerial photography so little has been said concerning this feature which is so fundamental and which, if in error, is quite impossible of correction.

Film.—Precise measurements have disclosed errors due to the use of film, which make it inappropriate for the purpose of aerial mapping. Various expedients, such as pressure plates and suction in different forms, have been tried with the same results. Films are distorted by the developing process, which in many instances frees the gelatine emulsion locally from its celluloid base in patches, adding further errors to the general shrinkage that takes place. Even without separation or "creep", the shrinkage is uneven in different directions. To quote M. André Carlier again, "However, we must state that at present for all work requiring great precision it is necessary to use photographic plates."* M. Carlier's conclusion is the more significant because he might be expected to have a partiality for film due to his experience with it in the war, when he was in charge of aerial photography at the front.

Plates.—The emulsion does not creep on glass plates, nor do such plates shrink in development. Ordinary photographic plates, however, may have considerable curvature, often as much as 15′ of arc. No means is available to correct the effect of this in a single photograph or to avoid its cumulative effect in a series. The use of flat plates is, therefore, essential.

Practical Operating Difficulties

The engineer, accustomed to *terra firma* for the legs of his transit or plane-table, expects some practical difficulties in making observations from a vehicle moving through air at a rate of more than 1 mile per min. The difficulty that has proved most troublesome is due to this lack of stability.

Overlap.—Adjoining photographs must overlap. Overlapping is needed if a gap is to be avoided as it is impracticable to take the photographs so as just to join end on. For the best mosaics, it is needed so as to permit the outer portions of each photograph to be discarded, as already explained. It is necessary in order to orient each picture with reference to those adjoining; for this purpose the two images of the same thing on the two views are utilized. Overlap is also needed so as to permit the stereoscopic treatment of the pictures.

Although overlap is essential for these purposes, it alone does not attain them. Overlap will not eliminate variation of scale nor assure a correct orientation of adjoining photographs.

Tilt.—Ground slope is not the only important cause of the variation of scale in the photographs. A variation is caused in practical operation by the changing inclination of the focal plane. Air pockets, wind changes, banking and tipping of the aeroplane, all tend to rock or tilt the camera. Fig. 6 is a

* "Cependant, nous devons dire qu'à l'heure actuelle, pour tous les travaux exigeant une grande précision, il faut se servir de la plaque photographique," *L'Illustration*, 13 decembre 1924, p. 585.

simple illustration of some of the results. It represents a vertical section through the axis of the lens of a tilted plate, and shows also the position the plate would have taken if it had been horizontal over the same position of the lens, that is, the axis of tilt is normal to the plane of the diagram through the common position, *L*, of the lens. The trace or intersection of the horizontal and tilted planes of projection is likewise a line normal to the diagram through the point, *m*. For convenience of illustration, the angle of tilt as shown is excessive, but this does not affect the principles involved.

For the horizontal position of the plate, the optical axis is vertical, intercepting the ground at *V*, whereas, for the tilted position, the optical axis is swung through the angle of tilt and intercepts the ground at a very different point, *C;* that is, the centers of the two negatives will receive images of different ground points and the distance and direction between them will vary with the varying degree and direction of the tilt.

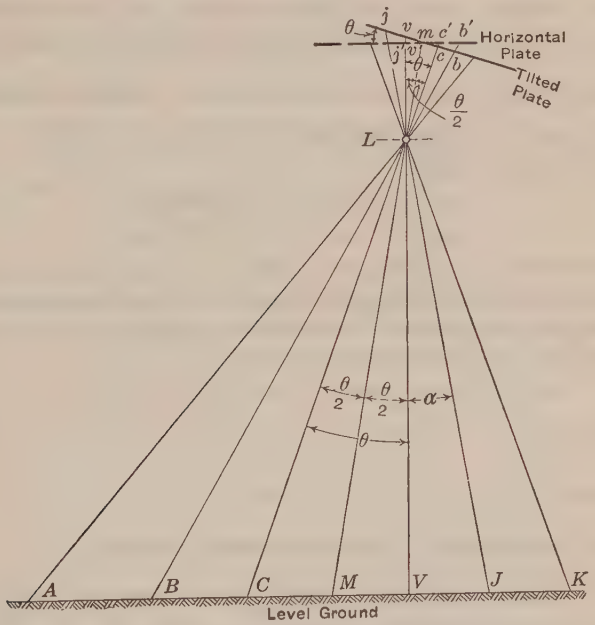

FIG. 6.—DISPLACEMENT DUE TO TILT.

The areas photographed will differ accordingly. Some light rays, such as *K L*, reaching the horizontal plate will not reach the tilted plate; some rays, such as *A L*, will reach the tilted plate, but not the horizontal plate; some ground dimensions, such as *M J*, will have a greater image on the tilted plate than on the horizontal; and the opposite will be true of some dimensions, such as *M B*. The trace of the horizontal and tilted planes of projection receives the same light rays in either case and the photographic scale in that one line is unaffected.

The axis of the lens for the horizontal position pierces the plate in its optical center, *v′*, and the same light ray intersects the tilted plate at *v*. The

axis of the lens for the tilted position pierces the plate in its optical center, c, and the same ray cuts the horizontal plate at c'. The angle of tilt is:

$$\theta = v\,m\,v' = c\,m\,c' = v\,L\,c$$

The focal length, $f = L\,v' = L\,c$, and hence the right triangles, $L\,v'\,c'$ and Lcv, are equal; that is, $L\,v' = L\,c$ and $v'\,v = cc'$; hence the the right triangles, $v'\,mv$ and cmc', are equal and $v'm = cm$. Therefore, the right triangles, $Lv'\,m$ and Lcm, are equal, and

$$\text{Angle } m\,L\,v = \text{Angle } m\,L\,c = \frac{1}{2}\,\theta$$

On the tilted plate, v is the image of the point on the ground vertically below the lens, vc is the image of the line connecting the vertical point with the optical center, and m is on that line at a distance from the center,

$$c\,m = c\,L \tan \text{Angle } c\,L\,m = f \tan \frac{1}{2}\,\theta$$

whereas, the distance from the optical center to the vertical point is $cv = f \tan \theta$. For small tilt, such as commonly occurs in practice, $f \tan \theta$ is virtually twice $f \tan \frac{1}{2}\,\theta$; that is, the point, m, is virtually half-way between v and c and hence it is sometimes called the mid-point. Thus, for 2° of tilt, with a focal length of 10 in.,

$$cv = 10 \tan 2° = 0.3492 \text{ in.}$$
$$cm = 10 \tan 1° = 0.1746 \text{ in.}$$

The displacement of images due to tilt is outward from m in the part of the tilted plane above the horizontal plane and inward toward m in the part below. Thus, for any image, j, formed by the light ray, $J\,l\,j$, at an angle, α, from the vertical, the displacement outward is:

$$d = m\,j - m\,j' = (c\,j - c\,m) - (m\,v' + v'\,j') = c\,j - v'\,j' - 2\,c\,m$$
$$= f \tan (\alpha + \theta) - f \tan \alpha - 2\,f \tan \frac{1}{2}\,\theta$$
$$= f \left[\tan (\alpha + \theta) - \tan \alpha - 2 \tan \frac{1}{2}\,\theta\right]$$

Similarly, for any point, b, the displacement inward is:

$$d' = f \left[\tan \alpha - 2 \tan \frac{1}{2}\,\theta - 2 \tan (\alpha - \theta)\right]$$

These values are in terms of dimensions on the photographs and must be multiplied by the scale, $\frac{H}{f}$, to determine the corresponding dimensions on the ground or datum plane.

For a few degrees of tilt a negligible error is involved in assuming that $2 \tan \frac{1}{2}\,\theta = \tan \theta$ and the equations become:

$$d = f\,[\tan (\alpha + \theta) - (\tan \alpha + \tan \theta)]$$
$$d' = f\,[(\tan \alpha - \tan \theta) - \tan (\alpha - \theta)]$$

In practice the tilt can usually be kept within 2 and need rarely exceed 3 degrees. This, however, causes substantial displacement of the points. Thus,

at a height of 12 000 ft., with a lens having a focal length of 12 in., producing a scale of 1 in. = 1 000 ft., in the case of a point so located above the midpoint that $\alpha = 25°$, a tilt of 2° would cause a horizontal displacement equivalent to a ground distance,

$$D = H\,[\tan 27° - (\tan 25° + \tan 2°)] = 99.5 \text{ ft.}$$

Serious errors also result in determinations of ground elevations from tilted plates. Thus, if the plate last defined were used for parallax readings in conjunction with a horizontal mate, with an exposure interval of 3 000 ft., or a base of 3 in. on the plate, the indicated elevation of the point, *B*, would be 20 ft. too high, that of the point, *J*, 17.5 ft. too low, or a total error of 37.5 ft. in the difference in elevation between the two points.

From the equations it will be noted that for a given scale the displacement in terms of ground measurements is proportional to the focal length. Thus, the device of flying at a greater elevation and increasing the focal length proportionally so as to retain the same photographic scale and reduce the displacement of points that is inherent in the conic projection of ground elevations, has the disadvantage of increasing proportionally the displacements due to the tilt. A photographic mosaic from original negatives will be subject to variations in scale from both causes (ground relief and tilt); for a given photographic scale any change in flying height and focal length will decrease the error from one cause and increase that from the other.

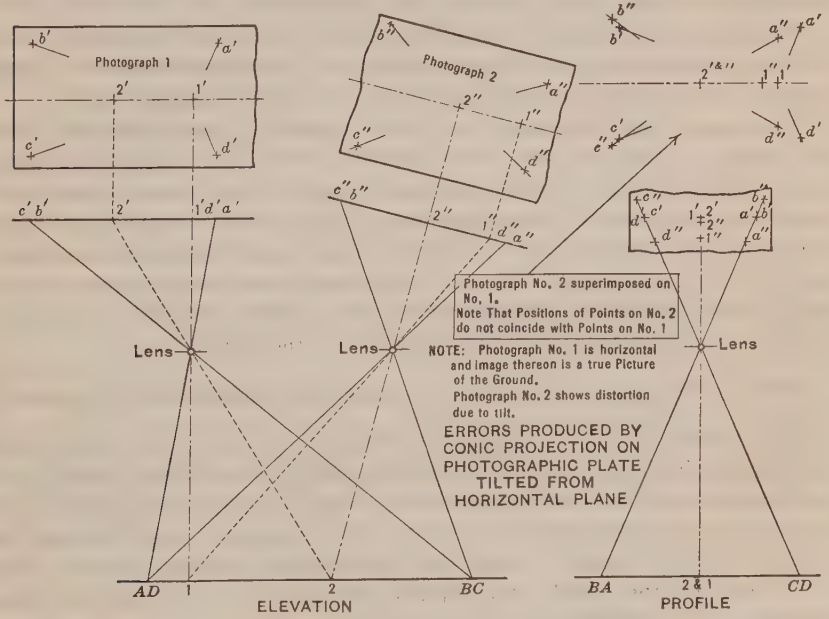
Photograph 1
Photograph 2
Photograph No. 2 superimposed on No. 1.
Note That Positions of Points on No. 2 do not coincide with Points on No. 1
NOTE: Photograph No. 1 is horizontal and image thereon is a true Picture of the Ground.
Photograph No. 2 shows distortion due to tilt.
ERRORS PRODUCED BY CONIC PROJECTION ON PHOTOGRAPHIC PLATE TILTED FROM HORIZONTAL PLANE
Lens
Lens
Lens
AD
1
2
BC
ELEVATION
BA
2 & 1
CD
PROFILE

Fig. 7.

Fig. 7 further illustrates the displacements in conic projection on the tilted photograph. The ground is here assumed to be level and four points, *A*, *B*, *C*, and *D*, are differently projected on the horizontal and tilted Photographs 1 and 2 at the left. The displacements on the tilted photograph are

radial. At the upper right corner are shown the two photographs superimposed so as to make the two images of the point where the focal axis of the second photograph intersects the ground surface coincide. Because of the tilt some of the points are displaced outward from the center and others are moved inward.

Variation in scale due to tilt alone is illustrated in Figs. 6 and 7, but the common condition is that this is combined with displacements due to elevation and other factors in uncertain and varying amounts. These diagrams show the effects in a single plane or direction; like effects occur in other planes and directions. Thus, the results in the photograph are complex. The addition of tilt to the other causes of changes in scale is a complication that has caused much difficulty to those attempting the translation of the photographic data into maps and has been the undoing of many such efforts.

Earnest endeavors have been made to prevent tilt—to avoid at the source the variation in scale from this cause. All such experiments with which the writer is familiar have served at best only to minimize it. The camera may be suspended above its center of gravity so that any tilt from the vertical immediately enlists the force of gravity to bring the camera back, but such action will not prevent slight tilting this way and that at the instant of exposure. Another simple device that suggests itself is to attach spirit-levels to the camera and to exercise positive control over its position in accordance with their readings; or to photograph the bubbles with the picture as a record of tilt and as a guide for its correction. The difficulty of this device is that the bubbles themselves are seriously affected by the accelerations and decelerations of the aeroplane. Their readings are useless for accurate work.

Gyroscopes have been used, but in the extended experiments with which the writer is familiar they have proved insufficient. Suspended pendulously, they tended to keep the camera vertical, but the corrective action was too slow to preclude tilt. Suspended neutrally, they tended to keep the camera in a plane according to their setting, but means were lacking for making the setting precisely horizontal. In an extended area, precession becomes an appreciable factor, that is, the gyroscope returns to a position parallel to its original setting, without compensation for curvature of the earth.

Height of Lens.—It has been shown that the photographic scale, S, equals $\frac{H}{f}$. As H is the vertical distance from the ground to the lens, for a given camera or focal length, the scale, S, is different for each different flying height; hence the importance of maintaining as near as may be a constant elevation of the aeroplane while photographing so as to avoid change of scale between overlapping views. Changes of scale due to this cause combine with the effects of tilt and ground relief, and require correction in converting the photographic data to accurate map form.

Transforming the simple formula for scale, $H = f\,S$; that is, the flying height necessary for a particular photographic scale is found by multiplying the latter by the focal length of the camera to be used. As H varies in any event with the ground elevation, the particular value of S used in this equation

to determine H, will apply exactly to only one datum. This datum may be chosen as approximately the mean elevation of the ground to be photographed, but, as already explained, a different scale will result for each other ground elevation appearing in the individual picture or mosaic.

Speed and Drift.—The speed of a plane in still air is partly under the control of the pilot, through his control of the motor and its fuel. Speed is also affected by the velocity of the wind with respect to the line of flight. If the flight is with the wind, the speed of the plane is greater than it would be in still air. If the directions of flight and of wind are opposed, obviously the speed of the plane is reduced. Under working conditions, the net or "ground speed" will ordinarily vary between about 60 and 120 miles per hour, or roughly between 90 and 180 ft. per sec. If the wind crosses the line of flight, the plane must be pointed so that the resultant of the forces carries it in the desired direction.

Ordinarily, flights in a strong wind will not be made, but to compensate for any cross-wind the plane must be pointed at a slight angle from the line of flight and the camera should in turn be swung back through the same angle; otherwise, a strip of pictures presents a serrated border, as indicated in Fig. 8. It is not imperative that this effect be eliminated, but for convenience and economy it should be minimized.

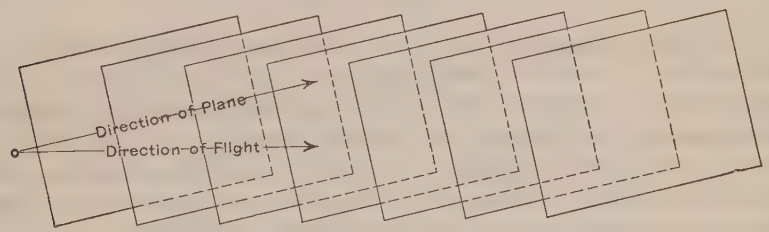

FIG. 8.—EFFECT OF DRIFT.

Intervals Between Exposures.—The frequency of exposure depends on the size of the useful image, focal length, height of lens, overlap, and speed of flight. Under theoretical conditions of level ground, with drift eliminated and photographic plates in one horizontal plane, if successive images were just to join end-on, the distance, d, between points of exposure would equal the ground dimension of a single photograph in the direction of flight. If the length of useful image is l, its width, w, and the corresponding ground dimensions, L and W, then,

$$L = l\,S = l\,\frac{H}{f},$$

$$W = w\,S = w\,\frac{H}{f},$$

and,

$$d = L \text{ or } W$$

according to whether the long or short side of the image parallels the flight.

Overlap is a direct reduction of the interval. If the overlap is expressed as a decimal, d, the advance in ground pictured for each overlapping view is,

$$(L \text{ or } W) \times (1 - d)$$

and if the net, or ground, speed of the plane is g, the time interval between overlapping exposures is,

$$t = \frac{(L \text{ or } W) \times (1 - d)}{g}$$

Thus, for a speed of 60 miles per hour, or 88 ft. per sec., at an elevation of 12 000 ft., with a 12-in. lens and an image 8 in. in the direction of flight, for a 60% overlap:

$$t = \frac{8 \times 12\ 000 \times 0.4}{12 \times 88} = 36\tfrac{1}{2} \text{ sec., approximately.}$$

Number and Area of Overlapping Exposures.—For a strip of pictures extending for a distance, D, on the ground the number of plates or exposures is,

$$N = 1 + \frac{D - (L \text{ or } W)}{(L \text{ or } W) \times (1 - d)}$$

Thus, for a flight as in the preceding example for a distance of 10 miles, or 52 800 ft.,

$$N = 1 + \frac{52\ 800 - 8\ 000}{8\ 000 \times 0.4} = 15.$$

With a useful image, 6 in. wide, the ground area pictured by such a strip would be $6\ 000 \times 52\ 800 = 316\ 800\ 000$ sq. ft., or 11.44 sq. miles. Simple computations of like nature will give the number of exposures needed for various distances, focal lengths, and heights of lens, to cover any combination of overlapping and tributary strips. The theoretical numbers thus found should be slightly increased to photograph the small excess area necessary to assure that the views will have at least the needed overlap and total area.

Optical and Mechanical Requirements

Instruments of precision are the foundation of modern surveying. The engineer of to-day takes his transit and level for granted, yet all his work of triangulation and precise leveling, his comparisons of stadia and plane-table, his other elections in methods of control, mapping, and location, rest on that little group of instruments that exercise so profound an influence on his work. The development of successful designs of the engineer's transit and other instruments demanded years of painstaking effort.

Aeroplane topographic surveys call for new instruments of precision. They are as essential to the new art as the transit and level to ground surveying. The design and construction of such new instruments, to equal or exceed the old in their accuracy and to meet the requirements of precision and practical workability, present a problem demanding similar effort and skill. Under pressure of other demands, the aeroplane has attained sufficient development and variety so that it may be taken for granted. It must be supplemented by instruments for accurate aerial photography, for the determination and

removal of tilt, for the interpretation and conversion of the conic projection in photographs to the orthographic projection and contour delineation in topographic maps.

The camera should be at least as accurate as the transit. As film is to be avoided, a plate camera is needed. To permit sufficient exposures on a single flight, magazines for the plates are required. The instrument used to "horizontalize" the plates must be essentially a photographic projector, with provisions for tilting the negative and copy board. The instrument for interpretation of the photographs must include provision for refined measurements of the displacements already mentioned, so as to determine the effects of tilt and change of elevation. In all the instruments, the lenses should be as free as possible from distortion, the different elements should be truly concentric, and, in general, the optical devices should be of high precision in design, assembly, and adjustment. A partial description of the instruments that have been designed and used for aeroplane topographic mapping in connection with the method under consideration, are here given.

Mosaics

Reference has been made to the mosaic as a development of the World War. Such a composite picture has many peace-time uses, including the preliminary consideration of rights of way for electrical transmission, railroads, and other lines; general studies of drainage basins, timber holdings, and other large areas; and city planning, tax assessment, and municipal zoning studies. Compared with maps made by old methods, the mosaic offers some marked advantages and some serious disadvantages.

One great advantage is speed. It is possible to obtain in a few days a mosaic of an area of such size and character as would require months or even years to map by old methods. Another advantage is its wealth of detail. Time and expense limit sharply the lengths to which the topographer can go in measuring stream meanders, timber outlines, and other details; conventional signs limit what the cartographer can show; but the camera gives a picture of the whole. While the transitman locates a point or line, the camera records a square mile.

The weakness of a mosaic, fatal to its use as a substitute for the engineering map, is its lack of uniform scale. It offers little or no indication of elevations. Its component parts are subject throughout to irregular changes of horizontal scale due to ground relief and tilt, as already noted. The assembly of these is an aggregation of the local errors, and offers opportunity for additional mistakes in orienting the fragments and fitting them together. A straight line on the mosaic will usually represent a broken or curved line on the ground and *vice versa*. Thus, the mosaic has been called "a caricature of the landscape."

An obvious method of merging overlapping prints into mosaic form is to superimpose conjugate images of two or more sharply defined points or features, tear or cut through the overlap, and waste the excess fragments, continuing the process with adjoining pictures to the limits of the area. Errors of orientation result from differences of elevation. Thus, if one point used is in the

datum plane and the other a high point, the result is as if the second print had been rotated about the first point and out of correct position an amount dependent on the elevation or resulting parallax of the second point.

Of the scale variations or errors in mosaics, those due to tilt can be eliminated if the tilted negatives can be so re-projected as to be "horizontalized" before taking prints for making into mosaics, but the errors, usually greater, due to ground relief, remain. The latter, being inherent in photographs, are inevitable in mosaics. They can, however, be lessened by shortening the interval between exposures, thus increasing the number of photographs and restricting the fragments used to smaller areas about the center points of the pictures. They can be lessened and localized if the centers of the pictures can be correctly located by platting and the fragments oriented; but the difficulty has been to identify precisely conjugate images of the center points. In any event none of these devices supplies a real map because of the lack of vertical scale and the inevitable variation in the horizontal scale. If the centers of the pictures are correctly platted to a chosen scale and correctly oriented, the photographs of ground between center points will have varying scales for different elevations of ground, and in trimming adjoining prints some of the ground will not be shown or will be duplicated in the mosaic.

A Solution

The first step in the method under discussion is to secure a series of overlapping aerial photographs on glass plates. Suitable pairs of plates are examined stereoscopically, and the tilt is determined and overcome by re-projection. Contours are drawn on the plates and all data are converted to orthographic projection, with such constant scale as is desired on the final tracing. By means of these successive steps there is produced from aerial photographs an accurate contour map in the usual form of tracings and blue-prints as long used by engineers. The instruments and technique used have been developed through continuous research since the conclusion of the World War.

Field Operations

Flying.—Aeroplanes of various types are used, according to the requirements of flying radius, ceiling, and weight-carrying capacity. Ordinarily, a field base is established near the area under survey and thus the flights are not sufficient to tax these requirements. A two-man crew is used, pilot and photographer. The area to be mapped is traversed in straight lines representing photographic strips, with overlapping strips and tributary lines as needed to cover the area with a simple network of nearly straight lines.

The effort is to fly on an even keel at a constant height, usually approximating 5 000, 10 000, or 15 000 ft. If it is necessary to determine the best program for covering the area under survey, a preliminary reconnaissance flight is made for that purpose.

Photographing.—Overlapping views are taken with an interval between exposures fixed so that each photograph will include the center point of any adjoining picture. Because of the inadequacy of film and of the curvature likely to exist in ordinary photographic plates, only selected flat glass plates

are used. To secure a record of sufficient accuracy and extent without excessive flying, a magazine camera of special design is used. Its details and construction are such as to handle glass plates easily and rapidly, producing overlapping photographic views of such character as can be utilized in the special apparatus necessary for the later processes.

Promptly after each photographic flight, the plates are developed and the prints are made and examined to assure that the desired area and overlap have been secured. In case of uncertainty regarding the exact area to be covered by the survey, a preliminary mosaic is used to define this area.

Ground Control.—To determine the effect of the tilt of the camera and interpret the data recorded in the photographs some ground measurements are made, but they are merely a small fraction of the work that would be involved in a ground survey for comparable results. For horizontal control, an isolated base line, preferably about 3 000 ft. in length, is measured at a convenient location near each end of a network of photographs, and every 15 or 20 miles in a long series. For vertical control, differences of elevation are measured on the ground for several points that appear on each plate; the distances between these points are not needed.

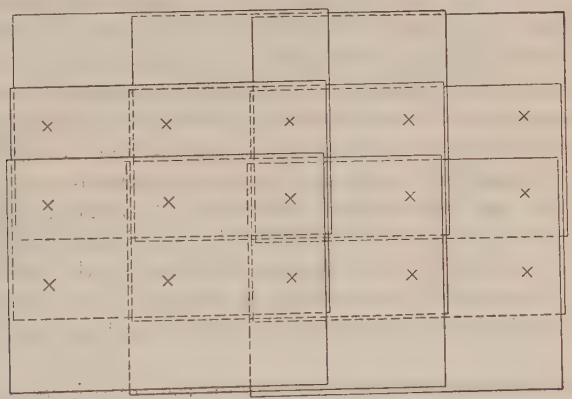

FIG. 9.—FIELD POINTS FOR OVERLAPPING PLATES.

Points used for horizontal and vertical control must be identified clearly not only on the ground, but in the photographs. Sharply defined points are selected from field prints of the photographs by use of a hand stereoscope. Although there is considerable latitude in the location of the points, the effort is to secure a point near the center of each picture and one near each end of a line through the center normal to the line of flight. For an overlap exceeding one-half the picture, this means that nine such points appear on any particular photograph within a strip, and that six of the points may serve also for adjoining photographs as indicated ideally in Fig. 9. Thus, the number of field points per square mile depends on the size and shape of the area as well as the photographic scale. As the scale, S, equals $\frac{H}{f}$, an increase of the flying height or a decrease of the focal length lessens the number of field points required

per square mile. For a flight at an elevation of 10 000 ft. with a lens of 8 in. focal length and a 6 by 8-in. image for three parallel strips of ten plates each, and an overlap of 60%, there would be 50 field points in about 22 sq. miles, or about 2 per square mile.

Office Operations

Centering Negatives.—The optical center of each plate (negative) is permanently marked. This operation takes little time and consists of simple details, but deserves description because of the effect on accuracy. The plate is laid on a glass table and a straight-edge is laid diagonally across it so as to join the fine marks it carries as a result of pin-holes in the aperture plate of the camera. A short mark along each diagonal is cut in the emulsion making an intersection that marks the center of the picture. The accuracy is enhanced by use of needle points, razor edge, and magnifying glass.

From this negative plate a positive is made by direct projection and used in subsequent operations. The glass positives carry all the attributes and markings of the negatives, including the center points.

Stereoscopic Examination.—The measuring stereoscope, shown in Fig. 10, consists essentially of systems of lenses for the stereoscopic examination of photographic plates mounted on turn-tables, with provisions for passing light through the plates from below and for certain measured movements in the plane of the turn-tables. These movements include the rotation of each table about its center, the movement of one table along the line joining their centers, and the joint movement of both along that line and normal to it.

In Fig. 10 the lens housing and eye-pieces (*a*) appear at the top and a pair of plates (*b*) appear on the turn-tables below. Hand-wheels control the various motions. Lying in racks against the front of the machine may be seen two rules (*c*) made of glass with metal frames. The glass parts contain horizontal and vertical cross-hairs carefully set in the long and short axes of the rules.

The rules fit recesses provided for them on the turn-tables and the intersecting cross-hairs of the rule then mark the mechanical centers of the tables. These are used to bring such centers separately in line with the optical axes of the stereoscope. This adjustment once made need not be repeated unless the machine is disturbed for some reason.

The two positive plates under examination are placed on the turn-tables, each separately centered under the cross-hair intersection of the eye-piece and clamped. This adjustment once made need not be repeated until a new pair of plates is mounted on the tables.

The tables are rotated so as to bring into alignment the four points on the plates marking the two views of the center point of each plate. This involves identification on each plate of the conjugate center or image of the center of the other plate. This can readily be approximated by observing the picture point with the naked eye, and can be accurately done by stereoscopic examination. This alignment once being established, the motion of rotation is not again used until a new pair of plates is mounted on the tables.

The center of one plate and the conjugate image on the other plate are separately centered under the eye-pieces. The bed carrying both turn-tables is then moved laterally (along the line of flight) until on the first plate the conjugate image for the center of the other plate is centered under the eye-piece. The other eye-piece will then not be over the center of the other plate. In other words, due to parallax and other causes the two images of the line between centers of the plates will usually be of different length. The micrometer dial (*d*) measuring the amount of divergence between the plates is set at zero. The other plate is moved independently until it is centered under the eye-piece. The micrometer reading will then show the "spread." Ordinarily, this will not be due solely to the difference in elevation (parallax), but will involve also the factor of tilt in one or both plates.

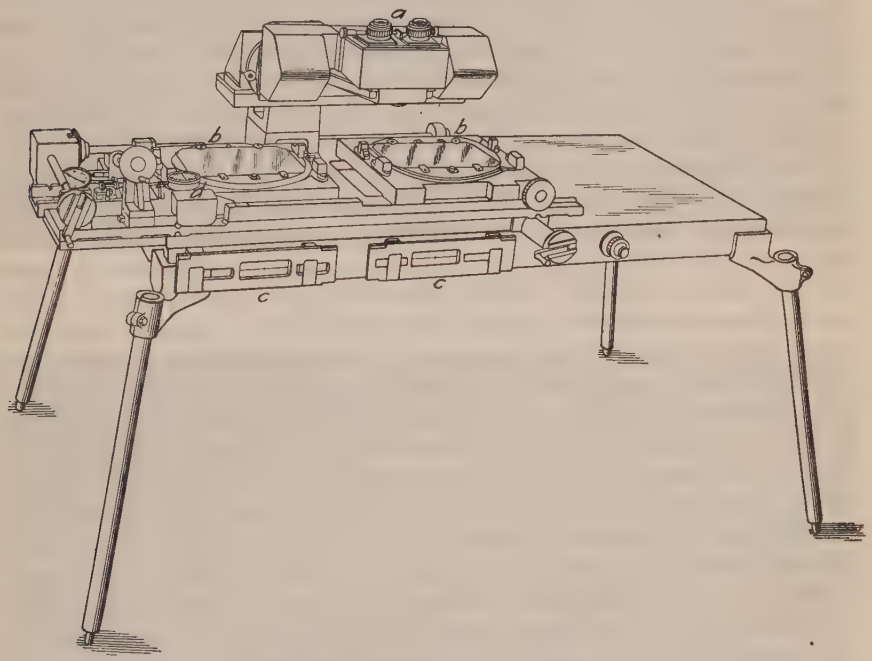

FIG. 10.—MEASURING STEREOSCOPE.

Similar parallax readings are taken for the few salient points previously mentioned. The correct (computed) parallax values based on the known elevations of the selected points as measured on the ground are compared with the readings. The differences, their direction (plus or minus readings), and their distribution over the plates indicate the direction and amount of the tilt in either or both plates.

Re-Projection.—With the tilt determined, each plate is "horizontalized" if necessary by re-projection. For this a machine is used having the usual facilities for re-projection of photographic plates, with special provisions for certain measured or controlled movements. The frame carrying the plate to be projected can be rotated about its center, moved vertically or laterally in the

plane of the plate, and tilted about a vertical or horizontal axis through its center. Another frame carrying the plate to receive the projection has like motions. The tilting motions are measured on graduating arcs. For use in the latter frame a ground-glass plate is provided, marked with vertical and horizontal axes.

The plate to be projected is put in position and its center is brought precisely into line with the focal axis of the lens. It is then rotated about its center until the conjugate image of the center of the other plate is brought into the horizontal plane through the center and focal axis. The tilting motions are then manipulated in the direction and degree indicated by the differences between parallaxes already computed from known elevations and those indicated by the preliminary stereoscopic examination. This adjustment is divided between the negative and the ground-glass plate so as to keep a sharp focus throughout the view, and then the ground-glass plate is removed, a sensitive photographic plate is substituted, and the re-projection made. The plate thus secured will approximate very closely the result that would have been obtained had the original aerial exposure been made on a horizontal plate.

The various plates needing correction are similarly re-projected and the accuracy of the results is verified by re-examining the plates in pairs on the stereoscope, comparing anew the parallax readings with the known parallaxes for the salient points. In rare cases a plate is re-projected a second time for greater accuracy. On the other hand, an original plate is at times such a close approximation to the horizontal as not to require re-projection at all. Thus, finally a complete series of overlapping views on horizontal planes are obtained which are used in subsequent steps of the process.

Contouring.—Two plates representing overlapping views are mounted on the stereoscope, centered and aligned as previously described. Starting with a low point of known elevation, the spread between the plates is set to correspond with the computed parallax for the elevation of the nearest contour line desired. The plates are then moved together and observed stereoscopically until a point is located under the centers of both eye-pieces, that is, a point having such a parallax. This is a point on the desired contour and is marked with a special pencil on one plate. Sufficient other points are similarly located to build up the contour line on the plate. The computed parallax for the desired contour interval is added to the reading of the machine and the next contour line is similarly located and drawn. This process is continued until the area common to the two plates is fully contoured on one of them. The plates are then removed and the pencilled contour lines are drawn through the emulsion with a needle point.

This process is greatly facilitated by the stereoscopic vision of the pictures. The cross-hairs seem to lie on the ground surface when a correct point is brought under their intersection. In case of a point too high the cross-hairs seem to bury themselves in the earth, and in the case of one too low they seem to float in the air above it. With this characteristic and the vivid picture of relief seen through the stereoscope, the skilled operator in practice can quickly and accurately trace any contour line. Fig. 11 shows a plate contoured

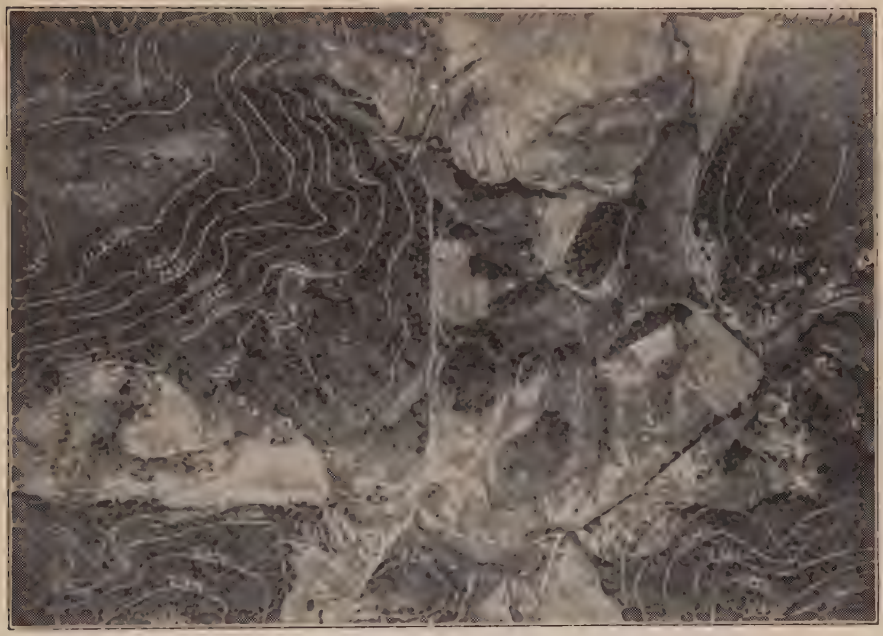

FIG. 11.—CONTOURED PLATE, GREEN RIVER SURVEY.

FIG. 12.—ORIGINAL PHOTOGRAPH, AEROPLANE CONTOUR SURVEY, MEDIA AREA.

throughout. In practice the contours are drawn on one view of each area common to an overlapping pair. The process is continued with the plates in pairs until by the aggregation of such parts the entire area is contoured.

Plotting and Tracing.—After contouring, the product is a set of plates representing the conical projection of the desired area on horizontal planes with at least 50% overlap in adjoining plates and contour lines of desired elevations and interval. As a contour line connects images of equal elevation, it marks a level plane and its conical projection on the horizontal plate has uniform scale. The scale varies, however, as between the different contours, but the scale of any contour line can be modified as desired by moving each point of the image radially.

To control the drawing, the centers of the pictures and other selected points are plotted separately. For selecting the control points the photographic plates are examined in pairs on an illuminated glass table and common images, sharply defined and located toward the edges of the plates, are chosen. At least six points are marked on each plate, usually more. For the plates containing the base line measured on the ground the plotting points include the ends of this line. The plotting of the map commences with the base line. Its known length is plotted with great care on drawing paper to the scale desired on the map. The centers of plates including the base line are then located with reference to it by means of direction lines taken from each plate. These "ray" lines from the centers of the plates to the ends of the base line and to other plotting points are correct in azimuth regardless of the changes in elevation or scale, provided the plates are "horizontalized" (corrected to the horizontal), just as horizontal angles or azimuths read by the transit are correct regardless of differences in elevation, provided the transit is properly adjusted to the horizontal. Similarly, the intersections of such ray lines from the different plate centers serve progressively to orient the succeeding plates and plot the various selected points. In this way, the points chosen on each plate are successively plotted, and, finally, the contour lines, drawn on the plates as already described, are readily transferred to the finished map with such corrections in scale as are necessary to make the final drawing correct to scale throughout.

Results

Aeroplane contour surveys are new, but have been subjected to some rigid tests. The originators of this method first sought an independent review of its correctness theoretically. A number of investigations and reports were obtained by them, giving expert opinion from the point of view of the physicist and the engineer, all of which were entirely favorable. Practical work was then undertaken, and a number of surveys have been, and are being, carried out. In all instances the results have been very satisfactory and the parties for whom the maps were made, or their engineers, have made highly favorable reports. The maps have been compared critically with independent ground surveys made by transit and other instruments at construction sites or more generally over the areas involved. In one instance the aeroplane surveys

disclosed a substantial error in the ground surveys, but in no case has the reverse occurred.

In 1921, the writer was assigned with other engineers to make a careful examination of the theories involved and to check the results obtained. One of the aeroplane contour surveys studied was for a district near Media, Pa., having the following characteristics:

Location: Delaware County, Pennsylvania.
Approximate area mapped: 10 sq. miles.
Character: Moderate slopes, small streams, much culture.
Approximate elevations: Maximum, 300; minimum, 87; difference, 213 ft.
Horizontal scale: 1:6952;
Contour interval: 20 ft.

Fig. 12 is one of the original photographs made in the Media survey and Fig. 13 is the resulting map.

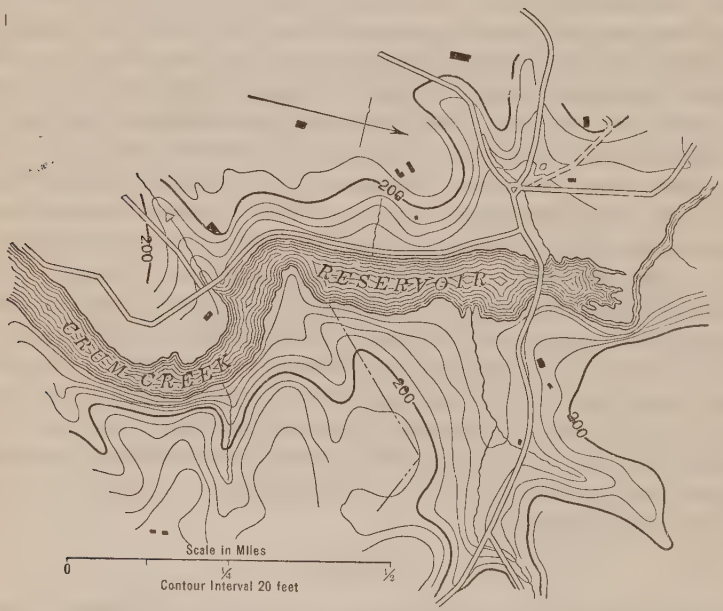

FIG. 13.—AEROPLANE CONTOUR MAP, MEDIA SURVEY.

Accuracy

The Media survey, made in 1921, was of an experimental nature, and was carefully checked by the writer. For this purpose, simultaneously with the photographic survey, a "stadia traverse," or transit survey, was made using the ordinary instruments and methods. This was not a complete topographic survey and no contouring was done, but the traverse followed generally the principal roads, locating also prominent landmarks, and closed or checked on its starting point with unusual accuracy, so that it offered a satisfactory ground control or reference line with which to verify the photographic results.

Elevations of numerous points readily identifiable were also taken along the base line by use of a wye-level and side shots from the base lines were made by vertical angle readings with the transit. In this check survey the level readings certainly, and the vertical angle readings probably, offered a satisfactory degree of accuracy for the purpose of the check.

Comparisons were made between the maps from independent sources, including the photographic work, the instrumental ground survey, and such other maps as were available. Study and comparison of these disclosed the fact that, to a highly satisfactory degree, the aeroplane photographic map coincided throughout with the instrumental survey. Such displacements as were contained in the photographic map were uniformly distributed and non-cumulative, gradually melting out over long distances, so that the relative positions of adjacent points were everywhere correct.

In the 12-mile circuit of the ground traverse, elevations were determined for about 400 points and compared with those shown by the aeroplane contours. In the subsequent report the conclusion was reached that aeroplane contours could be located with the assurance that none would be in error more than 2½ ft. The following is quoted from the report* dated June 4, 1924: "The accuracy with which this can be done with mechanical limitations of the present machines is a plus or minus 2½ ft."

In discussing the use of the stereoscope, the report continues:

"The same results could be secured by different men familiar with the operation of the stereoscope, proving that the operation was a mechanical one not influenced by the introduction of the human element."

Important improvements subsequently added to this process of making contour maps from aeroplane photography, should further increase the degree of accuracy. The theoretical considerations would point to an accuracy within 1 ft., plus or minus. This obviously is a very high degree of exactness for topographic maps of large areas.

Detailed checks were likewise made of two other surveys. One of these was along the Susquehanna River, near Conowingo, Md., and the other in the Green River Basin of the Blue Ridge Mountains in North Carolina. The topography elevations, scales, and other details differed, but the results disclosed the same degree of accuracy. The wide differences in character of terrain among the three cases offered an unusual opportunity for checking the accuracy and applicability of the method to different types of topography.

Time

One of the advantages of aeroplane contour surveys is the great saving in the time required to produce maps as compared to older instrumental surveys. In the original field observations the aeroplane and camera are so much more rapid than the transit, level, and plane-table as to preclude comparison. The plate correction, contouring, and other office operations connected with aeroplane surveys are also relatively rapid and are susceptible of partition to an extent that makes for maximum speed.

* Report by Day & Zimmermann, Inc., Engineers, Philadelphia, Pa.

This advantage in time is particularly marked for a large area of bold relief and forest cover, as illustrated in the Green River survey. To quote H. R. Faison, M. Am. Soc. C. E., the Resident Engineer who used the maps on that survey:*

"Maps can be completed in progressive sections, as desired, and are in the hands of the engineers for study in a fraction of the time required for ground surveys."

Costs

Unit costs of aeroplane topographic surveys vary with numerous factors, among which the following deserve special mention:

(*a*) Proximity of suitable landing ground and other facilities.
(*b*) Situation of the area, affecting the cost of flying or of transporting the aeroplane to the vicinity;
(*c*) Size of the area, over which such costs must be pro-rated;
(*d*) Shape of the area, affecting the number of flying hours and photographs;
(*e*) Character of the area affecting the labor required to depict the relief and other features;
(*f*) Contour interval; and
(*g*) Horizontal scale, affecting both field and office processes.

Obviously, these elements cover a wide range, within which costs vary with other factors, including the choice of equipment (notably planes), and the focal lengths of cameras. Detailed cost data have not been published, but are gradually being accumulated and indicate advantages for the aerial method, particularly in topographic surveys of large areas having bold relief and considerable growth of timber or brush. Under such circumstances the data available to the writer indicate that aeroplane topographic surveys may be made for a fraction of the expense required for comparable results by instrumental surveys on the ground. In connection with many projects the greatest savings that can be effected by aeroplane topography are the indirect ones of quicker returns and less interest on waiting capital by virtue of the great saving in time and reduction of the non-productive period devoted to preliminary investigation and survey.

Field of Application

Aeroplane topographic surveys have a broad field of usefulness. The resulting contour maps offer a superior basis for studies of water supply, power, irrigation, and other engineering projects. The unusual combination of speed, accuracy, and detail make the method ideal for use in comparing alternative plans for determining the best plan of development. The maps serve for preliminary location or project layout, for determining catchment areas, for capacity curves for reservoir sites, and for many other purposes for which topographic maps of engineering accuracy and suitable scales are used.

Accurate maps can be produced by this method with horizontal scales such as 200, 300, 400, 600, or 1 200 ft. to the inch. Contours with intervals of 5,

* *Engineering News-Record*, March 5, 1925.

10, or 20 ft., or more, can be located accurately. For more general purposes maps can be produced with horizontal scales of 1 in. to ½ mile, or more, and such contour intervals as 50 or 100 ft.

The advantages of this method are greatest in country of rugged topography and rough surface, but any type of ground can be mapped by this means. The instruments developed serve with the same facility to reduce the photographic information to true-scale maps in the case of flat, rolling, or mountainous terrain. Simple topographic shapes and the complex forms of erosion and glacial action are mapped with the same ease. Even a substantial growth of timber or brush does not defeat the process. The instruments magnify the vertical dimension and even where the ground is quite obscured in the perspective view of a single photograph, it may be clearly visible throughout the stereoscopic view. The contours of the ground surface are uniformly determined.

Origin and Personnel

In this discussion of aeroplane topographic surveys the attempt has been made to state some important elements of the problem which any effort at a solution must meet. Necessarily it sets forth the particular method with which the writer is familiar, that developed by the engineers at the Arthur Brock, Jr., Tool and Manufacturing Works, Incorporated, which is fully covered by patents issued and pending, both as to the method and the special apparatus invented for its convenient practice. As far as the writer knows, it is the only method devised in the United States for the development of contours directly from the data recorded in aerial photographs, and the one such method of any origin to date that is accurate and practical. The method is being utilized under the general supervision of F. E. Weymouth, M. Am. Soc. C. E.

DISCUSSION

HAROLD C. FISKE,* M. AM. SOC. C. E. (by letter).—The author states that "the object of this paper is to acquaint engineers with a method of making accurate contour maps directly from aerial photographs. The paper is accurate and complete in so far as it refers to the one (and only the one) method which the author has undertaken to present. It happens that, so far as is known to the writer, this method which requires certain patented apparatus, is used in the United States only by the individuals mentioned in the paper. On the other hand, aerial photographs are being used successfully and extensively as part of the process of map making by numerous agencies, governmental and private. The methods used in these cases are ones that do not require a measuring stereoscope nor even, necessarily, a tilt-removing camera, nevertheless the results are, from the engineering standpoint of reliability and economy, quite as satisfactory as those described in the paper. Where the costs and some other features of this special method are satisfactory, the engineer may expect most gratifying results. Whether or not this particular method seems to be applicable in any particular case, there are others from which a choice may be made. So great is the value of the aerial photograph as an adjunct to map making that its use should not be discarded until these other methods have had due consideration.

The sales agents of the various companies engaged in this class of work may be trusted each to set forth in convincing detail the merits of the system used by his firm. What seems to be needed now is not an explanation or a comparison of these different methods but rather a more general statement of the powers and limitations of the aerial photograph. Its use need not be in any sense difficult or intricate, although if the amount of map work to be done is large enough there is ample opportunity to develop skill and to utilize special methods for the elimination or minimizing of the errors inherent in these and all other photographs.

In approaching the subject of the use of aerial photographs the engineer must formulate clearly in his own mind a definite opinion as to what constitutes a map and the purpose which it is to serve. Is it a vehicle to express the skill of the man who made it or a document containing data required by the reader in such form that he may understand it clearly and use it with confidence? Is there any object in securing data and trying to incorporate it in a map with any higher degree of accuracy than can or will be employed by the user?

Maps have been made and used for many centuries. Until about two hundred years ago they were all pictorial and the artist was the map maker. The user recognized that the hills and streams depicted were substantially the same as those in the vista before him. If this picture guided him to the general vicinity of his destination he relied as a matter of course on his native intelligence to do the rest. The scaling of distances from such a map probably was never considered. In the Eighteenth Century it was realized that the principles of geometry and trigonometry could be applied to map

* Lt.-Col., Corps of Engrs., U. S. A., Camp Lewis, Wash.

making. If the distances and angles of a polygon were measured on the ground the mathematically similar figure could be constructed on a much smaller scale on a sheet of paper.

From this point to the modern map progress was rapid. The accumulation of error, due to unskillful work or inaccurate instruments, was promptly recognized and efforts were constantly made to improve both. With better instruments and greater skill larger areas were mapped and in order to keep down the effect of increasing accumulations of diminishing errors the nicest refinements of measuring, recording, and preserving have had to be devised and used. In primary triangulation work distances may be measured with an error of less than 1 in 1 000 000*, angles are read to seconds, and the results are engraved on metal plates that will not be distorted by temperature or humidity.

Suppose, however, that the engineer wants to know the distance from Triangulation Station A to Triangulation Station B with an accuracy of 1 in 100 000 and has before him the map on which are shown these two stations. He knows that he cannot scale this distance with anything approaching such accuracy and if the true distance is not printed on the map he must inquire of the map maker, who does not think of scaling the distance from his metal plate but takes it from a tabulation.

Modern maps are possible only because of the ultra-refinements that have been devised by their makers, but it is physically impossible to carry these ultra-refinements through all the processes of map making to the finished product, a printed or lithographed sheet. Maps now have two distinct phases, the mathematical and the pictorial. The mathematical map is a tabulation which, even in high-class work, cannot be shown pictorially. The map user thinks of the tabulation as such, and for him a map is a conventionalized picture on which he sees features of the landscape. On this map he expects to be able to use a scale with confidence in the results but, perhaps outside elaborate city surveys, he will accept an error that is unthinkable to the surveyor. The map maker may spend days determining whether the distance between Triangulation Stations A and B is 133 256 ft., or 133 257 ft. The map user will lay his scale on the map and say that the distance from Station A to Station B is 25.2 miles and will be perfectly satisfied when he finds that he has missed it by only a few hundred feet. When this distance from Station A to Station B is part of a mathematical map, the 1-ft. difference may be of vital importance; if it is needed only for the pictorial map, it is of no importance.

For more than four years the writer has been making maps by a method that includes the use of aerial photographs. Between 2 000 and 3 000 sq. miles have been covered and several thousand photographs have been used in the process. All experience to date points clearly to the conclusion that the aerial photograph will never enter the field of the mathematical map. Its use will probably be confined to the preparation of the pictorial map and in order to find its proper place in this field some analysis of such maps is necessary.

* "A Treatise on Levelling, Topography and Higher Surveying," by W. M. Gillespie.

This map is a conventionalized picture of several square miles of average open country on which all kinds of natural and artificial features have been vertically projected on a horizontal plane. It shows all the natural or artificial topographic and such other detail as in any particular case may be called for. The map is to be used by an engineer who has undertaken to lay out and to supervise the construction of an engineering work, such as a canal, a highway, a railroad, a storage reservoir, etc., involving the accurate knowledge of an area of substantial size. With the map before him, the engineer will shape his general ideas, he will scale distances along roads or streams, the areas of fields, lakes, ponds, or those bounded by contours at different elevations.

If this map is to be made by the standard methods which do not include the use of aerial photographs, two distinct steps are required, which have already been designated as the mathematical and pictorial. Under the first comes the establishment of the triangulation or traverse system for the control of the map. Here the permissible limit of error will be 1 in 1 000 to 1 in 100 000, or even less, and the field work is done by the most skillful man available. Such errors are much less than the width of the finest line that will appear on the finished map. Distances can rarely be scaled from a map with such a degree of accuracy, and in any event the user has comparatively little interest in these few accurately determined points.

In the second step, field work is done, frequently, by men less skillful than those required for the first step. A plane-table and field sketching methods are used. Horizontal distances are measured by odometers, pacing, and estimating. Vertical distances are measured by hand-levels, aneroid barometers, and by estimating. The odometer will give a limit of error of 1 in 100, or better; the pace, an error of 1 in 100, or worse; and no one can with certainty estimate distances to visible objects with an error of only 1 in 100. In contour work the position of an invisible contour is first estimated and then the distance to this invisible point. The result of this combination of estimates is often radically wrong (as will be illustrated by two examples subsequently). In the field work for the pictorial features of the map errors greater than 1 in 100 or the width of the finest line on this map are the rule rather than the exception and the office draftsman in smoothing out contours, bends in roads, etc., introduces still more. In other words, the pictorial data are put on the map with less accuracy than can be employed by the user.

A study of the methods of making ordinary maps will impress on the mind of the engineer three things: First, no matter how accurate may be the control, including the plane-table triangulation or even the road traverses, the engineering detail which most concerns him is that which by former methods is merely sketched in, frequently hurriedly, always (from an engineer's standpoint) inaccurately and usually incompletely; second, and of marked importance, no real check on detail is economically possible by former methods; and third, while the map maker doubtless does well to preserve his truly accurate control on metal sheets, where detail is concerned this is wasted effort. Engineering drawings are customarily made on tracing linen and reproduced on blue-prints, and map detail, as engineering data recorded in this manner, need suffer no more than any other.

The engineer does not use standard maps where details are essential but makes his own independent field measurements. This is true in smaller matters, but where large areas are involved the engineer does accept a standard map as an adequate source of technical information. The inaccuracy of such maps is unbelievable. Two examples will illustrate the point.

For a decade or more there has been under consideration by thoroughly competent engineers a project including an area bounded by a contour. This area was planimetered from a standard map and for ten years was accepted as being 66 000 acres. Recently, this area was re-mapped by a method including aerial photographs and when planimetered again was found to be 28 685 acres. Another project involves an initial investment of more than $8 000 000. Certain possibilities were rejected by the engineers primarily because an area bounded by a contour was planimetered on a standard map and found to be "at least 30 000 acres." This area as found from maps prepared with the use of aerial photographs is 9 149 acres. It is believed that all, including those who made the standard map, will agree that the area from the aerial photographic map is not in error by more than 1%, or 2% at the outside.

This is not a criticism either of the makers of standard maps, of their methods, or of their finished product. Given the same restrictions as to time, cost, and methods, no engineer could have made a better map in these cases. Many business men will hesitate a year before they will give a deserving clerk a $100 increase in pay, but will hand over $10 000 at an hour's notice for oil well stock if the salesman is sufficiently persuasive. Engineers and business men sometimes seem to display similar traits.

In standard map making the methods and instruments used are the product of about two centuries of effort by highly scientific men and these have made possible a type of map which for accuracy other than that of detail was never before equalled. In so far as the control or general framework is concerned, it is probable that future improvements can be made only in matters of a technical nature in which the Engineering Profession at large has little interest. In matters of detail present ground methods can produce results that will satisfy the most critical engineer but the cost involved is so high that outside of city surveys and some other analogous special cases these methods cannot be afforded.

In aerial photography, the methods and instruments now used are the product of about seven years of effort by scientific men and many important improvements are still to be hoped for. As matters stand, however, the aerial photograph has already made an important contribution to the science of map making and this perhaps is one of more value to the Engineering Profession than to any other. The camera cannot supplant the theodolite, the transit, or the spirit level, and probably it will not seriously compete with them; but the topographer with his pace, odometer, and skill acquired with practice cannot compete for a moment with the aerial photograph. He will not cease to exist but when aerial photographs are used he will be required to measure elevations for contours, check scales of photographs and such details on them as may be obscure, too small to be identifiable, concealed by overhanging vegetation, etc.

A map with no control or general framework would be of little value to an engineer. A map having perfect control is of little value to the engineer if the detail is all wrong or is unreliable. A moderate amount of control can be obtained from aerial photographs, sufficient in many cases for the engineer's needs, but their outstanding value is the fact that engineering detail can be taken from them and added to the map maker's control within such limits of time, cost, and reliability as to make the detail on maps true engineering data.

The one item of outstanding importance in making maps by the use of aerial photographs is to get the photographs. One engineering company, known to the writer, bought an airplane, camera, and photographic apparatus complete, and added aviators and photographers to its engineering staff. The results are understood to have been considered superior for all practical purposes as compared with former ground work. The special equipment and personnel, however, have now been disposed of and in the future aerial photographs will be obtained by contract from companies specializing in such work, and the maps will be made by the engineering company itself.

Several companies take aerial photographs, and most, if not all, of them are also anxious to make the finished map. Whether an engineer should contract for a finished map or make his own from photographs bought from others is a subject still under debate. The writer prefers to make his own maps but recognizes that the other method would free his organization from a vast amount of detail work. In any event, the engineer should have a fair understanding of the nature and value of the various steps involved.

It is to be understood that by aerial or vertical photographs or prints are meant those taken from an airplane with the axis of the lens vertical and the negative horizontal. Maps can be made from oblique aerial photographs. This method is much in favor in Canada because of special conditions obtaining there in the flat lake region, but there seems to be no reason to expect that this method will be used extensively in the United States. The scale of the photograph depends on the focal length of the lens and the height at which the plane is flying when the exposure is made. Ordinarily, from a height of 5 000 ft., the scale is about 12 in. to 1 mile. Exposures can be made from lower elevations but most work is done from elevations between 5 000 and 15 000 ft. Commercial work can be done at 20 000 ft. and experimental work has been done at 30 000 ft.; but 15 000 ft. is a good practical limit.

The cameras used are manufactured especially for this purpose and bear about the same relation to the ordinary camera as does a modern surveyor's transit to the old-fashioned compass. High-grade lenses, color filters, and hyper-sensitive negative are among the features now standard in this work. The company employing the methods given in the paper uses glass plates for both negatives and positives. Practically all others in private and governmental service use celluloid base negatives and certain specified grades of standard commercial photographic paper for positives. The reasons for this distinction follow from the fundamental differences in the methods used.

It will be recalled that, in the method described in the paper, a moderate number of carefully measured elevations are required for each positive used.

With these available and with the special apparatus mentioned, the print can be completely contoured. The element of critical importance in this method is the fact that when the exposure is made the negative must either be exactly horizontal or, if it is not, must be so rectified that the resulting positive is the same as if such had been the case. This necessity comes from the fact that the machine which indicates the contours gives points of equal elevation above a reference plane on the ground parallel to the negative at the moment of exposure. If this plane were tilted 1°, these elevations would be in error by 92 ft. at a distance of 1 mile from the optical center of the print—an error which is of course far beyond permissible limits.

Emphasis is also laid on the necessity for the use of glass plates because of the danger of warping in other materials due to temperature, humidity, or the creeping of the emulsion. The measuring stereoscope functions with two vertical prints taken close enough together to include the same area in each. The negative is horizontal in each case but the exposures are made so close together that the areas covered by each exposure overlap by 50% or more. It will be realized that objects on the terrain above or below the reference plane have a parallax which is different for the same object in each of the two prints. Also, if the exposure is made from a point 5 000 ft. or more above, this parallax or difference of parallax is small, particularly for differences of 10 or 20 ft. corresponding to the contour interval to be determined.

The measuring stereoscope is a device which measures, not simply the parallax of a given elevation, but rather the difference in parallax of the same object on two successive and overlapping prints. This difference is a minute quantity, observed and measured with microscopes and a micrometer and recorded by the machine in terms of elevation. Such being the case, obviously variations in the photographic work, even if they may be microscopic, must be avoided and hence the prominence given to this step by the author and his insistence on the use of glass plates. In other methods where contours are measured on the ground these small distortions of the negatives and prints are much less important and film negatives are sufficiently accurate, as has been demonstrated to the writer's satisfaction, in the work of his office where nearly 13 000 film exposures have been taken.

Turning now to other methods—these do not require the measuring stereoscope and do include the use of film and photographic paper. Contours are determined by measurements made on the ground and all other detail is obtained from the photographs subject to such checks made in the field as may be desired, these checks being made when the contour work is in progress. Errors due to temperature, humidity, film creeping, etc., in the photographic work must be guarded against by skillful and painstaking work in the photographic laboratory. However, if to this step are applied standards customary in other phases of an engineer's routine work there is no difficulty in keeping such errors within manageable limits. Although tilt is always undesirable and the aviators are especially trained to keep this to a minimum, nevertheless a tilt of 2° does not destroy the usefulness of a photograph, and those having greater tilt are better than none.

The pilot is especially trained to fly at a uniform elevation and to follow a straight course accurately in spite of cross-winds, etc. If the elevation is constant so will also be the scale of the photographs except for variations in the elevation of the terrain below. Due to this factor and to the unavoidable variations in the elevation of the plane, it is best to have the contour party make a stadia determination of the scale of each print used. With a linear scale on each print even subsequent warping of the prints can be compensated. Constant elevation and a straight course also reduce to a minimum a tendency to tilt. These factors and the laboratory conditions are well understood by the firms engaged in this class of work and the engineer who buys his photographs may expect that due effort has been made to minimize such errors, the degree of success achieved perhaps varying with the company that does the work.

When the photographs are received the engineer must constantly bear in mind that they correspond to the plane-table sheets, which, in the standard ground method, have been filled in around some form of control. Where ground sketches are used the location of all detailed information has been determined largely by sighting, pacing, and estimating. The amount and character of the detail shown depend on the judgment and skill of the topographer and his understanding of the instructions under which he is working. Since no two minds are exactly alike, it is certain that he will, in some measure, include unnecessary items or omit some that his superior wants, or both. In addition, there will be errors, usually of omission, due to the fact that important objects may be hidden from his view by some feature of the terrain. In one instance, comparison was possible between the work of an especially skillful and conscientious topographic party and an aerial photograph taken soon after the ground work was completed. In a square less than 2 500 ft. on a side there were nine reasonably obvious errors by the ground men in the form lines of streams, the correct interpretation of which was the object for which the map was being made.

It is in the delineation of contours that the ground topographer uses the greatest freedom in estimating distance, elevation, and form. He is required to prepare a vertical projection on a horizontal plane of imaginary lines on slopes which he is able to view with his line of sight, making only a small angle with the horizontal. The writer's conclusion is that, outside of city surveys, the most skillful ground topographer is unable to draw contours so accurately but that they can be materially improved by the use of aerial photographs and a stereoscope (not necessarily a measuring stereoscope). Gently sloping terrain would form an exception to any such conclusion.

If, instead of the topographer's plane-table sheet, the engineer has an aerial photograph, he will find there a mechanically complete record of every object visible to the eye of the camera. Some points may be hidden by shadows, overhanging trees, etc., or they may be too small to be identified, but otherwise everything is there and the question of human judgment and the ability to observe, estimate distances, etc., has been wholly eliminated. In fact, the chief difficulty is that there is too much detail; the engineer probably is not interested in 1% of what is shown. This is why, in the general case, a map

made from aerial photographs, which includes what is of value and omits the remainder, is to be preferred to the photographs themselves. Certain essential objects will be missing, first and foremost of which are contours. In addition it will not be practicable to distinguish monuments marking property lines, telephone and telegraph lines, etc., while such things as church spires, towers, high chimneys, etc., which may be important landmarks will be difficult and sometimes impossible to identify from the photograph alone. Such items, however, can be quickly and definitely located by a field contour party.

The work of the ground topographer is subject to the errors of the instruments used and to the personal equation of the man. The photograph is subject to instrumental errors but not to the human equation. There is a case on record where in an important primary traverse the ground surveyors inadvertently omitted 100 ft. and the map was completed without the error being discovered until it was checked later by aerial photography. In the location of detail, which is the operation of prime importance to the engineer, the human equation is the controlling factor. With the aerial photograph this essential phase of the work is done mechanically. If the finished map omits essential features they can be added later, in the great majority of cases, from the photographs on file without going back to the field.

The chief error of the camera parallax is governed by natural laws for which due allowance can be made, and this can in no sense be compared with the vagaries of the human equation. In addition, not only do successive exposures in the same flight overlap but adjacent flights are expected to overlap by 50 per cent. The practical result is that, in the great majority of cases, a given point appears on from two to ten prints. This gives from two to ten independent determinations of any point; it permits the elimination of any photograph made inconsistent with the others by tilt; and the mean of these positions gives a reliable final position for the point. As a matter of fact, few prints have to be eliminated because of tilt.

Having the photographs at hand, the engineer can proceed with the construction of his map along lines closely analogous to those for standard ground surveying. That is, he can start with his known points, traverse roads and railroads, run level lines and then, with the photographs taking the place of the topographer's sketches, add detail from them and contours from ground measurements as may be desired. If this method is followed, the amount of road traverse necessary may be greatly reduced by the use of photographs. Experience shows that a single accurately located highway, for example, is sufficient control from which can be built up the map of an area several miles to each side. The accurate location of single points (selected so that they can be properly identified on the photographs) if spaced ½ mile to 1 mile apart will do as well as a road traverse. If the method of radial control* is used, accurate base lines about 8 miles apart are all that are essential, although they are habitually obtained from the ground scale check of individual photographs at much smaller intervals.

There are two ways actually to handle the photographs in this process of map making. The first method is most useful when the scale of the photograph is much larger than that at which the field work is to be done, but it

* *The Military Engineer*, September–October 1924.

can be used, of course, whatever this relation is. In this method the control is first plotted on plane-table sheets, next, the photographs are properly oriented, and then, by means of a pantograph, roads, railroads, streams, etc., are transferred from the photograph to the field working scale on the plane-table sheets. The topographer takes both this plane-table sheet and the photographs from which it was prepared into the field and there completes the map, using the photographs or measuring on the ground as may be most advantageous. His work is all done on the plane-table sheet and the photographs are used only for reference. This method has been used extensively by the U. S. Geological Survey.

The other method is useful when the photographs have been taken at the same scale as that selected for the final map. It is applicable when the map scale is large enough for photographs at the same scale to show ground features distinctly without the use of reading glasses or other magnification. This is the method used by the writer, the map and photograph scale being 1 to 15 000. In this case, the photographic print itself, used as the plane-table sheet, is taken by the ground topographer into the field. His work consists of checking the scale of the print, drawing contours, and reinforcing such features as may be small, obscure, or hidden by foliage, shadows, etc.

In the first method, contours are located and drawn on the field sheets in the standard way with such assistance as may be obtained from the detailed information on the photograph. In the second method, the contours are located and drawn on the photographs themselves*. Its importance comes from the fact that since the contours appear on the photographs proper they can be examined in the stereoscope and thus give, first, a highly accurate method of determining form and horizontal locations of contours in the field; and, second, a thoroughly convincing office check on these two items. Prior to the advent of the aerial photograph and its stereoscopic use, comparable accuracy in contour location in the field was possible only in expensive city survey methods and even then an office check of this item was physically impossible. Now both a high degree of initial accuracy and the office check are possible at costs which should be less not only than those of city surveys but also than those of the former highly inaccurate method of the sketching board, pace, and aneroid barometer. The stereoscope needed in this process can be made by the average cabinet maker and costs only a few dollars.

There are now several companies engaged in the business of taking vertical aerial photographs and of making maps from them. To such companies engineers must look at least for the first step in the process, that is, the taking of photographs. If the Engineering Profession at large comes fully to appreciate the value of this method and takes due advantage of it, these companies will be able to continue in business and it will not be long before such photographs will be considered an indispensable feature of a wide variety of engineering studies. Cost, as always, will be an important item and, as in all other business, this depends on the demand. Quantity production is of controlling importance in the aerial photographic side of this work and costs cannot be kept low except where the work is large or several smaller jobs can

* *The Military Engineer*, September–October, 1924.

be completed at the same time and from the same base, thus giving the equivalent of one large work.

Although costs can and should be lowered somewhat by the use of aerial photographs, this is of little importance compared with other advantages to be obtained. The engineer is most interested in the detailed information on maps. Heretofore this has been obtained and recorded by methods which have permitted errors of any kind and magnitude to be included and to pass unnoticed and even unsuspected. By the use of aerial photographs, details are now obtained and recorded by a method that eliminates errors due to the human equation and permits instrumental errors to be minimized to any extent desired. At all stages, including that of the finished product, there is a highly satisfactory check as to the accuracy of the work, particularly as it deals with that vitally important factor, detail.

Arthur P. Davis,* Past-President, Am. Soc. C. E. (by letter).—The need of topographic maps as a guide to the planning of extensive engineering works, such as water systems, roads, transmission lines, railroads, and irrigation works, has been appreciated in recent years to a far greater extent than formerly. Although most engineers now realize the desirability of a general topographic map of a high degree of accuracy as a preliminary to the detail study of engineering works, the time required frequently has prevented the construction of such maps.

To some extent this need is supplied by the Federal Government in the general topographic map of the country, but this map is by no means complete, less than 40% of the country having been covered. From the nature of the case the scale of such maps is so small as to amount, for engineering purposes, only to a reconnaissance survey.

Such maps are usually adequate for determining the drainage area of the catchment basin; for indicating relative altitudes in different parts of the basin; for furnishing a rough guide for the location and selection of reservoir sites; and for indicating the most promising routes for canals or pipe lines, roads, and railroads, to be surveyed in greater detail before plans are made. In no case does the general map furnish the final data, except perhaps in the matter of catchment areas, which are useful in computing spillway capacities and in comparing different basins or parts of basins, as to their water yield.

The development of the aeroplane and the adaptation of aerial photography to the construction of topographic maps has brought into a new relation the element of time necessary for the construction of accurate topographic maps on a large scale. The necessary aerial photographs usually are obtainable in a few hours of flight in clear weather, provided the basic location for control has been furnished in advance. Much additional work in the office and laboratory is necessary to make maps from these photographs, but this work is independent of the weather, and a part of its results can be quickly made available provided the maps can be completed from the original photographs.

So many errors are possible in this work that it has been customary to use photographs only for rough guides in the form of mosaic and to re-occupy field

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points for the purpose of sketching the contours, which are controlled primarily by the preliminary triangulation, and in detail by the photographs themselves. This use of photography is of great assistance, both in the speed and accuracy of the work, and with this method a high grade of topographic maps should be turned out more quickly and cheaply than without it.

The ingenuity of the inventors of the Brock process has accomplished a distinct advance in the production of topographic maps by photography. By clever devices and by re-photographing the plates most of the mechanical errors of the process can be corrected in the manner explained in this interesting paper.

The greatest and most unique advance, however, and the one that most distinguishes this process from others is the translation of the photographs into contours without additional field work through the stereoscopic methods described in the paper. This great improvement, checked as it has been by direct comparison of results, will accelerate the introduction of such methods.

When the advantage and economy, both of money and time, is appreciated, all large enterprises will be examined by the prompt establishment of the tie-points for necessary horizontal and vertical control, and prompt steps will be taken to make comprehensive and accurate photographic maps of the region under study. This will eliminate a vast amount of expense and delay in preliminary surveys and effect still more economies in the elimination of errors of plan and of estimate, due to insufficient preliminary information.

Such preparation should be recognized as essential for the preliminary location of railroads, wagon roads, and transmission lines, and for the location of reservoirs and the determination of their capacities, as well as the canals, tunnels, and pipe lines necessary to utilize the water.

These methods are adaptable to photography of every description. Their greatest advantages obviously lie in regions where other methods are most difficult, as in extremely rough localities or those covered with timber or brush, or those for which the field season is short, as in the more northern regions of America, Europe, and Asia. Vast areas having oil, timber, and power resources are yet to be studied, topographically and geologically, for their development, and such regions offer an almost unlimited opportunity for using accurate mapping methods, which, in turn, will greatly expedite and stimulate their development.

Haywood R. Faison,* M. Am. Soc. C. E. (by letter).—The first reaction to the study of Mr. Bergen's paper is profound admiration for his courage in undertaking a written analysis of the aerial photographic process. Like the solution of most engineering problems, demonstration in the field, shop, and laboratory is much more spectacular and convincing. Unfortunately, hitherto such demonstration has been available to but few engineers. It is to be regretted that there is no more dramatic way in which to broadcast this revelation than by leading the reader, step by step, through a logical sequence of equations, from axiom to axiom. The significance of the diagrams is grasped at a glance, but there is risk of losing the attention of the casual

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reader before spanning that tiny gap between the commonplace and the remarkable.

Introduction to the actual workings of the Brock process duplicates the experience of the skeptical urchin, confronted by Houdini; one witnesses the astonishing result before comprehending the means of its accomplishment. Only when later opportunity offers itself to manipulate and to experiment with the instruments of precision, to assist in bringing the reproductions to true horizontal and vertical scales, are the fundamental principles actually grasped and assimilated. One then experiences that inevitable twinge of disillusionment at the total absence of the miraculous.

The Green River Gorge through the Blue Ridge in Western North Carolina may not be ideal terrain for aero-mapping, but it is certainly unsuited to mapping by any other known method. Maps of parts of the region, by private surveyors, and by the U. S. Geological Survey, made throughout the last three decades, clearly reflect the disastrous limitations affecting even the best of topographic personnel and methods. The elements of time and cost prohibit that consistent adherence to a uniform standard of accuracy possible in aerial work. Local observations are likely to be unduly refined in certain zones, at the expense of adjacent inaccessible areas. Contour control points must usually be connected from memory, or by the aid of sketches, at best, rather than from the continuous stereoscopic relief under the eye of the aerial map draftsman throughout the operation.

While the aeroplane survey is free from these drawbacks, there are peculiar limitations to which the process is subject. Through a wide range of flying altitude, the property of the reduction mechanism, which will be called precision-ratio, for lack of a truer designation, is equal to, or better than, that of an approved triangulation theodolite. It is evident that the errors to be corrected by this mechanism—those due to tilt, drift, and variations in the speed and altitude of the plane—may be assumed to be constant and independent of the "height-of-lens", although, in actual practice, flying near the ground tends to increase these errors. It would seem, therefore, that below a certain definite "height-of-lens" above the average ground elevation, the desired precision-ratio would be rapidly exceeded, tending to introduce serious relative inaccuracies into attempts at large-scale mapping for detail work. However, since the object of the Green River survey was to cover many square miles of difficult mountainous country, on a scale of 600 ft. to 1 in., showing possible sites for dams and other hydro-electric structures, for pond areas, and feasible routes for construction roads and transmission lines, the flying altitude allowed a scale-ratio well within the limits of the required accuracy, and permitted reasonable variation of contour interval as desired.

Among the many parts of the mapped region selected for detailed instrumental study, some were densely timbered. Nevertheless, the contours checked so closely as to bear out the author's contention that the ground surface was clearly visible "throughout the stereoscopic view". It is not clear whether the claim is made that magnification of the vertical dimension, under the stereoscope, increases the visibility through the interstices between the tree-tops.

This would seem to involve a compensating distortion in the horizontal plane which would offset any such advantage in visibility. In any case, it must be admitted that large areas of heavy timber do exist, through which it would be impossible to see the ground surface, even stereoscopically, for any considerable distance.

Ground work was not stressed in the paper, and quite properly so, because an elaborate system is unnecessary. A high degree of accuracy in the ground control, or better still, a systematic check to avoid serious errors, is essential to the smooth working of the subsequent phases of the process. Detection of errors in the elevations of field points, and of inconsistencies in base-line measurements, is certain during the reduction of the plates to scale, but much laborious computation and comparison is required to locate the discrepancy beyond question of doubt, and to eliminate all elements of uncertainty.

If the fall of the leaves in autumn is allowed to intervene between the time of the exposures and the establishment of the ground control much confusion results from the ensuing obscurity or complete obliteration of the selected field points. Conversely, identification of the points, by their visibility from overhead, is made difficult for the ground party by the untimely budding of the leaves in spring. Peculiar light-reflecting properties of some materials, as caught on the photographic plates, must be taken into consideration by the ground party. Dark granite boulders, painted a brilliant white, were not to be distinguished from their brethren of natural complexion on the plates of the Green River survey. Definite points on the plates often proved to be indistinguishable on the ground, such as gravel and sand patches of neutral hue. This always necessitated the choice of some other point in the immediate vicinity, sharply defined on both ground and guide print. Points on static shore lines, edges of solitary boulders, acute angles of road intersections, etc., afford ideal field points for elevation. The extremities, at least, of the base lines should be of approximately equal elevation.

As to the intricate shop and office work which embodies the principles of the new process, one must read between the lines of Mr. Bergen's orderly development of his analysis to sense the many disappointments, the groping in blind alleys, the discouragements which that determined group of pioneers finally surmounted. The subject touches activities in many branches of Civil Engineering, and the author's logic is sound and comprehensive. It is to be hoped that the paper receives the attention which it merits, because perfection and simplification of the process are being carried on continuously, and constructive comment from engineers at large may well be the basis for working out further improvements.

Gerard H. Matthes,* M. Am. Soc. C. E. (by letter).—The author has drawn a clear picture of the difficulties attending the utilization of vertical aerial photographs in the construction of maps by the particular process to which his paper is mainly devoted. The title of the paper and treatment of the subject lead to the inference that the statements are generally applicable to all forms of aeroplane topographic surveys. To correct any such impres-

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sions, and in the belief, also, that the profession is entitled to know the position which the particular process described by the author occupies with respect to other aerial mapping methods, an aspect that was not touched on by Mr. Bergen, this discussion is submitted.

There are at present two distinct schools, if they may be so styled, of aerial surveying. One school undertakes to make maps from aerial photographs, either in mosaic or in ordinary line map form, by comparatively simple methods involving the use of no patented appliances aside from copying and enlarging cameras. Its aim is to produce maps reasonably free from the various distortions described by the author, and based on the fewest possible number of control points established by ground survey. The general principles involved are so simple that—with proper training—any skillful engineer, surveyor, or draftsman can master them in a short time. In this school belong the aerial mapping activities of the U. S. Geological Survey, Topographic Branch, which have by this time reached an extensive state of development; the aerial mapping operations of the Dominion of Canada, the Topographic Survey of which now leads the countries of the world in the quantity production of aerial Government maps; the aerial surveys of the U. S. War and Navy Departments; and a host of lesser aerial surveys for corporations, municipalities, and private individuals. In most of these aerial surveys, scale errors and distortions such as are mentioned by the author and are normally inherent in aerial photographs, are either eliminated through correction by the radial orientation methods,* or reduced to negligible proportions through material reductions in the scale of the finished map.

In North America, to date, by far the majority of aerial maps has been produced by methods of this kind. Experience indicates that these methods have proved best adapted to surveys at the smaller scales, namely, those ranging from as small as 1:62 500 to as large as 1:5 000. Although adapted to larger scales than 1:5 000, these methods do not always effect material economies over mapping by ordinary ground surveys for the reason that at these larger scales the labor involved in correcting errors in scale, displacements, and tilt distortions, becomes considerable.

The other school of aerial surveying, in which belongs the process described by the author, makes use of highly precise optical-mechanical appliances and has for its chief characteristics: (*a*) The correcting of each individual photograph for tilt and scale errors by a process of re-photographing known as rectification or restitution (a term borrowed fom the French and widely used in Europe); and, (*b*) the determining of differences of elevation in the terrain through measurement of the parallax of photographic images common to overlapping exposures. The author has given the best description of the principles involved that has yet appeared in print. It is important to note that extensive ground control is required in order to insure accurate results, and that all exposures must be corrected for scale and tilt errors, even if the discrepancies involved are so small as to be inappreciable by the other methods of aerial mapping.

* For a description of the method of radial orientation, see *Transactions*, Am. Soc. C. E., Vol. LXXXVI (1923), pp. 792–799.

The process described by the author is the only one of its kind in actual commercial use in North America. It is not, however, the only process of its kind in existence. Half a dozen similar mapping methods are in operation in Europe, and as many more are in process of development in America and abroad. The total amount of inventive effort that has been and is being expended in this direction is surprisingly large in view of the high cost of the appliances and the rather limited commercial market for them.

As a class, optical-mechanical mapping processes have proved to be well adapted to the making of maps on scales ranging from 1:10 000 to as large as 1:1 000, and have attracted considerable attention because of the ease with which contour lines may be produced mechanically, thus eliminating topographic surveying in the field. How well any of these processes (all of them patented) are adapted to producing maps on scales as small as 1:62 500 (the standard scale of the U. S. Geological Survey topographic quadrangles) remains to be seen. In Europe much experimental work has been done in this direction, but to date no small-scale surveys of large areas have been undertaken with apparatus of this kind.

The author stresses the refinements necessary for correcting the photographic data before they can be utilized in the map, and for determining differences of elevation. These refinements are characteristic of all optical-mechanical methods. However, it should not be imagined that the degree of precision to be maintained in this part of the work is necessarily present or reflected in the finished map. Its introduction is merely incident to dealing with parallax, the measurement of which involves exceedingly small quantities.

A numerical example will make this clear. Referring to Equation (1), assume an ordinary case, namely, a distance, B, between centers of overlapping photographs of 2.5 in.; a camera altitude, H, of 4 000 ft.; and a scale of the photographic images of 1:4 800. Then, the difference in parallax, p, for a difference of elevation, h, of 10 ft., will be found to be 0.006 in., a very small quantity indeed. Even for an object 100 ft. in height, under the conditions assumed, the difference in parallax would be only 0.06 in.

Undoubtedly, the apparatus described by the author is fully capable of making measurements with this degree of refinement. The important point to observe is that after these precise parallax measurements have been made, the construction of the map—that is, the piecing together of the bits of information derived from the individual exposures—still remains to be attended to, as does also the correction of the contour lines and other features for displacements caused by perspective, and for scale errors caused by differences in elevation. In these respects, the process described by the author appears to be much the same as the other aerial mapping methods; and it is here that the real skill in producing an undistorted map from a multitude of exposures is required. It is here, also, that frequently much of the extreme precision involved in rectification and contour determination is lost again; for, as in other mapping processes, this part of the work is a hand-and-eye job subject to all the chances for error resulting from varying degrees of judgment,

skill, and "pep" of the individual who transfers the lines and symbols from the photographs to the map and fits them to the ground control.

The author dismisses this part of the map-making process in less than two hundred words under the head of "Plotting and Tracing", leaving the reader with an incomplete picture of how all this material that has been worked over with so much painstaking care finally finds its way into its appointed place in the map. It is highly desirable for the author to shed further light on this phase of the subject, which seems the more important since in the preparation of maps of all kinds the most prolific sources of error occur in plotting the detailed data, whatever their origin and accuracy may be, in their proper position. In some of the European optical-mechanical processes all map features, inclusive of contour lines, are drawn mechanically on the map by means of a pantographic attachment forming part of the stereoscopic apparatus, thereby eliminating, very largely, errors in drafting.

Owing to the extreme delicacy with which parallax measurements must be made, errors in the position of photographic images, however minute, due to film and lens distortions become sources of material error. As pointed out by the author, glass plates must be used instead of film, but his statement to the effect that film is "inappropriate for the purpose of aerial mapping", is entirely too broad, being applicable only to aerial mapping by optical-mechanical processes involving parallax measurements.

Film has been and is being used much more extensively than plates in aerial work. Most of the larger aerial mapping agencies in the world, including Government bureaus and commercial companies, are using film with a great deal of success. As far as the writer knows all the more recent designs of aerial cameras use films. On the other hand, all optical-mechanical mapping processes are committed to the use of plates because of the small emulsion displacements mentioned by the author, to which film is subject at times. The errors in the position of photographic images resulting from this source, as well as those due to lens distortion, are only of academic interest in the preparation of maps by non-mechanical methods, because these errors do not average more than the width of a line, which, after all, is the limit of accuracy in drawing a map.

The author speaks disparagingly of the mosaic type of aerial map, stating that it has been called "a caricature of the landscape", and that "the weakness of a mosaic, fatal to its use as a substitute for the engineering map, is its lack of uniform scale". The writer is fully aware of the shortcomings of the mosaic, but believes in looking the facts in the face. Briefly stated, the demand for mosaics on the part of engineers is far greater than for line maps. Engineers are not averse to using maps known to contain errors, having learned through experience to place but little trust in the scaleability of any kind of map. The discrepancies inherent in mosaics have proved, as a rule, trivial alongside the blunders and omissions inherent in maps made by ground surveys. Mosaics afford information for engineering as well as commercial purposes that the best line map cannot supply, and convey, as a rule, infinitely more to the engineer's client.

The author's remarks apply more particularly to topographic maps for reservoir sites and for other purely engineering purposes, which must necessarily be of the line-map type with contours. These form but a small fraction of the demand for aerial maps. Speaking from intimate contact with the aerial mapping industry, the writer has learned that the bulk of aerial maps are for land development, transmission line location, tax assessment, city planning, zoning, oil exploitation, lawsuits, timber surveys, and many minor purposes. Such maps are nearly always ordered through, or at the recommendations of, engineers and these orders, in most cases, specify mosaics.

The author states that "the ideal method * * * will make maps directly from the camera records, and dispense with ground measurements or reduce these to a minimum needed for control only." If this be true, then the process described by him still is far from ideal, for its characteristic in common with that of other optical-mechanical processes is that it requires a great deal of ground control as compared with the non-mechanical methods of aerial mapping. It does obviate ground surveying for locating contour lines, but contour lines do not appear to be essential in the production of maps to the extent that they did before the advent of aerial surveys.

The greatest demand for map-making exists at present in South and Central America, where vast areas are so densely covered with vegetation as to make ground surveying for the purpose of establishing control points, such as described in this paper, a practical impossibility. For the same reason, contour location by parallax methods becomes a doubtful expedient, because over large areas the ground cannot be discerned in the photographs. It would be interesting to learn from the author what success has been had with this method in such regions.

From a general survey of the use of optical-mechanical methods it would appear that they are best adapted to the production of large-scale contour maps in regions where an abundance of ground control can be obtained cheaply. This is the case in European countries, where cadastral maps of great accuracy are available for use as ground control. This feature has had much to do with the success attained by optical-mechanical mapping processes in those countries. To what extent such methods will be found economical in the Americas, where ground control as a rule is conspicuous by its absence or difficult to resurrect, remains to be seen.

George T. Bergen,* M. Am. Soc. C. E. (by letter).—Probably only limited comment is to be expected on a paper dealing with a subject somewhat new. The writer is highly gratified by the extent of the discussions in this instance and the notable sources from which they come.

Few engineers have had such extensive experience with aerial photographs as Major Fiske recites, involving 13 000 film exposures and the actual use of several thousand photographs in mapping, even if this does not include their correction for tilt and their use for contour location by means of the properties of the photographs themselves. To such a method of use the paper was expressly directed.

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It seems that the paper did not prevent some misconceptions regarding the method discussed. An illustration is Major Fiske's statement that the stereometer, or measuring stereoscope, gives points of equal elevation above a ground plane parallel to the negative at exposure, which would cause an error of 92 ft. in elevation 1 mile from the optical center if the plane were tilted 1 degree. In elevation determinations by the stereometer two overlapping plates are always involved, with relations as suggested in Fig. 5. The operation of this instrument cannot be expressed in terms of a single photograph, but, as nearly as the writer can show, the relations are as in Fig. 14, in which the parallax would be a measure of distance normal to the tilted plane. Thus, the error would be,

$$e = A\,N - h = M \tan\theta \cos\theta + h \cos\theta - h$$

For a point in the true datum plane, $h = 0$, and if θ is small, $\cos\theta = 1$ (virtually), and $e = M \tan\theta$, which, if M is 1 mile and θ is 1°, would approximate 92 ft., as stated.

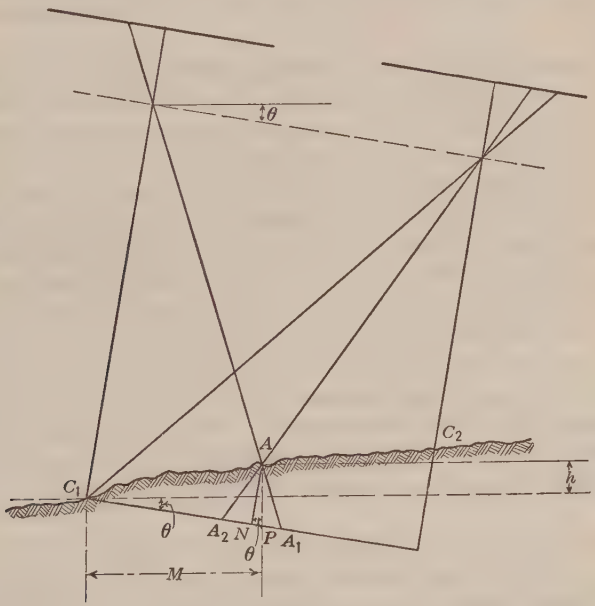

FIG. 14.—INCORRECT PARALLAX.

The assumptions involved, however, are faulty. They presuppose that the aeroplane and focal plane in effect glided down a 1° slope between exposures, so that both plates would be in one plane tilted 1° from the horizontal, a combination of circumstances not met in practice. They assume that the pilot lost 92 ft. in elevation from one exposure to the next, which is excessive for pilots trained in connection with the method discussed, with a modern altimeter in the cockpit and conditions suitable for photographic flight. However, in correcting the pictures for tilt and conversion of the photographic record to map form, allowance is made for a variation in the height of lens

as indicated by the stereometer and associated instruments from the photographs.

The most faulty assumption in Fig. 14, however, is that "the machine which indicates the contours gives points of equal elevation above a reference plane on the ground parallel to the negative at the moment of exposure." Only plates free of tilt or reprojected to the horizontal are used on this instrument for contour location so that the determinations, normal to such plane, are true elevations.

Major Fiske states that there are other mapping methods in which numerous agencies, governmental and private, use aerial photographs with results from an engineering standpoint quite as satisfactory as the one under discussion. In amplification he describes two ways of actually handling the photographs in such a process of mapping in this country. Both these methods are attributed to Government agencies, both involve all the work of contouring in the field, both compromise with the nature of photographs as perspective projections on tilted planes, and both are limited accordingly in their application.*

One of these methods is that used in recent years in some of the work on the series of topographic sheets that Major Fiske refers to as "standard maps." He finds such maps often radically wrong in contour location and of an inaccuracy unbelievable. The implication is that this criticism does not apply to the maps of this series made with the aid of aerial photographs. Yet, in making these maps, with or without the use of photographs, the contour location is accomplished on the ground, with essentially the same instruments and methods in either case. The fact is that this great series of standard maps has been in the making over a long period of years and that during that time there has been a notable improvement in their general accuracy due to the adoption of higher standards, better control, and improved organization, instruments, and practice, in the main quite independent of the use of photographs. This improvement with time is known to many engineers who, in judging the probable reliability of any particular sheet, will look at its date, rather than question whether photographs were used in its making.

Mr. Davis, however, points out the most important factor limiting the comparative utility of the standard maps for engineering purposes, namely, their small scales, usually approximating ½ in., 1 in., or 2 in. to the mile. The aerial method discussed is of general application in topographic mapping for engineering purposes, while the small scales of the standard maps limit them to reconnaissance surveys, as Mr. Davis believes. His conclusions are reinforced by unusual experience, including prominent connection with the making of the standard maps, some contact with the new instruments for the aero-topographic process, and a distinguished career of engineering service, public and private, involving topographic mapping and its practical uses. The clear distinction as regards scale is illustrated by the practice of engineers in ordering mapping by the new process. Such orders for work in the United

* *The Military Engineer*, September-October, 1924, p. 399; also pp. 628-629.

States are accompanied by the standard maps whenever available, which are used for the purpose of outlining the areas to be surveyed.

Mr. Davis clearly points out the great value of obtaining accurate topographic maps of large area in a shorter time than was formerly required. Such advantages of the aerial method have been well illustrated in surveys made for power projects.* This is a field of great opportunity for use of the new methods and instruments. In the successive steps of utilizing the power possibilities of a drainage basin, from the preliminary reconnaissance of a virgin field to the progressive determination of major links in the ultimate net of transmission lines, the aerial survey and its products offers advantages. The field engineer seeking sites for storage and development, the hydro-electric engineer weighing alternative plans, the transmission line engineer selecting routes, the public utility executive, the right-of-way department, and counsel may all gain in time, cost, or wealth of information.

The field engineer commencing investigation of a large drainage basin can save a tremendous amount of time by direct use of the aeroplane. Flying as an observer, he can get a broad view of the area with which he is to deal, such as is possible in no other way. He can traverse the area or follow a long watercourse in a small fraction of the time required on the ground. Flying relatively low, he can get a good idea of the power reaches of the stream and the sections where favorable dam sites and storage are to be sought. In short, he may get quickly that grasp of the whole problem that is needed to guide intelligently more particular examination and study, and to save the time and expense required in detailed investigations. It is impossible to fix in the mind all the detail seen from the air, but by photographing from considerable altitude with a short focus lens, a valuable reconnaissance record soon may be had in the form of a photographic mosaic. This may be made as small as 1 500 or 2 000 ft. to the inch, showing 5 to 10 sq. miles in a single photograph. A mosaic of these pictures will show the water-shed and drainage lines; the timbered, cleared, and cultivated sections; the towns, roads, and other features. On it can be marked the sections needing more detailed examination on the ground and from it can be determined the best routes and program to follow in visiting these localities. On the mosaic, rough measurements can be made of all these things. In the absence of an accurate map it is invaluable.

By proper choice of altitude and lens, or by enlargement from good negatives, the photographs can be made of almost any desired scale, subject always to the deviations due to camera height, tilt, and perspective, as already set forth. These deviations can be determined and a topographic map produced by the instruments of precision and the processes described. Such a map, coupled with a mosaic of the larger scale photographs, offers the hydro-electric planning engineer a record much more complete than he has usually had. If he is dealing with a large area the mosaic is useful in defining the parts to be mapped and the order of mapping. If the whole is suitably photographed while the aeroplane is on location and a moderate amount of ground

* *Electrical World*, Vol. 86, No. 7, August 15, 1925.

control is obtained, the files of photographs establish a resource from which additional topography may be developed, if and when later needed, without the necessity of re-organizing, transporting, and maintaining survey and mapping parties.

Mr. Faison is one of those engineers who can speak of maps made with such means and for such purpose from detailed experience with them and with location surveys by the older methods. There are numerous factors involved in what he aptly summarizes as the "precision ratio" of the new mapping instruments. In actual practice the "height-of-lens" has varied from 2 400 to 17 000 ft., without fixing absolute limits and without losing photographic sharpness. The "length-of-lens" is equally a factor in fixing photographic scales and widens the choice. Of two sets of photographs taken for large and small scale mapping, one ran less than 100 acres per negative, while the other had more than 9.5 sq. miles on a plate of the same size. Low flying for large scale photographs tends toward increased tilt because the air is less "smooth" and the camera has less time to return when displaced from the vertical, but there are compensations in the increased sensitiveness of parallax readings and, because of the larger images, in the ease of identifying the objects. There are, of course, both high and low limits for the elevation of flight for this purpose. Probably neither absolute limit will be a matter of accuracy, but the high limit will be physical and the low limit economic, with the prescribed map scale and attendant accuracy fixing the intermediate choice for any particular case.

Mr. Faison is doubtless correct in stating that through large areas of timber it is impossible to see the ground surface continuously for considerable distances; nor is this necessary. The slopes and contours can be fixed if only frequent glimpses of the ground can be had and clearly recognized. It is this recognition of the drop from tree-top to base that is aided by the vertical magnification, and the effect of distortion is avoided by using the center of the optical field. There are doubtless areas where vegetation completely hides the ground from above, but the fact is that none such has been encountered in the surveys made by the new method, which include parts of the Eastern United States that are mountainous and heavily timbered. It would not be possible to "run out" the contour lines in the equatorial jungles, nor is the writer acquainted with such work on the ground; but without penetrating the jungle the special aerial method could produce a contour map of such an area with at least barometric accuracy. Any control available at the margin of the jungle could be connected and extended by the photographs and correcting projectors. There are various interesting possibilities of applying the new methods to such country, which it is hoped may soon be initiated.

Meanwhile, Mr. Faison's statements regarding improvements of the process as already used in accurate work are very true. A number of marked improvements have been accomplished since the Green River survey that he describes. The special cameras necessary for accurate work have become available in larger number and with wider choice in focal lengths. Since

the camera is almost entirely depended on for the original record, except only a small amount of ground control, it must be an instrument of precision.*

One of the most important requirements is a lens free of distortion. This is not a common article of commerce, and such lenses were obtained in desired number and variety only after the extended process of search, test, and selection. With the lens used on the Green River survey it was necessary for exact correction and contouring of the photographs to make varying allowance for lens distortion. Subsequently, a series of lenses have been put in use so free of distortion that it can be found only by the most exacting optical tests. Thus, it ceases to be a factor affecting the accuracy of the mapping. In other words, the resulting image displacement cannot possibly be plotted and is so microscopic that it can be ignored in elevation determinations from stereoscopic photographs involving measurements down to 0.001 in. or less on the stereometer. The most satisfying improvement, however, has been made in the instruments used for such measurement and reprojection of the photographs to eliminate the effects of camera tilt. The newest instruments for this purpose have shown the most gratifying results in further labor-saving in the office processes, in minimizing the required ground control, even extending the mapping for considerable distances without additional control, and in producing accurate results in comparison with the most exacting ground check of completed maps.

In attempting to explain such operations and instruments in readable form and within reasonable limits, the writer is conscious of the difficulty suggested by Mr. Faison. For the engineer schooled in the old methods of mapping and unfamiliar with photography, analogies are useful, beginning with the simple one between the camera and the human eye. The pupil of the eye is a lens and the lid is analogous to a camera shutter. As the iris of the eye affects the amount of light admitted to the pupil, so the camera has an iris diaphragm by which the lens may be "stopped down". The retina of the eye corresponds to the focal plane of the camera and each receives its image without the re-erection familiar to the engineer in some of his ground instruments. In the case of aerial photographs taken for mapping purposes with the negative approximately level, such re-erection has no place, but there is the problem of orienting the views, correcting them for slight departure from the horizontal, and overcoming scale changes due to varying elevations of the camera and ground surface. F. E. Weymouth, M. Am. Soc. C. E., has pointed out the analogies and differences between ground and aerial surveys.† It is as if there were a series of plane-table sheets made without exactly leveling the table and using slightly different scales, but with characteristics permitting their correction and conversion to true scale maps in the office. As Mr. Faison states, it is only by actually seeing the office operations, by manipulating the new mapping instruments, that their relative simplicity is appreciated and the new method clearly understood.

Mr. Matthes offers the best evidence of this by being confused with European processes and instruments quite dissimilar. In comparison, the new

* *The Military Engineer* Vol. XVII, No. 96, November–December, 1925.

† *Engineering* Vol. CXXI No. 3138, February 19, 1926.

method minimizes the required ground work and simplifies its character. It is being applied to small scale mapping. Work at a scale of 1: 62 500 is under way for one Government, similar demonstrations begun for two others, and the expressed desire of numerous others awaits only the accumulation of sufficient instruments in excess of current requirements for commercial work on larger scales.

There is a demand for mosaics of air photographs, which are useful for land exploitation and numerous other purposes. Some of these purposes are of an engineering nature; the writer has cited the case of a preliminary investigation for power development. Mosaics may be especially useful in connection with transmission line location and the acquisition of related right of way. Where the population is relatively dense, property value high, and the right of way by far the heaviest item of cost, such photographic products of aerial surveys are almost invaluable. Interests with which the writer was connected had occasion to locate a high-power transmission line through about fifty miles of such country. The area is a network of improved highways, railroads, and other rights of way, connecting numerous towns, and the countryside abounds in highly cultivated farms. Within a zone or strip of this country the choice of final location was greatly influenced, if not entirely dominated, by the expense for right of way. Flying about 10 000 ft. above the ground an aeroplane traversed this zone and a series of overlapping "vertical" photographs was taken of the strip. The pictures included a few known ground distances, and using these and the precise office instruments already described, a graphic triangulation served to orient the photographs and approximately bring them to common scale for a datum plane. On mosaics of such pictures a line was laid down that would avoid the most expensive right of way and minimize crossings of a nature requiring special construction under the law. The county records served to identify the property lines on the photographs and made it possible to list the owners of rights to be acquired.

All this could be done without premature disclosure of plans. When the right-of-way man was ready, he proceeded unhandicapped by the publicity attending survey and location on the ground. The ground survey was limited to the final exact location, following the acquisition of rights. Here, too, the aerial survey may aid. The photographic plates are kept on file, available for determination of relief as described, and in any event copies of the photographs and mosaics remain in the office, continuously useful for reference on account of their vivid showing of detail. Coupled with the records or testimony of the aerial photographer as to the time of taking, they may serve later as evidence in settling certain kinds of disputes, evidence of a nature most compelling and understandable to a jury. Thus, from the stage of preliminary examination to the final day in Court, the aerial survey may function.

The writer is, therefore, well aware of such utility and the demand for mosaics, but believes in looking all the facts in the face. The mosaic should be called a mosaic and not a map. It is distinctly superior to any conven-

tional map in vivid nature and wealth of detail, but it supplies no elevations and is distinctly inferior to a good map with respect to scale, as those who have been tempted to rely on it beyond its powers have found to their sorrow. This statement needs one modification. If the ground is level and the photographs free of tilt a mosaic may closely approach a true plan map, but as the ground usually tilts and aerial photographs nearly always do, such modification is in the nature of an exception to prove the rule. Doubtless this is why the U. S. Board of Surveys and Maps refuses to call a mosaic a map, following the action of its Photographic Committee of which Mr. Matthes is a member.

As Mr. Matthes suggests, the engineer may find mosaics of most use to him through their value as a vehicle in conveying information to his client. In their hurry a board of directors or a busy executive will probably get a broader conception of a project from well explained pictures than from maps, profiles, and detailed drawings, but the engineer will probably not abandon the use of the latter. The writer confesses that he is thinking of maps from the engineer's viewpoint. He cannot agree that elevation contours are losing ground or share Mr. Matthes' rather gloomy view of placing little trust in the scaleability of any kind of map, although that is the safe rule to apply to mosaics.

Mr. Matthes disagrees with Major Fiske as to the completeness of the description of the new mapping process, and as to this the writer agrees with Mr. Matthes. The process had been previously described only to the extent possible within the limits of a brief magazine article.* The writer's attempt was to set forth clearly some of the essentials from the beginning. Thus, his effort goes further as a discussion of the problem than a detailed account of its solution. It is to be hoped that it will be followed, as it doubtless will, by others describing more fully the new process of topographic mapping, the precision instruments evolved, and the improvements in both that are being accomplished.

Nearly a decade of painstaking experiment preceded the present commercial application.† One instrument after another was designed, built, put to practical test, and succeeded by something nearer the goal. The persistence, ingenuity, and unfailing support of an ideal constitute an incident worthy of record in the history of American science. Sometimes it is said that in the hurry of the present tremendous industrial progress, technical research in this country lags in comparison to the applied science of engineering. It is a pleasure to record one of many instances to the contrary, to know that although this development has reached the point where it is possible to give it a place in the practical field and literature of engineering, the research and experiment are continuing and further advances materializing.

* *Engineering News-Record*, Vol. 94, No. 15, April 9, 1925.

† "Research Narrative," Vol. 5, No. 7, Engineering Foundation, New York, N. Y.

AMERICAN SOCIETY OF CIVIL ENGINEERS
INSTITUTED 1852

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Paper No. 1607

HISTORY AND PROBLEMS OF IRRIGATION DEVELOPMENT IN THE WEST*

By John A. Widtsoe,† Esq.

With Discussion by Messrs. Samuel Fortier, Arthur P. Davis, C. E. Grunsky, and F. H. Newell.

Within the Great West of the present United States, there are three periods of irrigation history—the prehistoric, the pre-pioneer, and the modern. Considerable information is available concerning each of these periods.

The Prehistoric Period.—Irrigation was practiced in the West before modern man took possession of America. Remnants of irrigation canals and structures in California, Arizona, New Mexico, Colorado, and Utah, bear witness to a dense population that throve under the irrigation ditch.

The Pre-Pioneer Period.—The pre-pioneer period represents the meager attempts at irrigation by those who came into the Great West as trappers, traders, ranchers, and missionaries before 1847. The Roman Catholic Fathers established missions in Arizona, New Mexico, and California, around which were built small irrigation systems. Occasionally, members of the great body of fur trappers and traders would divert water from a stream in the great unsettled West and raise a crop or more of grain or vegetables. The Spanish Government undertook, about 1796, to build an irrigation community in California, near the present site of Santa Cruz, under the direction of Lieutenant Alberto de Cordoba. Most of the principles at present advocated for the settlement of irrigated lands were used, but the venture ultimately failed. Until 1847 all Western irrigation ventures by white men were small and not on a permanent community scale.

The Modern Period.—The modern period began on July 24, 1847, when a company of Mormon pioneers, led by Brigham Young, entered the Great

* Presented at the Summer Meeting, Salt Lake City, Utah, July 8, 1925.

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Salt Lake Valley, and on that day spread water from City Creek over land which was being plowed. From this initial experiment have grown the vast irrigation enterprises of Western United States. The modern period is characterized by three classes of enterprises, which succeeded each other, although they frequently overlapped.

The pioneer enterprises were those in which the farmers themselves, chiefly by their own labor and always with limited capital, in the spirit of co-operation, set about to dig ditches and to bring water on dry land. The pioneer enterprises flourished mostly until about 1880, although, ever since the original Salt Lake Valley experiment, there have been occasional ventures conforming in method to the early enterprises. The pioneer enterprises were not comparable in extent with the irrigation ventures of the present day, but, in view of the difficulties and hardships of the pioneer days, they were remarkable. For example, in the State of Utah, in 1865—18 years after the coming of the first pioneer company—there had been dug, by co-operative effort, 277 irrigation canals of a length of 333 862 rods, costing $1 766 959, and capable of serving 153 949 acres at an average cost of $12 per acre. Besides, in that year, there was in progress the construction of other canals of an estimated cost of $877 730. The Union Colony, the forerunner of the great irrigated section centering about Greeley, Colo., is another famous example of a pioneer irrigation enterprise.

The prosperity that attended these pioneer enterprises was so marked that capital soon became interested. The capitalistic enterprises then followed the pioneer enterprises. It seemed as if the investment of capital in irrigation enterprises would be safe and profitable. Vast areas of land were brought under irrigation by the capitalistic enterprises. In fact, in 1900, 9 000 000 acres of land in Western America had been brought under cultivation, at a cost of from $15 to $20 per acre. It was discovered, however, that many difficulties and dangers attend an irrigation enterprise that is not principally fostered, supported, and managed by the water users themselves. It seemed to be demonstrated that, ordinarily, a capitalistic irrigation enterprise is not profitable. Finally, it became clear that the development of the irrigated section was lagging because the precarious financial returns of irrigation investments made capital reluctant to engage in irrigation enterprises. Meanwhile, it was fully admitted that the development of irrigation was highly beneficial to the country.

In 1902, Congress passed and President Roosevelt signed the famous Reclamation Act. Thus, the Federal-aided irrigation enterprises were inaugurated. This unique instrument for the conquest of the arid and semi-arid lands of the Republic by irrigation, provided that the proceeds from the sales of public lands in the Western States, later augmented by royalties from oil and other lands, were to be used for the building of irrigation works. The farmer was to pay no interest on the investment, but should return the cost in easy installments. Thus, a perpetual revolving fund would be created which, in time, would utilize all available water on the lands of the western part of the United States.

The United States Reclamation Service, in the two decades following the passage of the Reclamation Act, expended nearly $150 000 000 in the construction of magnificent irrigation works of outstanding merit, brought about 2 000 000 acres of land under the ditch, and contributed a volume of experience to irrigation practice, before unknown in the history of the world. The annual crop income from the Federal irrigation projects was upward of $75 000 000.

The passage of this Act, and the consequent activity in the construction of irrigation works, had the further effect of stimulating the interest of capital in irrigation ventures. From 1902 onward, therefore, there was another great increase in the irrigated area. In 1920, the irrigated section had expanded to 20 000 000 acres, at a cost of $30 to $50 per acre, and the crops harvested from the irrigated section in that year were valued at about $760 000 000. Numberless cities, towns, and villages covered the West as a result of this tremendous irrigation enterprise. Roads and railroads intersected the irrigated section and all the institutions and organizations of modern civilization had arisen on the reclaimed deserts of the West.

It would seem that, then, with the experience gained by the pioneer, the capitalistic, and the Federal-aided enterprises, the principles of irrigation would be so well understood as to insure the permanence and prosperity of the practice. Yet, at this very time, irrigation was, so to speak, placed on trial before the people of the United States—and the hearing is not yet ended. The older methods of founding and conducting irrigation enterprises appeared to be insufficient under the changed economic conditions of the day. Seepage and other insidious dangers had developed which threatened the permanence of the movement. The humid area called attention to its own lands not under the plow. Abnormal post-war conditions raised the specter of over-production. Colonization of the irrigation projects was slowed up. The newer projects carried an increasing acre cost, often beyond the reach of the water user. The water users on the Federal irrigation projects were not repaying the construction costs, in spite of the fact that several relief measures had been passed making it easier for the farmers to repay. It seemed at first that the Reclamation Service alone was on trial, but it soon developed that the whole irrigated area was under consideration by those who questioned the soundness of the movement. Many of those who raised questions relative to the future of irrigation were friends of the cause, who desired to save reclamation by irrigation, as they thought, from destruction. Others were frankly enemies to Western development. Since 1923 investigations have been made by the Federal Department and laws have been enacted by Congress which seem to promise that whatever changes are necessary to make the movement stable and successful will be made. Meanwhile, a new day in irrigation is before the people of the West. There must be a revision of principles and methods to conform to present needs.

The Problems of Irrigation

Irrigation is a complex art, involving the application of the best knowledge of agriculture, especially the relationships among soils, crops, and water; of engineering, especially of hydraulics; of rural economics and sociology; and of

the life and response of human beings under the unique conditions always prevailing under the irrigation ditch.

It is not surprising that so complex an art, at this time, when thoroughgoing changes are reshaping economic and social structures, should present many and serious problems. Such problems pertaining to the development of irrigation in the West do exist, and must be solved, if the great American experiment in irrigation shall become permanently successful. Yet these problems, essential to irrigation development, may be classified very simply into two groups:

1.—To establish on every irrigation project an acceptable economic and social environment.
2.—To extend gradually, as economic needs demand, the irrigated area by completing existing projects and by reclaiming new areas, until the water resources of the West shall be fully utilized.

An Acceptable Economic and Social Environment

The heroic figure in irrigation development is the water user—the man and woman who throughout the years live upon the land, till it, and make it yield enough for life's sustenance. Capitalist and engineer, statesman and tradesman, are all necessary, but compared with the water user they are of secondary rank in bringing irrigation development to success as measured by the standards of the age. The water user is as the great river that drives the wheels of the power plant; all other workers in behalf of the project are as the contributing side streams, higher up, that swell the volume of the river.

The tremendous importance of the water user in successful irrigation development has not been understood clearly; less important, although essential, factors, have been stressed. The result has been, too often, distress or failure on corporate and public irrigation projects. Experience has given its order, "right about face"—face to face with the needs of the water user. This is the large modern problem in the development of irrigation in the West.

Agriculture under any clime—humid, arid, or irrigated—will always remain a mode of living, a manner of life, rather than a business in which much money is made. Only those who are content with the normal rewards of rural life will succeed and be happy on the farm. Yet the dawn of the new agriculture gives promise that, as a result of his labors, the farmer may participate in the joys and satisfactions of progressive civilization. His income must suffice to provide the things that the majority of men find desirable; that is, a prime concern of irrigation development will be to strike a proper balance between the annual income and the obligations of the water user. The income must come from the lands; and the obligations must include all costs of a modest, thrifty, but comprehensive life in this day of the latest achievement; that is, irrigation farming must be made profitable. Any irrigation enterprise which does not make this thought fundamental is foredoomed to failure. Colonization, social development, and all other present needs of the irrigated West, depend on a readjustment by which the farmer can meet his obligations from his land income and yet live as men should.

This problem applies to all farming, but its solution varies with environment. The irrigation farmer lives under the ditch. He has a control over

water not possessed by his brother in the humid regions. He is not dependent on the rainfall. Herein lies a power which, properly used, may free him from useless competition and make his products desired by all markets. In addition, he lives under a clear sky. Climate and water re-act on crop production and composition. The peculiarities of irrigation environment must be capitalized to bring about full irrigation success. This phase of the problem is not beyond the field of the engineer. The trained mind is needed, however the training has been secured. The problem is worthy of the best thought.

Then, if the capitalist, whether private or governmental, will refuse to construct or support any irrigation project which cannot enable the water user to "make a living", the highway to the solution of this problem will be reached. Successful water users make successful projects. How to make water users successful is the first problem.

The Extension of the Irrigated Area

The East has need of the West; and the West has need of the East. The country must be developed as a whole, and not in sections. The activities necessary for the development of the vast resources of the West will cluster about, and depend on, the irrigated centers. The venture in irrigation will not be wholly satisfactory until the irrigated area has been extended to the full utilization of the water resources of the West. This cannot be done at once; nor should it be done hastily. Yet, if the work is to go on properly, there must be a constant movement toward the full realization of irrigation possibilities.

The Completion of Existing Projects

Few irrigation projects in the West are completed to meet present needs and possibilities. The newer of the existing projects are not yet well-seasoned, many of them not fully colonized, and few of them under an acceptable program of agriculture. They need much help before they can be viewed as established ventures.

The early projects were small, the water supply was relatively large, and the engineering devices were simple. As the population grew, the water was made to cover more land than was originally intended. As this spread became wider and the depth of water thinner, the danger of the dry year became more evident, until to-day there is a demand on most of the older projects for more water in the critical months and in the dry years. This condition requires that reservoirs be built to hold back the flood waters of normal years to supplement the present supply, and to store the excess of the wet years to serve as a hold-over supply for insurance against the dry year.

The older projects developed gradually. The first canal was followed by later canals that merely extended the original project. The result is that frequently the higher canals irrigate the lower-lying lands. On some projects this situation, the gradual result of irrigation development, is chaotic and expensive. There is need on many of the older projects of a complete re-organization of canal management, having in view lower operation costs, greater return flow, and a higher economy in water use.

Wherever water falls there is a power possibility. There is an increasing need of power on the farm. Irrigation by pumping is assuming a higher importance. The commercial demand for power is becoming larger. Power production on irrigation projects cannot be ignored. Provision must be made for power development, by the well-planned use of the water in streams and canals devoted to irrigation purposes, even if the power generation occur only during the irrigation season. Underground waters will be needed in irrigation development, and summer power will find large service in the use of such water supplies.

These and many other problems, such as that of full colonization, in connection with the completion of existing irrigation projects must be given early attention. There are hundreds of small private projects which, with a little help of the kind mentioned, would greatly increase in efficiency. We have been very prone of late years to value irrigation ventures according to size. The small venture is usually the most successful.

Irrigation Extension by Building New Projects

Although the completion of the older projects, small and large, should be the first concern in extending the irrigated area, yet the reclamation of lands remote from existing projects, for which water may be secured, must not be forgotten. In the main, such future projects require the construction of expensive and difficult irrigation works, and Federal aid, with interest-free money and easy terms of repayment, is indispensable. Difficult as these structures may be, high as the price may be, it will be found profitable to bring such lands under irrigation. In time, the country will need all its irrigation resources. Meanwhile, such projects should be undertaken slowly, gradually, as economic need is foreseen, and in accordance with a definite plan covering the whole West. There must be no haphazard irrigation construction if one intends to win success. The program of extending the irrigated area by building new projects should be preceded by an irrigation survey and stock-taking. Legislators and Courts must be willing to revise irrigation codes, on the basis of modern knowledge, for a full, efficient, and economical development.

The Means for Solving Irrigation Problems

Request has not been made for suggestions as to the solution of the irrigation problems. However, to mention some of the means at hand for such solution may not be improper.

First.—There must be established, within the Nation, a new faith in irrigation. It must be shown that the development of the arid and semi-arid section is a National need. The people must be made to understand that the vast resources of the West—mining, forestry, ranching, and dry-farming—can be made really successful only if they are in association with irrigated centers and with a wise program for the full, although gradual, development of its irrigation possibilities.

Second.—A new comprehensive study must be made of the total irrigation needs and possibilities of the West. This would include a consideration of

each irrigation project. It would be an irrigation stock-taking of the United States. Clear thinking would result from such knowledge.

Third.—The great treasures of irrigation knowledge won during the last few decades in research laboratories, on experimental farms, and from practical experience, should be organized and used more freely. Within a generation, materials for a science of irrigation have been gathered. Unfortunately, applications of this knowledge have not kept pace with the material gathered.

Fourth.—Reclamation by irrigation must be approached hereafter with a new appreciation of the need of economic and social emphasis. There has been a tendency to trust that, when water and land are brought together, successful irrigation enterprises will be brought forth. It is now known that this is not so. Economic and social principles as well as physical and biological laws must be brought into operation to make irrigation projects successful. Many or most of the disasters which have befallen irrigation in recent years appear to have been due to the failure to recognize these social and economic factors.

Fifth.—Training must be provided for the irrigation leaders who shall build upon the magnificent foundations laid by the present and earlier leaders. Such training must be made to conform with the best knowledge and views of the present day. Engineering schools may well accept the responsibility for such training, even to the extent of a partial re-organization of the traditional engineering courses.

For some years there has been a growing feeling that the standard college course in engineering is so highly technical and of so narrow a range that the graduate is not able to make the best application of his training in a world of men and women, touching many and varied activities of life. Certainly, those who in the future shall take and hold leadership in the irrigation development of the West, must know, in addition to the fundamentals of engineering, some agriculture, economics, and law. The engineering schools which desire to serve in the irrigation development of the West, may be obliged to formulate a new course of instruction based on the irrigation knowledge and experience gained during the last two decades.

Conclusion

It has been the speaker's good fortune during 1924-25 to spend nearly half his time in an exhaustive study of the irrigation conditions of the West, especially as pertaining to the Federal reclamation projects. He has come out of the work with renewed confidence in the value of irrigation to the West and to the country as a whole. The difficulties that have arisen, the failures that have been recorded, are not arguments against irrigation, but rather pointers to the way which we must tread in the future. Without question, the future of American irrigation is secure if the means at hand are used in the solution of the relatively few problems connected with the development of irrigation in the West.

DISCUSSION

SAMUEL FORTIER,* M. AM. SOC. C. E. (by letter).—What may be regarded as the most general irrigation policy of the United States is that the cost of building, maintaining, and operating irrigation works is a direct charge against the land benefited. The general character of this policy may be better understood by a brief reference to the methods followed by each of the several agencies engaged in irrigation development. These include (1) individuals and groups of individuals; (2) co-operative or mutual companies; (3) commercial companies; (4) irrigation districts; (5) the U. S. Reclamation Service, now the Bureau of Reclamation; (6) Carey Act companies; and (7) the U. S. Indian Service. It is obvious that the first two named, being composed of farmers acting singly, in partnerships, or in organized bodies, pay for their irrigation systems. The commercial company constructs and operates the canal system but distributes both costs over the land under it; formerly it was the custom to secure re-payment for the works by the sale of water rights and to maintain and operate the system by the annual collection of water rentals. The sale of water rights having been prohibited by the laws of many Western States, commercial companies now deliver water to farmers at rates fixed usually by some public authority. Under authority conferred by the State the irrigation district may bond the lands of the district and in this way secure money to buy or build works. Sooner or later, however, those who own the irrigable land within the district are obliged to pay all the indebtedness incurred in building and maintaining irrigation works.

The same principle is embodied in both the Carey and Reclamation Acts. In the former, the Federal Government donates the land to the State on condition that it be reclaimed. The State in turn makes use of the corporation as a reclamation agent to provide capital, construct works, and pro-rate the cost over the lands reclaimed. Under the terms of the Reclamation Act the Government takes the place of the corporation in the Carey Act in financing and constructing works, but the expenditures, less the interest charge, are assessed against the lands benefited and each entryman assumes his *pro rata* share. About the only exception to this rule is the Indian and even he, when possessed of the necessary funds, comes under this country-wide policy since a large part of the funds expended by the U. S. Indian Service on the construction of irrigable works is reimbursable to the Government.

This policy is not only general but fundamental, for so long as it is in effect irrigation works and irrigation systems, regardless of their cost or magnitude, should properly be classified with other property belonging to farmers. This conception of irrigation—and it is believed to be the true one—brings the construction of irrigation works into close relationship to agriculture and renders necessary as a preliminary measure a critical study of such questions as the property of the farmer; its character as regards cost, durability, and efficiency; the profits to be derived from farming; and the ability of the average farmer to redeem a farm mortgage.

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According to the U. S. Census of 1920 the total values of different kinds of farm property were, in round numbers, as follows:

Land	$54 830 000 000
Buildings	11 486 000 000
Implements and machinery	3 595 000 000
Domestic animals	8 013 000 000

There are nearly 6 500 000 farms in the United States and the mind cannot well grasp their aggregate value; but some idea of their true status is readily obtained in dealing with averages. Thus, the average value per farm in 1920 was $8 503 for land; $1 781 for buildings; $557 for implements and machinery; and $1 243 for live stock. The average mortgage indebtedness as accurately as it could be ascertained was $3 356, leaving an equity of $8 728 per farm. A further analysis of farm property shows that the farm mortgage debt nearly quadrupled from 1890 to 1920; that farm dwellings as a general rule are inconvenient and unsanitary, and are lacking in both comfort and taste; that the buildings which house stock and protect implements and machinery are inferior in quality and poorly maintained; and that, in appearance, character, and efficiency, farm structures generally are of a low order when compared with those of civic and industrial development.

In regard to the average net income of the farmer in the United States, the preponderance of evidence goes to show that it is low. According to data collected by the U. S. Department of Agriculture prior to 1917, on 266 farms in New Hampshire the farmer received on an average $337 for his year's work. In fairly prosperous rural communities in Chester County, Pennsylvania, the average yearly income of 378 farms was $789. In three districts located in Indiana, Illinois, and Iowa, respectively, the average farm income was $870, and the average income on 69 irrigated farms in the Salt Lake Valley, Utah, was $417. To these incomes should be added the food and fuel derived from the farm and consumed by the family living thereon as well as the shelter furnished by the farm dwelling. According to the same authority the average total farm income in 1922 on 6 094 farms located in different parts of the country was $1 211, made up of a net cash income of $715, value of food and fuel produced on the farm, $294, and an increase in inventory of $202.

Some idea of the gross returns per acre per annum from irrigated land may be had from the records of Government irrigation projects, chiefly for the ten-year period from 1912 to 1922. These records cover 26 projects in 14 Western States and average in gross value $45.66 per acre.

In contrasting the achievements in irrigation development during the past 75 years with the policy herein outlined, it is at first difficult to understand how so much could be done with funds derived from the profits of irrigated farms. A brief review of this development and its noteworthy achievements discloses the fact that irrigation farmers have received since the pioneer stage large amounts of capital and a large number of completed or partly completed canal systems at greatly reduced costs from outside sources.

The report of the Census for 1920 shows that more than 19 000 000 acres were irrigated at that time at a cost of about $700 000 000, but although this cost under the general requirements was to be paid by the profits from irrigation farming, as will be shown later, only a part was so paid.

The pioneer found fertile river bottom-lands that could be watered readily from tributary streams and both the stockman and the farmer were able through their individual efforts, or those of small groups of their class, to establish homes supported in part at least by the products of irrigated farming. The first 1 750 000 acres, approximately, was thus reclaimed without aid from outside sources. Those who owned the land controlled the water and as a rule large profits were derived from a union of cheap land and cheap water. The crude and inexpensive methods of the pioneer, however, were not adapted to the irrigation of the higher bench lands, which frequently involved high diversion dams and long lines of main canals through rocky canyons and along steep mountain slopes. Therefore, when the lower lands bordering the streams had been irrigated another agency had to be found to finance and construct the larger and more costly works for the irrigation of the higher bench lands. It was at this juncture in the late Seventies that the land and water corporation entered the field and for twenty-five years continued to be the main agency in the irrigation development of the West.

It was due to the successful efforts of the pioneers that the productivity of desert lands had been demonstrated and this, in turn, led to far-reaching possibilities along the lines of land settlement, rural and urban development, transportation, industry, and commerce—in short, the possibility of building an empire in a land that was thought fit only for the abode of the buffalo and the Indian. This was the lure that attracted capitalists, who thought they saw wonderful opportunities for profitable investment. Promoters organized companies, secured capital from America and foreign countries, filed on water supplies, acquired irrigable lands whenever they could be had at a low price, and proceeded to build canal systems, with the optimistic belief of attaining independence in a few years by the sale of water rights and the collection of water rentals. Those who invested money in these enterprises realized after it was too late that the bulk of the lands under these new systems were held by speculators who would neither use nor pay for water, that the *bona fide* settlers were few in number and possessed of little means, that the maintenance and operation of canal systems were costly, and that it was the work of a generation rather than a few years to establish prosperous communities in an irrigated region.

Few of these commercial enterprises returned any interest to the investors and in the large majority of cases much of the principal was lost, but the systems remained and a large percentage has since been acquired by the farmers under them at a valuation much below the original cost.

The financial failures of commercial irrigation enterprises which occurred from 1870 to 1895 were repeated by Carey Act projects from 1898 to 1914. The main provision of the Carey Act is that the Secretary of the Interior is empowered on request of the State to donate to the State not exceeding

1 000 000 acres of land, on condition that it be reclaimed and settled. The States which accepted this offer were either too poor to construct irrigation works, or they were barred from doing so by their Constitutions and they "side-stepped" the issue by contracting with companies to do the work. These companies agreed to build works supplying water for definite areas of land and to sell to settlers water rights carrying an interest in the works. Funds for construction were usually obtained from the sale of stocks and bonds, but all the security the company could offer to purchasers was a lien on the works to be built and on the notes of settlers covering deferred payments on water rights. This later security failed to materialize on many projects for lack of settlers and the result was heavy losses to the investors. It is claimed that only 4% of Carey Act projects were financially successful. Canal systems were built to water about 1 250 000 acres, but in time many of these systems were re-organized into irrigation districts and other enterprises and the land owners within them fell heir to the systems at prices far below the original cost.

By the terms of the Reclamation Act the Secretary of the Interior was authorized to construct works to provide an adequate water supply for the arable lands in each project and to return to the reclamation fund, in ten annual installments or less, by assessments levied on the land owners of each project, the costs incurred in building and maintaining the necessary works.

By the terms of an amendment to the Act passed August 13, 1914, the time of payment was extended to twenty years. These more liberal terms, however, do not account for the lag in refunding construction charges. After twenty-two years of operation of this Act one finds a construction debt of $148 827 844, out of a total construction cost of $152 566 386, still due the Government. Under this Act the liberality of the Federal Government in relieving the settler of the payment of all interest on the cost of his water right and its leniency in extending the time of repayment of charges have been of material aid to farmers on Government projects.

In turning from the past, with its long record of successes and failures in irrigation development, to a consideration of the future it would be folly to disregard the knowledge and experience that have been acquired at so great a cost in money, labor, and human suffering. One of the lessons taught by the past is that farmers unaided cannot establish homes, prepare their farms for successful irrigation and profitable crops, and, at the same time, pay for costly water rights. They succeeded during the pioneer stage when land was cheap and the ditches inexpensive, but in the later stages failures have been common notwithstanding the fact, as has been pointed out, that they have received from time to time, from various outside sources, a large amount in the aggregate of financial assistance. If the profits from irrigation farming have not sufficed to pay for water rights in the past when conditions were comparatively favorable there is no probability that they will do so in the future, under much more unfavorable conditions. It is generally conceded that the easy tasks in so far as irrigation development is concerned have been done, the difficult tasks await this and succeeding generations.

A reasonably accurate estimate of the water requirement of crops and stream flow made by the writer in 1924 showed that about 57 000 000 acres might be irrigated in the 17 Western States. According to the Census of 1919 the systems were capable of irrigating 26 000 000 acres. This leaves 31 000 000 acres yet to be reclaimed if one excludes what has been accomplished since 1919. A large part of the water supply for this acreage, providing it is ever reclaimed, will be derived from a storage of flood waters and this fact, coupled with long canals in difficult locations and the preparation of rough and uneven land, will increase the cost of water per acre far beyond that of recent undertakings.

The people of the West may elect to follow one of two courses: They may decide not to reclaim by irrigation any more arid land; or they may decide to grant the necessary State aid for a further extension of irrigated agriculture. The first course would have few advocates since it is not only opposed to the progressive spirit of the West, but it is believed to be unsound economically. Much of the material prosperity of the West is based on irrigated agriculture and if no more land is to be rendered productive, rural growth, industry, and commerce would be at a standstill. A number of favorable conditions and circumstances seem to demand that more food and clothing be produced in the Pacific and Rocky Mountain States. It is a healthy region and its invigorating climate will attract and hold settlers. About three-fourths of the total hydro-electric energy of the nation is to be found west of the Missouri River. Future generations will also look to the West to supply the bulk of the minerals needed in industries, and beyond all is the broad Pacific and commerce with the Orient.

It would seem necessary, therefore, to devise ways and means of establishing more farm homes under new irrigation enterprises and, at the same time, seek to better the condition of those living under the old systems by a more or less complete overhauling of their faulty methods of conveying and distributing water. In the majority of cases the building of new works will be closely linked with the re-organization, enlargement, and reconstruction of old enterprises; and if this truth becomes generally recognized in time it will greatly aid in the adoption of proper methods and policies for future development. A large number of Western communities in the aggregate have reclaimed as much land as they can with the use of the summer flow of streams. They have made little progress for years owing to the lack of storage of flood waters. With part or all of the available flood waters stored, most of their leaky ditches abandoned, and a much smaller number of good canals built, the irrigable area might be increased from 25 to 100 per cent. These are the nuclei around which much of the extension of the irrigated area is certain to cluster.

This building on the one hand and remodeling on the other calls for new boundaries of irrigation enterprises. By this is meant the water-sheds of streams. All the water users on the smaller water-sheds should be amalgamated under one organization, and those on the larger, under several, divided by natural and political lines of cleavage. At present, outside of California, there is no type of organization adapted to the inclusion of all the

irrigable lands of a water-shed when these lands are made up of non-irrigated, partly irrigated, and irrigated holdings. The irrigation district has been and is a wonderful aid in irrigation development, but its provisions are based on the assumption that the lands included come under the same classification of not being irrigated but susceptible of irrigation. What is needed is a more elastic organization that will include dry as well as irrigated lands, individuals as well as co-operative companies.

In providing water supplies for the 30 000 000 acres or more yet to be reclaimed it is doubtful whether the farmers who will eventually occupy these lands will be able to pay more than 50% of the cost of the water supplies. Placing the average cost at the conservative figure of $100 per acre would involve an outlay of more than $3 000 000 000. This amount, large as it may seem, is small compared with what the farmers would be required to invest in developing their lands. The land and its preparation, farm systems of irrigation, buildings, live stock, and equipment, for the area under consideration would cost fully $9 000 000 000. Judged by the past the income from this farm investment would be small, but it would place the West on a sound and prosperous foundation by increasing its urban population and wealth and by greatly enlarging its industries, transportation, and commerce.

During the past 23 years the Federal Government has expended, including interest, about $170 000 000 in aid of irrigation in fourteen of the Western States. The financial assistance given by any of the Western States during this period or before has been negligible.

The Federal Government should complete the projects it has begun and fulfill its obligations to settlers; but, from the standpoint of fairness to other sections of the country which are in as great need of National aid, it is difficult to justify a continuation of Federal aid for irrigation without full co-operation with the States concerned.

On the other hand the granting of National or State aid to farmers is attended with considerable risk. When done in the wrong way it tends to break down that spirit of sturdy independence which has been inherited from Anglo-Saxon and other fine lines of ancestors and to transform self-sustaining Americans into bounty-fed dependents. The part performed by the State or the Nation in the irrigation development of the future should be well defined and wholly apart if possible from that performed by the farmers. If this policy is to be followed it would be best for the State, acting alone or in co-operation with the Federal Government, to build the necessary storage works and to wholesale the water thus provided at reasonable rates to organizations of farmers. This was the policy advocated by the writer and many of his colleagues twenty-five years ago, before the Reclamation Act was passed. If the Government had begun at that time and continued until the present to expend the funds derived from the sale of public lands in conserving the water supplies of the West and contracting their use to private irrigation enterprises on terms favorable to the users, much greater progress would have been made and the West would have had fewer farmers spending their working hours writing Congressmen for payment extensions instead of milking their cows.

ARTHUR P. DAVIS,* PAST-PRESIDENT, AM. SOC. C. E.—The statement by Mr. Fortier, that of the total capital investment in Reclamation works—about $152 000 000—more than $148 000 000 is still unpaid, needs further explanation. From his figures it might be inferred that only $4 000 000 has been repaid to date. This is far from the truth, for as this money is repaid it is immediately re-invested, as provided by law. Normally, therefore, the difference between the capital invested and the amount unpaid will be merely a reasonable working fund as long as the law operates.

The time and place of the origin of irrigation are unknown. The earliest records of Assyria, Babylon, Egypt, Persia, India, and China, and practically every country of antiquity, bear testimony to ancient and well developed practices of irrigation. It is probably one of the oldest occupations of civilized man. It is a well established fact that civilization originated and developed in an arid region. At the time of the Spanish Conquest in Mexico, extensive irrigation systems existed, antedating the earliest traditions of the people using them. Traces of such works were found in South and Central America and in Arizona, New Mexico, Colorado, and California.

As to reclamation as a modern activity of the Anglo-Saxon race, one of the earliest examples is the Salt Lake Valley, Utah. The early settlers of California, Arizona, and New Mexico extended the previous practice of the Spanish and Indians in those States. Under those primitive conditions, only the simplest and easiest irrigation enterprises could be carried out. No others were justified by economic conditions.

Under the physical circumstances of the Salt Lake Valley, where the streams emerge from the high mountains, discharging most of their water, during the summer, in rivers and creeks with rapid fall, traversing wide fertile plains with considerable slope, irrigation presented advantages which were quickly recognized and successfully developed by the Mormon leaders. Under such circumstances anything that would divert a stream could be used to place the water in a ditch and convey it to lands in the immediate vicinity without any appreciable expense, except a moderate amount of human labor and enterprise.

The hardy pioneers carried out these methods of using the smaller streams on the lower valleys which they traversed, and, in the aggregate, cultivated a great deal of land and produced large quantities of forage. These enterprises extended to all the States in which favorable conditions for easy development could be found, but these sites were gradually exhausted. The unregulated water supply, declining during July, August, and September, would accomplish only a fraction of the total possibilities of the stream, and, to increase this supply, large storage reservoirs were necessary, generally in regions difficult of access and requiring large investments. Other streams flowing through canyons could be utilized only by high diversion dams, long tunnels, pressure pipes across other canyons, and expensive side-hill canal work, sometimes a long distance from the land to be irrigated; and these projects also required storage reservoirs for their full development.

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These enterprises, which required such heavy investments, could not be carried out by the individual efforts of the farmers, and private capital was invited to assist in the development. For a period the representations of promoters inspired investors with enthusiasm and confidence, and considerable areas were reclaimed; but the necessity of heavy investment with its promotion and interest burdens, and the long wait for settlers to develop the desert lands before production could begin, soon demonstrated that the rosy anticipations of promoters were not justified. It may be stated that, with few exceptions, investors in irrigation works as such were financially unsuccessful. Although the development produced might be of benefit to the country, those benefits were largely absorbed by the owners of the land, and, if they were not the same persons as the investors in the works, the latter were generally losers.

It took only a few years to demonstrate that such investments were hazardous and most of them disastrous. Then came the era of development under district enterprise. State laws were passed to encourage this method and much progress was made by combining land ownership with irrigation development. Again, however, the heavy interest charges, the long time and the large investment required for full development, made this method slow and burdensome and entirely unsuited to many of the larger and more expensive projects requiring greater capital and long time in construction and development.

The assistance of the Government, through its ownership of the public lands, was early invoked, as manifested by various laws, especially the Desert Land Act, conferring title on the individuals that reclaimed the land. This Act was, in the aggregate, a great assistance to the smaller enterprises, but did not remove the difficulty of development where the enterprises were large and costly, as the land could not, under the law, be turned over to the large interests necessary for its irrigation.

As the easier irrigation projects were selected and developed by one of the various agencies available, the problem became more and more acute as the physical difficulties of undertaking new projects progressively increased by the elimination of the smaller and easier developments. Recognizing these facts and the interests of the people at large in the irrigation development of the Western States, a movement was started about thirty years ago to adapt the laws to the handling of the proceeds of the public lands in such a manner as more effectively to encourage their development.

In the Middle States the Homestead Act was effective in settling and developing the country under the conditions there obtaining. The settler was given, as a resident, 160 acres of land on condition that he develop it and build a home. The clearing, fencing, and cultivation necessary were made effective by the climatic condition under which a single successful home could be made through the efforts of one family without the co-operation of its neighbors. This law, however, was found to be unadaptable to a country requiring irrigation, especially where extensive investment of cash and labor was required. So far as irrigation from smaller streams could be carried out by a small group of farmers through their combined labor, this

was done, but the attempt became impossible when applied to the difficult enterprises requiring large storage reservoirs, expensive dams, canals, tunnels, flumes, and pressure pipes. The same theory, however, could be adapted to this construction; that is, the values residing in the public lands could be devoted to the development of agriculture in the arid States, as well as in the humid States, if the laws were properly drawn for this purpose.

This was the intent in 1902 when the National Reclamation Act was passed appropriating the receipts from the sales of public lands in the sixteen Western States to the construction of irrigation works in that region. Its avowed purpose was to construct the larger and more difficult projects which were incapable of development through private capital or district agencies. In the selection of projects under this law many were rejected on the ground that they were small enough and easy enough for successful handling by private enterprise. In some cases this proved to be a mistake and projects rejected for this reason were undertaken after a trial had demonstrated that they could not be successfully developed by private enterprise.

Under this law, twenty-five projects have been developed, resulting in a full water supply being made available for nearly 2 000 000 acres of land, furnishing homes for 30 000 families, and a supplemental water supply for about 1 000 000 more, or nearly 3 000 000 in all.

To accomplish these results reservoirs have been built with an aggregate capacity of more than 10 000 000 acre-ft.; 15 000 miles of canals and ditches have been constructed; 103 tunnels, aggregating about 150 000 ft. in length, have been built; structures on the canals, such as drops and turnouts, aggregate about 126 000; more than 9 000 bridges have been constructed, aggregating 217 000 ft.; 11 000 culverts have been built, aggregating about 400 000 ft.; 3 160 000 lin. ft. of pipe have been provided; nearly 4 000 flumes, with a total length of 750 000 ft., have been supplied; and about 1 600 buildings for offices, power plants, pumping stations, barns, and storehouses have been constructed. In connection with this work, there have been built 83 miles of railroad, 1 100 miles of wagon road, and 3 284 miles of telephone lines. The work performed has involved the excavation of more than 200 000 000 cu. yd. of earth, and nearly 50 000 000 cu. yd. of rock and indurated material. It has required 3 500 000 cu. yd. of concrete and a somewhat larger number of barrels of cement. Incidental to this work, a few power plants have been built as by-products of the storage development, the proceeds of which were credited to the water users on the projects. These plants have developed 64 300 h.p. of energy and have required the construction of about 1 500 miles of line to transmit the electrical energy to the point of use. This power development, scattered over many projects, while small compared with the aggregate development, has been an important factor in the comfort and prosperity of the settlers, being mainly used in the homes and towns developed by the irrigation enterprises.

The total investment in National irrigation has been about $190 000 000, of which about one-third has been from re-payments by the settlers of the money invested for their benefit. It is believed to be the only National con-

struction, except the Panama Canal, which has made such large returns in proportion to the investment.

The cash value of the annual products of these twenty-five projects aggregates more than $50 000 000, even in this period of agricultural depression, and, in more prosperous times, has approximated $90 000 000, as in 1919; it is safe to state, therefore, that the product of three years exceeds the investment by the Government, and that the aggregate has been more than $700 000 000. The estimated increase in land values, due directly to irrigation works, is $600 000 000, or more than three times the investment. The law requires the return of that investment, with the exception of the money expended on projects which have not been built or completed, and under the liberal terms offered by the Government and the long time for re-payment, this, in most cases, could be done from the products of the land, except for the propaganda against such return on the part of opposing interests. Whether such propaganda will succeed in its endeavor to wreck the irrigation enterprise remains to be seen, but the values have been created by the combined influence of Government investment and the activities of the industrious farmer, and they have added to the wealth of the West what practically amounts to an additional State in value and in product.

From the small beginnings described, irrigation has become an important factor in nineteen States and is the dominant form of agriculture in eleven Western States. These eleven States contribute more than 20% of the value of all crops in the United States, producing an average value per acre of more than double that of the unirrigated farms of the country.

Probably the most serious weakness of the original National Irrigation Law was the requirement that the major part of the money be expended in the States in which it was received from the sale of public lands. This injected an entirely foreign element into the consideration of projects for construction and produced a still greater moral influence in favor of regarding the Reclamation Fund as a fund for distribution among the States, rather than as one to be allocated in accordance with the business merits of the various projects. This influence led naturally and inevitably to the selection of projects in States where the public lands were bringing in large revenues, even when those projects were of inferior merit. In the promotion of projects in such States, the promoters dwelt more on the rights of the State to use the fund than on the physical merits of the project advocated.

This provision of the law was repealed in 1910, but not until after most of the projects had been selected and construction started, and a feeling of proprietorship had grown up in each State concerning ownership of the Reclamation Fund. There have not been wanting those who argue that the moneys received in a given State from the sale of public lands naturally belong to that State to be invested there, irrespective of the merits of the proposed investment, and this spirit continues to prevail about as strongly as ever.

Recently, efforts have been made by public officials to induce or require the States to share in the expenditures and responsibilities of making the irri-

gation enterprises successful through the selection of competent settlers and of supplying them with the means of improving and properly equipping their farms so as to put them on a paying basis. Such measures have been successfully carried out in some foreign countries, and probably with proper supervision the same might have been true in America. The agitation for relief from payments has made such progress and received so much encouragement in official quarters that it is doubtful whether collection could be enforced, if such measures were adopted. The involvement of the State in the responsibilities for the success of the projects in itself is an excellent idea and would contribute greatly to their success.

C. E. Grunsky,* Past-President, Am. Soc. C. E.—That water is the nation's most valuable natural resource requiring conservation is coming to be generally recognized. In the matter of the volume of supply Nature has been generous, but as to distribution to the various sections of the land and as to dependability as regards the time of availability considerable improvement might have been suggested.

Wherever the human race has established itself in permanent settlements water has been needed; first, of course, for domestic purposes; for drinking by man and his domesticated animals; for washing and bathing; to facilitate transportation; for irrigation; and, where available in flowing streams, to generate power, which, in recent times, is made available at places remote from the source, by electrical transmission. The unequal distribution of rain and snow as to time and place, coupled with the orographic features of the surface of the earth, has resulted in great diversity of stream flow, and, therefore, as soon as the demand for water exceeded the supply of the local spring, stream, or well, the newer problems of control, regulation, and diversion of flow—in other words, of conservation—have been introduced.

The early problems of water utilization were simple. The wants of a sparse population could be supplied without regard to economy of use. Frequently, a small part of the supply has been, or still is, sufficient to meet fairly well the needs of the dependent people; but population is growing at a tremendous rate—in some countries at nearly a geometric ratio diminishing but slowly—and with this increase comes the insistent requirement, although not always clearly expressed, nor yet fully understood, that better provision be made by human agencies to get for mankind the greatest possible good out of Nature's bountiful supply of water.

Originally water, like air, was free to be taken by any one who had need of it. Because of the large supply as compared with the small demand the water right had no value. Because early settlement was of necessity at points where water could conveniently be obtained in the desired quantity no elaborate appliances were needed to modify natural conditions of occurrence and flow. As soon as permanency in the place of living was found to be desirable the advantage of stimulating plant growth by the artificial application of water was recognized. In some measure, first small and, later, larger quan-

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tities of water were brought under control, and service to the community was commenced.

It is self-evident that early works—whether early refers to the progress of civilization or whether in a more restricted sense to any particular region—must have been simple in their nature, easy of execution, and constructed to achieve an immediate purpose rather than to fit into the comprehensive plans of water utilization that later developed. It is only natural, therefore, that frequent occasion has arisen to find fault with the wasteful methods of control and use of water in those regions where the demand resulting from increasing population began to exceed the quantity that was conveniently accessible and obtainable. Correction of established customs and practices has not always been easy. The rights of the individual as against those of the State are not always clearly defined.

It seems fundamental that sooner or later the State—meaning any large political organization—must step in, and wherever demand for water exceeds the supply or where it is foreseen that such a condition is proximate, must assume control and define the features of the project for water conservation. The most obvious step to be taken in the case of the flowing stream is so to regulate its flow by storage and related works that it will render a maximum of service and do a minimum of damage.

In the United States, where the wise policy has prevailed of encouraging private enterprise, generous provision has been made for the early development of natural resources, including the use of water; and private rights have been established and are generally recognized, which do not, however, always comport, as they should, with the best interests of the country.

It is only in recent times that the privilege of utilizing parts of the public domain for water storage is coupled with conditions that make for the fair protection of public interests by limiting the granted right in the matter of time. The indirect control exercised by Federal authority over the waters of the lesser streams, however, frequently results in an invasion of the rights of the individual State. There is here sometimes conflict of authority that should receive timely attention. Just where, in other words, should Federal control stop and State control begin?

It seems that wherever a right of way, whether for conduit lines or for storage purposes, is required over lands of the public domain, the Federal Government has assumed a right of regulation and control which interferes in no small degree with the general conception of State control in such matters. There can hardly be any question that, when authority was first granted to the Secretary of the Interior to permit the use of lands in National parks and in forest reservations for the development of water resources, the functions of his Department were to be mainly ministerial. Such rights of way, it is fair to believe, should have been granted in all cases of merit and honesty of purpose, provided there was no material interference with the enjoyment by the public of the lands in park or forest for the purposes for which they were set apart. This view unfortunately has not prevailed. The Government conditions imposed in any grant of a right-of-way privilege may and fre-

quently do supersede and annul the rights supposed to have been acquired under State law.

Attention may be directed in this connection to the so-called Raker Law granting certain reservoir rights in a forest reservation to the City of San Francisco, Calif. There is a conditional grant in this law for the utilization of a few acres of ground for water storage purposes made by Congress on the failure of the Department of the Interior to do its plain duty in the premises. No one can read this law without deploring the interference of Congress in matters that should never have been brought before it, nor yet without feeling that San Francisco's representatives were unwise in accepting the imposed conditions.

Where then shall the line be drawn between control by the State and control by the Federal Government of the use of water in the flowing stream? No complete reply to this question will be ventured at this time, but attention will be directed to only one or two matters connected therewith. It has already been stated that regulation of stream flow is essential to obtain for the community the maximum service from the water. Whenever a stream is an international or interstate stream the comprehensive plan of regulation and control should originate with the Federal Government. When, however, the stream originates and flows in one State only, there should be no interference by Federal authority save only in the cases in which navigable waters are affected by regulation. The devising of far-reaching plans by the Federal Government and by the several State Governments should be accepted as a fundamental duty, and after the adoption of any such plan, development whether by the Federal Government, by the State, or by private effort, should conform to the adopted program, and, of course, to subsequent modifications thereof.

It would be well, wherever practical, for the State to take charge of stream regulation, to build the necessary reservoirs, and to wholesale the output. The State of Massachusetts may be referred to in this connection. In that State authority has been given to a Water Board to issue State bonds to an amount which reached a highest limit of more than $40 000 000. With funds realized from the sale of these bonds, storage works were constructed and main conduits were built. Water is delivered by them to Boston and to about twenty-eight or more cities and towns near Boston. The delivery is wholesale, each community being free to arrange for the distribution of the water to the consumer as it sees fit. The ownership of the local distribution plant may be either public or private. As the interest and sinking fund requirements to meet the State's obligation under the bond issue have been fully met from the revenue resulting from the payments at wholesale for service rendered by the Water Board, a tax or assessment has never been levied by the State in this water matter. Those who get the service have paid the bills.

No reason is apparent why the example set by the State of Massachusetts should not be emulated by the Western States where regulation and full use of the water supply is of vastly greater importance, because of relative scarcity and lack of rain, than in the New England States. Recognition of this greater need for Government help prompted the passage of the Reclamation

Law of 1902 by the United States. In this law, however (perhaps, of necessity), were embodied several provisions which, to say the least, were unfortunate. The appropriation for the use of the Reclamation Service was an indirect appropriation. The moneys realized from the sale of public lands were allowed to flow into the Reclamation Fund instead of going where they should go—into the General Fund of the United States Treasury. This gave to a number of the States a feeling that they were contributing inordinately to developments in other States. There should have been no reclamation law until the country was ready to make a budget allotment for a well-considered reclamation program.

Again, in another matter, the only additional point to which attention will be directed, the law set out to accomplish too much. Partial regulation of stream flow was to be combined with land settlement projects. No project could be undertaken unless coupled with the subdivision of large land holdings.

The Government, perhaps justified by the fact that it was giving a large bonus, in the shape of interest relinquishment to the land owner, assumed a large measure of direction as to the operation of the individual farms, although without material effect on the use of the land for speculative farming. Without expressing any opinion as to the timeliness of this Government entering on land settlement plans, it seems unfortunate that Government activity in the reclamation of land by irrigation was not restricted to the storage and wholesaling of water, leaving the construction of works for its distribution to the individual farm, private initiative, or district organizations, without any onerous settlement restrictions. It would seem wise now to take heed of the lessons of the past and to separate land settlement enterprises from the regulation of stream flow and the wholesaling of the water output.

F. H. Newell,* M. Am. Soc. C. E. (by letter).—Dr. Widtsoe has given a broad review of the situation, developing so many lines for discussion that it is necessary to choose among these, selecting preferably the one which is of most immediate interest to engineers and to economists concerned with the extension of irrigation development, namely, the use of public funds in reclamation.

The financing of irrigation development by private capital is covered by the other papers† and discussions presented with that of Dr. Widtsoe. These bring out certain advantages and difficulties, so that it is unnecessary to do more than refer to the fact that the great development of the country through irrigation always has been, and presumably always will be, through the wise use of individual and corporate funds, invested under State law. The Federal Government as such has relatively little concern with these private enterprises. In a broad way it may be said that, as pointed out by Dr. Widtsoe, irrigation was initiated by private enterprise and has been carried on with success; and it was only because of the existence of certain rather temporary conditions that the Federal Government was brought into the matter.

* Pres., The Research Service, Inc., Washington, D. C.

† See pp. 710 *et seq.*

For every acre reclaimed by the use of Federal funds, as shown by the Census reports, ten acres or more have been irrigated by private enterprises. At present a larger amount of money is being invested in irrigation or reclamation district bonds and in other securities than is being appropriated by Congress. Therefore, the question may be properly asked as to why any Federal money has been or is being diverted for this purpose, also whether the reasons that called for the imposing on the overloaded taxpayers of the additional burdens of the costs of Federal irrigation should be continued.

Public Lands.—The argument for Federal reclamation of arid lands, as pointed out by Dr. Widtsoe, was based primarily on the fact that, at the time agitation was started for this Act, the Federal Government was the owner of more than 500 000 000 acres of land, a small proportion of which, possibly 1% or 2%, was so located and of such character that it might ultimately be utilized in home-making. At that time, also, about 1895-1901, the larger capitalistic enterprises, including those initiated under the Carey Act, for storing and distributing water to the arid lands were passing through a period of depression. They had not paid, and neither the State laws nor the prevailing practices provided adequate safeguards such as to render feasible further reclamation on a large scale.

There were at that time also not only enormous areas of lands which might be irrigated (some of it in ownership by the Federal Government), but a strong demand for these reclaimed lands. Population was still pushing westward and there were insistent demands for the extension of the principles of the Homestead Act. The easily tillable lands in Kansas and in other States of the Great Plains region had been taken up, but the tide of emigrants still continued to pour into and through these States, with the corresponding demand that in some way or another the Federal Government make available more tillable farms. The only way to do this appeared to be by providing water for some of the vast extent of arid land.

Moreover, at that time there still remained unappropriated considerable quantities of water flowing from the mountain areas, especially in the streams that crossed State lines. State laws and the decisions of the Courts were still in a somewhat chaotic condition, and it was shown that unless the Federal Government would step in, build some of the large storage works, and make far-reaching plans, it would not be practicable to utilize effectively the great resources latent in the proper control and use of these streams.

Under the urge of these conditions, especially the demand for more lands which might be used as homes for citizens, and because there was recognition of the need of comprehensive plans for development, active propaganda, liberally supported by the Western transcontinental railroads, was kept up to the point where the majority of Congress was convinced that here was a necessity and a duty. It may be pertinent to state that this education of the public, initiated thirty years ago, has been carried on so thoroughly and effectively as to plant deeply in the minds of the present generation the fact that, somehow, the Federal Government is committed to a policy of reclamation, even if the conditions that required it have largely disappeared.

In these past thirty years, especially in the last decade, there has been practically a reversal of the conditions on which arguments were based for Federal intervention in these local or State enterprises. First and foremost, it may be said that there is no longer any considerable body of reclaimable public lands. Of course, it is admitted that here and there are many isolated areas, usually inferior in character, which have been passed over or rejected again and again as being unavailable. This condition has been pointed out* and it was shown that the total extent of public land entered from 1901 to 1922, in the reclamation or public land States, aggregated 344 000 000 acres, leaving as unappropriated and unreserved, possibly open to homestead entry, not quite 190 000 000 acres, practically all of which is rough mountain lands, mainly of scenic value, having little vegetation, although susceptible of use occasionally in wet years as open or free grazing.

The greatest change, however, has been in the cessation of demand for irrigated or reclaimed lands, accompanying the general decline in agricultural population. Instead of a steady pressure for more homes on the land there is a tendency in the other direction. Thousands of families are going from the country to the city or to centers of industry where the earning power of the individual is far greater. Millions of acres already reclaimed by irrigation and drainage are awaiting settlers. The owners or managers of these reclaimed tracts are moving heaven and earth to get people who will live on the lands either as owners or tenants; they are offering every conceivable inducement to attract farmers.

The main question brought out by this presentation of the history and problems of irrigation development in the West is as to whether it is now wise and desirable to continue these efforts to reclaim more land and to attract more settlers. The answer involves the further question as to whether all taxpayers should be called on to spend more money, adding directly or indirectly to the present burdens of taxes—such as must follow from efforts of the Federal Government to get more settlers to occupy lands in the West.

Subsidy.—It is assumed that it is a proper function of the Federal Government to use its money and influence to bring about the most complete development possible of the Western or public land States, even if most of the public lands have been converted into National Forests, or otherwise disposed of. This is a policy that has been accepted for a generation. Assuming, however, that this is correct, it is of great importance, to all engineers and economists, as well as to all citizens, to consider the limits to which the Federal Government, co-operating possibly with the State governments and local or district authorities, can and should go in offering subsidies or in giving bonuses to the land owners in the West, in order to secure the desired cultivation and use of their lands.

It may be urged in this connection that the Federal Government is not appropriating the money of the taxpayers but on the contrary is using special funds derived mainly from the sale of public lands. This, however, is merely "begging the issue", as Congress diverts from the Treasury to the Reclamation

* *Reclamation Record*, January, 1923.

Fund certain sums which otherwise would go into the Treasury and which would relieve the taxpayers from a corresponding burden.

The original Reclamation Act, signed by President Roosevelt on July 17, 1902, set the limit of this Federal subsidy or bonus at ten years' exemption from payment of interest. These debtors to the Government for the water which made their lands valuable, were to be stimulated or encouraged to take up the Government land, freely given to them, or to utilize other lands purchased by them. It was believed that this exemption from interest payment, together with possible freedom from taxes on the unpatented public lands, would induce the settlement and development of these lands.

As a matter of fact, this ten years was in addition to a considerable lapse of time during which the reclamation works were being completed or "tuned up," so that even under this ten-year exemption many of the land owners enjoyed the use of the water for, say, fifteen years, and at relatively small expense.

In fact, all large reclamation enterprises, with a few notable exceptions, must be subsidized directly or indirectly. This is because the land when ultimately reclaimed and settled does not have an economic value equalling the cost of reclamation and other work put on it. These subsidies coming to the land owner may be concealed under many forms, but they can always be found by careful analysis of the total original cost and accrued interest. Much of this subsidy in a case of private enterprise is in the nature of failure of the original investors. The stockholders and even the bondholders of most of the large irrigation enterprises lost heavily, the works built by them as "involuntary philanthropists" were ultimately paid for at less than cost, this loss being practically the equivalent of a bonus or subsidy to the land owners.

Knowing this condition, the Roosevelt Reclamation Act was adopted under the belief that this ten-year exemption would be an adequate reward to the land owners. It is now generally believed that this is not adequate and the real question that confronts the engineer and economist, when stripped of the emotional, political, and other side issues, is simply "How large a subsidy should be paid to induce reclamation cultivation and settlement of waste lands? What are the limits that should be set?"

With ample funds and no limits set to further bonus or subsidy, it is possible to carry out any of the projects now proposed. If, however, it is decided that the bonus or gifts to the land owners are to be limited, then only a few of the more desirable of the proposed Federal projects may be safely undertaken.

Infinite ingenuity and almost endless oratory have been used to obscure this simple fact. The politicians have succeeded in creating a smoke screen of sentiment, involving social and political considerations. Dust is thrown into the eyes of each investigator and every effort has been made to divert his attention from this, the real issue, namely, not what will be the first cost but what will be the ultimate contribution to the land owner on the project.

The benefits of reclamation, great in themselves, have been magnified or distorted. Sentimentality has run riot, distracting attention from the question which should be stated in the plainest language, namely, "How much

must be donated by the taxpayers directly or indirectly and how does this bonus compare with the ultimate real value of its benefits to the land owners?"

One of the elements tending to obscure this question is the failure to recognize the fact that the liberality of the Government in giving this bonus or subsidy is not conferred on the public or on a large group of people but on a limited number of land owners. These owners may or may not be settlers, water users, or farmers. The reclamation works built at Government expense provide water only for certain designated tracts. It is only through the wise use of these particular pieces of land that benefits can come to the public at large. In a sense the whole question is in the control of these land owners.

In the case of subsidies granted by the Federal Government for hard roads, it is usually understood that these roads can be used by any one. In the case of irrigation works, the reverse is true—the use of the water must be narrowly limited to carefully defined areas. If the owners of these lands do not utilize the water to the best advantage the investment of Federal funds is a corresponding loss. To prevent such loss, it is now proposed to exercise a still larger or more drastic control of these land owners.

Moreover, it should be kept clearly in mind that the actual cultivator of the soil, the man who is trying to make a home and to develop the country, may not receive any benefit from the liberality of the Government. If he is a tenant or if he is trying to purchase a farm from one of the men who obtained it practically for nothing from the Government, his burdens may be greatly increased for the reason that the original land owner, or the speculator who has purchased the land, places an exorbitant price on it because of the assertion that the Government may not collect the debts.

Few students of the subject contend that the Federal Government should not subsidize reclamation in some way. This policy is now well established. It is almost a waste of time to discuss its wisdom. It is, however, of the greatest importance, for the continuation of reclamation, that the fundamental facts be faced clearly and that engineers and economists join with other citizens in the attempt to define the limits and the methods of the necessary subsidies. It will then be possible to consider in connection with each of the proposed reclamation projects whether under present conditions it will yield as large or as valuable results to the public as a whole as might come from the investment of an equivalent sum of money in other or better considered undertakings.

In 1902, when the original Reclamation Act was passed, the Federal Government had an overflowing Treasury, it was not borrowing money to any considerable extent, and the taxpayers' burdens were moderate. It could well afford to let its surplus funds be invested without interest return for ten years in creating opportunities for homes or public lands as needed by its citizens. These conditions are now reversed.

Repudiation.—It was freely predicted in Congress and elsewhere when the Reclamation Act was under discussion that in one way or another the land owners who ultimately would become beneficiaries of this Act would endeavor to repudiate their debts to the Government in whole or in part. Enough

was said to make the advocates of Federal reclamation peculiarly sensitive on this point and to cause vigorous protests of good faith; nevertheless, within 10 years, and before substantial repayments had been made, a vigorous and successful effort was made to extend the time of non-interest payment from 10 to 20 years. It was followed by other demands for extension of time of repayment, without interest, to 40 years, 60 years, or even to 90 years.

When the first Extension Act was under discussion in Congress it was urged by a few far-seeing engineers and economists that, although it might be within bounds of reason to permit 10 years to elapse without interest payment, yet after 10 or more nearly 15 years of settlement of a project, then the land owners could and should pay a small interest, 3½ to 4%, or whatever the Government was paying at that time for the money, which, in a sense, was borrowed to replace the amount invested in these reclamation works. This suggestion was not accepted.

The success of the advocates of the Extension Act stimulated activities all along the line among the men who believed in whole or partial repudiation. They pointed out that the Government was expending millions of dollars, which it did not expect to recover, in the way of levees or protection works benefiting other agricultural lands; that it was spending hundreds of millions in hard roads; and that a single battleship built and destroyed without use cost more than a great irrigation reservoir or completed project. It was feelingly urged that the settler on the Government project had to endure all the hardships of pioneering and that, in a certain sense, he was performing a public function in aiding to reclaim lands and in making his home in a remote part of the country. For this, he should be rewarded by being excused from a part or possibly from the whole of the amount owed to the Government for the reclamation of his lands.

With every session of Congress these requests for further relief are pressed. The condition is such that although the immediate beneficiaries are in number an almost infinitesimal proportion of the farmers and land owners of the country, yet the petitions and cries for relief for these "wards of the Government" obstruct many other measures of importance to all citizens.

Limit of Subsidy.—This side of the picture is presented in order that consideration may be given to the limits which should be set for the time and amount of the subsidy or bonus to the owners of lands reclaimed by the Government. Assuming it is a settled policy that such subsidy must be given, directly or indirectly, in order to induce a larger settlement of the Government projects, the lowest limit of this bonus or subsidy was that set by the original Reclamation Act as 10 years' freedom from interest. The Extension Act increased this to 20 years.

The estimate of this subsidy or cost to the taxpayers of the Nation has been discussed by officials of the U. S. Department of Agriculture. For example, on Salt River, Arizona* (probably the most successful irrigation project of the Government):

"The announced charge for water is $90.00 per acre. Water was supplied to some farmers 11 years before payments began. In the beginning of the payments the accumulation of interest charges would have increased the

* *Bulletin 1257*, U. S. Dept. of Agriculture, August 23, 1924, p. 12.

charge to 187.84% of the announced charge, or to $169.06; and at the end of the 20-year period, during which the $90.00 will be repaid, without interest, the accumulated debt, with interest, will amount to 407.44% of the $90.00 charge, or $366.70. This represents the subsidy, per acre, to such a farmer in the Salt River Valley at the time when the Government will consider his debt discharged."

Enthusiasts may justify this expenditure and dwell on the benefits to the United States, but, while adorning these facts with flowers of rhetoric, they must not forget that behind the roses are the thorns of hard fact, namely, that the taxpayers of the country have had to pay, in real money, deftly taken from their pockets, this enormous sum. On less favored projects even larger amounts, as pointed out by the economists of the U. S. Department of Agriculture, must be paid if similar conditions are to continue. These facts should be presented in such a manner as not to alarm, but to stimulate action along lines which will bear the most careful scrutiny.

After facing squarely the fact that an enormous but partly concealed subsidy is being paid to a relatively small body of land owners and weighing the advantage to these land owners and to the public, the people can consider on their merits the new enterprises which may be presented and which demand still larger subsidies. In such consideration should be kept in mind the fact that private or corporate capital undertaking reclamation enterprises does not ask or expect any large bonus or subsidy from the general taxpayers. Are the new projects or extensions of existing projects likely to be of sufficient value to the taxpayers of the country to justify their construction, taking into account the fact that if built by public capital they must be heavily subsidized, also that there is a strong and increasing sentiment toward repudiation under various guises? Congress must answer.

Land Owners.—Dr. Widtsoe has pointed out that in all this work, whether by private or public initiative, the man to be considered is the man on the land. As he states:

"The heroic figure in irrigation development is the water user—the man and woman, who throughout the years live upon the land, till it, and make it yield enough for life's subsistence. Capitalist and engineer, statesman and tradesman, are all necessary, but compared with the water user are of secondary rank in bringing irrigation development to success as measured by the standards of the age."

This is true; all consideration should be given to the real home makers. Nothing should be too good for the man and especially the pioneer woman; but it is proper to ask whether the plans and policies proposed, if carried out, will result in real good to this relatively small class of people, selected somewhat arbitrarily from the great mass of other pioneers and farmers. Does not the interference, although well intentioned, the persistent offers of advice, of loans of money, tend to reduce the energy and independence of these people and to get them deeper and deeper into debt from which escape, during the present generation, or even in the next generation, seems possible only through the door of repudiation, so temptingly held open by the local politicians?

In all these discussions of benefits which could or should be conferred by a generous Government on this small class of pioneers, there is a tendency to confuse two entirely distinct groups of individuals and to include under the

name of "farmer" various kinds of people who may or may not be actual water users or residents on the land. The public appreciates that the farmer is at the base of civilization. Popular sentiment supports any measure that seems to be of benefit to him or that he thinks may help him. Everything he really needs must, of course, be given first consideration but, in the case of the Government projects, is it really the farmer who is most conspicuous in his demands? Not every land owner on a Government project is a farmer. The mere fact that a man owns one or more farms or farm units given to him or to his predecessor by the Government does not make him a real farmer.

The land owners on Government projects, who for the time are the immediate and direct beneficiaries of the liberality of the Government, theoretically may be called farmers. Through a strict enforcement of the law they should be actual residents and water users; but the fact has been brought out again and again that a large proportion of the persons owning lands on Government projects can hardly be called farmers in the sense that they are skilled or experienced in farming or expect to spend any considerable part of their lives on the farms, made tillable by the use of Government funds. These conditions have been discussed particularly in Congressional hearings,* which showed that these land owners represent every conceivable occupation running through the alphabet from actors, architects, and artists, through the "butchers, bakers, and candlestick makers", down to undertakers, watchmen, and yardmen of the local railroads. Some are absentee landlords. Tenantry has increased to upward of 40 per cent. Now the cry coming from every Federal project is: "For Heaven's sake give us less advice, less talk, but get us more and better tenants."

Dr. Widtsoe's summing up of the means of solving the irrigation problems are those which may well be applied to any capitalistic undertaking: First, faith in the enterprise; second, study of its needs; third, diffusion and better use of knowledge; fourth, a broader recognition of the values of the undertaking; and, fifth, training for leaders, with particular emphasis on economics and law. All must agree with these conclusions as thus broadly stated. In their practical application, however, they reach into the much debated questions of over-production, or rather of lack of provision of adequate transportation and marketing facilities. The consideration of these latter brings up the problem as far as the Federal Government is concerned as to the limit of these direct or indirect subsidies and of the extent to which it can properly or wisely interfere with private enterprise and go into business paralleling or rivaling that of corporations and individuals. Already a strong reaction has set in against the Government plunging into new schemes, because as yet no limits have been set to the subsidies that will be demanded. Moreover, it has not seemed possible to set any limits when once the original limit approved by President Roosevelt, that of a ten-year exemption from interest, has been exceeded.

State Co-Operation.—In their perplexity as to Federal duties and subsidies some of the members of Congress have taken refuge in the now fairly

* Before Committee on Irrigation of Arid Lands, H. R., 67th Cong., S. 4187, January 24–25, 1923, p. 24.

well established "fifty-fifty" rule. They frankly admit that they see no defense against the assaults of the subsidy seekers. In their despair they call for the help of the States. If the people of any State, its Legislature, and Governor, will go "fifty-fifty" with Uncle Sam, as in the case of hard roads, or will guarantee to do their part in any reclamation enterprise, then the Government can afford to go into it. As a matter of fact, however, privately they admit that there is little probability of any State doing this because the local people are too well informed to be willing to venture their hard-earned money on a Government reclamation scheme. They are perfectly willing that generous Uncle Sam should spend his money on their State but they are not venturesome enough to risk spending the money taken directly from the taxpayers of their own localities. Already they cry that the State and local taxation is too high and they cannot see why they should contribute money for the benefit of small groups of land owners, even through entering into a partnership with the Federal Government.

While engineers and economists may not go so far as to fully agree with the conclusions stated* by the U. S. Department of Agriculture, namely, that "there is no justification for a National subsidy for land reclamation", yet as citizens to whom the public may turn for advice, they should consider carefully and impartially the limits which may be set to such a subsidy, balancing the gains to the public by the development and use of waste lands against the cost or increased taxes to the public. The Federal Government, as before stated, is definitely committed to the general principle of subsidies given directly or indirectly to certain public needs, such as post offices, the building of post roads and other highways, the maintenance of navigation, and commerce between the States, as well as the building of levees in connection with such commerce, the main value of which, however, is in the protection of certain agricultural lands. It may well be argued that the providing of opportunities for homes on lands which are now waste and useless can and should be a matter of National consideration. All are agreed that plans should be prepared and funds made available within proper limits for reclaiming these lands as rapidly as there may be an actual need for them in providing prosperous homes for citizens.

The question of greatest concern is not whether a subsidy or bonus to the land owner should be provided by the taxpayers of the country but whether this particular way of subsidizing a small group of land owners is the best or will produce the largest benefits to the great body of taxpayers. Reclamation works, unlike waterways or highways, are not State-wide nor susceptible of use by the public in general. They are strictly local, and the direct benefit from them must necessarily go to a few land owners and local industries.

Because of such conditions, it is urged that local or State co-operation and support should be forthcoming. If the Government is to spend, say, another $150 000 000 in reclamation works to make available in the future more homes for citizens, it should be considered in the broadest possible way whether this $150 000 000, or more, might not otherwise be used to produce larger results in home-making and benefit to all the taxpayers. Is it

* *Bulletin 1257*, U. S. Dept. of Agriculture.

not a fact that lands can be reclaimed or made available for home makers throughout the United States at less cost and can be occupied and improved with less hardship and uncertainty than in the case of the new irrigation enterprises? Not only is this a fair question, but it is one which it is the duty of every broad-minded citizen to consider.

The history of the projects pointed out by Dr. Widtsoe is full of instruction and warning against continuing as at present. It illustrates among other things that although the Federal Government is a good spender, it is about the worst possible collector. In fact, it is generally recognized that Uncle Sam cannot get back from citizen debtors any very large part of the money owed to him, whether this is for public lands or for seed grain. In discussing plans for Federal farm loans or other advances, it is asserted that there would be little hope of getting back all the money advanced for loans if the public was not well informed of the fact that the money thus loaned did not belong to the Government but, in turn, was borrowed from people who must be repaid.

The plan which past experience shows may be most likely to succeed is that which utilizes the experience of the Federal Farm Loan Banks and similar institutions. Assuming that the Federal Government should subsidize the reclamation of waste lands, this subsidy may well be confined to the first ten years after settlement and be limited to loss of interest on the investment. At the end of ten years, when real property values have increased, the owners of the lands that have been reclaimed and cultivated during that time should then be required to give to the Government what may be considered a negotiable mortgage, for the repayment of the cost of reclamation, with a small interest charge—no more than that paid by the Government for the money it is borrowing—plus, say, a 1% installment on the principal invested.

The best way to carry out such a plan will doubtless be to have incorporated under State law in each locality a Reclamation District with power to make contracts and to levy taxes. The bonds of this District may be received by the Federal Government in full satisfaction of all expenditures; at a suitable time these bonds may be put on the market and sold for what they will bring, thus closing up the entire transaction as far as the Government is concerned and relieving Congress from the annually recurring demands.

As a compensation for the benefits to the whole country, the Government may well assume the loss of ten years' interest or more on its investment and also the discount in selling the bonds. Careful consideration of such losses will show that they will be far less than under the present system. History shows also that the dealings between the District and its bondholders are likely to be conducted on fairer and more business-like terms than between a Government bureau, the local voters, and their political representatives.

AMERICAN SOCIETY OF CIVIL ENGINEERS
INSTITUTED 1852

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Paper No. 1608

THE FINANCING OF IRRIGATION DEVELOPMENTS BY PRIVATE CAPITAL*

By R. E. Shepherd,† Esq.

With Discussion by Messrs. John E. Field, Elwood Mead, and F. H. Newell.

Synopsis

The substance and the conclusions of this paper are largely the result of experience in solving the problems of the Twin Falls (Idaho) North Side Project. This is a privately financed enterprise. The failure of the original promoters forced the property on the bondholders, for whom the writer has been engaged since 1914.

The Twin Falls North Side Project is one of the large projects of the West, covering 185 000 acres of irrigated land. In the course of this work nearly, if not quite all, the problems peculiar to irrigation have been encountered. Among them has been the question of a dependable water supply for the abnormal year.

The question of an adequate water supply not only affected the "North Side", but to a more or less degree most of the projects in the Snake River Valley of Idaho. The solution lay in building a large storage reservoir at American Falls on Snake River. This was a joint enterprise, participated in by many, including the U. S. Reclamation Service. To secure the required funds, the North Side and other near-by projects, covering about 450 000 acres of irrigated land, organized themselves into a quasi-municipal district, known as the American Falls Reservoir District. This District entered into a contract with the United States for the building of the reservoir, and for about 300 000 acre-ft. of storage capacity therein. It issued its bonds for

* Presented at the Summer Meeting, Salt Lake City, Utah, July 8, 1925.
† Mgr., Twin Falls North Side Land & Water Co., Jerome, Idaho.

about $2 000 000 to cover the contract and paid the money to the Government. This arrangement has proved very satisfactory.

It will be noted that the lands were already under irrigation and that no new canal construction was required. The whole object was to remedy a defective water supply. For this reason the bonds of the District—6%—sold readily at par and a small premium, the country served being already generally settled and productive.

Many other problems have presented themselves for solution in the course of the speaker's experience on the North Side; these, and his observations of other projects, have led to the conclusions contained in this paper.

Dependence on agriculture is recognized, as well as the fact that growth in population requires a constant increase in production of food and those raw materials which must come from the soil.

Practically all the public domain lying in the humid area suitable for agricultural use in its natural state has passed into private ownership and is now to a large extent devoted to farming. It is true that much of it is capable of increased production, the necessity of which has for some time engaged public attention. There are limits fixed by cost which for the present define the extent to which the ever-growing need of the country may be supplied from this source.

It has been found that the arid lands of many parts of the West possess wonderful fertility, and where irrigation was practicable large areas have been devoted to agricultural production. It would be interesting to inquire into the results already secured, the wealth produced, and the influence on the Nation as a result of the present reclamation of the arid lands of the West. It supports large systems of railroads, makes a market for manufactured products, supplies much of certain raw material, and contributes a large part of the food supply of the United States. Therefore, it may be taken for granted that the reclamation of arid lands is something to be desired, and that it should be done at such times and in such areas as will fit in with the changing needs of the country.

Recognizing all this, engineers are here concerned with the question of how shall irrigation development be financed. Shall it be by private capital or out of public funds? To what extent is it prudent for the Nation to embark in any business enterprise? Should the Government participate in, and cooperate with, private enterprise and capital in the work of land reclamation?

Where the land to be reclaimed is in private ownership, and where there is a near-by source of water supply and the problems of storage reservoirs and interstate streams are not involved, clearly such work should be left to the enterprise of the owners of such lands. Practically all the earlier reclamation work related to such conditions was undertaken by those directly interested and was financed by private capital. The growth of such work led to the enactment of laws by most of the States, providing for the creation of irrigation districts of a quasi-municipal character, for the construction, operation, and financing of irrigation systems to serve the lands within such districts. In this manner provision was made for the reclamation of larger bodies of

land in diverse ownership, but having a common need of irrigation and capable of being served by one irrigation system and from the same source of water supply.

Under this system, which has been gradually perfected by legal enactments as time and experience have shown the necessity, most of the work of reclamation under private enterprise and capital has been accomplished. The results under this system have not been uniformly successful, and much loss of capital and effort has resulted from lack of proper knowledge of the real merits of the project, or its true cost. The losses under this system, however, were not inherent in or peculiar to the system, but were due to the same preventable blunders and ignorance which produce failures in all misconceived enterprises.

Can private capital be reasonably assured of safety in any irrigation enterprise? There are certain questions that must be satisfactorily answered before any project should be undertaken. It seems strange to one looking back over the failures in this kind of work to find how little attention was paid or study given to the work undertaken, a most casual investigation of which often would have prevented inevitable loss, both to the investor and to the trusting farmer.

The order in which these questions will be presented does not signify their relative importance, as the writer believes that if any project will not at the time stand up under all of them it should be let alone until such time as it will. Some of these questions go to the very vitals of an irrigation project, whereas others relate to the time of the undertaking. Changed conditions may make desirable in the future that which would now be unprofitable. Satisfactory answers should be required and proof offered as to each of the following questions. If doubt exists as to any one, time should be taken to clear it up. Practically all the problems, formerly more or less hidden, have come to light in the disasters of existing irrigation projects, so that knowledge and information, once guessed at, are now available.

1.—Is the land to be reclaimed sufficiently fertile and its soil structure such as to produce valuable crops for an indefinite period without resort to excessive cost for fertilization?

2.—Is the surface of the land such as to permit of its irrigation without too great expense, having due regard to the class of major crops for which it is adapted?

3.—Can the land be readily drained at reasonable cost, so as to prevent its becoming swampy or alkalied, after repeated irrigation?

4.—Is there an adequate supply of water available within a reasonable distance sufficient at all times to supply all the land in the project with the quantity required to produce profitable crops?

5.—Can this water be diverted for this purpose through a canal system that can not only be built within reasonable cost, but can thereafter be operated and maintained without excessive expense?

6.—Is the project so located with reference to transportation and markets as to offer an incentive for its farm development?

7.—Is there a present market demand for the products for which such land is naturally adapted, that under ordinary conditions would make the use of such land profitable to a farmer of ordinary ability and means?

8.—Will the entire cost of the work, including the time required to secure settlement on the land, plus a reasonable profit, when spread over the entire area, be such that the acre cost to the farmer, including the original cost of the land and its improvement by him, compare favorably with the cost of a farm in the humid sections, having due regard to character of crops, yield, and cost of production?

9.—Is the project so financed to the farmer that it will attract the man of limited means to locate thereon with assurance that he can succeed and meet his obligations promptly?

There are other questions that may enter into each particular project not common to all, but satisfactory answers to each of the foregoing should be made before venturing on the enterprise, and when these are had, then capital may safely undertake such development.

The writer wishes particularly to dwell on a few of the fundamentals. The question of water supply is one of the most common errors and has caused more loss than all the others combined. The discharge of a given water-shed is not uniform. There are years of high stream flow and years of low; many factors enter into it. The information on a particular stream may be unreliable. Then, again, there may be prior rights on the stream not fully understood or fully developed, which may later reduce the supposed right of the project. Too great care cannot be exercised in obtaining accurate information on this subject. Many old projects, which were assumed to have good water rights, are now finding it necessary to build storage reservoirs in order that flood waters may be accumulated for later use.

Then, again, experience has shown that no man can with exactness foretell what the transmission losses in a canal system will be. So much of the water diverted from the stream may be lost in transmission, that the quantity reaching the farm may prove inadequate, requiring the area of the project to be reduced with the attending loss of investment. Peculiarities are likely to develop in the canal system, which could not be reasonably anticipated, limiting the farm deliveries. The question of farm duty of water cannot well be accurately predetermined.

On the other hand, it is known that as a farm is developed and reaches a stage of good cultivation, as the humus content in the soil increases by vegetation, as the farmer himself acquires greater knowledge in the use of water on his land (which can only come with time), and as a greater diversification of crops is grown on the project, less water is required to overcome losses.

Therefore, the writer has long since reached the conclusion that in some respects an irrigation project is more or less a case of evolution. It should be undertaken in units wherever possible, having always in mind that successful farming is the end and aim of irrigation development. No irrigation system should, or can, carry its maximum load during its early years; it will grow,

but the exact limit of its growth cannot be known at the start. It, therefore, is good judgment to finance irrigation development with a due regard for these inherent characteristics.

Another class of irrigation projects and their relation to private capital and enterprise, and some of their problems will now be discussed. In most if not all the Western States, there are large bodies of land suitable for farms, but involving the use of so much capital and the solving of so many new problems that private capital and enterprise hesitate to undertake their reclamation. Without taking the time to review the history leading up to the passage by Congress of the Reclamation Act, the Carey Act, and the Warren Act, a brief statement of the reclamation work undertaken in consequence of these acts and some of the results may prove helpful in the present discussion.

Under the Reclamation Act the Government undertook, through a bureau created for the purpose and now known as the Bureau of Reclamation, to construct irrigation works for the reclamation of certain parts of the public domain in the arid West. The funds for this purpose came from certain revenues of the General Land Office and have since been increased from other sources. The details were left largely to the discretion of the Secretary of the Interior. The reclaimed lands were made subject to entry, conditioned that the entryman should assume and pay the cost of construction, to be thereafter ascertained and published by the Secretary of the Interior, as well as the annual cost of operation and maintenance. Wide publicity was given to this new field of reclamation work. Each State was active in securing projects, and a general invitation was extended to take up farms thereon. The applicants were many, and a system of drawings not unlike a lottery was provided.

The first noticeable result was that a large part of the lands were drawn by those not expecting to make a so-called farm home. Then came the realignment. As the construction cost was to be paid in small installments, without interest, commencing after the cost had been ascertained and announced, and as there was a general expectation that this cost would be much less than that of a private enterprise, entrymen were able to, and many did, sell at good profit to those unlucky at the drawing. So the unearned increment was guessed at and quickly capitalized, and the final holder of the land got it at about the going price of other lands, and subject to, in many instances, an unascertained construction cost.

All this, however, is aside from the real work of reclamation undertaken by the new Bureau, and much of the criticism growing out of it should rest on other shoulders. Great care was exercised in the study of each project, and in the water supply therefor. The work in most, if not all, these projects was well done and a credit to those in charge. However, the cost frequently exceeded that expected by the entryman, or subsequent purchaser, and is now the subject of much controversy. The usual consequences of land speculation induced by the World War are now in evidence; whether this could have been prevented does not enter into the question being considered. It will be noted, in passing, that if the Government does finally absorb some of these losses,

it must do so for other reasons than that the lands and irrigation works acquired from the Government were not worth the charge.

The real question is, "Was the work worth what it has cost?" There may be some projects where it is not. If, so, the writer is not familiar with them. It should cause no surprise if in a work so extensive, covering such great diversity of conditions, there have been some errors of judgment.

At about the time of the Reclamation Act, another known as the Carey Act was passed, having the same general object in view, but under which the work was to be undertaken by State supervision. By this Act any State having arid land of the public domain within its borders capable of irrigation, might have a limited part of it segregated to the State, to be thereafter conveyed by Government patent to the State on a showing that irrigation works had been completed and a water supply obtained for its reclamation. The State, in turn, was to enact suitable legislation to secure this desired result. As part of this program the State was authorized to enter into contracts with such construction companies as might desire to engage in the work of building irrigation systems for the reclamation of certain designated areas thus segregated by the Secretary of the Interior for the purpose. These lands in turn were to be open to entry, conditioned that the entryman must first have purchased a water right for use on his entry from the construction company. The water right consisted of a certain proportionate part of the canal system and water secured for the project. The State, by its contract with the construction company, fixed in advance the price per acre to be paid by the entryman for such water right and the terms of payment. The so-called water-right contracts were made a lien upon the land they served, and were by the State contract made assignable to a trustee, as security for an issue of bonds out of the sale of which the funds were to come with which to construct the works. The essential difference, it will be noted, as far as the entryman was concerned, being that under the Carey Act the price was fixed, whereas under the Reclamation Act it was indefinite and left for determination after entry. The result was that many Carey Act projects were started, particularly in Idaho. No end of trouble ensued, as may well be imagined. It was something new. Many contracts were loosely drawn, and endless litigation finally resulted.

At the start the State used every endeavor to induce capital to embark in these enterprises. Engineers readily pointed out the great opportunities. Certain bond houses saw a chance at profitable financing, and soon, without the careful study which has been pointed out, numerous projects were started and people rushed to them as to a new gold field. These Carey Act projects have been more or less of a disappointment and loss to many.

The root trouble in most of these projects grew largely out of two causes: First, under-estimating the construction cost, which fixed in advance the sale price of water rights; and, second, over-estimating the available water supply. These companies were held to their contracts on the question of price and the Courts required the areas to be reduced to fit the water supply, where it could not be improved.

This was the first big venture of the general public in irrigation investment. It proves nothing for or against the broad question of the legitimate development of arid land by irrigation, but it does lead to the inevitable conclusion that left entirely to the direction of either State or Nation it has not so far been a financial success.

Growing out of the experience of the work of the Reclamation Bureau and the Carey Act companies, and the fact that they were often both looking to the same source of water supply, came the Warren Act, which permitted of joint undertaking in building reservoirs. This Act was followed by others encouraging co-operation between all irrigation districts and companies with the U. S. Bureau of Reclamation in the work of conserving the waters of the great water-sheds, and in the correction of many of the original mistakes. A notable example of such co-operation may be found in the building of the American Falls Reservoir on Snake River. The past mistakes are gradually being corrected and thousands of acres of fertile land are now being successfully farmed as the result.

Although there are some areas of land in private ownership yet to be reclaimed, that may be undertaken safely by private capital, the largest area is that of the public domain. Therefore, any effort to reclaim these lands must in some way be connected with the Federal Government. It, therefore, remains to point the way whereby this may be successfully done, and these great waste places be made a source of wealth to the people of this Nation and help supply their need as the time may come therefor. This question has engaged the attention of Congress for many years and numerous measures have been introduced, but as yet no one measure has met with general approval.

In view of all this, it may appear presumptuous to offer a suggestion, but the writer will take the risk. The Federal Reserve Banks, the Federal Land Banks, and the Joint Stock Land Banks are all examples of business enterprises fostered by the Federal Government, and they have functioned well. Can a way not be found in them to meet the reclamation problem? In them a certain public service is performed under private direction and by private capital, supported as necessary by the Federal Government.

Why should Congress not create a corporation, the capital stock, at least in part, to be contributed by the Government, and its general powers, rights, and duties to be fixed by the Act? Its board of directors or managers, or a part of them, could be appointed by the President. Such corporation should be independent of any bureau, but connected with the Department of the Interior or Department of Agriculture in much the same way as the Reserve Banks are connected with the Treasury. The business of such corporation should be the reclamation of the waste lands of the country, either those of the public domain or State, or in private ownership, and whether by irrigation, drainage, or other means, the work should be so ordered and carried on from time to time as the demands on the agricultural resources may require, and not to the prejudice of the present farmed areas, by unduly increasing the lands available for cultivation, and under such terms and conditions as to assure a reasonable return on the undertaking. The lands should be disposed of in such a manner as to pass directly to actual farmers, on such

terms as can be easily met by the proper use of the land. It would necessarily follow that no work should be undertaken where these conditions could not be reasonably met. Such corporation should have the power to issue its own bonds which should enjoy the same privileges and relation to the Federal Government as the bonds of Federal Land Banks, and should take over, complete, and liquidate the existing Government projects.

The advantages of such a corporation would be to take the reclamation problem out of politics. It would assure the orderly working out of these problems as the growth of the Nation demanded, and not to the harm of the agricultural population. It would secure the advantages of private skill and enterprise. It would lead to a continuing policy, improved and developed with time and experience, regardless of changing administrations, yet it would enjoy Government support and restraint in case of need. It would work great economy over the present method. It would continue to function as long as the opportunity existed to add to, or improve, the areas capable of agricultural production, and would carry on the work when and as the needs demanded.

The foregoing suggestion is presented, without attempting to go into details, in the hope that it may set in motion some satisfactory solution of this problem. The conclusion is, therefore, that for the present private capital can only venture in assisting those projects in private ownership which come under the general rules previously stated; also in the refinancing of proved going projects, or in financing their improvement and further development. Should Congress see fit to provide for a corporation such as that suggested, or something better, a way may be opened for the further safe utilization of private capital.

Let us profit by past experience; it has been an expensive teacher, but out of it has come the knowledge of the weakness of the methods employed to put the desert to profitable use. Just as surely as the day will come when these lands will be put to beneficial use will engineers be able to outline a safe and sane, orderly, and timely way to do it. Until this is done, except to the limited extent noted, irrigationists are not in a position to invite private capital to invest in this great work.

DISCUSSION

JOHN E. FIELD,* M. AM. SOC. C. E. (by letter).—The author has selected for his subject the most vital and all-controlling problem of irrigation development—that of financing. To finance a project necessitates that it be sound in every respect, regardless of the method or of the source from which the funds must come. To be sound financially requires that the interest and principal be paid regularly and surely and that, at the allotted time of maturity of the obligation, the principal be paid.

The author has suggested many difficulties which for the benefit of those who have had a limited experience might well have been enlarged on and further details given.

In conversation with several friends, financiers and attorneys with experience in irrigation development, when the writer suggested financing by private capital, the concensus of opinion was suggestive of the backwoodsman who, seeing a dromedary for the first time, studied it carefully and on finally moving away remarked, "They ain't no such animal." Just as the backwoodsman was wrong, so were these financiers wrong. The dromedary is particularly fitted to arid regions, he requires a modicum of water and an attendant who understands his moods, his limitations, requirements, and possibilities. The same applies with equal force to financing an irrigation project, and particularly financing by private capital.

In reviewing the paper the writer was struck first by the statement, in connection with the desirable reclamation of arid lands wherever possible, that "it should be done at such times and in such areas as will fit in with the changing needs of the country."

That there is now under cultivation in the United States a sufficient area to meet all present needs is demonstrated by the presence of a surplus of nearly every agricultural product, even in years of low production, by the prices obtained by the producer, and by the marked drift from rural to urban life. No "back to the farm" propaganda—no "stay on the farm" slogan—will avail so long as the farm is unprofitable, or so long as there is the present marked difference in comforts, luxuries, hours of labor, and net earnings between the farmer and the mechanic. When a man with $12.50 can buy an automobile, buy a home "just like rent", and clothe himself and wife in silk on the installment plan, the acquisition of a farm on the amortization plan will not appeal to the young farmer unless he sees, in addition, a definite promise of profit and financial independence within a reasonable time.

The Bureau of Economics of the U. S. Department of Agriculture states that there is now under improvement in the United States an area which, if properly cultivated, would meet the needs of the population, with due regard to the probable increase, for the next fifty years. Whether or not this estimate is correct, it is certain that there is no present or crying need for more land. Twenty years ago there was a slogan, "Land for the landless man and men for the manless land". Now this slogan must be discarded in both its phases. To-day there is much land in the arid West under canal, with good water

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supply, markets, railroads, and other necessities crying for men. Farms are being offered at less than the cost of the irrigation works. It seems, therefore, that to discuss financing the development of new irrigation projects is a waste of time, or at best an academic question. Not so, however, with the financing of existing projects, of finishing and of perfecting existing systems. Irrigation engineers of the older generation take something of a morbid pleasure in analyzing their failures and mistakes.

There is a time for all things, and the author in limiting development to the changing needs of the country has struck the keynote. He asks to what extent is it prudent to embark with private enterprise and capital in the work of land reclamation. In answer to that question the writer would remind the older men and inform the younger ones that back in the early Nineties the idea fostered by the Irrigation Congresses was that the Government should build reservoirs to supplement the water supply of existing systems, that this and this only was to be the program. How far has been the departure from that idea and what troubles and failures that departure has wrought are too well known to require recital here. Next came the suggestion that main canals covering public lands should be built. This development was probably wise, but still no hint, even at the time of the passage of the Reclamation Act, had been given that laterals would be built, much less that the systems would be operated and maintained by the Reclamation Service, nor was there any suggestion that the Service would build drainage systems.

As the Project Engineer of the North Platte Project the writer opposed the building, first of any laterals, then of any except those involving great difficulty and expense, and never as long as he remained in the Service did he countenance the idea of Government maintenance and operation. Referring specifically to that project, one can easily imagine the limiting of Government activity to the construction of the Pathfinder Reservoir.

Private capital was ready and anxious to build the canals, and indeed it was necessary to take measures to prevent private enterprise from building both the Interstate and Fort Laramie Canals. The Tri-State Canal, covering much of the land under the Interstate Canal, was actually built by private capital, and now irrigates land originally embraced in the area intended to be served by the Government Interstate Canal. The first mistake was made in extending the Government work to include the lateral systems, but the great and serious mistake was made when the Service undertook the work of maintenance and operation.

The story goes that two travelers down in Arkansas, driving along a road through the woods, were startled by seeing a wild-eyed woman with streaming hair break out of the underbrush into the road and then, like a startled deer, plunge into the opposite thicket. Hard on her heels came a man with scraggly beard and disreputable appearance. He inquired of the strangers if they had seen a woman cross the road. They denied that they had and asked what he was doing, and he replied, "She's my mammy and she's trying to wean me." The way the settlers are acting and the way the Reclamation Service has treated them in the past reminds one of that story. Mr. Mead and Secretary Work evidently have concluded it is weaning time.

Answering the question specifically, the co-operation of the Government should be limited either to the construction of reservoirs for supplemental supply, or if it is deemed best that this Government become paternalistic, socialistic, or communistic, then let it go ahead and do everything of every kind and nature. Personally, the writer much prefers that American initiative and ingenuity be preserved and developed, even if some hardship is entailed. The lack of responsibilities imposed, the absence of sure and certain penalties for shiftlessness and procrastination, is responsible for much of the trouble on irrigation projects. Most of the trouble is peculiar, not to Government projects, nor to irrigation projects, nor to farming in the arid West, but to the condition of farming itself.

The author has listed nine questions all of which must be answered in the affirmative if an irrigation project can be pronounced sound. The first three deal with the land; the fourth and fifth with the water supply; the sixth and seventh with markets; and the eighth and ninth with the cost of the project and revenue from the land. All irrigation engineers worthy of the name have in the past thirty years considered carefully in their reports the first seven and many have considered the last two. Unfortunately, some engineers have considered it outside their province to pass on the financial aspects of a project, leaving that matter to the promoter, the financier, and the colonization agent. Some have even limited their activities to the cost of construction, avoiding the vital matter of character of soil, climate, products, water supply, and markets to some one else. In this, the engineer failed to grasp an opportunity to do much useful, valuable constructive work, and to rise above the level of a surveyor, and of a "slip-stick artist". In consequence, many of the failures are chargeable to the engineer.

With the nine conditions imposed by the author the writer is in accord, but he would enlarge the list. The value of a farm is divided among the land, the water, the improvements, and the expense of making it a going concern. These four elements on the public land with ordinary costs would be: Land, $1.25; water, $35; improvements on 80 acres, $4 000, or $50 per acre; and five years to make it a going concern which might be put at $50 per acre; total, $136.25. There must be added to this figure some bonus or unearned increment, in order to induce the farmer to break off his old association and to suffer the hardships of the pioneer and settler; he must believe that within a reasonable time he will be considerably better off by making the change. It would seem that at the end of 10 years his 80 acres should be worth $200 per acre. However, as the author states, "the unearned increment was guessed at and quickly capitalized." The final holder paid the following:

To the original entryman	$63.25
Improvements	50.00
	$113.25
Interest on $113.25 for 5 years at 6%	33.97
Going concern	50.00
Water	35.00
Total	$232.22

This is $32.22 more than the land fully developed was worth. At first, a profit is taken by the holder of the land when he sells, but later a loss is taken, of necessity, by the settler and the bondholder. These figures indicate that the cost of the water should not greatly exceed $35 for land worth $200 when fully developed.

The history of an irrigation district bond is interesting and informative—taken by the broker or contractor for 60% of its par value, sold by the broker at par and the profit divided between the broker and the contractor. Interest is paid for a few years from the proceeds of the sale of bonds, and then the broker is no longer interested. The interest becomes delinquent due to lack of settlement and the fact that the settler paid out all his available resources in the purchase of a relinquishment for land, and for improvements, if any, and in living expenses. The bonds drop to 60% of par. A new series of bonds is issued and exchanged for the old at 60 per cent. Thus, the project becomes a success, and the settler gets his water at cost. Often the exchange of old bonds for new ones at less than 60% of par is effected and the process of refinancing is often repeated until the indebtedness is reduced to a point which the settler can pay. The Government is in effect a bondholder. Will it, too, take a loss just as all others have done before?

So much for the past; now for the future. The choice is between Socialism and Government aid or private financing and independence. Assume that the nation is not yet ready for Socialism, then how shall private financing be accomplished for the purpose of completing present projects and for building those which may be found feasible as time goes on? The author proposes a form of Government corporation similar to the Federal Land Banks, to finance the reclamation of waste lands, whether privately owned, State owned, or in the public domain. His suggestion is a good one.

There being no need, however, of adding to the total farm area, the business of such a corporation should be limited to taking over, completing, and liquidating existing Government projects. To give it greater scope would be to perpetuate the scramble for public money, to inject politics into its operation, and to subject the corporation to importunities and influences, as was the Reclamation Service. This would lead to similar unwise, unnecessary undertakings. If its activities were so limited and if it proved a success, then, with the experience gained and with the possible development, say, in 25 or 30 years, of a healthy demand for more farms and for the development of areas and projects impossible to private enterprise, a new corporation could be created; but the salvaging of existing and the undertaking of new projects should be separate and distinct. It is to be anticipated that new work would be so much more attractive that the salvaging of the old projects would be neglected and become secondary.

Having been the salvaging engineer on many projects in the last fifteen years, the writer knows how hard that work is, how unsatisfactory, how devoid of glory, and how poor the remuneration. As compared with the enthusiasm, the publicity, and the advertising on new projects, such work would be like the "cold gray dawn of the morning after." To improve and complete projects where the land is principally in private ownership the landholders should give

individual mortgages on their land and raise the necessary funds. If the land is already encumbered either by mortgage or irrigation district indebtedness, the creditors must join the owner of the fee title in the new mortgage. There is no reason for relieving at public expense either the land owner or the mortgagee of the results of his mistakes. Government aid should be confined to:

First.—The gathering of data and research work necessary to a proper analysis of proposed irrigation projects.

Second.—The construction of reservoirs for supplemental water supply for existing projects.

Third.—The completion of projects already under way on which the future expenditure is less than the value of the enterprise, and which, when completed, can be bonded and the bonds sold at par and at not more than 6% interest, thus definitely getting rid of Government control and imposing the burden of responsibility where it belongs, on those most vitally interested—the water users.

The Government evidently hopes to induce the States to take over its projects. Such a step would simply be "jumping from the frying pan into the fire." When a change is made it should be from the Government to the water users' own organization. Corporation control and management is bad; State control and management would be disastrous. Government control has had a trial over two decades and is in a mess, to say the least, and, as far as successful financing is concerned, could not well be worse.

The engineer should pass on the nine conditions mentioned by the author and, in addition, show the cost of improvements, going concern, value, and add an estimated bonus or unearned increment; if colonization is necessary add 25% for that item. If these items aggregate less than the value of land already developed, then the project could go ahead. However, if expense for colonization is necessary, the greatest care should be taken to verify all other costs. Quick colonization and development are essential to success. More good projects have failed through lack of proper and quick colonization than from all other causes. A project simply cannot stand years of accumulating interest charges and operation and maintenance costs.

The writer's recommendation to those contemplating irrigation construction at this time is like that of Punch to those contemplating matrimony, namely, "don't"; but, like Punch, he does not expect to have his advice taken, nor would he wish it to be unless his own money were involved, for on the dreams of the optimist and his courage is progress founded. The optimist and the promoter and those of whom it is said there is one born every minute, will keep development from ten to twenty years ahead of its time. It can no more be stopped than can matrimony on the part of those whose heart is in it. It is possible, however, to "give them a run for their money," and to see to it that the projects are sound, although possibly premature.

The criticism of all proposed developments and the methods suggested is that a false stimulant is almost invariably proposed in the form of money without interest, long-time payments, houses furnished, land plowed and

planted, livestock sold on credit, in fact, a proposition which any "ne'er do well" will accept, the shiftless will try, and the incompetent will undertake. It simply makes more difficult the ultimate and necessary weeding out of the undesirables; it puts the sincere, earnest, and competent in competition with the notionate itinerant, the man who has failed in everything else, who is willing to try something where there is nothing to lose and everything to gain, especially when there is a prospect of selling out and moving on, leaving extra burdens on his successor, the burdens of increased cost, of discredited effort, and of disillusion and disappointment to overcome.

If this false stimulant is necessary, then the project is dangerous. If there is no demand—no insistent demand—for land and farm and home, then the time is not opportune and the project should wait. In these circumstances, any project that cannot secure private capital at a reasonable rate of interest is unsound financially. Non-taxable municipal securities are bringing 3½ to 4% interest; the tax-free bonds of an irrigation district will find ready market if the interest payments are secure and certain. Let the saleability of the bonds be the final test, and let the policy be not to sell bonds below par, nor to give bonds for construction, not to employ a colonization agent, and to offer only reasonable inducements; then it will be certain that no unwise promotion will be attempted.

Furthermore, let each individual land owner assume a definite obligation and a personal risk and responsibility; let him know that the debt to the last farthing must be forthcoming; let him join the enterprise calmly, soberly, and in the fear of failure; let him know and feel that success depends on his personal effort and industry. Then responsibility will develop his judgment and his ability to meet and overcome difficulties. Those projects already started should be classified and those which require an expenditure greater than their value should be definitely abandoned until they can pass the acid test here proposed.

Westerners wish the West to grow and to prosper, they are impatient, they are selfish and care little whether these developments hurt other sections, they believe in the "survival of the fittest," in the immutable laws of Nature, of finance, and of political economy, and they believe in every project standing on its own merits. If they cannot grow as fast as they wish without a stimulant then they are willing to wait, preferring to grow slowly, surely, and safely rather than by alternate booms and depressions. They want no settlers who have to be bribed to come and live among them.

Let Westerners pray for patience and work for sound progress; pray as if there was no help on earth or from Washington; work as if there was no help from Heaven. Then they will return in the spirit to that time when they fought their way against desert and drouth and redskin and remoteness, fought with hope in ultimate victory and confidence in their own efforts and capabilities, when Uncle Sam was more of a myth than a reality, or rather when they themselves were a helpful part of Uncle Sam in all his majesty, power, and glory.

ELWOOD MEAD,* M. AM. SOC. C. E. (by letter).—This is an interesting discussion of a feature of reclamation about which there is marked difference of opinion. The author lays down nine conditions which must be met to enable an irrigation project to be financed by private capital. Condition No. 8 requires the entire cost of the work, with a reasonable profit, to be paid and still have conditions compare favorably with the cost of a farm in humid sections. Condition No. 9 states the project must be so financed that it will attract the man of limited means to locate thereon.

Both these conditions must be largely decided by the judgment or bias of those in charge of the enterprise. No mathematical rules can be laid down. No doubt the promoters of nine out of ten of the private projects that have failed believed they would succeed; but their judgment was faulty. The paper is not clear as to how such mistakes are to be averted in the future.

What this irrigation planning lacks in the United States is attention to the costs which must be incurred in changing the raw land under an irrigation project into improved productive farms. Under the cheaper projects of twenty-five years ago this was not necessary, although it would have been valuable. When, however, construction costs of canals are more than $75 per acre, as practically all canals of any size are at present, and an equal or greater sum has to be spent leveling the land and putting up the necessary buildings and fences, the total investment runs so high that the settler of small means fails before he gets started. The future success of private and Government irrigation depends on the measures adopted to overcome this obstacle.

The plan proposed by Mr. Shepherd is a corporation, created under an Act of Congress, part of the members of the board of directors being appointed by the President, with the Government subscribing for part of the capital stock, this corporation to be connected with the Department of the Interior or the Department of Agriculture. This is not private enterprise because in the end the Government would be the sole moneyed member.

It is not clear how such a complex organization would have any advantage over the present Reclamation Bureau; it would have a much greater overhead expense, the risk of loss would be equally great, and the responsibility of management less because the private directors would be operating largely with public funds. The main advantage urged for such an organization is that it would be an independent board, free from political control. Speaking from some considerable experience with both public and private development, the writer is doubtful whether this freedom would be secured. The Tariff Commission has not taken the tariff out of politics.

The only way that reclamation can be taken out of politics is to carry it on without Government money and that is precisely what cannot be done under present conditions. There are now (1926) pending in Congress appeals from about twenty private enterprises (that is, enterprises started with private capital and as private undertakings) that are not completed, are financially embarrassed, and are appealing to the Government to be a financial life-saver.

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There are many more that are not paying interest on bonds. The reasons are not the mistakes in design or construction, but solely that the profits of agriculture have not been sufficient to enable farmers to make a living and pay irrigation charges, or the opportunities of the projects have not been sufficiently attractive to enable the land to be settled and brought under cultivation. It is the farmer who determines whether or not an irrigation enterprise is a financial or economic success.

This paper, therefore, does not touch on the fundamental weakness of all past development, public or private, namely, the failure to include in the plans for development a study of the economic questions connected with land settlement and the improvement and equipment of farms. Until there is a definite land settlement policy, and unless the question of how raw land is to be changed into farms is thought out as clearly as the size of canals and the design of engineering structures, there will be delayed development and uncertainty as to the financial outcome.

About one-half the Government reclamation projects are meeting their payments. No equal number of private enterprises in the United States make a better showing. They have no financial problems, but they do have an economic problem in the increase in tenancy and non-resident ownership of land. The other half of the reclamation projects are in arrears to the Government in their payments, but this is not due to bad design of works or, in many cases, to poor land. On one-half the projects that are in financial difficulties, the whole trouble is lack of settlement—there is good land, there is plenty of water, but this land fell in the first place into the hands of people unable to develop it, or who acquired it for speculative purposes and have held it for that purpose ever since.

The adjustment now under way has as its fundamental principle the closer settlement of land, the selection of settlers who will be first of all good farmers, and then the establishment of a type of agriculture suited to the markets and the local climatic conditions. These are as important for private enterprise as for Government projects, but it looks as if the Government were destined to be a leader in the introduction of the new principles and new methods needed to bring reclamation into accord with the economic conditions created by the World War.

F. H. Newell,* M. Am. Soc. C. E. (by letter).—The author goes to the foundation of the vexing problems encountered by engineers and economists in the financing of irrigation development by private capital. Collateral with this he gives an answer to a question of National concern, as to the relation the Federal Government should have to irrigation enterprises. He suggests that Congress create a corporation, with continuity of policy, to take over all the duties and responsibilities of irrigation now imposed on the Secretary of the Interior and to conduct future operations through a board of directors, with full authority to carry on the business with reference to the larger interests of the public as well as of the people concerned. He also suggests that:

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"Such corporation should have the power to issue its own bonds which should enjoy the same privileges and relation to the Federal Government as the bonds of Federal Land Banks, and should take over, complete, and liquidate the existing Government projects."

To any one who has been intimately concerned with the initiation, construction, and operation of the Federal enterprises, this suggestion has a peculiar appeal. It is in accord with the experience of the past not only in irrigation enterprises dependent on private and public funds but is in line with other transactions involving the efficient investment and recovery of large sums of money. The need for a corporation separate from purely governmental functions or political control to handle Federal reclamation funds with efficiency and economy arises from the fact that departmental machinery with its many restraints growing up from ancient checks and balances has been evolved with reference to enforcing laws, mainly restrictive. It does not readily permit nor stimulate the exercise of sound judgment to fit present-day exigencies. It reflects past rather than future needs.

This is exemplified by the striking losses in amounts due the Reclamation Fund from the land owners (not necessarily farmers) whose lands have been increased in value from practically nothing to sales prices of more than $100 per acre. The inefficiency and losses in making collections from these land owners are in striking contrast to the activities of the Farm Loan Board. This is due to the fact that this Board, virtually a Government-owned or controlled corporation, has been able to pursue a persistent policy, one which recognized that collections must be made and has not tried to exempt groups of land owners but has proceeded with its work, not as a side issue or incident to some other more important operations, but as a business by itself, one worthy of exclusive attention.

It is now practically admitted that if, in place of the transient personality of the Secretary of the Interior, a permanent board or small continuing group of experts (chosen for special ability and experience) had determined on the policy of requiring prompt payment from land owners of the funds due under the terms of the Reclamation Act, speculation in reclaimed lands would have been checked and the present unfortunate situation would never have developed. One of the best informed experts who has devoted years to the study of the situation on the ground asserts that the present situation would now be largely overcome if a business-like fiscal method of collection had been adopted and the persistent idea of repudiation of debts had been discouraged.

The greater part of irrigation development, with its benefits to individuals, communities, States, and less directly to the nation, as noted by Mr. Shepherd, has been made by private investment. Moreover, this condition is continuing; more land is being brought under cultivation by the use of private or corporate funds than can be developed by any conceivable appropriations of public funds. There is no doubt that as a whole the public funds were wisely and effectively expended in the earlier enterprises, and the public has gained thereby. The real reason public funds should not be more largely appropriated lies in the conditions recited by Mr. Shepherd; but these obstacles

to such further wise investment of public funds may be removed by a National reclamation corporation.

To appreciate the situation fully it is necessary to emphasize the peculiar condition which has resulted with reference to Federal investments in irrigation enterprises. This work, involving the expenditure of many millions of dollars, was entrusted by Congress to a single official. No provision was made for any organization, bureau, or group of advisers. Upon one man alone, a Cabinet Officer, usually a lawyer chosen for political acumen, was imposed this responsibility, to fill a position to which he could devote only such fragments of time as might be taken from innumerable other more obvious or pressing demands.

It is true that, by inference, the Secretary of the Interior, the official named in the law, could employ whom he pleased, when he pleased, at any compensation, and without hindrance from civil service or any other restriction, except the will of the President, or such laws as might later be enacted by Congress. Such unlimited opportunity and discretion has hardly ever been given to a single official. He held in his hand the future of large communities.

At the same time this single official was charged with a multitude of other and most exacting duties in connection with public lands, National parks, Indian affairs, patents, pensions, education, geology, hospitals, insane asylums, a list of which would require a page even to enumerate the titles and a volume to describe the duties. It is true that for most of these certain responsible assistants were provided, but in infinite detail he is practically the court of last resort and must give some personal attention to each and every item.

The condition would not be so bad if this official and his principal assistants were educated or trained for the work or remained in office long enough to understand what it was all about. For two or more decades Secretaries of the Interior have held office on an average only a little more than two years. With few exceptions they have not been selected with reference to the work in hand; traditionally, they must come from the West; above all things they must be effective party men. An enumeration of the names of those who have had personal charge of the Reclamation Fund—Hitchcock, Garfield, Ballinger, Fisher, Lane, Payne, Fall, etc.—is sufficient to indicate the wide variation in temperament and experience. With notable exceptions, these men were not skilled in economics or finance.

All these men, although differing widely, have agreed in one respect, namely, that immediately upon assumption of office each has taken a lively interest in irrigation, and has promptly reversed the policies of his predecessor so far, at least, as these pertain to finances. Each has entrusted the technical or engineering problems to the Reclamation Service, created by the Hon. James R. Garfield, but each has reserved to himself everything which had to do with finances and particularly the collection of the money, for the reason that, as administered by them, this involved many intricate political considerations.

With continually shifting personnel and policies regarding finance, it is indeed a wonder that so much money has been collected from the land owners

to whom water was sold on installment. For analogy, imagine what would happen to the income tax if the Secretary of the Treasury announced that he did not believe that any considerable number of the taxpayers would pay and that, as far as he was concerned, he would not oppress any man!

A volume might be written upon the psychology or politics involved in this experiment in finance; it is sufficient to point out that Mr. Shepherd sees the difficulty and offers the solution, namely, that of taking the responsibility out of the hands of temporarily appointed politicians. He would entrust the enforcement of the policies, determined in a broad way by Congress, to a corporation in such manner as to permit the exercise of a sound discretion and a continuity of policy. This can be made possible by entrusting the powers to a small number of directors, preferably appointed for long terms, which overlap in such a way as to insure this necessary continuity.

Another point brought out by Mr. Shepherd is that success in all these enterprises, whether financed by private or public funds, must be a matter of evolution, that is, of slow growth based on experience, year after year. The greatest losses of public funds have been due to the anxiety of each successive Secretary of the Interior to push the maximum of construction during his term of office.

The original idea embodied in the Roosevelt Reclamation Act of June 17, 1902, was to provide a small revolving fund, utilize this in building large works, reservoirs, and main-line canals, the magnitude of which were such that they could not then be built by private capital; then to permit a natural evolution, re-investing the money, as it slowly came in, on the basis of the experience already acquired.

Preliminary to this investment authorized in 1902 the most careful studies possible were made (beginning in 1888) of the water supply, the topographical conditions, and the related physical facts; the best experts available were obtained to consider soils and economic conditions. The unknown factors which could not be determined in advance were to be ascertained by the steady development of the projects for which these main structures were built.

Unfortunately, this gradual progress was not acceptable to the short-term politicians. The otherwise orderly sequence of events was interrupted; a loan of $20 000 000 was urged to expedite work. It is significant to note that none of this money was expended for five years after the loan was authorized. This was because of the fact that the then conditions of securing rights of way, of establishing water rights, or economical construction, did not justify this expenditure. The same motives which dictated the securing of this loan were closely akin to those of indefinite extension of time of repayment and practical repudiation. Had the rule been rigidly adopted of limiting expenditures to the amounts which might be obtained from past investments much, if not most, of the criticism now rampant might have been avoided.

This relatively reckless attitude contributed to the inflation of land prices and to the spirit of speculation in governmentally reclaimed lands, so extensively commented upon. An analysis shows that the excessive speculation in lands, irrigated by the use of non-interest bearing public funds, lay in the fact that these speculative values resulted not so much because of the pos-

sible productive values of the land, but were based on the assumption that the owners of these lands would continue "to get something for nothing" from the Government. That is, they would get not merely exemption from interest payment on the cost of reclamation for a period of 10, 20, or more, years, but it was asserted that ultimately a considerable part of the debt, if not all of it, might be repudiated.

This avowed policy of repudiation was a plank in the platform of many a politician running for office; under this stimulus excessive land prices were agreed upon and in some cases paid; namely, on the claim that the Government would never dispossess any land owner. It was asserted that liberal Uncle Sam, who could afford to build and throw on the scrap heap a $10 000 000 battleship, or put millions of dollars in diking swamp lands, or building hard roads, would never stoop to collect the little driblets from a few thousand owners of irrigated lands.

With this lesson fresh in mind and with the advice of experienced men like Mr. Shepherd it would seem as if the people of the country and Congress would see to it that these intricate financial matters were entrusted to a small body of men especially chosen and that they would bring into line with practice the methods which experience everywhere is showing to be necessary in the investment and recovery of large sums of money.

AMERICAN SOCIETY OF CIVIL ENGINEERS
INSTITUTED 1852

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Paper No. 1609

PRESENT POLICY OF THE UNITED STATES BUREAU OF RECLAMATION REGARDING LAND SETTLEMENT*

By Elwood Mead,† M. Am. Soc. C. E.

With Discussion by Messrs. B. A. Etcheverry, Thomas H. Means, George C. Kreutzer, and O. L. Waller.

Twenty-three years ago, Congress passed an act for the reclamation of arid lands by irrigation. Since that time, about $200 000 000 has been spent in the construction, operation, and maintenance of canals and reservoirs, and the building of drains. This money has come principally from the proceeds of sales of public lands, leases of oil land, and repayment of construction, operation, and maintenance costs by settlers. For all practical purposes it has added another State to the Union. Homes have been made available for 130 000 people. The farms created out of land once worthless are valued at $300 000 000 at least, and the towns and cities have had a great increase in taxable wealth. The crops produced in 1924 are valued at $66 500 000 and this annual value will be increased when advancing technique in agriculture and further subdivision of land have persuaded the reformed cowboy that irrigation on horseback is out of date.

The first conception of the Act made reclamation mainly a matter of engineering and construction. It was the popular opinion that if water were provided, irrigated agriculture would follow, that settlement of land and farm development under canals could be left to take care of themselves. No funds were provided to aid in such settlement and the help which could be incidentally extended was limited.

In the line of its main activity, the Reclamation Service has made a record which entitles it to the grateful appreciation of the people of the United States

* Presented at the Summer Meeting, Salt Lake City, Utah, July 8, 1925.
† Commr. of Reclamation, Dept. of the Interior, Washington, D. C.

The loftiest dams in the world are on these projects. Many had to be built in remote localities and numerous difficulties had to be overcome in handling torrential floods and deep foundations. Speaking generally, the engineering works on the twenty-four projects of the Bureau form an enduring testimonal to the zeal and capacity of the engineers of the Reclamation Bureau. The writer does not believe there is any service in any country which has a more competent staff of designers than that at work in the Engineering Headquarters of the Reclamation Service in Denver, Colo.

The operation of these projects has been difficult because water has had to be delivered to beginners in irrigation, to whom its methods and practices were new. They had to overcome great obstacles in the preparation of their land, and do this in too many instances without adequate capital and without credit suited to their needs. To these harassed individuals, the project superintendent has been a conspicuous and patient target for bricks hurled when they had to hit some one. Few bouquets are bestowed.

As far as the plan in the construction of works is involved, the organization of the Reclamation Bureau has taken a permanent and satisfactory form. These features of the Bureau's operations have passed beyond the sphere of evolution and need not be further considered, but social and economic forces render it necessary that present ideas as to what the term, reclamation, should include, should be revised. The increase in construction costs growing out of the World War, the greater expense of everything that goes into the development and operation of a farm, and the depression in agriculture, have made the question of farm development and the productive value of water far more important than was the case when the original Act was passed. Together they render it inevitable that if future reclamation is to be made a solvent enterprise, plans for new projects must include settlement and farm development and they must be made an important part of the Reclamation program.

On five projects authorized by the last (68th) Congress, but on which construction has not yet begun, and on two on which works have just been completed to supply water to new areas, the estimated and known construction costs vary from $100 to $160 per acre, with an average of more than $125 per acre. The land that is to pay these costs is unimproved, unpeopled, unleveled sagebrush, most of it remote from markets, and in some instances having such broken surfaces that preparation for irrigation will be expensive. The investigations preliminary to the undertaking of these projects, included the determination, by competent boards familiar with local conditions, of the probable cost of changing this raw land into habitable farms, and on these seven projects these boards fix the minimum expenditure necessary at from $4 000 to $10 000 per farm, or, roughly, $100 per acre. Combining the cost of the water right and the development cost of the farm, the settler will face an expenditure or indebtedness of from $200 to $250 per acre, before he has grown a crop or has established the productive value of land and water in irrigation.

The question which has to be considered is: How will settlers regard such an outlook? Under the Reclamation Act as it now is, which makes no provision for the organization of these settlers, for giving them expert advice and direction in farm development, and provides no means of advancing money to

supplement their own capital in farm development, will they consider it an opportunity? If left to carry out this hard and costly work where they have to shift for themselves, will they be attracted to these projects, and if they come, will they succeed? The experience on the older projects is overwhelmingly in the negative.

On one of the projects which has been in operation for fifteen years, where the soil is good, the water supply ample, and where skilled cultivators are growing high-priced crops, half the land to-day is unoccupied, 20% is cultivated by renters, and only 30% by owners, and they grow crops of low acreage value. The money that has been lost by the Government on this project in operating an undeveloped property, would have financed the development of the farms.

In one State where more than $15 000 000 has been spent in the construction of works, the total income since the outset—and that on some projects embraces a period of fifteen years—has not paid operating expenses. On the contrary, if repayment for construction is ignored, there is a deficit of $4 000 000. The reasons are delayed farm development and lack of an agricultural program. These examples might be multiplied. They furnish a weighty reason for determining in advance how the land is to be settled and how the farms may be made productive before the project is approved and construction begins.

There is another important reason for doing this. Two-thirds of all the land on the older projects was in private ownership when they were authorized. Inflation of land prices, misstatement of conditions and improper use of the arts of modern publicity led to these lands being unloaded on uninformed, over-sanguine people, many of whom had no farming experience and who broke down when confronted with the obstacles which they had to surmount.

The greater part of the land in four of these new projects is privately owned. In its dry state it has little value. It is doubtful whether the income from it more than pays the taxes. The owners are widely scattered. Nearly all lack money to improve and cultivate, and a still larger number lack any desire or intention to do so. What they seek is to have the Government close its eyes to the future, trust to blind chance, and spend millions of dollars in building reclamation works. If this is done, however, there is danger that it will be followed by an inflation of land prices similar to that which occurred on the old projects.

The relation of private ownership of land to the social and economic value of these works built with Government money cannot, therefore, be ignored, and there has recently been conducted an economic investigation in which many of these land owners have been interviewed to ascertain what they believe their land is worth, what they will sell for if the irrigation works are built, and what terms they will give the purchasers. They have also been asked to indicate, in case they do not sell, their ability and willingness to farm, or to turn their land over to the Government and form a part of a co-ordinated scheme of colonization.

The responses confirm past experience that unless land prices are fixed in advance and a program of settlement decided to its final result, the building of these works will involve the Government in heavy losses. Farm development will be so slow that not only will the Government be kept out of the return of the money invested for many years, but operation and maintenance expense will either be a ruinous burden on the settlers, or have to be paid out of the public treasury.

It would seem, therefore, that one of two courses should be adopted. Either the building of irrigation works should cease, or aid and direction in settlement and farm development should be made a part of the Reclamation program. If this latter is done along lines that are in any sense expedient, no doubt is felt that nearly all these projects will be made more quickly productive than in the past, and that they will prove solvent enterprises. If this is to be done, the program must include the following:

1.—The farm must be small and must be intensively cultivated. Only in this way can returns be secured that will justify the large acreage investment.

2.—The people who cultivate these farms must be selected. They must have industry and thrift. Only the superman could succeed if he had to bear the whole burden of this indebtedness. He could not do it at all unless money were advanced from some source not now available, and a credit scheme applied to settlers without capital would be financially unsound. The possession of some money is in most instances an indication of the earning and saving ability required in this work.

Any development scheme will break down that is based on the idea that settlers can be secured who have sufficient capital of their own. The records of the Reclamation Bureau show that 70% of the applicants for farms have less than $1 500 capital, and only about 10% have more than $2 500. The number of settlers who have from $5 000 to $10 000 is so small that it must be ignored. If the settler with $2 000 capital is accepted, who takes a farm that will require an expenditure of $4 000 to make it a going concern, the additional $2 000 that is needed must be provided from some source on terms entirely different from those which local banks or private enterprises can and do give. Money for changing the raw land into farms must be advanced on long-time payments at a low rate of interest.

If this is done, there must be on these projects, an agricultural or economic superintendent who will oversee these advances and watch the expenditure of the money, because they will not be made as loans but as service, not turned over to be carried around in the pocket of a borrower, but used to pay for definite and necessary improvements. If such a plan as that is put into operation and the money is furnished by the Government, then the Government must own the land on which the improvements are made. In such a case, this advance for improvements is underwritten (1) by the capital provided by the settler, and (2) by the fact that it enhances the value of the land which is owned by the Government.

With such advances, to be repaid under an amortized plan, with an interest rate of, say, 4%, and with from twenty to forty years' time for repayment of

the principal, the man with from $2 000 to $3 000 can see his way through. If he loves rural life and wants to become a member of a prosperous community, these projects will attract him. If this aid and direction is given in order to provide an opportunity for a home-seeker who is an actual settler, then the condition of cultivation by the owner should be maintained. It would be little less than a farce to furnish all this assistance with the idea that it was to enable a desirable family to be planted permanently on the land, and then permit the farm to be sold to a non-resident land owner, who would install a tenant.

The last (68th) Congress passed a bill providing for the selection of settlers and empowering the Secretary of the Interior to fix the amount of capital they must have and their other qualifications. The Irrigation Committees of both Houses held many hearings on the question of aid and direction which should be included in a settlement program. The result was a bill which provided for the employment of competent persons to act as advisers to settlers in farm development, for the subdivision of lands into farms and farm workers' homes, and for making advances to settlers to enable them to complete their farm improvements up to $3 000, the settler to furnish 40% of the money for all improvements on which advances were made.

This bill passed the Senate and was favorably reported in the House, but encountered opposition from members of the House who believed that the State should take charge of the subdivision and settlement of the land and furnish the money needed for advances in development. Provisions requiring this were incorporated in appropriations for three new projects, and the Secretary of the Interior was authorized to incorporate such provisions in a fourth. This legislation was opposed, in part, because it applied only to a few projects and not to others, and by those who do not believe the State has the resources to enable it to carry out this part of the development.

Whatever authority takes charge, no one who is familiar with conditions on the older projects and who realizes how much greater the expenses will be in the future, believes that a success can be made of reclamation under the methods of the past. Not only must the settler have financial aid, but there must be introduced into the social and economic life of these new communities things that will enable people, by working together, to make farming more profitable, to cut down expenses, and to give them a sense of permanence and security that is too often absent. Long-time payments, community and cooperative organizations, tend to lessen the migratory and speculative spirit that has played too large a part in the development of new farming communities.

DISCUSSION

B. A. Etcheverry,* M. Am. Soc. C. E.—There is no doubt that the financial results of Federal Reclamation as far as reimbursing the Government is concerned, have not been successful. The Fact Finding Commission found that $18 861 146 will never be recovered and that there will be a probable loss of an additional $8 830 000. The total of these amounts is 20% of the money spent by the Government and includes no interest. It also assumes that payments will be made on the remainder, without additional relief measures, which past history would indicate to be a rather optimistic assumption unless a different policy for repayment is to be enforced by the authorities.

The failure in repayment has been largely attributed to the inability of the land owners to pay; and this has usually been charged to (1) the cost of construction, exceeding the original estimates of cost; (2) the inflation of land prices, or the excessive prices the settlers have had to pay for lands held in private ownership; and (3) the inexperience of the settlers and their inability to finance themselves. These conditions are in general not more unfavorable than those found on irrigation projects built by irrigation districts and other agencies. On the contrary the land owners on the Federal projects have had the great advantage of having no interest to pay on their obligations and no cost of financing, such as discount on bonds. In addition, through the Extension Act of 1914 and several leniency acts, they have had a part of their payments due on the capital and on operation and maintenance deferred.

Critics have stated that some of the engineers of the Reclamation Service have been more concerned in building monuments of engineering than in the economic problems of successful land development. Even if this were true the water users have generally had more adequate water supplies and have demanded better service than on projects built by other agencies. In many irrigation districts the land owners have had to be satisfied with only a partial season's supply during the early period of settlement and development, up to the time when they could finance the storage works necessary to develop a full water supply. In other words, they have built at first what they could afford and not what they desired.

This was the economic solution when these districts, before they had become established, had to discount their bonds 10 or 15%, or more, and pay interest at 6 or 7%, or more. It is true that many failed, but on the other hand many were successful and the failures would have been much decreased had the projects been financed with no interest and no discount on bonds. The past policy for repayment to the Federal Government has been largely adjusted to fit the land owners who are either incompetent, unsuccessful, or unwilling to meet their obligations to the Government, with the result that the more successful and those who were able to pay and, in many cases, willing to do so, have either accepted the advantages of blanket reliefs or have made their payments because of their sense of pride in meeting just obligations.

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That non-payment has often been due to unwillingness to pay is indicated by the records of the Reclamation Bureau.

The Commissioner of the Bureau states* that in 1924 one of the good projects paid only 7% of what it cost to operate it; another paid only 15 per cent. One project that owed $440 000, only paid $25 000, or 6%; another project that owed $16 000, paid only $69; and another that owed $112 000, paid only $6 000. The records show that many of the larger good projects received water for 11 to 14 years before any payments were made on the construction charges, whereas on other less desirable projects payments had to be made immediately after water was first applied. It is difficult to find any good reason for these inconsistencies in repayment. On one of the largest and best projects, enjoying a large income from hydro-electric power, water was first supplied in 1907 and the first payment on construction was reported in 1918. The net construction cost on June 30, 1923, was more than $10 000 000 and the amount paid by the water users on construction to that date was less than 10 per cent. The economic conditions on this project are as favorable, or even more so, than on many irrigation districts where the land owners have had to pay in discounts and interest an amount nearly equal, if not equal, to the entire cost of construction.

Other projects have apparently showed a greater willingness to pay or have not taken full advantage of the leniency of the Government. The Orland Project in California is reported as the only one of the twenty-five Federal projects that has made without deferment its regular payments to the Government for annual costs and for operation and maintenance. Although the land owners suffered a 60% loss in crop production in 1924, due to the extreme shortage in water, they have maintained their record of full payment. There is no doubt that on some of the projects leniency was necessary because of economic conditions making the projects more or less unfeasible. For these projects it would seem that the best policy is for the Government to recover only that part of the cost represented by its useful value to the land owners. In the consideration of a new policy for Federal Reclamation the following fundamental questions arise:

(1) What shall the Government do with its existing projects?
(2) Should the Government undertake new projects for bringing new lands under irrigation?
(3) If the Federal Government is to continue to use its funds for reclaiming lands, how can they be used to the best advantage, with reasonable assurance of their repayment?

The answer to the first question is that the fair value of each project to the water users should be determined and the project turned over to an organization of the water users which will give as good guaranties as possible for the repayment. This, apparently, is in accordance with the new policy of the Federal Government.

The answer to the second question is that, for the present at least, there is no real demand for such new projects. From the standpoint of the Federal Government there is no profit and from the standpoint of the settlers there

* *New Reclamation Era*, June, 1925.

should be no demand, if the complaint arising from many of them properly represents true conditions. Disregarding these complaints, however, there is now much land under irrigation which only lacks settlers, and in some localities there are lands fully developed which can be purchased at a cost less than that of creating newly developed farms. Mr. Mead has stated that on the five new projects authorized by the last Congress each settler will face an expenditure of $200 to $250 per acre before he has grown a crop or has established the productive value of land and water in irrigation. In the Annual Report of the Commissioner of Reclamation of June, 1924, the statement is made that well-improved irrigated land, fenced, equipped with buildings and growing crops, can be purchased in many of the Federal and private projects of the Northwest for $150 per acre, including a paid-up water right.

With this condition existing there is apparently no general need of bringing new lands under irrigation for new settlers, to be financed by the Government or the State. New development should be postponed until the agricultural demands make it necessary. In some localities agricultural and economic conditions are favorable to success and there is now, or soon will be, demand for additional lands that will justify the undertaking of new projects.

The answer to the third question is that the Federal Government should not limit the use of the Reclamation fund by favoring a relatively small number of land owners at a prohibitive cost to the Government, especially when it apparently has resulted in creating a number of dissatisfied citizens. It should extend its help to the most meritorious projects throughout the West and adopt a plan or policy which will give reasonable security for, or even assurance of, repayment to the Government.

The sentiment has been expressed by some settlers that the Reclamation fund does not belong to the United States, but to the West. Many will differ with this view, but even if it is admitted as correct the West does not consist entirely of the land owners on a few projects. It certainly would be much more equitable to apply the fund to all the land development projects or to as many as possible.

It is well known that important causes of failures are the high cost of financing new projects and the large development cost due to the long period of time necessary to approach substantial settlement or development, especially on large projects. As previously stated, new irrigation districts are financed by discounting their bonds 10%, or as much as 15%, and they must pay interest on the bonds at 6 or 7% from the time the project is started. Operation and maintenance costs must also be met. Later, the payments to retire the bonds must be started. The burden is excessive especially during the first ten years, or even longer for large projects, because the area brought under irrigation increases slowly.

The largest results should be obtained by using the Reclamation fund in accordance with the following policy.

The Federal Government should take the bonds of the Districts and hold them, either for a fixed period of ten years or more, or until the projects have been established to the extent reasonably required to meet future obligations, when it should sell them to reimburse itself. These bonds should bear a low

rate of interest or even no interest (if the Government is to continue this practice) during the period that the Government held them. Should a project be only partly successful, or not economical, or not feasible because of engineering, agricultural, or economic mistakes, beyond what may reasonably occur on any project, the Federal Government should absorb the loss, cancelling some of the bonds so as to sell only the amount representing a fair value of the project.

The Government could protect itself by extending this aid only to projects which it considers sound, and in which it can obtain satisfactory guaranties against land speculation during at least the period that it held the bonds.

This plan could be applied not only to irrigation projects built by irrigation districts or other agencies, but to reclamation districts such as those in California where the cost of reclamation for levee protection, drainage, and irrigation has placed greater burdens relatively on the land owners than on the average irrigation projects. Many of these reclamation districts have had to be financed with 15% discount on warrants and 7% interest, followed by a discount on bonds.

It may be urged that Federal aid to the amount suggested will extend over a larger area than if it is applied to the more intensive aid required to build a project and also finance the settler, and therefore may not be sufficient to produce success. The fact remains that there are many irrigation districts where the land owners have had to carry much heavier burdens than on the Federal projects without deferments on payments and have generally done so with less complaints. This has resulted in the weeding out of the more incompetent so that the remaining land owners are good citizens who have retained their pride and sense of obligations. The larger and more difficult projects yet remain to be constructed; although they should not be constructed until the demand for agricultural lands has exhausted the supply of available good lands already under irrigation and awaiting settlers, they will have to be constructed, some of them in the near future, and Federal aid during the period of settlement will be most needed.

Thomas H. Means,* M. Am. Soc. C. E. (by letter).—Mr. Mead's frank statement of the settlement problem confronting the Reclamation Service brings out sharply the difference between economic conditions in the early days of Government irrigation and present-day conditions. The idea then prevailing was that all the Government needed to do was to build the reservoirs, or where reservoirs were not needed, to construct the main canals. Everything else could be done by the farmer's own labor. The reason back of this opinion was that the part of the works requiring any particular skill or necessitating the expenditure of money in large amounts, could not be well done by the farmer, but the lateral canals and the preparation of the land could be cared for by farm labor, thus placing the land on a producing basis.

In those days, when a farmer estimated the cost of bringing a farm into cultivation, he considered the cash outlay necessary to do the work; his labor and that of his family were not considered as a part of the cost. Ditch

* Civ. Engr., San Francisco, Calif.

building could be done by his own labor and that of his teams. The provender for the team was raised at home with little cash required. Purchases of lumber and hardware in small quantities were the only cash outlays. Groceries and clothing for the family were the principal items of expenditure and in those days included very few articles. Vegetables, potatoes, beans, and other principal items of food were home-raised—the first effort was toward the raising of food for the family and the animals on the place. Cows produced the milk, butter, and cheese supply and the farmer knew how to slaughter a calf or a pig. The art of home-curing of meats was understood and the wife usually raised chickens and looked after the production of eggs and of many other things now bought from the store.

Nowadays all is different. The prospective farmer inspects the site of his new farm in an automobile. This vehicle consumes gasoline, oil, and tires, none of which can be produced on the farm but must be purchased for cash. The annual outlay necessary to keep a $1 000 automobile in running order to-day would have paid all the cash expense of an 80-acre farm twenty years ago. The upkeep of a Ford at the present time often exceeds the water bills and taxes on a quarter-section of land. Other habits of the present-day farmer require much cash. A diet of bacon and beans and home-made bread does not sustain life as it once did, and the farmer's table to-day is supplied from the ends of the earth. Many food products once thought to be luxuries are now considered essential to life and happiness.

A great deal of the difficulty of the present-day farmer and his inability to make good on the farm without a large amount of money comes directly from this fact. There does not seem to be any way of getting around the matter; automobiles, radio, movies, canned fruits, and foods from distant sources are necessities nowadays and no one seems willing to do without them. The fact is that more cash is required now than twenty years ago. The standard of living is higher. The day of the pioneer is gone. Few men now seem inclined to go out in the desert and carry on the hard work necessary to make a producing farm out of sand and water. Not only is the inclination lacking, but the ability to perform such a feat is possessed by fewer people. So far as any large percentage of American stock is concerned, it is poor material to use in conquering the desert. About the only recourse is to hand-pick men of special training for settlers and coddle them until they can be safely left to stand alone.

In addition to Mr. Mead's paper, the writer has had the opportunity of seeing some of the reports of the technical committees of the Bureau of Reclamation on unfinished projects and new proposals for extensions of projects. In none of these proposed new works is the cost of irrigation works less than $100 per acre and in some cases it goes to nearly $150 per acre. This cost is only for irrigation works, to which the farmer must add the cost of improvements and land preparation on the farm. Mr. Mead estimates the minimum expenditure to be from $4 000 to $10 000 per farm, or roughly, $100 per acre. The total cost of the farm will then exceed $200 per acre without interest. The particular projects referred to are in Washington, Oregon, and

Nevada, in sagebrush regions. In addition to all this, roads and schools and all the other facilities for community life must be provided.

These statements emphasize the fact that the prospective farmer on one of these projects must be a man with considerable means. Men who have had experience with several settlement projects say they have never seen any considerable number of settlers possessed of nearly the amount of money required. It is incredible that many men with $10 000, or even $4 000, in cash, would want to undertake the hard labor, the privations, which even in these days cannot be entirely eliminated, and the uncertainty of the final outcome of the attempt to make a home out of a tract of desert land. A man who has had the ability to earn and save $10 000 before he becomes old and decrepit seldom wants to attempt any such enterprise. The man who possesses $10 000 which he did not earn usually thinks too much of the "bright lights" and the creature comforts to undertake to reclaim a desert farm. Among men with sufficient money the choice is limited, therefore, to a small number of country life enthusiasts who truly believe that the country is the best place in which to live and raise a family. There are still a few people of this sort who have enough money to reclaim a farm, but there are not enough of them to fill up a very large tract of desert land.

On the other hand there are a large number of men and women with small means, but with energy, enthusiasm, ability, and love of country life, who would be willing to wager a few years of hard work if there was a chance of winning a little money. Few people are willing to spend half a lifetime in paying for a farm unless it is possible to sell out at a profit. The farmer who spends $200 per acre or more in making a farm out of raw sagebrush land in Nevada, Washington, or Oregon, is paying all the farm is worth. He is taking much risk and buying the land at its full value. It is not an attractive undertaking, but it does offer a method of acquiring a farm home that will enable many men to succeed. If the prospective settler has the ability to make a desert farm productive and has the hope of selling the farm for more than it has cost him, he will work, and work hard. The only hope in the future in reclaiming the arid West lies in the farmer with the hope of making money as the driving force behind him.

There are in America few men so constituted mentally as to be happy when struggling to pay off a debt which stretches out thirty years or more in the future—few who are interested in working a lifetime to earn a farm home. The hope of reward in the not too distant future cannot be eliminated entirely; something must be done to make it possible for the farmer to pay up in a comparatively short time.

There are two problems—either to reduce the cost of reclamation or to subsidize the new settler in some way. There seems to be little hope of reducing the cost of irrigation works. All the cheap irrigation projects have been built. Construction costs are not likely to decrease much during the present generation. The farmer's expenditures on the ranch are more important than the construction costs and the tendency is toward the need of more cash rather than less.

Direct subsidy of reclamation farmers, beyond the freedom from interest, has not been seriously considered, but many efforts have recently been made to connect the Reclamation Service to enterprises in which direct subsidies would finally have to be made. On such projects as those where construction and farm costs equal or exceed $200 per acre, there is no possibility of paying for the farm out of proceeds in any reasonable time. If reclamation is to continue the Government is faced with the necessity of selling water rights or farms at their agricultural value and not for the cost of reclamation works. It would be better for every one if, on such a project as the Spanish Spring Project in Nevada, the water rights were sold at $50 per acre and the payments collected in 20 years, rather than have the full cost of $100 or more per acre spread over 50 or more years. Raw sagebrush land is seldom attractive at costs of more than $50. Of course, no fixed sum can be set for all localities, for it will vary with the location, but there is everywhere a limit of value. That limit should be the maximum charge for water right no matter what the cost of the works may be.

To appropriate money for irrigation works in excess of the probable return would be a radical departure for Congress to take, but it would be in line with other Federal endeavors, such as leveeing along the Mississippi, road appropriations, and harbor work. This should be the next step if it is intended to complete the reclamation of the West. There is no likelihood of private enterprise doing much more irrigation work, except in the better settled areas and along the Pacific Coast. This work is of sufficient importance to call for the help of the nation. There is no reason for restricting future work to the arid regions. Many swamp and overflow projects are of great importance.

In many places the burden of this cost above the sale value of water rights can be collected from the people of the immediate vicinity and not from the National Treasury. Reference is made to places where power development is possible; a small charge on such development would pay for storage and flood control and not be a burden on any one. Such conditions exist on the Colorado River, on the Columbia, and on other streams. A mill or two per kilowatt-hour would not add any considerable burden on industry or the population of the cities, but would furnish enough money to provide for reclamation works of great magnitude. It is the writer's opinion that the people of Los Angeles, Calif., and the surrounding territory would find it good business to pay 0.25 cent more per kilowatt-hour for power if that resulted in bringing into cultivation 1 000 000 or 2 000 000 acres of irrigated land in Arizona and Southeastern California. In the Sacramento and San Joaquin Valleys irrigation districts are finding it possible to build storage works and sell the power to public utilities at a price large enough to pay the carrying costs on the reservoirs. Many reclamation projects have very little power of value. If, however, the power could be pooled it would go a long way toward carrying out reclamation enterprises, wherever they may be located, as fast as there is need for the land.

A rather close contact with both Federal Irrigation District and private irrigation enterprises, extending over a period of twenty-five years, has

caused the writer to hope that the day is not far distant when Government operation of irrigation works will cease. Nearly five years of operating one of the first Reclamation Service projects and afterward the operation of a privately owned project for three years, and for the past ten years a close contact with a number of irrigation districts has led to the belief that the operation by the Government is the least desirable of all methods. The reason for this belief is not that there is any particular superiority in the methods or personnel, but on account of the attitude of the water users. Where the Government is operating there is no question but that a large and increasing percentage of the water users are hoping that some means will be devised whereby they can escape the payment of the debts which they have contracted to pay. The result is that the local politicians have an opportunity to work and something to work on. They, in turn, work on the larger politicians until enough noise is created to attract attention. The belief is prevalent that if this noise can be made loud enough and continued long enough, relief will be offered them no matter if there is no justice in the cause or any need of relief. The result is bad for the morals of the community. There is a tendency to complain, to ask for relief, for more facilities, to display little desire to get along with what has been provided until it is paid for, and almost no desire to pay off these debts to the Government.

The management of the projects is directed from Denver, Colo., and Washington, D. C., which is a bad thing, for long-distance management is never satisfactory. Rules and regulations for the governing of all projects alike have to be promulgated and in the course of time the project manager becomes little better than a chief clerk. The management should be turned over to the people as soon as responsible officers can be selected, and the Government rule from distant points should be conspicuous by its absence. To this end the irrigation district organization is being used with considerable success. One feels that, in the past, Reclamation Service operation has probably been more efficient, more fair-minded, and altogether better than could be expected from local managers, but the writer is thoroughly convinced of the final necessity of local management and assumption of responsibility by the water users.

Mr. Mead believes in the selection of settlers and the establishment of advisers to encourage and foster scientific farming and business-like selling of farm products. Selection of the settlers is certainly advisable. It is not altogether clear what formula has been devised to assure the selection of the most competent men. Knowledge of agriculture, is not the only requisite. Energy, a large amount of "stick-to-it-iveness," and, most important of all, the mental quality which prevents discouragement in the face of failure or disappointment, are as essential as money. Records kept of the cause of success or failure of about 1 200 settlers on irrigation projects led to the conclusion that lack of money was a small cause of failure. Plain discouragement, often before there was any reason for discouragement, caused 50% of the failures.

It may not be possible to select men with the requisite qualities of success. It has been tried, but not successfully so far. Certainly, selection can accomplish something toward raising the quality of settlers above that of those who have heretofore acquired homes on reclamation projects. There is, however, a great deal to be said about the equal rights of all citizens to share in the benefits of reclamation. A large percentage of men are incompetent and fail wherever they go and in whatever they undertake. Sometimes, it seems that it would be better to allow the incompetent to eliminate himself through the natural processes rather than to appoint a committee to do it for him by examination in a preliminary way. Either process is a painful one for the one who is eliminated, but when the cause of elimination is his own act, his appeals for consideration do not echo so loudly.

Many other factors than those which can be examined by the proposed committee for selection of settlers enter into the cause of success or failure of a farmer. There are in California two interesting land settlement colonies carried out by State funds on a plan credited to Mr. Mead. One of these colonies is a success, the other, a turmoil of agitation. Apparently many of the settlers on the latter must be eliminated before this colony can ever be made a success. No amount of capital would make successful farmers out of some of these men. There is a great difference in the physical conditions of these two colonies, but neither those differences nor the slump in prices are altogether responsible for the difference in the two colonies. Some of those subtle changes came about in the mental attitude of these people, and some of them have changed from hard-working and hopeful home-builders to hard-working appellants for further aid from the State. This sort of thing can happen anywhere, but it is most likely to happen when the State or Government is the party to whom the debt is due.

There is little tendency to avoid paying the kind of taxes that are collected by the county tax collector. If Reclamation debts are placed on the same plane as county taxes, a much larger percentage of collections will be made. It is questionable, therefore, whether or not selection of settlers will prevent failures. Failures occur in any business and it is probable that reclamation farmers are no more likely to fail than grocers or any one of many occupations. One must not expect the future of reclamation to be free from the frailties of human beings, nor is it fair to place the blame of present-day failures on the farmers of present reclamation projects. In the case of the California land settlement colonies a great deal of unfairness has been evident in the discussion of the situation, but this, being a State colony, came within the realm of State politics.

Following the selection of the settler, Mr. Mead urges the necessity of aid and direction in farm development and of co-operation in disposal of the products. At present, farmers receive such aid in nearly all parts of the country. There is greater need for such aid and organization in the West where markets are distant than in many other parts of the United States. The writer has always believed that such aid was necessary and has found it very helpful in many ways. In California, there are farm bureau organizations in nearly all the counties which are effective in spreading informa-

tion about methods of farming and in forming marketing organizations. There is little need for any other organization although special problems are being effectively studied by the U. S. Department of Agriculture through its various bureaus, particularly that of Western Agricultural Extension. Every reclamation project can well afford to support an experiment farm.

The field of reclamation does not seem bright when future development is considered. The difficulties which have centered around the Reclamation Service have been felt by every organization that has built Reclamation works. There is little encouragement to go ahead except in regions where there is a decided demand for land. The warmer sections and the coastal valleys close to large cities are able to go ahead without Government aid. In other parts of the country development is not going on. Many years must pass before there is sufficient demand for new farms to make it possible to proceed without direct subsidy.

In the last twenty years much has been learned about how reclamation should be carried on. It is necessary to select the land carefully so that none but the best be placed in the hands of farmers. It is necessary to select the farmers in order that the farms be placed in the hands of none but the most experienced men. It is then essential that these choice farms in the hands of selected men be organized and advised and directed until they become profitable, and until good habits, in an agricultural sense, become established. Even with these matters all attended to, there is no hope that the reclamation projects of the future will be free from failures. The number of failures, however, will be much reduced.

Direct subsidy of future reclamation when there is a demand for more land seems essential. Works cost more than the land is worth in many cases, at least more than the farmer can hope to repay in any reasonable time. The question of the future is whether work is to be stopped or whether Congress will provide funds in excess of the amount which can be collected from the water users. The extension of agriculture in the West is so important that Congress will be justified in direct appropriation.

George C. Kreutzer,* Esq. (by letter).—The author points out that the cost of construction of Federal irrigation projects has been increasing and now varies from $100 to $160 per acre for new projects approved by Congress. With this increased cost of construction a corresponding increase has taken place in the cost of developing and equipping irrigated farms. A 40 or 80-acre farm with suitable buildings and fences, including leveling and farm ditches, live stock, and farming equipment, will cost the farmer, exclusive of the land, from $5 000 to $10 000.

Experience has shown that settlers with such amounts of capital are scarce and when found are inclined to buy improved farms in established communities. The majority of settlers who take raw land on new irrigation enterprises have capital varying from $1 000 to $2 500.

The cost of water and farm development, high taxes, and labor costs have precluded the growing of low priced crops at a profit and have made large

* Director of Reclamation Economics, U. S. Dept. of the Interior, Washington, D. C.

farms with few exceptions unprofitable. The small irrigation farmer, largely doing his own work, cultivating intensively and being free from burdensome debts bearing high rates of interest, has been able to pay fixed charges, including those for water, and to get ahead. His crop values per acre are relatively high, his labor bill small, and he has no idle land. He grows a large part of the food consumed by his family. These advantages over the larger operator gives him security and freedom from the other's difficulties. When the gross income from a 40 or 60-acre farm reaches $2 000 to $3 000 per year its owner is usually prosperous.

The selection of settlers is a step in the right direction. It is not expected that these settlers know how to do every farm task efficiently and well but they should have had experience enough to realize that to operate a farm calls for hard work and to know that they enjoy this means of making a living. In addition, they should have character and industry and a reasonable amount of capital. These qualifications when possessed by a settler indicate that he is adaptable for rural life and has a good chance of succeeding. Farming demands frugal expenditures and the practice of economies seldom experienced by the city dweller. The settler who has had previous experience knows these principles and practices them from the beginning. Selecting settlers in accordance with desirable characteristics will not preclude failures but will greatly reduce them.

The only kind of irrigated farms that pay are those that are completely developed and well equipped. If settlers have not the capital nor the credit to do this it results in delayed development and the piling up of debts. The loans required to complete development must be repayable over a long period and with a low rate of interest. The returns from farming demand this.

The weakness of Federal reclamation and also of private irrigation enterprises is that the construction of the irrigation system is acomplished by the use of borrowed money and as such is a loan, yet the only security given is raw land of low value having no immediate earning power. The land itself usually is not worth more than a few dollars per acre. Earning power must be created. This can be done only by putting labor and capital into the raw land to convert it into profitable farms. The enterprise is not complete until then. When it is not completed there is a default in the construction payments. This is like building a factory and omitting the machinery. It leaves too much to work itself out. Certainly it has neither of the two conditions demanded by any other financial agencies in making a loan—security for protection and income to pay the debt. In irrigation developments these two conditions must be created. The safest way to do it is to put on the land a good settler who has some capital. His investment of capital and labor will create the security. Then money may be loaned him on terms he can meet to create income.

Settlers come from regions where the climatic and agricultural conditions differ from those of their new locations. To many, irrigation farming is new. They do not know how to lay out land for irrigation, which field to prepare first, or what kinds of crops to plant. Mistakes can be avoided if

settlers are given friendly assistance and guidance from the beginning. They require counsel and help in the preparation of sound financial and agricultural programs. Co-operative institutions are brought into being where this service is extended by a competent leader. The services of such a leader is doubly required and his duties multiplied if the selection of settlers becomes a part of his duties and if credit which requires supervision is included in reclamation development.

The principles advocated by the author are entirely sound and if put into practice would alleviate many of the difficulties of reclamation.

O. L. Waller,* M. Am. Soc. C. E. (by letter).—Irrigation development rests not only on the cost of water delivered on the land, but also on the productive value of the water and on the price the crop will be likely to bring. The net value of the crop is a major element in determining the feasibility of a project. Before lands are developed, there should be an agricultural economic survey. Such a survey should assemble data showing the productivity of the soil, the adaptability of the climate to the production of crops, the length of the growing season, the kind of crops that would be likely to be grown on the project, the transporting and marketing facilities, and the probable market value of the crop. Without these facts, the prospective settler cannot make a rational estimate of the income that he may reasonably expect to receive from his farm.

The history of irrigation shows that it has been the custom to make a serious study to determine whether water could be delivered to the farms and at a reasonable cost. The author states that:

"It was the popular opinion that if water were provided, irrigated agriculture would follow, that settlement of land and farm development under canals could be left to take care of themselves."

The agricultural, economic, and social viewpoints have largely been omitted. Fortunately, the Federal Reclamation Service is now adding to its program a study of the economics of the irrigated farm and has extended its researches to include settlement and farm development, and is treating these as important parts of the reclamation program.

As in the past, studies must include not only the water supply, the necessary engineering treatment of the project, and the cost of delivering water on the land, but in addition the cost of settlement and farm development. Formerly, projects were financed, but now the settlers must be financed. The day when small irrigation projects could be developed by a single farmer or a group of farmers at a minor cost is past. Future development will cost much more; and ordinarily men of established financial credit are not buying such farms. Only settlers qualified by farming experience are to be permitted to undertake to farm irrigated land on Federal projects. Usually they will be men of moderate means who will need financial help to make it possible for them to improve their farms. Shall this new policy be accepted, these additional studies made, and credit arranged for the development of the farms? Or shall the statutes providing for Federal irrigation be repealed

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and arid lands allowed to remain barren, to raise sagebrush, and to breed horned toads and jack-rabbits; or shall the Federal Government be forehanded and develop its natural resources, irrigate its arid lands, and thereby provide homes and food for its rapidly increasing population?

In coming to a conclusion as to whether unproductive lands are to be developed or to remain barren, it must be kept in mind that the last great West has passed into history, as far at least as cultivable lands are concerned, and that future tillable soils must come from irrigating arid lands, from draining swamps, or from clearing logged-off areas. The world's population is constantly increasing, while the limit of tillable areas has nearly been reached. O. E. Baker, Agricultural Economist, U. S. Bureau of Agricultural Economics, states:* "The agricultural production of the United States is no longer keeping pace with our increasing population." In the rural districts of the United States, the yearly increase of population is about 600 000. These young people know farming, and farming ought to be made profitable enough to induce them to remain on the farms. The demand for agricultural land has constantly been increasing, as is shown by the increase in land values. The Hon. Henry C. Wallace, former Secretary of Agriculture, has stated that:†

"The average value of farm land (including buildings) in the United States increased between 1850 and 1920 at a rate equivalent to compound interest on the 1850 valuation of 2.65 per cent. The increase from 1900 to 1920 was at the rate of 6.47 per cent."

Since 1920, there has been a slight drop from war prices.

When arid lands can be made to carry the financial load for water with which to irrigate them; to pay for the lands, for their development and improvement; to pay a reasonable interest on the farmer's investment and pay wages for his labor and that of his wife and children who work on the farm, the Government should be interested in their development. The nation needs the homes thus made possible for its rapidly increasing population.

President Coolidge, in his letter to the Farmers' Conference of November 17, 1924, pointed out that the United States already imports foodstuffs that it should raise. At present, farmers are raising too much of some kinds of food and not enough of others. Probably the yearly fluctuations in yields will always produce temporary over-production in one crop or another. It is not a question of permanent over-production, but of raising the right kinds of crops and of proper selling organizations. The remedy is not to be found in importing food produced by under-paid foreign labor.

In determining whether new lands are needed, the really important matter is whether the new lands may be profitably worked by farmers living on them. In that case, the more farms, the better. Otherwise, the more people that fail in the business of farming, the worse off will the nation be. The real effort should be to get as much land as possible into continuous and profitable use.

* Year Book, U. S. Dept. of Agriculture, 1921, p. 408.

† *Loc. cit.*, p. 4.

As much farming land is annually being abandoned and going back to the jungle state as is reclaimed by irrigation and drainage. The poorer lands are constantly being exchanged for more fertile lands. Business men quit poor locations and move to better ones. Why should not the farmer leave his worn-out non-productive land that is not yielding a fair return for his investment and his labor and move to more fertile lands? To do this, the arid and swamp lands must be developed. They are the nation's most fertile land asset. People talk about abandoned farms; they ought to be abandoned—many of them ought never to have been deforested. Mr. Baker says:*

"The trend of land utilization in the United States is toward the more intensive use of the more fertile or more favorably situated land * * *; and toward the less extensive utilization of the less fertile or less favorably situated lands."

Practically all the land that is easily available for agricultural purposes is now under the plow. Only by draining and reclaiming the wet lands, by clearing the cut-over lands, and by irrigating the arid lands can the producing area be increased. Soon all the National land resources will be needed for domestic food production. Mr. Wallace estimated that the population of the United States in 1950 would be 150 000 000. Dr. Raymond Pearl, Specialist in Vital Statistics at the Johns Hopkins University, estimated† a population of 150 000 000 in Continental United States in 1950. This is an increase of 45 000 000 over the 1920 Census. The same thought is ably discussed‡ by Dr. Edward M. East, of Harvard University.

To provide food for the normal increase in population, not counting immigration, Mr. Wallace estimated that it would be necessary to bring under cultivation 8 000 000 acres per year, or approximately 250 000 000 acres between 1920 and 1950.§ Mr. Baker states that improved land increased less than 5% from 1910 to 1920, as compared with 15 to 50% for the previous decades. This 5% increase was practically confined to the precariously productive semi-arid lands of the Great Plains region.

"The land in the United States suitable for agricultural use without irrigation, drainage, or heavy fertilization is nearly all occupied. Consequently, one of the great questions before the American people is how to maintain the supply of foods and fibers for the increasing population at that high level to which we are accustomed."‖

A study of population and food statistics would convince one that the Federal Government should continue the policy of developing its non-productive lands; that all such areas should be carefully studied and plans should be made for their improvement. These plans should contain a careful examination of all features pertaining to the soil, climate, crops, availability of water, and engineering treatment, and should provide for the selection and financing of home builders. A plan should be provided also for the elimination

* Year Book, U. S. Dept. of Agriculture, 1921, p. 409.
† In an article on "World Over-Crowding."
‡ "Mankind at the Cross Roads."
§ See Year Book, U. S. Dept. of Agriculture, 1921.
‖ *Loc. cit.*, p. 408.

of the land speculator. Otherwise, the cost to the settler will be much increased and the development of irrigation projects will be greatly delayed.

The development of the land resources of the United States should be advanced as rapidly as the new lands can be made to provide proper guaranties for the return to the Treasury of the costs of developing them. The writer most heartily concurs in the opinions of the author, but, in addition, suggests that a systematic study should be made of all available areas that may ultimately be irrigated, cleared, or drained. In other words, let a Federal inventory of the nation's land resources be obtained.

AMERICAN SOCIETY OF CIVIL ENGINEERS
INSTITUTED 1852

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Paper No. 1610

LAND SETTLEMENT OF IRRIGATION PROJECTS*

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With Discussion by Messrs. W. G. Swendsen, Charles H. West, R. A. Hart, R. K. Tiffany, F. H. Newell, Walter E. Packard, and Augustus Griffin

There are many different aspects and phases to the problems of land settlement on irrigation projects. The subject is treated with the intention rather to suggest ideas for consideration and discussion than to attempt any thorough treatment, even of one phase of land settlement. The word "promoter" has been used in a broad sense to include a Government, district, railroad, or private colonization agency. It is realized that the manner of financing, sale of land and water rights, collection of charges, and handling of settlers, varies so widely that in a brief paper suitable distinctions and qualifications of statements cannot be made to conform to all conditions.

There are probably very few irrigation projects under which closer settlement is not possible and desirable, but many of the settlement problems have in these cases been solved or have become modified by local conditions. This paper, therefore, will be directed more particularly to the large isolated projects, developed on desert or sparsely settled range or dry-farmed areas, where the existing population is a small factor in necessary settlement and where the settler is a pioneer.

As soon as the first dollar is spent on a project, the annual charges begin. First, there is interest, then maintenance, and the annual charges of management and operation of the system. A large part of, and possibly all, the original capital investment must be made before any returns can be had and the investment has been increased by accruals of interest and annual costs.

* Presented at the Summer Meeting, Salt Lake City, Utah, July 8, 1925.

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Only when water is available can settlement start. Settlers are a necessity, for the returns on the invested capital must be produced from the land through the medium of farmer settlers. Money, energy, and art are required to make a "new settler" out of an ordinary man, but the art has been well developed and when the money is available the energy is easily commanded and by well-tried methods settlers are obtained whenever economic conditions are favorable. The foundation is laid for one of the first and most important settlement problems, namely, "dissatisfaction". A family which possibly had no intention of moving has been persuaded to sell, move a thousand miles or so to a new district, of which it knows nothing, to engage in business wholly new to it, under new surroundings, and among strangers. Possibly the finances were inadequate or the personalities and training of the members of this family such that they were foredoomed to failure or to a long struggle against difficulties. Yet the project has to be settled in order to pay the annual costs and return the investment (if possible), in addition to which such settlement is necessary or desirable as a step in National development, to provide traffic for a railroad, or to sell somebody's surplus land. The promoter then may have two definite objects in view: First, to secure a return of the investment; and, second, to populate and develop the project as a National enterprise or to create traffic or other kind of business. The pioneer settler often feels that the secondary objects, which do not directly benefit him, are so important that he, as an agent in accomplishing them, is entitled to special consideration.

If the settlers on a project are prosperous, have their farms paid for, or can see that they can pay in a reasonable time; in other words, if they are making money, there is no serious settlement problem. The problem, in fact, is to bring about this highly desirable state of affairs.

It is necessary, therefore, first to have settlers, and the proper kind to get and the proper way to get them are large subjects in themselves which will not be discussed in this paper. It is sufficient to repeat that the art of getting settlers is well developed. Most new settlers find themselves on the project that some colonization organization has chosen for them without any very clear consciousness of how they got there. Fortunately, many of them feel that they arrived there by their own choice and initiative and this undoubtedly reduces the percentage of dissatisfied ones. Within certain limits the newer the project and the less development it has, the easier it is to get settlers, as there is generally the talking point of large areas of cheap land to choose from, with chances for speculative increases in land values, and the absence of actual trials and failures permits a freer scope in painting the possibilities.

There is undoubtedly much room for greater care and judgment in selecting settlers, but under the most favorable conditions there will always be misfits and others who, for various reasons, are foredoomed to dissatisfaction or failure. It has become a saying that the first two crops of settlers will fail and that the third and following crops may achieve success on the pioneer work of the first two. This is not strictly true, and it is probably more nearly correct to say that the successes of the first two crops, even if comparatively

few, do the pioneer work which makes a greater percentage of successes among the later comers.

As the first settlers come to a project, they face a complex and critical set of conditions, although more often than not they do not realize this. They are full of hope and confidence, based mostly on the rosy tales of salesmen. They are strangers in a strange land, among neighbors as new as themselves. Even if they have farmed before they are facing a new type of agriculture, and this is usually true even if they have been irrigation farmers before. Previous irrigation experience may not be much of an asset if it has been with a different irrigation practice, under different conditions. The writer has come to the opinion that the man who will succeed is the man with tenacity of purpose, adaptability, good common sense, and with the ability and inclination to work hard. It is likewise essential that he and his family be of one mind, for many failures are traceable to dissatisfaction or homesickness of the wife or other members of the family. The previous occupation or finances are of lesser importance, although other things being equal, there is no question that a man with previous farming experience is a better prospect. A certain minimum amount of capital is necessary for any particular set of conditions and larger capital will undoubtedly enable a settler to get on a paying basis sooner if it is judiciously used. Too often, however, the settler with ample capital dissipates it and sets himself back by the amount of time consumed in so doing.

Conditions vary greatly on different projects. On some, the land is native sod which has to be broken; on others, it is desert land, possibly covered with brush. In others, the land lies so nearly perfect for irrigation that water may be spread satisfactorily without preparation; in others, an expenditure of capital, of time, labor, and money must be made in smoothing, leveling, grading, or checking the land before a crop can be planted. Expenditures equivalent to use of the land for a year or more, and up to $50 per acre, are not uncommon. If the settler purchases 40 to 160 acres of land it may easily be a number of years before he can have all of it prepared for irrigation. The proper sequence and proportions of the various preliminary operations and the economical use of time and money are very important and this is where many failures start.

On large projects it will take a number of years to complete settlement and it is desirable to settle it by units, preferably on the best or most favorably located land, giving, also, due consideration to operating economics. Scattered settlement increases annual costs and hampers the development of the social structure.

The crops which can be raised successfully and marketed profitably are among the most important considerations. Alfalfa is the one staple common to all projects in the United States and Canada, which can be grown successfully and for which markets can be most easily developed, but even alfalfa growing usually goes through a period of depression when the supply exceeds the local demands and before the live stock industry has caught up with it. Generally speaking, irrigated lands are adapted to a wide variety of crops which become a drug on the market after the local demand is satisfied and until the community is sufficiently organized to develop markets, or until

the supplies are large enough to induce outside buyers or industries to come to them.

The new settlers must get acquainted with their neighbors, adapt themselves to the new country, learn its climate, amalgamate themselves into communities, create a social structure of schools, churches, business centers, etc., study agricultural practices, and learn to prepare land for irrigation and to practice the art of irrigation. In many cases they must evolve the practices of land preparation and irrigation adapted to their conditions of soil, topography, climate, crops, markets, etc. These usually tend to uniformity on one project, due to uniformity of conditions, patterning after practices on similar projects, and to the necessity of adapting them to the design of the irrigation system. For example, they will differ very materially on a system designed to deliver irrigating heads of 1 or 2 sec-ft. continuously from a system designed for heads of 15 sec-ft., or more, in rotation. The evolution may extend through a long period, gradually changing from a continuous-flow system to a rotation system.

There is a natural and usually unavoidable tendency to excessive use and misuse of water in the early years, for which poor land preparation due to lack of knowledge, time, and capital, inexperience in irrigation, excessive water supply, and ignorance of the dangers of over-irrigation are contributing factors.

The next step in irrigation development then following is the appearance of seepage, a rising water-table, water-logged land, and alkali. These frequently appear on land that has just previously produced exceptionally good crops with little or no irrigation and may cause widespread consternation. Probably, in most cases, these conditions are not actually caused by avoidable waste of water, but the date of their appearance is advanced by it. Most distressing conditions may result. Farmers whose equity in the land is small may move away, others sell at a sacrifice, while others must stay and face conditions. Remedial measures are necessary, but there may be serious obstacles in the way. The price of land was probably based on the construction of the irrigation system only, no allowance being made for drainage and reclamation of injured land. The settlers place the onus on the promoters; the promoters disclaim it. If the land is largely paid for by the settlers, the promoters can wash their hands of responsibility. It may develop as a problem not directly associated with the irrigation system. A loss of time almost surely results during which the ability of much land to produce crops is impaired or destroyed. Eventually, it may require an organization of a district with the raising of additional capital for drainage purposes. Extensive studies are required and at the best there are many unsatisfactory and obscure features and many questions still unsettled with respect to drainage and reclamation of alkali lands on a paying basis. On many existing projects the water-logging and alkali problems have been solved, some fortunate ones have never had them, whereas on others it is so serious as to impair the soundness of the project, or to result in a lower type of agriculture. There are cases in which serious errors of judgment were made by the promoters with respect to the suitability of land for irrigation or to the economic soundness of the development.

Given sufficient money and authority, engineers can build an irrigation system, and, also, to many engineers in the past the design and building of the system has seemed an end in itself. It has given them opportunities to express their personalities and theories in design and execution. They have not recognized, or have ignored, the fact that the diverting dam, the canals, and the structures are only a few among the means to an end, and that every dollar they spend, must be (or is expected to be) returned with interest by the settlers who come after. These payments must be produced from the soil, using the irrigation water as an instrument in production. They must not only pay interest on the construction cost and eventually extinguish the principal, but they must also make payments on the land if it is priced separately, pay the annual operation costs, taxes, and farming costs, but must themselves live, with such degree of comfort, and with such of the social and material conveniences and luxuries of life and with such margin of income over expenditure as will make them feel that they have bettered, or stand to better, the previous condition and opportunities of themselves and their families. It is unfortunately true that too many expect more than is reasonable in the pioneer days and those who win soonest and surest the comforts and luxuries of life are those who practice frugality and self-denial during this period and who build up a capital reserve of productive land, equipment, and bank account.

The irrigation system then should be designed as an instrument of service to the farmers who will come after, and as cheaply as practicable so that the farmers, who must eventually pay for it, will be burdened as little as possible. The design and construction of an expensive or untried type of structure to gratify the designer's vanity may prove to be a costly error. No irrigation system is used to full capacity in its early years, and cheap or even makeshift construction may well be justified, leaving enlargement of the system and permanent construction of many structures to be carried out in connection with maintenance operations as the need arises. It is often possible to enlarge canals by promoting scour, possibly causing the coarser material to deposit where it can be used to advantage in strengthening banks. Much of the finer material may be automatically disposed of by being carried to the irrigated land. It requires more courage, self-restraint, and good judgment to build a cheap system than to build an expensive one, especially if ample funds are available.

An irrigation system should be designed and built for service and it should also be maintained and operated for service. The watchword of every operating organization should be "service" for twenty-four hours every day—personal service to every irrigator. System is nowhere needed more than in delivering water. Rules, policies, and records are necessary, but these should be drawn to serve the best interests of the irrigators, individually and collectively, and not to suit the convenience or swell the pride of the operating organization or its head. The system should be operated so as to contribute in the greatest degree possible to the productiveness of the land and the prosperity of its purchaser. Every member of the operating organization, from the head downward, should be "ordinary folk" with the farmers so that they can achieve a sym-

pathetic understanding of their personalities, problems, and aspirations, and, at the same time, show the farmer that certain rules and restrictions are necessary to an orderly conduct of business and in the interest of the community and the individual. One of the troublesome features of irrigation farming is that each farmer must give up some of this individuality and freedom of action to satisfy the relations of irrigators to each other.

An organization, or organizations, either established by the promoters or by public agencies for experimenting with and introducing new crops, educating the new farmers, developing markets, and guiding the evolution of farming and irrigation practices, is very useful if not absolutely necessary. Such an organization should keep in close touch with farmers, not only to disseminate new ideas, but also to gather and distribute the cumulative experience of the farmers themselves. The operating staff can, with advantage, be organized to perform such functions, but if an independent organization is developed, it should be closely associated with and its work correlated with that of the operating staff.

If land is sold on small payments down and small annual payments thereafter, the farmer has, for some years, a very small equity in the land and, therefore, but little incentive to stay, unless he is getting good returns. If land is cheap, settlers will incline to purchase too much. If times are bad, and the settler gets discouraged, he has no anchor to hold him and can abandon his land with but little loss. In such cases it is more than ever necessary to pick settlers carefully, to be sure they are the right type, that they have at least a minimum of capital, and to have an educational organization to help them and supervise them carefully.

Settlers will cost the colonization department anything from a few dollars each up to several hundred, or even more than $1 000 each, and as little money as possible should be wasted in unproductive expenditures for this purpose. It is probable that in the first years many of the settlers will not meet their contract obligations. Good judgment is required in dealing with them. It is a distinct loss in money, time, and prestige if a settler who has cost several hundred dollars abandons his land. Therefore, each settler should be dealt with individually on the merits of his personality and his situation. It may be a good investment to make concessions to him, or, on the other hand, it may be the best kind of business and a real kindness to him to close him out. Every effort should be made to have each farmer increase his equity in the land by payments, even of small amounts, on his contract and by productive improvements to the land. The building up of attractive home surroundings is a strong tie in holding a settler, but the construction of buildings out of proportion to his means, or the purchase of unnecessary or high priced equipment and live stock is a drain on his resources and is the cause of many failures.

There are times when the promoter feels that it is to the interest of the settler and himself to forego payments when due, in order that the settler may use the money for farm development. It is the writer's opinion that this is almost always unwise. It increases the farmer's debt, leads him into ill-advised expenditures, and tends to dull his conscience with respect to his con-

tract. At any time that a settler's contract obligations are increased by accruals of unpaid interest, operation, and maintenance charges, or taxes above the original principal amount, his affairs and, therefore, the affairs of the promoter, are in an unhealthy condition. The seller of the land is protected only by the margin between total contract obligation and the net price which he can realize from the land if it returns to him, and, in this, account must be taken of the time and cost of abrogating an existing contract, the time and cost of securing a new settler, and the risk that the new settler will fail, even as his predecessor. It is, therefore, desirable to keep the sale to each settler in good standing and one means to this end is to exercise definite, but not irritating, pressure to reduce the settler's obligation at regular intervals. It is the writer's opinion that the agency for this purpose should be located on the project with full authority to deal with each individual farmer, under the limitations of any general rules or policies that may be laid down.

Where land is sold on contract and the settler has no title to the land, or where it is covered by a prior lien, it may be difficult for him to secure legitimate financing for his annual needs. This is a matter which may deserve careful attention. Generally speaking, if he has title to the land and the lien is in the form of a bonded debt he will have less trouble with financing. If and when each of a large proportion of the settlers acquires the major equity in his land, the settlement problems are well on the way toward solution. If and when land values increase, land seekers become plentiful, and loans exceeding the outstanding contract obligations against the average parcel of land are readily available, the problems are largely solved. When these conditions have been attained they will be evidence of the facts that the combination of soil, water supply, irrigation system, location, transportation, and other contributing factors have been favorable, that industrious, wide-awake, and intelligent farmers and business men are located on the project, that economic conditions have been favorable or adverse conditions have been overcome, that crops have been produced and marketed profitably, and that what was originally a group of strangers in a strange land have become a community of friends and neighbors, with schools, churches, and attractive homes. When such a stage has been reached, the problems of the pioneer, which are the problems of land settlement, will be largely settled.

DISCUSSION

W. G. SWENDSEN,* ASSOC. M. AM. SOC. C. E. (by letter).—This paper is a valuable contribution to the meager information concerning this subject. Any discussion of it must be based, necessarily, on experience and observation since it is abstract, not to say complex, and cannot be made to yield to any concrete or scientific analysis. It would be surprising in a discussion of such a subject if wide differences of opinion did not develop, but, notwithstanding this condition, the writer finds himself largely in accord with the author, and will only endeavor, therefore, to elaborate on one or two matters of major importance.

It is no easy task to fabricate a farm from the desert. To accomplish the feat, a person must be equipped either with ample funds or with an unlimited supply of physical strength, endurance, courage, and with a mental development capable of directing these forces; after this is accomplished, the farm provided and improved, the larger problem—its successful operation and the enjoyment of life during the three hundred and sixty-five days each year that must be spent upon it—remains to be solved.

The matter of colonization, after all, is but mere traffic in human beings, and a full realization of the seriousness of the business must be appreciated if success is to be obtained. Success is used here in its broader sense and implies not only financial gain or prosperity, but happiness and contentment as well. To the extent that Government agencies, including States, cities, and municipalities, engage in colonization, it must be assumed that the purpose is to build good citizens, promote public welfare, happiness, and contentment, and add to the well-being of humanity and of the nation, generally. While other agencies are engaged in the business, and quite properly, for financial gain or business advantage, the real seriousness of the matter—traffic in human beings—should not be overlooked. Families should not be encouraged to abandon cities or other homes to take up life on the farm until it is definitely known that they are equipped and fitted for such a vocation. They should be equipped not only financially but, in addition, have general fitness for the new life they are to undertake. Education or understanding of one's work and sympathy or co-ordination with one's surroundings are essential to both financial success and happiness. After all, these are the real essentials in farm life as in other businesses.

Primitive man, by reason of his limited mentality, was contented and happy with a full belly and a gratification of his physical passions. In the present highly intellectual age, understanding of one's surroundings and work are essential to success and happiness. There is no more interesting or wholesome work than that of farming under irrigation, dealing as it does with the natural elements of sunshine, soil and soil fertility, atmosphere, water, etc., and the bringing of these together in maturing plant life. The greatest return cannot be had from such a vocation unless the participant is equipped with, at least, a fundamental scientific knowledge of agriculture and the proc-

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esses which the seeds and plants must undergo to reach maturity. It would, indeed, be a dull and uninteresting life to the engineer or the man in other professions if he were obliged to subscribe to mere formula in the building of structures and the doing of other tasks, and thus proceed without a definite knowledge of the conditions and things with which they necessarily deal. Likewise, it must be a dull life for the farmer, isolated as he is from extended contact with his fellow man, if he is obliged to devote himself to the mere physical task of getting results and is not equipped with the proper training to consider the interesting scientific end of crop production.

The irrigated farm, offering as it does conditions well adapted to intensive farming, the rotation of crops, and with it, the continued building up of soil fertility, and the adjustment of crops to meet market and other conditions, offers advantages not enjoyed in agricultural territories where irrigation is not practiced. With these advantages, however, the farmer also must assume added responsibility and, if success is had, must be equipped to farm under real scientific methods.

So long as the farmer must sit on his plow from day to day with no other thought than that of how nicely the furrow turns over, or how difficult his daily manual tasks are, farming will be a drudgery; but when he is able to understand and analyze the simple, but very interesting and beautiful processes through which plant and animal life pass to reach maturity, drudgery will cease and his daily tasks will become a real pleasure. When this condition is fully realized and satisfactorily solved, the greatest problem in colonization will, likewise, have found a satisfactory solution.

To cite just one glowing example: In 1910 a settler was observed building his house on a piece of land under an irrigation project in the West. The site was on sloping ground. He had not even taken the trouble to level up the foundation on which he was building, and was so inexperienced in matters of this kind that this necessity did not occur to him. After it was called to his attention, he readily yielded to the suggestion and with some assistance provided a level foundation. Later, in the building of the superstructure, this settler made the mistake of putting the weather-boarding on his house upside down. Similar experiences were had in the improvement of the farm and, finally, in its operation, until after a comparatively short time, his funds and courage were exhausted. He had wasted two or three years of his life and that of his family only to meet with dismal failure and to return to his previous work in the city, that of operating a street car. This man was equipped with physical strength and mentality to perform the tasks on the farm, but neither of these energies were trained for such a pursuit. Had the two years wasted on the farm been spent in an agricultural college, or other institution, where the fundamentals of farm mechanics, agriculture, agronomy, soils, and other subjects, are taught, this farmer could have moved on a farm with even much smaller financial resources and probably would have succeeded.

In other words, if colonization on irrigated areas is to succeed settlers must be selected from persons who are trained and skilled in the art of irri-

gation and farming, and who are adapted by natural inclination and environment to the work which they are undertaking.

Assuming that the natural agencies exist, for example, that the soil, climate, market, and other conditions are favorable, that the water supply is sufficient, the irrigation works and systems are adequate and so permanent as to insure a continuity of service in the delivery of water, then the colonization, if done with the right kind of settlers, will succeed. Obviously, this is the only means by which the investment in any irrigation enterprise can be recovered.

It is rather unfortunate that, because of the selfish ambitions of promoters, citizens, communities, and other institutions, the tendency is to promote the reclamation and colonization of new or unoccupied areas in advance of the time when economic conditions, including a demand for increased farm production, really warrant such development. In view of present conditions, the policy should be to promote the interest and prosperity of persons already on the farm to the end that each farm may be made a good one and be put on a dividend-paying basis; if and when this is accomplished colonization of other areas can be had with little, or no, difficulty.

Charles H. West,* Assoc. M. Am. Soc. C. E.—With the exception of Southern California and the foothill fruit areas of the great interior valley, irrigation development in California is for the purpose of increasing the agricultural production of lands already farmed. Most of the small projects are now completed so that construction generally is for the extension of projects or the consolidation of several projects into one larger unit. The problems, therefore, are different from those of the projects of the arid region built for reclamation, for in California the proper type of agriculture is established and the best methods of irrigation practice are well understood. A change from dry farming to irrigated farming involves the subdivision of the land, the leveling, checking, and growing of general crops, a task less difficult than producing a farm out of virgin land.

The pioneer type of project settler is now nearly a thing of the past. A few years ago settlers could become established with little capital and could get along on a small cash income. They produced much of the food they consumed and the standard of living was simple. To-day, the family garden and home-cured meat are seldom used; the neighborhood grocery store and meat market provide the farmer's food; the standard of living is much higher; the family must have automobiles and a radio, and must go regularly to the "movies". Taxes are also higher due to the paved highways, the many modern school buildings, etc. Because of better transportation facilities and better marketing agencies, farm products are brought into competition with a much larger producing area. The keener competition with older producing areas and the increased standard of rural life demand better farming methods, more efficiency in farm operation, and better business methods.

It is almost impossible for a man with no capital to improve and pay for a farm. He must have money enough to improve the land when he buys it or capital enough to pay for his household and operating expenses while he is

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developing the farm. He cannot compete with others if only one-third to one-half his land is improved, except by greatly reducing his standard of living. To-day, the farmer's family is not willing to remain on the land if too much hardship is experienced, for it is generally only a short distance to town where easier work can be obtained.

Owners of land for subdivision, bankers, and real estate men have continually refused to recognize the amount of capital needed to develop farm land. This is emphasized by the lack of interest shown in considering planned farm settlement. If raw land costs $50 per acre and when improved acreage is worth $200 per acre, where is the settler to get the $150 per acre to put into the farm? The new settler usually has about 15% of the capital needed to purchase and improve his land, and, hence, his credit with the bank will be quite limited. Only through help and co-operation from the former owner can he hope to succeed. The former owner often rents (to the settler) farm equipment or additional land and by introducing him and speaking a good word for him to the local banker helps him to obtain credit, and often foregoes payments due him on the purchase of the land in order to help the new settler to become established. If the owner is not willing to co-operate to this extent he should at least make sure that the purchaser either has sufficient funds to see him through to complete development or is discouraged from the undertaking. To see the venture to half completion and there encounter privation and discouragement is unfortunate for both the former owner and the purchaser. Many land owners have found that failures on their projects are poor advertising and, therefore, expensive. It is being realized more and more that settlers who have a possibility of success must be selected and not merely those who have a strong desire to succeed. The principal requisite to success, aside from a strong ambition and an inclination to work hard, is the necessary amount of capital to develop the farm to full production, or the equivalent, an amount of credit at reasonable rates of interest to see the venture through.

To give a more definite idea of the capital involved in a typical farm development in California, the following illustration is presented. This example has been selected at random from the records of numerous farms that have been studied. This farmer purchased 34 acres of raw land in June, 1921, and now has 26 acres of the 34 developed. His investment in the farm consists briefly of the following:

34.23 acres of land at $250 per acre	$8 557.50
Pipe line distributing system	2 379.40
14½ acres of alfalfa, at $33 per acre	478.50
4 acres of pasture, at $25 per acre	100.00
4½ acres of vines, at $235.05 per acre	1 057.71
2 acres of trees, at $142 per acre	284.00
Buildings and barns	3 458.75
Farm equipment	739.60
Household goods	681.00
Live stock: 18 dairy cows, 5 heifers, 1 pure bred bull, and 1 team of horses	3 100.00
Total	$20 836.46

The total expenditure represents $585 per acre and a capital expenditure for improvements of $335 per acre.

Because of refusal to recognize the large capital required to develop farm land no effort has been made to provide financial machinery to bridge the gap between the limited capital of the farmer and the first mortgage credit available when the land is fully developed. This problem merits earnest consideration.

Trouble also has been experienced from failure to appreciate the difficulties and the expense involved in improving raw land. The planning and developing of farm land can be done most economically by those experienced and trained in land development. The trained and equipped organization does the developing in a thorough way the first time, eliminates all waste of time in performing the necessary operations, and does away with the losses caused by doing over things that were imperfectly done the first time. Few farmers do their developing efficiently. The type of checking used may not be satisfactory, or the ditch system is inconvenient, or some of the leveling has to be done over. This is true with all farmers. It takes much more to get the farm layout into final form than was expected. This is only natural, as a farmer usually develops but one farm in a lifetime.

Compare the status of the ordinary tenant farmer, acquiring land, with that of the settler on the new project. The former usually rents until he knows the peculiarities of the farm and has saved sufficient money to make a down payment of 10 to 15% on the purchase price of the land. A contract of sale is obtained and for several years the operator applies his earnings on the purchase price. He works and saves his earnings from a plant the layout and capacity of which are familiar to him. Returns from farming are small. It takes a lifetime for a man to acquire and pay for a farm. A farm represents a large capital. It should take years to acquire such an estate.

The irrigation project settler with no more capital begins the purchase of a farm. The first year he has no income. His funds are exhausted when the farm is about one-third to one-half developed. He tries to develop the remainder of the farm and pay for the land with his plant working at one-third to one-half its capacity. No wonder hardships and misfortune overtake him and his family. In the past, lack of capital has been paid for by privation and hardships. It has been an expensive sacrifice to all but those who get joy in subduing and conquering a difficult situation. Those who persevere and conquer have the pioneer spirit.

From his experience in studying irrigation projects and interviewing farmers to learn of their financial condition and to determine their financial requirements for complete development, the speaker has been deeply impressed with the enormous waste in land improvement and the gross miscalculation, by those improving land, of the capital needed for development. Possibly organizations promoting land settlement have refused to consider the capital needs of settlers because most settlers had less capital than their rough calculations showed to be required.

Besides making due allowances for the wastes and losses in developing the proper farm layout, in order to determine the capital requirements certain assumptions must be made of the income that may be expected from the land

during the period of development. Paper computations of capital requirements can be made by those experienced in land development and colonization work, but to a large extent these figures are arbitrary and the assumptions far from accurate. Engineers make their estimates of the cost of construction based on yardage calculations, current costs of construction, and the usual allowances for contingencies based on past experience. Yet more of these estimates fall short of the costs than equal or exceed the cost of construction. It is highly desirable to determine from actual figures (1) the degree of success usually attained in ditching and leveling farm land; (2) the losses sustained in using temporary installations until capital is obtained with which to install permanent improvements; (3) what are the usual household expenses, incidental and contingent expenses; (4) what are reasonable yields; (5) how often should crop failures be expected; (6) of the crops produced for how much will the producer find a market; (7) what are safe prices to assume; and (8) how much outside employment can the settler find while improving his farm, etc. These data are essential to the project settler. It is easy to determine leveling, checking, and ditch costs for a large outfit of trained men using good equipment and having plenty of capital on which to operate, but the results for the farmer of limited experience in land development, working with poor equipment and with much less capital than he needs, is a far different consideration.

If the owner of farm land to be subdivided will plan and improve the farm units before sale he will eliminate the usual waste in land development, but in order for the farmer of limited capital to make good it will be necessary to sell him land on small initial payments, charge a low rate of interest, and extend the payments of principal over a long period of years. Not many land owners have the capital or the desire to undertake such a program. For owners of farm land who have not sufficient capital to improve and subdivide their property, and in reclamation projects where the land is owned by the Federal Government, some provision should be made for obtaining financial assistance for the settler who undertakes the development of farm lands. This financing agency must be associated with the management of the project and both must be kept free from politics. State universities, the Federal Reserve Board, and the Federal Farm Loan Board perform public service free from politics. This agency must be kept as free from molestation from local bankers, politicians, and settlers' committees as possible, and must have authority to deal firmly with each settler as the merits of the case require. The management must be thoroughly business-like and each settler under contract should be compelled to follow out an approved program of farm development. The agreement must be flexible, but it must also be definite and binding and should not be changed except by authority from the financing agency and the project management. It is evident that assistance should be given only to those who have enough capital to put into the enterprise to assure their interest and close co-operation. Representatives of the management must check up the operations of the farmer to see that he lives up to his contract and performs in a thorough and proper manner the work he agreed to do. Competent agricultural advisers are a necessity for such develop-

ment. Past experience has demonstrated that for successful operation of a project the settlers must have representation in the management.

There are no more misfits in agriculture than in any other industry, but due to a lack of consideration of the problems here mentioned, which are highly essential in planning land settlements, and also due to lack of appreciation of proper financial facilities to finance farm development, it has become a saying that "it takes two crops of settlers to make it possible for the third to stick".

R. A. Hart,* M. Am. Soc. C. E. (by letter).—The author points out the almost universal need for artificial drainage on irrigation projects. The possibility—nay, probability—that such need will arise should be kept in mind from the beginning on every project and the settlers should be educated to the idea. It has been suggested even that the drainage works should be constructed at the same time as the original irrigation works. In few cases would such procedure be practical and in no case would it be economical, since the annual charge against the additional investment would run for years before the works actually were needed.

Drainage works should be installed, however, before the inhibitive condition becomes serious. Not only would the cost of installation be less, but loss of crops would be avoided and the danger of losing settlers would be lessened. No one is better fitted to redeem injured land than its original owner, and the cost of securing a new settler will be saved. It has been found expedient by drainage officials to go to considerable length to keep the present owners on the land after reclamation by drainage.

What is full colonization at one time or under one set of conditions may not be full colonization at another time or under a different set of conditions. Drainage reclamation is a case in point. With the drainage charges added to the original cost, it may be necessary to change the type of crops or, at least, operate on a more intensive scale in order to make the enterprise pay. This means that the average ownership must be reduced by a considerable margin. The new colonization represents a process of infiltration—the most successful and satisfactory kind of colonization. If this sort of colonization could have been applied to irrigation projects generally many of the difficulties encountered in the past would have been avoided. The seed of these difficulties lay in the fact that the supply of new lands greatly exceeded the real demand for them.

R. K. Tiffany,† M. Am. Soc. C. E. (by letter).—This able and comprehensive paper shows a knowledge of the subject that can be gained only by direct and intimate contact with all its various phases. If the paper and the interesting discussion following it appear to wander from engineering into broader fields, it only emphasizes the fact that the engineer who would serve his clients and the public in planning development projects must make a comprehensive study of the social and economic features involved.

Had such a study been made of all the present irrigation projects, public and private, it is quite certain that many of them would not have been built.

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They did not at the time of construction and do not to-day meet the requirements outlined by Mr. Griffin for a successful project, or the requirements that Elwood Mead, M. Am. Soc. C. E., Commissioner of the U. S. Bureau of Reclamation, has indicated* to be essential to his approval of a new project.

It may be admitted that for such projects—some of them ill-conceived and without intrinsic merit, some inherently sound but built before their time—an artificial aid to land settlement will be necessary, and that in order to save present investments and assure the success of the settlers already on the ground, Federal subsidy of such aid on these Federal projects is necessary and justified by the conditions. To establish State or Federal subsidy of land settlement as a permanent policy for the future is, however, quite another question and opens up the much broader field of future State and Federal reclamation policy.

The country as a whole will eventually need the production of all its arid lands. Most of the remaining projects, with their high cost, large area, and, consequently, their long development period, will require a subsidy to the extent at least of freedom from interest charges during the construction and development period; but further than this it will not be necessary to go if the projects are built only as there is shown to be an economic need for their products.

It will be said, however, that the nation should plan for the future. This is granted; it should perhaps even build for the future within certain limits. Herbert Hoover, Hon. M. Am. Soc. C. E., and other distinguished engineers and economists have suggested that some great public works, such, for example, as those that will be required in the development of the Colorado River and Columbia Basin Projects, should be planned years in advance and their construction undertaken by the Government during the periods of industrial depression to neutralize the slackness of employment in private enterprise at such times.

No attempt should be made by Government subsidy to force into production new areas of land in competition with that of tax-paying farmers who are not receiving fair returns on their labor and investment. Further irrigation development in the West should be steady and gradual to keep pace with the natural growth of its markets. The settlers to provide for such extension can be largely found in the natural increase of the present farm population. The experience of the past twenty-five years has shown a steady supply from this source for new units of projects that are economically sound.

As to artificial selection of settlers, the writer would question any further requirement than these—a love for the land and for growing things, demonstrated habits of thrift and industry, and a willing and able helpmeet. Rejection of the inexperienced for that cause alone would have eliminated many of the outstanding successful farmers on the various projects with which the writer has been connected. In an experience of 25 years involving close personal contact with more than 5 000 water users, it is his observation that the

* See p. 733.

percentage of success is greater among those without previous farming experience, but having these qualifications, than it is among those who had farm experience, but lacked one or more of the other requirements.

What of organization, guidance in selection of crops, stock, etc.? The value of such service has been amply demonstrated; but cannot this service be better rendered by existing agencies, such as the State College or University, the County Agent, and representatives of the State and Federal Departments of Agriculture, rather than by the agency that builds the works and must collect their cost from the settler?

Is it not obvious that such advice and assistance is far more effective when it comes from a disinterested agency? If furnished by the creditor agency, every failure will be blamed on the adviser and the very effort to be helpful becomes the basis of personal and political appeals for financial relief, postponement, and reduction or cancellation of the debt. If, in addition, the building agency becomes banker to the settler for a home-building loan, a chattel loan, and possibly a loan for seed and equipment, it would simply mean that the sources of friction and the difficulties of collection increase in geometrical ratio with the number of loans.

Admittedly, the problem of making a success of many Federal irrigation projects is a most difficult one. The experiment of land settlement financed, aided, and directed by the Federal Government may be worth while. Undoubtedly it will be if it is in friendly and capable hands; but for future reclamation development is it not feasible to work out a policy and a program that will be more simple and more economical in administration; that will eliminate the necessity of the settler dealing in his most intimate business and financial affairs with an agency 3 000 miles away; that will make of the new irrigation projects not a specially favored and pampered community but a self-reliant unit of the State, served, developed, and supported by the same agencies as those on which the other sections of the State depend?

The experience of the past, the unfortunate condition of many projects at present, the confusion and utter lack of a definite program before the present (69th) Congress, all join to make the moment opportune for serious consideration of the whole reclamation problem and for an earnest effort to formulate a sound and permanent State and National reclamation policy.

F. H. Newell,* M. Am. Soc. C. E. (by letter).—Mr. Griffin presents for discussion certain ideas which, if translated into action, might enable the "promoter" (Government, district, railroad, or colonization agency) to secure settlement on irrigation projects. These problems of land settlement arise primarily because the land to be settled is not worth what it costs; no really experienced farmer of his own motion will purchase it at the price asked. Thus, the promoter must resort to extraordinary tactics to attract and hold families, promising advice and financial aid such as may enable them ultimately to gain a foothold.

The existence of a settlement problem is *prima facie* evidence of poor land. Really good irrigated land, offered on reasonable terms, needs no special

* Pres., The Research Service, Inc., Washington, D. C.

inducements to attract and hold good farmers. It is the marginal or doubtful tracts that call for this special treatment.

The danger, if it may so be called, of the exploitation of "high-pressure" methods of settlement lies in the fact that promoters are encouraged to urge development of other tracts of similar marginal land. To this extent they create or perpetuate unstable conditions. There is plenty of good farming land to be had throughout the United States, particularly in the arid West. There the quantity of water is limited and the available supply should be kept for and applied to this really good land and not be developed for land of such character that a settlement problem arises. When brought to the good land at a reasonable cost per acre there has not been, nor is there likely to be, any especial difficulty in the matter of settlement.

Under ordinary conditions, the margin of profit in farming is small; on the marginal or inferior lands there is frequent loss at least of the time or equivalent wages of the man who attempts to cultivate it. He is naturally attracted to the larger wages paid by "quantity production" industries. As a result thousands of acres of marginal lands are lying idle or are being abandoned, especially in the older settled parts of the United States. Efforts are being concentrated on the better lands. With improved systems of farming and of marketing, also with a larger use of cheap power and machinery, the more skillful farmer, concentrating his efforts on the best lands, is making a living. Even after the abandonment of these inferior lands, the farmers can increase the total crop production whenever there is a sufficient demand to assure them of a fair profit.

In face of these conditions it is questionable wisdom to pour more money and more effort into schemes to settle lands which in the open competitive market are not profitable, and especially to do so by the use of public or corporate funds taken directly or indirectly from the unwilling or uninformed taxpayers or stockholders. Well-informed farmers and their organizations throughout the country are beginning to view with distrust these attempts to interfere with the action of the laws of supply and demand.

There is a tradition or belief that any dry land that may be irrigated, or for that matter any wet land that may be drained, must necessarily have good productive soil. It has been argued that any such land that has remained idle up to the present time and now can be reclaimed must necessarily be good land because it has not been cropped or exhausted and that, having incurred large expenses directly or indirectly in reclamation, it is the duty of some one to see that it is settled.

Granting that the degree of moisture in the soil can be controlled by irrigation or by drainage, it does not follow that this soil under the new or changed conditions will continue to be tillable even if some crops may be had for several seasons in succession. It has been shown by Mr. Carl S. Scofield* that "soft water makes land hard and hard water makes land soft". The soil after reclamation is not in a stable or fixed condition. The good farmer appreciates this and is becoming more and more cautious of risking his time

* *Journal of Agricultural Research*, Vol. 21, p. 266.

and money on these newly reclaimed lands. The failure of three or more successive owners of such lands has been popularly attributed to some defect in the methods of selecting, directing, or financing the settlers. Too little attention has been given to the fact that fundamentally the cause has been in the inferiority of the land. Recognition of these defects has properly deterred the best farmers from attempting settlement, leaving the field open to those less skilled or competent. "Good land attracts good farmers—poor land remains for poor farmers."

Mr. Griffin notes the saying "that the first two crops of settlers will fail and that the third and following crops may achieve success on the pioneer work of the first two"; he shows that it is not the failures but the early successes that insure a greater percentage of success among later comers. To put it in another way, may it not be that these few early successes were due rather to the fact that the better lands were selected by better settlers and that the superior qualities manifested by these settlers were associated with their ability to select the better lands?

When attention is given to the subject, it is comparatively easy to show that among the pioneers in Kansas, Nebraska, Minnesota, and on through the irrigated West, the good farmers who selected good lands remained to enjoy such success. On the irrigation projects, while there has been as a whole a notable shifting of population, it has been shown that this shifting or these successive waves of settlement have been confined almost wholly to those settlers who took up the inferior lands, some of which, it now appears as soil changes have proceeded, should not have been reclaimed since they are not worth the cost of reclamation and cultivation.

The obscuring of this fact and the special emphasis placed on the beneficial results that may flow from the solution of these "problems of settlement" has its baneful effect in stimulating promoters to urge on the Government the continued reclamation of lands of doubtful value such as would not be selected by good farmers. It is asserted that by careful selection, by aided and directed settlement, by loans, by continual supervision, it may be possible to hold settlers on certain areas. The consideration of the really productive value of these areas, the possibility of marketing the products, and the other fundamental conditions which should be given weight, are pushed into the background by the enthusiastic discussion of what is termed "the human element". "Ignorance is bliss" and, as pointed out by Mr. Griffin, "the art of getting settlers is well developed"; also "within certain limits the newer the project and the less development it has, the easier it is to get settlers."

Skillful practice of this art of getting settlers is being relied on by promoters to offset serious deficiencies in the quality of land. A precedent is sought in the fact that selection is exercised and largely depended upon by an employer, or by the Civil Service, or when enlisting men in the Army. If selection is then necessary or desirable, why not in selling irrigable or reclaimed lands?

A wide difference exists; in the case of the selection of employees or even of farm tenants there is in mind the fact that the persons thus selected will remain indefinitely under the direction of the employer or owner. Every action taken by these selected people will be guided for the benefit of the employer and only indirectly for the benefit of the person selected.

This idea is diametrically opposed to the basic conception of Federal reclamation, which seeks to provide homes on the land for Americans—independent voters and citizens—working first, last, and all the time for the immediate benefit of themselves and their families. People of that sort resent paternalism and do not willingly remain under the direction of an agent or employee of the promoter even if it be the Government.

Under a system of "aided and directed settlement" the assumption is made that the selecting agency must and will dominate and that the promoter or his agents will continue to direct and finance the settlers in their operations, practically assuming the responsibility for their acts. Such selection and direction may be necessary with renters but the theory of American land settlement is to discourage tenancy and to develop an independent land-owning community free from bureaucratic control. Experience has shown that where such aid and direction has been undertaken, it has been followed by the demoralization of the people aided, producing a condition dangerously approaching peonage.

This is touched on by Mr. Griffin, who comments that, in his opinion, it is almost always unwise to forego payments when due as it increases the farmer's debt and dulls his conscience. Striking examples of this unfortunate condition are all too prevalent where, chiefly because poorly informed people have been located on sub-normal lands, there has been a resultant breaking down of the morale. This breakdown has been made more complete through well-meaning efforts to "aid and direct" the land owner by liberal extension of credit and then by permitting a partial repudiation of his debts. It is inevitable that these groups of dependents located on inferior land will call each year for more and more concessions. They are well aware that the promoter, having made a large investment in them, does not dare to let them go. He has already put up a bonus directly or indirectly* and they are firmly convinced that a further subsidy is due them as a partial recompense for their disappointments or for lack of success on these lands. In their minds the promoter has assumed a moral responsibility for their success.

While all this is going on, while the promoter is trying to select and hold his people, he is struggling against a general movement of agricultural population. Under the inexorable law of supply and demand the marginal or inferior lands are being permitted to go out of production, steadily reducing the area of cropped lands in many of the States. This is due to the fact that with improved methods, increased use of machinery and power, crops can be produced on the better lands in excess of needs or markets. The officials of the Department of Agriculture have long since recognized this, and have not been disturbed by the abandonment of inferior acreage. They have steadily admonished against a rapid increase of tilled land until market

* See p. 704.

conditions demand, and then only of the best lands. Economists show that there is already more than enough land under the plow and that the farmers' earnings should be increased not so much by bringing more land under cultivation as by wisely selecting the best areas and steadily improving these.

It is incumbent on the engineer and economist to view with a keenly critical eye the question as to whether in any particular case the probable productive value of the land to be reclaimed will justify the risk and cost of aided and directed settlement. Imagination and sympathy warm toward the man who "made two blades of grass grow where one grew before" but some one must consider whether the value of these two blades justifies their cost. The question properly arises as to whether the promoter, even if this happens to be the Government, is now justified in adopting high-pressure methods of settlement accompanied by loans. No one should be swept away by sentiment and popular appeal to undertake the salvage or settlement of lands regardless of ultimate costs and values simply because money has been spent in their reclamation.

Walter E. Packard,* Esq. (by letter).—The following statement has been made by the author:

"If the settlers on a project are prosperous, have their farms paid for, or can see that they can pay in a reasonable time; in other words, if they are making money, there is no settlement problem. The problem, in fact, is to bring about this highly desirable state of affairs."

There is plenty of evidence to indicate that people will buy land on any project where money can be made. Success breeds success. If farmers are making money raising wheat in Canada, beans in California, cotton in Arizona, or apples in Washington, a flow of settlers will immediately start for those fields of profit. If they succeed, others will follow.

The experience of a settler on a project in California that has been successfully settled since the World War, will serve to illustrate the importance of profits as a factor in promoting land settlement. A university graduate with $50 cash, some implements, a team of horses, and sufficient experience to keep him from making too many mistakes, contracted to purchase 100 acres of land in 1920. He rented 150 acres more and farmed all of it himself, with occasional help during the rush seasons. He made $10 000 the first year with a barley crop that yielded 58 sacks to the acre. He has not made less than $10 000 per year since and is now raising beans on land that will readily rent for $35 to $50 per acre. Needless to say, an opportunity for making profits of that kind is attractive. There is no serious settlement problem under such conditions. Stampedes for land would occur in the future as they have in the past if opportunity for real profits was offered. The problem arises because "that highly desirable state of affairs" does not exist on the average project.

Questions of advertising, promotion, selection of settlers, low rates of interest, and long-time payments are all affected by the primary problem of securing a profit from the land. If that problem can be solved for any project the other problems become relatively unimportant.

* Care, Presidente Comision Nacional de Irrigacion, Chapultepec, Mexico.

Too little attention has been paid to the soil in considering the possible profits from the land on a project. Good soil, plenty of water, adequate drainage, and a climate which allows for a wide range of crops, are essential factors if money is to be made. Compare those projects that are having trouble with those on which some progress is being made and it will be found that some one of these basic factors is lacking. If the soil map is compared with a map showing the density of the farming population one will find that the rich loams are in the center of the settled areas, and that the poorer soils are more sparsely settled. Projects including good soil, plenty of water, and adequate drainage will develop into prosperous communities. It is the project with poor soil, little water, and poor drainage that presents the problem. Long-term credits or low rates of interest will not make a garden out of a gravel pit or prosperous farms on goose land areas.

Even with good soil a man must have sufficient land to produce an adequate income. A French peasant may be satisfied to go by foot to his little 2-acre strip of land at sunrise, with some bread and sour wine to keep him nourished, but such a prospect would not settle a project in this country. A man cannot have the comforts of life, educate his children, or be content on a peasant's income. The acre and independence idea has not worked out. A profit of $25 per acre is all right if there are sufficient acres. It is the total return that is important. The settler on the project mentioned previously would not have remained if he had been limited to 40 acres. The Imperial Valley dairyman with 80 acres is more content than the man with 40 acres, even where yields per acre are as high as they are in Imperial Valley. On the other hand, a man does not want to be land poor; carrying unproductive land is certainly no successful pastime.

The success of a project often hinges on the part of the engineer in the program. His responsibility is to construct a project that will have an adequate supply of water at a cost which the farmer can pay. Unit construction and the utilization of temporary works are mentioned by Mr. Griffin as important factors in this respect. The irrigation system is for service to the settler and anything that can be done to improve that service should be done. Drainage should be provided for in the budget and every effort should be made to carry out the drainage program in time to prevent unnecessary loss to individuals whose land may become affected by a rise of water level.

The price of land is usually the big barrier the settler must meet. The price of land includes more than the purchase price. The cost of leveling, of putting the land into crop and of waiting for returns, and, in pioneer communities, of allowing for an adjustment to market conditions, must all be included in the price the settler pays. An arbitrary purchase price is usually set, which is supposed to permit of a profit to the owner. In the long run, however, the price of land is a residual item and depends on the demand for the products as well as on how much of these products the land will produce. Advertising will dispose of land ultimately, but if the price is not right the first settler will likely fail on account of inability to pay interest and make a living.

Terms of payment are, of course, helpful in enabling a settler to meet the price he has to pay. They also allow a man of meager means to purchase

land when otherwise he might not be able to do so. Long-term payments, however, are frequently used to justify high prices for the land, which constitutes a burden rather than a benefit. Long-term payments do not always appeal to the best class of buyers for they conclude that, if such payments are necessary, they prefer to go where the land can be paid for in a shorter time.

A discussion of the problems of settlement naturally leads to an analysis of the motives behind settlement. Mr. Griffin mentions three: First, to secure returns on the investment, whether it be in land or irrigation construction; second, to populate and develop the project as a National enterprise; and, third, to create traffic and other business.

If the development creates a real opportunity the promotors are genuine benefactors and all concerned prosper. If, however, three generations of settlers will be required before the land is successfully developed, the settler has some justification in considering himself the real benefactor. The settlers in Palestine carry this feeling so far that they consider themselves an army of occupation, preparing the way for those to follow, who are supposed to pay the bill. In reality, it often resolves itself into a struggle for all concerned—a struggle in which the settler is not the only loser.

There is a middle-ground or average project on which the settler has a good chance to acquire independence, if he is willing to work, deprive himself of some comforts, and undergo some hardships. Legitimate advertising, with helpful assistance for the settler who may not be familiar with the locality, will bring results. Each family is an asset and although the growth may be slow, the ultimate result will be the establishment of a community.

Pioneering depends very largely on the individual. A man with tenacity of purpose will pull through and create a home for himself where a weaker man will fail. If the successful settler could be selected in advance, it would simplify matters considerably. It is impossible, however, to select settlers satisfactorily, except in a very general way. Often the man who appears to be weak has the willingness and ability to succeed where the good prospect may fail. The establishment of a financial standard is helpful, although not always best. Men with money are often losers, as Mr. Griffin indicates. In a going community the failures take care of themselves. In a pioneer section, however, the failures are conspicuous.

Projects should be carefully selected before they are launched. The poorer land will come on to the market in time, when the margin of production is extended by the pressure of population. That time will come soon enough. Only the best land should be developed, especially if the water supply is not sufficient for all the land available. When good land is offered at a reasonable price the settlement problem will not be serious. The present temporary lack of demand for land will change when agriculture as a whole is on a better paying basis. When an industry such as agriculture receives National advertising because of the low price of products and the consequent suffering of some communities, it will take some time to bring enthusiasm for that industry back to a healthy basis. The establishment of "that desirable state of affairs" will do it.

Augustus Griffin,* M. Am. Soc. C. E. (by letter).—The writer is pleased with the frank discussion of this paper and wishes to express his thanks to those who have contributed to it. It is not a simple subject either to outline or to discuss. The details of land settlement and its attendant problems vary widely on different projects.

The one fundamental fact brought out is that the problems are comparatively simple if the land is worth the price. Quality of soil, price of land, annual costs of operation and maintenance, and costs of land preparation are relative and are of greatest significance in relation to products and markets. Time is an important factor; the poor project of to-day may be the good project to-morrow.

One of the highest costs paid by the pioneer settler may be isolation—the lack of social intercourse and the lack of educational and cultural opportunities for a growing family. The most pathetic regret the writer has heard was expressed by a man who had lost the substantial savings of his active life in an agricultural venture in a pioneer section and was reduced to day labor as a means of livelihood—a regret that his children had passed through the formative period without schooling and without the cultural advantages of a settled community. The same regret has been expressed by others who achieved a measure of financial success under similar conditions. For the same reasons others have given up before financial considerations became a factor. Undoubtedly, any settlement should be "close" settlement in small units.

The prevailing opinion of the day is that ample land is now available to supply present demands and that new projects should be undertaken with great caution. It must be recognized, however, that there are strong inducements for owners of large tracts of land to undertake colonization. The pressure of accumulating interest and increasing taxes may be very great. There is genuine need for National development whether or not it be actually as pressing as it may seem. There are good reasons why the pioneer settler may consider himself the benefactor.

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AMERICAN SOCIETY OF CIVIL ENGINEERS
INSTITUTED 1852

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Paper No. 1611

IRRIGATION DEVELOPMENT THROUGH IRRIGATION DISTRICTS*

By E. Courtland Eaton,† and Frank Adams,‡ Members, Am. Soc. C. E.

With Discussion by Messrs. Richard R. Lyman, J. B. Lippincott, O. V. P. Stout, and E. Courtland Eaton and Frank Adams.

From the earliest settlement, co-operation has been the dominant factor in irrigation development in Western America.§ In one form or another, it has been the means whereby at least 50% of the lands now irrigated in the seventeen Western States have been provided with an irrigation water supply. It has occupied the great middle ground in irrigation between individual and partnership effort on the one hand, and commercial and Government effort on on the other. Furthermore, throughout irrigation history, co-operation has been the stabilizing influence in the creation of American irrigation institutions and policies, for through it men have learned that in irrigation, the interests of the community far transcend the interests of individuals.

The irrigation district is the most effective agency for co-operation in irrigation development. To the will to co-operate it adds prescribed forms and methods of government, the power of taxation, and the guiding hand of the State. By these means it has established itself as the most generally accepted agency for financing and operating the works of irrigation too large or too costly for successful individual effort.

Nature of Irrigation Districts

In all essential particulars the irrigation district is an agency of the State. It has a political status comparable to that of a town or a county, that is,

* Presented at the Summer Meeting, Salt Lake City, Utah, July 8, 1925.

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§ For a history of irrigation districts in California, 1887-1915, see *Bulletin 2*, California State Dept. of Eng. For an account of irrigation districts in all the Western States, see *Department Bulletin 1177*, U. S. Dept. of Agriculture, published in 1923.

legally established boundaries, an elected governing board, the power to tax and to create bonded debt, and the power to construct and operate works, save only that its authority is limited to matters relating to irrigation and to such coordinate affairs as drainage and hydro-electric power.

Present Status of Irrigation Districts

Irrigation districts are found in all the seventeen Western irrigation States except Kansas and Oklahoma—more than 500 in all, of which 244 were operating, 37 under construction, and 159 in preliminary stages, according to a Government tabulation, in 1921. The total area embraced in irrigation districts in that year, omitting those inactive, exceeded 11 000 000 acres, and the irrigation district bonds outstanding on December 31, 1921, approximated $105 000 000. Since then irrigation district activity has been very considerable in a number of the States, particularly in California and Idaho. In these two States alone 48 additional districts have been organized since 1921, embracing more than 1 000 000 acres and involving proposed expenditures in excess of $20 000 000. For ten or twelve years the United States Bureau of Reclamation has favored the irrigation district as the agency through which it shall deal with the water users on its irrigation projects, and many of the Government projects have been organized into irrigation districts. It is now the settled policy of the Bureau of Reclamation that all Government projects shall be so organized. Furthermore, outside the Government projects, practically no new irrigation development of great importance is now attempted on any other basis of organization, that is, the irrigation district in some form, whether of the type and name now most prevalent, the "conservation" district of some States, the larger water storage districts recently organized in San Joaquin Valley, California, the water improvement districts of Texas, or the irrigation or conservancy districts formed under certain special legislative enactments.

State Control of Organization and Financing

When the first irrigation district law was passed—the Wright Act, enacted by California in 1887—the thought uppermost in mind was to give communities the power to carry forward irrigation development against the opposition of strong minority land owners. Apparently little thought was given to the possibility of unwise action by proponents of irrigation enterprises. Consequently, no adequate safeguards were included in the legislation. The experience of the first ten years in California established the need for a more conservative procedure and this was provided in 1897. Further experience in California, as well as in several of the other Western States, confirmed this need and also the need for State regulation not only of irrigation district organization, but also of financing and construction of irrigation district works. Such State supervision is now general. To cite California alone, 90% of the time of the State Engineer, all the time of one irrigation engineer, and occasional part time of several others, is consumed in investigation and supervision of irrigation and water storage districts. The results of this State supervision have been gratifying in all the States providing it. Although the State authorities

charged with such supervision have the difficult task of safeguarding the interests of the communities, the States, and the investment market for irrigation district securities, they have succeeded so far as to restore irrigation district bonds as accepted investments not only in Western investment centers, but also in the larger markets of Chicago, Ill., New York, N. Y., and several other cities entirely outside the irrigation field. To be sure, these markets are conservative, but they should remain so in order that only sound issues shall find a sale and that the influence of a discriminating investment market shall be joined with that of supervising public officials in holding irrigation district development to enterprises that are physically feasible and economically justified.

FIG. 1.—DISTRIBUTION OF IRRIGATION DISTRICTS IN THE UNITED STATES, DECEMBER 31, 1921.

TYPICAL IRRIGATION DISTRICTS

To those members of the Society who have not had contact with irrigation, it might be profitable to describe very briefly a few typical irrigation districts. As the irrigation district activities of the writers have recently been in California, it is most convenient to cite in these brief descriptions several districts in that State. In California, with minor exceptions, irrigation districts have been formed to improve land already in production, or to

re-finance enterprises initiated and carried to some considerable development by private capital, rather than, in the first instance, to reclaim desert areas. To that extent the California districts do not fully present all the conditions found in some of the Western States. California, however, by virtue of its varied climate in different areas, has widely different types of agriculture, and, consequently, a number of different types of irrigation systems. Fig. 1 shows the distribution of irrigation districts in the United States as of December 31, 1921.

Hot Springs Valley Irrigation District.—This district is in the extreme northeastern county of California at an elevation of about 4 500 ft., in the region of the "horseback" farmer, using the crudest methods of irrigation. Here, water is applied to the wild meadow hay, or alfalfa fields, simply by flooding directly from Pit River, or through inexpensive channels, diversion from the river being made by means of temporary earth, brush, or timber check dams thrown across the stream each year. The district organization was formed to finance the construction of an earth dam in Big Sage Valley to impound water for supplemental summer use. Table 1 gives some statistics on the Hot Springs Valley District.

TABLE 1.—Hot Springs Valley Irrigation District.

Area, in acres	9 640
Bonded debt per acre	$15.85
Water supply	Pit River and tributaries
Type of system	Gravity
Storage capacity of reservoir, in acre-feet	77 000
Principal crops	Meadow hay, alfalfa, grain
Duty of water, in acre-feet per acre	2 net
Average cost of water per acre per year	$1.53

Turlock and Modesto Irrigation Districts.—In the central part of the State in San Joaquin Valley are the Modesto and Turlock Irrigation Districts, organized in 1887 immediately after the passage of the Wright Act, the first two to be formed under it. Although separate districts, they obtain water from the same stream and by means of the same diverting structure—La Grange Dam. Together they embrace 259 973 acres. Production embraces alfalfa, grain, and the full range of vines, nuts, and deciduous fruits grown in California. At first without a late summer water supply, the need for storage later became pressing, and the districts jointly financed and constructed Don Pedro Dam and Power House, a few miles up Tuolumne River from La Grange Dam, and now have nearly a 100% water supply. These districts are perhaps chiefly of interest to engineers as having demonstrated the possibilities for obtaining revenue from the sale of power as a by-product, with the ownership and full control of this power in the hands of the districts, which are themselves distributing it to the individuals, towns, and communities within their boundaries. Under agreement, surplus power is sold at the sub-station at 4½ mills per kw-hr. Don Pedro Dam, 279 ft. above the stream bed (Fig. 2), forming the highest structure of the kind in the world, has made this distinctive feature of these districts possible. In Tables 2 and 3 some statistics on these districts are presented.

Lindsay-Strathmore Irrigation District.—Still farther south, in San Joaquin Valley, is a small but highly developed irrigation district—the Lindsay-Strathmore—representative of high annual costs and typifying the length to

Fig. 2.—Don Pedro Dam and Power-House, Modesto-Turlock Irrigation Districts.

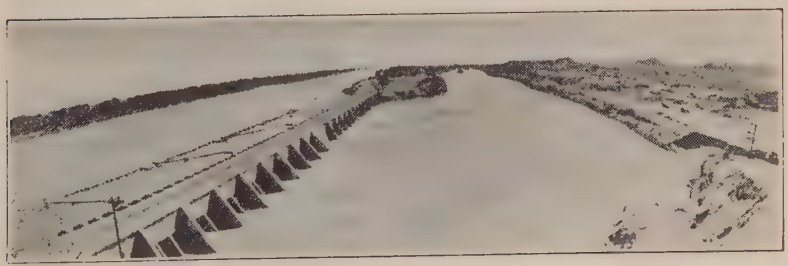

Fig. 3.—Rockwood Diversion Dam, Colorado River, Imperial Irrigation District.

FIG. 4.—RUTH DREDGER REMOVING SILT FROM CANAL, IMPERIAL IRRIGATION DISTRICT.

FIG. 5.—VIEW OF IRRIGATED LAND, IMPERIAL IRRIGATION DISTRICT.

which the successful fruit grower feels he can go to obtain an adequate water supply. The development is an old one, principally citrus, commencing by irrigation from private wells. At first, centrifugal pumps set in shallow pits were effective, the lowering water supply later necessitating the substitution of deep

TABLE 2.—DON PEDRO DAM, MODESTO AND TURLOCK IRRIGATION DISTRICTS.

Location	On Tuolumne River
Type	Concrete "gravity" arch
Height above stream bed, in feet	279
Crest length, in feet	980
Concrete, in cubic yards	278 810
Storage capacity of reservoir, in acre-feet	250 000
Cost per acre-foot of storage capacity	$12.55
Power plant	15 000 kv-a.
Cost of power plant per kilovolt-ampere	$61.00

well pumps. For a number of years the water-table dropped an average of 8.8 ft. per year, finally standing at 150 ft. below the ground surface when the district was organized. The rapid depletion was due to the location of the area close to the foothills where there was little annual replenishment. To make matters worse, at a depth of about 150 ft., salt water made its appearance, and in some cases as much as 22 lb. per tree per year was carried in the irrigation

TABLE 3.—MODESTO AND TURLOCK IRRIGATION DISTRICTS.*

Combined area, in acres	259 973
Bonded debt per acre, Modesto District	$60.39
" " " " Turlock District	$40.66
Water supply	Tuolumne River.
Type	Gravity distribution with storage.
Principal crops	Alfalfa, vines, deciduous fruits.
Duty of water, in acre-feet per acre	2.5, net
Average cost of water per acre per year:	
Modesto District	$5.86
Turlock District	$3.73

* Hydro-electric power is generated and distributed by these Districts.

water, with disastrous results. Therefore, 1 100 acres of water-bearing land in the delta of the Kaweah River was purchased by the District and water was pumped from thirty-nine wells through concrete lined canals and a concrete flume (of special interest because built with a Gunite machine) and steel pipelines, from which it is metered to each consumer. The entire lift is 285 ft.

TABLE 4.—LINDSAY-STRATHMORE IRRIGATION DISTRICT.

Area, in acres	15 285
Bonded debt per acre	$107.95
Type	Pumping from wells with pipe distribution
Principal crop	Citrus fruits
Duty of water, in acre-feet per acre	1.6, net
Average cost of water per acre per year	$26.24

Imperial Irrigation District.—Farthest south in California, next to the Mexican border, is the Imperial Irrigation District (Figs. 3, 4, and 5), in the land of the real desert, where absolutely no economic production would be pos-

sible without irrigation. This is the largest irrigation district in the United States. Here, rainfall is hated, because it is never sufficient to have value and is a nuisance in gumming up the roads. Irrigation is practiced all the year round. After the desert hummocks were removed and the land was leveled, grain was planted as the breaking crop, followed by alfalfa. In the World War period, cotton was the chief single crop and is still largely grown, but the Valley is in the main one of diversified production, with cantaloupes, lettuce, and asparagus for the early markets perhaps outstanding. The most interesting engineering features of this enterprise are associated with its control of the Colorado River and its unusual silt problems. The water, diverted from the Colorado, contains 0.6% by volume of silt and sand which are removed from the canals by dredging and sluicing. The sluiced water is passed into Salton Sea, 250 ft. below sea level, as is the drainage water also. The process presents a duplication in miniature of the building up of the deltas at the mouth of the Colorado. The district maintains more than 60 dredges and 1 300 men in the annual clearing of its canals and laterals from Colorado River silt deposits, which, if not removed, raise the canal beds from 6 to 12 in. per year. Statistics on the Imperial Irrigation District are given in Table 5.

TABLE 5.—Imperial Irrigation District.

Area, in acres	605 000
Bonded debt per acre	$26.45
Water supply	Colorado River
Type	Gravity distribution without storage
Principal crops	Alfalfa, cotton, cantaloupes, lettuce
Duty of water, in acre-feet per acre	3, net
Average cost of water per acre per year	$5.50
Annual cost of silt removal	$1 000 000

Some Conclusions

Engineers are interested in irrigation districts because at present, and apparently in the future, those who are working in irrigation, whether on investigations, construction, maintenance and operation, irrigation machinery and equipment, and to a very considerable and increasing extent, on hydroelectric development, will find themselves dealing with irrigation districts. Some of the obvious conclusions from irrigation district history, therefore, should be of interest.

First, the irrigation district cannot safely be used as the irrigation agency in uneconomic land developments or speculation. This has been tried over and over again and in every case failure has resulted. Although in some of the States the irrigation district form of organization must be resorted to in order to initiate projects involving the reclamation of arid lands, the proceeding is exceedingly difficult and hazardous. Until 1921, inclusive, 58% of all districts formed were for supplemental construction or the acquirement of existing systems.*

The ideal conditions for the organization of an irrigation district is to find the enterprise already under way, although not adequately financed; sufficient

* *Department Bulletin 1177*, U. S. Department of Agriculture, p. 54.

settlement and sufficient voting population to insure a *bona fide* decision on the organization and financing program on the basis of real merit as contrasted with speculation; and established agricultural possibilities. State regulation of irrigation district organization is an absolute requirement if unsound district ventures are to be stopped at their inception—projects with inadequate water supply, unproved agricultural possibilities, uneconomic costs, and unpromising opportunities for settlement. In deciding upon the feasibility of an irrigation district project agricultural and economic factors of production and settlement are equally important with good engineering.

In planning an irrigation district system the only safe and fair course is to have the plan comprehend not only the immediate needs of the district, but also those that will be required for the completion of the system even if not to be constructed for some years to come. Experience has proved it to be far safer to tell settlers and irrigators the whole truth about what irrigation water will cost them when the system, including storage, distribution features, and necessary drainage, is finally installed.

Although the market for irrigation district bonds has been gratifyingly receptive to sound issues up to several millions of dollars, there is not yet a satisfactorily competitive market for the larger issues such as those recently put out by some of the districts. Lack of competition among bond dealers recently made one irrigation district take a discount of 4% on a $10 000 000 issue, and part of it brought a 6% profit to the dealers. This is more than irrigation can stand, and some way must be found to supplement the credit of districts proposing such large issues.

Although the ultimate authority in the management of irrigation districts is placed by law in the boards of directors, very properly subject in most States to a measure of State control of financing and construction, the real leadership must be furnished by the firm, intelligent, and tactful engineer.

It is the engineer who, through his skill, must hold the irrigation district in safe bounds and make it go; who must supplement the lack of business experience of the average board of directors by his judgment; who above all must keep his equanimity and carry on in spite of the all too frequent ill-support and lack of understanding by land owners. It is the engineer who must ever have in mind that, however efficient his services, the professional relationship between the engineer and his irrigation district client is still a more or less unstable one, because the human beings who constitute irrigation districts, like the rest of us, have not yet learned all the lessons of co-operation.

DISCUSSION

RICHARD R. LYMAN,* M. AM. SOC. C. E.—This paper covers the subject so thoroughly that little concerning the character and importance of irrigation development through irrigation districts can be added.

The farmer in the humid regions is perhaps the most independent and, therefore, the most individualistic citizen in the United States. He plows and plants and harvests, independent of his neighbors. This immunity from the necessity of help from others has given these farmers an independence which under certain conditions may be a serious handicap.

Farmers produce life's necessities. They could control the food supply. Before a man will endure long the pangs of hunger, he will gladly give all he possesses for food. With this advantage if farmers were as shrewd and, shall it be said, as selfish, as many business men, they would "sit tight" and demand prices for foods that would make them the multi-millionaires—the great "captains of capital".

Will they do it? They will not. Why not? Because they cannot; they are too independent. They have not had to co-operate, they have had no experience working with others, they will not combine. In fact, their natures and training are such that they cannot work together—they cannot trust their fellow farmers. Where, however, irrigation is a necessity and the ditches to be built are greater than one farmer can construct, he is compelled to get the help of his neighbors.

Now that all the simpler irrigation systems in the West have been built by individuals, partnership enterprises, and private corporations, the day has come when public corporations must be called on to furnish for long periods the great capital needed. So stupendous are the dams, canals, power plants, and other parts of the irrigation systems remaining to be built that their construction demands not only the co-operation of the States, but that of the Federal Government.

Thus has been thrust on the farmers of the West the necessity of co-operation, the idea of working together, and the irrigation district is the latest and, thus far, when organized under favorable and proper economic conditions, the best co-operative institution for farmers that has been devised. It has in it the municipal idea. Practically the same stable machinery that controls and operates counties and cities also operates and controls these districts.

These organizations have many advantages: They operate under popular control; they can be formed without the distressing delays required when private contracts have to be made; no abstracts of title need be secured; no strong, dissenting minority can delay progress indefinitely; theoretically, at least, no profit is paid for promotion; the assessments for these districts are made the same as those for general taxes, and they are collected by the same agency. The obligations of the district have priority over private mortgage liens on the property in the district, and with the approval of the Secretary of

* Civ. and Cons. Engr. (Lyman & Pack), Salt Lake City, Utah.

the Interior the district secures control of even the public land that is located in it.

Such an organization necessarily has a standing in the financial world that individuals and private corporations cannot possess. Districts can co-operate readily with the U. S. Reclamation Service. They eliminate undesirable speculative features, and are able, because of their stability, to continue successfully when organizations without such standing might fail. Land operated under an irrigation district can be put under cultivation sooner, can meet assessments earlier, will have a more rapid development, and will, in a shorter time, reach the profitable stage.

While these are some of the advantages of an irrigation district, there are disadvantages. In a district definitely specified benefits are assessed against specified areas. Once made, this assessment, however unfortunate, however unjust, can be changed, it seems, only with very great difficulty.

In one district in Utah, so the speaker has been informed, the land owners organized, thinking that water allotments could be increased or decreased from time to time. It seems that many allotments were increased without the knowledge or consent of the land owners. Allotments were improperly and perhaps hurriedly made.

For example, the original cost per acre-foot of water was roughly $100. The original area to be benefited was given as more than 7 000 acres. The actual area proved to be less than 4 000 acres. The bonds were sold at $84. The required annual payment per acre is, in round numbers, $16 for each acre-foot of water, and, therefore, for land having an allotment of 4 acre-ft., the annual cost per acre (and these payments will cover a period of twenty years) is $64. It has been stated that 4 acre-ft. have been allotted to some land on which no water at all can be used. No one under these conditions would make such payments.

It looks as if all concerned in this district had in mind in the beginning that amounts allotted could be changed from time to time and that the final adjustment would be to the satisfaction of all interests. The Utah law, however, at once fixed these allotments so that the grossest injustice, it seems, cannot be corrected. The law reads:*

"The Board of Directors * * * shall * * * make final revision and allotment of available water * * * provided that * * * such final allotment may not thereafter be decreased as long as there may be any outstanding indebtedness in excess of 2% of the assessed valuation on the lands within the said district."

Then, again, if assessments in this particular district are not paid, the land, it is stated, cannot be sold because the metes and bounds of the areas on which the assessments have been made, have not been given in any case. The wording of the allotment is somewhat as follows: "Ten acres of a specific 40 acres has been allotted 3 acre-ft. of water." How can a deed for such a 10-acre tract be prepared?

* Session Laws, 1921, p. 191, Chapter 73, Sec. 11.

Confusion, misunderstanding, and ill-feeling—these are the results thus far of the organization of this irrigation district. Tremendous financial losses also must come sooner or later to all who have a financial interest in it.

Certainly farmers are entitled to know the complete facts concerning a proposed irrigation district before they assume the financial obligations it imposes. The State or the Government, or the two together, should provide competent engineering service in all cases of this kind. The Mapleton and Springville Irrigation Districts, in Utah County, Utah, which are operating under the Strawberry Valley Reclamation Project, seem to be pronounced successes.

The irrigation district is a boon to irrigators. Experience will soon disclose the imperfections in the present law and present methods of organization. With these corrections, irrigation districts will be powerful agencies for promoting co-operation among irrigators and for advancing agricultural interests generally.

J. B. Lippincott,* M. Am. Soc. C. E.—The State of California now has had thirty-eight years' experience in organizing and operating irrigation districts. Of this period the first ten or fifteen years were unsatisfactory because the law was made the medium of speculation in land and water. For the last twenty-two or twenty-three years there have been a series of amendments to that Irrigation Act that are particularly directed to the enforcement of the collection of the assessments and obligations of the irrigators. Probably the most effective factor has been the appointment of a State Bond Certification Board composed of the State Engineer, the Attorney General, and the Superintendent of Banks of the State, who, with their respective expert knowledge and abilities, are able to pass on the legal status, the engineering features, and the financial features of the irrigation districts for presentation to State authorities.

It has been stated that 2 000 000 acres have been irrigated by the Federal Reclamation projects. California has irrigated more than twice that acreage under the State Irrigation District laws. There are now 98 irrigation districts in California. The breaking up of large land holdings has come automatically so that, in 52 districts of which the speaker has records, the average area of a farm is 58 acres, which is a small farm unit; but the small farm unit is the ideal one and the one which the Federal projects are endeavoring to approach or reach.

The enforcement of the collection of the assessments for interest and operating expenses has been made most effective. The primary authority is the Board of Directors of the District, but if it fails in carrying out its duty, the County Board of Supervisors and the County Attorney are required under their bond to carry out and enforce the necessary acts for the collection of these moneys. If the County authorities fail, then the Attorney General of the State is, under the law, obliged to proceed in the collection of the necessary funds for the payment of these charges. If the taxes are not paid, the lands are sold by these authorities. As a result the irrigation district bond in

* Cons. Hydr. Engr., Los Angeles, Calif.

California to-day is considered a more secure investment than the ordinary municipal or school district bond.

In California only 1.47% of the area of the irrigation districts—of more than 4 000 000 acres—is in any way delinquent in the payment of their obligations; but 1.94% of the bond issues are temporarily delinquent. According to the bondholders and the State authorities these small adjustments are well under way toward payment.

The irrigation district bonds in California are on a 5½ to 6½% basis whenever they are approved by the State Bond Commission. If they are not approved by the Commission, they are not marketable. As far as irrigation problems are concerned, the State Irrigation District is a satisfactory and effective institution. It has its defects just as other large institutions have, but it is a success. All this refers now to "The California Irrigation District Act". There are other broader Acts that provide for co-operation between the various districts, political units of the State or subdivisions, or with the Federal Government if desired.

This record is of especial interest in connection with the various reports of the failure to make adequate and prompt collections on these Federal and Reclamation projects.

In the selection of these Federal projects, the engineer was there, of course, with his reports and advice; but the Congressional delegations—the delegations of Chambers of Commerce, delegations of farmers, delegations of citizens who went to Washington and pressed their claims on Congress and the Secretary of the Interior—they were largely the controlling factors in obtaining the decision of the Secretary of the Interior, which is the final decision in the "adoption" of the project. The engineers were all "good fellows" as long as they had money to spend, but when the money was gone, these same delegations were back in Washington with all their reasons why the bills should not be paid and why these contracts should not be carried out on the part of the irrigators—reasons why they should not pay for these projects that they have had built, without interest. It was that pressure of "statesmen" and of politicians that was instrumental in encouraging this delinquency. The engineer has been a very convenient "goat".

While this "Fault-Finding" Commission has gone throughout the country giving a volume of its reasons why these payments have not been made, the real reason is because they have not been insisted upon. California has the same sort of farmers with much the same crops, but they have paid their bill because they have had to pay it. If the same policy were adopted throughout arid America by the Reclamation Service it would have had the same experience as California. The engineer is not responsible for this situation.

The worst conceivable way to go before Congress or State authorities to obtain additional money for the expansion of irrigation enterprises in arid regions is with a vast number of repudiated contracts. Before getting more money for big enterprises, like the Colorado River or the Columbia River projects, the thing to do is to pay the bills for the work that has already been done.

O. V. P. Stout,* Esq. (by letter).—This paper is a remarkably able and concise summary of a large subject. It leaves little in the way of discussion except an elaboration of the points summarized. It informs those who have not had contact with the subject and orients those who have had that contact.

Some emphasis may be added to the statements at the close of the paper, relating to a phase of peculiar personal interest to the average engineer, but scarcely unique or peculiar to the irrigation engineer or with irrigation districts. The writer has had some opportunity to observe that "the professional relationship between the engineer and his irrigation district client is still a more or less unstable one," and to speculate on the causes and the outcome. The cause is undoubtedly as stated, that "the human beings who constitute irrigation districts, like the rest of us, have not yet learned all the lessons of co-operation." The irrigation district, however, should be one of the best schools in which to learn co-operation, because it is imperative to the success of the district and to that of the individuals who compose it that the lesson shall be learned. Unfortunately, most or all of them are slow to learn. Fortunately, however, there are some indications that in time they do at least begin to learn.

In California, some of the older and larger irrigation districts seem to have reached that stage of progress in their learning at which a competent engineer-administrator of their affairs can be recognized, the affairs placed largely in his hands, and his tenure of position made fairly secure. Other districts, however, not all of them small nor young by any means, having for a period employed competent engineer-managers, have not only failed in appreciation of the work done by them, but also, on the basis of the little learning acquired in connection with the enterprise, have concluded that lay management of irrigation works is entirely adequate, and have dispensed altogether with engineering advice and assistance. Others have not gone so far, but have placed the engineering, even on projects of some magnitude, in the hands of either very young men or of those whose engineering knowledge and training do not extend beyond the routine of the particular project in question.

In the absence of any immediate issuance of securities or the undertaking of important construction operations the State has no authority to compel engineering service to such districts.

E. Courtland Eaton† and Frank Adams,‡ Members, Am. Soc. C. E. (by letter).—The discussions by Messrs. Lyman, Lippincott, and Stout are much appreciated by the writers. These discussions have helped to illuminate a presentation that was necessarily much compressed. No divergent views have been advanced, but emphasis has been added to some of the characteristics of irrigation district development suggested or implied in the paper.

Mr. Lyman is correct in stating that there are disadvantages as well as advantages in the irrigation district. Nothing could be more true, and perhaps

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these disadvantages were not sufficiently stressed. The fact remains, however, that for the large majority of cases, the irrigation district in some form is the best type of irrigation organization now in use, a fact on which all, as far as the writers are aware, agree. In many situations it has fully and satisfactorily met the need and accomplished a high type of agricultural development. This can be said of all the fifteen Western States having irrigation district laws. In all these States, some districts have been organized and some have issued bonds and built works in the absence of full justification—economic as well as physical. In between these two extremes most of the States having these laws can show some districts that are progressing only fairly well, either because of unfavorable physical conditions—scant water supply, mediocre engineering, and questionable soil—lack of settlers, or general agricultural depression. The gratifying thing, however, is that, given a situation that physically and economically justifies an irrigation enterprise, irrigation district laws and operations under them have advanced so far as to leave no question as to the adequacy of the irrigation district as the agency through which the development can successfully be carried forward.

One important feature of irrigation district development on which the discussion has not touched is the extent to which State authority should govern and carry moral responsibility in the organization and financing of irrigation districts and in carrying out their construction. Originally, State authority was not involved. Districts were organized as men saw fit and most of them went merrily to their destruction without let or hindrance on the part of any agency of the public. This costly experience fully justified the amendments to the district laws which gave the States (usually through the State engineers) the right of at least partial veto on the organization of irrigation districts. Experience under this procedure has given still further justification and no one questions the practice. The next important forward step, which brought the State into the examination of construction plans and into the certification of irrigation district bonds for certain purposes, was also one that was warranted by the conditions that had developed out of costly experience and this, too, is fully accepted as sound procedure.

The specific questions that might profitably be raised about this whole matter of State authority in irrigation district development are:

(1) Conceding an adequate irrigation water supply, satisfactory soil and growing conditions, and a feasible plan of works, should the State withhold approval of organization or of bonding and construction programs if, in the opinion of the State official (or State officials) charged with responsibility for action, there is doubt as to the economic success of the project at the time proposed?

(2) Should the action of the State bond certification commissions with reference to irrigation district bond issues, when favorable, be of such nature as not virtually to constitute State "approval", since every "approved" project is not certain to succeed and since in the minds of most investors the "approval" now given carries at least moral State responsibility?

(3) Should the State, through appropriate administrative officials, hold a helpful and friendly attitude toward irrigation districts or proposed irrigation districts, particularly during the planning and construction period of organized districts, in order to insure a higher efficiency in the district management and to give it the benefit of the larger experience of the State?

(4) Should the State undertake to supervise the accounting of irrigation districts, both during and after construction, in the first instance to insure sound use of funds supplied by the investing public, and in the second instance to accomplish the highest efficiency in the management of these enterprises, which are really agencies of the State?

Since these questions were not raised in the paper or in the subsequent discussion, the writers will not argue them in their closing discussion. They feel free to state, however, that they would answer at least the last three of these questions in the affirmative.

In conclusion: The writers do not wish to advance the thought that the irrigation district as described in their paper is the final step in irrigation organization, although it is a permanently established form of organization for situations similar to the many in which it is successfully and satisfactorily operating. Several other types of districts were referred to in the paper, the most important of which are the so-called "super" water storage districts already formed and the conservation district under formation in San Joaquin Valley, California. These have been developed out of the necessity of building expensive storage works that embrace from two or three to fifteen lesser units that already have partial irrigation supplies, and each is organized around entire or practically independent stream systems.

There are some still larger problems of irrigation organization ahead, problems involving re-distribution of water from areas of excess to areas of deficiency and these, in turn, will demand even more comprehensive forms of organization and procedure. Whatever turn these take, however, they are not likely to depart very far from the principles of co-operation and community ownership and management exemplified in the irrigation district of to-day, except as wider State authority is found necessary to accomplish more economic utilization of still larger units of water and land, and as some financial participation by the State is required to bring the financial burden involved in these larger movements within a figure that the farmer on the land can pay.

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Paper No. 1612

CORROSION OF CONCRETE*

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Synopsis

Portland cement does not form compounds insoluble in water corrosive to calcium carbonate. The solubility of calcium carbonate depends on the alkalinity, hydrogen-ion concentration, and other salts in the water.

Some of the generally assumed compounds of calcium and silica may not exist in cement. It is possible that the excess of calcium in concrete over that theoretically required to combine with certain proportions of silica is attributable to surface adsorption or to a solid solution. Finely ground hydrated Portland cement gives off calcium hydroxide approximately to the point of calcium hydroxide saturation. The saturation equilibrium decreases by the addition of successive quantities of water, forming a curve that does not indicate definite high calcium compounds of silica.

When mixed with water the finer particles of cement tend to coagulate around larger particles of cement or aggregate, producing a structural formation somewhat similar to a meshwork of minute fibers filling the space between the larger particles. The hydration of cement probably follows the general law of the chemical changes of solids, namely, that a solid does not change in chemical composition without first going into the soluble state. In the case of Portland cement the solution and re-precipitation apparently takes place with-

* Presented at the meeting of May 5, 1926.
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out any material change in the shape or size of the individual cement particles, except that the particles actually unite in the newly formed solid where they touch each other.

Porosity is a very important factor in determining the life of concrete exposed to water or to the weather. It largely governs the rate of diffusion of soluble compounds from the interior to the surface. Water molecules apparently are oriented when they are against any solid surface. This is the orderly arrangement of the molecules in such a manner that they do not move freely with the molecules somewhat distant from the surface. Such a phenomenon in engineering is called surface friction. When two solids are submerged with part of their surfaces close together the oriented layers of water may join, and there is little movement of the water molecules between the surfaces. If all surfaces are so close together that the oriented water layers unite throughout, there is little communication of the water in the interior with the outside surface. This is the ideal condition for resisting corrosion.

A method of measuring the voids in concrete is given in this paper. It is suggested that the No. 8 sieve be the dividing line between fine and coarse aggregate.

Changes taking place in concrete exposed to water or to the weather are usually the liberation of calcium hydroxide, and its combination with carbonic acid to form calcium carbonate. If the water is corrosive to calcium carbonate it will be dissolved gradually, but the rate at which it goes into solution is slow. The gelatinous compounds of alumina and silica remaining after the calcium has been dissolved greatly aid in decreasing the solution rate as the diffusion through such compounds is slow. Freezing or considerable surface friction hastens the rate of corrosion, largely by removing these gelatinous compounds.

Moisture evaporating from a concrete surface tends to concentrate destructive compounds, if present, at various points. The surface of most of the concrete exposed to water corrosive to calcium carbonate should be waterproofed.

Structures valued at many millions of dollars are now showing deterioration where they are exposed to water and freezing weather. In many instances the deterioration has progressed to a point where it is jeopardizing the structures, and millions of dollars will have to be spent on repairs and replacements within the next decade to keep them in serviceable condition. This should make the problem of concrete corrosion of great interest. Engineers have been too ready to place all the blame for disintegration on poor construction, and have not fully realized that concrete has its limitations. It is the purpose of this paper to point out a few causes for disintegration when good materials have been used. In some instances the causes are of such a complex nature that it will require years of patient research to fully explain them, but in most instances there is a similarity suggesting a common cause. It is hoped that these few fundamental facts that have been established will be a start in the solution of this important problem.

It seems essential to use one chemical term that may not be understood by engineers. The symbol, *pH*, is used to represent the concentration of dissociated hydrogen-ions in the water. Pure water dissociates slightly into positively charged hydrogen-ions ($H+$) and negatively charged hydroxyl-ions ($OH-$), which bear a certain ratio to each other. The addition of acid or alkaline compounds changes this ratio. Pure water at a *pH* of 7 has approximately 0.0001 parts per million of dissociated hydrogen-ions. At a *pH* of 8 the hydrogen-ions are only one-tenth the amount at a *pH* of 7, and at a *pH* of 9 only one one-hundredth the amount. A *pH* of 6 has ten times as many dissociated hydrogen-ions as a *pH* of 7. The *pH* has a very important bearing on the saturation equilibrium of certain compounds.

Corrosion as used in the paper is distinguished from disintegration in that it is a weakening of the concrete by the dissolving of certain compounds from the cement. Disintegration of weak concrete may be caused by freezing when there may have been no dissolving or changing of the cement compounds.

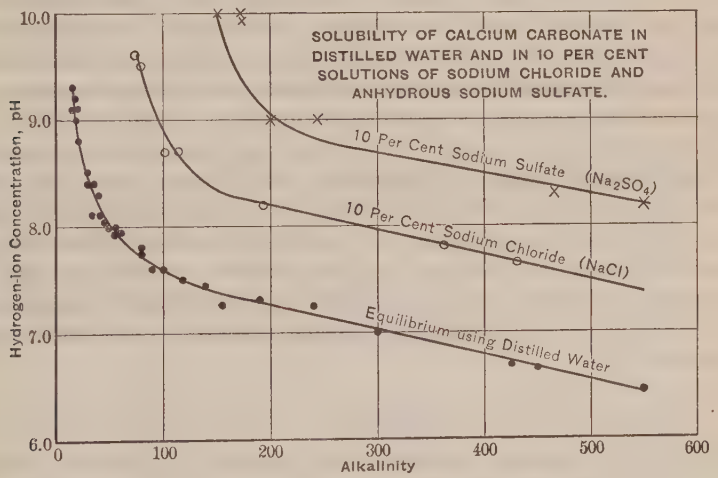

FIG. 1.

SOLUBILITY OF MOST OF THE COMPOUNDS IN CEMENT

Heretofore engineers have assumed that Portland cement formed compounds insoluble in water. It will be shown that this is not the case. The fact that there is usually an intermediate change to calcium carbonate makes the problem, in effect, one of the solubility of calcium carbonate plus the protective action of certain gelatinous compounds.

As shown by the equilibrium curves in Fig. 1 the solubility of calcium carbonate depends on the alkalinity and hydrogen-ion concentration of the surrounding solution. Certain salts in solution effect this equilibrium as shown by the two upper curves, for 10% solutions of sodium chloride and sodium sulfate. For conditions represented by points above the curves, calcium carbonate will be precipitated; for those below, it will be dissolved.

These equilibriums were established at room temperature, and the hydrogen-ion concentration, (pH), determined calorimetrically.

The procedure in obtaining these curves was as follows: For the determination of the curve without neutral salts about 50 grammes each of calcium carbonate from three different sources was pulverized in a laboratory mortar and put into 300-cu. cm. glass-stoppered bottles. Chemically pure calcium carbonate, pulverized limestone, and the surface crystals from a saturated solution of lime water were the three substances. Both distilled water and water from the city supply of Baltimore, Md., was used; it was changed several times before the first results were recorded. High alkalinity concentrations were obtained by passing carbon dioxide through the solution. The bottles containing the calcium carbonate were filled with water, the glass stoppers inserted, and then the specimens were allowed to stand in the laboratory for one week with an occasional shaking. Later experiments have been run in which pyrex flasks were used and more than a month allowed for solubility equilibrium to become established. The same general procedure was followed for establishing the curves where neutral salts were present.

The rate at which reactions (such as the dissolving or precipitating of calcium carbonate) take place near the curve may be extremely slow, but as the distance increases the rate of solution or precipitation increases rapidly. Calcium carbonate solubility equilibrium might be called one of the fundamental forces of Nature, and it is the real key to concrete corrosion under many conditions of exposure. From the fact that rain water has practically no alkalinity and a pH usually less than 6.0, it is readily shown by Fig. 1 how corrosive to concrete it may be. Nearly all surface waters will dissolve calcium carbonate, but the rate is quite variable. More than 90% of the total calcium in concrete will be dissolved by water slightly corrosive to calcium carbonate. The compounds remaining are usually soft and of a gelatinous nature.

Chemical Compounds as Generally Assumed May not Exist

Much has been written regarding the chemical composition of Portland cement. This paper does not propose to take issue with the generally accepted theories as to the compounds formed, except to show either that there are a number of compounds formed in concrete, or that stable compounds of high calcium content do not exist. The gradual change in equilibrium, as shown by the curves in Fig. 2, points strongly to the fact that if there are definite stable compounds of calcium and silica in concrete, they are probably compounds of lower calcium content than tri-calcium silicate, and that any additional calcium above these stable compounds may be merely in the nature of a surface adsorption, or, as has already been suggested, a solid solution. Any soluble solid in contact with water will dissolve until saturation is reached. There are several factors that may vary the rate of solution, but, all conditions being equal, the saturated solution contains the same quantity of the dissolved compound. Fig. 2 shows that this is not the case with hydrated Portland cement or concrete. There is a gradual change in the saturation point as

calcium is removed by extraction until at least two-thirds of the calcium has been extracted.

While the amount of alkali in solution, as shown by the curves of Fig. 2 and by Tables 1, 2, 3, and 4, is expressed in parts per million of calcium carbonate as determined by the ordinary alkalinity test, most of it is in the form of calcium hydroxide with probably just a little sodium and potassium hydroxide or carbonate present for the first few extraction tests. To obtain the parts per million of calcium oxide, the alkalinities should be multiplied by 0.56. The saturation point of calcium hydroxide at a temperature of 60° Fahr. is approximately 1 340 parts per million of calcium oxide (CaO), or, expressed in terms of alkalinity as used in the diagrams and tables, about 2 400. Any material showing an alkalinity of more than 2 400 probably contains a more soluble alkali, such as sodium or potassium, or contains a supersaturation of calcium hydroxide. Most of the tests in Fig. 2 were made after the specimen had stood from 10 days to 2 weeks with frequent agitation. Solubility equilibrium is not quite reached in this time; a few check tests were made in which from 2 to 4 months were allowed. The longer periods of standing do not materially alter the shape of the curves. All material was thoroughly pulverized to the point where it could pass a 100-mesh sieve. A steel

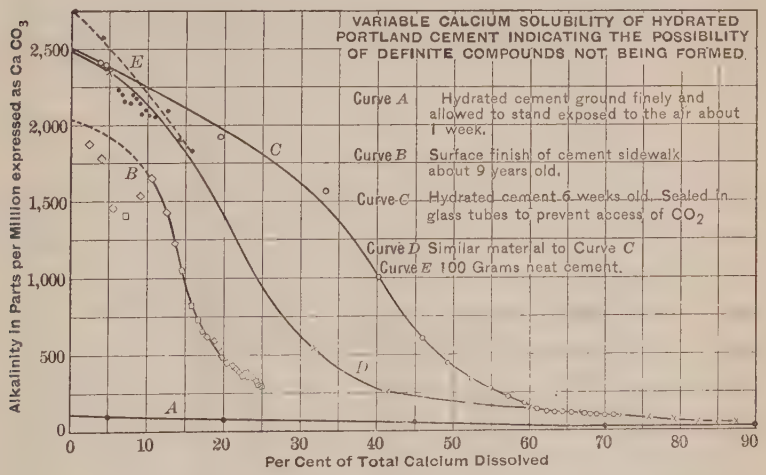

FIG. 2.

ball was inserted in the flasks to keep the sample thoroughly crushed, and the flasks were tightly stoppered with rubber stoppers. In some instances they were sealed, but the rubber stoppers seem to have been perfectly air-tight. Should definite compounds of silica with high percentages of calcium be formed after hydration, which now seems doubtful, the most significant fact demonstrated by these tests is that such compounds are not stable in the presence of most natural waters. Compounds of calcium and alumina are unstable in most surface waters, as chemists realize.

TABLE 1.—EXTRACTION OF THE ALKALIES IN HYDRATED PORTLAND CEMENT.*

Date.	Time standing.	Alkalinity, in Parts per Million.		pH.
		Phenolphthalein.	Methyl orange.	
1924:				
6-20	16 hours.	49	60	....
6-21	1 day.	50	64	....
6-23	2 days.	55	65	....
6-25	2 "	52	64	....
6-28	3 "	57	66	....
7- 2	4 "	57	65	....
7- 4	2 "	50	55	....
7- 7	3 "	40	52	....
7- 8	1 day.	30	43	....
7-12	4 days.	48	59	....
7-14	2 "	38	47	....
7-16	2 "	33	44	....
7-18	2 "	31	40	....
7-19	18 hours.	23	38	....
7-21	2 days.	26	41	....
7-22	1 day.	22	36	....
7-24	2 days.	22	36	....
7-30	6 "	24	35	....
7-31	1 day.	14	32	....
8- 8	8 days.	22	35	....
8-11	3 "	12	31	....
8-14	3 "	18	30	....
8-16	2 "	11	29	10.0
8-18	2 "	10	28	10.0
8-20	2 "	10	28	10.0
8-22	2 "	9	31	9.8
8-25	3 "	9	28	9.8
8-27	2 "	9	31	9.8
8-29	2 "	9	28	9.8
9- 1	2 "	9	27	9.8
9- 5	4 "	9	26	9.8
9- 7	2 "	7	28	9.7
9- 9	2 "	6	26	9.7
9-11	2 "	6	25	9.6
9-13	2 "	5	27	9.6
9-21	8 "	7	29	9.7
9-23	2 "	5	24	9.6
9-25	2 "	5	24	9.6
9-27	2 "	5	24	9.6
9-29	2 "	4	23	9.4
10- 1	2 "	4	24	9.3
10- 3	2 "	4	24	9.2
10- 5	2 "	5	25	9.4
10- 7	2 "	5	25	9.3
10- 9	2 "	5	25	9.2
10-11	2 "	4	25	9.2
10-13	2 "	4	24	9.3
10-15	2 "	5	23	9.3
10-17	2 "	4	23	9.2
10-18	1 day.	3	23	8.9
10-19	1 "	3	23	8.9
10-20	1 "	3	21	8.9
10-21	1 "	2	21	8.9
10-22	1 "	1	19	8.5
10-23	1 "	1	19	8.5
10-24	1 "	1	20	8.4
10-25	1 "	1	18	8.5
10-27	2 days.	1	21	8.4
10-28	1 day.	1	19	8.5
10-29	1 "	1	19	8.3
10-30	1 "	0	16	8.0
10-31	1 "	0	13	6.9
11- 1	1 "	0	11	6.7

*A cement mortar, 1 month old, was pulverized very finely, and allowed to stand 1 week exposed to the air. A 2-gramme sample containing 1.19 grammes of CaO was used. The total CaO extracted, as calculated from the alkalinity tests, was 1.12 grammes. The residue at the end of the test contained 0.082 gramme of CaO.

TABLE 2.—EXTRACTION OF THE ALKALIES FROM OLD CONCRETE.*

Date.	Days standing.	Amount of solution withdrawn from flask, in cubic centimeters.	ALKALINITY, IN PARTS PER MILLION.		Total CaO withdrawn, in grammes.	Percentage of total sample.
			Phenol-phthalein.	Methyl orange.		
1924:						
8-30	10	200	1 840	1 878	0.210	2.10
9- 9	10	200	1 728	1 770	0.408	4.08
9-19	10	200	1 420	1 460	0.572	5.72
9-29	10	200	1 372	1 400	0.729	7.29
10- 9	10	200	1 500	1 532	0.901	9.01
10-19	10	200	1 632	1 660	1.087	10.87
10-29	10	180	1 396	1 432	1.230	12.30
11- 8	10	180	1 220	1 236	1.355	13.55
11-22	14	180	1 012	1 040	1.47	14.7
12- 6	14	200	800	820	1.57	15.7
12-20	14	200	708	728	1.65	16.5
1925:						
1- 3	14	200	624	648	1.73	17.3
1-17	14	200	600	620	1.80	18.0
2- 1	16	200	556	588	1.87	18.7
2-16	15	200	488	536	1.92	19.2
3- 2	14	200	460	480	1.98	19.8
3-16	14	200	446	464	2.02	20.2

* 50 grammes of a pulverized sample of the top finish of a cement sidewalk more than 9 years old was used. Sample tested 20% CaO. A 300-cu. cm., tightly stoppered flask was used. An iron ball was inserted in flask on October 9, 1924, in order to pulverize thoroughly the particles that had lumped together. Complete equilibrium probably was not reached in some of the tests before October 19.

TABLE 3.—EXTRACTION OF THE ALKALIES FROM PORTLAND CEMENT.*

Date.	Days standing.	Amount of solution withdrawn from flask, in cubic centimeters.	ALKALINITY, IN PARTS PER MILLION.		Total CaO withdrawn, in grammes.	Percentage of total sample.
			Phenol-phthalein.	Methyl orange.		
1924:						
10-26	13	900	1 932	1 944	0.98	16.3
11- 6	11	800	1 544	1 576	1.69	31.6
11-24	18	800	952	988	2.13	39.9
12-11	17	885	584	612	2.43	45.7
1925:						
1- 4	24	800	424	448	2.63	49.4
1-19	15	800	324	344	2.79	52.4
2- 2	14	900	264	284	2.93	54.9
2-16	14	950	216	232	3.05	57.3
3- 2	14	960	172	188	3.16	59.3
3-16	14	950	146	160	3.24	60.8

* Neat cement mortar sealed in glass tube for six weeks. Pulverized, and 10-gramme sample used. Cement mortar tested 53.32% CaO. One liter flask used.

TABLE 4.—EXTRACTION OF THE ALKALIES FROM PORTLAND CEMENT.*

Date.	Days standing.	Amount of solution withdrawn from flask, in cubic centimeters.	ALKALINITY, IN PARTS PER MILLION.		Total CaO withdrawn, in grammes.	Percentage of total sample.
			Phenol-phthalein.	Methyl orange.		
1924:						
9- 2	10	200	2 720	2 752	0.308	0.47
9-12	10	200	1 968	2 012	0.533	0.82
9-23	11	200	2 043	2 046	0.762	1.17
10- 3	10	200	2 244	2 296	1.019	1.57
10-13	10	200	2 400	2 428	1.291	1.98
10-23	10	200	2 376	2 408	1.661	2.56
10-27	4	200	2 156	2 192	1.806	2.78
11- 2	6	200	2 312	2 340	2.068	3.18
11-12	10	200	2 536	2 572	2.356	3.67
11-24	12	200	2 524	2 568	2.643	4.07
12- 8	14	200	2 420	2 452	2.918	4.48
12-22	14	200	2 352	2 384	3.185	4.90
1925:						
1- 5	14	200	2 300	2 324	3.445	5.30
1-19	14	200	2 264	2 288	3.701	5.19
2- 2	14	200	2 228	2 260	3.954	6.08
2-16	14	200	2 192	2 220	4.203	6.48
3- 2	14	200	2 168	2 192	4.448	6.83
3-16	14	200	2 128	2 156	4.689	7.22
3-31	15	200	2 124	2 148	4.936	7.58
4-14	14	200	2 168	2 208	5.182	7.98
5- 3	19	200	2 160	2 188	5.428	8.36
5-17	14	200	2 124	2 156	5.670	8.73
5-31	14	300	2 036	2 064	6.016	9.26
6-28	28	300	2 072	2 104	6.370	9.81
7-12	14	300	2 040	2 074	6.723	10.35
7-26	14	300	2 040	2 074	7.071	10.87
8- 9	14	350	2 056	2 092	8.184	12.54
8-23	14	900	1 876	1 896	9.139	14.04
9- 6	14	900	1 804	1 840	10.066	15.48
10-18	42	900	1 428	1 456	10.800	16.62
12-20	63	900	1 204	1 228	11.419	17.55
1926:						
1- 3	14	900	928	948	11.897	18.27

* 100 grammes of neat cement containing 65.0 grammes of CaO was used. Prior to November 12, 1924, the sample tended to form lumps. An iron ball was inserted in the flask and all lumps were crushed.

SURFACE ADSORPTION

Surface adsorption is a force almost unknown to engineers. Many, if not all, solids have the power of attracting certain compounds to their surface; that is, the surface molecules form some kind of a loose chemical combination with certain other compounds. The concentration of the attracted molecules may not be limited to the thickness of one molecule. The amount of a compound which will be adsorbed depends on the surface area, the attractive force of the solid, the temperature, the concentration of the compound being adsorbed in the surrounding phase (solution or gas), and probably on other forces. The presence of compounds in the solution which are not adsorbed frequently affects the amount and rate of adsorption. It is possible for certain greatly expanded gelatinous compounds to hold by surface adsorption large quantities of other compounds.

The usual conception of gelatinous compounds is that they are fairly soft, yet actually they may be composed of a network of minute chains of crystals, frequently sub-microscopic in size, which are as hard as the original compact

solid. Surrounding these crystals, or any solid which is submerged, is a layer of water that has lost its power to flow. When the solid is exposed to the air there also appears to be a surface layer of air which does not flow. These are probably oriented layers, that is, there is an orderly arrangement of the molecules next to the surface.

The force with which adsorbed compounds (assuming that water or other liquids and gases are adsorbed) are held attracted to a surface is greatest next to the surface, gradually diminishing until a point is reached where the surface has no effect. Some gelatinous compounds are able to hold more than fifty times their weight of water, agar being such a compound. Water does not flow from the gel regardless of how long it may be left with unsupported sides; yet there is no impervious membrane holding the water back. It will evaporate if the moisture content of the air is below the saturation point, but it will not flow away from the fibers of the gel. When pressure is applied it tends to overcome the attractive forces of the solid, so that materials impervious at a low pressure may not be so under high pressure. Clay may be nearly water-tight, yet have more than 50% of its volume in voids. Particles of the same shape as those composing the clay, but about 1 mm. in diameter, may arrange themselves so that the total percentage of voids is much less than in the clay, yet water will flow through them hundreds of times as fast as through the clay.

The fact that the water molecules may be adsorbed or oriented against a solid surface and thus prevented from flowing gives a good illustration of the power of solid surfaces. Orientation and adsorption are by no means confined to the water molecules, for the concentration on solid surfaces of molecules dissolved in water covers a wide range of compounds and conditions. When a solid has attraction for both acids and alkalies there is a certain hydrogen-ion concentration at which practically no acid ions are adsorbed, and another concentration at which practically no alkaline ions are adsorbed. Between these points both ions appear to be attracted to or near the solid surface in variable quantities, depending on the hydrogen-ion concentration and a number of other variables.

Hydration of Portland Cement

Chemical compounds probably do not change from one compound to another without first going into the liquid stage. Certainly this is the case with most changes, and future research will probably prove it to be true for practically all compounds. Most chemists now believe that there are exceptions to this rule, although every indication points to the fact that cement does follow the general rule. In the hydration of Portland cement the dissolving and re-precipitation as the cement compounds unite with the water takes place with little change in volume or form from that in which the particles of cement are left after they have been mixed with water. Where the particles touch, or are extremely close together, the dissolving and re-precipitation builds them up together. It is this that gives cement its binding power. The reaction progresses slowly, and possibly requires considerable time before it reaches the center of fairly large particles, if it ever does. Re-ground cement

mortar several years old usually has a slight tendency to set when it has been mixed with water, indicating that the original reaction might not have been complete for all the particles.

A nearly saturated solution of calcium hydroxide tends to coagulate finely divided particles of many compounds, and when the particles of cement liberate caustic lime it, in turn, will partly coagulate the finer particles of cement around the larger particles, giving a structural formation somewhat as shown in Fig. 3. The full effect of this tendency is not realized in concrete when only sufficient water is added for proper handling, for the fine particles do not become greatly dispersed in the water. It is not possible to produce neat cement mortar with less than about 25% of voids when it is first mixed. The hydration does not reduce this percentage materially. It is only by the addition of some other compound, such as carbonic acid, which forms calcium carbonate with the liberated calcium hydroxide, that the voids are reduced. The formation of calcium carbonate tends to build up another structural formation within the voids of the cement, greatly reducing them and usually adding strength to the concrete.

There is a point, however, long before all the voids are filled, where the stagnated layers of water or air close the pores against rapid communication with the surrounding water or air. Then, the concrete becomes water-tight or air-tight, yet it may have more than 20% of its mortar volume as voids. It is the formation of calcium carbonate within concrete that gives it the greatest protection from the weather, and this protection is largely the result of reducing the mass to the impervious or stagnated stage. The gelatinous aluminum hydroxide and silica offer the greatest protection to concrete submerged in water corrosive to calcium carbonate, but the intermediate change to calcium carbonate greatly aids in retarding the rate of corrosion for most natural waters. This applies to conditions in which there is nothing tending to remove the gelatinous compounds from the outer concrete surface.

Structural Formation of Cement Mortar

The addition of water to cement, and the mixing, leaves it in a porous structural formation with a slight tendency to assume the characteristic meshwork of fibers as shown in Fig. 3. This is from a very porous section of concrete, largely composed of the coagulated finer particles which have re-precipitated into solid fibers. The drawing is partly camera lucida. It is from a greatly expanded part of the mass, and was selected because it would be difficult to show clearly the more compact part. There are numerous large particles varying in size from those shown to others as large as the circle. A sand grain 0.5 mm. in diameter would be about twice the diameter of the circle.

A true picture of the structural formation of hydrated cement mortar would show a structure with fibers much shorter, thicker, more irregular in shape, and more compact than shown in Fig. 3, with some particles as large as the entire drawing.

Immediately after mixing with water every particle of cement touches or is extremely close to another particle, and usually to several others. A pile of crushed stone will probably give some idea of the arrangement of the cement

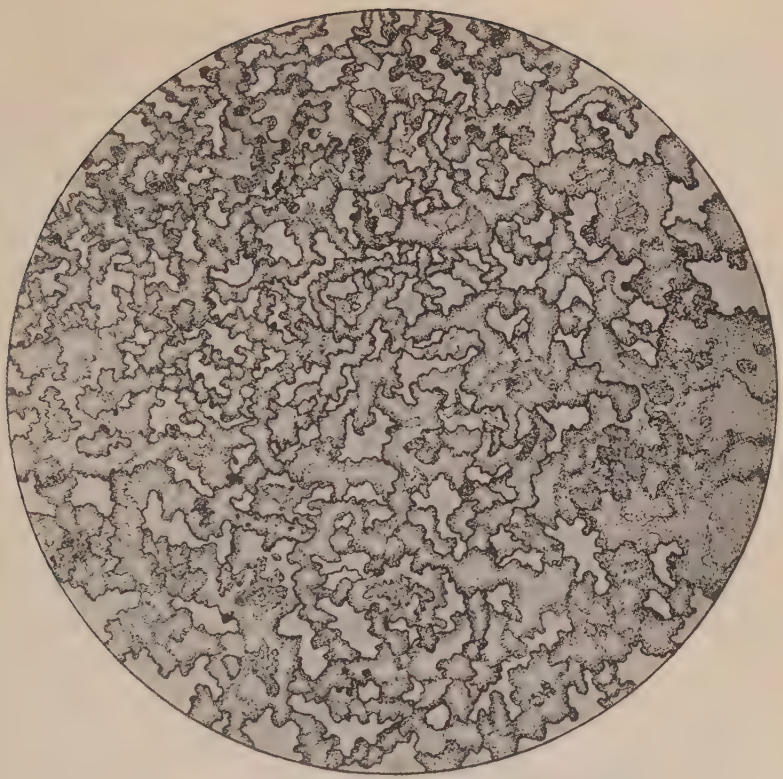

FIG. 3.—STRUCTURAL FORMATION OF PORTLAND CEMENT REPRESENTING A FIELD 0.25 MILLIMETER IN DIAMETER.

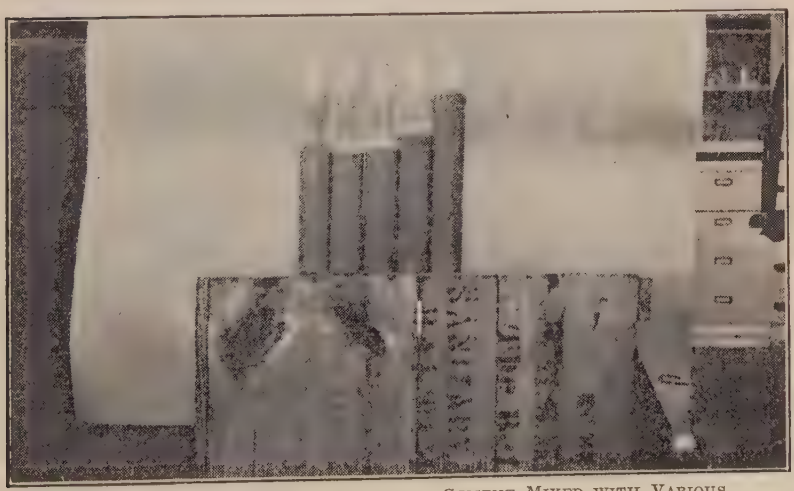

FIG. 4.—VOLUME OF 100 GRAMMES OF CEMENT MIXED WITH VARIOUS QUANTITIES OF WATER.

grains; however, this is not a correct picture, for the finer particles of cement tend to coagulate around the larger particles of cement or aggregate somewhat as shown in Fig. 5. This applies to the finer particles of dirt or silt as well as to the cement. To obtain Fig. 5, a small particle of sand, rejected for use on the addition to the Montebello Filters in Baltimore, Md., was agitated in water and then allowed to stand about 30 min.; 1 cu. cm. of the dirty water was placed in a counting cell and a few particles of cement were added. The water was slightly agitated for about 5 min., and the camera lucida drawing was made about 15 min. afterward. A gelatinous coating had formed around practically all the particles of cement, as shown in Fig. 5.

Using the pile of stone to complete the illustration of the action of the cement—the outside of the stones becomes plastic and adheres, and then solidifies again. The plastic part, representing the change from the oxides in cement to the hydroxides, gradually moves toward the center of the mass, being followed closely by the solidification. In the case of cement there is no plastic layer, but it is likely that an actual solution is taking place, probably within oriented layers of water. This is not to convey the idea that all the cement particles undergo this change, for, in cement manufacture, in heating several compounds that may have different melting points, it is probable that insoluble compounds of the nature of glass may be formed. It does apply, however, to all particles undergoing hydration, which for some cements constitutes practically all the material.

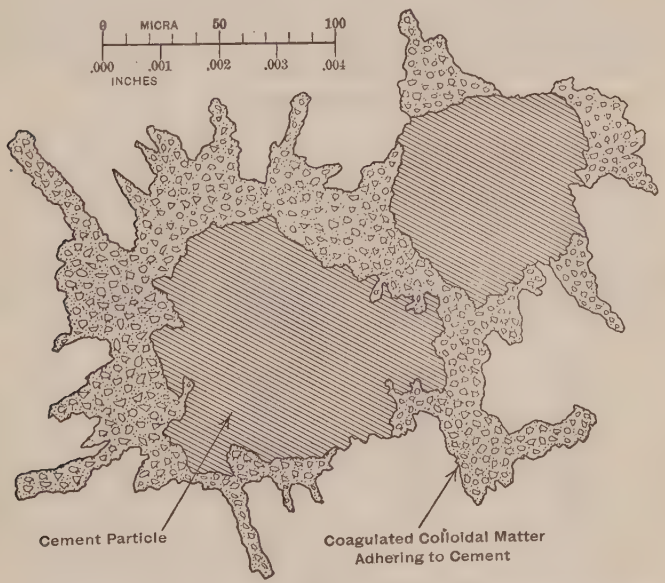

Fig. 5.—Adherence of Dirt to Particles of Cement Submerged in Turbid Water.

Importance of Porosity

When good materials have been used and properly mixed, the chief factor in determining whether the concrete will withstand the weathering agents is its

porosity. Air-bubbles, which under ordinary practice are not excessive, have little effect. It is the spacing between the fibers or solid particles that determines its corrosive resisting power.

This is best illustrated by comparing the volume of equal weights of cement mixed with variable quantities of water as shown in Fig. 4. In this experiment five batches, of 100 grammes each, of neat cement were mixed with different quantities of water placed in identical glass tubes. The photograph was taken one week afterward. From left to right 20, 25, 30, 40, and 60 cu. cm. of water were used. The volume is a minimum when approximately 25% by weight of water is used. This is the point of minimum voids and probably the approximate point of maximum strength. The addition of more water to a maximum of about 60 to 70% increases the voids, not by forming pockets of water, but by an expansion of the structural formation. Cement is composed of particles varying from almost sub-microscopic size to more than 0.2 mm. in diameter. If the total mass is expanded 50%, actually it will be much more than 50% for the finer particles that fill in between the larger ones.

If a piece of loose cotton is submerged in water the volume of water around the fibers may be quite large in proportion to the actual volume of the cotton, yet the fibers may be fairly uniformly distributed throughout the mass. If this same piece of cotton were compressed to about one-tenth its original volume the fibers would still be fairly uniformly distributed throughout the mass, but much closer together. In the loose piece of cotton, water from the interior of the mass would communicate much more rapidly by diffusion than in the compressed mass; that is, a certain salt in solution may diffuse through 1 in. of the loose cotton within 10 min., whereas it might take 100 hours or more to diffuse through 1 in. of compressed cotton.

TABLE 5.—VOIDS IN NEAT CEMENT MORTAR USING VARIABLE QUANTITIES OF WATER SEALED IN GLASS TUBES FOR TWO MONTHS.

Percentage by weight of water.	Volume, in cubic centimeters.	Dry weight, in grammes.	Weight after being submerged 24 hours, in grammes.	Water absorbed, in cubic centimeters.	Percentage of voids.
20	30.0	60.9	67.6	6.7	22.3
25	29.0	56.5	64.2	7.7	26.5
30	25.5	47.6	54.6	7.0	27.4
35	28.0	48.3	56.7	8.4	30.0
40	21.0	34.0	41.3	7.3	34.7
45	16.5	26.4	32.2	5.8	35.2
50	16.2	23.4	29.7	6.3	38.9
55	15.5	22.0	28.1	6.1	39.4
60	20.7	28.1	36.7	8.6	41.5
70	27.0	32.8	46.0	13.2	48.9

Table 5 shows that the voids in neat cement mortar may be varied quite widely according to the quantity of water used in mixing. The range from a minimum of 22% of the volume, with 20% by weight of water, to a maximum of 49%, with 70% of water, represents a considerable variation in the expansion of the cement particles. If the diffusion rates varied only in proportion to

the voids, the quantity of water used would make little difference, but on account of the stagnated layer of water on the solid surfaces this variation in voids may result in increasing the diffusion rates throughout the mass a hundredfold. It has been shown that cement is soluble in water, that is, that most of the calcium is soluble; now if this calcium could diffuse readily to the surrounding solution the life of concrete submerged in most natural waters would be quite short.

Neglecting air-bubbles, all voids in the cement mortar were filled with water when 25% by weight of water was used, yet the mortar did not separate from the water when 60% was used. The only explanation is the force of the stagnated layers of water holding the particles apart. It is difficult to realize the force required to remove this stagnated layer from solid surfaces. A good illustration is the oil in a bearing. It may be under hundreds of pounds pressure to the square inch, yet a film of oil keeps the two metal surfaces from touching. If the oil acted as a liquid next to the metal the heavy pressure would force it out.

Water is not as good a lubricant as oil but the tendency is the same. When water is added to cement to such an extent that there is an actual separation of the solids after it becomes still, the distance between the particles is so great that the stagnated layers are not joined together continuously throughout the mass, and if it were not for a decrease in voids by the formation of calcium carbonate while or after setting, this wet concrete would deteriorate rapidly when exposed to the weather. Notwithstanding this reduction of voids it does disintegrate much more readily than when the optimum quantity of water is used.

Density is wilfully sacrificed for economy of construction in modern concrete work. Under some conditions this may be justified, but any concrete exposed to water or to the weather on an important structure should be of the densest material possible. Concrete wet enough to run in a chute with the angles of slope now generally used, should never be exposed at a water's edge, especially if the water is corrosive to calcium carbonate and if the climate is cold enough to cause freezing. In most instances such wet mixed concrete will begin to disintegrate within 20 years, and frequently in less than 10 years.

The examination of disintegrating concrete on a number of structures under such conditions has almost invariably shown a large percentage of voids—not that it was honey-combed, but that there was an expansion of the cement fibers. This applies not only to construction in which good materials have been used but to that in which it is not exposed to sea water or sulfate waters. Such waters bring about conditions that even the best concrete may not withstand for more than a few years.

It is believed that a void test and the inclusion of maximum limits would be desirable additions to the specifications for concrete on important exposed structures. It is realized that any addition of laboratory tests is not regarded with great favor in construction work; however, when present methods under the best of inspection do not always produce durable concrete, the small cost of such additional tests would be insignificant if they aided in producing better

concrete. One procedure for making the tests that gives satisfactory results will be described.

Measurement of Mortar and of Voids in It.

Pour a sample of the concrete into a mould 4 in. or more in its least dimension, covering it to make it as nearly air-tight as possible. Allow the mixture to stand over night, or until it has hardened sufficiently so that it may be removed from the mould without breaking. Leave it for at least 24 hours in a drying oven having a temperature between 212° and 250° Fahr., then cool it to room temperature, weigh it, and submerge it in water of approximately the same temperature. Weigh it again after it has been submerged successively 5 min., 2 hours, and 4 hours. If the 4-hour weight is appreciably higher than the 2-hour, submerge it for another 2 hours, repeating until the weight is practically constant. Remove the outside water with a dry cloth each time before weighing. Measure the volume by displacement of water while the sample is wet, and, without drying, crush the sample, preferably with a hand hammer, using care not to injure the pieces of coarse aggregate. Striking the pile of crushed concrete repeatedly with a light hand hammer will crush the mortar very fine. Sift the material through a No. 8 standard sieve, and repeat the crushing and sifting until all the mortar has been removed from the aggregate. From five to ten sievings will usually be sufficient. Weigh the moist aggregate, dry it at the same temperature as was used for the concrete, and weigh it again. Determine the actual solid volume of aggregate by submerging it in water. This should be done while it is moist. Few air-bubbles within the concrete will be filled with water by submergence if the sample is not heated. It is desired to measure the voids around the cement fibers only, and no effort should be made to fill the bubble spaces with water.

The following is a very convenient form for record:

Test for Mortar and Voids in Concrete

Date.—October 21, 1924.
Sample.—Addition to Montebello Filters. Settling basin floor.
Age.—1 day. Theoretical proportions in mix.—1 : 2 : 4.

Weight of dry concrete	1288 grammes.		
Submerged 5 min	1359 grammes	5.5%	water absorbed.
Submerged 2 hours	1390 grammes	7.9%	water absorbed.
Submerged 4 hours	1390 grammes	7.9%	water absorbed.
Percentage of total absorption within 5 min		70	
Volume of concrete		590	cu. cm.
Weight of wet gravel		867	grammes.
Weight of dry gravel		860	grammes.
Volume of loose gravel		560	cu. cm.
Percentage of voids in loose gravel		40.7	
Actual volume of solid gravel		332	cu. cm.
Mortar volume (590 — 332)		258	cu. cm.
Water absorption of mortar (1390 — 1288) — (867 — 860)		95	cu. cm.
Percentage of voids in mortar (95 ÷ 258)		36.8	
Percentage of mortar volume to total volume		43.7	

Remarks.—Voids in mortar are excessive, caused by very wet mix.

This procedure does not give the total voids in the concrete, for there are numerous air-bubbles not displaced by the water. In fact it is believed that

few air-bubbles are filled with water unless the specimen is boiled or subjected to a vacuum. Voids that might be filled so easily probably would not exist in the concrete. The test gives a fairly accurate measure of the voids surrounding the cement fibers, which is the information desired, for it is the spacing between fibers or the structural expansion of the cement that determines its power to resist the elements. Air-bubbles have little effect on the life of concrete, except possibly when they become filled with water and freeze.

This proposed test is not to supersede the slump test, but to aid in interpreting it. The aggregate has considerable influence on the slump, and, although a fairly good idea of the porosity may be obtained in this manner, it is not as accurate as the water absorption test. The requisite slump for concrete that is to be exposed to water and freezing weather is so small that considerable variation in density will not greatly affect it. The void test is applicable to old concrete, but it is more difficult to remove the mortar from the coarse aggregate. Table 6 shows the voids in various concrete samples.

Proportioning Aggregates

The writer does not believe in changing customary practice as long as the old method gives satisfactory results. The present 1 : 2 : 4 proportion of cement, fine aggregate, and coarse aggregate, especially the ratio of fine to coarse aggregate, does not give the best results over a wide range of aggregate sizes. That there is need for a more scientific proportioning of aggregates in concrete is generally recognized. Fine aggregate is quite variable, even if it is taken from the same locality, and frequently from the same source. In one locality an excellent quality of sand may be found varying in size from fine to ¼ in. in diameter, and in another locality sand and gravel may be found mixed in various proportions. The engineer is confronted with the problem of how to utilize local materials to the best advantage.

Specifications for fine aggregate usually state that it shall vary from fine to coarse, with a certain percentage passing a No. 4 sieve. The recommended specifications* of the Joint Committee on Standard Specifications for Concrete and Reinforced Concrete suggest that not less than 85% pass a No. 4 sieve. These specifications give little idea of what should pass a No. 8 or No. 16 sieve, except in an Advisory Appendix (XVI)† that recommends varying proportions of aggregates for variable sizes and slumps. It is not uncommon to find fine aggregate that will comply with these specifications with from 25 to 35% retained on a No. 8 sieve, or with more than 90% passing a No. 16 sieve. Curves of the percentages by weight of sizes determined by sieving are plotted for various sands. One characteristic of the curve is the rapid increase in size for the coarser particles. This is shown in Fig. 6, which is a plotting of several characteristic sands.

All evidence points to the fact that the effectiveness of sand for concrete work depends largely on the quantity passing a No. 16 sieve, providing it is properly graded below this size. It requires less than one-half as much sand by

* *Proceedings*, Am. Soc. C. E., October, 1924, Papers and Discussions, p. 1153.

† *Loc. cit.*, p. 1277.

TABLE 6.—COMPARISON OF VOIDS IN CONCRETE MORTAR.

Sample.*	Proportions in mix.	Age.	Dry weight, in grammes.	Hours submerged.	Wet weight, in grammes.	Volume of concrete, in cubic centimeters.	Coarse Aggregate: Wet weight, in grammes.	Coarse Aggregate: Dry weight, in grammes.	Coarse Aggregate: Loose volume, in cubic centimeters.	Coarse Aggregate: Percentage of voids.	Coarse Aggregate: Actual volume, in cubic centimeters.	Mortar: Total volume, in cubic centimeters.	Mortar: Percentage of voids.	Ratio of mortar volume to total volume.
1	1 : 2 : 4	1 day	1 108	4	1 177	495	746	741	470	39.3	285	210	30.5	42.3
2	"	2 days	1 386	4	1 522	682	751	746	480	40.6	285	397	33.0	58.2
3	"	1 day	1 288	4	1 390	590	867	860	560	40.7	332	258	36.8	43.7
4	"	"	736	4	777	332	512	507	320	39.1	195	137	26.3	41.3
5	"	"	766	4	802	348	492	488	320	42.2	185	163	19.6	42.2
6	"	6 months	800	24	835	353	520	515	335	38.2	207	146	20.5	41.3
7	"	1 day	1 584	4	1 708	750	1 067	1 059	680	36.0	435	315	36.8	42.0
8	"	22 days	1 231	4	1 300	553	845	839	525	38.5	338	225	28.0	40.7
9	"	1 day	1 456	4	1 536	657	933	925	620	38.8	379	278	25.9	42.2
10	"	24 days	1 150	4	1 205	515	823	816	524	38.1	324	191	25.1	37.1
11	"	3 months	491	24	527	209	325	320	200	36.5	127	82	37.8	38.2
12	"	1 day	1 450	4	1 547	662	1 034	1 027	680	37.8	423	239	37.6	36.1
13	"	11 years	444	24	461	186	236	233	165	40.0	99	87	16.1	46.7
14	1 : 2	10 "	577	24	602	258	.....	.....	...	....	...	258	12.0	....
15	"	10 "	124	24	135	55	.....	.....	...	....	...	55	20.0	....
25	1 : 2 : 4	11 "	766	24	806	349	475	472	310	41.6	181	167	22.1	47.8
24	"	6 "	1 725	24	1 824	784	1 128	1 123	...	....	432	352	26.7	44.9
26	"	11 "	955	24	1 025	480	512	507	...	....	195	285	22.8	58.9
27	"	9 months	110	24	135	70	53	52	...	....	20	50	48.0	70.1
28	1 : 2 : 4	12 "	1 246	24	1 320	535	811	806	...	....	310	225	30.7	42.1

*Samples 1, 4, 5, 8. 9 and 10 were mixed in the laboratory, the mixes varying from fairly dry for Sample 5 to wet for Sample 1. All other samples were from concrete on important work. Sample 27 is given to show what sometimes happens to concrete on an important structure. Samples 4, 9, and 10 were mixed with about the right proportion of water for proper handling in forms, but not wet enough to run in a flat sloping chute.

volume passing a No. 8 sieve for gravel that is uniformly graded from the No. 8 size to 1½ in. in diameter. The desire to maintain the 1 : 2 ratio of fine to coarse aggregate probably has done more to place the dividing line between them at the No. 4 sieve size than has any scientific reason. It is not a good dividing line, and the tendency now is to establish other ratios, depending on the aggregate sizes. Since a change is essential, it is believed that much good will be done by establishing the dividing line at the No. 8 sieve size. This will be especially advantageous for the smaller jobs where the necessary fine sieves and men to operate them and calculate theoretical proportions are not available. All that will be needed is a No. 8 sieve and scales for weighing, or some means of measuring the volume. The computation necessary for proper proportioning is then simple.

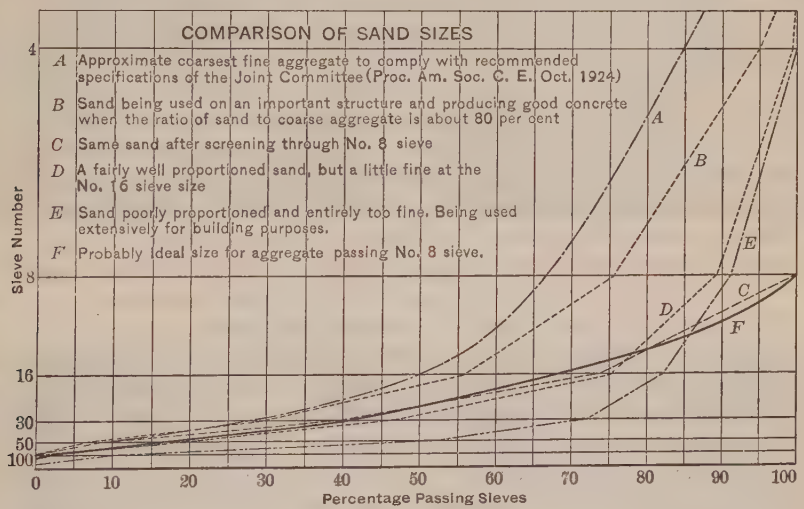
COMPARISON OF SAND SIZES
A Approximate coarsest fine aggregate to comply with recommended specifications of the Joint Committee (Proc. Am. Soc. C. E. Oct. 1924)
B Sand being used on an important structure and producing good concrete when the ratio of sand to coarse aggregate is about 80 per cent
C Same sand after screening through No. 8 sieve
D A fairly well proportioned sand, but a little fine at the No. 16 sieve size
E Sand poorly proportioned and entirely too fine. Being used extensively for building purposes.
F Probably ideal size for aggregate passing No. 8 sieve.
Sieve Number
4
8
16
30
50
100
0
10
20
30
40
50
60
70
80
90
100
Percentage Passing Sieves

FIG. 6.

To establish this size as the dividing line between aggregates will not confuse those who wish to use the proportions recommended by the Joint Committee on Standard Specifications for Concrete and Reinforced Concrete, or the Abrams' fineness modulus. If all material passing the No. 8 sieve is classified as fine aggregate and all that is retained on the sieve as coarse aggregate, and if the proportion of fine to coarse is from 2 to 5% more than the voids in the loose coarse aggregate, the variation from the fineness modulus as recommended by Abrams* will be little for the materials ordinarily used. This is not suggested to take the place of the more accurate methods of proportioning on large jobs, or where skilled supervision is available, but to urge the need of a change in the dividing line between aggregates. The No. 8 sieve is practically an essential in the proposed void test; it more nearly coincides with the abrupt change in aggregate sizes; and it is the limit of size usually specified for brickwork and plaster mortar.

* "Design of Concrete Mixtures," *Bulletin 1*, Structural Materials Research Laboratory, Lewis Inst., Chicago, Ill.

Accordingly, the following is suggested as a desirable addition to the customary specifications for 1 : 2 : 4 concrete:

The ratio of cement to aggregate shall be.....(this will depend on the character of work. The ratio, 1 : 6, is a good proportion) by volume, the fine and coarse aggregate being measured separately. All aggregate passing a No. 8 standard sieve shall be classified as fine aggregate, and all retained on this sieve as coarse aggregate. The coarse aggregate shall be graded from..... (insert largest size desired) to the No. 8 sieve size. Not more than 15% by weight shall pass a No. 4 sieve. The fine aggregate shall be graded from the No. 8 sieve size to the finest, with not more than .... (about 20) per cent. and not less than.....(from 5 to 10) per cent. passing a No. 50 sieve. The proportion of fine aggregate to coarse will be computed on the basis of the voids in the loose coarse aggregate. The percentage of voids plus.....(this should vary from about 2 for very coarse aggregate to about 10 for ½-in. aggregate; 4 is about the proper figure for 1-in. aggregate) will be the ratio of the loose fine aggregate to the loose coarse aggregate. In case the specified proportions do not give a mortar volume in the concrete at least 2% in excess of the voids in the coarse aggregate the proportions shall be changed to give at least this amount. The voids in the mortar of the concrete 1 day old shall not exceed 28% of the volume of the mortar.

It is difficult to state the maximum allowable porosity for the mortar in concrete. Certainly, when it may be exposed to water and freezing weather the percentage of voids should not amount to 35 or 40 as is now frequently the case. This may be suitable for protected buildings where great strength is not necessary, but it is not suitable for outside exposures. It is believed that 25% of voids is the minimum practical quantity in cement mortar as it is first poured, except in the top finish of sidewalks and floors, where the excess water is allowed to escape and the mortar afterward is worked together to its maximum density. The treatment results in the most durable concrete now being constructed. For concrete at the water line, or under water pressure, the maximum of 28% of voids suggested for the mortar should be lowered if it is practical to do so. This will give concrete that is too dry to be chuted in the flat sloping chutes now generally used, and will require the exercise of extreme care in placing to prevent honeycombing. However, the additional expense is more than justified on important structures.

What Happens to Concrete

When concrete is exposed to the air but protected from water, the formation of calcium carbonate is probably the only change that takes place after the hydration of the cement. If the mortar is very porous, that is, if the structural formation of the cement within the mortar is greatly expanded, one of two things will happen: The change to calcium carbonate will gradually fill the voids until they are reduced to the point where the stagnated layers of air practically prevent further diffusion of the carbon dioxide into the mass; or the calcium solubility of the concrete will gradually be reduced and the hydrogen-ion concentration will be increased until calcium carbonate equilibrium is nearly reached. This latter change takes place with no apparent weakening of the strength of the concrete; in fact, there is usually an actual increase in strength.

Fig. 7.—Stirrups Corroding in a Reinforced Concrete Girder About 10 Years Old.

Fig. 8.—Slow Diffusion Rates of Calcium Hydroxide Through Alumina Hydroxide.

The only danger where the structure is protected from the weather is the possibility of the steel reinforcement rusting and breaking the concrete as shown in Fig. 7. This construction was about 10 years old and was protected from the weather, but was on the inside of a building where the air was moist most of the time. There was a layer of concrete, $\frac{5}{8}$ in. thick, over the steel. The concrete that flaked off was tested by pulverizing it and submerging it in distilled water in a closely stoppered flask for 20 days. The solution at the end of this time had a phenolphthalein alkalinity of 28 and methyl orange alkalinity of 52. These figures are considerably below the point necessary to prevent rusting of iron. The rate of penetration of carbonic acid will be slowed up considerably as the depth increases, but it may eventually penetrate the entire depth of the girder. The materials used in construction probably were good, but mixed too wet.

The hydrogen-ion concentration, *pH*, and the alkalinity necessary to prevent rusting have not been definitely determined, but when the alkalinity is about 200 parts per million the *pH* should probably be as high as 11.0. Some wet mixed concrete is so porous that the CO_2 penetration will eventually reduce the alkalinity and *pH* below these points for some distance from the surface if not throughout the entire mass. Iron rust, however, cannot progress without space for expansion. If the expansive force is not sufficient to crack the concrete, rusting will not seriously injure the steel, but it does leave the concrete under a strain, which, combined with other stresses, might eventually cause failure. The hydrogen-ion concentration and alkalinity of a solid mass are virtually the same as the water within the pores if any is present.

If concrete is very dense the pores are filled practically to the point of preventing further entrance of CO_2 gas before much of the calcium has been changed to calcium carbonate. No rate of penetration for CO_2 can be set, for it will depend on a number of variables, the most important of which is the structural expansion of the cement within the concrete. It will never penetrate very deep into dense concrete, probably less than $\frac{1}{2}$ in., but in concrete so porous that the voids are not reduced to the stagnated stage the penetration will eventually reach the center of the mass. This has already happened in many instances, and there will probably be a great increase in the cracks produced by rusting reinforcement within the next 10 to 20 years; however, the pressure necessary to crack the concrete may give protection to deeply embedded steel.

Concrete exposed to the weather is subject to alternate wetting and drying, which tends to hasten the final equilibrium, which in this case occurs when the voids have been reduced to the stagnated stage, or when the concrete has nearly reached calcium carbonate equilibrium. This latter condition will never be reached when the concrete is very dense; only a thin layer on the outside will reach final equilibrium. Concrete $\frac{1}{8}$ to $\frac{1}{4}$ in. from the surface of a well constructed sidewalk many years old will give up calcium when submerged until it reaches an alkalinity of 1 600 to 1 800 parts per million. This is the desired condition for resistance to the elements.

If the concrete is so porous that calcium carbonate equilibrium is nearly reached before the stagnated stage, every wetting of the concrete with water

corrosive to calcium carbonate will wash away a little of the calcium, for there is no caustic lime present to combine with carbon dioxide forming a precipitate. Gradually the pores will be increased. Diffusion rates will be increased at a much greater rate than the pores. The final stage for such concrete is disintegration, which will be greatly hastened by exposure to freezing weather. Sometimes concrete that has withstood the weather for a number of years suddenly begins to disintegrate. Such cases, where good materials have been used and where the results are not affected by certain neutral salts or where there is no concentration of other detrimental compounds due to moisture travel, are almost invariably due to failure in reduction of the voids to the stagnated stage. There is probably a gradual weakening beginning a few years after construction, but it requires years to reach the point of disintegration.

In concrete submerged in water corrosive to calcium carbonate the porosity of the cement is again the main factor affecting corrosion. From a fresh exposed surface, calcium hydroxide is set free in the water. If there is considerable CO_2, free or half-bound, in the water, calcium carbonate forms and precipitates with much of it adhering to the concrete surface. This soon reduces the outside pores to the stagnated state and prevents rapid diffusion of the calcium from the interior of the concrete to the surface. The water adjoining the surface then approaches its original *pH* and alkalinity, which has been assumed to be corrosive. In this manner there is a gradual diffusion of calcium hydroxide to the surface, a change to calcium carbonate, and a gradual dissolving of the calcium carbonate.

When this action has progressed a short distance into the concrete, probably less than 0.001 in., another force is brought into play. The dissolving of the calcium from the aluminum compounds leaves aluminum hydroxide, which builds up a gelatinous structure around the outside surfaces, tending to close the voids, and make the mass more impervious. Gelatinous silica also aids, but there is no definite proof that silica goes into solution when most of the calcium is dissolved.

These gelatinous compounds, even if apparently loosely formed, greatly reduce the diffusion rates. Some idea of the slowness of these rates may be obtained from Fig. 8. In this experiment about ½ gramme of hydrated lime was placed in the bottom of the tube and aluminum hydroxide coagulation was added. The hydroxide was obtained by precipitation from aluminum sulfate and was washed nearly free of sulfates before being used. A small quantity of phenolphthalein solution had been added previously. The diffusion of the calcium hydroxide could be followed quite readily by the change to a pink color, as shown in Fig. 8. It required 30 days to penetrate the entire depth of about 2½ in. Had the aluminum hydroxide been more compact the rate of penetration would have been much slower.

Unless the surrounding water is considerably below calcium carbonate equilibrium and there are no other forces aiding, such as freezing or detrimental soluble compounds, corrosion will be slow. The free access of water into the interior of the concrete, by cracks or honeycombed concrete, greatly aids in extending the corrosion into the interior.

Diffusion Rates and the Concentration of Destructive Compounds Within Concrete

The rate of seepage through concrete due to capillary attraction is usually many times the diffusion rates of most fairly soluble compounds ordinarily found in water or concrete. The finer the pores within a solid the slower are the rates with which soluble compounds within the solid will diffuse to the surface, and also the slower are the rates of water travel by capillary attraction. Evaporation of moisture from a concrete surface near the point where water is in contact with the mass, usually causes water travel within the concrete at a greater rate than the diffusion rates backwards for many of the fairly soluble compounds. For a concrete wall partly submerged, as shown in Fig. 9, the surface above the water line gives off moisture and forces by capillary attraction water into the concrete below the water line. This not only concentrates the soluble salts originally in the water, where the moisture is evaporating, but also dissolves soluble compounds from the cement near the point where it is entering and concentrates them near the evaporating point.

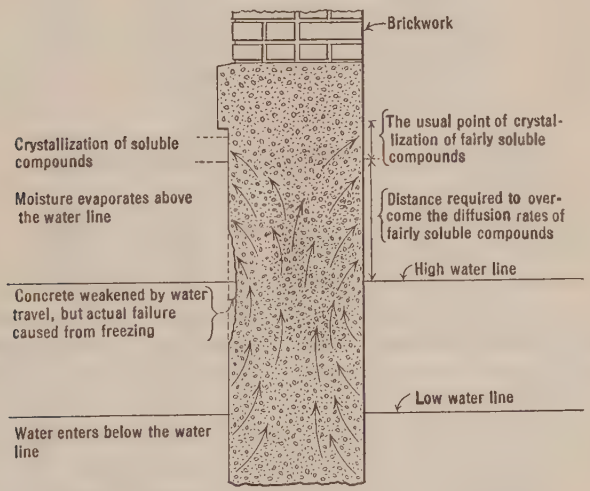
Brickwork
Crystallization of soluble compounds
Moisture evaporates above the water line
The usual point of crystallization of fairly soluble compounds
Distance required to overcome the diffusion rates of fairly soluble compounds
High water line
Concrete weakened by water travel, but actual failure caused from freezing
Low water line
Water enters below the water line

Fig. 9.—Water Travel in Concrete Wall Partly Submerged.

Soluble compounds, such as sodium sulfate, sodium chloride, sodium carbonate, magnesium sulfate, magnesium chloride, and others, may be concentrated near the surface where the moisture is evaporating, unless removed by rains or frequent washing of the surface. Neutral salts, such as sodium chloride and sodium sulfate, have the power of base exchange when the concrete has reached certain stages; that is, the calcium combined with or adsorbed by the silica and possibly the alumina is exchanged for sodium. This exchange frequently takes place when the quantity of salt in solution is at low concentration, probably less than 25 parts per million, and when the calcium in the silica and alumina has been reduced to a certain stage. Under such conditions the compound concentrating near the surface where evaporation is

taking place may be sodium carbonate. Moisture evaporates from the surface of sodium carbonate and sodium sulfate nearly as readily as from a water surface, consequently, the concentration of these salts to the crystallizing point probably does not greatly reduce the evaporation rate as in the case of

TABLE 7.—THE CALCIUM SOLUBILITY INDEX OF CONCRETE FROM VARIOUS SOURCES.*

Sample.	Alkalinity, in Parts per Million.					
	2 Hours:		24 Hours:		10 Days:	
	Phenol-phthalein.	Methyl orange.	Phenol-phthalein.	Methyl orange.	Phenol-phthalein.	Methyl orange.
Concrete Exposed to the Weather and Disintegrating.						
3	46	76	94	116	174	196
4	54	98	64	100	98	128
12	210	228	246	272	228	256
14	52	68	28	52	20	40
15	.....	.....	20	40	16	36
16	226	246	330	344	376	394
17	6	36	12	40	18	40
18	24	48	30	46	44	58
27	.....	.....	10	33	12	32
Concrete Exposed to the Weather and in Good Condition.						
2	418	448	1066	1100	1838	1874
13	450	480	890	922	1800	1850
23	1150	1194	1440	1476	1840	1878
24	1364	1400	1466	1500	1822	1850
29	524	568	908	940	1218	1244
30	300	336	360	394	410	438
Surface of Concrete Exposed to Water Corrosive to Calcium Carbonate.						
1	28	60	32	48	36	46
19	0	40	0	50	0	56
31	24	52	26	54	30	50
32	32	48	28	56	28	58
Concrete Inside of Buildings and not Exposed to Water.						
6	.....	.....	12	52	32	52
10	1 170	1 222	1 252	1 302	672	760
20	4	40	8	42	16	44
21	124	228	180	288	432	548
22	546	582	720	760	640	684
25	52	60	60	90	130	156
26	560	600	588	640	720	738
33	1 150	1 190	1 466	1 480	1 840	1 862

* All concrete except Sample 4 is more than 9 years old. Samples 2 and 13 were not crushed very fine.

calcium carbonate and probably of some of the other compounds. Some salts, when confined, crystallize under pressure and may induce failure by causing the concrete to swell and crack. Actual failure is usually greatly aided by freezing weather.

Calcium Solubility Index

A number of disintegrating structures have been visited during the course of this investigation and many samples have been collected. It has been found that the quantity of calcium going into solution gives a good idea of what has been taking place within the concrete. This is shown in Table 7. Neat cement mortar or concrete, sealed in air-tight containers immediately after mixing with water and allowed to stand for a number of years, will, when pulverized and submerged in water, give up considerable calcium. When the ratio of the weight of concrete to that of water is fairly large, calcium will go into solution usually to the saturation point of calcium hydroxide. Occasionally, the total alkalinity will exceed the saturation point, but this is due to the presence of alkalies more soluble than calcium hydroxide, or to certain neutral salts. Tests indicate that, unless free lime is present, calcium saturation is never reached. By submerging pulverized samples in well stoppered containers and agitating them frequently, the saturation or equilibrium point appears to be reached within less than thirty days. The following procedure has been used in determining the calcium solubility.

Crush the sample in a small rock crusher or with a hammer, and then pulverize it in a laboratory mortar or ball mill. Discard as much of the coarse aggregate as may be removed conveniently. If a ball mill is used, sift the product through a No. 16 sieve after pulverizing. Do not allow the pulverized concrete to stand exposed to the air longer than is necessary before sealing. Mix approximately 60 grammes of the pulverized concrete with 300 cu. cm. of distilled water in a 300-cu. cm. flask. Stopper tightly and shake frequently for 1 hour. After an additional hour filter a part through a washed filter paper, discarding the first 20 or 25 cu. cm. Test for alkalinity as soon after filtration as possible, also at the end of 24 hours, 10 days, and 30 days. It is not necessary to filter before making the latter determinations if the solution is perfectly clear. The sample should be agitated several times daily.

If possibly water has passed through the concrete concentrating certain soluble salts, such as sodium carbonate, the calcium determination should be checked by a gravimetric test. As a rule, if the phenolphthalein alkalinity is only slightly less than the methyl orange alkalinity the alkali will be in the form of calcium hydroxide.

Laitance

Laitance is composed of a greatly expanded and partly coagulated formation of the finer particles of cement, or of any other finely divided material that may be present. Frequently, it contains numerous fine air-bubbles, but not necessarily. The voids are usually from 60 to 75% of the mass. The extremely fine particles are expanded to the limit of the stagnated stage. The mass also possesses free waterways. The communication with the outside air or water is so rapid that the concrete is reduced to approximate calcium carbonate equilibrium before hydration is complete. If laitance is placed in a sealed container and allowed to stand indefinitely, it will maintain as high an alkalinity as good concrete, but will develop little strength because it is so greatly expanded.

It is not possible to expand cement to such a high percentage of voids because of its larger particles, but if all the cement were ground to the size of the smaller particles and sufficient water and agitation were provided, the expansion would be about as great. To a certain extent, this is what happens to the smaller particles of cement between the larger ones when an excess of water is used; that is, the smaller particles are expanded to the laitance stage. There is a possibility that a small amount of aluminum hydroxide is liberated when the water is added, and the formation of an aluminum hydroxide coagulation greatly aids in keeping the small cement and dirt particles much farther apart. Fig. 4 shows that dirt particles are readily coagulated by the calcium liberated from cement. Laitance is not cement of a different character or composition from the remainder, but it usually contains a larger percentage of dirt that has been washed out of the aggregates.

What to Do

The investigation shows that the measurement of voids in the mortar gives an idea of what may be expected of concrete, when good materials have been used. It indicates that the voids in concrete 1 day old should not exceed about 28% of the mortar volume when the concrete is to be exposed to water or freezing weather. For concrete more than 1 year old, the voids probably should not exceed about 22 per cent. The alkalinity test, however, is the better criterion for judging old concrete.

The problem of how to take care of existing structures is a serious one. If a conduit or wall with water pressure on one side shows a leak immediate steps must be taken to stop it, for the concrete is weakened as the calcium is removed. In case of delay, until an exposed structure begins to disintegrate before making an effort to prevent it, it may be too late. Progress in making durable concrete has resulted from using good materials and exercising care in mixing and placing; yet with all these precautions many structures begin to disintegrate after a few years of exposure. Many of these structures become strong enough a few months after construction for the load they are to carry, and if this strength could be maintained they would remain safe. Structures disintegrate, even when the inspection has been good and every care has been exercised to see that good materials are used, as in the cases illustrated in Figs. 10 and 11.

Probably the same materials were used in the disintegrating slab (Fig. 10) as in the surrounding ones. This poor concrete when pulverized and submerged in distilled water gave a phenolphthalein alkalinity of 20 and methyl orange alkalinity of 40 parts per million. One of the good blocks tested showed corresponding alkalinities of 1 800 and 1 850, respectively. Too much water in the mixing may have been responsible for the disintegration. If the surrounding blocks do not suffer any material change from their present alkalinity they will never disintegrate.

These conditions force the realization that present-day knowledge of concrete is still quite meager. If good materials were used in the bridge abutment (Fig. 11) it seems that only one thing can account for the disintegration—a

FIG. 10.—CONCRETE FLOOR EXPOSED TO THE WEATHER.

FIG. 11.—BRIDGE ABUTMENT SHOWING CONSIDERABLE DISINTEGRATION.

great structural expansion of the cement fibers. Tests of a sample from the abutment showed a phenolphthalein alkalinity of 16 and a methyl orange alkalinity of 36. The voids in the mortar were 44 per cent. Such a greatly expanded cement structure, if not the direct cause for disintegration, certainly is a contributing cause.

The alkalinity of concrete measured ¼ in. from the surface for structures exposed to the weather or to water should not fall below about 200 parts per million. For a depth of ¼ to ½ in. the alkalinity should not fall below about 1 200. Poor sand may produce concrete which will have a high calcium solubility, but which because of its weak strength will disintegrate in freezing weather. However, it is not the purpose of this paper to consider disintegration except where good materials have been used.

It seems that the most logical procedure is to examine existing structures. If the calcium solubility index at a depth of ½ in. or more from the surface is low, the concrete should be protected. It is believed that, when the safety point of alkalinity is exceeded, reliance should be placed only on some impervious surface coating, such as paint. After cracks have developed, protection is very difficult. Millions of dollars in future repairs and replacements probably could be saved by protecting exposed structures as they fall below the safety limit of calcium solubility. Many exposed structures apparently in good condition will be found to be below the safety limit, but it is likely that a gradual weakening is taking place and that eventually this may cause failure.

The writer wishes to express his appreciation of the aid given by other engineers, especially by Thaddeus Merriman, M. Am. Soc. C. E., Chief Engineer, New York City Board of Water Supply.

DISCUSSION

THADDEUS MERRIMAN,* M. AM. SOC. C. E.—The author has done a great and real service to the science of concrete. No contribution of which the speaker is aware has so thoroughly developed the many phases involved.

Like all materials, concrete has its limitations. The author pointedly speaks of "corrosion". Limestone and steel corrode from the surface inward. When, however, "corrosion" is active not only on the surface but also within the mass, then it is that "disintegration" begins.

Perhaps the most potent influence which has operated to build up in the minds of people everywhere the idea that concrete is an imperishable material is the advertising slogan "Concrete for permanence". The author, however, has clearly shown that, in the absolute sense, no concrete is permanent and that the word, "concrete", is only a term which embraces everything from the best down to the poorest.

All other things being equal, the densest concrete will probably be the strongest and the most durable. The author, however, has forcibly pointed out that the rate at which water diffuses through a concrete is not necessarily a function of the absolute density; and therefore it clearly follows that concrete made from those cements that will develop a structure of such nature as to permit a minimum of water diffusion will be the most durable.

Those concretes through which water diffuses least readily contain, as has been indicated by Colony,† a maximum of "the amorphous constituent". These resistant concretes, as pointed out by the author, are also those that show a high alkalinity. The conclusions drawn by Colony from his microscopical examinations thus seem to be confirmed by the author's investigations.

If all cement were of the same definite composition, then all concrete made under like conditions should show similar characteristics and equal resistance to both corrosion and disintegration. Such, however, is not the case. Cement is a variable product with respect to its content of those constituents that are necessary to the development of a structure which will act to prevent the diffusion of water. One cement, which, by way of example, contains only 10% of the compounds that go to develop the amorphous constituent, cannot possibly make a concrete which will be as resistant as that made from a cement containing 25% of these compounds.

The speaker has indicated‡ some of the great differences among present-day cements. These differences are not disclosed by the standard specification under which all cements meeting the minimum requirements are labeled as equals. The science of cement has lagged far behind in the general march of progress and the almost universal demand for better concrete will not come to be satisfied until better cement is available for its making.

* Chf. Engr., Board of Water Supply, City of New York, New York, N. Y.

† "Judging the Quality of Portland Cement," *Transactions*, Am. Inst. Min. and Metallurgical Engrs., 1921.

‡ *Engineering News-Record*, April 22, 1926.

The consumer must learn to specify his cement just as he now specifies his steel, his coal, his wheat, and his oranges. A single standard for any commodity can do no less than promote mediocrity of quality.

JOHN G. AHLERS,* M. AM. SOC. C. E.—From the contractor's viewpoint, the speaker disagrees with the author's opinion, which seems to indicate that much of the concrete being produced is bad. It is poor policy for the Engineering and Contracting Professions to illustrate disintegrating concrete and not to show a proportionate amount of good work.

After sixteen years of practical building experience, the speaker offers to go back over the structures constructed either directly by him or under his supervision and challenges critics to find anything like the conditions described. The reason for this is neither because a better quality of cement was used nor a better kind of aggregate, but merely because the concrete was made with reasonable care under good standard conditions; because standard brands of Portland cement were used and, what now becomes increasingly clear and more important every day, because not too much water was used in mixing the concrete. The cause of every case of bad concrete mentioned in the discussions, probably may be traced back, not to inferior cement, but to improper mixing, placing, or exposure to freezing during the setting period, with a poor grade of aggregate. It is entirely too much to expect the cement to overcome such obstacles and, in spite of them, to make durable concrete.

When the water-cement ratio is kept constant for any definitely desired high strength, uniform, durable concrete will result. To illustrate the simplicity of principles involved, Fig. 12 shows how the concrete mixed by the speaker's Company is controlled.

Every job has its own "job constant", fixed by the water-cement-ratio of 1, from which increases or decreases in strength can be obtained by automatically moving the water-measuring tank; the strengths will follow the curve, known as a "job curve". Only the job constant is affected by the quality of the aggregate and the cement used.

Engineers, therefore, do not need different standards of testing cement or different gradings of cement. If any one believes that one brand is better than another, he can buy the grade that he believes is better and, by combining it with the aggregate available, establish his own job constant. Innumerable tests made by the Lewis Institute and by many others show that the quantity and grading of the aggregate have no effect on the strength as long as the concrete is workable and plastic; also, the strength, and therefore the lasting quality of the concrete, is merely controlled by the water-cement ratio used in producing the concrete.

Judging the paper from the angle of field experience since 1924 in controlling concrete by the water-ratio theory, the speaker feels that the chemical value of the author's work may be good and can neither be criticized nor discussed. This phase presents a viewpoint that is interesting to a contractor and may explain surface disintegration or weathering. Such weathering, however, is slight in properly made standard concrete. Therefore, the paper is

* Secy.-Treas., Barney-Ahlers Constr. Corporation, New York, N. Y.

perhaps more applicable to work showing lack of control during the process of making the concrete. Where good contractors do the work and proper supervision of the water-cement ratio takes place, there need be no danger of disintegration. In fact, the author really affirms the importance and necessity of water-ratio control of concrete. There would seem to be no need of such a special test for void as he asks for if water-cement ratio is controlled.

The speaker does not believe in specifying proportions of sand to gravel or in changing the size of the maximum grains in the sand. Both the water-ratio theory and the paper itself indicate that the size and grading of the sand grains have no effect on the strength or the density of plastic concrete. Size and grading affect only the economy of the mixture. If a change is made, it should be as the author recommends, to have sand of $\frac{1}{8}$ in. and less, and two other sizes to combine with this; this is almost the same as the speaker's practice—to buy commercial "ready-mix" and check the quantity of fine sand contained. The remainder of the aggregate, as Mr. Baylis points out, is of no material consequence except as a filler for the resulting concrete.

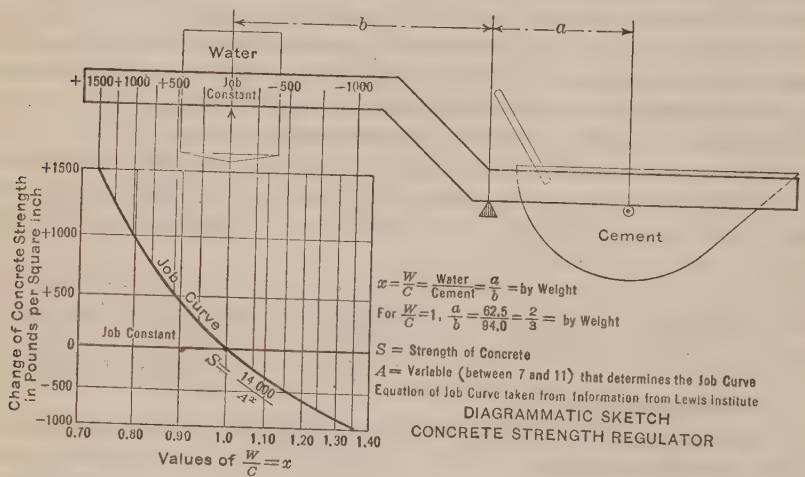
b
a
Water
+1500+1000 +500 Job Constant −500 −1000
Cement
Change of Concrete Strength in Pounds per Square inch
+1500
+1000
+500
0
−500
−1000
Job Curve
Job Constant
S = 14,000 / A+7
0.70 0.80 0.90 1.0 1.10 1.20 1.30 1.40
Values of W/C = x
x = W/C = Water/Cement = a/b = by Weight
For W/C = 1, a/b = 62.5/94.0 = 2/3 = by Weight
S = Strength of Concrete
A = Variable (between 7 and 11) that determines the Job Curve
Equation of Job Curve taken from Information from Lewis Institute
DIAGRAMMATIC SKETCH
CONCRETE STRENGTH REGULATOR

FIG. 12.

It would be better, if "ready-mix" is not used, to have three sizes of aggregate—from $\frac{1}{8}$ in. down, from $\frac{1}{2}$ in. to $\frac{1}{8}$ in., and from $\frac{1}{2}$ in. up—with which a very economical concrete can be made. It is a question whether there is any advantage in cost to the contractor by using three sizes, having in mind that the size and grading of the aggregate have no effect on the resulting strength.

Thus, there need be no fear of poor concrete providing a standard cement is used in a proper way, and providing the water-cement ratio is controlled by some approved device. There are a thousand examples of good work to every instance of disintegrating or poor concrete.

Contractors as well as engineers know good clean aggregate and can obtain it, test it, and use it, if they are willing to pay the price. This quality of the aggregate affects the job constant and the job curve, but is a separate factor

from that of grading, which is not important, as it affects only the yield or economy in volume of concrete made.

The effect of the paper is valuable in calling attention to the importance of control by the water-cement ratio and in emphasizing the need of hiring contractors who are equipped and know how to handle the production of concrete under such control.

M. N. Clair,* Assoc. M. Am. Soc C. E.—In his method of "Measurement of Mortar and of Voids in It" (page 806), the author mentions that he heats a specimen to a temperature of about 212° Fahr. for about 24 hours. If the specimen is broken and heated, the moisture content will be very different from that obtained by the unbroken specimen in a drying oven.

In analyzing a number of samples of concrete, the specimens were dried in the manner described by Mr. Baylis. The specimens were 4-in. and 6-in. cylinders of various heights, and consistent results could only be obtained by first breaking them and then drying them.

R. H. Bogue,† Esq. (by letter).—This paper is of interest because the author reports data on chemical experiments and interprets these data in terms of generalizations on the nature and behavior of Portland cement.

In the case of all scientific papers, and more especially those in which generalizations are formulated which may affect profoundly the treatment of a most important structural material, it seems imperative that the author exercise the utmost caution in his statements and make no assertion whatsoever for which he does not give either his authority or his experimental evidence. Even a conjecture on matters of such importance is impermissible unless the bases for an opinion are expressly stated. This is the more compulsory when such statements or conjectures appear to be at variance with established chemical or physical law.

The author has presented almost no references to published investigations, and confirming data on many statements are lacking. In many cases statements are made which leave the impression that they represent established truths while as a matter of fact they are the author's interpretations on observed phenomena. These interpretations may or may not be justified, but in every case it should be made clear that they represent only the author's views.

Herewith are given a few typical instances that will serve to illustrate these points.

1.—It is stated:

"When a solid has attraction for both acids and alkalies there is a certain hydrogen-ion concentration at which practically no acid ions are adsorbed, and another concentration at which practically no alkaline ions are adsorbed. Between these points both ions appear to be attracted to or near the solid surface in variable quantities, depending on the hydrogen-ion concentration and a number of other variables."

* Testing and Designing Engr., The Thompson & Lichtner Co., Inc., Boston, Mass.

† Research Director, Portland Cement Association Fellowship, U. S. Bureau of Standards, Washington, D. C.

This statement seems not to be in accord with the usually accepted principles of electrolytic equilibrium and requires supporting data or authority.

2.—"In the hydration of Portland cement", it is also reported, "the dissolving and re-precipitation as the cement compounds unite with the water takes place with little change in volume or form from that in which the particles of cement are left after they have been mixed with water." It seems that the statement must be qualified, for it is hardly conceivable that a conversion of the several anhydrous crystalline compounds of clinker to the partly amorphous and partly crystalline and usually hydrous compounds resulting from reaction with water could be effected without change in form. Evidence supporting the author's opinion is necessary to establish these points.

3.—The statement is made that "stagnated layers of water or air close the pores against rapid communication with the surrounding water or air". Since this statement constitutes the basis for the development of a method for testing concrete structures, it should be fully supported by authority or experimental data.

4.—The setting of Portland cement is referred to in various places in the paper as concerned with or brought about by a change in the oxides to the hydroxides and the presence in the product of both aluminum hydroxide and hydrated silica. No authority or data is offered for these statements, and since they are not in accord with recent published investigations, they require supporting evidence.

5.—It has not been explained what is meant by the "hydrogen-ion concentration of a solid mass". Such a conception is not recognized in physico-chemical science.

6.—The author states that diffusion is greatly retarded by the presence of gelatinous materials, and quotes an experiment in which 30 days were required for diffusion to proceed 2.5 in. Similar slow diffusion may be demonstrated in pure solutions with no gelatinous material present. For example, copper sulfate solution placed in a jar below water may take years to diffuse through a distance of a few inches to form a homogeneous solution.*

In addition to these statements of the author, which lack supporting evidence, there are a few general points that require detailed attention.

Surface Adsorption.—A few experiments are described on the leaching out of the "calcium" from various cements. From the description presented it is inferred that the solvent was distilled water and that after each interval of several days, when a part of the solution was removed for test, a similar quantity of water was added. The data are plotted with "alkalinity" in parts per million expressed as calcium carbonate as the ordinate against "percentage of total calcium dissolved" as the abscissa (Fig. 2). The curves obtained show a more or less regular drop, indicating that as the total "calcium" dissolved becomes greater, the "alkalinity" of each fraction tested decreases. The author regards the smooth nature of the curves (the points on Curve *B* show a distinct break that is not explained) and the drop in "alkalinity" as evidence that "either there are a number of compounds formed in concrete, or that stable compounds of high calcium content do not exist". He adds,

* Smith's "College Chemistry."

"The gradual change in equilibrium, as shown by the curves in Fig. 2, points strongly to the fact that if there are definite stable compounds of calcium and silica in concrete, they are probably compounds of lower calcium content than tri-calcium silicate, and that any additional calcium above these stable compounds may be merely in the nature of a surface adsorption, or, as has already been suggested, a solid solution."

These conclusions leave it optional whether there are a number of compounds formed in concrete, or surface adsorption complexes, or solid solutions. The author, however, makes it apparent in the following section on "Surface Adsorption"* that he purposes to champion the surface adsorption idea. The main point seems to have been missed entirely. The curves presented are typical curves for hydrolysis. Any salt which contains either a weak acid radical, or a weak basic radical, or both, will undergo in water a decomposition into its acidic and basic constituents.*

The case in point involves the tri-calcium silicate, di-calcium silicate, and other aluminates, ferrites, etc. To deny the existence of such compounds in clinker is to lay aside the well-known work of the Geophysical Laboratory on the system, CaO-Al_2O_3-SiO_2. Of course, errors may have been made in the attempt to carry over this work on pure components into the much more complicated polycomponent cement, but at least any denial of such work must be accompanied by convincing evidence. By the principle of hydrolysis, therefore, these silicates, aluminates, and ferrites of lime would be decomposed in water into calcium hydroxide and either the oxides of silica, alumina, and iron, or a silicate, aluminate, and ferrite of lower lime content. If the water were continuously or intermittently removed, the end products would probably be hydrated silica, alumina, or ferric oxide. If the water were not removed, the hydrolysis would be expected to continue until an equilibrium condition were reached at which the rate of recombination of acid and base just equalled the rate of hydrolysis. This condition, however, is determined by the dilution and the nature of the remaining solid material. This latter has changed continuously by becoming less basic, so that the equilibrium concentration of hydroxyl-ion to just counterbalance hydrolysis of the remaining material becomes less at each test. That is, the "alkalinity" should become less at each test, which is the condition found by the author. This is the state of the reaction at equilibrium. If equilibrium is not attained, the departure from a constant hydroxyl-ion concentration would be even more pronounced.

Thus, in the data presented by the author, there is nothing that is not altogether comprehensible and completely explainable on the basis of the recognized laws and theories of chemistry. The injection of a theory of surface adsorption seems unnecessary and unwarranted.

Significance of the Void Test.—In the paper much significance is attached to the void test. It is imperative, however, that an adequate method for the measurement of voids be at hand before conclusions may legitimately be made on the significance of the voids. The method suggested consists in taking

* Publications of A. A. Noyes, or any modern textbook on Physical Chemistry.

a special 24-hour old sample, drying at 210 to 250° Fahr., cooling, weighing, submerging in water for periods of 5 min., 2 hours, and 4 hours, and reweighing. Omitting details, the percentage of voids is then calculated as the ratio of the water adsorbed to the volume of the sample.

It must be realized that by drying to constant weight at temperatures up to 250° Fahr., a considerable quantity of water will undoubtedly be lost by dehydration and desorption, and that the water so lost will probably again be taken up on submergence. Furthermore, the wetter the original mix, the greater, presumably, will be the surface exposure, and consequently the greater will be the quantity of water taken up in a limited period of time by these processes, which is the condition observed by the author.

Thus, the void test as described appears to be a measure of a number of factors including the water of wetting, hydration, and hydrolysis, as well as that which actually enters the voids. Methods previously described* avoid these difficulties to a large extent, and Talbot has discussed at length the usefulness of the test, but none of these is referred to in the paper.

Calcium Solubility Index.—The author suggests a method for determining whether any given concrete structure is continuing in a satisfactory state, or whether it is weakening and becoming unsafe or subject to rapid disintegration. The method consists of a determination of the "alkalinity" of a sample. In general, this becomes a calculation of the uncarbonated lime in the sample. High "alkalinity" means that the lime has not become carbonated. The author states that the "alkalinity" $\frac{1}{4}$ in. from the surface should not be less than 200; and $\frac{1}{4}$ to $\frac{1}{2}$ in. from the surface, not less than 1 200. The reason given for this is that a high value indicates a concrete of such imperviousness that carbon dioxide cannot enter, while a low value indicates a porous structure.

This suggestion is interesting and should be properly tested. The experiments quoted are inadequate for basing any conclusions, but they seem to be sufficiently suggestive to warrant a carefully planned and comprehensive series. Such a series of investigations, however, should consider the testing not only of structures being built, but structures subjected to exposure over a period of years, in order to observe any changes that may be taking place.

Conclusions.—The foregoing remarks are intended to bring out the necessity of presenting searching confirmatory data whenever new theories or ideas are expressed. Progress in science is best served by the announcement of such theories only after sufficient evidence has been gathered to justify the formulation of clearly defined conclusions. By following such a procedure, there will result a better understanding of Portland cement and a warranted confidence in the soundness of the theories presented. From this understanding and confidence will follow a more intelligent application of its use by engineers.

G. M. Williams,† M. Am. Soc. C. E. (by letter).—No doubt many engineers using cement and concrete for structural purposes believe, or at least

* By A. N. Talbot, Past-President, Am. Soc. C. E., and, also, F. E. Richart, Assoc. M. Am. Soc. C. E., *Proceedings*, Am. Soc. for Testing Materials, Vol. 21 (1921), pp. 940 and 964.

† Prof. of Civ. Eng., Univ. of Saskatchewan, Saskatoon, Saskatchewan, Canada.

have had no reason to question, the idea that Portland cement in hardening forms compounds insoluble in water. That Portland cement is soluble has been known for many years to engineers of the Western States and Canada who have been building, and observing the behavior of, concrete structures exposed to both fresh and alkali waters. The greater part of the rapid deterioration of concrete in these regions has been due to the soluble salts, principally the sulfates, found in the ground-waters, but there have been numerous examples of the removal of lime from the mass and its deposition at the surface in the form of the carbonate by waters that have low salt contents and would not be expected to attack concrete of fair quality under normal conditions of exposure.

Since 1906 numerous investigations have been conducted and experiments made to determine how concrete of known quality is affected by exposure to these abnormal conditions. Although broad and final conclusions cannot yet be drawn, the results of most experiments are in fairly close agreement. It has been determined (*a*) that Portland cement concrete of good quality may be disintegrated, due to the breaking down of the cement; and (*b*) that for concretes made from fair to good quality aggregates the cement content is the greatest factor in prolonging the life of concrete. Field and laboratory experiments together have indicated that concretes exposed to water, either fresh or alkaline, should have low permeability and that permeability is most greatly influenced and controlled by change in the cement content. As the cement content increases, permeability decreases. Generally these most nearly impermeable concretes are high or highest in percentage of voids and lowest in density. The mixture least permeable, and most resistant to fresh or alkali water action, yet lowest in density, is neat cement. The mixture most permeable, least resistant to fresh or alkali water action, but highest in density, is a mixture, such as 1 : 2.5 : 5 or 1 : 3 : 6. Mixtures leaner than these will usually show a decrease in density and a great increase in permeability depending on the sand gradation, since the cement content is no longer sufficient to fill the pore space to the same proportional degree as in the richer mixes.

Many technical papers on concrete refer to the necessity for obtaining high density or low porosity if water-tightness or high strength is desired. The criterion for both strength and water-tightness is assumed to be density, and it is generally supposed that these desired qualities will be attained when cement, aggregates, and water are so proportioned as to have the highest density.

The writer believes that both laboratory studies and field experiments are in accord in pointing to the conclusion that concretes, or the mortars in concretes, of highest densities are not those of highest strength or water-tightness, with consequently greatest resistance to deterioration, and that there is little or no relation between density of concrete or mortar and strength and impermeability.

The data to be presented seem to indicate, (*a*) that the method described by the author for determining the density of mortar is unnecessarily laborious and involved; and (*b*) that density of concrete, or the density of mortar in a concrete, cannot be accepted as a criterion for either strength or water-

tightness. The data reported apply to the particular cement and aggregate gradings used in these tests.

Determination of Density of Concrete and Mortar.—The following method, generally used in concrete laboratories for determining density, is suggested as being simpler and more accurate than that based on absorption, as outlined by the author.

Assume that it is proposed to make a concrete of the proportion 1 : 1.5 : 3 by volume and that materials having the physical properties given in Table 8, are available. It is desired to determine the quantity of cement required per cubic yard of concrete, the density of the concrete, and the density of the mortar in the concrete.

TABLE 8.

	Weight per cubic foot, in pounds.	Specific gravity.
Cement	94	3.12
Sand	98	2.36
Gravel	92	2.66

Since in the laboratory it is more convenient to use the metric system, all weights will be determined in grammes (Table 9) and volumes in cubic centimeters.

A vessel having a capacity of 0.05 cu. ft. = 1 417 cu. cm., held 3 410 grammes net weight of this wet concrete.

$$\frac{\text{Weight of 1 417 cu. cm.} = 3\,410}{\text{Weight of batch} = 6\,050 \text{ grammes}} = 0.563$$

Then, in 0.05 cu. ft. there are:

$$0.563 \times 1\,000 = 563 \text{ grammes of cement}$$
$$0.563 \times 1\,550 = 873 \text{ grammes of sand}$$
$$0.563 \times 2\,940 = 1\,660 \text{ grammes of gravel.}$$

TABLE 9.

Proportions by volume.	Proportions by weight.	Batch weights, in grammes.
1	1	1 000
1.5	1.55	1 550
3	2.94	2 940
Water for required flowability	....	560
Total weight	....	6 050

Computation of Cement per Cubic Yard of Concrete.—

$$\frac{563 \times 20 \times 27}{453 \text{ (grammes per pound)}} = 672 \text{ lb. of cement per cu. yd. in wet concrete.}$$

This quantity should be slightly increased because of the shrinkage during hardening, dependent mainly on the flowability. A good average value would be 2 or 3 per cent.

Computation of Density of Concrete.—

$$\frac{563}{3.12} = 180.5 \text{ cu. cm. occupied by cement in 0.05 cu. ft.}$$

$$\frac{873}{2.66} = 328.0 \text{ cu. cm. occupied by sand in 0.05 cu. ft.}$$

$$\frac{1\,660}{2.66} = 623.0 \text{ cu. cm. occupied by gravel in 0.05 cu. ft.}$$

$$1\,131.5 \text{ cu. cm.} = \text{volume of solids in } 1\,417 \text{ cu. cm. space}$$

$$\text{Density or solidity ratio of concrete} = \frac{1\,131.5}{1\,417} = 0.798$$

Computation of Density of Mortar in Concrete.—

Voids in concrete = voids in mortar = 1 417 — 1 131.5 = 285.5 cu. cm.
Volume of mortar in concrete = 1 417 — 623 = 794 cu. cm.

$$\text{Voids in mortar} = \frac{285.5}{794.0} = 0.359, \text{ or } 35.9 \text{ per cent.}$$

Density of mortar = 1.000 — 0.359 = 0.641

The density of concrete or mortar may always be computed as here outlined whenever the quantities of materials used in a batch as well as the weight of a known volume of the mixed concrete can be determined or closely estimated. This may always be done with materials in the laboratory, but the application of the method to field operations is dependent on the method of controlling proportions.

In Fig. 13 the densities of neat cement and mortars are shown for different proportions of cement to aggregate and for the range of consistencies needed in practice. Flowability was measured by means of the flow-table. A flow value of 150 represents a sticky plastic mass that spreads sluggishly, while a flowability of 250 represents as wet a mixture as generally need be used in construction work where a reasonable amount of spading and tamping is done. It will be noted that the porosity increases or the density decreases as the cement content increases. The rich mixtures are most porous but stronger and more impermeable than the lean mixtures. These results would indicate that in mortars containing no coarse aggregate, strength and impermeability, and consequently durability, vary inversely with density.

In concretes the density variations are not so great as in mortars and the values are influenced by other factors, such as relative quantities of fine and coarse aggregate. In Table 10 are grouped concretes varying in cement content per cubic yard from 356 to 667 lb.—as great a range as is usually found in practice. Two sand-gravel ratios were used, 1 : 2 and 1 : 1. Densities of concretes and mortars were computed and specimens were moulded for determining densities by the absorption test as outlined by the author. As would be expected the absorption method gives higher densities and lower voids since it is not possible to fill all the pore spaces.

For ease in making a comparison the concretes of 1 : 2 sand-gravel ratio are grouped together as shown in Table 11.

Comparing the computed voids, the increase of flowability from 206 to 231 in the 1 : 1.5 : 3 mixture makes a far greater increase in voids than does the reduction of cement content to that of the 1 : 3 : 6 mix. Voids determined by absorption indicate that the same change in flowability affects the voids as greatly as the reduction in cement content to that of the 1 : 3 : 6 mix. Permeability and strength tests in the laboratory as well as field exposure tests would indicate the 1 : 1.5 : 3 proportions to be the best mixture, yet the density results indicate little choice.

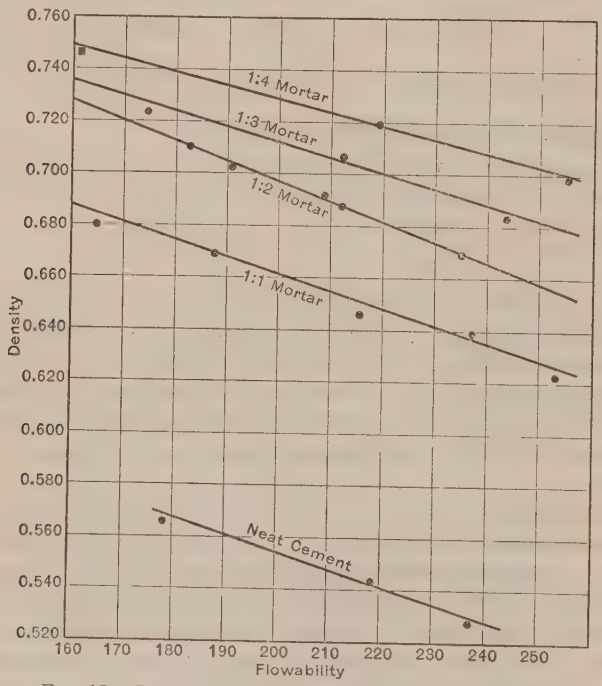

FIG. 13.—DENSITIES OF MORTARS AND NEAT CEMENTS, PROPORTIONS BY VOLUME.

Grouped values for the 1 : 1 sand-gravel ratio are given in Table 12. Here, again, selection based on mortar voids would suggest that the lean mixtures are as desirable as the rich, contrary to tests and experience.

A third comparison, that of 1 : 2 sand-gravel concretes with those having 1 : 1 ratios, indicates wider differences in voids for equal cement contents than is found in any one group between the richest and the leanest. Laboratory tests of concrete of these sand-gravel ratios indicate little difference between permeabilities and strengths of concretes of equal cement content and flowability.

The void contents of the mortars listed in Table 10 indicate little variation between the proportions, 1 : 1 and 1 : 3, based on absorption, while for com-

TABLE 10.—Computed Densities and Densities Based on Absorption.

Proportions by Volume.			Cement per cubic yard of concrete, in pounds.	Flowability.	Densities.				
					Computed.			By Absorption.	
Cement.	Sand.	Gravel.			Concrete.	Mortar.	Voids in mortar.	Mortar.	Voids in mortar.
Concrete.									
1	3	6	356	203	0.815	0.638	0.362	0.711	0.289
1	4.5	4.5	372	209	0.803	0.694	0.306	0.756	0.244
1	2	4	520	193	0.805	0.638	0.362	0.702	0.298
1	2.9	2.9	527	234	0.790	0.683	0.317	0.758	0.242
1	1.5	3	650	206	0.802	0.642	0.358	0.741	0.259
1	1.5	3	647	231	0.782	0.615	0.385	0.710	0.290
1	2.2	2.2	667	190	0.792	0.691	0.309	0.764	0.236
1	2.2	2.2	638	262	0.757	0.647	0.353	0.750	0.250
Mortar.									
1	3	0	795	About 210	.....	0.702	0.298	0.773	0.227
1	2	0	1 050	" 210	.....	0.685	0.315	0.785	0.215
1	1	0	1 555	" 210	.....	0.656	0.344	0.765	0.235
1	0	0	2 850	" 210	.....	0.545	0.455	0.699	0.301

puted values the leaner mixtures would be favored. Neat cement, based on the void content, is less desirable than a 1 : 3 mortar mixture. It would also be less desirable than a 1 : 3 : 6 concrete.

TABLE 11.

Mixture.	Cement per cubic yard, in pounds.	Flow.	Voids in Mortar.	
			Computed.	By absorption.
1 : 3 : 6	356	203	0.362	0.289
1 : 2 : 4	520	193	0.362	0.298
1 : 1.5 : 3	650	206	0.358	0.259
1 : 1.5 : 3	647	231	0.385	0.290

Test data obtained in the writer's concrete laboratory since 1920 have consistently pointed out that little or no relation exists between density and concrete quality. The richest and best concretes from every viewpoint have slightly lower densities than lean mixtures, which are low in strength and which have a short life when exposed to alkaline soil waters. All evidence has pointed toward the importance of impermeability or low permeability as the quality to be sought where the best concretes must be used. Fortunately, permeability decreases, although not necessarily proportionally, as the cement content and strength increase and, within the fairly wide limits necessary in practice, variations in aggregates and flowabilities appear to be secondary factors.

TABLE 12.

Mixture.	Cement per cubic yard, in pounds.	Flow.	Voids in Mortar.	
			Computed.	By absorption.
1 : 4.5 : 4.5	372	209	0.306	0.244
1 : 2.9 : 2.9	527	234	0.317	0.242
1 : 2.2 : 2.2	667	190	0.309	0.236
1 : 2.2 : 2.2	638	262	0.353	0.250

To eliminate much of the difficulty which was experienced with some of the earlier types of permeability apparatus the writer devised the arrangement shown in Fig. 14. In operation the placing and adjusting of the test specimen is as easy and rapid as the placing of a book in the old style letter-copying press. The test specimen may be removed and replaced as often as desired for further tests without injury. Fig. 14 shows a specimen being tested qualitatively, the time interval required for moisture to show on the upper face being a measure of the permeability. If quantitative results are desired a second base casting and gasket may take the place of the four-legged casting on the upper face. The water is applied on top and the seepage

may be caught below in a graduated container that takes the place of the water feed pipe in Fig. 14. The U-shaped rubber gasket is effective in preventing all leakage. Test specimens are always cast on edge with the large surfaces in contact with the sides of the form, which eliminates errors and variations due to excessive troweling.

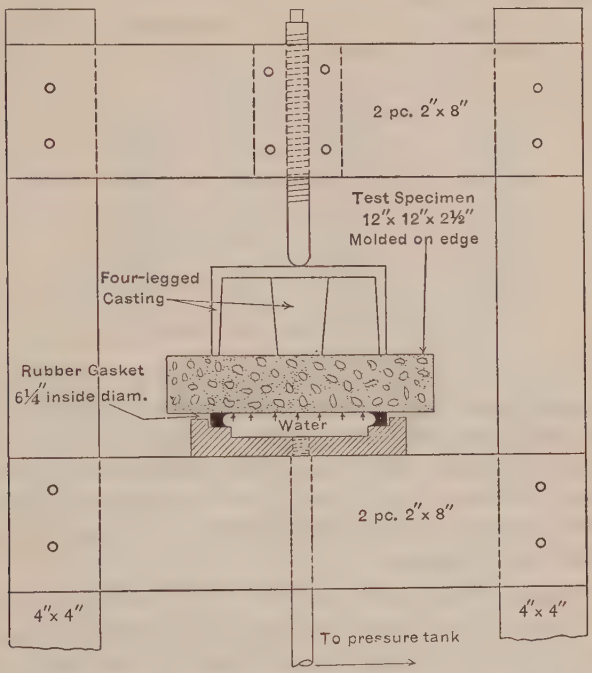

FIG. 14.—APPARATUS FOR PERMEABILITY TESTS.

Density of Cement Paste in Mortar and Concrete.—The foregoing test results would indicate that density of concrete and density of mortar in concrete do not vary directly with their strength and impermeability, so that density cannot be utilized as a criterion for such qualities. It would appear from Table 13 that there may be a relation of importance and value between the density of the cement paste in a mortar or concrete and the qualities of strength, water-tightness, and resistance to deterioration. Evidently, density of cement paste varies directly, although not proportionally, with these qualities. Whether or not slight changes in this density will always accompany changes in strength, even for the same materials, will require many more tests than the writer has made, but the few values given in Table 13 show how the factor varies for the particular aggregates used. It is easy to rank these mixtures in order of strength and impermeability without further tests so far as the individual groups are concerned. The neat cement should head the list and it has the highest cement paste density. The mortars should rank in the order given but the strength values would bear a greater ratio

to each other than the ratio of the densities. The concretes with the two sand-gravel ratios would rank as listed in each group.

TABLE 13.—DENSITY OF CEMENT PASTE IN MORTAR AND CONCRETE.

Proportions by Volume.			Flowability.	Density of cement paste.
Cement.	Sand.	Gravel.		
1	0	0	About 210	0.545
1	1	0	" 210	0.463
1	2	0	" 210	0.389
1	3	0	" 210	0.358
1	1.5	3	206	0.389
1	1.5	3	231	0.360
1	2	4	193	0.340
1	3	6	203	0.276
1	2.2	2.2	130	0.402
1	2.2	2.2	190	0.346
1	2.2	2.2	220	0.330
1	3	3	234	0.334
1	4.5	4.5	209	0.266

It is possible that further study will show no correlation between the density of cement paste and the strength and other properties of concretes and mortars, which will make this relation of practical use or permit of its general application; but it would seem that this is the one "density" of the three which may prove to be of value. This density can only be computed as here shown for the assumed batch of concrete; it can not be obtained practically from the absorption test.

JASPER O. DRAFFIN,* ASSOC. M. AM. SOC. C. E. (by letter).—In this excellent paper the cause of the corrosion of concrete is analyzed and a method of minimizing this action is proposed in the form of a new test, a mortar-voids test for porosity. It is claimed that porosity is the most important factor in the deterioration of concrete. The paper should attract the attention of two classes of engineers, those who are interested in a greater knowledge of the basic action of the different compounds in Portland cement concrete, and those who are interested in knowing why one piece of concrete will go to pieces and another, apparently quite similar, will give good service. Relatively more progress has been made in the study of the strength of concrete and of the factors which affect that strength than of the elements which affect its durability. Many structures have suffered serious deterioration as a result of this lack of knowledge. This paper, which treats of the durability of concrete, and of the different conditions which affect it, should challenge the careful thought of engineers and investigators.

The theory presented is based on the hypothesis that the hydration of Portland cement produces calcium hydroxide which is later changed into calcium carbonate by the carbon dioxide in the air. This calcium carbonate increases the strength of the concrete but under certain conditions it may be dissolved by water and thus leave the concrete porous and weakened. The

* Asst. Prof. of Theoretical and Applied Mechanics, Univ. of Illinois, Urbana, Ill.

two sources of protection against the formation and subsequent solution of the calcium carbonate are the aluminum and silicon hydroxides and the stagnated layers of air, both of which prevent the entrance of carbon dioxide and water. For this purpose the aluminum hydroxide is more effective than the silicon hydroxide. The degree to which these protective layers have been effective may be measured by the alkalinity of the concrete.

This hypothesis at once raises a question regarding the compounds formed during the hydration of Portland cement. The author makes no dogmatic statements concerning the hydration of Portland cement and in view of the present state of knowledge this is probably wise. It is not stated which of the various compounds in Portland cement produce calcium hydroxide although it may be inferred from the context that all of them do so. It may be well to state the present fairly well accepted theory concerning the composition and hydration of Portland cement.

Normal Portland cement is generally believed to consist principally of tri-calcium aluminate, β-di-calcium silicate, and tri-calcium silicate. Klein and Phillips,* working with pure constituents, concluded that these compounds re-acted as follows:

Tri-calcium aluminate hydrates to hydrated tri-calcium aluminate. This forms first as an amorphous hydrated aluminate and later crystallizes; the crystals have the approximate formula, $3CaO\ Al_2O_3\ 9H_2O$. Calcium hydroxide is not formed.

Tri-calcium silicate hydrates to calcium hydroxide and hydrated silica.

β-di-calcium silicate hydrates very slowly and to an amorphous substance, but no calcium hydroxide is formed.

Messrs. Klein and Phillips further state that the hydration of Portland cement gives the same products in a general way and it also gives calcium sulfo-aluminate crystals, due to the calcium sulfate in the cement.

The writer has made some microscopic studies of the hydration of the various pure constituents and in a general way his observations agree with those of Klein and Phillips just outlined. β-di-calcium silicate showed only slight hydration after fifteen months and no calcium hydroxide could be detected. Tri-calcium silicate gave abundant crystals of calcium hydroxide; and the calcium aluminates, both low and high calcium, gave no calcium hydroxide.

These studies were made by placing a small quantity of the substance on a glass slide, mixing with an excess of water, covering with a cover glass, and sealing around the edges with Canada balsam. The calcium hydroxide which formed from the tri-calcium silicate began to go back in solution in about ten days and the re-solution was complete in about two months, leaving only the outlines to show where the crystals had been. As the hydrated tri-calcium silicate was sealed from the air, there could hardly have been enough carbon dioxide in the mixing water to change the calcium hydroxide into calcium carbonate, and continued observation showed that no calcium carbonate was

* "Hydration of Portland Cement," *Technologic Paper No. 43*, U. S. Bureau of Standards.

formed. Therefore the calcium hydroxide went into solution directly from the hydroxide form.

In view of these observations, there might seem to be some question regarding the validity of the assumption concerning the formation and solution of calcium hydroxide. This would not change the view of the mechanism of corrosion presented but only the degree to which it might occur. It might affect, however, very markedly, any conclusions regarding the durability of high alumina cements.

The emphasis on the importance of a dense concrete is timely. Writers in the past have pointed out the same thing but it needs reiteration from different points of view. Nathan C. Johnson, Assoc. M. Am. Soc. C. E., in a study of defective concrete,* attributes the "disintegration" to the formation of "crystallin dikes" which by their growth disrupt the concrete. Although the character of the action which produces the disruption is not the same, according to the views held by Mr. Johnson and Mr. Baylis, they agree in one important particular, that of the need for dense concrete. Mr. Johnson states that "the problem of making durable concrete for all services is essentially one of making concrete impervious".

An interesting question arises in trying to reconcile the different ideas propounded in recent discussions. It is asserted† that the proportion of tri-calcium silicate in cement largely governs the quality of the cement; the cement will be better with a large proportion of tri-calcium silicate. The quality in this instance is judged by the strength developed. If it is true that only the tri-calcium silicate produces calcium hydroxide, as stated by Klein and Phillips,‡ it would appear that the condition for maximum strength is that for minimum resistance to corrosion, so far as chemical composition is concerned, because high strength requires tri-calcium silicate, which would produce much calcium hydroxide to change into the carbonate and dissolve. It is evident that there are many factors to be considered in determining the conditions which shall produce the maximum value of any given property. It would not be surprising if further study developed the fact that a cement with high values of certain properties, which make it well adapted to one use, gives a material poorly adapted to some other use; which is to say that future development may bring out a series of cements each suited to a specific purpose.

Charles A. Newhall,§ Esq. (by letter).—Not so many years ago an air of mystery was thrown about the manufacture and composition of Portland cement. To-day, although much research is under way, probably less is known about the chemistry of Portland cement than of any other of the important materials of construction. This lack of knowledge of the reaction and properties of Portland cement has led engineers to put the material to many impossible uses, or to proper uses without the obvious safeguards. Many types

* "The Microstructure of Concrete," *Proceedings*, Am. Soc. for Testing Materials, Vol. XV (1915) Pt. 2, pp. 172–213.

† "Lime-Silica Index as a Measure of Cement Quality," by Thaddeus Merriman, M. Am Soc. C. E., *Engineering News-Record*, April 22, 1926, pp. 648–650.

‡ *Technologic Paper No. 43*, U. S. Bureau of Standards.

§ Chemical Engr., Seattle, Wash.

of steel are easily rusted. No engineer would think of exposing unpainted steel of a bridge, for instance, to the elements, yet for years engineers have given no thought of protecting concrete surfaces from the numerous corrosive agencies that obviously will bring about serious weakening or even destruction of unprotected structures.

Mr. Baylis' paper comes at an opportune time when engineers and chemists all over the world are being confronted with problems arising from the failure, or threatened failure, of structures made of modern Portland cement concrete. His advice, given in the section, "What to Do" (page 818), should be considered by all who have anything to do with Portland cement.

As far as the writer is aware, Mr. Baylis is the first to set forth clearly the part played by carbon dioxide in the hardening and aging of Portland cement concrete. It is evident that up to a certain point the formation of calcium carbonate, through the interaction of carbon dioxide and calcium hydroxide, is beneficial to concrete; for as Mr. Baylis indicates, this results in a decrease of porosity. Other reactions, not discussed by Mr. Baylis, are not beneficial and are known to take place in concrete under certain well recognized conditions.

When a concrete structure is exposed to moisture and carbon dioxide, a condition develops whereby normal calcium carbonate is first formed, followed by the change to the bicarbonate of calcium, the latter being relatively very soluble. The solubility of calcium in the form of bicarbonate is greatly increased if the water carries dissolved organic matter. All natural ground-waters, sewage, and even rain water are highly charged with carbon dioxide and thus are corrosive to concrete through the bicarbonate reaction.

Calcium carbonate can occur in three forms, each differing from the other in density and in resistance to corrosive agencies, these forms being calcite, aragonite, and chalk. The first two are crystalline and the last amorphous. It is probable that "hair-cracking" and spalling of concrete surfaces are caused by volume changes in the surface layer of the concrete, resulting from the formation of calcium carbonate and the subsequent reversion of an unstable form to the stable form. It is known that after 15 to 20 years, stresses of thousands of pounds per square inch develop in the surface layers of concrete structures when such surface is exposed to the action of carbon dioxide under conditions not yet clearly formulated.

The writer's experience of nearly twenty-five years in the manufacture and use of hydraulic cements indicates that special formula cement concrete can be made, which will not be seriously affected by the numerous corrosive agencies that are causing damage to Portland cement concrete. Such special formula cements are now being made in a small way and will be widely marketed if long-time service tests are consistent with the short accelerated tests.

The chemical formula used in the manufacture of present-day Portland cement is a development of the last twenty-five years and has been devised with the end in view of securing the maximum physical strength in the shortest possible time. No thought has been given to the resistance of the cement to the corrosive agencies to which all exposed structures are subjected. Engineers

have assumed, as pointed out by Mr. Baylis, that modern Portland cement concrete was made up of compounds that are insoluble in water and like natural agencies. Benson and others* were the first to point out the solubility of modern Portland cement in water. Old formula Portland cement, natural cement, pozzuolana cement, etc., all give compounds that are largely insoluble as compared with modern formula Portland cement.

Engineers long ago gave up the idea of limiting their specifications to one type of ferrous compound. Nowadays, cast iron, wrought iron, high and low carbon steels, alloy steels, etc., are specified for the particular class of service to which each is adapted. In the same manner, the writer is certain that in the future several distinct types of calcium compounds will be specified for different classes of work that call for the use of a hydraulic cement concrete.

Troy Carmichael,† Assoc. M. Am. Soc. C. E. (by letter).—One of the principal uses of Portland cement concrete is in street and highway construction; consequently, any paper on the corrosion of concrete has a very direct bearing on this phase of engineering.

Heretofore, it has generally been supposed—the idea having been blatantly shouted from the paid pages of the press—that the strength of Portland cement concrete increases as it ages. In recent years, however, an extension of the use of this material as a road surface has been accompanied by a series of failures, the failures taking place in a comparatively short time after the roads were open to traffic. Sometimes they have been attributed to causes not inherent in good concrete, the principal ones being: (1) Inadequate thickness; (2) insufficient cement; (3) improper aggregate as to quality or grading; (4) too much or too little water in the mixture; (5) the presence of organic matter in the aggregate; (6) poor workmanship; and (7) improper working conditions such as subsoil and temperature.

Wherever first-class materials are used and the pavement has been laid to adequate thickness, the ability of properly constructed Portland cement concrete to continue to maintain its original strength during successive years of road traffic has seldom been questioned by investigators into these failures, for apparently they entered into their studies with the premise that Portland cement concrete in a road, when constructed according to approved specifications and of sufficient thickness, will not deteriorate as it ages.

There are two main reasons why such a premise is erroneous:

First.—Portland cement concrete is subject to a change in its structure under impact of road traffic—the phenomenon known as "fatigue".

Second.—There is a chemical change in Portland cement concrete as it ages (which is ably described in this paper).

As to fatigue, independent of the well-known chemical change and deterioration due to the presence of sulfur or alkaline salts (of common occurrence in aggregates or seepage water in many localities) which are drawn into the concrete by capillary action, there is the phenomenon of fatigue which

* "Solubility of Portland Cement in Weathering Agents," by H. K. Benson, J. S. Herrick, and T. Matsumoto, *Industrial and Engineering Chemistry*, Vol. 16, 1924, p. 1063.

† City Engr., Helena, Mont.

causes rigid masses to become weaker in structural strength under continual jarring and impact. (The well-known crystallization of steel in a car axle is akin to the breaking down of the rigid mass of Portland cement concrete in a road surface by "fatigue".)

The phenomenon of "fatigue" is touched on in an important series of tests* made in collaboration with the U. S. Bureau of Public Roads on 638 cores or test cylinders which had been drilled from apparently sound sections of California State highways, the sections from which these cores were taken totaling 800 miles. These roads had been under traffic from 1 to 7 years and the cores were tested for crushing strength. The record of tests indicated that both 1:2:4 and 1:3.5:5 concrete mixtures when laid to thicknesses of 4 to 5 in., became weaker as the pavement aged, which fact is shown by the crushing strength of the cores tested, as follows:

A.—1:2:4 Mixtures.—

1	year	old	crushing	strength,	3 800	lb.	per	in.
2	"	"	"	"	3 500	"	"	"
3	"	"	"	"	4 300	"	"	"
5	"	"	"	"	3 100	"	"	"

B.—1:3.5:5 Mixtures.—

1	year	old	crushing	strength,	3 800	lb.	per	in.
2	"	"	"	"	3 400	"	"	"
3	"	"	"	"	3 400	"	"	"
4	"	"	"	"	3 200	"	"	"
5	"	"	"	"	3 200	"	"	"

These recorded tests indicate that notwithstanding the fact that concrete is known to increase in strength when it is undisturbed, the actual tests of the cores of concrete taken from sound sections indicate a weakening as it ages. Except for deterioration due to alkali in the soils, or due to chemical action within the mass, this reduction in strength can only be accounted for by impact, causing the concrete to approach each year nearer the "point of fatigue" and final breaking down of the structure.

In drawing conclusions from these tests, the report makes two important deductions:

"1.— * * * average strength as determined by cores tested from concrete laid during the various years indicates that there may be a slow progressive deterioration in the concrete itself or 'fatigue' in a thin slab subject to excessive flexure.

"2.—There is * * * a question as to whether or not the concrete may be slowly deteriorating, but this condition has not been conclusively proven. More study of this phenomenon is required before any conclusion can be reached."

The author's scientific investigations are confirmed by the analysis of road conditions by disinterested road engineers. W. Calder, Chairman of Roads Board, Victoria, Australia, in his report (March, 1925) to the Minister of Public Works, Australia, on his investigation of road problems in Europe

* Report of Study of California Highway System to California State Highway Commission, by U. S. Bureau of Public Roads, 1922.

and America, makes the following pertinent statements as to the life of concrete in road construction:

"1.—While a great deal of knowledge has been gained during the past decade as to Portland cement concrete, a great deal has yet to be learned and it cannot be definitely stated that the science of concrete road building has yet passed its experimental stage or that approved standards of design are finalized.

"2.—Concrete roads designed and carefully constructed * * * are not suitable for roads subject to any considerable amount of horse-drawn traffic, under which their surface will rapidly disintegrate.

"3.—The exaggerated statements of their (Portland cement concrete pavements) durability are not justified by their service value. * * * of concrete pavements completed up to 1924 only eight (8) per cent. were over ten years old and only one-quarter ($\frac{1}{4}$) of one (1) per cent. were fifteen years old * * *. Concrete roads are prone to temperature changes which would warrant the conclusion that their greatest utility is that of a foundation for other surface pavement.

"4.—While considerable mileage of concrete roads has been and is now being constructed in Great Britain, it cannot be said that concrete has been adopted as a standard road surface material in that country and the majority of British engineers consider that the proper function of concrete is to serve as a base for other surfacing materials.

"5.—Impact is relatively non-important, but continued impact or repetition of heavy traffic blows in great numbers will disintegrate any slab—particularly where flexure occurs. Traffic on concrete roads should be rubber-tired."

This paper opens up an important field for further research in order that the limitations of concrete may be realized and that "millions of dollars which have to be spent on repairs and replacements within the next decade" as stated by Mr. Baylis, may be minimized.

Dalton G. Miller,* Esq. (by letter).—The method for the "Measurement of Mortar and of Voids in It", as suggested (page 806) to determine the quality of concrete for resistance to "corrosion" is interesting and the results obtained by this method, when carefully interpreted, should have considerable value. The method is, however, rather tedious of application and in this respect, when applied to freshly made concrete, has perhaps little advantage over a straight permeability test made on specially prepared specimens.

It is believed that a permeability test, properly made, has an advantage over any aggregate test of furnishing a more direct index of the actual resistance of concrete to "corrosion"; for the desired end, after all, is to produce a concrete as nearly impermeable as possible.

Regardless of how dense a concrete may be there are at least two other factors to which too little attention is given in practice, each of which may have a very far-reaching influence on the durability of concrete, regardless of quality, under special conditions of exposure. One of these is the durability of the cement used in the mix and the other, which will only be mentioned here, is the curing of the concrete.

Work in the Co-Operative Drain-Tile Laboratory at St. Paul, Minn., has shown that there is a wide difference among Portland cements in resistance

* Drainage Engr., Bureau of Public Roads, U. S. Department of Agriculture, in Charge of Co-Operative Drain Tile Laboratory, at University Farm, St. Paul, Minn.

to the action of the sulfates of magnesium and sodium. This difference is so great as to warrant the conclusion that certain standard Portland cements, passing all physical and chemical requirements, may be wholly unsuited for use in sulfate waters although satisfactory for ordinary purposes of concrete making. Such being the case, it may make little difference in the final results whether the concrete is well made, or otherwise, if exposed to sulfate waters, unless the particular cement used is at least of average resistance. This point is brought out in Table 14, showing results of tests of cylinders in which twelve different brands of cement were used.

The mix in all cases was 1:3 by volume and the aggregate was graded so as to have a fineness modulus of 4.67, the largest pebbles passing a $\frac{3}{8}$-in. screen. The water-ratio was kept at 0.60 in all cases except in Series Nos. 268, 269, 271, and 273, in which it was 0.62, and Series No. 275, in which it was 0.67. All cylinders were cured for 24 hours in the moist closet followed by 20 days in distilled water, and when 21 days old were stored in the solutions, except the check cylinders, which were stored in tap-water.

TABLE 14.—Strength and Absorption of Concrete Cylinders Made of Different Brands of Portland Cement and Relative Resistance to 1 Per Cent. Solutions of Sodium Sulfate (Na_2SO_4), as Indicated by Strength Tests at 1 Year.

(Each Result is the Average for Five, 2 by 4-Inch, Concrete Cylinders Made on Different Days.)

Series No.	Cement No.	Absorption at 21 days.	Stored in Water. Compression Tests, in Pounds per Square Inch.			Stored in 1% Na_2SO_4.	
			7 days.	28 days.	1 year.	Compression, 1 year.	Percentage of strength ratio at 1 year.
273	62	5.62	2 480	4 470	5 760	0	0
274	33	5.74	3 170	4 880	5 950	0	0
270	60	6.07	2 300	3 850	6 380	0	0
272	63	5.75	2 390	4 610	5 950	1 900	32
269	55	6.37	2 960	4 290	5 650	3 510	62
264	39	5.84	3 280	5 020	6 080	4 790	79
268	42	6.26	2 450	3 580	6 220	4 900	79
263	37	6.26	3 180	4 820	5 740	4 600	80
262	35	5.85	3 010	4 630	6 250	5 070	81
267	40	5.60	3 310	4 600	6 480	5 310	82
275	34	6.20	3 160	4 970	5 730	5 040	88
265	41	5.51	3 460	4 790	5 860	5 930	101

The mortar of the aggregate when tested for voids by the method outlined by Mr. Baylis gave 23.9%, based on tests of a 2 by 4-in. cylinder, and 21.7% based on tests of a 4 by 8-in. cylinder, both of these results coming well below the suggested maximum of 28 per cent. These results, taken in connection with the results of absorption tests at 21 days and compression tests at 7 days, 28 days, and 1 year, would indicate that the concrete itself when made was satisfactory and that the difference in the results of tests of these cylinders after one year's exposure was due entirely to differences in resistance among the cements used.

No 1-year tests are recorded for the cylinders of Series Nos. 273, 274, and 270 stored in the sodium sulfate solutions, for the reason that these cylinders had completely disintegrated. Cylinders from Series No. 273 when tested at 32 weeks had 31% of normal strength.

The cylinders in this group are now (August, 1926) 19 months old and those of Series Nos. 262 to 265, 267 to 269, and 275 show little more evidence of action than they did at 1 year. At 2 years all the cylinders may show a loss of strength as compared with the 1-year tests, but at present there is every reason to believe that at least some of the series will test fairly well. When compared with 31% of normal strength at 32 weeks, as was the case with Series No. 272, in which Cement No. 62 was used, the wide difference in cements is evident.

Attention is called to the fact that none of these cylinders had any air hardening previous to exposure to these sulfate solutions. This accounts for the fact that deterioration has been so rapid. As they were all treated similarly, however, the comparative results form a fair basis of judging the relative resistance of the cements used in this work. Such being the case the necessity of considering the cement itself, as well as the grading of the aggregate for all concrete exposed to sulfate waters, becomes evident.

F. R. McMillan,* M. Am. Soc. C. E. (by letter).—This paper is very thought-provoking on several lines and is deserving of careful study by all engineers interested in permanent concrete. The author's emphasis on the significance of porosity in the resistance of concrete to weathering is to be commended particularly.

The writer believes, however, that the value of Mr. Baylis' contribution on this latter subject will be somewhat obscured by the proposals for a new sieve size as the dividing line between the fine and coarse aggregate and for a new test for density.

The matter of the dividing size between fine and coarse aggregate does not seem to be of major importance and emphasizing it, as Mr. Baylis has done, detracts from his more important theme—porosity. The proposed test for density, offered as a solution of an important problem, can be shown to be quite unnecessary, as the information desired can be obtained in a simpler and more direct manner.

The writer believes that the author has been led to both his proposals through the conception of concrete as consisting of coarse aggregate, in which the voids are completely filled with a mortar of desired quality.

For a long time, the writer held this same conception of concrete and found it very useful in clarifying some of the problems of concrete mixtures. A real understanding of the significance of density and porosity was not had, however, until the aggregates were thought of as a single mixture thoroughly incorporated in a cement paste. Under this conception, the quality of the concrete is seen to be determined by the quality of the paste that binds the particles of aggregate together. Also, the proportions of fine and coarse aggre-

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gate and the relative amounts of aggregate and paste are at once recognized as purely questions of economy when concrete is considered in this light.

Abrams' studies leading to his law based on the relation between the water-cement ratio and strength and his subsequent investigations, as well as those by other investigators, have clearly established that the strength of concrete from a plastic mix is a function only of the water-cement ratio for given materials and conditions of test. A study of the results of Professor Withey's* tests of permeability shows unmistakably the preponderant influence of the water-cement ratio on the resistance of concrete from plastic mixes to the passage of water under pressure.

Fortified by these indisputable test results, the writer has studied the question of density and porosity in the hope of finding a conception of the interrelation of the various factors that would not only correlate present knowledge, but would make clear the paths for further improvement in concrete practice. The conception already given is the result of this study.

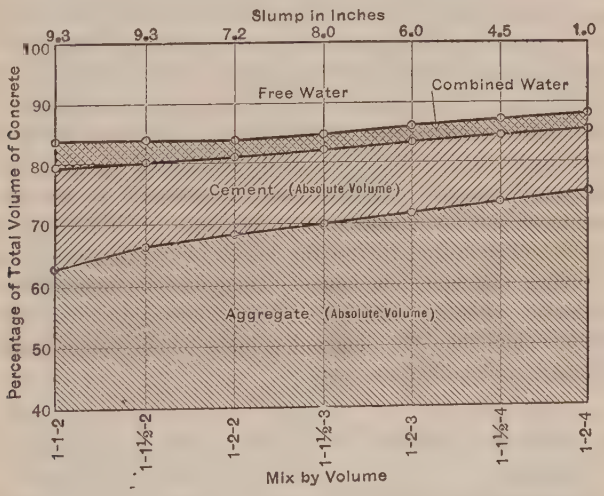

FIG. 15.—DENSITY OF CONCRETE OF CONSTANT WATER-CEMENT RATIO, 6½ GALLONS PER SACK, ALL MIXES BEING ON BASIS OF DRY COMPACT AGGREGATES.

To illustrate what this conception means in the study of concrete mixtures, Figs. 15 and 16 have been prepared. The data for these figures are taken from some recent tests made at the Structural Materials Research Laboratory, Lewis Institute, Chicago, Ill. In Fig. 15 are shown the data for a number of mixes in which the water-cement ratio was 6½ gal. per sack of cement. The scale of abscissas is of no significance, the mixes being arranged at equal intervals in the order of their densities. The ordinates to the diagram represent proportionate parts of the whole volume of concrete occupied by the different ingredients. These proportions are in terms of absolute volumes. Fig. 16 represents in a similar manner the data for two mixes, 1:1.5:3 and 1:2:4, carried through several water-cement ratios.

* "Permeability Tests on Broken Stone Concrete," by Morton O. Withey, assisted by Christopher A. Wiepking, *Bulletin 1245*, Univ. of Wisconsin, Madison, Wis.

The data used in the construction of these diagrams are all directly from the test results, except as to the quantity of combined water. This is an arbitrary quantity selected only as an illustration. The test specimens had been broken some months before this method of analyzing mixtures was conceived so that it was not possible to determine this factor. The proportion of combined water shown on Figs. 15 and 16 is taken as 0.3 of the absolute volume of the cement or about 10% by weight. It can be shown that the combined water can vary anywhere from little or nothing to two or more times the quantity indicated in these diagrams, the exact quantity depending on the length and condition of curing.

It will be apparent at once from the foregoing that the essential quality of concrete is governed by the relative quantities of cement, combined water, and free water in the paste, the part played by the aggregates being largely that of reducing the cost. In what follows brief mention will be made of the relative importance of grading or proportioning of fine and coarse aggregate.

It is not so obvious, but none the less true, as can be shown by ample test records, that for plastic mixes the sum of the absolute volumes of materials used, including the water (corrected for absorption and loss in manipulation), exactly equals the quantity of concrete produced. In all the mixes in Figs. 15 and 16 and in many others tried in this study, when the absorption in the aggregates was taken into account the yield computed in this manner agreed within a very small margin with the yield determined in the usual manner from the weights of the test specimens at 24 hours. This small difference in every case could be accounted for by the losses in manipulation and by water settling out during the first few hours. It should be noted that, for mixes that are not plastic, when the absolute volumes are added in this manner the sum is found to be less than the original volume of the mixed aggregate, indicating that there is not sufficient paste.

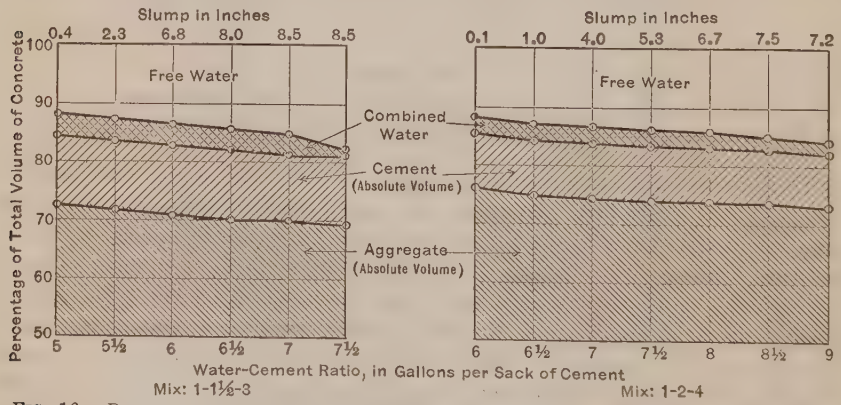

FIG. 16.—DENSITY OF CONCRETE OF VARIABLE WATER-CEMENT RATIO. MIXES ARE ON BASIS OF DRY COMPACT AGGREGATES.

The fact that the yield of concrete can be so accurately computed from a knowledge of the material entering the mixture is of importance in this discussion for two reasons: First, it materially clarifies the conception of concrete

mixes, as from the very nature of this computation it is apparent that concrete consists of an aggregate mass thoroughly incorporated in a cement paste, and, further, that the quality of the cement paste is seen to be intimately associated with the quantity of water; and, second, it is this fact that makes the density test proposed by Mr. Baylis an unnecessary requirement.

If one of the mixes indicated in Fig. 15 be subjected to the proposed density test the combined water at 24 hours perhaps would not exceed 5 or 6% by absolute volume of the cement. During the drying preparatory to the absorption test the remainder of the free water would be driven off and the quantity would represent the actual voids. The re-absorption of water to determine the voids, if completely carried out, would require this same quantity of water. All that the test accomplishes, therefore, is, in effect, a determination of the quantity of combined water. The quantity combined at the end of 24 hours probably does not represent the total to be expected when the structure is put into service, if the curing is at all favorable. This test, therefore, may be wholly misleading, and the simple computation of density, assuming a probable value of combined water, would be more reliable.

Unfortunately, the author did not give the quantity of water used in the example illustrating the application of his test so that a comparison cannot be made. In the data in Table 5, however, a similar comparison can be made as follows: Take, for example, the data for the first mix given in Table 5. Assume 100 grammes of cement $\left(\text{absolute volume} = \frac{100}{3.10} = 32.3\right.$ cu. cm.$\left.\right)$ which with its 20% of water gives an original weight of 120 grammes and an absolute volume of 52.3 cu. cm. Table 5 does not show the respective amounts of material used, but by proportion it is seen that the 30 cu. cm. of hardened paste (Column (2)) would have an original weight, as mixed, of $\frac{30}{52.3} \times 120 = 68.8$ grammes. This weight, it will be noted, is substantially that shown for the hardened sample after being submerged for 24 hours. Thus, all the test has accomplished is the extraction and re-absorption of 6.7 grammes of water and the useful information obtained is that, of the 11.4 cu. cm. of water used (in 30 cu. cm. of paste), 4.7 cu. cm. has gone into combination with the cement and the remainder is free to evaporate, leaving its volume in voids. This combined water is 8.2% of the cement by weight. This actual test is no doubt more accurate in this instance (after 2 months of curing) than an estimate as to the quantity of combined water, but if performed at 24 hours as proposed this would not be the case. Similar computations for the other mixes in Table 5 give the same close agreement between the original weight of fresh paste and the weight after being submerged 24 hours. In like tests made for the writer a similar small discrepancy was found. This was largely accounted for by the loss due to a slight film of water forming at the top of the mixture.

The real significance of the analysis of concrete mixes shown by Figs. 15 and 16 goes far beyond a mere test for density. Consider the almost endless

studies of concrete mixtures that have occupied the attention of many individuals for several years past. In all these studies grading of the aggregates or subjects closely related to grading have been the central theme. In Fig. 15 it is seen that for a fixed water-cement ratio the sum of the absolute volumes of cement and aggregate varies only from 0.79 to 0.85 for a wide range of mixes. This range of mixes, it should be stated, includes seven out of fifteen mixes tried for this water-cement ratio. The other eight are arbitrarily excluded either because they were too stiff to puddle (slump less than 1 in.) or because the quantity of coarse aggregate was less than the fine. It might be pointed out that a diagram including all fifteen mixes is exactly similar and only extends the range of densities from 0.75 for the 1:2:1 mix at one end to 0.85 for the 1:2:5 at the other.

It is the narrow range in possible densities (sum of the absolute volumes of aggregate and cement) that has impressed all students of the problem, the desire being, of course, to increase the density and thus reduce the porosity. In his discussion Professor Williams has pointed out what has been well known, that increased cement content contributed both greater strength and impermeability, and yet gave lower density. Professor Williams has failed to point out, however, why increased cement content increased both strength and resistance to the passage of water. The reason is that, for a given workability, the richer mixes require lower water-cement ratios. The differences in workability of the mixes in Fig. 15, indicated by the slumps at the top, should not be allowed to escape notice.

TABLE 15.—Density of Concrete of Constant Slump (2.5 to 4.5 Inches).

Mix, dry compact aggregates.	Water-cement ratio, in gallons per sack.	Proportion of Total Volume of Concrete Occupied by Solids.		Slump, in inches.	Sacks of cement per cubic yard of concrete.	Compressive strength at 28 days, cured damp, in pounds per square inch.
		Absolute volume of solids in aggregate.	Absolute volume of solids in aggregate plus cement.			
1 : 1 : 2	4.5	0.66	0.82	3.0	9.0	6 100
1 : 1 : 3	5	0.72	0.85	2.7	7.3	5 400
1 : 1.5 : 3	5.5	0.72	0.84	2.3	6.6	4 780
1 : 2 : 2	6	0.68	0.81	4.6	6.8	4 320
1 : 1.5 : 4	6.5	0.75	0.85	4.5	5.5	3 870
1 : 2 : 4	7	0.75	0.84	4.0	5.1	3 520
1 : 2.5 : 3	7.5	0.70	0.80	4.0	5.3	3 100
1 : 2.5 : 4	8	0.74	0.82	4.0	4.6	2 730

By reducing the water-cement ratio these richer mixes would give somewhat higher densities, thus improving the concrete both through the lower water-cement ratio and the increased density. Table 15 shows the data from this same series of tests for mixes having about the same workability. Since in these tests the water ratios and mixes were fixed it is not possible to compare mixes having exactly the same slump. It will be seen from Table 15, however, that for all mixes having about the same degree of plasticity the densities are nearly the same.

In view of the foregoing considerations, it must be apparent that the hope of further improvement in concrete practice does not lie in minute considerations of aggregate grading, but rather in the selection of proper water-cement ratios, in increased curing, and in extracting in some manner the excess water, this after the concrete is in place. It needs only a glance at Figs. 15 and 16 to visualize the possibilities of increasing both the strength and density, first by taking out as much of the free water as possible while the mass is still plastic and in a condition to consolidate as the water is drawn out; and, second, to bring into combination through extended curing a large proportion of that water which cannot be extracted.

Neither of these proposed methods of improvement is visionary. Both strength and impermeability have been shown to be greatly improved by longer curing. Professor Withey's conclusions are particularly emphatic in this respect. Abrams* has shown the importance of curing on both the strength and absorption. The following comparison is taken from these test results. For a 1:4 concrete tested at 4 months, a water-cement ratio of 8.25 gal. per sack when cured wet for 21 days gives about the same strength as a water-cement ratio of 5.75 gal. per sack cured wet for only 3 days. Also, the absorption in a period of 24 hours (age of concrete, 1 year) was almost identical for these same two concretes, 8.25 gal. cured wet for 21 days and 5.75 gal. cured wet for 3 days. When both these concretes were cured wet 21 days the 5.75-gal. mixture was 65% stronger and absorbed 38% less than the 8.25-gal. mixture.

For an example of the extraction of water from the freshly placed concrete to increase both strength and resistance to wear or weather, there is the sand-moulded artificial stone and the absorption process of finishing cement floors. An outstanding example of concrete from which water was extracted as it was placed is the Fountain of Time at the entrance to Washington Park, Chicago, Ill. This famous piece of statuary by Lorado Taft was built of concrete by Mr. John J. Earley in 1922. It is remarkable in that the thin sections and sharp protruding members and edges are as perfect to-day as when they were built. The essential feature of Mr. Earley's method† is the use of a porous back form which extracts the water from the concrete as it is being placed.

To summarize these remarks it seems that the conception of concrete as an aggregate mass thoroughly incorporated in a cement paste well explains present-day knowledge of concrete mixtures and gives a clear understanding of the paths open for future progress. By this conception it is seen that the strength and endurance of concrete as it is now made are largely determined by the water-cement ratio of the paste and the extent of the curing. The grading of the aggregates, when the concrete is mixed with a fixed water-cement ratio, is principally a question of economy, offering little opportunity for further improvement; and future progress on an important scale lies largely along

* "Effect of Curing Condition on the Wear and Strength of Concrete," by D. A. Abrams, M. Am. Soc. C. E., *Bulletin 2*, Structural Materials Research Laboratory, Lewis Inst., Chicago, Ill.

† See "Building the Fountain of Time," by John J. Earley, *Proceedings*, Am. Concrete Inst., 1923.

the lines of increased curing and the extraction of excess water from the freshly placed concrete.

NATHAN C. JOHNSON,* ASSOC. M. AM. SOC. C. E.—This paper on the "corrosion" or disintegration of concrete is timely and the subject is of the utmost importance. The literature of concrete has been saturated with papers on design and reports of laboratory procedures with deductions therefrom, but there has been a dearth of papers on things as they really are in commercial structures.

It is regrettable that all corrosions or disintegrations of concrete are loosely termed "failures". The study of disintegrations in commercial structures is possibly the best and most informative of all studies of concrete as a material that could be undertaken. Yet the subject has been neglected, partly because of the persistent use of the undeserved term "failures" and partly because, in reflex, any dissertation on failure offends the innate philosophy of the American mind, which demands successes as a steady diet, both in its work and in its play.

Yet a sane study of disintegrations in commercial structures is of the utmost importance. Such a study brings to light a wealth of constructive comment on the art as it is and on the necessary improvement of that art if repetitions of unfortunate behaviors are to be avoided in the future. Further, such a study shows how defects may be effectively repaired in existing structures and how they may not be successfully repaired. And not least, it points the way to economies in construction that will place concrete as a structural material on hitherto unrealized heights of efficiency and economy within a few years.

"Failures" as failures are not attractive, any more than people sick in body or in mind are attractive. Failures as failures are merely unpleasant comments, bill-boarded for all to see, on the contrast between the wealth of assurances an owner receives before he builds and what he actually gets for his dollars.

"Failures" as a clinical study, however, are of an importance and interest that cannot be under-estimated. The study is difficult and it must be prolonged, else false conclusions will be drawn and a phenomenon of little importance will be given rank over a matter of real and vital significance.

To classify the different types and kinds of failures, to show their origin, to evaluate their importance, and to point out the remedy for them, would be beyond the proper scope of any discussion of this paper; but a few classifications of disintegrations and their causes may be briefly made, which will be of benefit and of help to those who are interested in concrete, either as owners, who wish to know of a surety the permanent security of their investment, or as professional men or constructors, who wish actually to be in a position to secure for an owner this stability of invested funds.

Any breadth of experience in concrete makes evident the fact that disintegrations in concrete, however severe they may be, rarely mean or threaten actual collapse or total failure of the structure. The reason for this is that present standards for design rest on assumptions of such low unit stresses that

* Cons. Engr., New York, N. Y.

even plaster of Paris as a cementing medium would bid fair, considering the masses and sections involved, to be capable of supporting any direct load likely to be imposed.

One cannot go far in a study of disintegrations without being forced to the conclusion that the construction industry to-day abuses cement rather than uses it; and that the simple and inevitable fact that Portland cement when wetted with water, sets, hardens, and gains strength with time is the actual basis of the art, rather than the skill or ability of any person or persons or organization to bring about this reaction.

This reaction and its ultimate effect always happen, and produce, when this cement and water mixture is placed in forms, the outward appearance of concrete. The significance of this comment is not lessened by recognition that construction in concrete is complex as a business, but complex only in so far as every business is complex. In the evolving of this business the art of managing men and the art of materials handling have been stressed as the science of construction, rather than the production of a material of definite qualities, definite properties, and definite value.

The more commercial concretes are studied, the more is the conviction forced that artistry and architectural skill, or wizardry in mathematics and design, or great executive ability as the head of an organization, do not necessarily qualify any one as an expert in concrete; and that specifications and the work executed under various specifications, so long as the commonly omitted necessities for enduring construction are unprovided for, cannot be expected to endure beyond a limited time without deterioration so long as skill in design alone is assumed as the sole necessity.

Disintegrations of concrete as found in commercial structures and the conditions that gave rise to them cannot be reproduced in the laboratory. Likewise the conditions surrounding the commercial use of cement cannot be had in the laboratory, nor can laboratory usages or dicta be reproduced in the field. Further, there is a danger that the fruits of laboratory experiment, valuable *per se,* when given without caution to the field man, result in their being warped for commercial ends, as in the case of the successful and profitable marketing of unfit materials under the doctrine that so long as water and cement are in a certain ratio, one to another, nothing else matters. Even the water-boy on a job knows that this is not fact; and that materials, both in kind and in quality and in quantity (as well as the manner in which they are used), make a world of difference in result, regardless of this one ratio.

The art has suffered further from a conjugate of laboratory work, that is, from the fostering of the theory that strength is all important; and that strength at twenty-eight days is an indication of fitness and quality.

And as a further mistake, but contributed by the commercial side, the mixing machinery used universally to-day is incapable of fully utilizing the remarkable abilities inherent in Portland cement, so that, to illustrate by the one property of strength alone, a mixture of materials which might be reasonably expected to give 10 000 lb. per sq. in. of compressive strength, is considered remarkable if it renders one-third that strength in the prime of its life.

From one and all of these factors arise certain unsatisfactory behaviors in concrete. Some of the various kinds and types of such behaviors have been shown and commented on by Mr. Baylis. Others have not been mentioned and some of those omitted are of the greatest practical interest and importance.

A brief classification of the various kinds and types of commonly found disintegrations is:

(*a*) Three-branched crackings, or crazings, visible in nearly all surfaces of concrete. This is the commonest type of visible disintegration and will be commented on more fully.

(*b*) Dusting and scaling on exposed surfaces—phenomena of the hydrolysis of cement itself.

(*c*) A gradual loss of coherence in the mass due to slow reaction with elements contained in the aggregates—more common than is realized.

(*d*) Loss of coherence through lack of uniformity in the mix—a common occurrence.

(*e*) Softening, followed by loss of coherence, due to the entrance of plain water.

(*f*) Softening, followed by loss of coherence, due to alkaline waters.

(*g*) Softening, due to entrance of sea waters and accompanied by various phenomena, some simple, others complex. The total loss-and-repair bill annually on sea-water structures runs into millions.

(*h*) Disintegration by bacterial action—very rare.

(*i*) Disintegration by actual electrolysis—very rare, although many disintegrations of other origins are confused with electrolysis.

To deal with these various classes and types fully would require a fair-sized book on each. Further, frost and freezing may aggravate and accentuate as a secondary influence any disintegration arising from any one of these causes, thus giving rise to other conditions which complicate any brief discussion of this general subject.

Even aside from the disintegrations listed as to type, there are other factors which relate to cement alone and to its natural behaviors which must be known and taken into consideration if there is to be a reasonable degree of value inherent in any job. Portland cement has a right to these behaviors; and it is the obligation of the users of cement to learn what these behaviors are, and to abide by them, or else to be prepared to suffer the penalty.

For instance, Portland cement, if undiluted by sand and stone, may "kick" itself to pieces without outside influences, because of the enormous energy liberated by its reaction with water and the continuance of that liberation of energy after set has taken place, through the perpetual thirstiness of cement for water.

This is well known, yet it has been a habit for decades to make over-rich in cement all outer surfaces of concrete; and to add pure cement where it is desired to unite old and new concrete. No permanent success has ever attended these ventures, nor can there be hope of success, yet tradition dies hard; and as the procedure is reputed to be "practical", it is reputable, even if it is the very essence of the theoretical—and of the mistakenly theoretical at that.

In somewhat the same manner, cement is responsible, not through possessing improper qualities, or lack of possession of proper qualities, for that commonest of all disintegrations—the three-branched, or "map" cracking of concrete surfaces. These map cracks are commonly the first outward evidences of disintegration. There must be and is a cause for them; and, inasmuch as strength tests, or any of the usual routine tests for cement and for concrete, tell nothing as to causes or as to how to control them, it behooves engineers as a start to learn all there is to be learned about this one phenomenon and even to the devising of new tests and of apparatus for tests, if these should prove necessary for a determination of fact.

It is often stated that "map cracks" are "merely superficial". Even if this is granted, what happens in this peculiar and unruly superficial layer of any concrete when it sets? And if these cracks are merely superficial, why do deep disintegrations follow exactly the lines of these map cracks? And since cement, when set, is strong, how much stronger must certain forces be to overcome this innate strength and to bring about fracture? And how great are these forces and what is their total significance?

These are difficult questions to answer—unless more light is shed on the behavior of cements than can be extracted or cajoled from the hundreds of thousands of strength tests, recorded and available.

To the end of obtaining answers to these questions, the instrument shown in Fig. 17 was devised. Inasmuch as it measures tensions and behaviors due to tensions, it has been named the tensometer. In its elements, the tensometer comprises a scale-beam, having a balance pan on one end; and on the other end a pair of clips, between which latter is put a sheet of thin rubber, about 5 in. square.

By putting weights into the scale-pan, an initial stretch, or rigidity, can be created in the rubber. An initial tension on this rubber of 3 kg. will give a satisfactory rigidity. If then, this stretched rubber be coated with some substance, such as a paste of Portland cement and water, it can be determined whether or not anything is happening in the Portland cement film which might find an outward manifestation in the behavior of concrete, as pictured in photographs by Mr. Baylis.

Suppose that this stretched rubber membrane is (in commercial work) the side of a building, or the sub-base of a concrete sidewalk, or the under-slab of a concrete floor. Just as in these members of any structure, the base is fixed, so it may likewise be assumed that, due to the weights in the pan and the consequent initial rigidity in the rubber sheet, this sensitive base is also fixed, in so far as the behavior of a thin superficial coating of cement put on the rubber is concerned.

If, now, this rubber be coated with Portland cement grout in a thin coating and permitted to set and to harden, a rather remarkable behavior manifests itself. In the first place, there is undoubtedly evaporation of water taking place. Evaporation means loss of water, and loss of water means change in volume. It soon appears in the upward movement of this scale-beam as shown by its upright indicator, that loss of water (induced by a little electric fan

in the experiment) or some other cause, has caused a contraction and has lifted the weight in the scale-pan.

This lifting by contraction must, therefore, be evidence of a force greater than the force initially induced in the stretched rubber. This extra force can be measured by adding weights to the scale-pan, until the scale indicator returns to its original position.

Until these extra weights are added, however, it is observed that the cement film on the rubber is intact, although the edges of the rubber have curled inward. To many, it will be at once evident that the shape of the rubber sheet is now an exact reproduction of a "dished" cement floor, and "dishing", of course, precedes cracking, in many cases of failure in concretes.

If, now, other weights are introduced into the scale-pan, at a certain point, rather suddenly, the cement film on the rubber sheet abruptly ruptures and the pointer returns to zero position (Fig. 18).

If the pattern of these ruptures is examined, it is to be noticed that they are always three-branched. That is, if a circle is drawn around the junction point of these cracks, one with another, it will be found that they are always at 120° one from another.

What has happened then, is that with the additional weights in the scale (4 kg.) the rubber sheet has been made to correspond more nearly than it did before to a rigid base, such as that on which cement toppings and cement washes are daily placed. Therefore, a rupture in the cement film or cement covering has been induced by opposing its contraction with a greater force—the force in this case being 4 kg. plus 3 kg., or 7 kg. minus. The opposing force in a commercial floor is many times 7 kg. so that something has to give way when a commercial topping is applied.

This trial means, then, that the tension induced in this thin surface film by evaporation or by other causes is so great that it lifts between 3 kg. and 7 kg. before the inherent strength of the Portland cement is overcome, but that once it is overcome, a rupture of the surface is brought about.

If the pattern of this rupture on this rubber sheet as shown in Fig. 18 is compared with the pattern in a concrete sidewalk topping or other concrete structures (Fig. 19), it could be seen that each is identical in form; and that all this "crazing" or whatever it may be called, is at 120°, one crack with another and that, by and large, it is a serious fault in concretes in commercial work.

This tension is occurring on every concrete job done in the usual manner, where cement is left to its willful and uncontrolled way, or aggravated to a worse behavior by those who use it. What must be the tension in a surface that is 50 sq. ft. in area in commercial work? Many times concrete is reinforced "against cracking," when cracking arises from a wholly different cause; and needless to add, concrete cracks regardless of this reinforcing.

Is it to be wondered, then, with this enormous tension induced in such a thin film, that rupture occurs in the usual floor or sidewalk toppings, which of course may be 1 in. or more in thickness? Or is it to be marveled that oftentimes this rupture goes deeper and actually tears apart the concrete and

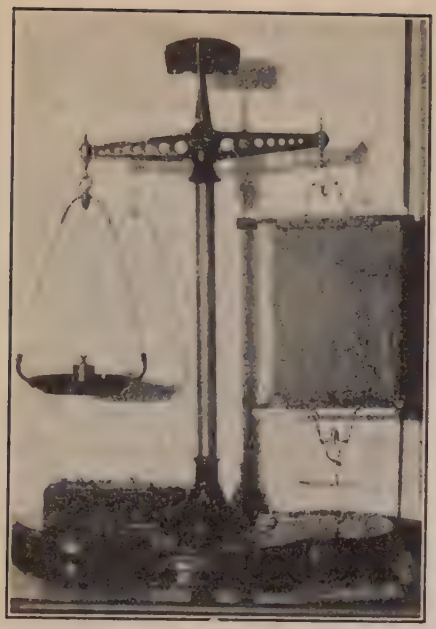

FIG. 17.—TENSOMETER, SHOWING A FILM OF PORTLAND CEMENT AND WATER ON A STRETCHED MEMBRANE, BEFORE SETTING.

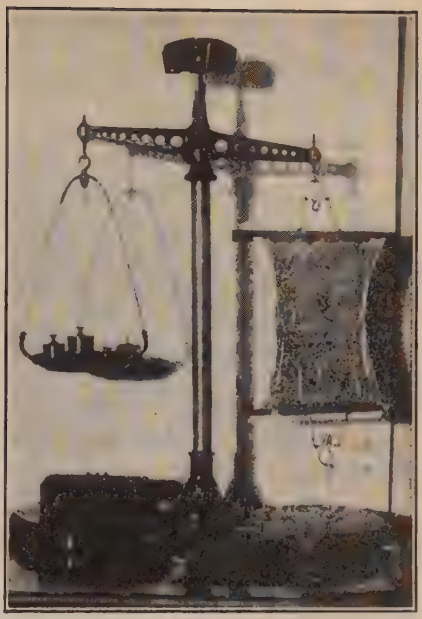

FIG. 18.—TENSOMETER, SHOWING THE FILM AFTER SETTING AND BACK-WEIGHTING TO PRODUCE CHARACTERISTIC RUPTURE LINES, BUT STILL SUPPORTING A HEAVY LOAD.

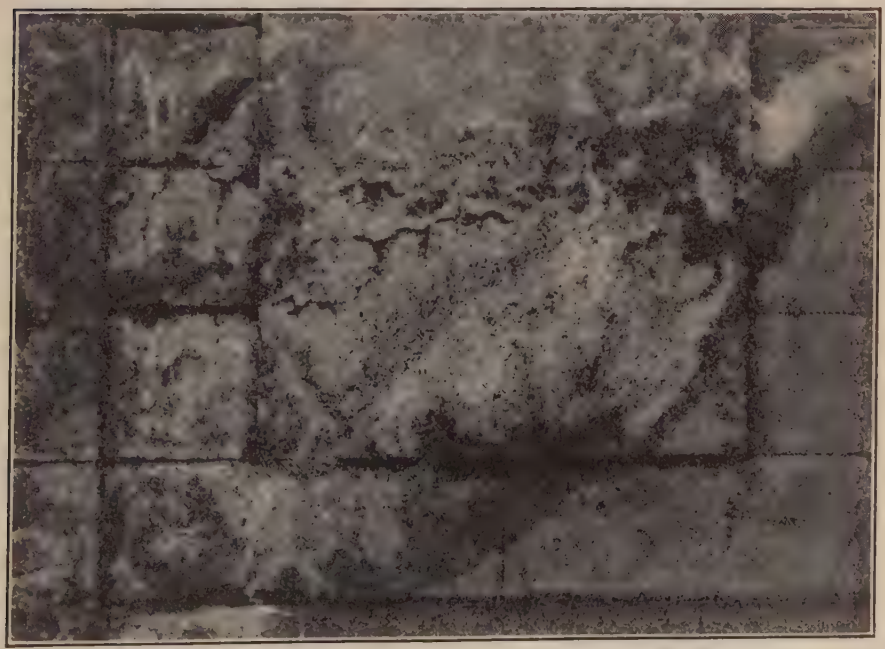

FIG. 19.—SURFACE OF CONCRETE WALL PANEL, SHOWING THREE-BRANCH CRAZING AT 120 DEGREES.

thus brings about other appearances shown in the paper in their final stages of an active disintegration?

By careful experimentation, it is proven that evaporation alone is not responsible for this tension, but other elements enter in; and among these elements is the reaction of the carbon dioxide of the air with the lime hydrate of the cement to form calcium carbonate. This calcium carbonate has a tension of its own entirely different from, and quite capable by itself of exceeding in value, the tension caused by evaporation alone.

This means that sprinkling to cure these evils is not, and cannot be, wholly effective, tradition to the contrary notwithstanding; but it also means that, having found out this new-old behavior natural to cement, the devising of a remedy becomes almost automatic. All these troubles are easily prevented and any concrete can now be made free of such cracks or crazings. Similar "mysteries" about concrete may be dispelled in like manner by a little sane research and independent thought.

Evidently, then, more accurate and better knowledge of concrete is needed and the more one amasses of that knowledge and the more skilled and able one becomes in embodying that knowledge in useful, commercial constructions, the more amazing seem the every-day assertions about concrete—assertions that make up in vehemence what they lack in truth or in basis on fact.

It has been asserted that "no one cares about anything except price." That will prove a costly doctrine to those who practice it, or sanction its practice in commercial concrete work. The speaker does not believe this is true and has found, on the contrary, that those who practice the art are, in greater and growing numbers, vigorously determined that the constructions of this year and of next year and of the years to come shall prove by their unquestionable quality that the old-time, 28-day art—the trade governed by tradition and rules of thumb and uncollectable guaranties—may not and shall not stand as the best that the American nation of to-day can show to posterity.

Walter D. Binger,* Assoc. M. Am. Soc. C. E. (by letter).—In the winter of 1919 and spring of 1920 a river structure was built in the Housatonic River at Shelton, Conn. This structure consisted of a beam-and-girder deck, supported by bents comprising four piles. Reinforced concrete was the only material used except for the piles, which were of wood. During low tide several feet of the wood piles were exposed, while during high tide the concrete columns were submerged for several feet. The Housatonic at this point has a velocity of approximately 3 miles per hour.

Between April and August, 1922, a second smaller section, 366 ft. long, was built, similar in all respects to the previous structure, and of identical materials. Before starting work on the second section the first was examined and found to be in good condition. The materials consisted of new billet structural grade reinforcing bars, standard Portland cement, a pre-mixed sand and gravel dredged from Long Island Sound near Port Jefferson, N. Y., and city water. The cement and steel were tested by a public testing laboratory

* Pres. and Treas., Thompson & Binger, Inc., New York, N. Y.

and found to conform to the specifications of the American Society for Testing Materials.

What have been called the first and second sections were built by the same construction company with the same general superintendent, but under different resident superintendents. The river structure in question runs along property occupied by a textile manufacturing plant, which included a large dye-house emptying its waste into the river. Between February and April, 1923, this dye-house was wrecked and a new one was built in an adjoining location also at the "river driveway".

On April 15, 1923, the writer was notified that both the first and second sections had deteriorated. Inspection showed that the concrete had become so soft that large pebbles could be picked out by hand with little effort. It appeared to be badly abraded at the corners where the reinforcement had become exposed.

It was desired to determine as nearly as possible the true cause of this deterioration, and much money was spent for tests, examinations, and expert opinion. A chemical analysis of the river water in the vicinity of the structure was obtained, based on about one hundred samples taken at different times. A chemical analysis was made of the deteriorated concrete, with results as herein given. However, no analyses could be made of the water as it was between February 2 and April 15, which is the period during which the concrete in both sections failed.

Apparently, the gray Portland cement had been converted into a white powdery substance with no binding qualities. The appearance of the reinforcing steel at the time the corrosion of the concrete was discovered, was such as to preclude the possibility that the damage was caused by corrosion of the steel. Furthermore, in each case the deterioration of the concrete was from the outside in and never in the form of smooth plates of hard concrete spalled off from the inside in the typical manner of a concrete failure due to steel corrosion.

The concrete was mixed in the proportions of 1:1.5:3 in a small standard mechanical mixer. The water was measured in pails. No effort was made to grade the aggregate for a particularly dense mix, but the ready mixed aggregate was well graded from fine to coarse in accurate proportions of one of sand to two of pebbles.

The steel was placed 1½ in. from the surface with a fair accuracy which in a few cases brought it quite close to the surface, and in others as far as 3 in. The forms were made of tongue-and-grooved lumber, well oiled, and tight enough to hold mortar. The pile-cap forms were of 2-in. planks, open at the top and giving access to the river water as soon as the tide covered the forms several hours later.

The deterioration (Fig. 20) was at its worst in some places where the steel was even 3 in. from the surface. At the date of last examination, in the fall of 1925, the superstructure, namely, that part above high water, was in essentially perfect condition (Fig. 21).

While deterioration of the concrete was rapid during the period of several months after its first discovery, it subsequently became less noticeable, and

FIG. 20.—VIEW OF STRUCTURE SHOWING DETERIORATION OF PILE CAPS AND COLUMNS.

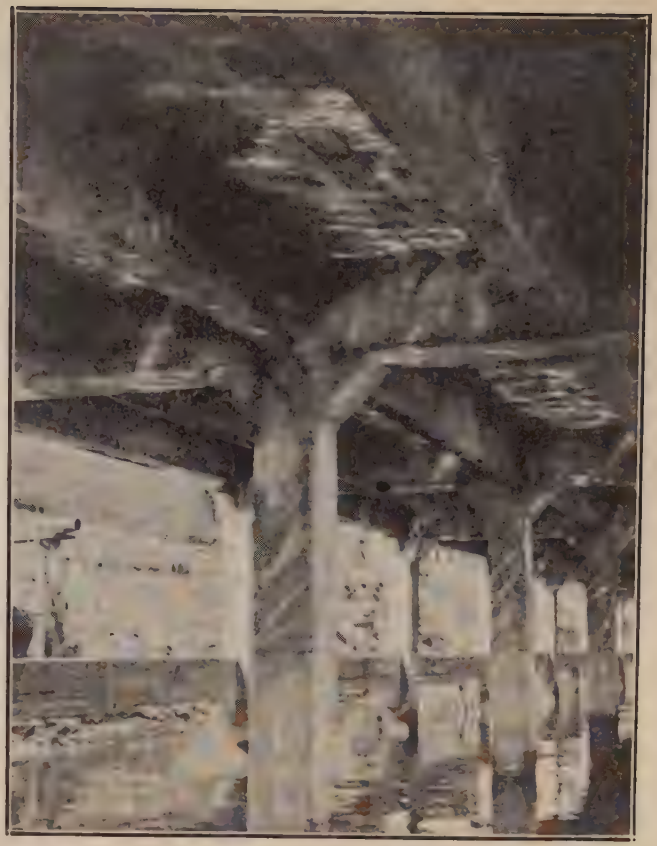

FIG. 21.—VIEW OF STRUCTURE ABOVE MEAN HIGH WATER, SHOWING GOOD CONDITION.

during the early part of 1925 seemed to have ceased. The deteriorated concrete was easily affected by frost and floating objects. There seems to be no possibility that frost and the abrasion by floating objects were the primary causes of the deterioration, as the first section showed no ill effects of the cold winters of 1920 and 1921, while it corroded in the same period of two months during which the second section deteriorated.

The structure was provided with several expansion joints extending from top to bottom. In at least two cases this was not sufficient to take up the total expansion and some of the concrete at the edges of the abutting bents was cracked. Pieces so broken off showed sound concrete, in no way resembling the soft, almost mushy, part of the structure deteriorated in the water. The most minute examination of the entire structure and the study of several hundred photographs failed to disclose a single instance of a crack due to strain, overloading, or lack of steel; in fact, the only cracks in the entire structure were at a few "day's work" joints where the beams of one bent had been continued into the adjacent bent half-way between columns. For architectural reasons, the columns and most of the members were made much larger than the light live load required, and the steel had very low stresses. In fact, none of the steel in the pile caps figured in the computations.

It is the writer's opinion, based on his own observations and a long and careful study of the facts, that the cause of the deterioration is the destruction of the cementitious qualities of the Portland cement, due to a chemical reaction with the substances carried into the water for the two months in which the dye-house was rebuilt, during which time the manufacturer failed to neutralize his acids. This belief was arrived at after having access to the studies made by the following experts, all of whom concur in this opinion: Bryan C. Collier, Benjamin A. Howes, E. DeV. Tompkins and Watson Vredenburgh, Members, Am. Soc. C. E., and Louis Weisberg, Consulting Chemist. The report of Dr. Weisberg includes the following statements:

"A typical sample of the deteriorated concrete taken from below high water in June, 1924 (the year after the deterioration was first reported) showed a white furry deposit, which could be crumbled easily between the fingers and which was found to extend through the sample. Upon analysis this white material dissolved in hydro-chloric acid with production of carbon dioxide and was found to be mainly calcium carbonate. The analysis of this concrete sample shows that the cement has been replaced by a material in which calcium carbonate is one of the principal constituents. Approximately, 80% of the calcium has been transformed into calcium carbonate.

"Analyses of the water samples show that practically every sample contains bicarbonates. A certain number also contained carbonates. Either carbonates, bicarbonates, or carbon dioxide in the water would be capable of bringing about this transformation of the calcium in the cement to calcium carbonate. The water samples for the most part also show the presence of acid which is capable of dissolving the calcium carbonate formed within the mass and removing it therefrom, thus bringing about disintegration of the whole mass."

Regarding the water, he says:

"Of the samples taken Saturday and Sunday, 33% showed an alkaline reaction, whereas those taken on working days only 10½% showed alkaline reaction, and 89½% showed an acid reaction.

"In short, it is my opinion that the disintegration is due to the transformation of the calcium in the cement, to calcium carbonate by the action of carbonates and bicarbonates in the water, followed by a solution of this calcium carbonate by the action of acid in the water."

Dr. Weisberg also made a comparison of a complete chemical analysis of the sample of deteriorated concrete taken from below the high-water line with one of undeteriorated concrete taken from above that line. The piece previously mentioned as being broken off at the expansion point, served as the latter. He found only 24% of the calcium in this air-set concrete to have been transformed to carbonate, while in the sample taken from below the high-water line, calcium in the form of carbonate is nearly 80% of the whole. In other words, not only can the acid attack the vulnerable part of the concrete, but three times as much of the concrete is vulnerable.

Fig. 20 shows in detail one of the worst deteriorated pile caps and columns, the front face of the column being corroded up to a horizontal line which coincides very closely with mean high water as evidenced by a horizontal deterioration on the adjacent face. At this time the structure was carrying a load of broken stone equalling approximately twice the designed load per square foot. Although this load remained for a long period, the structure showed not the slightest sign of suffering from this severe test.

A close view of the structure above mean high water is given in Fig. 21 which plainly shows excellent concrete. This photograph was taken at mean high tide.

E. E. R. Tratman,* Assoc. M. Am. Soc. C. E. (by letter).—An examination of several concrete structures made a few years ago for the purpose of ascertaining the appearance of exposed concrete surfaces, is called to mind by the extended discussions of this paper, and of other recent papers on various aspects of the concrete problem.

The structures in question included railway and street bridges, retaining walls, river-front or bulkhead walls, and buildings of different kinds. Most of these structures were only a few years old, but only a very small proportion of them could be classed as satisfactory in appearance, owing to the development of craze marks and streaks and patches of discoloration which marred the surface. At several bridges and walls, water had seeped through the joints and the concrete mass, in spite of water-proofing on the back, and thus had added to the unsightly appearance. The defects noted were those inherent to concrete, exclusive of the effects of dirt and smoke which may accumulate on any kind of surface. They were independent also of structural cracks, such as seem to be almost inevitable in long retaining walls. The examination was not concerned with structural disintegration of the concrete, but in some cases there may be a close relation between surface defects and general disintegration or corrosion of the concrete.

Although concrete buildings are being erected in large numbers, few of them have exposed concrete surfaces, a veneer of brick, stone, and terra-cotta being applied as a rule for the purpose of giving a pleasing and permanent finish. Such exterior coverings are sometimes cleaned or renovated

* Western Editor, *Engineering News-Record*, Wheaton, Ill.

at intervals. In other cases concrete surfaces are relieved by inserts of colored tile or by grooves and recesses to break up flat surfaces. Treatment of this latter kind, however, is utilized for decorative effect rather than to prevent or remedy a poor appearance of the finished concrete. The comments on the surface treatment of concrete by Mr. Newhall are in line with suggestions offered after the examinations mentioned, to the effect that it might be advisable to study the practical and economical possibilities of giving to exposed concrete a surfacing that would produce an appearance as good and as durable as that of a veneering with other material. Such treatment might result in considerable saving as compared with the usual veneer covering, while it would also avoid the unsightly appearance of structures which do not warrant the expense of such veneer.

Surface grinding and grouting of fresh concrete has been used in a number of cases, but does not appear to be an entire assurance against crazing and discoloration. Painting of exterior surfaces has been used less extensively and with rather doubtful results.

The two questions or problems involved are: First, the possibility of developing a treatment or manipulation of concrete that will give a surface of permanently good appearance with reasonable economy; and, second, the possibility of further developing such treatment for the protection of concrete exposed to water and wave action. As far as appearance is concerned the following appropriate statement* is quoted: "There is an open field for developing concrete construction that can honestly show its own face without causing pain because of its unsightliness."

JOHN R. BAYLIS,† ASSOC. M. AM. SOC. C. E. (by letter).—The writer is grateful for the very interesting discussions of the subject, and feels that much of value has been added to the paper. There are only a few points where additional comment seems desirable. In most instances where there is disagreement with the writer's conclusions, it is believed to be due more to a misunderstanding of the subject than to differences of opinion. When more details of the problem of concrete deterioration are published it is believed that the value of some of the facts discussed in the paper will be more readily appreciated.

On a subject of this nature it is expected that there will be some differences of opinion. Dr. Bogue submits no evidence in his criticism, and the writer does not agree with his views on the points discussed. A criticism of this kind brings to mind a passage from a recent editorial in a well-known technical journal on the art of writing:

"No one has any right to expect that the writer's thoughts should agree with the reader's; it should suffice that the writer has given us his own thoughts expressed so far as possible in his own order and his own words. Here and there should be found a few lines worth reading, an idea or two worth considering, even a phrase which is illuminating or apposite."

It is gratifying to know that a few contractors are producing good concrete. The writer, however, does not agree with Mr. Ahlers that the bad

* *Engineering News-Record*, December 16, 1920.

† Chf. Filtration Chemist, Bureau of Eng., City of Chicago, Chicago, Ill.

qualities of concrete should not have received greater consideration in the paper than the good qualities. He knows that a great deal of good concrete is being manufactured, and that there is much in the engineering literature telling of its good qualities. It is also known that there is much concrete that has been manufactured with the greatest care, that is deteriorating in strength. It was the writer's intention to point out a few reasons for this deterioration, and it was assumed that the good qualities of concrete had already been fairly well advertised. It is true that some of the evils to which reference was made can be avoided by the use of the water-cement ratio, and this is a step in the right direction, but it is not the solution of all deterioration problems.

Mr. Ahlers, as well as Professor Williams, has failed to grasp the purpose of the void test. It was intended to be used in interpreting what to expect of concrete after it has been mixed, or more especially after it has been placed in the forms. No idea of the quantity of cement used can be obtained from the test as outlined. If it is desirable to know this it may be determined by carrying the test a little further, providing silica sand is used for the fine aggregate. Knowing the calcium content of the cement used, any variation of this constituent in the mortar sieved from the coarse aggregate is readily detected. Such a test is not reliable unless all aggregate larger than the No. 8 sieve size is removed, also if any aggregate finer than this size contains calcium soluble in acid. The writer has followed such a procedure for examining concrete in which the quantity of cement used was in doubt and finds that it gives fairly accurate results.

In his density test Professor Williams assumes the volume and specific gravity of all materials used, to be known. It was the writer's intention that a testing engineer should collect samples as the concrete is being poured into the forms, or, even better, after it has been placed. In instances where it is possible to do so, it might be best to collect samples by cutting out small pieces of the concrete after it has set a day or more. Professor Williams' method includes all voids, whereas that proposed by the writer does not include voids of more than about $\frac{1}{100}$ in. in diameter. Voids $\frac{1}{8}$ in. in diameter, or even larger, caused by air-bubbles, do not materially affect the power of the concrete to resist internal chemical changes, yet such large voids may materially affect the total percentage of voids. Heretofore, test methods have tried to measure all voids, a procedure which is practically useless for estimating the power of the concrete to resist the weathering agents.

The proposed void test is not an absolute measure of the durability of the concrete, but it seems to give better indications than almost any other test. Because a test is not absolutely positive in its indications does not signify that it is of no value. The measurement of aggregate by loose volume is not accurate, but it is certainly better than no measurement. With the same materials used and the same condition of exposure, the void test gives a better indication of the durability of the concrete than may be predicted by any other test. The flow of water through a piece of concrete may be influenced by so many factors that it is not a good criterion of the

power of the concrete to resist deterioration. The tendency may be in the same direction in most instances, but the passage of water through the concrete is affected quite considerably by the tendency to form channels. In other words, practically all the flow may be passing through a very small section of the concrete.

Those who are still of the opinion that all troubles can be eliminated by adherence to a certain water-cement ratio, or any other test that is made in advance of placing the concrete in the forms, will do well to read a recent article by Greene.* Attention is called to the following:

"Assume a column or wall, large enough for a man to work in. Place concrete in this wall of a consistency to give 1-in. slump at the mixer or as sampled out of a buggy. Run 7 ft. in depth of the concrete in three hours with men spreading the concrete. At the end of the run there will be loose water on top of the concrete and the material tested in the hole may show a slump of 4 in. * * * It is the judgment of the writer that samples for testing should be taken only in the hole and from the wettest concrete in the hole, not from the mixer or from the buggy. * * * The concrete should be bought 'f.o.b. the hole', not at the mixer, the dumping hopper, or in the buggy."

The proposed void test would be very helpful for such instances as that described. It will show quite readily that the top concrete which has the excess water is too porous for durable concrete under many exposed conditions. Any test that takes the sample in advance of the last disturbance of the concrete in the forms will not give conditions as they are. Some kind of a test must be adopted that may be applied to concrete after it has been placed in the forms, or even after it has set, if engineers ever expect to get one indicative of actual conditions.

The new sieve size, to which Mr. McMillan takes exception, is a necessity for the proposed void test, and it will be more helpful than the $\frac{1}{4}$-in. size in proportioning aggregate. It will be impossible to make an intelligent interpretation of the void test unless the quantity of aggregate larger than about $\frac{1}{10}$ in. in diameter is shown. For a test to be applicable to fairly small samples of concrete the dividing line between fine and coarse aggregate must be set at a point where there is practically no possibility of the particles separating from the mortar. It is a fact that $\frac{1}{4}$-in. aggregate will separate in much of the concrete now being manufactured especially with a little disturbance, such as working the concrete in the forms. The separation may be slight and not enough to affect the strength of the concrete in many instances, but it spoils the void test.

No one should attempt the void test unless the No. 8 sieve is used. A few samples from concrete after it has been placed, or after it has set, will impress one with the value of using this size.

In discussing the void test, Mr. McMillan makes the statement that, "all that the test accomplishes, therefore, is, in effect, a determination of the quantity of combined water". How any one can conceive of this being the

* "Some Lessons Learned in Building a Long Concrete Bridge," by John F. Greene, *Engineering News-Record*, Vol. 97, No. 20, November 11, 1926.

case, the writer is unable to understand. After being dried at room temperature (60° to 70° Fahr.) concrete will show considerably more than 50% of its total water absorption, except possibly for very dense mortar. The writer feels sure no one believes that the hydrated cement compounds are dehydrated at this temperature. Perhaps it would be helpful if both Dr. Bogue and Mr. McMillan made a few void tests in order that they might better understand its limitations.

It was not intended in this paper to cover the entire field of concrete deterioration. Porosity was discussed more as a means of aiding chemical changes than as an attempt to give the changes that actually are taking place. One of the most serious problems in concrete deterioration was not even mentioned. It is hoped that additional research work will be made public in the near future, and that this will be of some aid in leading to a clearer understanding of the limitations of Portland cement concrete.

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Paper No. 1613

THE WORK OF THE QUEBEC STREAMS COMMISSION*

By O. O. Lefebvre,† M. Am. Soc. C. E.

The Quebec Streams Commission is asked by numerous correspondents from outside the Province what the Commission has done, and especially for data relating to its storage reservoirs. It is the intention, in this paper, to answer these questions in a general way.

Until 1910, there were practically no data regarding the streams of the Province, except those on height of rapids and falls and flow measurements undertaken by private companies, the latter being limited to a very few streams.

In 1910 it was felt that something should be done to determine the value of the water power sites which the Province held for lease. The value of the property the Government was leasing should be known before disposing of it. Other information regarding streams in the Province was also desirable, and as it was felt that this work could be carried out best by a commission, the Quebec Streams Commission was instituted by the Act 1, George V, Chapter 5, 1910. By this Act the Commission is authorized:

1.—To devise just and practical rules respecting the flow, the drawing off, the disposal, the distribution, the storage, and generally respecting the preservation and management of running waters in the Province of Quebec.

2.—To determine whether the laws of the Province governing watercourses, as regards floating of timber, the flow, drawing off, disposal, distribution, and storage of water, ought to be amended.

3.—To make any suggestion tending to the adoption of practical rules adapted to protect the forests of this Province belonging to the Crown or to private persons, and to encourage and facilitate the utilization of water powers, while reconciling the interests of agriculture, of industrial pursuits, and of the forests with respect for property rights.

* Presented at the Fall Meeting, Montreal, Que., Canada, October 14, 1925.

† Chf. Engr., The Quebec Streams Comm., Montreal, Que., Canada.

4.—To consider whether it is expedient to have the rivers of the Province classified as navigable and floatable rivers, and rivers which are not navigable or floatable, by an administrative commission or otherwise, according to uniform rules, and to submit such rules, if it be advisable.

5.—To make any inspections and examinations of the lakes, rivers, ponds, creeks, and watercourses in this Province, which it may think necessary.

6.—The Commission shall proceed with all possible diligence to carry out the purposes for which it is constituted. In matters not provided for by this Act, it shall be guided by instructions from the Lieutenant-Governor in Council.

7.—The Commission shall report to the Lieutenant-Governor in Council as to its researches, as well as its recommendations and suggestions. This report shall be laid before the Legislature during the first ten days of the next session.

Members of the Commission, however, were not appointed until December, 1911. It was the good fortune of the Government to appoint as the first Chairman of the Commission, the Hon. S. N. Parent, who had previously been Prime Minister of the Province of Quebec, a man of great business ability, who realized fully the responsibility of the task which was entrusted to him and his associates. He was ably assisted in his work by two engineers, W. I. Bishop, M. Am. Soc. C. E., and the late Mr. Ernest Bélanger.

The 1925 Commission has at its head the Hon. J. A. Tessier, Arthur Amos, Chief of the Hydraulic Service, Department of Lands and Forests, and S. F. Rutherford, Engineer. The Commission commenced its work in 1912, and has issued annual reports giving details of its activities.

In 1912, the Commission was in the organization stage and made numerous investigations as to conditions of different streams, more particularly flood conditions, log-driving difficulties, and the storage possibility on the St. Maurice River.

In 1913, the Commission's work was becoming quite important, and an Engineering Staff was appointed, with the writer as its Chief Engineer. Immediately, systematic flow measurements of the principal streams of the Province were organized, together with the collection of more meteorological data, as well as numerous investigations of storage possibilities here and there.

Stream Measurements

In 1913, a systematic organization for flow measurements was initiated. On the principal streams gauges were established and read daily, sometimes three or four times a day. The flow of the streams was measured and related to the gauge height, wherever possible. The number of stations and the number of streams investigated have increased from year to year and data are now available for nearly all the principal streams of the Province. Since 1922, the work of stream measurement has been carried out by the Dominion Water Power Branch, Department of the Interior, Léo. G. Denis, District Engineer, with headquarters in Montreal. This was made possible through an agreement with the Commission, whereby the two services co-operate with each other, to the advantage of the Province.

Data on gauge heights are received monthly at the office of the Commission, tabulated, and made available for distribution to those who may be interested. A copy of this information is furnished to the Dominion Water Power Branch, which supplies the Commission with the results of the meterings.

Generally speaking, it may be stated that the average yearly run-off from the streams is about 1.6 sec-ft. per sq. mile of drainage area. The ordinary maximum run-off usually occurs in the spring and varies from 6 to 10 sec-ft. on the larger streams to 20 and 25 sec-ft. on the smaller streams, the ratio between minimum and maximum flow being about 1 to 25. The minimum flow takes place in winter, usually in March, and reaches 0.25 to 0.40 sec-ft. per sq. mile of drainage area, depending on the characteristics of the streams.

Flood flows much larger than the maximum just given have been recorded, especially during 1924, and the figures which prevailed during this period should be of interest. On the Ste. Anne de la Pérade River, a discharge of 43 000 sec-ft. was recorded at St. Alban; the drainage area is 672 sq. miles, which is a rate of 64 sec-ft. per sq. mile. On the Ste. Anne de Beaupré River, at the Laurentian Power Company's plant, where the drainage area is 400 sq. miles, the discharge was 32 000 sec-ft., or 80 sec-ft. per sq. mile. At Lake Kenogami, in the Saguenay District, the run-off was 20 sec-ft. per sq. mile for a water-shed of 1 400 sq. miles.

Meteorological Data

The collection of data regarding rainfall and temperature has been followed for a number of years. For the Cities of Montreal and Quebec, data are available from 1871 to date.

The work of collecting meteorological data has been initiated by the Dominion Meteorological Service of Canada—a branch of the Department of Marine and Fisheries at Ottawa—with headquarters at Toronto, under the direction of Sir Frederick Stupart, who is known as the Director of Meteorological Service of Canada.

The Quebec Streams Commission took great interest in this work, and suggested a large number of new stations. It has offered its full co-operation to the Federal Service. There are now 78 stations throughout the Province, where rainfall and temperature are recorded. The data are sent monthly to the Commission and transmitted to the Federal Office. This information is tabulated and put in proper form for distribution to engineers. The Provincial Government, through the Department of Lands and Forests, pays the observers, and many stations are in charge of power companies and manufacturing firms, the co-operation of which is greatly appreciated. The Streams Commission supervises the collection of this information and follows closely the work of the observers who are being impressed with its value and the importance of recording it properly.

It may be of interest to state that the average annual rainfall in the Province is about 36 in., including snow. The average snowfall is about 100 in., which is considered to be equivalent to 10 in. of rainfall. The run-

off may be assumed as averaging 50% of the rainfall. This percentage may reach 70 in very wet years and 35 to 40 in dry years.

Great difficulty is experienced in having the snowfall properly recorded. The best method is to have a small platform laid horizontally in the bush, protected against the action of the wind. This condition is one that is not always possible, and, in many instances, the figures given are not reliable, when the snowfall is accompanied by wind.

Power Sites

The Quebec Streams Commission has investigated, for the Department of Lands and Forests, a great number of water power sites covering all districts in the Province, from Abitibi in the northwest, to the Gaspé Peninsula in the Gulf of St. Lawrence, including the lower sections of the streams on the north shore of the River St. Lawrence. Numerous surveys and plans have been made of the power sites on the principal streams, namely, the Quinze River in the Ottawa District, the St. Maurice and St. Francis Rivers in the central section, the streams tributary to Lake St. John, and the principal streams of the north shore. In other cases, only sketch plans have been made.

In every instance, a plan for the proper development was submitted with an estimate of the power available and the probable cost. This information is given to the Department of Lands and Forests, at Quebec.

One point should be made clear: A great many people have an idea that the Quebec Streams Commission has the control of water powers in the Province. That is an error. The Commission investigates the value of power sites; it does not negotiate for the lease of such sites. Its report is placed in the hands of the Minister of Lands and Forests, and the Hydraulic Service is the body through which one must negotiate for the acquisition of any power site from the Crown.

To July 1, 1925, the Commission had spent $646 942.11 for these investigations and studies. A large part of this sum will eventually be refunded by the parties leasing power sites. A clause of the lease stipulates that the money spent by the Commission for surveys, plans, etc., must be refunded. All special problems regarding extraordinary conditions on any river have been referred to the Commission for investigation and report, such, for example, as the great floods on the Chaudière River, and their possible prevention.

The principal work of the Commission, however, is the construction of storage reservoirs, in order to regulate the flow of streams. A system of storage has been constructed on the following streams: the St. Maurice River; the St. Francis River on the south shore, in that section known as the Eastern Township; the Ste. Anne de Beaupré River; the Mitis River; and Lake Kenogami. Storage propositions have been investigated and authorized on other streams, such as the North River, north of Montreal, River du Loup (upper), and the Ste. Anne de la Pérade River, but no work has been commenced.

Legislation

Although the Act appointing the Commission gives that body authority to investigate conditions on any of the streams, a Special Act of the Legis-

lature has to be passed giving the Commission the authority to construct dams and regulate the flow of certain streams and tributaries. A separate Act has to be passed for each individual plan. Such an Act gives the Commission authority, subject to approval by the Lieutenant-Governor in Council, to construct dams, acquire existing dams necessary for its work, to buy all lands to be flooded, either by mutual agreement or by expropriation, to build new roads, if required, and to replace flooded highways. To pay the cost of all these works, the Provincial Treasurer is authorized to supply the Commission with the necessary funds, the total amount not to exceed a figure mentioned in the Act. He may borrow money for the purpose, or he may take it out of ordinary revenue. All these Acts are practically worded in the same way, and a copy of that for Lake Kenogami, is attached hereto. (See Appendix I.)

Before any storage project is authorized by the Lieutenant-Governor in Council, the Commission has to supply the Government with a complete report relating to the proposed works, their estimated cost, the companies likely to benefit by the works, and, if possible, the extent to which each company will benefit.

The Act gives the Commission authority to collect annual dues from the benefiting companies. It may impose a tariff. However, it is preferred that the annual charge be agreed to between the parties who bind themselves by long-term contract.

No storage plan is undertaken until the Commission is assured of a yearly revenue sufficient to pay the interest charges, the sinking fund in thirty years, and the cost of operating and maintaining the dams, plus a profit whenever possible, which introduces the question of the determination of rates.

As previously stated, all companies, associations, or persons benefiting from the storage works of the Commission are called upon to pay an annual charge proportionate to the amount of benefits derived. In many instances, the determination of this amount is quite a problem owing to lack of information on conditions prevailing before any storage work is undertaken.

In the case of the St. Maurice River, complete flow data were available for fourteen years, thanks to the initiative of the Shawinigan Water and Power Company, and the benefits could be calculated exactly at each power site. The usual procedure of the frequency curve was followed, and the additional power reduced to a basis of the horse-power year.

For the St. Francis River, no flow data were available and it was not possible, therefore, to establish with certainty the number of additional horse-power years each company could generate under the controlled conditions. It was then thought that each company should be charged according to the head under which its plant was operated. However, under the storage plan comprising two separate reservoirs, all companies do not receive the same quantity of water, one of them being located between the two reservoirs and getting water from the upper reservoir only.

Another method, therefore, had to be adopted. It is called the "second-feet-head" method, which has been in use on the Wisconsin River. Multiplying the number of cubic feet per second supplied at each plant by the head under which the water is used, gives the "second-feet-head". The annual

charge which is intended to be levied is divided proportionately to the number of "second-feet-head". On the St. Francis River, the rate is $0.50 per sec-ft.-head. On the St. Maurice River, when new power sites are developed, the owners have to pay for the storage, and this means added revenue to the Commission. In the case of the St. Francis River, the rate may be revised every five years, if new developments are carried out, and the companies should then enjoy a reduction.

On the other streams, the revenues of the Commission are limited to the percentage of the capital cost, and any new development means a reduction in the amount paid by the present companies. This condition is to prevail for thirty years.

Storage Dams

General information will be given on the different storage dams which were built by the Commission, as follows:

Gouin Dam.—Gouin Dam, shown in Fig. 1, is on the St. Maurice River, 240 miles from the mouth. The dam permits a total control of the water supplied by the drainage area of 3 650 sq. miles. A series of large lakes, at different levels, have been combined into one lake forming a reservoir of a total capacity of 160 000 000 000 cu. ft., or 5 725 sq. mile-ft. The dam, which is of the gravity section type, was completed in December, 1917, and has been operated since the beginning of 1918. The stored water is used to regulate the minimum flow of the St. Maurice River, at Shawinigan. When the dam was built, the minimum guaranteed to the power companies was 12 000 sec-ft. Since 1922, the minimum flow has been held to 16 000 sec-ft.

Under natural conditions the minimum flow of the St. Maurice River was about 6 000 sec-ft. It is seen, therefore, that the primary power of the hydro-electric plants on the St. Maurice River has been increased in the ratio of 6 to 16, or by 166 per cent. The cost of this plan was $2 500 000, and the annual revenue therefrom is $234 000.

The ratio of low-water to high-water flow at Shawinigan, under natural conditions, was 1 to 26, and under regulated conditions, is 1 to 10. During high-water periods the storage dam is completely closed, and water is supplied to hold the minimum required at Shawinigan. All power sites between the dam and the regulating point have a minimum flow which is variable, depending on the storage required at Shawinigan. The power site near the dam loses its practical value, as no water is available when the dam is closed. At that point, what was high water at flood times is now practically no flow, as illustrated by Fig. 2. The St. Maurice River, under regulated conditions, affords power possibilities of more than 1 000 000 h.p., which is twice what it was under natural conditions. The duration curves for flow at Shawinigan are shown on Fig. 3.

St. Francis Storage.—At the head-waters of the St. Francis River, a storage dam, called the "Allard Dam", has been built and operated since 1918. It is a hollow concrete dam, built on a hardpan foundation with cut-off walls beneath a concrete floor-slab properly reinforced, which supports a series of

FIG. 1.—VIEW OF GOUIN DAM, UPPER ST. MAURICE RIVER, CANADA.

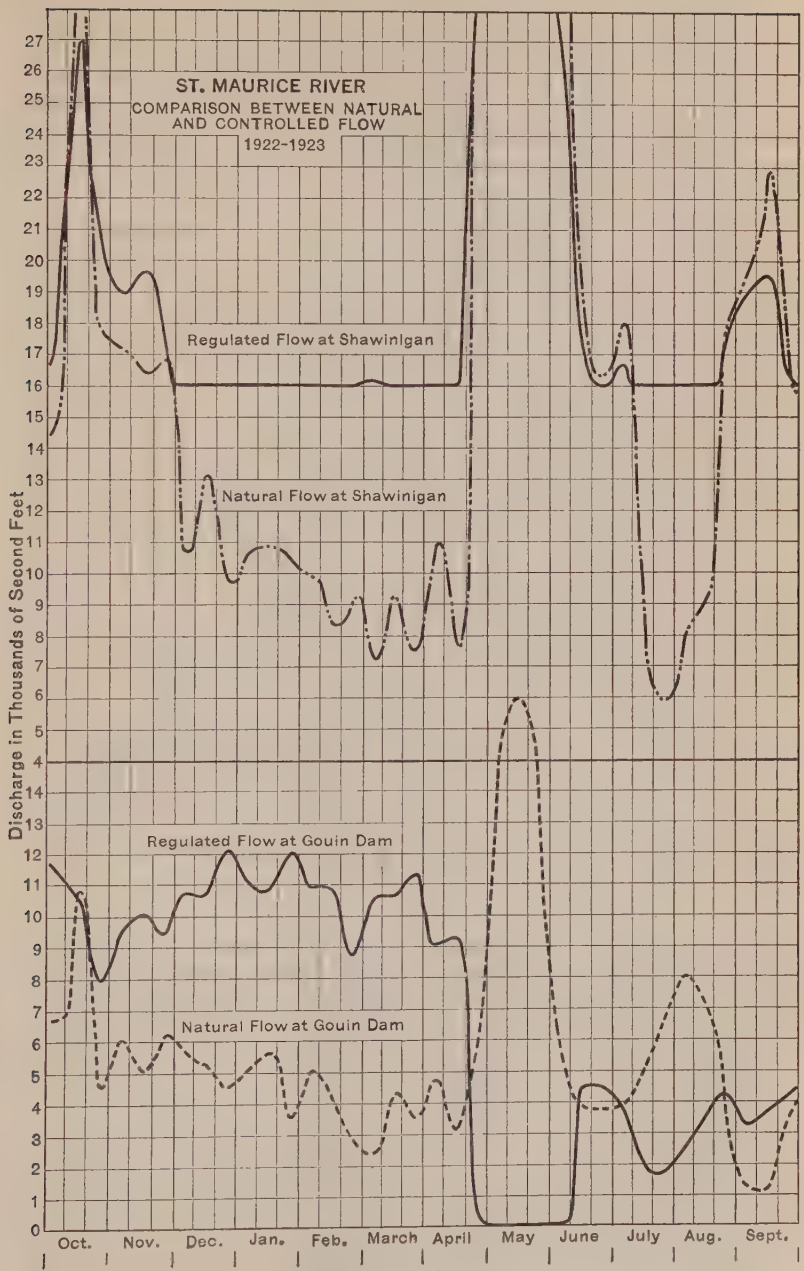
ST. MAURICE RIVER
COMPARISON BETWEEN NATURAL AND CONTROLLED FLOW
1922-1923
Regulated Flow at Shawinigan
Natural Flow at Shawinigan
Regulated Flow at Gouin Dam
Natural Flow at Gouin Dam
Discharge in Thousands of Second Feet
Oct.
Nov.
Dec.
Jan.
Feb.
March
April
May
June
July
Aug.
Sept.

FIG. 2.

piers, all connected by a sloping deck slab. This dam affords a complete control of the water supplied by a drainage area of 472 sq. miles. Lake St. Francis was raised 18 ft. above its natural high level, the total depth of storage being 27 ft. The capacity of the reservoir is 12 000 000 000 cu. ft.

Another storage dam located at the outlet of Lake Aylmer was acquired from the Brompton Pulp and Paper Company. It affords a partial control of the water supplied by a drainage area of 135 sq. miles. The depth of storage on Lake Aylmer is 12 ft., and the volume of water stored is about 110 sq.-mile-ft.

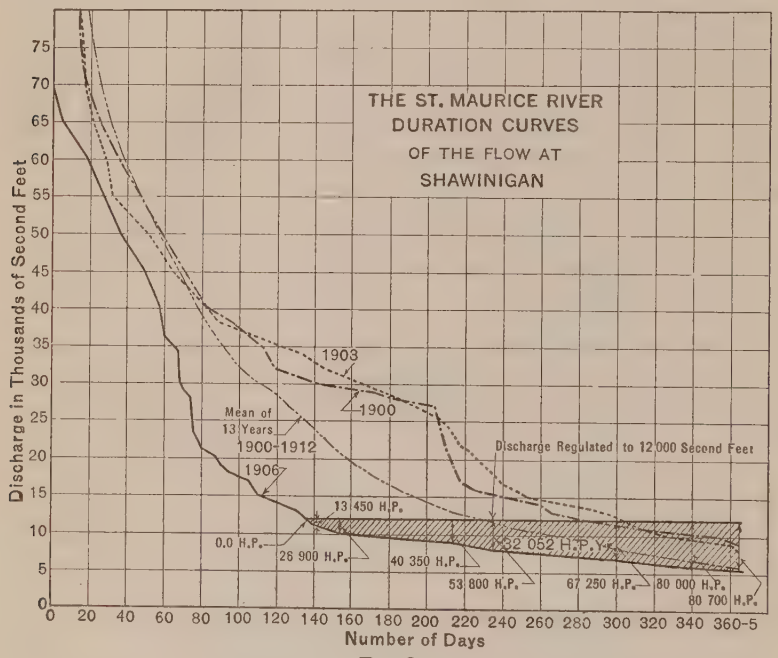

FIG. 3.

The quantity of water discharged from the St. Francis Dam is 900 sec-ft., although the minimum guaranteed by the Commission is only 600 sec-ft. The quantity released from Lake Aylmer is 1 050 sec-ft., although only 750 sec-ft. is guaranteed by the Commission. This means that the minimum flow of the St. Francis River is increased by 650 sec-ft., at every point below the dam. The total cost of the storage on the St. Francis River was $750 000 and the yearly revenue of the Commission is $70 000.

River St. Anne (de Beaupré).—Two wooden dams have been built on the River St. Anne de Beaupré, at a cost of $344 000. The two storage reservoirs, namely, the Lake Brûlé and the Savane River, have a capacity of 12 and 20 sq.-mile-ft., respectively, which is sufficient to regulate the minimum flow of the St. Anne River at the Laurentian Power Company's plant, at St. Ferréol, to 260 sec-ft. The revenue of the Commission, from this storage, is $34 000 yearly.

Fig. 4.—View of Lake Kenogami Reservoir, Chicoutimi Outlet Dam.

Mitis River.—On the Mitis River, the Commission has built a storage dam to regulate the flow of the river to a minimum of 350 sec-ft. at the plant of the Lower St. Lawrence Power Company, at Mitis Falls.

The dam, a stone-filled crib structure, has been completed at a cost of about $120 000. The cost of clearing the flooded area and paying for land damages will bring the total up to $225 000. The Commission will receive $22 500 yearly.

Lake Kenogami Dams.—Lake Kenogami, in the Saguenay District, has a drainage area of 1 400 sq. miles. The lake itself has an area of 12 sq. miles, and lies at Elevation 510. It has two outlets; one at the Chicoutimi River flowing into the Saguenay, at sea level, 15 miles distant. On this stream are located the plants of the Chicoutimi Pulp Company. The other outlet, the River au Sable, flows through the Towns of Jonquières and Kenogami to sea level in the Saguenay, 7 miles from the outlet. On this stream are located the plants of Price Brothers and Company, Limited.

Under natural conditions, the plants on both streams could be operated at full capacity for only seven or eight months of the year. With the additional storage provided by the Commission, the plants can be operated at full capacity for the entire year. The Chicoutimi Pulp Company's plant has a capacity of 300 tons of pulp per day, and all its power (32 000 h.p.) is generated from Lake Kenogami water.

The plant of Price Brothers Company, at Kenogami, has a capacity of 500 tons of paper per day, and only a small proportion of the power required is derived from Lake Kenogami waters. The cost of the Lake Kenogami dams will be about $4 000 000 and the revenue to the Commission, $320 000 yearly. The regulated flow of Lake Kenogami is 1 800 sec-ft., which is distributed directly to the power plants. At no time is the dam closed. The flow, however, is considerably reduced on Sundays, the water thus saved being distributed the following week. Fig. 4 is a view of the dam at the Chicoutimi River outlet.

Summarizing, the Commission has spent to date (1925) for storage $7 500 000, from which it receives an annual income of about $685 000.

Operation

It has, no doubt, been noticed from the foregoing that the storage reservoirs are operated under two different methods. One calls for a regulated minimum flow at a certain point, such as Shawinigan, in the case of the St. Maurice River, and under that method of control the storage dam is, at flood times, completely closed.

The other method consists in releasing from the storage dam at all times of the year a determined minimum flow. This is the method on the St. Francis River and on Lake Kenogami. In the latter case, the outlets from the lake have no drainage area and no other method of control could be satisfactory.

Which of the two methods is the better is the question to be considered on each stream. In the first instance, such as the St. Maurice River, the power sites lying in the vicinity of the storage dams are greatly decreased

in value, whereas those near the controlling point are greatly enhanced in value. The best method can only be determined after a complete analysis of the conditions at each power site.

The importance of regulating the flow of rivers is fully realized and admitted by all. It is an insurance against factory shut-downs and loss of time and business. During the winter of 1923, low-water conditions were so bad in the Province of Quebec that industry would have suffered greatly except for the large quantity of water which could be drawn from the Government Reservoir on the St. Maurice River. As it was, the districts served by St. Maurice power did not suffer from the lack of water; neither did the St. Francis District. In other districts, however, plants were closed and trade and labor suffered a heavy loss.

Although the revenue derived by the Commission is important, it is insignificant when compared with the advantages which the Province at large derives from these works. This policy will be followed also for other streams which the Commission is investigating.

APPENDIX I

ACT 8, GEORGE V, CHAPTER 13:

AN ACT TO GRANT CERTAIN POWERS TO THE QUEBEC STREAMS COMMISSION, RELATING TO THE STORING OF THE WATERS OF LAKE KENOGAMI

Whereas the Quebec Streams Commission recommends the construction of works required for the storing of the waters of lake Kenogami and its tributary lakes and rivers in order to better regulate the flow thereof; and

Whereas it is in the public interest to carry out the said recommendation, and to grant the Commission the powers necessary for that purpose;

Therefore His Majesty, with the advice and consent of the Legislative Council and of the Legislative Assembly of Quebec, enacts as follows:

I.—The Quebec Streams Commission, after having deposited in the Department of Lands and Forests all the plans and specifications necessary to precisely indicate the work which it wishes to undertake under the authority of this act, and the probable cost thereof, may be authorized by the Lieutenant-Governor in Council to do the work necessary for the erection of storage dams or other works in lake Kenogami and in the Chicoutimi and au Sable rivers and of their tributary lakes and rivers, and of regulating their flow, both from the point of view of the lessening of floods, and for the better utilization of the water powers to be obtained from such lakes and rivers and their tributaries.

II.—The Commission shall give out the works authorized by this act, by means of tenders and contracts, after public advertisements and notice that the plans and specifications have been deposited in the office of the Commission for examination.

The giving out of the work shall be evidenced by a contract which shall be awarded to the contractor making the lowest tender, and who also, in the opinion of the Commission, has sufficient ability, experience and means to do the work properly.

Nevertheless, no contract shall be entered into by the Commission except with the approval of the Lieutenant-Governor in Council.

III.—

1.—The Commission may, with the approval of the Lieutenant-Governor in Council, acquire, either by mutual agreement or by expropriation:

(*a*). The existing dams and works at lake Kenogami in the Chicoutimi river and the river au Sable;

(*b*). All the immovable property required for the carrying out and upkeep of the work it is authorized by this act to execute;

(*c*). All the immovable property that may be flooded or otherwise affected through the execution and upkeep of the said works;

(*d*). All the immovable property required for the making of public or private roads to give access to the said works or to replace roads flooded or otherwise destroyed or damaged;

(*e*). All the immovable property required for establishing the necessary servitudes;

(*f*). All the servitudes, real rights, rights conferred by the Legislature, rights resulting from contracts, and all other rights of any kind whatsoever.

2.—Every expropriation under this act, shall be effected in accordance with the Quebec Railway Act, and the Superior Court of the district or a judge of that Court may grant immediate possession on such conditions as it or he may deem advisable.

3.—When it is necessary to expropriate only a piece of a lot, whether cadastral or not, the Commission, with the consent of the Lieutenant-Governor in Council, may acquire the whole of such lot and afterwards sell such portions thereof as it does not require.

IV.—So soon as the plans and specifications mentioned in Section 1 are deposited, it shall be the duty of the Commission, after hearing the parties interested, to submit to the Minister of Lands and Forests:

(*a*). All forms of contracts to be entered into between the Commission and any person, company or association, who may benefit by the works for the storing and regulating of the waters coming within the scope of this act;

(*b*). A general tariff fixing the rates, prices and conditions which may be demanded from or imposed upon any person, company or association, as regards any matter not governed by the contract mentioned in Paragraph (*a*), for the use of the said waters.

No contract passed in virtue of Paragraph (*a*), shall have any force or effect until approved by the Lieutenant-Governor in Council.

The tariff to be established in virtue of Paragraph (*b*), after having been approved by the Lieutenant-Governor in Council, shall not come into force until published in the *Quebec Official Gazette*.

V.—To ensure the construction of the works and the acquisition of the immovables coming within the scope of this act, the Lieutenant-Governor in Council may authorize the Provincial Treasurer to contract, from time to time, such loan or loans as he thinks necessary, but the loan or loans so contracted shall not exceed the sum of one million eight hundred thousand dollars.

The said loan or loans may be effected by means of debentures or inscribed stock issued for a term of not more than fifty years, and at a rate of interest of not more than five per cent. per annum.

The said debentures or inscribed stock shall be in the form and for the amount that the Lieutenant-Governor in Council may determine, and shall be payable, in principal and interest, annually or semi-annually, at the place mentioned by him.

The debentures or inscribed stock issued under this act shall not be subject to the duties imposed by the Quebec Succession Duties Acts.

VI.—Sections 11, 12, 13, 14, 15, 16, 17, 18, 19 and 20 of the Act 3, George V, Chapter 6, shall apply, *mutatis mutandis*, to the present act, as if they had been specially enacted therefor.

VII.—Nothing in this act shall authorize the Commission to acquire by expropriation any of the property or rights mentioned in Section 3 of this act, belonging to the companies called "The Jonquière Pulp Company" and "Price Brothers & Company, Limited".

VIII.—This act shall come into force on the day of its sanction.

SECTIONS 11, 12, ETC., OF ACT 3, GEORGE V, CHAPTER 6

11.—The Lieutenant-Governor in Council, pending the negotiation of the said loan or loans may also authorize the Provincial Treasurer to advance from time to time out of any unappropriated public monies, the sums necessary to carry out this act on complying with the requirements of Section 12.

Special accounts of such advances shall be kept in the Treasury Department, and the proceeds of the loans shall first of all be applied to pay them off.

12.—The sums required for the doing of the works authorized by this act shall be paid from time to time by the Provincial Treasurer upon a certificate of the Minister of Lands and Forests stating that such sums have been applied for by the Commission, that they are necessary and that the payment thereof may be made in accordance with this act.

13.—A sinking fund sufficient to pay off the loan or loans authorized by this act shall be established. The annual instalments to form part of this fund as well as the interest accrued thereon shall be invested or deposited by the Provincial Treasurer under the direction of the Lieutenant-Governor in Council.

14.—The monies received for rent or other dues in respect of works constructed under the authority of this act shall be paid through the Minister of Lands and Forests into the Consolidated Revenue Fund to form part of the sinking fund until the loan or loans have been paid off in full.

15.—The Commission shall require from each tenderer a deposit in money or an accepted cheque, sufficient to ensure, to the satisfaction of the Commission, the due execution of the work. The contracts shall also contain a stipulation that a part of the contract price shall be retained for such time and upon such conditions as may seem necessary for the due carrying out of the work.

16.—No money shall be paid to a contractor until the engineer in chief has certified that the work in respect of which the monies are claimed has been properly done and that the sum claimed is due and payable in virtue of this act; and until such payment has been recommended by the Commission.

17.—The properties which the Commission may hold in virtue of this act are exempt from taxation, and shall be held in trust for the Government exclusively.

18.—The members of the Commission shall not be liable to be sued for acts done, carried out, or omitted in the performance of their duties under this act.

The Commission can be sued only with the consent of the Attorney-General.

19.—The Commission shall within the first fifteen days of each session, make a report to the Legislative Assembly of the work done under its control during the year next preceding.

20.—The Lieutenant-Governor in Council may make, amend, or repeal such regulations as he thinks necessary to carry out this act and as he thinks proper to make to provide for cases which may arise and for which no special provision has been made.

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Paper No. 1614

DEVELOPMENTS OF THE HYDRO-ELECTRIC POWER COMMISSION OF ONTARIO*

By Frederick A. Gaby,† M. Am. Soc. C. E.

Synopsis

This brief description of the power developments of the Hydro-Electric Power Commission of Ontario presents its general historical and economic relationship to the program of public service as expressed by the work of the Commission. This will lead to a more correct understanding of that work and will explain the continuity of policy that has actuated the Commission in obtaining and supplying electrical energy to meet the rapidly growing municipal, industrial, commercial, and domestic needs of the citizens of the Province of Ontario. An organization beginning in 1910 with a small initial load of less than 1 000 h.p. and, fifteen years later, distributing about 800 000 h.p. and operating twenty-three plants, which, when fully developed, will have a potentiality of more than 1 000 000 h.p., could not have progressed to its present magnitude and stability without following some well-defined policy.

The hydro-electric enterprise of the Province of Ontario has sometimes been referred to as a Government-owned undertaking; but the phrase, "Government ownership", as frequently used elsewhere—and so often in the United States—has a significance radically different from any conception that should be attached to the operations of the Hydro-Electric Power Commission of Ontario. Ontario's great hydro-electric undertaking is a co-operative municipal-ownership enterprise, operating through the agency of independent Commission control and administration, and is free from political influence.

Such public ownership, administered with honesty and for the circumstances existent in the Province of Ontario, is eminently satisfactory, and on

* Presented at the Fall Meeting, Montreal, Que., Canada, October 14, 1925.
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behalf of the citizens of the Province renders a service that is unequalled in any other comparable territory.

Administration

Broadly speaking, the Hydro-Electric Power Commission is entrusted with the duty of supplying the electrical needs of the citizens of Ontario municipalities at the lowest possible cost. At first, this end was achieved most efficiently by purchasing the power from existing sources of supply. Later, however, the source of supply itself was, in some instances, purchased outright by the Commission; while, sometimes, a number of generating plants have been purchased which together with an existing electric network have constituted the basis from which extensions have been made. Finally, the Commission has made its own hydro-electric developments, and is considering the desirability of obtaining a substantial quantity of electrical power by means of steam.

Financial Structure.—Certain principles which the inaugurators of this undertaking believed sound, and which have since proved to be so, were laid as a basis for administering the various assets in which the municipalities are concerned. The general system of administration may be summarized briefly as follows:

First.—The generation and transmission of power on a wholesale scale is managed by a Commission which, although appointed by the Government of the Province, acts independently in the capacity of trustee for the partnership of municipalities.

Second.—The local distribution of electrical energy within the borders of each municipality is, in general, under the administration of a Public Utilities Commission appointed under the provisions of the Public Utilities Act.

Third.—Capital required for plant to generate and transmit power is loaned by the Government on receipt of formal requisition from the Commission. Contracts are executed between the Commission and the municipalities under the terms of which the municipalities undertake to repay, over a period of thirty years, the monies thus loaned by the Government, with interest in full.

Fourth.—The local distribution system is financed by the issue of municipal debentures. Provision is made in the rates charged to the ultimate consumers for revenue with which to retire these bonds in from twenty to thirty years.

Fifth.—The Commission supplies power to the municipalities, charging each municipality the actual cost. To do this, an interim charge, based on the estimated cost, is made monthly and, at the end of each year, credit or debit adjustment is made of the amount charged in order to make up the actual total cost. The "cost of power" includes all the usual costs of operation and maintenance of the generating, transforming, and transmission plant and equipment, and, in addition, the annual interest charges on the monies borrowed for the initial cost of installation, also provision for renewal (depre-

ciation) and sinking fund reserves, as well as a special reserve fund for contingencies.

Sixth.—Each municipality sells electrical energy to its own local consumers at rates and under conditions approved by the Commission. The rates charged to its consumers by a municipality are made sufficient to take care both of the cost of distribution within the municipality, and of the estimated cost of power to be paid to the Commission by the municipality. The cost of distribution is ascertained in a manner identical with that used by the Commission in arriving at its wholesale costs.

Seventh.—Under the Power Commission Act, the Commission is required to determine, annually, the actual cost of service supplied to the municipal corporations by the local commission for such strictly municipal purposes as street lighting and the operation of water-works' pumps and street railways, and if any profit has accrued through the charging of the rate used throughout the year, this surplus is returned to the municipality.

The basic conception of the whole municipally-owned, electrical undertaking as administered by the Hydro-Electric Power Commission of Ontario is a partnership of municipalities formed to obtain power at cost, each municipality paying its proportion of the cost for the service received. The Commission, acting as agent and trustee for the municipalities, exercises both administrative and constructional functions, and by application of the principles adopted, has evolved a well-defined and successful working policy for the development, transmission, and distribution of hydro-electric power under municipal ownership.

Sources of Power Supply

In 1908 the Commission, in view of the vested interests in power developments at Niagara, secured its first electrical energy for supplying the municipalities of the Niagara System by contracting with the Ontario Power Company for a supply of electrical energy up to a maximum of 100 000 h.p., at a price of $9.40 per h.p.-year, until a load of 25 000 h.p. should be reached, after which the price would be $9.00 per h.p.-year. This 100 000 h.p. was completely utilized by 1915. The Commission arranged for additional power from other companies, and in 1917 purchased outright the Ontario Power Company with its plant capacity of more than 160 000 h.p. In 1920, the plant of the Toronto Power Company with a capacity of more than 125 000 h.p. was purchased. At present (1925), the Commission is completing its own Queenston-Chippawa plant with a capacity of about 550 000 h.p. All these provisions were made to supply power to the Niagara System.

From time to time various municipalities in districts adjacent to other sources of hydro-electrical energy requested the Commission to take action with a view to supplying their power needs. Thus, separate systems were formed in localities in general proximity to various sources of power supply.

Concurrently with its efforts to keep pace with the continually increasing demand for power on the Niagara System, the Commission found it necessary to construct power developments for many of the other systems, which were

experiencing corresponding growth in demand for electrical energy. These developments differ not only in physical characteristics, but also in their economic relationships to the various systems which they supply, as well as to the Commission's undertaking viewed in the large.

There is a tendency to reduce the number of separate systems by a process of consolidation, brought about by their expansion and the extension of their transmission networks to a point where they interlock with those of other systems. When this stage in their growth is reached, it becomes advantageous to connect the adjacent systems by tie-lines.

In some cases, special tie-lines have been run to transfer energy from one generating station to another. Thus, two or more systems became physically one from an operating standpoint. Finally, when this stage was reached, it was found advantageous to extend the consolidation to include the financial and administrative features. Both from the operating and administrative viewpoint the consolidations have effected substantial economies.

In other cases the Commission purchased the properties of companies operating within the confines of a system already established. For a time these properties were operated as separate entities, but as it became convenient they were absorbed into, and co-ordinated with, the larger systems. These consolidations also effected economies in operation and administration.

Systems Operated by the Commission

The systems operated (1925) by the Commission (Fig. 1) will be described briefly.

The Niagara System.—This embraces all the territory between Niagara Falls, Hamilton, and Toronto on the east, and Essex and Goderich on the west, as served with electrical energy generated at the several plants on the Niagara River. These include the plants formerly owned by the Ontario Power Company and the Toronto Power Company, also the new Queenston-Chippawa development. The area served by this system is about 13 500 sq. miles, with a population of 1 800 000. The load supplied is about 700 000 h.p.

Georgian Bay System.—This system, which is a consolidation of four systems previously existing—the Severn, Eugenia, Wasdells, and Muskoka Systems—serves that part of the Province of Ontario surrounding the southern end of Georgian Bay and to the north of the territory served by the Niagara System. It includes the district surrounding Lake Simcoe and extends as far north as Huntsville in the Lake of Bays District. It is served by five generating stations, the Wasdells and the Big Chute, on the Severn River; the Eugenia, on the Beaver River; and the South Falls and Hanna Chutes developments, on the Muskoka River. Of these, the Big Chute plant was purchased from the Simcoe Railway and Power Company and subsequently enlarged by the Commission. The South Falls plant was purchased from the Town of Gravenhurst and is also being enlarged, while the other three plants were constructed by the Commission. In addition, power is purchased from the Corporation of Orillia and from the Niagara System. The area served by the Georgian Bay System is about 8 900 sq. miles, with a population of 115 000. The load supplied is about 20 000 h.p.

MICHIGAN
LAKE HURON
GEORGIAN BAY
LAKE ONTARIO
LAKE ERIE
NEW YORK
Saginaw Bay
Goderich
Owen Sound
Georgian Bay System
Nottawasaga Bay
North Bay
Lake Nipissing
Nipissing
Nipissing System
Huntsville
South Falls
Gravenhurst
Wasdell's Falls
Durham
Eugenia
Lake Simcoe
Bradford
Tottenham
Central Ontario System
Stratford
St. Mary's
Kitchener
Guelph
York
Toronto
Cooksville
Niagara Preston System
Dundas
Brant
Hamilton
London
Woodstock
St. Thomas
Lake St. Clair
Essex
Kent
Niagara Falls
Campbellford
Ranney Falls
Picton
Kingston
Brockville
River St. Lawrence
St. Lawrence System
Cornwall
High Falls
Perth
Rideau System
Carleton Place
Smith's Falls
Merrickville
Ottawa
Ottawa System
Kemptville
Chesterville
PROVINCE OF QUEBEC
Ottawa River
MANITOBA
Lake Winnipeg
English R.
Albany R.
JAMES BAY
PROVINCE OF QUEBEC
Lake Nipigon
Lake of the Woods
Nipigon
Port Arthur
Thunder Bay System
LAKE SUPERIOR
Ottawa R.
North Bay
Nipissing
Georgian Bay
NORTHWESTERLY PART OF ONTARIO
HYDRO-ELECTRIC POWER COMMISSION OF ONTARIO
TRANSMISSION LINES AND STATIONS
SCALE IN MILES
0 10 20 30 40 50 60
LEGEND
High-Tension Lines
Proposed H-T. Lines
Low-Tension Lines
Proposed L-T. Lines
Generating Stations
High-Tension Stations
Low-Tension Stations
Proposed L-T. Stations

FIG. 1.

St. Lawrence System.—This serves the district immediately to the north of the St. Lawrence River between Brockville and Cornwall. The supply of power for the System is purchased under agreement from the Cedar Rapids Power Company which generates its power at Cedar Rapids on the St. Lawrence River, delivery being made at a point near Cornwall. The area served by this system is about 2 100 sq. miles, with a population of 28 000. The load supplied is about 6 000 h.p.

Rideau System.—This system serves the district in the vicinity of Smiths Falls, Perth, and Carleton Place. Power is available from two generating plants, one at Carleton Place and the other installed by the Commission at High Falls. Both plants are situated on the Mississippi River. The Commission also purchases power from the Rideau Power Company, of Merrickville. The area served by this system is about 1 500 sq. miles, with a population of 16 350. The load supplied is about 3 000 h.p.

Thunder Bay System.—This system serves the district at the head of the Great Lakes, including the Twin Cities of Port Arthur and Fort William, with power from the development at Cameron Falls on the Nipigon River. The area served by this system is about 4 000 sq. miles, with a population of 17 000. The load supplied is about 45 000 h.p.

Ottawa System.—This system serves the City of Ottawa, and an extensive rural power district adjacent thereto. Power is purchased from the Ottawa and Hull Power and Manufacturing Company under an agreement whereby 20 000 h.p. is provided. The present load is about 16 500 h.p. The population served is about 134 650.

Central Ontario and Trent System.—This system serves the district bordering the north shore of Lake Ontario between the territory on the west served by the Niagara and Georgian Bay Systems and that on the east served by the St. Lawrence and Rideau Systems. The nucleus of the Central Ontario and Trent System was the group of properties formerly controlled by the Electric Power Company, Limited, and operated by it through the agency of twenty-two subsidiary companies. All these properties were purchased by the Province of Ontario on March 1, 1916. Since June 1, 1916, they have been operated by the Commission as Trustee for the Province and the system has been greatly enlarged to meet the constantly growing needs of the district.

The power supply for the Central Ontario and Trent System is obtained from nine power developments situated on the Trent and Otonabee Rivers, which have been made in conjunction with dams required for navigation purposes. Two of these are new developments now (1925) under construction at Dams Nos. 8 and 9. The area served by this system is about 4 700 sq. miles, with a population of 145 000. The load supplied is about 45 000 h.p.

Nipissing System.—The Nipissing System comprises the Town of North Bay and certain small municipalities south of Lake Nipissing. It was purchased together with the Central Ontario System by the Province in 1916 and has since been operated by the Commission. It is supplied with power from two hydro-electric developments on the South River at Nipissing and Bingham Chute. The area served by this system is about 700 sq. miles, with a population of 13 500. The load supplied is about 2 750 h.p.

Plants Constructed by the Commission

In addition to the systems purchased several noteworthy hydro-electric developments have actually been constructed by the Commission. Early in 1912, on the application of a number of municipalities in Ontario County situated on the east side of Lake Simcoe, the Commission made a thorough survey of the available water powers in the district, and recommended a site at Wasdells Falls (Fig. 2), on the Severn River, about three miles below Lake Couchiching, as being the most economical for power development. This marked a new step in the progress of the policy of the Commission in that it was the first development constructed by it on behalf of municipalities. Work was commenced in July, 1913, and the project was completed in September, 1914.

Wasdells Development

The detailed hydraulic investigations showed the available head to vary considerably, owing principally to back-water. However, taking into consideration the effect of forebay levels, the maximum and minimum heads available after construction were determined to be approximately 15 and 9 ft.,

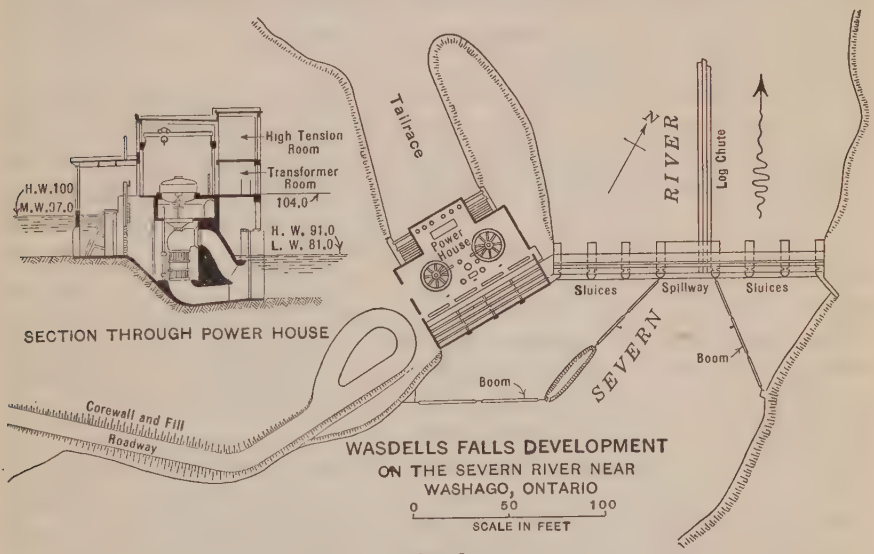

Fig. 2.

respectively. Except for difficulties encountered due to this extremely low head, all the inherent features of the site were favorable for development. The natural formation is solid granite, affording an excellent foundation for the superstructure.

The dam substructure and superstructure are constructed of mass and reinforced concrete, and offer no special features of design. The two main turbines are of the vertical double-runner type in open-flume settings, and each runner is provided with a separate draft-tube. The units are designed to operate normally at a speed of 90 rev. per min. under a head of 12 ft., with

a guaranteed capacity at three-quarters gate of 600 h.p. When operating under a 9-ft. head at full gate, the guaranteed capacity is 500 h.p. The guaranteed efficiencies vary from 75 to 85%, for gate openings of from 50 to 100 per cent. The main generators are of the vertical, 3-phase, 60-cycle type, and each is of 400 kv-a. capacity with the usual guaranties and specifications.

Eugenia Development

The second development undertaken by the Commission, situated on the escarpment near Georgian Bay, in the County of Gray, and designed to serve the municipalities in the Owen Sound-Durham District, was commenced in July, 1914, the initial development being completed in November, 1915. The unique features of this plant, known as the Eugenia Development, are the arrangements for maximum economic utilization of the run-off from the drainage area, and the high head under which reaction turbines are used.

The development consists essentially of a storage dam on the Beaver River, about ½ mile above the Eugenia Falls, forming a reservoir, a canal, 5 000 ft. long, a forebay or settling basin, two wood-stave pipes and steel penstocks, and the power-house. The gross head under which the plant operates is 552 ft., of which 50 ft. is obtained from the storage dam and the remainder from the natural fall of the river.

The drainage area is 74 sq. miles, much of which is tamarack and cedar swamp; the run-off is remarkably constant, due partly to this fact, and partly to the geological formation. The escarpment is Lockport dolomite underlying Cataract limestone, the whole overlaid with thick beds of morainal boulders, gravel, and clay, and this top covering forms a vast natural equalizing reservoir. The rainfall on the drainage area (about 39 in. per year) is above the normal for Southwestern Ontario, since the water-shed lies on the high plateau between Georgian Bay and Lake Ontario. The storage provided at the dam, together with the natural regulation of the stream, will furnish a run-off of about 1 sec-ft. per sq. mile throughout the year. As recorded from 1920 to 1925, the maximum flood run-off is only about 7 sec-ft. per sq. mile, while the minimum is 0.27 sec-ft. per sq. mile.

The main storage dam, or Dam No. 1, is a reinforced concrete structure of the Ambursen type. Its total length is approximately 1 900 ft., of which 1 260 ft. is reinforced concrete and 640 ft. is earth embankment with concrete core-wall. The reinforced section is approximately 51 ft. high from the bottom of the cut-off to the top of the crest at the highest point. The crest of the spillway is at Elevation 646, and its length is 103 ft. The maximum to which the water is raised is Elevation 649, at which point an area of about 1 650 acres is flooded. This upper 3 ft. is controlled by flash-boards, and the capacity gained by this means is 190 000 000 cu. ft., while the total storage capacity of the dam is 780 000 000 cu. ft., or about one-third of the total flow of the river in a normal year.

The canal connecting the reservoir with the head-works is constructed as an open trench protected by a wooden box-flume at points subject to earth slides. At its lower end a natural basin was made by placing an earth-fill

dam across an opening in the valley. The length of this dam is about 800 ft., its maximum height, 30 ft., and its width, 10 ft. at the top, with slopes of 1 on 3 on the water side and 1 on 2 on the lower side. The two wood-stave pipe lines are 46 in. in diameter and about 3 350 ft. in length, ending in surge tanks located at the brow of the escarpment above the power-house.

The wood-stave and steel pipe lines are joined by head-blocks, from which risers, 40 in. in diameter, connect with surge tanks, which are designed to handle without spilling a complete shut-down of the plant when all units are operating.

The steel penstocks are 52 in. in diameter, 1 557 ft. in length, and vary in thickness from $\frac{5}{16}$ in. at the upper end to $\frac{27}{32}$ in. at the lower end. The girth seams at the top are single-riveted and at the lower end double-riveted; the longitudinal seams are triple-riveted, double-butt strap joints, with rivets varying from $\frac{3}{4}$ in. to $1\frac{1}{4}$ in. The penstock rests on concrete saddles 20 ft. apart, and rides on steel channels bent to the outer radius of the pipe. Four concrete anchors are placed at changes in grade below the head-blocks; at the ends of the first three anchors are expansion joints of the stuffing-box type. The penstock is housed for about one-half its length as protection against frost.

The penstocks supply three spiral-casing, single-runner turbines, two of 2 250 h.p. capacity, at 540-ft. head, operating at 900 rev. per min., and one of 4 000 h.p. capacity.

Two of the generators are rated at 1 411 kv-a. and one at 2 800 kv-a., all being 3-phase, 60-cycle, and 3 800-volt. Based on the machine ratings the capacity of the whole development is 7 500 h.p.

Queenston-Chippawa Development

General Description.—The most important development that the Commission has made is the Queenston-Chippawa Project on the Niagara River. As already mentioned, the supply of power made available by purchase from the Ontario Power Company at Niagara was fully in use, and the Commission was faced with the menace of an oncoming power shortage. This shortage materialized about 1918, and was pronounced until relieved in 1922 from this new plant.

To meet the growing demand for electrical energy for industrial purposes, the Commission, in 1917, commenced the Queenston-Chippawa Development. This plant was designed originally to utilize the surplus water that was available from Canada's allotment of 36 000 cu. ft. per sec. under the Boundary Waters Treaty, but when the Commission purchased the Niagara plants of the Ontario Power Company and the Electrical Development Company the size of the projected new Queenston-Chippawa plant was increased from less than 200 000 h.p. to about 450 000 h.p., and eventually to about 550 000 h.p. This enlargement was possible because a greater quantity of water became available for use under much more efficient conditions; that is, in the Queenston-Chippawa plant the available water could be used at nearly double the head.

The general scheme of development (Fig. 3) comprises an intake structure in the Niagara River at Chippawa; the deepening of the Welland River for a distance of 4½ miles; the construction of a canal, 8½ miles long, from Montrose to the forebay and screen-house, which are located on the cliff about 1 mile south of the Village of Queenston, and the construction of the power-house in the Niagara Gorge.

Throughout the period of preliminary study and, later, as the design progressed, continuous use was made of models of the various structures in order that the mathematical analyses might be augmented by actual demonstrations under the assumed conditions. Such models were made for studying the intake, the bends in the canal, the transitions (that is, points where change of section occurred in the canal), the diffuser at the mouth of the forebay, and the draft-tubes and station substructure. It is believed that the beneficial results of such studies and of the care taken in the design of what are often considered minor elements of a power development, have been demonstrated by the results obtained. The tests already made indicate conclusively that the over-all efficiency secured is at least as high as has ever before been obtained.

True conservation of the waters of the Niagara River for power purposes demands that practically the whole fall of about 327 ft. between Lake Erie and Lake Ontario be utilized. The various power plants now operating at Niagara Falls utilize heads of from 130 to 210 ft. only, and with widely different degrees of efficiency. The Queenston-Chippawa power development, the first unit of which was placed in operation during December, 1921, will have a normal operating head varying from 294 to 305 ft. when the installation is complete. The conservation of head effected by the reduction of hydraulic losses to a minimum (Fig. 3), and by refinements in the design of the various essential elements of the project as a whole, has resulted in a power development which is believed to represent the best in modern engineering practice.

The present installation will consist of nine 55 000 to 65 000 h.p. turbine units operating at 187.5 rev. per min. The generators are of 45 000 to 55 000 kv-a. capacity at 12 kv., 3 phase, 25 cycles, with an ultimate aggregate capacity of from 450 000 to 550 000 h.p., depending on the water levels. The electrical energy is stepped up from 12 to 110 kv. for transmission.

Intake.—One of the great obstacles to the continuity of service is the entry of ice into the canal. Great fields of ice, formed in Lake Erie with its shallow bays and sloping shores, are discharged down the Niagara River every spring, and at frequent intervals during the winter. The river itself never freezes over, but considerable anchor and frazil ice are formed at times of low temperature.

The site of the intake of the Queenston-Chippawa power development, at the mouth of the Welland River, is favorable in that floating ice does not ordinarily follow the shore lines at this point; but the smooth gradient of the river surface, and the comparatively shallow water with its low velocity, are unfavorable to the proper handling of ice.

The removal of water in large quantities from a river heavily charged with ice is always a difficult problem, but is greatly simplified when a natural break

FIG. 3.—BIRD'S-EYE VIEW OF THE QUEENSTON-CHIPPAWA CANAL.

in the river surface, accompanied by a sudden drop, gives a source of power for the separation of floating ice, and for its continuous disposal. The use of a horizontal diaphragm to skim the surface water, with its burden of ice, from the lower strata, thus permitting the upper water layer to be accelerated and removed clear of the intake without objectionable eddies, while the lower layer, free of ice, is changed in direction and flows through the intake into its new channel, gives a positive and satisfactory solution.

When, as in the present case at Niagara, the natural conditions do not permit such an arrangement, radically different measures must be taken. To confirm certain ideas developed as a result of many years of experience and observation by the engineers of the Commission in connection with the plant operating on the Niagara River, an extended series of tests and experiments on large-sized models was made. These models duplicated to scale the topographical conditions at the site of the intake and resulted in a design which, it is expected, will operate in such a manner as to keep the plant free of this ice menace.

The complete intake structure (Fig. 4) is about 1 100 ft. in length and is made up of an entrance with lock-gates for navigation, a bulkhead section, and the intake proper, the latter combining two forms of intake. The conventional or surface intake consists of a concrete barrier or boom with fifteen openings each 18 ft. wide, having normally 8 ft. of submergence, which, however, by means of drop-gates can be increased to any amount up to the full depth of water, or 35 ft. A submerged intake—which has not yet been constructed—as designed consists of six gathering tubes or draft distributors, aggregating 675 ft. in length. Water will enter the tubes on the up-stream side through slots along a distance of 500 ft. (Fig. 4, Section *A-A*). These tubes will be controlled by gates similar to those on the surface intake—air tubes comprise an outer tapering section wherein the velocity is maintained constant, with a longer inner section 20 ft. in diameter, wherein the velocity increases regularly with respect to the distance along its axis. Diffuser sections are placed at the inner end to reduce the velocity, with as little loss as possible, to correspond with that in the Welland River section. These diffuser sections have already been constructed as part of the intake works.

The submerged tube section of the intake was designed as an ultimate protection against ice, and its installation will be undertaken if it is found that the existing construction proves inadequate; to date (1925), it has satisfactorily taken care of ice diversion.

The Canal.—For many miles above its mouth the Welland River is a sluggish stream, meandering slightly in a depression that can hardly be called a valley. For 4½ miles this stream forms the first reach of the canal (Fig. 3), and its low banks provided a suitable disposal area for much of the material excavated in the process of straightening and deepening the channel. Leaving the river channel near the crossing of the Michigan Central Railway, the canal takes a course almost due north for more than 3 miles. The earth overburden is quite heavy on this stretch, the bottom grade of the earth section of the canal meeting the rock surface 1 mile from the Welland River. The

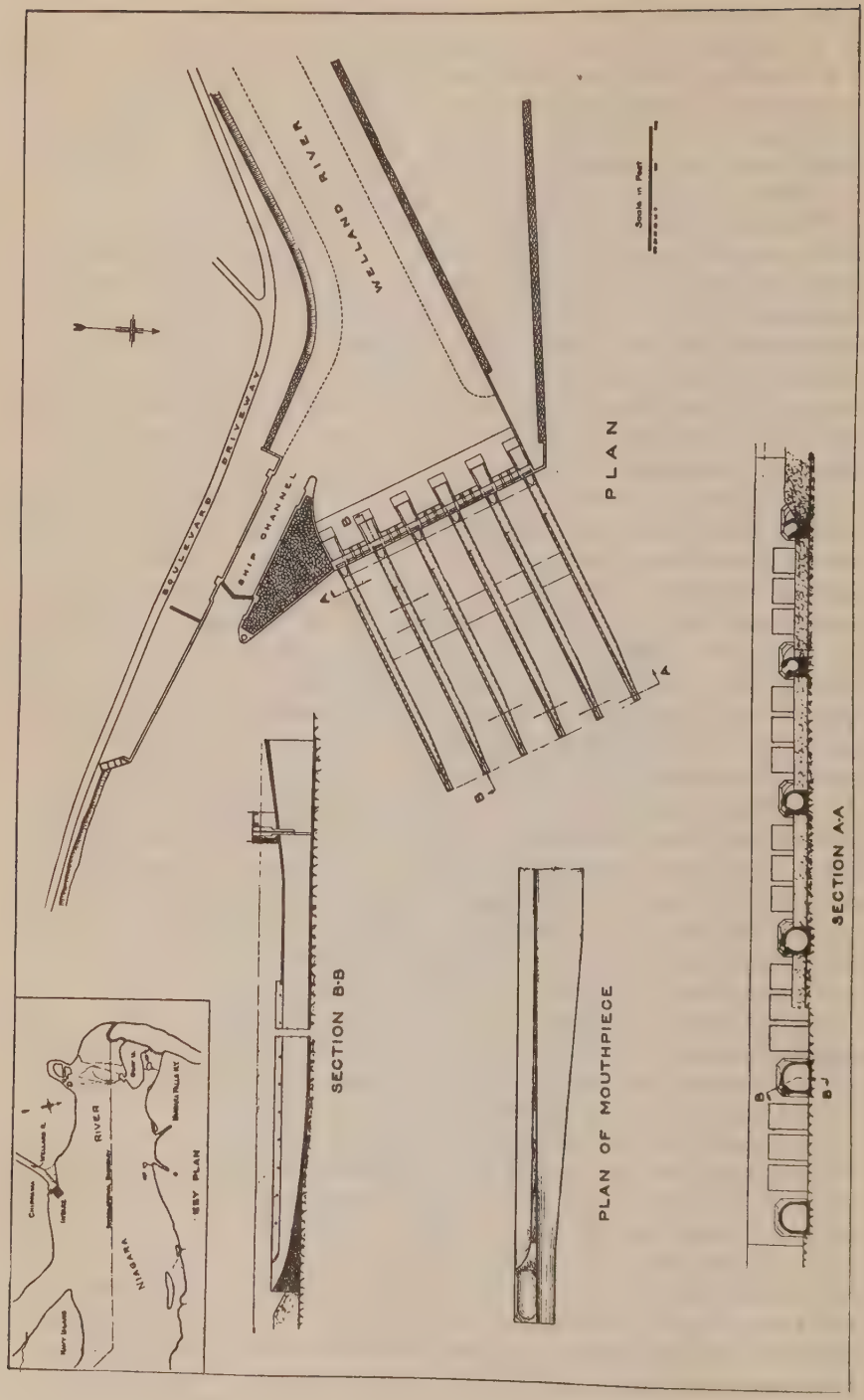

FIG. 4.—PLANS OF INTAKE STRUCTURE, QUEENSTON-CHIPPAWA DEVELOPMENT.

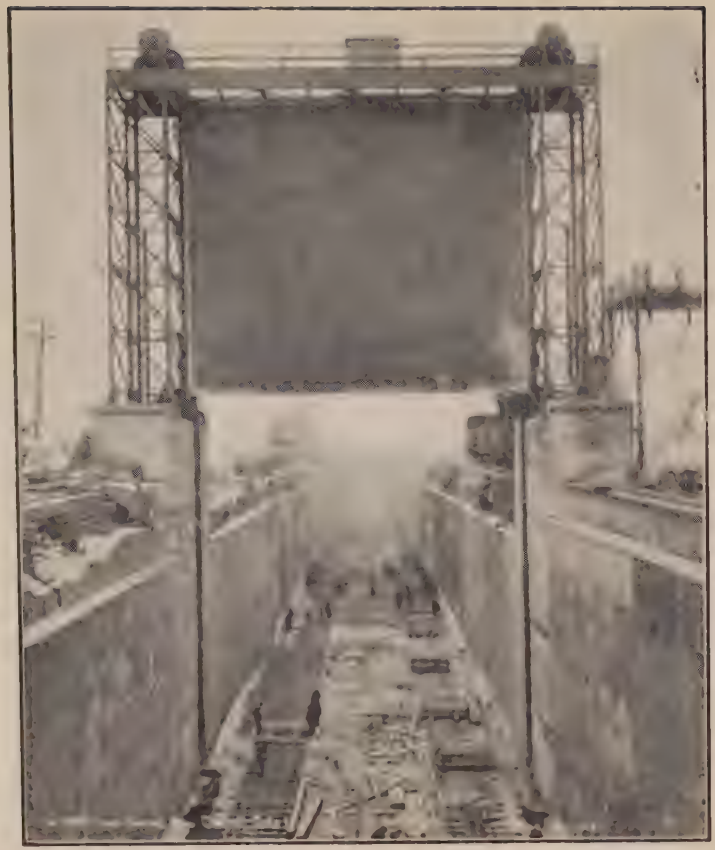

Fig. 5.—View of Electrically-Operated Roller Sluice-Gate, Queenston Plant.

Fig. 6.—Cross-Section of Canal, Whirlpool Section, Queenston-Chippawa Development.

maximum rock elevation is not coincident with the maximum earth surface elevation, but the profiles of rock and earth surface are roughly parallel to each other. The five bends in the canal vary in curvature from 27 to 51 degrees.

One depression that is crossed (Bowman's Ravine) is apparently an old river channel through which, in pre-glacial times, the Niagara River flowed toward Lake Ontario. At the time the construction started this old channel was the course of a small stream having its outlet at the Whirlpool. The ravine crossing is made on a fill and the valley itself proved a convenient disposal area for about 1 500 000 cu. yd. of excavated material.

The Design of the Earth and Rock Sections.—Long-continued investigations were made to determine factors of roughness for large canals in earth and rock, with and without concrete lining. One of the conclusions reached was that the Kutter formula should be used. The roughness factors used in the hydraulic studies were 0.035 for the river section and 0.012 for the concrete-lined rock section.

Control Works.—An electrically-operated roller sluice-gate (Fig. 5) of 48 ft. clear span—the full width of the rock section—is located at Station 97 + 00, near the beginning of the rock section. The use of two gates with a central pier was considered, but the single gate was found to be the more advantageous, as it provided an unobstructed waterway with a consequent reduction in friction losses. The gate, which is supported on steel towers with a concrete substructure, weighs about 100 tons, and is provided with two hoisting mechanisms and two counterweights. When the gate is at the top of its run it is high enough above the water surface in the canal to permit a tug to pass beneath.

Whirlpool Section.—Bowman's Ravine, situated west of the Whirlpool, was crossed on a rock-fill, the cross-section of the canal being changed from a rectangular section 48 ft. in width to a trapezoidal section with a bottom width of 10 ft. and side slopes of 1 on 1½ (Fig. 6). This was designed to give as great a cross-sectional area as the rock section has at the extreme minimum water level. The Whirlpool Section is lined with reinforced concrete. In order to withstand the pressure of the back-water when the canal is emptied, the lining is provided with vents of sufficient size to drain the fill as quickly as the water can be drawn down in the canal.

Concrete Lining.—Economic considerations prompted the lining of the rock section of the canal with concrete (see Figs. 5 and 6). The height of the lining was fixed slightly lower than the profile of the water surface existing when the load conditions on the plant are at a maximum and the flow in the Niagara River is at a minimum. Thus at all times the lining will be protected by submergence against the action of frost. The thickness of the lining varies according to the rock over-break, but averages about 20 in. Where necessary, steel dowels were used to anchor the concrete lining to the rock.

Extreme smoothness of surface is not the only determining factor, precise alignment being also a most important element in the reduction of hydraulic losses. Great care was taken to obtain a smooth surface by the use of steel

forms and for this purpose a positive and rigid method of form-setting was devised. This insured almost perfect alignment.

Screen House.—At the lower end of the forebay, serving as a dam, is the screen house (Figs. 7, 8, and 9). This structure forms the entrance, and the control, for the penstocks. The entrance to each of the main penstocks is a modified bellmouth consisting of three openings, each 12 ft. 8 in. wide and 29 ft. high, at the rack supports. These three openings gradually converge into one opening, 16 ft. in diameter, at the point of connection to the penstocks. In designing these water passages, particular care was taken to secure smooth stream lines and consistent changes in velocity. The bellmouth entrances are sealed by a concrete curtain-wall extending down to Elevation 542.0, which gives a depth of 28 ft. above the floor of the forebay. Immediately behind the curtain-wall, steel-lined gate-checks are provided to support structural steel gates. These gates provide a means of unwatering in case it is necessary to inspect the lower sections of the racks, or the penstocks. The intake is divided into three waterways to reduce the spans for the gates to a convenient size and also to permit the use of racks of a somewhat new design. The entire rack structure is removable and is split horizontally into two sections for convenience in handling. A specially designed rack follower with an automatic latch arrangement facilitates the removal of the racks.

The screen house, as constructed, provides for nine main units, a service unit, and an ice chute, and is arranged so that a further unit entrance may be added at the north end. It is near the edge of the escarpment, only a narrow ledge of rock being left between it and the gorge. Owing to the disastrous results that would follow a failure, the screen house substructure was designed to resist the full head exerted by water in the forebay.

Penstocks.—From the screen house, the water is carried to the turbines in steel penstocks (Fig. 9). The first five main unit penstocks are 16 ft. in diameter for approximately two-thirds of their length, and then reduce by a taper section to a diameter of 14 ft. The remaining penstocks are 16 ft. in diameter for their complete length. The penstocks have bends only at the top and at the bottom. These elbows are held in massive concrete anchor-blocks, the one at the upper bend forming a foundation for the piers supporting the sidewalk and roadway extending along the edge of the escarpment (Figs. 8 and 9).

Each penstock ring consists of two plates with longitudinal joints on the horizontal center line. The joints are all double butt, varying from double riveted at the top to quadruple riveted at the lower end. The circumferential joints are single butt, double riveted, with the butt strap on the outside. The longitudinal joints are caulked on the inside, but the circumferential joints are made water-tight by electric welding. This type of circumferential joint gives a much better alignment to the inside of the pipe than can be obtained with the usual outside and inside course with lap joints. The plates vary in thickness from $\frac{1}{2}$ in. at the top section to $1\frac{1}{4}$ in. at the lower section. In the erection of the penstock, an innovation was tried, namely, the use of electric rivet heaters. Each penstock is covered throughout its length with

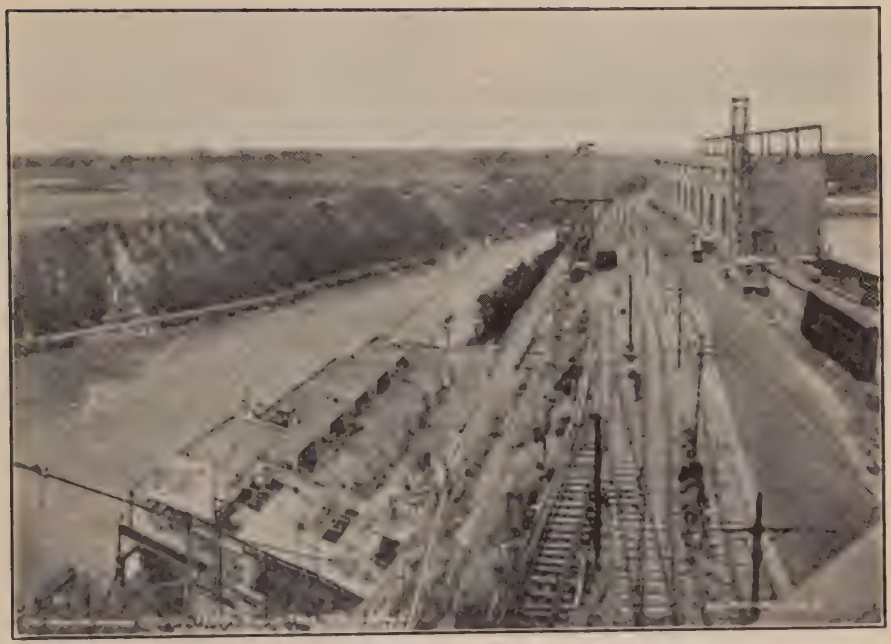

Fig. 7.—Power Plant at Queenston from Level of Head-House.

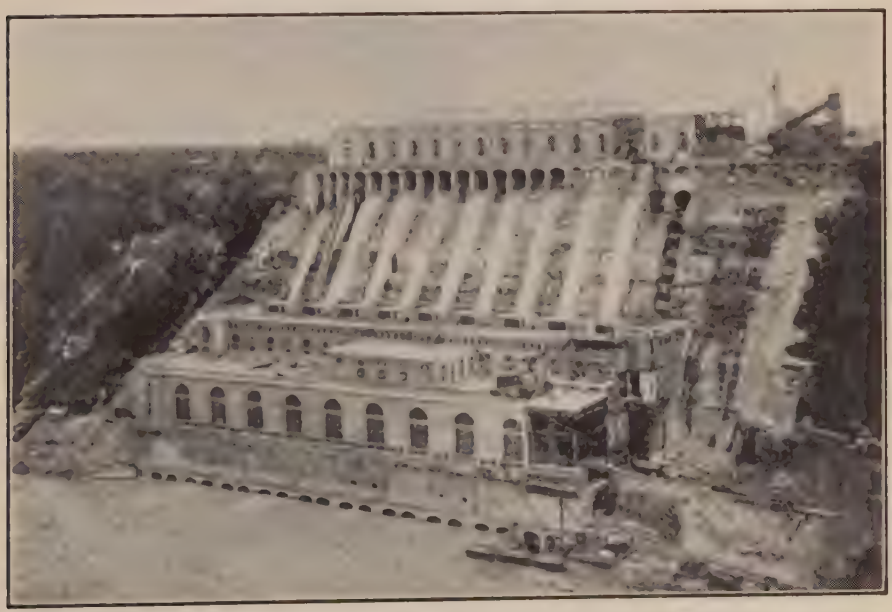

Fig. 8.—View of Plant at Queenston from Cliff on East Side of Gorge.

a concrete envelope having a minimum thickness of 24 in., a protection which will prolong the life of the steel.

Generating and Transformer Station.—The generating and transformer station is situated below the escarpment and close to the river's edge, extending about one-half the distance to the top of the escarpment (Figs. 7, 8, and 9). The structure required to house nine main units and the service equipment is 650 ft. long. The substructure is of massive concrete carried down to rock foundation. The superstructure consists of a structural steel framework with reinforced concrete floors and roofs, and concrete, brick, and tile walls and partitions.

Turbines and Generators.—Each turbine has a capacity of from 55 000 to 65 000 b.h.p. under a 305-ft. head, at 187.5 rev. per min. The draft-tube for Unit No. 1 is of the common curved type modified at the elbow, whereas each of the other units is equipped with a Moody spreading tube.

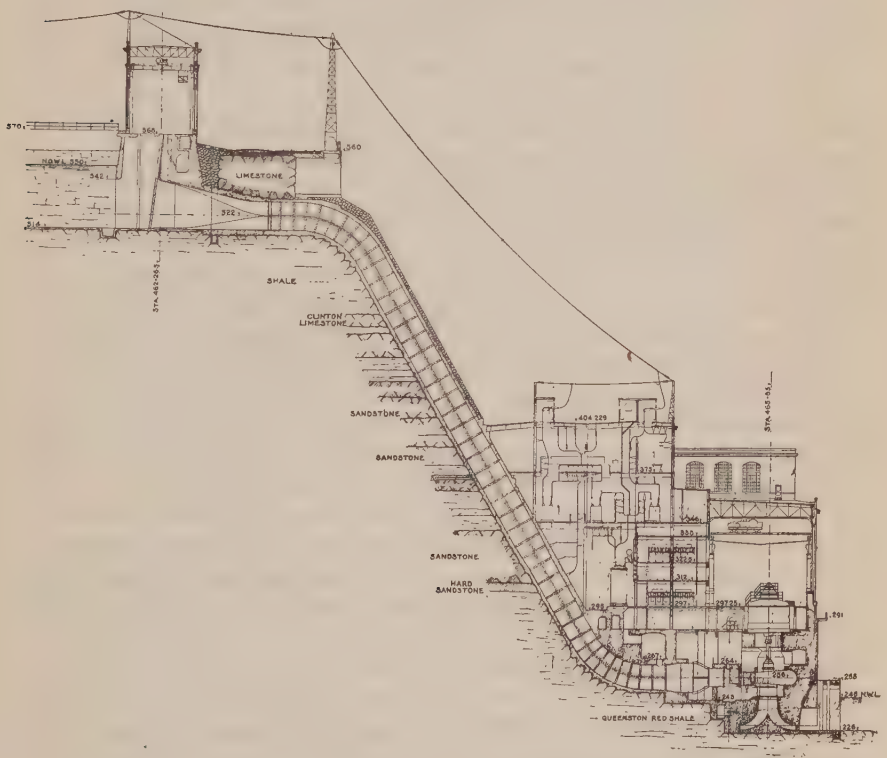

Fig. 9.—Cross-Section of Steel Penstocks, Queenston-Chippawa Plant.

Generators.—Each of the present units is rated at 45 000 kv-a., 80% power factor, 12 000 volts, 3 phase, 25 cycles, at 187.5 rev. per min. They are capable of being operated continuously at 49 500 kv-a., with either voltage or current 10% in excess of the rated values. The units are vertical (Fig. 9), with direct-connected shunt-field commutating-poles, and 250-volt, 150-kw. exciters.

The over-all efficiency of the generators is slightly in excess of 97% at a power factor of 80 per cent. The thrust bearing is designed to support a load of 1 000 000 lb., which is slightly in excess of the weight of the rotor plus the hydraulic thrust imposed by the turbine. Upper and lower guide-bearings are provided, the latter on account of the length of the shaft and in order to make the generator a self-contained unit.

Gratifying results were obtained in testing the turbine and generator units. An efficiency test of Unit No. 7 at the Queenston generating station of the Commission was made on March 21 and 22, 1925. The electrical measurements were made by carefully calibrated electrical instruments. Sufficient readings were taken to compute the generator losses and the turbine output was taken as the sum of the generator output plus generator losses. The measurements of water consumption were made by the Gibson pressure-time process, a detailed description of which has been published.*

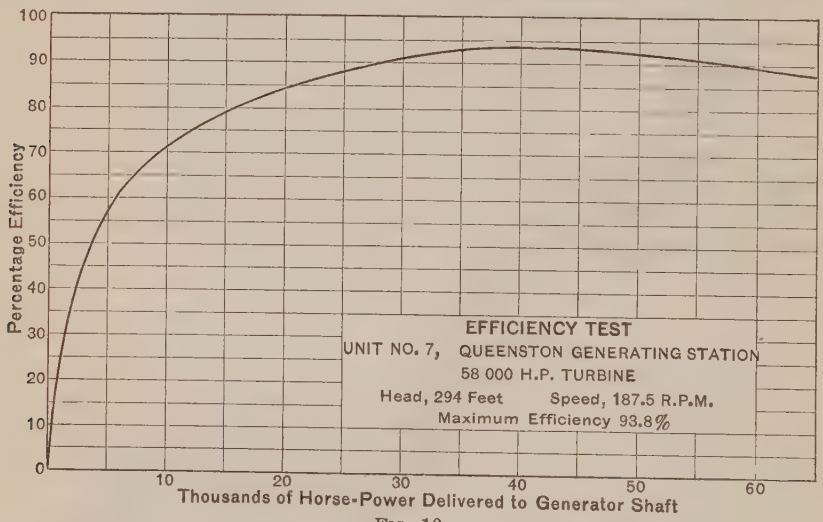

FIG. 10.

The outstanding points in the results of the test (Fig. 10) are as follows:

1.—A maximum turbine efficiency of 93.8% at 55% of full gate-opening and 42 210 h.p. turbine output. This is probably the highest efficiency yet attained.

2.—A turbine efficiency of 93%, or greater, for outputs between 36 000 and 49 000 h.p.

3.—A turbine efficiency of 90%, or greater, for outputs between 28 000 and 59 000 h.p.

4.—A maximum turbine output, at full gate and at 294-ft. head, of 64 800 h.p.

Nipigon

History.—At the head of the Great Lakes lies the Thunder Bay System of the Commission. In this territory are the important cities of Port Arthur

* *Transactions*, Am. Soc. Mech. Engrs., Vol. 45, 1923.

and Fort William. Tributary to these communities is an extensive territory rich in natural resources, the development of which only awaited the furnishing of adequate supplies of electrical energy. The district possessed valuable undeveloped water powers.

As early as 1901, the City of Port Arthur had embarked in the business of municipal power development by constructing a plant on the Current River. The growing demand for power, however, soon made it necessary to provide for further supply. The results of the activities of the various municipalities in the Niagara District prior to 1906 and the creation of the Commission inspired the city officials of Port Arthur to seek a solution of its power problems along lines similar to those adopted by Niagara municipalities. Subsequently, the Commission purchased power from a power company operating in the district and, later, when this source of supply was beginning to prove inadequate, the municipalities of Fort William and Port Arthur entered into agreements with the Commission for additional power, with the result that the Commission made a thorough canvass of possible new sources of power supply.

The large quantities of power available from the Nipigon River, if developed, would permit the conversion of great supplies of pulpwood into paper at satisfactory costs and the opening up of the mineral and other resources of the Thunder Bay District. In view of this, and as a result of the investigation, Cameron Falls on the Nipigon River was selected as the site of operations, and construction was started in 1918.

Hydraulic Data.—The elevation of Lake Nipigon is 850, and that of Lake Superior is 602, giving a fall of 248 ft. between the two lakes, along the Nipigon River, in a length of about 32 miles. There are four main power sites on the river, the upper two of which may possibly be concentrated at Pine Portage, where a head of slightly more than 100 ft. could be secured. At Cameron Falls the head is 78 ft., the head-water level extending about 12 miles back to Pine Portage. About 2 miles down stream from Cameron Falls, at Camp Alexander, is the remaining power site, where a head of 60 ft. is available between the tail-water of the Cameron Falls plant and the level above Lake Helen at Elevation 606.0.

The area of Lake Nipigon is 1 530 sq. miles and that of its drainage basin 9 125 sq. miles. The discharge from Lake Nipigon is controlled by a ridge at the outlet about 150 ft. wide, marking the beginning of the Nipigon River. As would be expected with such a large controlling reservoir, the river has a remarkably uniform regimen, the flow varying from a minimum of about 4 000 cu. ft. per sec. to a maximum of 18 000 cu. ft. per sec. From records taken to date the mean flow is approximately 6 500 cu. ft. per sec.

By the erection of a concrete control dam, now almost completed, at the outlet of the lake, the total run-off can be stored in the lake and used as required to meet the variable seasonal load on the plant. The capacity of this huge reservoir for each foot of depth is equivalent to a flow of 16 000 cu. ft. per sec. for 1 month, so that a variation of 4 ft. is sufficient to give complete regulation of the flow; this variation can be obtained between the natural high and low-water levels.

Cameron Falls Site.—The regulated flow of 6 500 cu. ft. per sec., under a 78-ft. head, at Cameron Falls, with a 60% load factor, gives a peak capacity of 75 000 h.p. The plant has been laid out for this installation. As it is situated at the lower end of Lake Jessie, which is merely an expansion of the Nipigon River that floods back through Lake Maria to Pine Portage, a distance of 12 miles, a large still reservoir is created. This area freezes over in the early winter, and, by thus providing an ice cover until the spring break-up, prevents the formation of the troublesome frazil and anchor ice. This head pond has an area of more than 4 sq. miles, and gives sufficient storage to provide for the weekly load factor on the plant, with a variation in the head-water level of less than 3 ft.

At the location chosen, Nature has provided a convenient site where the power works may be concentrated within a small area. The power plant is made possible by the construction of a dam across the Nipigon River, and the excavation of short head-race and tail-race channels across the point of land formed by a bend in the river.

Power-House.—The main dam is a concrete spillway structure, provided with eight sluice-ways. Regulation is obtained by stop-logs, and wing-walls on either end extend the crest to the required contour level. The stop-logs in the sluice-ways are raised or lowered by an electrically-operated spud-winch, which moves along an operating deck over the top of the spillways. The total length of the dam is approximately 300 ft., and the maximum height in the center is 67 ft. In addition to the main dam, five auxiliary earth-fill dams and one timber crib dam were required.

The forebay occupies an opening in the natural rock walls of the river approximately 300 ft. long by 263 ft. wide. The power-house, containing six 12 500–h.p. units, is of the concentrated type of reinforced concrete construction in which the head-works, supply pipes, and power-house all form part of the same structure. The inlet for each unit consists of three sections 35 ft. high to the under side of the curtain-wall, by 10 ft. wide. Integral steel racks in sections to permit easy handling are installed in these openings, inside the head-works; through them the maximum velocity is approximately 2 ft. per sec. Behind the racks are the head-gates, three gates for each unit, the two outside ones being of the sliding type and the middle one a free roller gate which acts as a by-pass in filling or emptying the supply pipes. Each gate is 11 ft. wide by 18 ft. high. The sliding gates have bronze bearing strips, while the roller gates are of the Stoney type and can be opened against full water pressure. Ahead of the racks and head-gates, stop-log checks are provided for convenience in unwatering the racks or gates, should this be found necessary. It was considered essential in this climate to house the racks and gates in a building that could be heated in cold weather. It is due to this feature, in combination with the large ice-covered forebay, and with the low velocity, which latter not only prevents ice being drawn down under the curtain-wall, but also gives a negligible loss through the racks, that freedom from the operating difficulties usually experienced by power plants in the more northern latitudes has been found, even under severe climatic conditions.

The supply pipe for each unit is composed of three rectangular sections each 10 by 13 ft. The distance from the head-gates to the center line of the units is 86 ft. The supply pipes, which are of concrete, are reinforced for internal water pressure, and for loads from above when empty. The turbine scroll cases are also of reinforced concrete. Although the reinforcement required for this head was necessarily heavy, no difficulty was experienced either in the design or the construction, and by proper care at construction joints, the pipes are remarkably free from leakage. Ample drainage is provided in the turbine case to handle the leakage from the head-gates. The units are spaced at 45-ft. centers. The draft-tubes are of the elbow type, the upper section having a steel-plate liner.

The tail-race, about 1 000 ft. long by 125 ft. wide at the water line, is an open cut across to the smooth water in the river below the falls. The upper section near the power-house is in rock and the lower section in earth, a rock-filled timber crib protecting the down-stream end from débris carried down by flood water.

Turbines.—The turbines are of the Francis, single-runner, vertical type, rated at 12 500 h.p. under a 72-ft. head, and operate at a speed of 120 rev. per min. The guide-bearings for Units Nos. 1 to 4 are of the Lignum-vitæ, water-lubricated type, and those for Units Nos. 5 and 6 of the oil-pressure, babbitted type. The governors are supplied with pressure from a central pumping plant having duplicate centrifugal pumping units with capacities of 650 gal. per min. The water used for the operating fluid, containing a small quantity of potassium bichromate, has proven quite satisfactory.

Construction.—Due to the location of the development considerable work of a preliminary nature was found necessary. This included the construction of a temporary power plant, of about 1 mile of railway, including a bridge across the Nipigon River, as well as of various camp buildings. The temporary power plant consists of two turbines obtained from an abandoned plant, which were utilized to drive one 350-kw. generator and three 300-h.p. air compressors.

The Nipigon River is justly famous for its trout fishing, and to help preserve this sport a fish-ladder has been provided around the west end of the power-house. This is of timber construction, about 400 ft. long. The design of the ladder was approved by the Provincial Game and Fisheries Department, and, it is believed, provides a satisfactory passage for the fish in their annual up-stream migration.

Central Ontario System

The Commission, in compliance with applications from municipalities in what is known as the Central Ontario System, negotiated the purchase of the Electric Power Company's system operating on the Trent River, including six generating plants with a total capacity of about 27 000 h.p. To take care of the continued increase in demand for electrical energy in this district the Commission has since constructed additional developments.

The first of these new plants was located at what is called Dam No. 10, or Ranney Falls, on the Trent Canal, near Campbellford (Fig. 11). In the construction of the Trent Canal, below Dam No. 10, the Dominion Government

installed intake sluices in the canal wall immediately above Locks 11 and 12. It is through these sluices that the water is obtained for operating the Ranney Falls plant.

An open forebay, 124 ft. long, connects the sluices with the power-house. The regulated canal level at this point is 477.0 (sea-level datum). From the power-house, the tail-race, 30 ft. wide, extends 275 ft. to the Trent River, the level of which at its junction with the tail-race is 429.0. The forebay is 116 ft. wide by 12 ft. deep, at the intake. The side-walls converge, and the forebay increases in depth, until at the gate-house the dimensions are 71 ft. wide by 24 ft. deep. This design is conducive to low velocity at the racks.

The gate-house substructure (Fig. 12) is a block of reinforced concrete, 84 ft. by 30 ft. in plan. There are four entry chambers, two for each unit, 14 ft. wide. The racks are at the head of these chambers, and are built of structural steel in two sections. In the center pier of the gate-house a 5 by 4-ft. chute is provided to carry off any ice or other floating material from the forebay.

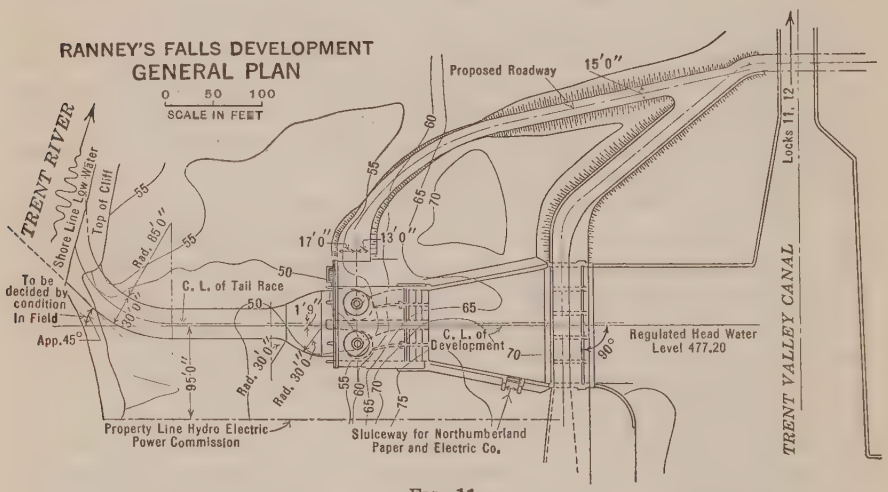

FIG. 11.

The double supply pipes connecting the gate-house to the turbine cases are each provided with a Stoney gate, one gate for each unit being of the sliding type, and the other of the rolling type. The supply pipes are of reinforced concrete. The double supply pipes converge at the bottom, leading the water to the turbine scroll cases in the proper direction, with minimum eddy loss.

The two turbines are of the vertical single-runner type, each direct-connected to a 4 500-kv-a. generator, and have a rating of 5 000 h.p. at 120 rev. per min., when operating under 47-ft. head. The two generators are each normally rated at 4 500 kv-a., 3-phase, 60-cycle, 6 600-volt, 120 rev. per min., at 80% power factor; but are capable of carrying 5 300 kv-a. at 80% power factor, with cooling air at an ambient temperature of 15° cent.

Developments at Dams No. 8 and No. 9

General.—The second and third developments recently placed in operation by the Commission at Dams No. 8 and No. 9 near Campbellford, contain some interesting features, in that they are automatic and operated by remote-control equipment. This enables their operation to be carried out from the central station at Ranney Falls, which is located about 1 mile from Dam No. 9 and 3 miles from Dam No. 8. It is felt that this remote-control feature will effect a considerable saving in the cost of operation for these two plants, which cost, in the case of smaller developments, is an appreciable factor in the cost of power.

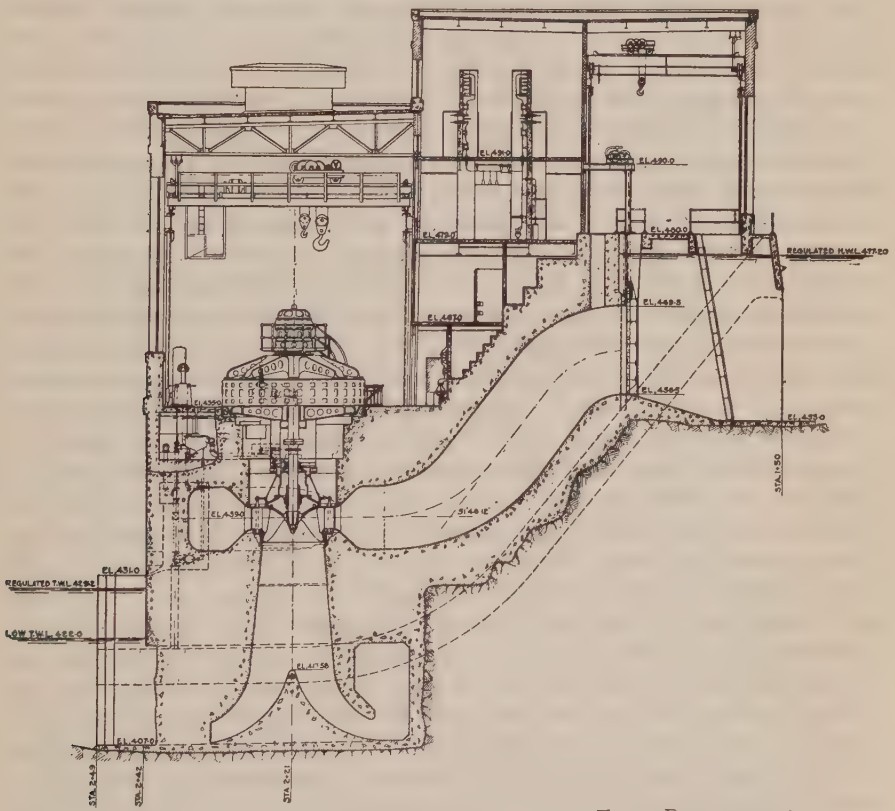

FIG. 12.—CROSS-SECTION, POWER-HOUSE, RANNEYS FALLS DEVELOPMENT.

The Trent River is a canalized stream forming part of the inland waterway between Georgian Bay and Lake Ontario by way of the Kawartha Lakes. The three main interests making use of the river—navigation, lumbering, and power—conflict considerably in their flow requirements, but a regimen for the control of the flow has been established, which serves as a basis for the installation of generating equipment. The regulating dams were constructed several years ago by the Federal Department of Public Works and the various power

developments, of which there are a number, are installed in the neighborhood of these several dams and take advantage of the fall at the canal locks.

A short distance below Campbellford the river is divided by Meyer's Island into two branches, known as the East and West Branches. Dam No. 9 extends across the river immediately up stream from the Island, being connected thereto by a concrete core-wall. Dam No. 8 is situated on the East Branch about 2 miles down stream from Dam No. 9 and a short distance from the lower end of the Island.

The minimum regulated flow down the East Branch at present (1925) available for power is 1 200 cu. ft. per sec., about 200 cu. ft. per sec. being allowed down the West Branch to satisfy riparian owners, but it would be possible to increase this flow by means of storage, to 1 650 cu. ft. per sec. The net head at Dam No. 8 is 32 ft. and at Dam No. 9, 22 ft., and a flow of 1 650 cu. ft. per sec., at the load factor under which it is intended to operate these plants, gives a capacity of 6 600 h.p. at Dam No. 8 and of 4 800 h.p. at Dam No. 9.

Dam No. 8.—The generating plant at Dam No. 8 consists of three 2 200-h.p. units situated in the three most easterly sluice-ways. These sluice-ways are 20 ft. wide, the piers being 6 ft. thick, thus fixing the spacing of the units 26 ft. center to center. A considerable saving was effected by using the existing dam structure as a head-works, cutting slots for the racks in the sides of the piers, and extending the piers down stream to form the draft chests. In case one or more of the units are unwatered, the original piers would be subject to side pressure, and, although the piers were considered to be safe as regards bending (on account of the low ratio of span to thickness), it was necessary to take care of the end reactions. This was accomplished at the bottom by pouring a concrete pad in the floor of the four end sluice-ways, and at the top by drilling through the piers and tying them together with 2-in. tie-bolts and steel plate washers, which were afterward concreted in. The heavy loads on the turbine floor were carried on a structural steel framework composed of two parallel girders spanning between piers and two I-beams framing into them on either side of the draft-tube. This method of construction, it was felt, would give added rigidity and, besides facilitating construction, would be cheaper than the usual flat-slab system of reinforcement. The same method was used to support the generator floor, which also carried heavy loads, and was subject to considerable vibration. This floor had to be reinforced also for upward pressure. The draft-tube bell was constructed of steel plate bolted to an upper cast-iron section and fastened rigidly in place at the lower end to the tail-race piers. A concrete cone, 6 ft. 6 in. high, securely anchored into the tail-race floor, was provided for each unit. Stop-log checks for unwatering the draft-tubes were provided in the tail-race piers.

The tail-race, which is an open cut in the river bed, 33 ft. wide by about 17 ft. deep in the average, extends down to the foot of Meyer's Island, more than 3 000 ft. from the power-house. Ordinary methods of rock excavation were used in taking out this cut, which was in solid limestone, and, although the sides were necessarily rough, the bottom was comparatively smooth, as the limestone strata were approximately horizontal. The roughness factor as deter-

mined from tests was about 0.37, which was slightly less than that used in the preliminary computations for tail-race losses.

Surplus and flood waters are diverted through the westerly sluices of the dam across Meyer's Island to the West Branch of the river, instead of allowing them to flow down the East Branch, thereby reducing the deposit of material in the tail-race cut, as well as avoiding some loss of head. This diversion necessitated the construction of a gravity retaining wall from the west corner of the power-house over to the Island, a distance of 220 ft., and a rock-filled timber crib and rock-fill section from the end of this retaining wall along the east side of the Island for a distance of about 1 500 ft.

The hydraulic equipment consists of three vertical units, in open-flume setting, each unit being rated at 2 200 h.p. under 32-ft. head. These turbines are reaction runners, and operate at 150 rev. per min. The governors are of the oil-pressure type, and as the piping for the three units is interconnected, and as one pumping unit is of sufficient capacity to operate two units, it is felt that no trouble will be experienced from this source under operating conditions. The noteworthy feature about this station is the installation of automatic and remote-control equipment to enable the units to be operated from the Ranney Falls Station. The additional attachments to the governors include a solenoid which may be energized or de-energized, and a synchronizing motor which records the speed of the units, with the necessary automatic features such as valves, dash-pots, etc., to enable the operations of starting, synchronizing, putting load on the machine, closing the gates, and applying the brakes all to be controlled from the remote station.

Dam No. 9.—The layout at Dam No. 9, where also the generating plant is in three units, while similar to that at Dam No. 8 in many ways, has some essential points of difference. Five power sluices, each 20 ft. wide, had been built in the side of the canal, just up stream from the canal lock, to accommodate the turbines. However, as the piers were only 4 ft. thick and the concrete showed some signs of disintegration, it was considered inadvisable to adopt a power-house layout adjoining these piers as at Dam No. 8. An inner forebay therefore was provided by constructing concrete wing-walls, and the power-house was located about 25 ft. down stream from the existing sluices. This layout necessitated the construction of a head-works section to contain the racks, immediately adjoining the power-house proper.

The power-house design adopted and the method of carrying the floor loads were somewhat the same as in the sister plant at Dam No. 8, with the exception that the turbine floors were reinforced concrete arches. This type of construction was made possible by the fact that the existing rock surface was above the floor level. The units were spaced at 30-ft. centers, which gave a longer span for the generator floor slabs, but the loads from the weight of the machines were lighter than at Dam No. 8. The draft-tube bells are steel plate as at the other installation, but without concrete cones. Stop-log checks were also provided in the tail-race piers for unwatering.

The tail-race was excavated in the bed of the river and extends down to the slack-water which is also the head pond for the Dam No. 8 development. This

cut is 1 400 ft. long by 40 ft. wide and averages about 12 ft. in depth, branching out near the end into two channels, one on each side of a small island.

TABLE 1.—Hydro-Electric Power Commission. Distribution of Power in Province of Ontario, Canada.

System.	Dimensions of District, in Miles.			Number of municipalities.	Population.	Estimated system demand, in horse-power, 1925.
	Area served, in square miles.	East and west.	North and south.			
Niagara	13 500	236	101	254	1 800 000	700 000
Georgian Bay:						
Eugenia Division	4 050	76	76	30	46 500	
Severn Division	2 300	50	62	21	44 500	20 000
Wasdells Division	1 300	25	59	14	19 100	
Muskoka Division	1 250	42	30	2	3 900	
St. Lawrence	2 100	85	30	18	28 000	6 000
Rideau	1 500	60	36	5	16 350	3 000
Ottawa	400	30	16	5	134 650	16 550
Thunder Bay	4 000	90	50	2	17 000	45 000
Central Ontario and Trent	4 700	126	67	41	145 000	45 000
Nipissing	700	27	30	4	13 500	2 750
Total	35 800	....	....	396	2 268 500	838 300

TABLE 2.—Hydro-Electric Power Commission of Province of Ontario, Canada. Summary of Capacity of Plants, in Horse-Power.

System.			Capacity.	Ultimate capacity, present plus undeveloped.
Niagara:				
Ontario Power plant	183 500			
Toronto Power plant	145 000			
Queenston-Chippawa plant	440 000	(8 units)	788 500	898 500
Power purchased from Canadian Niagara Power Company	20 000			
Georgian Bay:				
Severn Division	5 800			
Eugenia "	7 400		19 450	29 950
Wasdells "	1 000			
Muskoka "	5 250			
St. Lawrence:				
Purchased power			10 000	.....
Rideau			3 450	6 550
Ottawa:				
Purchased power			20 000	.....
Thunder Bay			50 000	75 000
Central Ontario and Trent			47 028	53 128
Nipissing			3 600	5 600
Total			942 028	1 068 728

The turbines are of the open-flume, single-runner, vertical propeller type, and are rated at 1 600 h.p. under 22-ft. head, operating at 180 rev. per min. The governor and piping layout is almost a duplicate of that at Dam No. 8, the governors themselves having the same automatic and remote-control features.

Possibly it is too soon to state that these two plants, which are unique in that they are the largest of the remote-controlled type yet constructed in America, have been an unqualified success, as they have been operating only a comparatively short time. Nevertheless, the conclusion to be drawn from operating experience thus far is that the development of many power sites hitherto considered commercially impracticable, will probably be brought within the range of economic feasibility, due to the saving that can be effected by the use of remote-control equipment.

Statistics

In addition to the salient features of the more important of the generating plants serving the systems of the Commission already described Tables 1 and 2 present further details. When completed, the plants under development by the Commission will aggregate 1 000 000 h.p. The whole undertaking has been carried forward on the basis of municipal co-operation. The municipalities are financially responsible for the undertaking; the Commission on behalf of the municipalities, and the municipalities in connection with their local utilities, have invested $275 000 000 in order to obtain electrical energy at cost.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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Paper No. 1615

INCREASING THE EFFICIENCY OF PASSENGER TRANSPORTATION IN CITY STREETS*

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With Discussion by Messrs. J. C. Stevens, Harry R. Miller, Carl W. Stocks, John C. Long, W. G. Strait, Harold M. Lewis, William E. Thompson, Lewis A. De Blois, E. N. Johnson, Charles Gordon, J. R. Slattery, William J. Shea, Harold H. Dunn, Max Miller, William T. Lyle, T. Kennard Thomson, J. P. Snow, Theodore T. McCrosky, F. Lavis, and John A. Miller, Jr.

Synopsis

Many plans for the relief of traffic congestion are based on the general theory of providing more roadway area to be used in exactly the same way that the existing street area is used. Cutting through new thoroughfares, widening present roadways, arcading sidewalks, double-decking arterial routes, and placing the electric railways underground are among the proposals made. Enormous expense would attend the execution of any of these plans. While it cannot be denied that substantial benefits would result, it is extremely doubtful whether permanent relief from traffic congestion would be attained. Experience has shown in a number of instances that new streets cut through to relieve overcrowding on older streets soon become congested themselves, without any marked diminution of traffic on the other streets. It seems, therefore, that the problem might well be approached from a different angle. By stimulating the more intensive use of present streets, a far larger volume of traffic could be moved through them without the necessity for huge expenditures on improvements.

* Presented at the meeting of June 2, 1926.

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This paper shows that the general practice of using roadway space for the storage of private automobiles is in reality neither a convenience to the automobilists nor a benefit to the merchant. The elimination of parking in congested districts would greatly increase the roadway available for moving vehicles. The use of attractive common carrier vehicles can be developed to handle a large part of the traffic now requiring single passenger or two-passenger vehicles. A material reduction in congestion will thereby be effected without excessive cost. The movement of all traffic will be facilitated to a degree that will be advantageous to the users of public and private conveyances.

Introduction

The congestion of traffic in the streets of the business districts of practically all large American cities has become acute. The problem thus created is threefold, involving the transportation of passengers and of merchandise, and the movement of pedestrians. Particular interest attaches to the subject of passenger transportation, however, because it affects directly the comfort and convenience of a larger number of people than either of the other phases of the problem.

Despite the serious traffic congestion the actual capacity of existing streets for the transportation of passengers has by no means been reached The volume of traffic that can be moved over a particular route in a given time depends on the effective size of the existing roadway and the efficiency with which the available space is utilized. Broadly speaking, both can be increased with comparative ease. To accomplish this, all the roadway of important arteries should be made available for moving vehicles and should be put to the most intensive possible use. In other words, the most efficient vehicles should be used in the most efficient way.

Increasing the Effective Width of Streets

By the elimination of automobile parking, the effective width of existing streets can be increased for moving traffic from 30 to 200 per cent. A transportation survey of Washington, D. C., disclosed that 29% of the roadway area in congested districts was reserved for automobile parking. In other cities, where the streets are narrower, an even larger proportion of the roadway is used for parking. Often parking is permitted on both sides of streets having a width of 60 ft. or less between building lines. Deducting two 12-ft. sidewalks, about 36 ft. of roadway is left. A row of parked automobiles will occupy an 8-ft. strip on each side. Thus, there remains for moving vehicles only a 20-ft. roadway, capable of accommodating one stream of traffic in each direction. For all practical purposes, 50% of the street is being used for parking. On one-way streets it happens not infrequently that there is room for only a single stream of moving traffic between the lines of parked vehicles. Under such conditions, the area devoted to parking would approximate two-thirds of the total.

The use of the public streets for the storage of private automobiles is tolerated largely because of an exaggerated idea of their importance in the general transportation plan. Traffic counts made in various large cities, however, show that public transportation vehicles are carrying upward of 75% of the total traffic, although they constitute only about 15% of the total number of vehicles. A check made of 13 leading stores in New York, N. Y., during a busy shopping hour showed that of 35 252 customers only 813 came by private automobile or taxicab. The others came by street car, subway, or bus. To the users of public transportation agencies, who thus greatly outnumber the users of private conveyances, relief from congestion by the elimination of parking would be a great boon. For example, one of the effects of the passage of a no-parking regulation in the down-town section of Pittsburgh, Pa., was to reduce by several minutes the running time of street cars through this area. The average savings are shown in Table 1.

Opposition to an absolute ban on parking on a particular street, however, is likely to be made by retail merchants whose stores are located there. Usually they contend that their business will be injured unless an opportunity is given for customers to park their automobiles in the street in front of the stores. A preferable plan, it is asserted, would be to fix a time limit of 1 hour for parking.

TABLE 1.—Time Saved in Operation of Street Cars, Pittsburgh, Pa., by Parking Elimination.

Loop Number.	Number of cars during maximum hour.	Average decrease in time, in minutes and seconds.
1	26	1:01
2	84	2:47
3	81	3:51
4	27	5:21
5	57	2:55
6	23	0:25
7	20	0:56
8	52	0:35
9	159	3:17

Such an idea is a fallacy. With parking parallel to a curb, a frontage of 100 ft. would accommodate 5 or possibly 6 automobiles at a time. If these 6 automobiles were replaced by another 6 each hour throughout the business day, a total of 48 automobiles would have been accommodated. Probably they would not carry more than 1 actual customer per vehicle, and the net result of 1-hour parking would be to accommodate about 50 customers during the day. For a retail store with a frontage of 100 ft. on an important street this number of customers would be a comparatively unimportant part of the daily business. Analysis thus confirms the census taken in the New York stores, already mentioned, showing that the number of customers arriving by automobile was only a small fraction of the total.

Moreover, from the point of view of general traffic congestion, 1-hour parking is just as detrimental as unlimited parking, perhaps more so. Limited

privileges of this kind are far more difficult of enforcement than an absolute ban. The capacity of a street for moving vehicles suffers a continuous reduction on account of the limited parking and there is also the added confusion due to automobiles pulling into and out from the curb line.

Elimination of Parking Will Aid Business

Contrary to a prevalent opinion, prohibition of parking in the congested districts is likely to aid business rather than injure it. After the enactment of the no-parking ordinances in Pittsburgh, already mentioned, retail trade in the down-town district showed an increase in comparison with that of the outlying commercial centers.

Experience of Pittsburgh.—Comparisons of the buying power of the Pittsburgh district in 1924 and 1925 as shown by bank payrolls, the total retail sales, and the down-town retail sales, as represented by seven large department stores, are given in Table 2 by 3-month periods.

Industrial depression caused the slight decrease in all these figures for 1925 as compared with 1924. It is significant, however, that the retail sales for the entire district showed a decrease of 2.80% for the year, whereas the down-town sales decreased only 0.8 per cent. This indication that the down-town business suffered less than that in the outlying sections appears to refute completely the claim that parking restrictions injure retail trade.

TABLE 2.—Change in Pittsburgh Business Conditions in 1925 Based on 1924.

Three months ending	Percentage change in buying power.	Percentage change in district retail sales.	Percentage change in down-town retail sales.
March 31, 1925	−9.01	−1.87	−2.53
June 30, 1925	−1.77	−5.90	−2.37
September 30, 1925	−8.67	−6.20	−2.77
December 31, 1925	−8.13	+1.50	+3.47
Total for year	−5.72	−2.80	−0.80

Effects of Congestion.—In a large number of cases analysis would undoubtedly show that the volume of trade in down-town stores has not kept pace in recent years with the development of a community. The 1924 population of a certain district, including a city of about 500 000 people, with perhaps 250 000 more in the adjacent suburban territory, represented nearly 130% of the 1914 figure (Fig. 1). Meanwhile, the wages received by the average individual had increased to approximately 205% of those received in 1914. Thus, the total purchasing power of the district in 1924 was 266.5% of that before the World War.

Investigation of the volume of business of a number of representative down-town retail stores in this particular city showed that, in 1924, it was only 240% of that 10 years earlier. Thus, it will be seen that the growth of retail trade in the down-town section lagged considerably behind the in-

crease in the purchasing power (Fig. 1). On the other hand, in the case of one particular high-grade store, about 3 miles from the business center, the present volume of business is approximately 480% of that in 1914.

Banks as an Index of Business.—A realization of the remarkable increase that has occurred in the outlying commercial centers during the decade may be had from a study of the locations of banks of deposit (Fig. 2). In 1924, the number of such banks in the entire area was 49 as compared with 37 in 1914. During these 10 years, however, only one new bank was opened in the center of the city; all others were in the outlying districts. The zone from 1 mile to 2 miles from the center showed an increase of 4, as did also the zone from 3 to 4 miles. A summary of this is given in Table 3.

Figures of this kind do not show with mathematical accuracy the comparative growth of business in the city center and suburban areas. Nevertheless, a bank of deposit is rarely opened in a purely residential section, and its presence may be taken to indicate fairly the existence of a business community. The surprising increase that has occurred in the outlying sections illustrates clearly how business has sprung up during the past ten years.

TABLE 3.—Summary of Business Development as Indicated by the Location of Banks of Deposit.

	1914.	1924.	Increase.
Inside 1-mile zone	12	13	1
Between 1 mile and 2 miles	6	10	4
Between 2 miles and 3 miles	3	4	1
Between 3 miles and 4 miles	6	10	4
Between 4 miles and 5 miles	6	7	1
More than 5 miles	4	5	1
Total	37	49	12

Down-town traffic congestion undoubtedly is the most important reason for this process of decentralization. Transportation in public conveyances has become relatively slow and subject to irritating delays. It is a task of considerable difficulty and some danger to drive a private automobile in the down-town streets. As a result, people do their buying near home. This tendency will increase as congestion increases. Retail merchants, therefore, should favor rather than oppose the elimination of parking on important streets.

Substitutes for Parking.—If the parking of private automobiles in the public streets down town is forbidden, a question arises as to what can be done with them. One answer might be to provide parking spaces either in the centers of the blocks, which are usually not built upon, or beneath some of the existing buildings, or in many storied garages. The number of automobiles which could be accommodated in this way is more or less limited, however, and the charges are likely to be rather high.

In New York, one large department store provides storage space for the automobiles of its customers and drivers to take the vehicles to the garage and bring them back when the owners are ready. In another city, a large department store has arranged a garage some blocks from the business center to

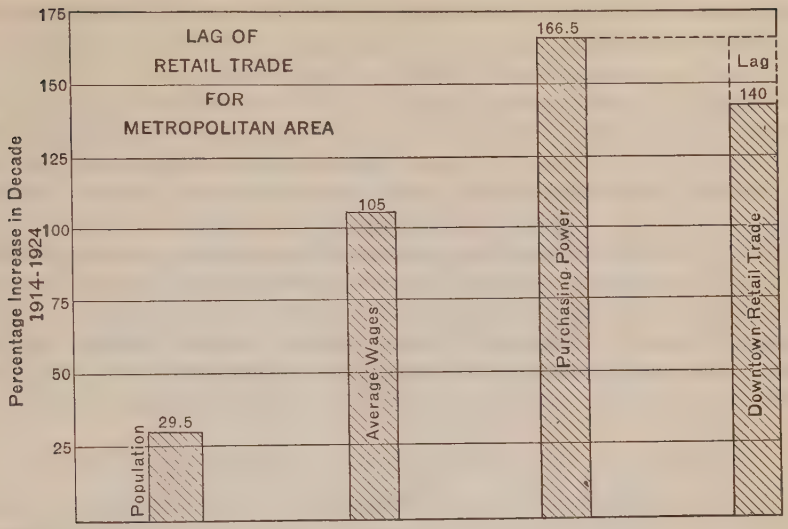
LAG OF
RETAIL TRADE
FOR
METROPOLITAN AREA
Percentage Increase in Decade
1914-1924
175
150
125
100
75
50
25
29.5
Population
105
Average Wages
166.5
Purchasing Power
Lag
140
Downtown Retail Trade

FIG. 1.

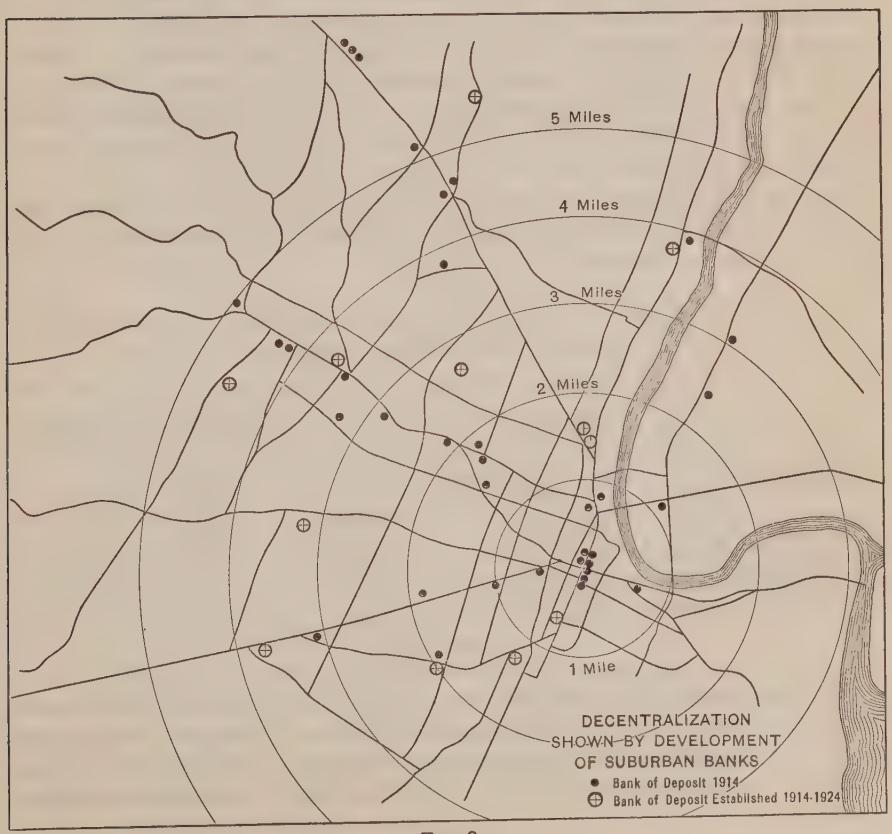
5 Miles
4 Miles
3 Miles
2 Miles
1 Mile
DECENTRALIZATION
SHOWN BY DEVELOPMENT
OF SUBURBAN BANKS
Bank of Deposit 1914
Bank of Deposit Established 1914-1924

FIG. 2.

accommodate the automobiles of the customers. The patrons are carried from the garage to the store by bus. Obviously, such remedies are applicable in only a few cases, as the smaller stores could not afford them.

The establishment of parking spaces on the outskirts of the city and transportation of passengers down town by public conveyance is a solution that is being tried in a number of cities. Several such parking spaces are operated by the Philadelphia Rapid Transit Company which carries the automobilists from the outlying sections into the business district by rail. A number of other electric railway companies have adopted similar methods, making the charge for parking to include transportation back and forth by street car. An arrangement of this kind not only solves the problem of where to store the private automobiles without wasting valuable roadway area, but it also promotes the efficiency of passenger transportation in the congested district by substituting one large vehicle for many small ones.

Economizing Roadway Space by Use of Large Capacity Vehicles

Advantage of Buses.—The substitution of public transportation vehicles of fairly large carrying capacity in place of individual automobiles holding on the average not more than two persons can be made advantageously in many instances. For example, *de luxe* bus service often will prove an entirely acceptable substitute for the use of numerous private automobiles. In Washington, D. C., the Capital Traction Company operates luxurious buses from the high-class Chevy Chase residential district on express schedules to the shopping centers of the city. Fares are higher than on the street cars because of the superior facilities provided. The net transportation expense to the Chevy Chase resident by bus, however, is smaller than when he uses his own automobile, and with existing conditions of congestion the greater convenience of the bus service is marked. This service has proved very popular.

TABLE 4.—Average Loading of Private Automobiles as Shown by Recent Transportation Surveys.

City.	Time of survey.	Number of vehicles.	Total passengers.	Passengers per vehicle.*
Washington, D. C.	April 30 (in) 15 hours	100 284	171 051	1.71
	" (out) " "	92 666	154 394	1.67
	June (in) " "	114 047	223 000	1.96
	" (out) " "	112 957	218 988	1.94
Chicago, Ill.	1 hour	2 600	4 700	1.81
Atlanta, Ga.	All day	27 540	49 600	1.8
Los Angeles, Calif.	11 hours	261 974	393 322	1.50
Detroit, Mich.	1 hour	12 180	21 200	1.74

* Includes driver.

Similar transportation facilities are provided in Chicago, Ill., between the Edgewater Beach Hotel and the shopping district. Although intended primarily for the guests of the hotel, patronage of this *de luxe* bus service is not confined to them, but includes residents of the entire fashionable neighborhood in that vicinity. Not only are the buses a cheaper and more convenient

mode of transportation than private automobiles or taxicabs, but they have come to be considered a more distinguished mode. In other words, it carries greater prestige to be seen alighting down town from the 35-cent Edgewater Beach bus than from a taxicab at twice that fare.

More general use of such collective transportation vehicles would go far toward increasing the efficiency with which the available street space is used. Traffic counts made in various cities show that private automobiles carry an average of less than 2 passengers per vehicle (see Table 4). This is equivalent to about 30 sq. ft. or more of street area per passenger. A bus of ordinary size can comfortably accommodate at least 20 persons. Assuming that it would average only three-quarters of a full load (15 people), about 16 sq. ft. of street area would be required per passenger. Inasmuch as the speed of the "limited stop" bus and the private automobile would be approximately the same in the congested district, the use of the bus to replace the automobile would double the efficiency with which the existing street area is utilized.

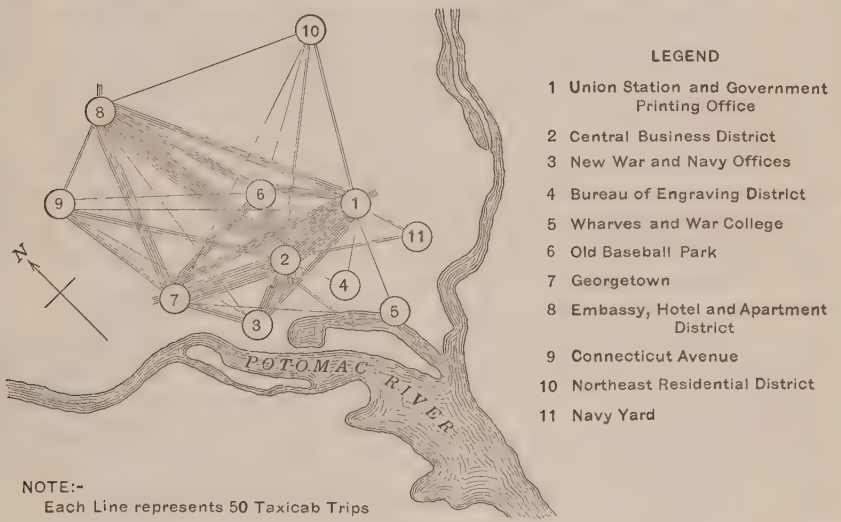

FIG. 3.—TAXICAB MOVEMENTS DURING A TYPICAL BUSINESS DAY IN WASHINGTON, D. C.

Theoretically, an even greater increase in efficiency might be attained by the substitution of the electric rail car, requiring only 3.5 sq. ft. of street space per passenger carried, for the private automobile. In actual practice, however, it is found that the character of the service rendered by these two modes of transportation is so different that it is impracticable to try to replace the automobile by the trolley. Well-operated bus service with commodious vehicles, on the other hand, can be made an entirely acceptable substitute for the use of private automobiles.

Taxicabs Inefficient.—Most wasteful of all vehicles used in passenger transportation is the taxicab. Investigations made by the writer at various locations in New York indicate that the average load is only 0.83 passenger. Approximately half the taxicabs were cruising without passengers; of those in use, however, many carried three and four passengers. As this vehicle

occupies about 64 sq. ft. of street area, the space required per passenger carried is about 80 sq. ft. When it is realized that there are more than 17 000 taxicabs in New York, and correspondingly large numbers in other cities, the importance of this cause of congestion is evident.

Channels of Taxicab Traffic.—Surprisingly enough, the greater part of the taxicab movements follow definite routes. In the transportation survey of Washington, D. C., previously mentioned, movements of 334 taxicabs operated by four taxicab companies were analyzed for 1 week day. The manifests made out by the drivers were abstracted to secure for each cab the origin and destination of each trip, the time at which the trip was commenced, and the time the trip was completed. These data were then analyzed as to the point of origin, point of destination, and time of day.

During this typical day, 7 924 taxicab movements were classified. Of this number about 5 000 movements were made along only 33 main routes. The three heaviest routes were between the Union Station and the business district, 600 trips, between the Mt. Pleasant residential district and the business district, 400 trips, and between Georgetown and the business district, 350 trips. Fig. 3 shows the number of taxicab movements on each of the 33 routes. On no other route in the entire city were there as many as 50 taxicab movements during that day.

These figures indicate that in Washington about five-eighths of the taxicab movements are along certain definite routes. Substantially the same condition no doubt prevails in other cities. Why not replace these taxicab movements along definite routes by bus service requiring 16 ft. of street area per passenger carried instead of 80 sq. ft.?

Obviously, it would not be feasible to try to replace entirely the taxicab by the bus, any more than to try to replace altogether the private automobile. It is clear, however, that so much of both of these kinds of traffic flow along a comparatively few main routes, that a real opportunity exists to effect an important improvement in passenger transportation in congested districts by thus using one vehicle of high carrying efficiency to perform the work now being done by numerous smaller vehicles of low carrying efficiency.

Conclusion

The foregoing considerations point to a possible solution of street traffic problems by (1) the reduction or elimination of parking; and (2) the substitution of buses in many instances in place of the extravagant private automobile, and the taxicab. Such a solution would go far toward increasing the efficiency of passenger transportation in city streets without injury to the mercantile interests. It would also possess the advantages of small cost and speedy relief of present congestion.

DISCUSSION

J. C. Stevens,* M. Am. Soc. C. E. (by letter).—The author has touched on a subject that is fast becoming critical in many cities. The continued and increasing traffic congestion will inevitably result in a gradual decentralization of business activities as pointed out in the paper.

It is doubtful, however, if the elimination of parking in retail zones will have the wholesome effect assumed. It is not clear why the change in business conditions in Pittsburgh between 1924-25 should be linked up with the enforcement of the no-parking ordinance. Many other factors could account for these changes. It seems purely gratuitous to assume that a 2% less reduction in down-town retail sales over district retail sales is due to "no-parking" in the down-town district.

As far as the movement of traffic is concerned, it is a fallacy to assume that the elimination of parking will materially relieve congestion. It will, temporarily at least, reduce the number of cars in daily use, because many cars will be left at home if they cannot be parked.

The point the writer wishes to make is that the delays to vehicular movement that often result in congestion originate at street intersections, and cannot be remedied by providing more space in the middle of the block. If a street with both curb spaces full of parked cars is still wide enough for a line of moving vehicles in each direction, the vehicles cannot pass through the street faster than they can cross at the intersection, hence there is nothing gained by removing the parked cars.

The efficiency of an intersection theoretically cannot exceed 50% and actually is much less, so that the space for moving traffic approaching a grade crossing need not be more than one-half that which would be required if the grades were separated. By "efficiency" of an intersection is meant the ratio of the number of vehicles that may pass a grade intersection to the number that could pass the same point if the grades were separated.

As a simplified illustration, suppose it were practicable for cars to move at uniform and equal speeds on two intersecting streets. It is possible so to space the cars in the moving lines that those in one line will pass between those in the other line without interference. They might be imagined as fixed to intersecting cable lines being drawn at uniform and equal speeds. There may be one line, or two or more parallel lines, of cars in each street. The required spacing of cars to permit passing in this manner is surprisingly large, and this spacing for grade crossings compared with the spacing for separated grades is a measure of the optimum efficiency of the grade crossings, which, it is readily shown, can never exceed 50% even when cars are passed in large fleets.

Let l = average length of cars;
w = average width of cars;
c = desired end clearance between cars;
D = end clearance between fleets of cars;

* Cons. Hydr. Engr. (Stevens & Koon), Portland, Ore.

s = end spacing center to center of cars in a line of traffic;
d = distance center to center of parallel lines of traffic;
n = number of ranks of cars in one fleet; and
m = number of parallel lines of traffic in each street.

If the two intersecting lines are moving at uniform and equal speeds—the actual speed being immaterial—from Fig. 4,

$$s = 2\ (l + w + c) \qquad (1)$$

If the grades were separated, however, the cars could be spaced a distance, $l + c$, center to center, hence the efficiency of the grade crossing is,

$$E = \frac{l + c}{2\ (l + w + c)} \qquad (2)$$

The majority of cars measure 15 by 5.8 ft. Based on an end clearance between cars equal to their width, the spacing, center to center, must be 54 ft. for grade crossings and 21 ft. for separated grades, hence the optimum efficiency for a grade crossing is 39% for a single line of such cars on each street, crossing one by one.

If there are two parallel lines of traffic in each street, it being immaterial whether the lines are moving in the same or opposite directions, the spacing between cars is found to be,

$$s = 2\ (l + c + w + d) \qquad (3)$$

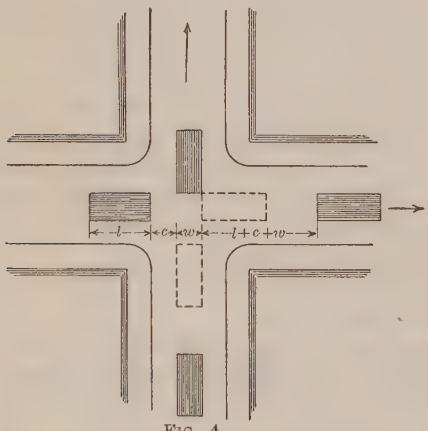

Fig. 4.

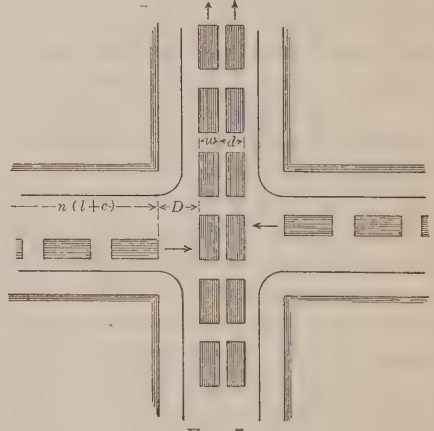

Fig. 5.

Two cars in parallel are equivalent to a single car the width of which is $d + w$. If grades were separated, the end spacing is the same as for a single line, hence the optimum efficiency for cars passing in ranks of two is,

$$E = \frac{l + c}{2\ (l + c + w + d)} \qquad (4)$$

Using the same average dimensions as before and fixing d at 15 ft., the required spacing is found to be 83 ft. between ranks and the efficiency of the crossing is only 25 per cent. If there were four lines of traffic in each street, the spacing would have to be 143 ft. and the efficiency drops to 14 per cent.

Where there is considerable traffic, vehicles are usually passed at intersections in fleets, which tends to increase the efficiency considerably. In fleet movements all traffic in one street is held up for a time while that in the other is passing. The clearance between fleets must be not less than the width of one sidewalk (to permit pedestrian travel) plus the distance from the curb to the near edge of the traffic line in the other street.

A fleet of cars is equivalent to one car the length of which is $n(l+c)$ and the width, w for a single line, $w+d$ for two parallel lines, $w+2d$ for three lines, etc. The general expression for the spacing of fleets then becomes (Fig. 5),

$$s = 2\ [n(l+c) + w + d + (m-1)d] \ldots\ldots\ldots\ldots\ldots (5)$$

If grades were separated the spacing is $n(l+c)+D$, hence the efficiency of the crossing is,

$$E = \frac{n\ (l+c) + D}{2\ [n\ (l+c) + w + D + (m-1)\ d]} \ldots\ldots\ldots\ldots\ldots (6)$$

If the average distance center to center of parallel lines of traffic is fixed at 9 ft. and the average fleet clearance, D, at 22 ft., the values for the efficiencies of grade crossings will be as shown in Table 5.

TABLE 5.—Efficiencies of Grade Crossings.

Ranks of cars per fleet (n).	Number of Traffic Lines (m).		
	2	4	6
2	40.6	30.7	27.9
5	44.7	39.7	35.6
10	47.0	43.8	41.0
100	49.6	49.2	48.8

The purpose of this analysis is to show that delays, and consequently congestion, originate mainly at grade crossings. Vehicles do not travel in an orderly and uniform manner as here assumed. They are of all sizes from "flivvers" to trucks and triple-header street cars; they "bunch up" at a crossing; and they go in every direction available. The clearances may be any distance from zero up. This traffic complex has the effect of reducing the efficiency of grade crossings over what would obtain if traffic could be systematically and orderly dispatched.

Even with the best ordered system of traffic control it is doubtful whether the traffic that can pass grade crossings is much more than one-fourth that which could pass were the grades separated. If then it is impossible to pass through the streets more than one-fourth their capacity, it is obvious that widening them by eliminating parked cars cannot be advanced as a promising relief for traffic congestion.

Most cities permit limited time parking in the business district, provide safety zones for street car passengers, and prohibit any parking that will interfere with such zones. In many cities all parked cars must be moved off the

streets of the business section at about 4:30 P. M. with the object of providing more space for moving traffic during the home-going rush. Wherever streets are wide enough to permit a line of traffic between street cars and the line of cars parked at the curb it is quite obvious that the later requirement defeats the very object it is intended to accomplish.

Just before the evening rush, from one-third to one-half the total number of cars in the business district are parked. If all such parked cars be required to move out at this time, they only add greatly to the congestion at street intersections without providing more space where space is needed, since the room they formerly occupied at the curb is taken by slowly creeping lines of traffic. Parked cars cannot add to congestion at street intersections, but moving cars, occupying the same space, can. It is only when parked cars prevent moving cars from reaching an intersection as rapidly as they can cross it that the parked cars hinder traffic.

Moreover, if there is only one parked car in the block it prevents a line of traffic from moving next to the curb just as effectively as if the entire curb were filled with parked cars. Thus, a street from curb to center line may be wide enough for three lines of traffic, but a single car parked at the curb in that block reduces the street's capacity to two traffic lines. It were better then that the remainder of the block be filled with parked cars rather than that those cars be adding to the congestion at the intersection.

It is the writer's opinion that money expended in the enlargement of street intersections to double or quadruple their present width will give far greater relief from congestion than an equal amount expended in any other manner.

HARRY R. MILLER,* ESQ. (by letter).—This paper is a concise and clear exposition of a matter that is of vital importance not only to the merchants in the congested areas of cities, but to every taxpayer and rent-payer therein.

It is patent to every observer that the automobile, particularly the passenger automobile, when used to provide rapid and comfortable transportation in the congested districts, has brought about a condition whereby enormous sums of money must be spent for street widenings (with a doubtful permanent solution of the problem) and with the ultimate result of compelling merchants in those districts to spend large sums of money in establishing and maintaining branch stores.

From the standpoint of economics, it would seem that money spent for intensive street widening and for merchandising establishments is only defensible as a last resort, that is, after all other potential solutions have failed. This is particularly true if among these plans there is one of a revenue-producing character.

Subways do not fall under such heading because of the expense both of construction and operation, and neither do trolley cars or buses when used in local service alone because of the comfort and speed learned from the use of the passenger automobile.

* The Miller-Schorn System, Detroit, Mich.

The schedule speeds of trolley cars in certain American cities have been reported,* as follows:

	Miles per hour.
Chicago, Ill. (Loop District)	6.21
Pittsburgh, Pa.	9.42
Buffalo, N. Y.	9.95
Cincinnati, Ohio	9.95
Detroit, Mich.	10.13
Washington, D. C.	10.21
Boston, Mass.	10.50
St. Louis, Mo.	10.90
Chicago, Ill. (Outside Loop)	11.63

and the schedule speeds of the buses of the Chicago Motor Bus Company are given as:

	Miles per hour.
Chicago, Ill. (Loop District)	8.57
Chicago, Ill. (Outside Loop)	11.87

In Chicago, the difference in speed between the trolley cars and buses is not sufficient to make either type of unit attractive as a competitor of the space-wasting passenger automobile, and neither can the space-saving trolley car or bus ever become attractive until its schedule speed is comparable with the passenger automobile—a condition not possible of attainment when trolley cars and buses are making from five to ten stops per mile.

However, increased schedule speed can be procured (without raising the maximum speed) by reducing the number of stops, but in so doing, consideration must be given to the fact that surface transportation is not a terminal-to-terminal proposition, but depends on intermediate business for existence. Such being the case, the answer is obvious—install on the same street an express and local service, using the trolley cars as express units, stopping approximately once to the mile, and buses as local units, stopping five to ten times per mile, as the business warrants.

Coupled with this combination service is the plan of making the vehicular traffic more "fluid" by removing the necessity for vehicles stopping with or behind standing trolley cars, such as is proposed in Detroit. This plan utilizes, at this express station, a passenger depot located between the tracks with a pedestrian subway leading thereto from the sidewalk. With such a scheme the trolley car rider is protected in going to and from the car, and, at the same time, vehicular traffic flows on uninterruptedly.

To summarize: The passenger automobile with two passengers occupying approximately 40 sq. ft. of street space per passenger, as against 7.57 sq. ft. per seated passenger in a "Peter Witt" street car, and approximately one-half as much in the case of the double-deck trolley car or double-deck bus, is the principal agent in producing traffic congestion in the business districts of American cities.

* *Bus Transportation*, February, 1923.

Whereas the individual should have every consideration in his attempt to provide rapid transportation for himself in the use of the passenger automobile, nevertheless when the use of such a vehicle places on the community as a whole an inequitable burden, the individual's interest should be made subordinate to the interest of the community. To compel the individual to do anything invariably meets with violent opposition; to provide him with something which adds to his convenience through an appeal to his love of new methods, is an entirely different matter.

The combination express and local service herein mentioned will eliminate the ruinous competition between trolley cars and buses, and will aid materially in solving the congestion problem, in that it provides a new and novel means of transporting the individual through the city streets in vehicles that are economical as to space required. As they may operate at higher schedule speeds and in combination with express stations located between the tracks protection is given to the trolley car riders in going to and from the car—not to mention the removal of obstruction to the flow of vehicular traffic by eliminating the stopping of vehicles with trolley cars.

This combination express and local service is a revenue producer, which cannot be said of money spent in intensive street widening or for building branch stores.

Carl W. Stocks,* Esq. (by letter).—In his analysis, Mr. Miller points out how inefficiently street space is now used. To feel, however, that the only way of increasing the efficiency of streets is the substitution of buses for taxicabs and private cars falls short, in the writer's opinion, of the real solution.

This plan can be considered as only one phase of the problem after due consideration has been given to proper routing. Mr. Miller fails to carry his analysis far enough to mention that at specific hours of the day traffic conditions change, and what can be considered acceptable as good public transportation during normal hours will not be satisfactory at other times of the day and evening. It is this ever-changing condition that plays havoc with any fixed public transportation system. In order that such a system be worth while it must be routed to meet traffic conditions.

The greatest difficulty in the handling of city traffic in mid-town shopping and business areas is the personal selfishness of a few people during shopping hours. This class includes the business man who wants to use public streets for private parking while he transacts his daily business.

In the evening, of course, the theatre and "joy-riding" traffic must be handled, and this is something entirely apart from the day traffic. It is true that many taxicabs and private cars can be replaced with the parlor car type of bus when operated over carefully selected routes in the larger cities from such points as road terminals to hotels and department stores. In some cities these bus lines would serve more efficiently if they were run on a taxicab basis instead of as common carriers. Such routes, however, when operated on a common carrier basis must be able to make as good time as is possible

* Editor, *Bus Transportation*, New York, N. Y.

with the more inefficient street-space-occupying vehicle—the taxicab—which has now become a necessity in every-day life. There are 25 000 taxicabs in New York, 1 800 in Washington, 1 500 in Philadelphia, and 5 400 in Chicago. Even so, it is most difficult to find an empty one on a stormy day in these cities. Whether the time is morning, afternoon, or evening, "try and get one". It would seem to the man in the street, then, that there are not enough.

One of the first steps toward increasing street efficiencies is to determine on a definite plan for handling various volumes of vehicular traffic. This requires first the keeping of a continuous and comprehensive census of the origin and destination of all classes of vehicles using city streets, especially those that pass through the mid-town areas or, in the larger cities, certain fixed areas. Then this traffic must be separated to expedite its movement by definite routes so that it will reach its destination in the easiest and most direct way. It becomes then merely a problem of transportation engineering, one of schedule-making to determine how best to handle each stream of traffic according to its density at different hours of the day.

To have such a plan adopted the public must be taught that all traffic need not use the same arteries of the city. In the past, cities and towns along the principal highways were insistent that all tourists pass through "Main Street". That desire is fast changing on account of the congestion thus created, and it is now possible to travel around many of the moderate sized cities over specified routes without being involved in purely local traffic. It is education that is needed, along with proper route markings by signs or numbers, following somewhat the plan that is used on the inter-city highways.

Purely local traffic can be handled in somewhat the same manner if traffic volumes are first analyzed. This phase of the work must be carried on by trained men who appreciate the manner in which it is done, and why, and should be followed up by observers whose business it is to lay out routes.

The parking idea fits into this plan, for it becomes necessary to keep these fast moving arteries clear to handle their full quota of traffic in the peak hours. Parking must be prevented to afford the street capacity necessary to handle the volume. At other times traffic will not be interfered with if parking is permitted.

The use of control signals must, of necessity, be fitted into the proper handling of traffic over these main arteries, but more attention must be paid to their operation than is done on the north and south highways in New York City. There must be clear east and west through streets as well as the north and south avenues. This means a close study of cross traffic over the longer avenues so that more than one can be crossed at a time.

The volume of traffic according to the lanes of travel must be kept as nearly equal as possible over different arteries if the time of travel on one street is to be kept in unison with its neighbor.

With all large cities charted, signed, and properly advertised for the benefit of local traffic it is the writer's opinion that many of the difficulties of highway travel can be obviated and streets can be called on to carry a greater volume

of traffic without the necessity of extraordinary widening programs. It will mean, however, that streets selected for main arteries will have to be repaved with a suitable smooth surface.

A practical slogan for street efficiency would be: "Make each street carry its full quota of traffic according to its available space", so as to eliminate congestion and bottle-necks on a few streets.

JOHN C. LONG,* ESQ. (by letter).—With only two points in this paper would the writer fundamentally disagree:

1.—It is the belief of the National Automobile Chamber of Commerce, representing the automobile industry, that the leadership of engineers is the major solution for traffic difficulties in cities.

2.—Popular opinion will not support complete restriction of parking even if this might permit the movement of a greater volume of persons over city streets.

Engineering Guidance.—The time is at hand when the engineer will have an added dignity and be more highly appreciated by the public because he can be the Moses to lead it out of traffic congestion. Certainly things are not at such a pass that cities must be resigned to the fact that more facilities are too costly or too elaborate, and that therefore the main remedy is to attack a parking issue. The chief trouble is that the public has been slow to realize what an engineer is and what he can do. It is a popular fallacy to look on him as some one wrapped about with blueprints, having his nose tied down to insignificant detail.

As a matter of fact the engineer is fundamentally an organizer. He knows that detail is important, that results do not come about by virtue of "ballyhoo". He handles detail, however, in the same manner that the general organizes masses of troops, food supplies, and ammunition. Present events foreshadow the birthday, or more accurately, the re-birth, of the engineer in public affairs. The engineer's importance in the World War was universally recognized. Here, again, is another great possibility needing organizing brains. The automobile industry, in which engineering is of utmost importance, would be the last to concede that the city traffic situation can be met primarily by ordinances rather than by the generalship of the engineer.

Every city should have a traffic engineer, and he should have responsible authority. Since 1916 State Governments have come to realize that highway control and development is primarily an engineering proposition, so that now State highway projects are laid out and carried through from start to finish by engineering brains. Apparently the cities are at the very start of their possibilities in traffic improvement. While, for a number of years, city planning commissions have been functioning, in many instances these commissions have been fostered by a few forward-looking citizens, but have not been taken with a great deal of seriousness because the need was not immediately pressing as far as the public was concerned. To-day, traffic is a public issue.

* Mgr., Educational Dept., National Automobile Chamber of Commerce, New York, N. Y.

Before admitting that the solution lies with the police force, why not permit the engineers to have a try at it? It is typically an engineering job. The basic facilities are not used to anywhere near capacity, and the first immediate step in every city is an engineering analysis of the situation with adequate publicity to tell the public what needs to be done. At the same time, the engineers will have the job of indicating what should be the immediate and permanent improvements.

Force of Public Opinion.—The engineering world will doubtless welcome this mandate, but even if this were not so, the public demand for assistance is likely to be overwhelming. The American spirit is to seek results and not restrictions. The man who owns an automobile can be convinced of the reasonableness of limitations on his parking, but he is not likely to submit to absolute prohibition of the use of the streets which is what the "absolutely no-parking" rule would mean. If a man cannot park his motor car for a short space of time in down-town areas, it means that the usefulness of his machine in such sections is practically nullified, and the man who has put his investment into motor transport is not going to see its value curtailed without a protest. He will say that if the engineering brains of the country have been able to build the Panama Canal and the Roosevelt Dam, they can work out some method of doing away with the "bottle-necks" in city traffic and arrange for other provisions that will permit a greater use of motor travel.

The author, to some degree, covers this comment in expressing the belief that various physical improvements in traffic in the long run invite more vehicles with the result that the situation becomes acute again shortly after relief has been introduced. He also suggests that the elimination of parking is a scientific method of making street spaces more useful. His approach is from an engineering standpoint. The difference of opinion lies essentially in a greater belief on the part of the National Automobile Chamber of Commerce in the capability of the engineer as an organizing factor. Rightly planned, traffic improvement measures will not lead to a repetition of the same difficulties; and drastic restrictions are not necessary until the engineer has been given a much freer opportunity to handle the job and see what he can do with it.

W. G. Strait,* Esq.—It is said that Napoleon made a practice of leaving letters unopened for one week on the theory that those matters which needed attention received it anyway within that time so the trouble of replying was avoided. It is also said that America wasted enormous sums of money in the World War by constructing ships before properly equipped shipyards could be built or economical organizations established.

The foregoing are matters of hearsay; but it is a matter of fact that much of the labor expended by the Traffic Committee of the Manhattan Surface Line Operators (New York City) during the past five months (January to June, 1926), could have been saved by waiting for this able paper. This Committee has collected data and reached conclusions closely paralleling those of Mr. Miller.

* Vice-Pres., New York Rys. Corp., New York, N. Y.

This Traffic Committee of the Manhattan Surface Line Operators was organized in January, 1926, for the purpose of studying traffic conditions in New York City and devising some practical measures for their relief. As a result of its study and the collective experience of its members in the transportation field, it is clear that no appreciable relief can be afforded until the unlimited use of private automobiles in congested districts is checked.

The Committee does not recommend any such drastic measure as the creation of areas into which private cars cannot go. It does advocate, however, the enactment and rigid enforcement of ordinances which will so restrict parking that the movement of private cars into the streets where those ordinances are in force is discouraged. Specifically, parking should be confined to one side of one-way streets and it should be prohibited on thoroughfares south of 65th Street where street cars and permanently established bus lines operate.

There is a tendency on the part of people in public and private life to regard the automobile as a sacred vehicle of transit whose owners are endowed with certain special privileges superior to the rights of the general public. This probably accounts for the extravagant makeshift remedies proposed from time to time as a means of relieving congestion without eliminating the cause.

Upon analysis the traffic problem becomes clarified. An application of the old law of supply and demand will go far toward solving it. Streets that were built before the automobile age are called on to carry a traffic far beyond their capacity. Roadway space is woefully inadequate for the requirements of the hundreds of thousands of motor vehicles in the Metropolitan Area, and it always will be, no matter how much additional space is provided, with the habit of riding in private cars unchecked. If every person who owns a motor car is to be encouraged to continue to use an excessive amount of roadway space while traveling in the congested districts, and, in addition, is to be permitted to clutter up the already over-crowded streets with his automobile even when it is empty, the supply of street space will never catch up with the demand.

The only reason why there are not two or three times as many private automobiles down town as there are at present is the lack of room for them. Only a small proportion of automobile owners now are able to use the down-town streets, and if new streets could be constructed overnight no change would occur in the traffic situation except that a larger proportion of private car owners would be able to use them. The congestion would not be relieved any more than it has been relieved by various street openings and street widenings in the past.

Congestion will be relieved materially and immediately when parking restrictions are put into effect and rigorously enforced. In the first place, two additional lanes of travel will be opened up at once on every thoroughfare where parking is prohibited, thus increasing the carrying capacity of those thoroughfares between 25 and 50%, depending on their width. In the second place (and this is perhaps the more important consideration) thousands

of motor-car owners who now use the most crowded streets in the most crowded sections unnecessarily, except in so far as they fancy their own personal comfort is promoted, would no longer do so.

The purpose of the Surface Line Operators in seeking parking restrictions is not to force the private car owner to use street cars. They would gain but a small part of this patronage; most of it would go to the taxicabs and to the subway and elevated lines, rather than to the surface roads with their relatively shorter haul. What they are interested in is the speeding up of traffic, enabling them to maintain schedules and provide the hundreds of thousands of their daily patrons with a better service and thus encourage a more general use of vehicles that engage in mass transportation. The speeding up of traffic, of course, would be as advantageous to moving automobiles and taxicabs as to moving street cars.

The merchants, or some of them, are the principal champions of parking. They are so solicitous of the luxurious comfort of the lady who drives up to their doors in a private sedan that they inadvertently are doing their utmost to make their stores and in fact the entire mid-town section inaccessible to the thousands of her sisters who travel by foot and in public conveyances. Their attitude, if continued, will result sooner or later in causing a general decentralization of retail business, which perhaps from the standpoint of public interest is to be desired; but the mid-town merchants and property owners surely will not welcome such developments. Already there is an amazing growth of shopping and amusement centers in residential sections. Bigger and better neighborhood stores and neighborhood theatres are springing up constantly and each year a larger proportion of people patronize these local enterprises instead of subjecting themselves to the loss of time and inconvenience incidental to a trip to the congested "Thirties", "Forties" and "Fifties".

Harold M. Lewis,* M. Am. Soc. C. E.—That there is a considerable inefficiency in the present use of the principal streets of American cities is clearly shown by evidence. First, it is necessary to consider what the efficient use of a street is. One cannot claim any degree of inefficiency unless he has some standard with which the traffic being carried on a street can be compared.

The Regional Plan of New York and Its Environs has made some theoretical studies of the vehicular traffic that streets of various types should carry under different conditions. One of the results of such studies is shown in Fig. 6. While the curves given were derived theoretically they are based on observations of velocity and spacing for various groups of vehicles occupying a length of several hundred feet of roadway. Rates of acceleration and deceleration were determined from observations of vehicles on Fifth Avenue, Borough of Manhattan. A formula was developed corresponding to the curve marked "Uninterrupted Traffic", which shows the number of vehicles per lane per hour as a function of the velocity, in miles per hour, assuming that all grade crossings were eliminated and that the lanes of traffic considered were entirely given over to moving vehicles. It may be seen that the capacity of

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such a street increases up to a velocity of about 15 miles per hour and then decreases slightly with higher velocities.

Few streets provide "uninterrupted traffic" as here defined, and therefore another formula was developed for conditions such as exist on Fifth Avenue, and other principal avenues, that is, a "stop-and-go" system of traffic control. The time schedule (90 sec. for motion, out of a complete cycle of 160 sec.) in operation in 1925 on Fifth Avenue was used. The curve showing this, marked "Tower Traffic Regulation—Running Velocity", indicates capacities approximately one-half those that might be obtained for uninterrupted traffic. The running velocity in this case means that maintained by vehicles after they had reached full speed, corresponding to the speed limit on the street. The average velocity is considerably less than this due to the periodic stopping for cross traffic. A curve showing the relationship to the average velocity permitted under such traffic control was added therefore to the diagram. It lags considerably behind the curve based on running velocity.

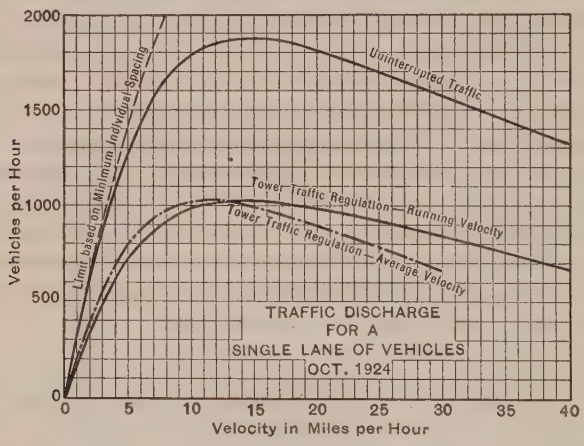

Fig. 6.

If each moving lane on a street were used to 100% efficiency the capacity of the street under different running velocities could be obtained by multiplying lane capacities obtained from Fig. 6 by the number of moving lanes. The upper curve should be used for uninterrupted traffic and the lower curve marked "Running Velocity" where a stop-and-go system of traffic regulation is in existence. No machine, however, is expected to maintain 100% efficiency and likewise no street could be expected to carry on each of its moving lanes the traffic shown in Fig. 6. It is, however, reasonable to state that a street with four moving lanes could carry on each lane about 80% of the single-lane capacities shown.

Fig. 7 indicates conditions as they existed on typical streets of Manhattan in the fall of 1924. It is based on 983 different observations of the relations between volume and velocity of vehicular traffic over stretches of roadway from 0.4 to 1.0 mile in length. The figures given within the squares represent the number of observations which come within the limits of that part of the dia-

gram. For example, there were nine observations which showed a use of between 300 and 350 vehicles per moving lane at average velocities of between 9 and 10 miles per hour.

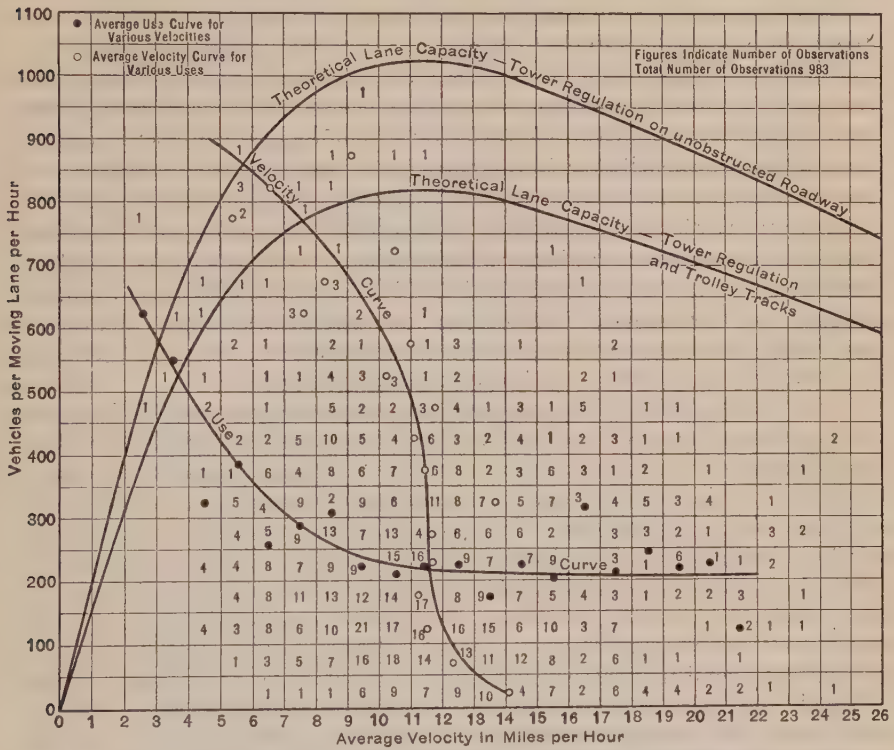
Average Use Curve for Various Velocities
Average Velocity Curve for Various Uses
Figures Indicate Number of Observations
Total Number of Observations 983
Theoretical Lane Capacity — Tower Regulation on unobstructed Roadway
Theoretical Lane Capacity — Tower Regulation and Trolley Tracks
Velocity Curve
Use Curve
Vehicles per Moving Lane per Hour
Average Velocity in Miles per Hour

FIG. 7.—OBSERVED RELATIONS BETWEEN VOLUME AND VELOCITY OF VEHICULAR TRAFFIC ON TYPICAL STREETS IN THE BOROUGH OF MANHATTAN, NEW YORK CITY, OCTOBER AND NOVEMBER, 1924.

The upper curve shows the theoretical single-lane capacity for tower traffic regulation as given in Fig. 6. This represented conditions on streets with unobstructed pavements; as only part of the observations made were on such streets there has also been added a curve showing corresponding capacities for streets with four moving lanes of vehicles and two trolley tracks within those moving lanes. If all the streets on which observations were made were being used to full capacity, the points plotted would have approached within about 80% of the figures represented by the single-lane capacity curves. It is obvious that they come far short of this.

An average use curve for various velocities is also shown. This indicates that increased use was obtained only by the sacrifice of a very considerable amount of velocity and when the latter exceeded about 11.5 miles per hour the average curve shows a practically constant use of only about 200 vehicles per lane per hour. This corresponds with only one-third to one-fourth of the traffic which those streets could have carried. The average weighted velocity of all the observations made was 11.3 miles per hour. What is the explanation?

It is not due to any general condition existing all over the Borough of Manhattan but to congested intersections and delays caused at the most serious friction points within the city. These have restricted the movement of vehicles throughout the Borough and have made impossible the efficient use of streets not only at the points of congestion but at some distance from them.

Probably it is impossible to eliminate all congestion in large cities. This can best be relieved by carrying out comprehensive traffic plans which would tend to separate through from local traffic and concentrate the former on routes planned primarily for its accommodation and laid out to furnish more direct communication between separate business and population centers. The business streets will still be called on to handle a heavy traffic, but the speaker believes that much can be done to obtain better traffic conditions on them. What is needed is not a greater running velocity or speed limit but a greater average velocity maintained between any two parts of the city. It avails a man little to be able to travel for a few hundred feet at the rate of 30 miles per hour if he must stop often because of traffic delays.

What really counts is the average speed which he can make. This can best be increased on main avenues by trying out a system of progressive traffic control signals designed so that as a motorist proceeds along a street at a reasonable running velocity each light signal would change to the "Go" signal as he arrived at its location. This is being done successfully in Chicago and Washington and it is understood that the New York City Police Department plans to experiment with such a system in New York City. It would be necessary to plan for a velocity which could be maintained by a large majority of the vehicles using the street. Where trolley cars are involved Chicago has found that this velocity must be about 12 miles per hour in order to enable the trolleys to keep up with the light signals. Where a street is used mainly for passenger traffic and is free from trolley tracks, the system could be designed for a much higher velocity. Such a condition exists on the Grand Boulevard and Concourse in the Borough of the Bronx and in other parts of New York City. If two of Manhattan's avenues could be cleared of trolley tracks and vehicles permitted to maintain a speed of 25 miles per hour the saving in time of the motor vehicles using those avenues would represent several million dollars per year.

On the main avenues coming into a city nothing can reduce delays more than the separation of grades at the most important highway intersections. Because of the great expense such a procedure is not feasible in the Borough of Manhattan except at a few of the most serious points, but the speaker believes that it is advisable and could readily be done on the main highways in suburban districts. In many such cases it would require relatively small expense.

Mr. Miller has pointed out clearly the obstruction caused by standing vehicles to moving traffic. He could not, however, obtain the full extra lane capacity suggested, as this would require the elimination not only of parking but of all stopping of vehicles. While this might be done at certain hours of the day and at certain restricted localities the speaker does not believe

that it can be applied over congested areas of considerable size. It could be accomplished in front of large office buildings, stores, warehouses, etc., which have a considerable amount of traffic to and from their doors, by compelling them to provide, within their property lines, space for the unloading of both freight and passenger vehicles. This should be done, but it is not reasonable to expect a property owner with only 20 or 25 ft. of street front to be restricted from stopping a vehicle occasionally in front of his building. There are always some such sections of property within the areas of congestion and for that reason it is not possible to apply any regulation forbidding the stopping of vehicles throughout such areas.

Mr. Miller's views of what can be done in the way of substituting buses for taxicabs seem somewhat optimistic. Conditions in Washington are probably much more advantageous for such a procedure than those in New York City. The taxicab and private automobile are far more flexible than the motor bus can ever be and will undoubtedly continue to exist in increasing numbers. Frequently the rider desires a door-to-door service. This the taxicab gives and the motor bus can never provide on any such scale.

About the only real substitute for the taxicab and private motor vehicle, in such sections as down-town Manhattan, is rapid transit service. It may become necessary to carry a large number of the people who now come in automobiles to the center of the city in modern underground rapid transit facilities for mass transportation. The transfer, at the edges of congested districts, between taxicabs or private motor vehicles and such rapid transit lines should be encouraged.

William E. Thompson,* Esq.—The outstanding point in the paper is that to create new lanes of traffic, whether it be by widening existing roadways, arcading sidewalks, cutting through new streets, or double-decking existing routes, will not afford the same permanent relief from traffic congestion as by the more economical use of existing street space by units of larger carrying capacity and by the prohibition of parking.

There are numerous proofs in New York City that the author's conclusions are sound. Some years ago Fifth Avenue was widened. More recently, First Avenue has been converted into a wide, well-paved street. Lafayette Street, Park Avenue, and Varick Street have been improved. All these betterments were highly desirable, but collectively they have failed of their primary purpose of relieving congestion. Without proper regulation, a sufficient number of the thousands of pent-up automobiles all around the congested area will trickle into and congest new arteries as fast as they can be opened.

When limited parking or no parking was first suggested as a remedy for traffic congestion, merchants insisted that such regulations would be disastrous to their business. That this viewpoint is changing and changing rapidly, is clearly indicated in the replies of about 2 000 merchants to questionnaires sent out by the U. S. Department of Commerce in connection with an exhaustive study of "Vehicular Traffic Congestion and Retail Business".†

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† *Trade Information Bulletin No. 394*, U. S. Dept. of Commerce, April, 1926.

These replies indicate that merchants are realizing more and more that vehicular congestion, caused largely, as Mr. Miller has pointed out, by parked automobiles, has reached the point where it is destroying their business by forcing decentralization and diversion of trade to sub-centers. When they do realize that, they, too, will insist on "no parking" in congested areas.

The relief to be gained from "no parking" is, of course, that it would leave the entire roadway for the use of moving vehicles except for such obstruction as is caused by loading and unloading at the curb. A distinction should be made between passenger automobiles and trucks as regards the effect of their loading and unloading at the curb. Passenger automobiles load and unload so quickly and in most cases at such infrequent intervals that it is not necessary to prohibit their use in this manner in order to obtain a large measure of advantage from the "no parking" rule. With trucks the case is entirely different and neither loading nor unloading is possible without destroying the benefits expected from "no parking". Therefore, traffic regulations must take into account the nature of the business in the street or its enforcement will be impracticable.

In the Borough of Manhattan the flow of travel in the morning is south and in the afternoon north. At least some measure of relief would be derived from "no parking" by prohibiting parking on the west sides of the main north and south arteries during the morning rush, say, from 8:00 to 10:00 A. M., so as to provide an unrestricted flow of south-bound traffic and in the afternoon the same regulations applied to the east sides from, say, 4:00 to 7:00 P. M., would provide an unrestricted flow of north-bound traffic. Certainly, in retail business and office building districts, such regulations would create an additional lane in the direction of the heaviest flow of traffic, now obstructed by parked cars. The fairness of the regulation, by shifting the "no parking" from one side of the street to the other during different portions of the day, should overcome any objections from merchants while the plan is being tried out.

As to the more economical use of existing street space, the speaker would like to add that various observations made at heavy traffic centers by the Third Avenue Railway System corroborate, in every essential detail, the facts that have been fully set forth in the tables accompanying the paper.

LEWIS A. DE BLOIS,* ESQ.—It is a question whether the entire elimination of parking, even if it were practical, would actually add another lane of traffic. The minimum practical width for a traffic lane of moving vehicles is greater than the width of a line of vehicles parked against the curb. Furthermore, drivers can not be expected to drive their cars as close to the curb as vehicles can be parked, nor would it be safe for pedestrians.

Although favoring the elimination of parking, in so far as it is practically attainable, the speaker desires to emphasize the fact that the chief causes of congestion are at "bottle-necks". In one case of interurban motor-vehicle traffic on a well-known highway, the average speed had been reduced from 30

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to 18 miles per hour because of an "S" turn in the road. The throttling of traffic caused by this was felt for a distance of almost 9 miles back from the constriction. Similar, though less clearly marked, cases occur throughout city streets, due to constriction at intersections and throttling caused by safety zones, trucks backing to the sidewalk for loading or unloading, and similar obstructions. It does not much matter how fast the traffic can be run through a city block if it cannot get out at the other end at a reasonable rate.

A parked bus throttles traffic, as shown by the congestion caused by buses collecting at street intersections to load or unload passengers, as on Fifth Avenue in New York City and Michigan Boulevard in Chicago. As for taxicabs, it is obvious that there would not be 17 000 taxicabs in New York City if they did not supply a real need. One such need is the convenient handling of personal baggage. This may account for the extreme use of taxicabs between railroad stations and hotels. It is, of course, true that the traveling public pays for empty taxicabs just as it pays for empty seats in buses and trolley cars.

E. N. JOHNSON,* ESQ.—In addition to automobile parking which Mr. Miller discusses so fully, there are other factors bearing on the efficiency of passenger transportation in city streets, which the speaker will outline briefly as follows.

Routing or By-Passing.—A considerable part of the through traffic entering the business area of any city does so not because of a desire to go there but because it does not know how to get through the city any other way. Provide circumferential streets or by-pass routes well marked with directional information, and through traffic without an errand down town will avoid the busy section. Of course, markings indicating the route to the business area should be provided for those who desire to stop there. Every through-bound automobile by-passed from the busy area helps a little to reduce the congestion and thereby benefits all forms of passenger transportation.

The "Skip Stop" Plan for Street Cars.—The adoption for street cars of the "skip stop" plan of picking up and discharging passengers has speeded street traffic in many cities by avoiding delays incident to the operation of such carriers and private cars on city streets.

One-Way Streets.—In cities having narrow streets, the adoption of a plan of one-way streets has generally had a beneficial effect on congestion. One would not think of attempting to put water through a pipe in two directions at the same time.

A protest will be raised at whatever is done toward improving traffic conditions. However, to-day's protest which seems reasonable to-day becomes absurd to-morrow. About seventy-five years ago it was proposed to widen Broadway, New York, by setting the sidewalk line back. The Broadway business men protested, and the basis of their protest was that they would lose space on which to pile cordwood.

CHARLES GORDON,† ESQ.—The whole traffic problem is a community problem and one of the difficulties with its solution has been that each individual

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† Editor, *Electric Railway Journal*, New York, N. Y.

approaches it from the standpoint of his own interests. The merchant objects seriously to restrictions on parking because he thinks it is likely to injure his business. Traffic movement is thought of as the speed with which private cars can be driven through the city streets. The public attitude toward the traffic problem is based on personal experience with the automobile. People still maintain the right to use their own personal transportation under the intensely complicated conditions of modern metropolitan cities, and opinions on transportation are sometimes slightly biased owing to that peculiar personal viewpoint.

The truckman, the man who is handling freight and commodities in city streets, thinks of traffic problems in the terms of his own business. The same is true of the trolley man, and others. The taxicab owner would seriously object to Mr. Miller's suggestion regarding the restriction of taxicabs.

A few are beginning to have a new conception of local transportation as a whole. It is apparent that the development of modern cities is dependent on the growth of transportation. The very life blood of a great city depends on its transit facilities. Interrupt the transit facilities and a most striking example of the dependence of mankind on transportation is demonstrated.

Co-ordination is the keynote of this problem. The trolley and bus interests have discussed co-ordination of the handling of passengers in city streets. Add to this the handling of freight and other services, and one begins to obtain a new conception of local transportation problems.

Co-ordination might mean the handling of freight and commodities in off-peak periods. Perhaps that might be at night. With only casual observation one realizes what a large factor is the handling of freight and commodities in and out of large buildings on narrow streets. Whether it would be best to handle these commodities in off-peak periods, to restrict the rights of individual property owners to the use of streets on the principle of the greatest good to the greatest number, are all factors in this general situation.

Street railway officials are broadening their viewpoints. The railway executive is seriously interested not only in moving the vehicles for which he is responsible, but also in the problems of the taxicab and bus operators, the private automobile operator, and all the other factors that enter into the problem. Only by this kind of an approach to this most serious problem of modern cities can hope for solution be obtained.

J. R. Slattery,* M. Am. Soc. C. E.—There are several factors affecting traffic in New York City that have not been mentioned: One is the loss of time caused by the promiscuous left-hand turning out of busy streets, such as Fifth Avenue. New York restricts the left-hand turn less than almost any other large city. For example, a car coming from the financial district to the Engineering Societies Building, 33 West 39th Street, loses, on account of delays caused by cars ahead turning to the left, twice as much time as by going beyond to 40th Street, and making three turns to the right, around the whole block.

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Another factor that slows up traffic is the failure of pedestrians to obey the signals. Pedestrians are difficult to control. Many cities do control them, however, and it could be done in New York. In driving down Lower Broadway an enormous amount of time is lost on account of the general "jaywalking" which prevails.

Another thing that delays traffic seriously in New York City is the practice of turning in the middle of the block. Traffic regulations provide that cars may turn in the middle of the block, if they do not interfere with other cars. Of course, under New York conditions they always interfere.

WILLIAM J. SHEA,* M. AM. SOC. C. E.—In his excellent paper, Mr. Miller has certainly placed his finger on the high spots in the congestion situation, and he has outlined several obvious improvements. The speaker, however, is inclined to take the view that the problem is not ripe for solution, and that it might, in a measure at least, work itself out, perhaps in the not distant future.

Acute surface congestion has been caused by the extraordinary increase within the past few years in the number of motor vehicles operating on the streets; it is reported that in the City of New York the registration of motor vehicles on May 1, 1926, exceeded that of 1925 by 14 per cent.

Undoubtedly there are a great many non-essential vehicles on the streets of New York, as well as elsewhere, due to the extravagant use of private motor cars. This applies to the commercial vehicle as well as the pleasure car. It is not an unusual thing during rush hours at a certain terminus of a subway line, to see several hundred people standing on the sidewalks waiting to cross the roadway while fifteen or twenty vehicles pass up or down the street, each with one passenger. Nor is it at all unusual to see on the street a 5-ton truck with one 500-lb. barrel as a load. Meanwhile this unrestricted and inefficient use of motor vehicles in congested areas is creating problems that appear to defy solution.

The situation is also complicated by the prevailing senseless operation of motor vehicles. For example, in a certain park in the city, which has a circumferential road, the traffic system was recently changed from two-way to one-way traffic. This is obviously the safer system and permits the most efficient use of the lane capacity of the roadway. However, numerous motorists are not content to travel in one lane but must "weave" in and out, cluttering up the whole roadway and materially reducing its capacity.

Manifestly it is impracticable to provide adequate facilities for a transportation service involving an ever-increasing number of separate units, many of which are extravagantly used and inefficiently operated much of the time.

It is probable that drastic regulations respecting the operation of motor vehicles would provoke much opposition. However, it is quite possible that the evils of street congestion and the consequent increased costs on all commercial activities will force owners and users of motor vehicles to realize the necessity of curbing the present tendency toward their unlimited use regardless of the economic factors involved in such operation. The elimina-

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tion of needless and useless operation of motor vehicles within congested areas should simplify regulation and insure a more efficient use of existing streets, while the co-ordination and efficient use of motor transportation would also improve existing conditions and permit the intelligent planning of increased facilities to meet future requirements.

Motor vehicles have become an important factor in the transportation system of the community, and it may be that owners, users, and the public generally need to be educated in the use and value of this facility. It is difficult to understand, for example, why commercial vehicles continue day after day and year after year to assemble *en masse* along the water-front streets with the certain prospect of hours of waiting before they can obtain access to the piers or approach the bulkhead sheds for the loading and unloading of freight, or why the carrying capacity of many motor trucks is so wastefully utilized as the result of under loading and empty movements. It seems that much might be done to educate the public in the use of definite traffic routes between certain points within the city, and as to by-passes around the more congested areas. The newspapers might assist just as is now being done frequently by at least one city paper in connection with automobile tours.

It is not surprising that numerous abuses attend the extensive use of motor vehicles, such as the parking nuisance. There is no precise definition for parking despite its serious interference with street traffic, consequently its regulation is difficult and the streets have become public garages for many vehicles. Probably the conflict between moving and standing traffic will continue until the slowing up of motor transportation and increased costs will make it economical to provide loading and unloading facilities within the building line of all large commercial establishments. Already tail-board loading or unloading is prohibited on three streets in New York City and it would seem that this prohibition could be made general with substantial advantage accruing to street traffic. The abuse of the parking privilege is illustrated by the Long Island motorist who parked his car in the vicinity of Queens Plaza of the Queensborough Bridge early in the morning; when he claimed it in the late afternoon he got into a discussion with the police officer who had had the car under observation for several hours and the motorist cheerfully admitted that he had spent the day on a business trip to Newark. One wonders what rights, if any, to the occupancy of a public street for hours at a time are inherent in the ownership of an idle or temporarily abandoned motor vehicle.

It appears to the speaker that out of existing conditions in congested areas will grow reforms from within the ranks of motor vehicle owners and users collectively, that will involve the repression of other than necessary traffic, the elimination of inefficient loading and empty movements, the scheduling of movements so as to avoid concentration of standing vehicles at points of commercial activity, the provision of parking garages and "off the street" facilities for transferring freight, the more general use of night trucking, and the co-ordination of all activities requiring the extensive use of motor

transportation. With this accomplished the problem of traffic congestion should resolve itself into definite terms, regulation will be simplified, and the planning of increased traffic facilities can be based on a more accurate estimate of what future requirements will be, rather than on what appears to be the present tendency, namely, to attempt to provide at public expense street and roadway facilities for the continued unlimited use of motor vehicles.

HAROLD H. DUNN,* ESQ.—Cities as a rule have been laid out to conform to the needs of a horse-and-pedestrian age when centralization of business and housing were necessary in order to conserve time and energy. No one foresaw the phenomenal growth of automobile use and the development of rapid transit facilities that has taken place in recent years and has so increased the radius of activity as to cause cities, through unforeseen expansion, to outgrow their centralized business centers.

Reliable data show that the day population of the business center of most large cities is equivalent to one-half or two-thirds of the population of the whole city. In Boston, Mass., for example, it is estimated at 500 000, or two-thirds of the total city population; in Pittsburgh, Pa., it is estimated at 355 000, or one-half of the city population; and the Regional Plan of New York estimates that the business day population of the 8.35 sq. miles in New York City's business center south of 59th Street is 2 940 000, equivalent to one-half the city's population.

Many of these people use means of individual transport, such as automobiles and taxicabs, and countless others are only deterred from so doing by the increasing traffic congestion. The registration of automobiles in the United States has doubled since 1921, and there is now an average of 1 automobile to each 1.5 families. It is fast approaching an average of 1 to each family. What the public obviously wants, and rightly so, is the greatest freedom economically possible in the use of the automobile; but unrestricted use is impossible if one-half or two-thirds, or even one-fourth, of the city's population wants to use the limited street space of the ordinary business center at the same time.

Replies to a questionnaire recently sent to several thousand women shoppers from Westchester County (adjacent to and north of New York City) indicate that only 9.1% of those who shop in New York City do so by automobile regularly; an additional 16.7% shop infrequently by automobile; and a great many find it necessary to park their automobiles in garages outside the business district. The replies showed, however, that even a much larger percentage of these customers of New York stores desired to use their automobiles for shopping purposes but were unable to do so because of traffic congestion. A similar survey reported by the *Electric Railway Journal* as made by several department stores in Chicago, Ill., among customers whom they thought from appearances would be likely to have arrived by automobile, shows that only 11.1% shopped by automobile and that 17.8% of this number left their automobiles outside the Loop District. Obviously, there is a veritable flood of automobile traffic which is only held back because of the difficulties presented by traffic congestion.

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The author has presented a convincing argument for the adoption of two means of increasing the efficiency of passenger transportation in streets, and thereby relieving the serious conditions now prevailing in business centers. The economic losses due to automobile parking in the streets of a business center are enormous and are wholly unjustified by the benefits accruing to the small proportion of the general public that is able to make use of this privilege. The reduction or elimination of parking and the substitution of buses for individual conveyances on much traveled routes should result in marked improvement of conditions.

The attempt, however, to overcome street congestion, while at the same time retaining the present centralized form of city development, is quite similar to the proverbial attempt to lift oneself by his boot-straps. Each increment of relief only opens the way for new increments of population, traffic, and building in the central district, and no one can foresee where and when a balanced development will be reached.

In recent years there has been a decided trend toward enlargement of the business area and decentralization, a trend which is coincident with the rapid development of the automobile and the consequent enlargement of the radius of activity. Decentralization opens the way for more efficient use of the central areas and particularly of its streets. It would make it unnecessary for thousands of workers to spend hours of their time each day in going to and from their work in the central area, would relieve the morning and evening congestion of transit facilities, and by making their load curve more uniform would permit them to operate more efficiently.

Sub-centers have been developed in many cities, but they have followed the old conventional lines and, as they grew, their usefulness has been throttled by traffic congestion. The modern business center must be planned to suit modern conditions, chief among which is the apparent determination of people to shop by automobile. Sub-centers will fall short of their purpose and possibilities if they are not planned to avoid the difficulties and drawbacks with which their "parent" in the heart of the city is beset. Off-the-street parking or garage space must be provided for customers' and employees' cars. It seems probable that the time is coming when this will be generally recognized as an element of city building relatively as important as adequate stairways, elevators, and heating plants. In fact, progressive merchants, singly and in groups, in various localities have already given recognition to this necessity by making such provision; and they are profiting from it. The shopper's questionnaire previously mentioned shows that the New York City merchants who make provision for customers' cars are obtaining the largest proportion of the Westchester County automobile trade and presumably, therefore, of all the automobile trade.

Max Miller,* Assoc. M. Am. Soc. C. E.—The speaker is one of that large and condemned class, the parkers. When he had to go twice within a fortnight through the unpleasant ordeal of paying a fine and losing half a day in the Traffic Court, he thought of an idea for making parkers (who are a necessary evil) sufficiently inoffensive to be tolerated.

* Gen. Contr., New York, N. Y.

The total elimination and ruling out of all parking within business zones is impracticable, because an ignorant person like the speaker (one of a class numbering 5 000 000, or thereabouts) will be likely to ask a very foolish question, "What is the use of an automobile, if a person is not to be allowed to go anywhere with it within the business zone?" Many business and professional men have to make fifteen or twenty stops during the day; and the automobile has come to be a necessity to them in the transaction of their business.

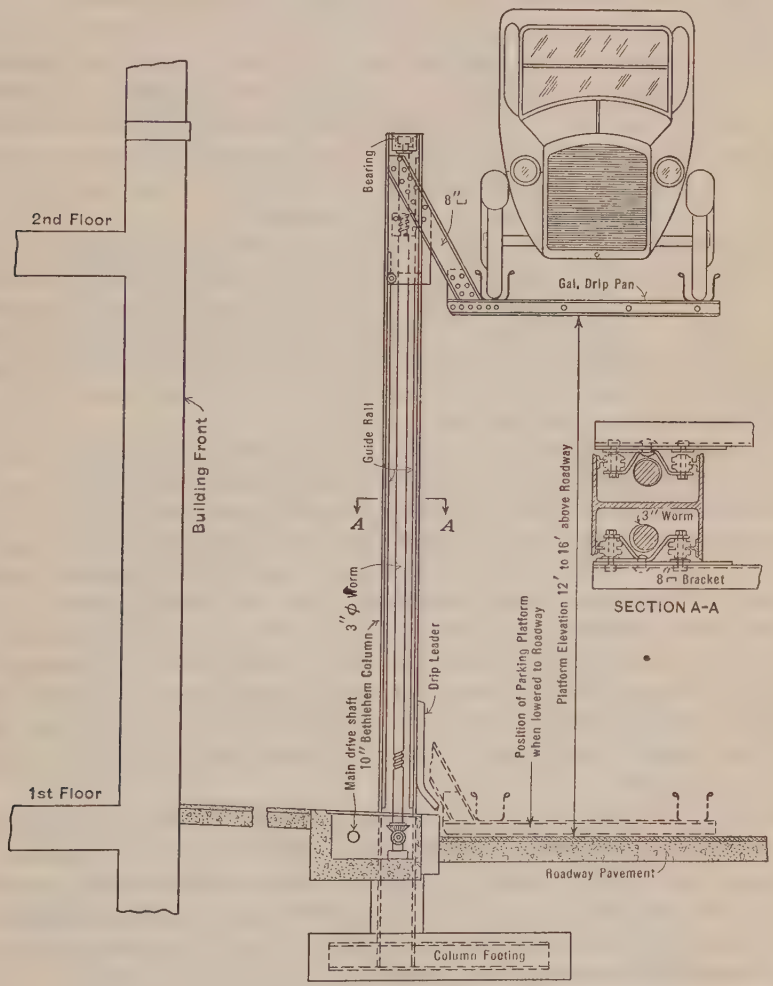

FIG. 8.—GENERAL SECTIONAL VIEW OF PROPOSED PARKING SYSTEM.

The parking system herein proposed, Fig. 8, consists of a vertically moving platform extending over the roadway at each side, adjacent to the curb. The platform is subdivided in lengths such that each subdivision is just sufficient to accommodate one automobile and may travel independently.

A series of steel posts about 16 ft. apart, located on the sidewalk, just inside the curb line, support and guide the platforms. The power and drive for the system is provided by a motor and shafting located in a small trench immediately under the sidewalk at the curb. A single motor can be arranged to operate all the platforms on one side of a block.

The platform, being essentially an elevator, is provided, of course, with all the safeguards so well developed in elevator construction, support being furnished directly by the solid steel driving shafts acting in tension.

The operation of the system would be as follows: The operator employed to attend the block lowers the platform to the roadway. The driver brings his car to position, moves forward on the platform and alights. The operator sends the platform up to its normal elevated parking position, the entire procedure taking no more than 1 min. The raising of the cars can readily be done in moving traffic, the lowering, however, will be accomplished with greatest safety when traffic on the block is stopped.

The advantages of this system of parking automobiles in busy sections of cities are numerous and self-evident. First and foremost, would come great relief of present traffic congestion by almost doubling the capacity of streets now available for moving traffic. Furthermore, the greater freedom afforded traffic by an increase of possibly 67% over present available roadway width would accelerate the movement of this traffic, making the actual increased traffic capacity about 100 per cent.

Car owners will not be limited as to parking time. Moreover, the car, its contents, and accessories will be absolutely proof against theft. It will not be mud-bespattered or injured by passing cars, or those getting in or out of parking position next to it. The ease, simplicity, and directness of parking accommodations, together with acceleration of travel by this parking system, will save the traveling business man from 1 hour to 2 hours per day.

The parking platforms and automobiles do not cover or obstruct the sidewalk so that the entire building fronts will be visible as now. Although the third-story windows will be obstructed from view, the first and second story window displays, which are the most important, will be clearly seen from across the street, whereas they now are obstructed from view by automobiles parked at the curb.

The plan here offered has the following favorable features: (1) Reasonable first cost; (2) it can readily be made self-supporting and even revenue-producing under municipal operation or private operation under a public franchise; (3) it would not alter or in any way affect present or future buildings, or any construction existing under city streets; and (4) rapid installation with little interference with present street traffic.

With this plan crimes, depending for their execution on a car waiting at the curb, would be made practically impossible, for a still-standing car at the curb would immediately be "spotted." The system would not interfere with the Fire Department's work and may, in fact, be of some assistance to it. The shed effect of the platforms would keep two strips of the roadway clear of snow. It would release a considerable police force now engaged in parking

regulation work to the more useful and urgent duty of mobile traffic regulation. In giving thought to the question of the possible adoption of this plan approximate computations indicate that a charge of 10 cents per hour for parking could be made to pay well for the installation, operation, and profit on the investment involved. Any one having to put up with the inconvenience of parking in Lower Manhattan would gladly pay this charge.

William T. Lyle,* M. Am. Soc. C. E. (by letter).—The author makes a clear case for the elimination of automobile parking from the business districts of large cities. In order that an anti-parking ordinance may be effective, however, the limits and complexion of the business district must be clearly established. Evidently, in a city where warehouses and light industries are interspersed among banks, drygoods stores, etc., such an ordinance would lose much of its potency, since the necessary loading and unloading of merchandise and raw materials would block the streets to a considerable degree.

Thus, the need of a zoning ordinance becomes evident. Without it a clearly defined business district is impossible, but with it, by the regulation of the "use" of buildings, the regulation of traffic becomes greatly facilitated.

Objectionable loading and unloading of trucks can be eliminated by the proper provision of alleys and loading spaces to the rear of the buildings. Such provision can be made in the city plan, as has been done in several American cities.

Thus, the intimate relation of the traffic problem to zoning and the city plan is evident. Passenger transportation is improved by proper parking regulations and the elimination of cruising taxicabs, but an even greater improvement can be effected by a more comprehensive regulation in connection with the general city plan.

In certain communities parking should not only not be forbidden but should be positively encouraged. Such communities are found along the great National arterial highways. The income from tourists every year is becoming greater. It is a hardship to compel the tourist to park his car at a distance from the place where he is stopping; in fact, he will not do it, but will drive on to the next town where greater courtesy is manifested. The tourist must be encouraged to park in the business district. Proper provision for this can be made by street widening. There are many ways of doing this, and these methods are quite generally understood by engineers.

T. Kennard Thomson,† M. Am. Soc. C. E.—The speaker would like to illustrate two projects which would give enormous relief to Manhattan. The first (Fig. 9) is a belt-line railroad around the Island, with an elevated street, 100 ft. wide, above it, and an automobile speedway, of the same width (it is shown as 36 ft. wide in Fig. 9), above the elevated street. The speedway should only come down to the elevated street level, say, every 2 or 3 miles.

Obviously, the present street level is the most economical place to put the railroad tracks, which should be at least six in number. The outside tracks

* Prof. of Civ. Eng., Washington and Lee Univ., Lexington, Va.
† Cons. Engr., New York, N. Y.

would be used only as unloading sidings; the next two as switch tracks to feed the sidings; and the center tracks for express trains—all for freight only.

An elevated unloading platform, 20 or 30 ft. wide, should extend all around the belt line, on the inside. This would enable "store door" deliveries, and save from two days to two weeks, or more, of the time now required to deliver much of the freight in New York City.

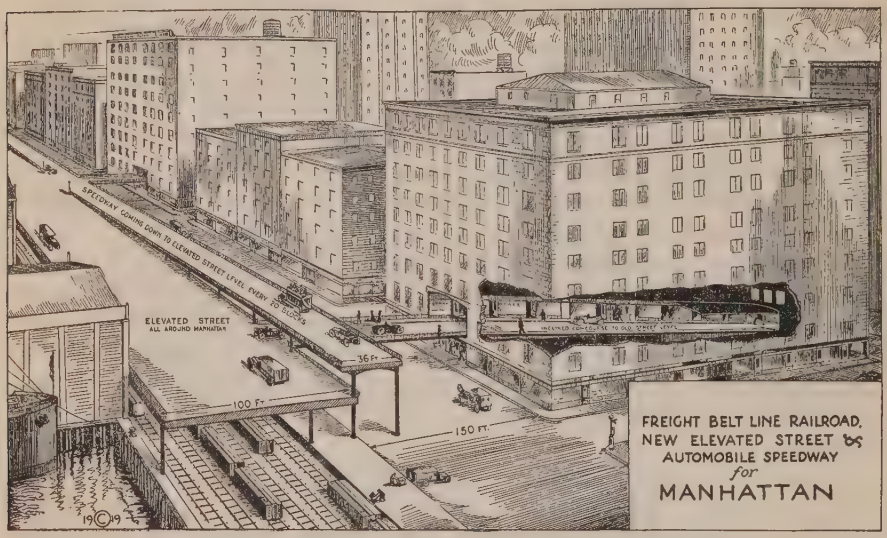
SPEEDWAY COMING DOWN TO ELEVATED STREET LEVEL EVERY 10 BLOCKS
ELEVATED STREET ALL AROUND MANHATTAN
36 Ft.
100 Ft.
150 Ft.
FREIGHT BELT LINE RAILROAD, NEW ELEVATED STREET & AUTOMOBILE SPEEDWAY for MANHATTAN

Fig. 9.

The key to this problem is how to get from the elevated street to the ground. To run inclines down West Street, or down the present cross streets would ruin them, as they are not even wide enough for the present traffic. The solution would be in building inclined ramps through the center of the blocks. If at first these ramps were built every 2 or 3 miles, the property owners in every block would soon demand similar ramps in their buildings. Not only would this enormously reduce the time and cost of handling freight in Manhattan, but it would enable automobiles to go down town with speed and comfort, relieving the congestion on other streets. In winter it would have a further advantage of providing two streets unhampered by snow.

The second project is to run 42d Street under Fifth Avenue. As Fifth Avenue at 42d Street is 10 ft. higher than Madison Avenue (on the east) and Sixth Avenue (on the west) its grade would be lower than it is now. The same solution, of course, could not be used at all Fifth Avenue crossings, but it would be quite possible to treat each crossing as a separate part of the whole problem in such a way that traffic up and down Fifth Avenue would never have to stop for the cross-town vehicles, any more than the latter would be delayed by the Fifth Avenue business.

These two solutions would increase the taxable value of New York property, and save an amount of time and money almost beyond computation.

J. P. Snow,* M. Am. Soc. C. E. (by letter).—The introduction of motor vehicles on highways is giving transportation a "jolt" comparable in some degree to that given it one hundred years ago when George Stevenson drew a train of cars on iron rails by a steam locomotive. The jolt affects people in many ways, and severe street congestion in cities is one of them. The author suggests the elimination of parking and goes so far as to recommend keeping private cars off certain streets, with the idea of confining the use of these streets to buses and electric cars.

Public highways were designed and built for free use by private vehicles, and owners of these vehicles have a natural right to drive them into town if it is profitable for them to do so; they, and not the public, are the judges as to this profitableness. The right of any person to park a vehicle on the highways so as to interfere with their legitimate use by others is another matter, wholly subject to public regulation. It is probably useless to consider prohibiting private cars from being driven over streets on which buses and other carriers are allowed. Without doubt efficiency in the number of passengers accommodated in a given time could be obtained by buses; but efficiency of this sort is not the only factor to be considered.

The elimination of parking will not prevent congestion because this occurs at intersections where there is no parking. The chief objection to parking anyway is that it prevents the objector from committing the same offense himself. Other means of handling the problem than by these two suggestions are certainly pertinent to the discussion.

The worst congestion occurs at intersections, due to the crossing of streams of traffic at grade. Separating grades is awkward, unsightly, and expensive. The present stop-and-start method causes long delays when long queues of vehicles have to be accommodated. In fact, the volume of traffic is reduced by this method to less than one-half the possible clear-street capacity if both streets are equally used.

What may be called the zigzag system will permit constant movement by vehicles on all streets except an occasional short stop to allow pedestrians to cross. A large flock of pedestrians can be accommodated in a fraction of the time required when strings of vehicles must pass, as now.

The first requisite under the zigzag system is that all streets must be rigidly restricted to one-way traffic; the second is that streets in one direction be selected as principal thoroughfares or arteries and those crossing them be considered as cross-streets. Movement on the arteries shall be in one direction throughout their length, while that on cross-streets shall reverse at each crossing of an artery. Fig. 10 shows a checker-board layout comparable to several of New York's avenues and streets.

This system allows no direct crossing of the path of one car by another car. Merging into a line of moving cars is far different from crossing that line. The cross-town car merges into an artery lane, slides over into the next lane in traversing the block, turns into the next cross-street, and at the next artery turns back to enter again the street that it just left. If the blocks are square the distance traveled by the cross-town car is twice the direct dis-

* Cons. Engr., Boston, Mass.

tance actually accomplished; but if stops for pedestrians are avoided, the time will be less than is required by the stop-start system. In the New York street layout (Fig. 10), the distance traveled by the cross-town car will be 1¼ times the air-line distance.

Pedestrian crossings under this system will be more efficient than at present because all vehicles will be stopped and the whole intersection will be clear, so that the four crosswalks and the two diagonals can be used simultaneously. Under this system and with one-way traffic under any system, only one-half the street corners need to be rounded. The rounding should be generous, with radii of at least 12 or 15 ft. which should be insisted upon whenever a corner is rebuilt.

FIG. 10.

In truly congested sections of a city where the zigzag system of cross-town traffic is required no surface rail traffic can be tolerated. In a congested section rails must be either overhead or in subways. Surface rails in streets are suitable only in suburbs.

With continuous movement an enormous traffic can be accommodated. Where pedestrian stops cannot be tolerated, overhead crossings with escalators can be installed at a cost that would not be prohibitive. Speed and other details of regulations should be established and enforced. A single lane of moving vehicles will be ample for some streets, a double lane will be needed in others; in but few streets will all the roadway between curbs be needed for legitimate movement. At reasonable speed it should always be possible for a car to stop for one or two people to enter or leave; longer stops should be prohibited. Lanes for traffic should be plainly defined by paint lines on the pavement. In many streets these lanes can be arranged to leave parking space either at one or both sides against the curbs and in exceptional cases in the center.

A distinction in terminology should be made between "ranking" cars parallel to the curb and "parking" them at an angle. From the curb to the traffic boundary 8 ft. is sufficient for ranking and 18 ft. for parking. The latter is preferable, and if on only one side of the street it should be the right-hand side as traffic flows, with regular stall lines painted on the pavement. These stalls may be at angles of 30 to 45° with the curb, and should be entered head on from the traffic lane whichever side of the street they may occupy.

If ranking or parking space is provided there is no logical reason why the city should not derive a revenue from it. The highways are provided for moving vehicles and not for storage purposes; but when storage can be allowed without sensibly delaying traffic it is wise to allow it under regulation and

at a price. No car owner can claim an inherent right to occupy street area for free storage of his car; hence he should pay for the privilege or stay away. Police are now employed to tag cars that stop too long at a given place, and they could as well collect the prescribed toll. Bus lines could rent standing room permanently for their regular stopping berths at which to discharge or receive passengers and abutting merchants could rent space for the shipping and receipt of goods.

The one-way feature here recommended, the adoption of which the writer believes to be imperative in order to get the greatest efficiency in street service, reduces the necessary width of streets materially. Sidewalks should be widened where needed. When streets are repaved the space outside the traffic lanes may well be surfaced with cheaper pavement than is required by traffic. Good bituminous macadam is ample for these areas.

Where real rapid transit systems are in operation it would seem rational for a driver to park his car in some field arranged for storing near a suburban station of the system and go into town on a train. The fares plus the charge for outdoor storage could hardly be more than a fair charge by the city for street parking, and the time and risk should be materially lessened. Buses or other carriers operating on the surface cannot properly be called rapid transit.

Theodore T. McCrosky,* Jun. Am. Soc. C. E. (by letter).—This paper is an excellent presentation of the present traffic situation in the larger cities, and suggests valuable methods of aiding in the solution of the problem of congestion.

The writer believes that the discharge capacity of city streets is determined not so much by their width as by the size and complexity of their intersections. Street intersections must be enlarged. In busy sections, a great enlargement in most cases would be prohibitive in cost, even were it physically possible. Fortunately, there is a means of increasing the size of an intersection which comes under the head of minor changes.

It was found by experiment that the turning radius of a certain automobile having a 132-in. wheel-base was 17 ft. 4 in. Many cars have an even longer radius. Furthermore, when a car turns a corner, it does not follow a circular curve, since the steering wheel must be turned gradually. In order to avoid the necessity for the car to swing out toward the center of the street it is entering, and thus waste roadway space in the intersection, not only should the curb corners be of long radius, but also, when feasible, of varying curvature. Such curves would be difficult to locate, and a simple but satisfactory substitute plan is illustrated in Fig. 11. The 25-ft. radius has been chosen for purposes of illustration, as representing the ideal when sidewalk widths and intensities of pedestrian traffic permit its use.

In the case of heavy pedestrian traffic, this type of curve would cut off too much sidewalk space, but in no case should the curb corner be cut back on a radius less than the width of the narrower sidewalk; and a radius equal to the width of the wider sidewalk is usually practicable. When pedestrian traffic is fairly light, the method illustrated can always be used; and it can

* Instr., Sheffield Scientific School, Yale Univ., New Haven, Conn.

also be employed when it is possible to pass pedestrian traffic under the corner of the building. This is done by cutting back the corner of the building on the first floor, and supporting the upper stories on a column. A great many shops now provide this facility in order to gain the advantage of having their door directly on the corner.

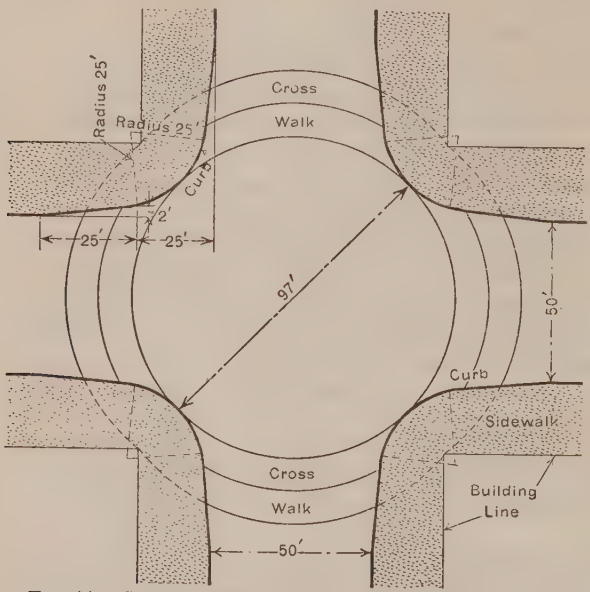

FIG. 11.—STREET INTERSECTION. PROPOSED METHOD OF LOCATING CURB CORNERS AND CROSS-WALKS.

With curb corners cut back as illustrated in Fig. 11, cars making right-hand turns would be able to hug the curb all the way. It would be advisable to prohibit the ranking and stopping of vehicles for an appreciable distance back from the curb line of the cross street. This restriction gives an extra traffic lane on each side of each street at the intersection, which is precisely where extra space is most needed. At the intersection of two 50-ft. roadways, the diagonal distance between curb corners cut on 6-ft. radii is 75.6 ft. When the radii are increased to 25 ft., this diagonal distance becomes 91.3 ft. This, again, shows how the size of an intersection may be increased by this comparatively simple means. By the curve and tangent method shown in Fig. 11, the diagonal distance is increased to approximately 97 ft.

It has been argued that long radius curves make it harder and more dangerous for pedestrians to cross the street. The writer feels that this objection does not hold when cross-walks are marked out in curves (Fig. 11). Furthermore, observation at the corners in the vicinity of Times Square, New York City, indicates that small radii are more pernicious than large. The pedestrian traffic in this district is colossal in the evenings on account of the theater crowds. Instead of waiting on the sidewalks, where they should, pedestrians stand in the roadway, about 3 ft. from the curb, knowing from

experience that most automobiles have to swing the corner at least that far from the curb. Then an automobile approaches which can turn on a 12-ft. radius. The pedestrians try to back up and get back on the sidewalk, but are unable to do so on account of the crowd behind them. Many accidents result. If the corners were cut back so that the majority of automobiles could follow them closely, in a short time the pedestrians would realize that vehicles practically always turn close to the curb, and would soon learn better than to stand in the gutter as they do now.

Cross-walks should be curved as illustrated in Fig. 11. All arcs are circular. In this way, the pedestrians are kept a little farther from the congestion in the intersection, are less in the way, and run less danger of being hit by cars turning the corners. At intersections at which is installed the rotary system of traffic regulation, it is particularly desirable to lay out the cross-walks in this form.

The writer is of the opinion that it would not be fair to prohibit all ranking along street curbs, nor would it be possible. A driver would still retain the privilege of stopping for the instantaneous picking up or depositing of passengers, or for the expeditious handling of goods; and such stopping, as has been pointed out by numerous authorities, would rob the street of a lane of moving vehicles for some distance in front of and behind the standing car, as operators will not drive next to the curb for fear of getting "boxed" behind a stationary car. It is none the less perfectly fair to refuse ranking space to the selfish driver who wishes to leave his car on the public street all day long. If a pedestrian likewise elected to stand on the sidewalk at a busy corner, a policeman would tell him to move on. All drivers must have the privilege of stopping at the curb for a reasonable time, except close to street intersections, and this privilege becomes impossible of realization when certain drivers elect to block the curb all day with "dead" vehicles.

The relief of traffic congestion is an engineering problem, and the solutions of it must be economic solutions. Such ultimate means of relieving congestion as sidewalks arcaded on the second story, and, eventually, buildings standing on stilts, thus providing running and parking space beneath them for vehicles, are bound to come. Engineers and architects would do well seriously to consider providing some such facilities in future buildings. The writer agrees fully with Mr. Miller that, in the meantime, traffic engineers can do much to increase the efficiency of transportation in city streets without materially altering the existent curb and building lines.

F. Lavis,* M. Am. Soc. C. E. (by letter).—The writer is much more nearly in accord with Messrs. Stevens, Lewis, Slattery, and De Blois, than with the author, in so far as they point out the fact that the capacity of many streets or highways is limited to a far greater degree by obstructions such as cross traffic, etc., than by parking.

There may be some exception to this (in degree) in the case of certain "Main" streets in country towns or villages, where main-line highways pass through and where the cross traffic may be light and parking the real obstruc-

* Cons. Engr., Jersey City, N. J.

tion, but in city streets, especially in New York—around which the discussion seems to have centered—the cross-traffic interruptions are much the more serious factors in decreasing the efficiency and capacity.

In connection with the study of certain economic factors governing the design of the location of the main-line, through highway that the State of New Jersey is building through the congested Metropolitan Area on the west side of the Hudson River, calculations were made by the writer and S. Johannesson, M. Am. Soc. C. E., of the costs of delays at a street intersection of two main-line highways, each capable of carrying four lanes of traffic.

It was assumed that the main through highway would have a traffic of 20 000 000 vehicles per year and that at the proposed crossing the main-line traffic could proceed for 3 min. and the cross traffic for 1 min., alternately. It was further assumed that there would be measurable loss during only 15 hours each day, when traffic was regulated, and that the loss on two lanes in one direction would be only about one-third that of the other two lanes on the highway moving in the direction of maximum flow of traffic.

Without going into details of the computations, it may be stated that the actual loss in car-minutes per day was calculated to be:

On the main highway	14 532	car-minutes
On the cross street	9 000	" "
Total	23 532	car-minutes

Assuming an actual loss of 2.2 cents per car-minute (an average for the proportion of various types of vehicles assumed to use the main highway), the total monetary loss was calculated to be $156 000 per year, equal to a capitalized value of about $3 000 000.

Inasmuch also as in this particular case the State of New Jersey is building, at a very considerable cost, this new highway to take care of the traffic increase in this territory, the question of traffic capacity is an element of importance, and it was found that the delay at this assumed crossing had a monetary value of another $3 000 000 due to the decrease in capacity on account of the delays. These calculations are believed to be conservative, and can be amply justified. They tend to confirm the statement made at the beginning of this discussion in regard to the importance and cost of the delays due to cross traffic.

The only solution of the traffic congestion problem which appeals to the writer is that now being put into effect in New Jersey on the highway referred to, namely, the entire separation of through from local traffic. It involves large expenditures of money, but the most conservative estimates of losses to the operators of motor vehicles will show in many, if not all, of these cases that there is hardly any expenditure for this purpose which may be considered at all within reason, but what can be justified from an economic point of view.

John A. Miller, Jr.,* Assoc. M. Am. Soc. C. E. (by letter).—Elimination of parking on Fifth Avenue, New York City, during the 1926 subway strike resulted in an increase of more than 50% in the speed of vehicular movement between 32d and 52d Streets. On June 17, 1926, the writer found

* Associate Editor, *Electric Railway Journal*, New York, N. Y.

by careful investigation that the average speed of north-bound private automobiles in this area between 3:00 P. M. and 6:00 P. M., was about 4.3 miles per hour. On July 28, with anti-parking regulations in effect, a second check showed an average speed of approximately 7.2 miles per hour. Other conditions on these two dates were so similar that the elimination of parking must receive the major part of the credit for the improvement.

It has been stated by Messrs. Stevens, Snow, McCrosky, and Lavis, that the capacity of streets is determined by the intersections. That theory is only partly true. Of course the elimination of all crossings at grade would greatly increase the capacity of any street; but such elimination is a dream of the future. For the present cross movements are unavoidable, and the problem is to move the greatest possible amount of traffic during the period that the intersection is open.

If vehicles move across an intersection in six lines instead of four, the number crossing in a given time is increased 50 per cent. That was what happened on Fifth Avenue when parking was eliminated. On the day when the second of the two observations was made, north-bound vehicles crossed the 42d Street intersection in three lines, while on the first day they crossed in only two lines. The same space was available at the intersection on both occasions, but on the earlier date automobiles parked along the curb between 42d and 43d Streets created a bottle-neck on the far side of the crossing.

Undoubtedly much can be done to improve traffic conditions at street intersections. The full width of the roadway should be available for moving traffic. Left-hand turns prevent the most efficient utilization of space at an intersection and should be prohibited at heavy traffic points. This was brought out by Mr. Slattery. Cutting back the corners to permit vehicles to turn on a longer radius while yet hugging the curb, as suggested by Mr. McCrosky, might prove advantageous in many places. Better control of pedestrian movement certainly would help. All this, however, will do little good if the roadway space between intersections is not used to the best advantage—a condition that does not now exist. Detailed consideration of these matters, however, lies outside the scope of this paper.

The argument has been advanced by Mr. Stevens that elimination of parking will not create an additional traffic lane because a certain number of vehicles will always be stopping along the curb to pick up or discharge passengers. This argument assumes that these vehicles are in the habit of drawing up close to the curb, but such is not the case. Often a solid line of parked automobiles renders the curb inaccessible. Consequently, vehicles picking up or discharging passengers stop in a second line in the middle of the roadway. Thus, the elimination of parking will actually create an additional lane for moving traffic despite the necessary stopping of occasional vehicles.

It cannot be denied that there is some curtailment of the rights of the individual when parking is restricted, but Mr. Long goes rather far in saying that this is contrary to the American spirit. Restriction of individual rights for the common good has been practiced since the beginning of civilization.

The "no parking" rule is only a new example of a well-established principle. Most cities have forbidden sidewalk obstructions. As Mr. Johnson points out, to-day's protest becomes absurd to-morrow. If more people benefit by regulation than suffer thereby, it is clearly justified. Of this the public and not the individual must be the judge.

TABLE 6.—Passengers Carried by Public and Private Vehicles.

Location.	Number of persons using public transportation vehicles.	Number of persons using private automobiles.	Percentage by private automobile.
Fifth Avenue, New York, N. Y.	19 224	3 220	13.9
Broad and Market Streets, Newark, N. J.	38 000	6 000	13.6
Business district, Detroit, Mich.	57 150	18 281	24.1
Business district, Baltimore, Md.	58 859	12 509	18.1
Business district, St. Louis, Mo.	69 768	17 601	20.1

From all available evidence it appears that the beneficiaries of parking restriction would far outnumber the sufferers. A careless assumption often is made that nearly every one has an automobile and wants to park. The fact is that only a small fraction of the users of the public streets ride in private automobiles. Investigations made by the writer concerning traffic conditions on Fifth Avenue, New York, showed six times as many passengers riding in public transportation vehicles as in private automobiles. Traffic checks made elsewhere show similar conditions. Even in Detroit, the greatest automobile manufacturing center in the world, more than three-quarters of the users of the street ride in public transportation vehicles. Figures from a few typical cities are given in Table 6.

TABLE 7.—Means of Transportation Used by Store Customers, Pedestrians not Included.

City.	Number of stores.	Number of persons using public transportation vehicles.	Number of persons using private automobiles.	Percentage by private automobile.
New York, N. Y.	13	34 439	815	2.3
Chicago, Ill.	4	13 549	1 680	11.0*
Detroit, Mich.	27	158 801	36 272	19.1
Cleveland, Ohio	22	69 419	20 313	22.7
Baltimore, Md.	3	38 433	1 712	4.3
Brooklyn, N. Y.	1	28 800	2 640	8.4
Los Angeles, Calif.	1	17 759	2 058	10.4
Total	71	361 200	65 488	15.3

* Customers questioned in this investigation were specially selected as being of a type likely to have come by automobile.

An even smaller proportion of private automobilists is shown by investigations made to determine the means of transportation used by customers to

reach retail stores. Of 426 688 customers checked at 71 stores in various cities only 15.3% came by private automobile. Results of these counts are shown in Table 7.

Not all the users of private automobiles are "parkers". Many are en route and do not wish to stop. Possibly this is why the store counts show a smaller percentage of automobilists than the street counts. Many others leave their automobiles in garages. It is difficult to tell exactly what proportion of the users of private automobiles actually are "parkers". Figures on this phase developed by the four Chicago department stores mentioned in Table 7 show 452 automobiles parked free in public streets, as compared with 369 using paid storage space. It is safe to say that not one person in ten really profits by the parking privilege.

No one has suggested that elimination of automobile parking on important streets offers a complete solution of the problem of traffic congestion; but experience has shown conclusively that it does afford relief. It can be done at once. The cost is practically nothing. A large majority of the public will benefit by the greater amount of roadway space available for moving traffic.

AMERICAN SOCIETY OF CIVIL ENGINEERS
INSTITUTED 1852

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Paper No. 1616

STREAM REGULATION WITH REFERENCE TO IRRIGATION AND POWER*

By J. C. Stevens,† M. Am. Soc. C. E.

With Discussion by Messrs. Edward H. Sargent, W. G. Hoyt, F. F. Henshaw, Barry Dibble, Ralf R. Woolley, Arthur P. Davis, J. W. Swaren, William B. Landreth, J. D. Galloway, and J. C. Stevens.

Synopsis

The regulation of the water of a stream to render it available for both power and irrigation is largely a local problem.

Power and irrigation demands are usually so widely different that a decision must be reached as to which use shall be given the preference, and rarely do the two demands dovetail so completely that there is no conflict.

In some sections, notably in California, a source of power is being rapidly developed that is purely a by-product of irrigation. Where large reservoirs are constructed for the storage of irrigation water, plants are being built to develop power when and as irrigation water is released from them. On some streams by reason of the law of priority of water rights the opposite is true and irrigation use is secondary to power, agriculture by irrigation thus becoming a by-product of industry.

The purpose of this paper is to inquire in general terms into the varied conditions where power and irrigation uses are of importance and to ascertain whether it is possible to aid in providing against a conflict of interests where new developments are in prospect, or in harmonizing to some extent conflicts that already exist.

Stream Flow Characteristics

In general, the more uniform the flow of a stream the more useful it is in its natural state, whereas the greater its variation the greater the necessity

* Presented at the meeting of the Power Division, Salt Lake City, Utah, July 9, 1925.
† Cons. Hydr. Engr. (Stevens & Koon), Portland, Ore.

for reservoir regulation. Uniformity of flow is, however, of secondary consideration in water-sheds where there are feasible reservoir facilities to conserve the entire supply.

In the arid West there is much more land suited for irrigation than there is water to supply it. Practically all land that can be irrigated from the unreserved supply has been under irrigation for many years. The expansion of irrigated areas, therefore, must depend in the main on reservoired water supply. On this account it is safe to state that in the arid West the important river characteristic is total annual water yield rather than uniformity of flow.

It is obvious that if the total annual yield of a water-shed is to be used for irrigation, sufficient storage capacity must be provided to make that supply available during the irrigation season, and it makes no great difference whether that supply accrues gradually throughout the year or within a few months.

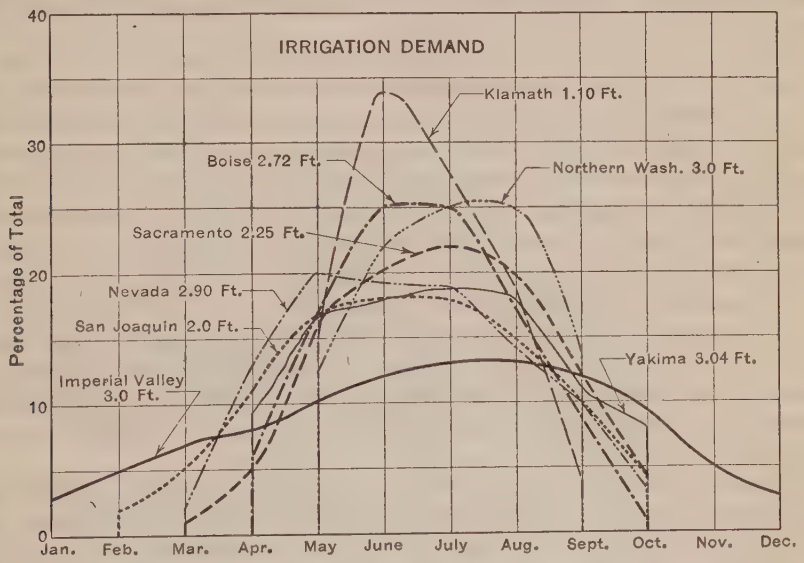

FIG. 1.

IRRIGATION DEMAND

Passing from the southern to the northern limits, or higher altitudes, of the arid West the irrigation season becomes shorter, with correspondingly greater variation in the monthly irrigation demand. In order to illustrate this point the water requirements for typical irrigated sections of the West are shown in Fig. 1. It will be noted that the maximum monthly demand varies from 13% of the yearly total in the Imperial Valley to 34% in the Klamath District. It is obvious, therefore, that for equal land areas and equal yearly water yields, reservoir capacities and distributing canal capacities must be relatively greater going toward the north, if the same yearly total of irrigation water is applied. There appears to be no relation between the net yearly duty of water and the intensity of its application, this being governed by factors other than climatic. For example, the water requirements of Impe-

rial Valley with a 12-month irrigation season are no greater than those of Northern Washington with only a 5-month season; but the intensity of application in the northern district is double that in the southern, hence for equal water supplies, reservoir and canal capacities per acre in Northern Washington must be twice those of Southern California to give the desired degree of regulation.

Power Demand

The only common characteristic between irrigation and power demands is the irrigation pumping load. Although considerable power in the aggregate is used for irrigation pumping the proportion of total generated power so used in the West is so small that its effect on the total power demand is hardly noticeable. That power for irrigation pumping is destined, however, to become a much more important factor than at present is evident from the continually increasing pumping demand.

There are about 19 000 000 acres under irrigation at present (July, 1925), in the United States, of which 16 000 000 acres, or 84%, are supplied by gravity, and 3 000 000 acres—only 16%—by pumping. It has been estimated that there are approximately 15 000 000 additional acres that can be irrigated by gravity through reservoired water supplies, and 10 000 000 additional acres that can be watered by pumping. Thus, the proportion of acres irrigated by pumping may be expected to increase from 16%, as at present, to 38% as the ultimate limit.

An analysis of thirty-five typical pumping plants of the West with lifts from 2.5 to 200 ft. showed a total of 608 000 acre-ft. lifted an average height of 40 ft. An acre-foot of water falling 1 ft. represents the same amount of energy as a kilowatt-hour. The average power consumption of the plants referred to was 2.1 kw-hr. per acre-ft. lifted 1 ft., hence their average over-all efficiency is 48 per cent.

The total electric power generated in the United States in 1924 was 60 000 000 000 kw-hr. If all irrigation pumping in the United States had been by electric power on a water-duty basis of 2 ft., the total energy so used would be 500 000 000 kw-hr., or only 0.8% of the total power demand of the United States. Compared with the Mountain and Pacific States alone, wherein 11 000 000 000 kw-hr. were generated in 1924, the pumping load would amount to 4.5 per cent.

The growth of industrial power demand in the West is probably greater than the growth of power demand for irrigation pumping, hence it seems safe to assume that in the Mountain and Pacific States the power demand for irrigation pumping will not exceed 4 to 5% of the total power demand in those States, even if all susceptible areas are ultimately brought under irrigation by electric pumping. Although this statement, applying as it does to the entire West, indicates that irrigation pumping will not be a controlling factor in power development, it is, nevertheless, a very important factor in certain localities.

Fig. 2 shows the monthly power output of five large power systems of the West. The Idaho Power Company shows the greatest relative demand for

irrigation pumping, with the Southern California Edison Company second. For comparison a typical lighting load is shown on the same diagram. Note that the load characteristics of the Idaho Power Company are quite symmetrically opposite to those of a lighting load, due almost exclusively to irrigation pumping.

Relation of Water Supply to Irrigation and Power Demand

Practically none of the rivers of the country has flow characteristics that coincide with either the power or the irrigation demand. It is obvious, therefore, that if the natural water supply is to be utilized as fully as practicable, extensive artificial regulation is inevitable.

Figs. 3 to 9, inclusive, show the water supply characteristics in seven representative districts, in relation to the irrigation and power demands therein. Quantities are expressed in percentage of yearly totals, hence equal areas are enclosed by each curve.

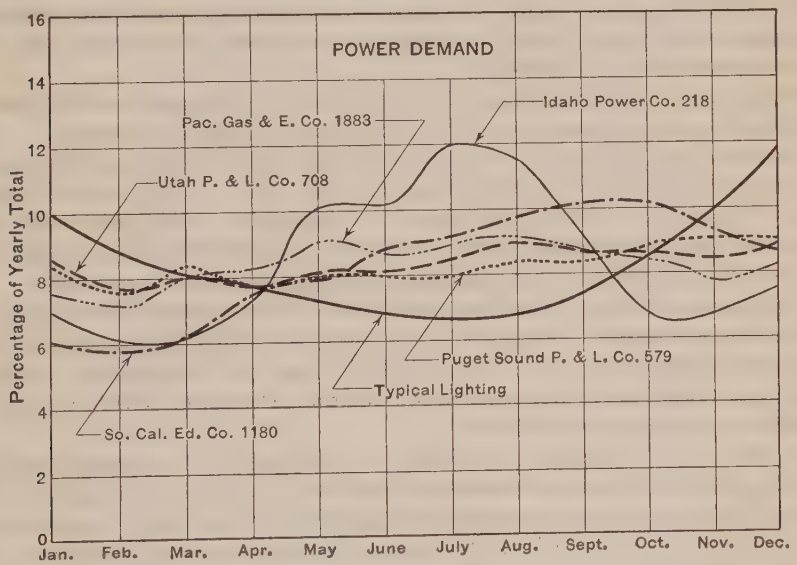

Fig. 2.

Irrigation is generally recognized as the superior use of water, and rightfully so. Water supply alone governs the extent of irrigation, but does not govern solely the amount of power that may be developed, as both head and supply are factors in hydro-electric developments. There are many substitutes for water in power generation, but there is no substitute for water in irrigation. It may as well be conceded, therefore, that the water of the West is destined to be used for irrigation primarily and that power generation may utilize what is left after irrigation has been supplied.

On each of the diagrams, Figs. 3 to 9, the extent of artificial storage required for complete utilization of the supply for irrigation is indicated in

percentage of the yearly supply, also the proportion of the annual water supply that can be used for power after irrigation requirements have been satisfied.

Fig. 3 shows the characteristics of the Lower Colorado River Valley. Of the flow of the Colorado River, 26% will have to be stored to render the entire supply available for irrigation. This would amount to 2 800 000 acre-ft. for the average low-water year. The U. S. Bureau of Reclamation has placed 2 300 000 acre-ft. as the limit of storage required for all lands susceptible of irrigation in both the Upper and Lower Basins, as there is not quite sufficient land to utilize the entire water supply. If this river were completely regulated by storage for irrigation, 85% of the water supply could be used for power without interfering with irrigation. This would mean the possible distribution of 700 000 000 kw-hr. for every 100 ft. of fall utilized.

Fig. 4 shows the characteristics in the San Joaquin Valley. The storage required is 34% of the yearly supply. Of this supply 65% could be used for power without interference.

Fig. 5 shows the characteristics of the Sacramento Valley and indicates that 63% of the annual supply will have to be artificially stored for complete utilization of the water supply for irrigation. Only 58% of the supply may be used for power without interference.

In the Salt Lake Basin, Fig. 6 shows that 64% of the water must be stored for irrigation and that only one-half the yield may be used for power.

As shown in Fig. 8, 44% of the supply in the Yakima Valley must be stored for complete irrigation, and power use may be made of 58% of the yearly supply. Reservoirs already constructed or planned for early completion in this water-shed will have an aggregate capacity of 1 285 000 acre-ft., which is 90% of that just indicated as required for the average low-water year.

The characteristics of the Deschutes River Valley, shown in Fig. 9, are particularly interesting. The flow of the stream is remarkably uniform, but irrigation demands that the entire supply be used in five months. If any stream merits preference for power over irrigation, Deschutes River may legitimately lay claim to that distinction.

In spite of the remarkable uniformity of flow, 64% of the annual supply must be artificially stored for complete irrigation. On the other hand if the water of this stream were reserved for power the entire supply could be so used by storing only 6% of it. In terms of acre-feet passing Bend, Ore., a small reservoir of 60 000 acre-ft. would completely regulate the stream for power purposes, whereas the storage of 640 000 acre-ft. is required to accomplish the same purpose for irrigation. If so regulated for irrigation, only 40% of the upper river supply will be available for power without interfering with irrigation.

In this basin it is proposed to construct reservoirs having an aggregate capacity of 525 000 acre-ft. which will furnish complete regulation for the irrigation of all available land during the year of least supply. It is also proposed to construct a reservoir of 100 000 acre-ft. capacity on its principal tributary, Crooked River, to be used entirely for power on the lower river, thus regaining to some extent the power possibilities lost by giving preferential irrigation use to Deschutes River water.

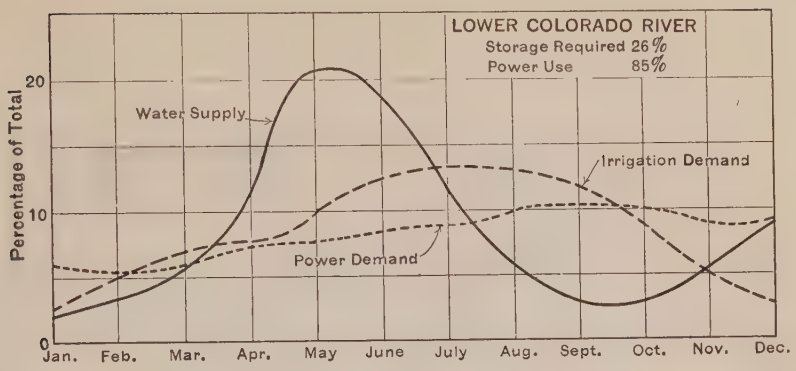

FIG. 3.

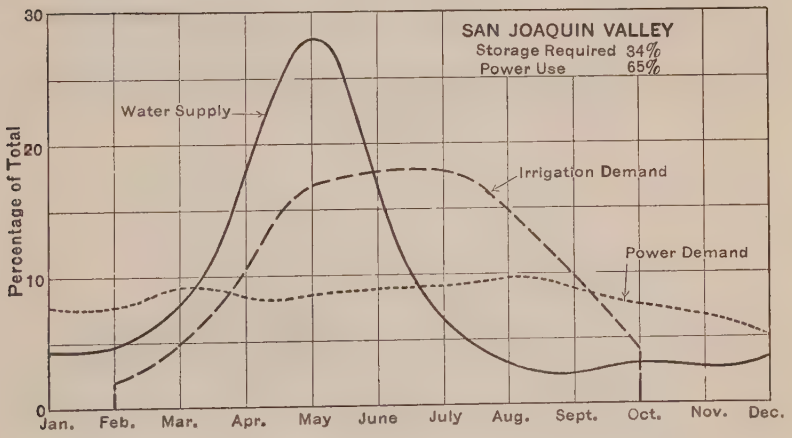

FIG. 4.

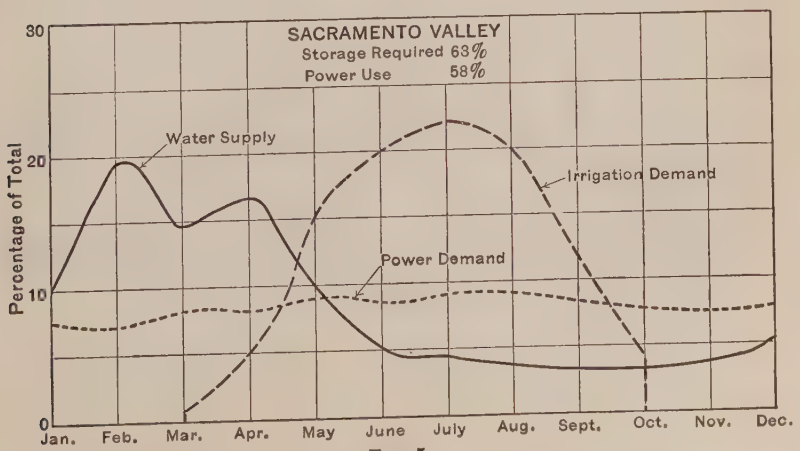

FIG. 5.

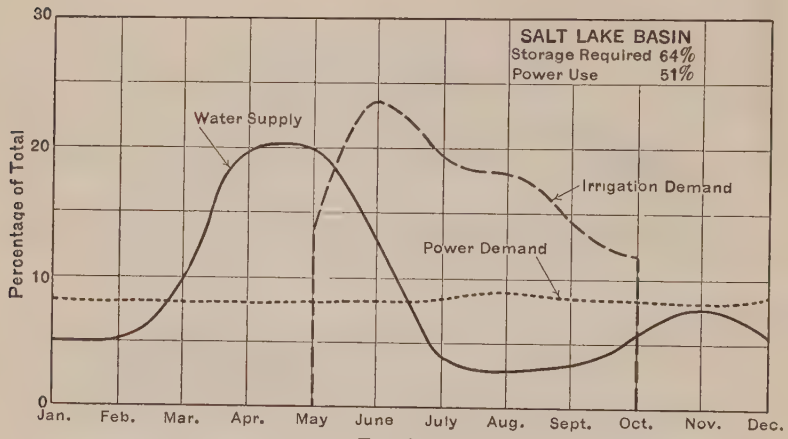
SALT LAKE BASIN
Storage Required 64%
Power Use 51%
Water Supply
Irrigation Demand
Power Demand
Percentage of Total
30
20
10
0
Jan.
Feb.
Mar.
Apr.
May
June
July
Aug.
Sept.
Oct.
Nov.
Dec.

FIG. 6.

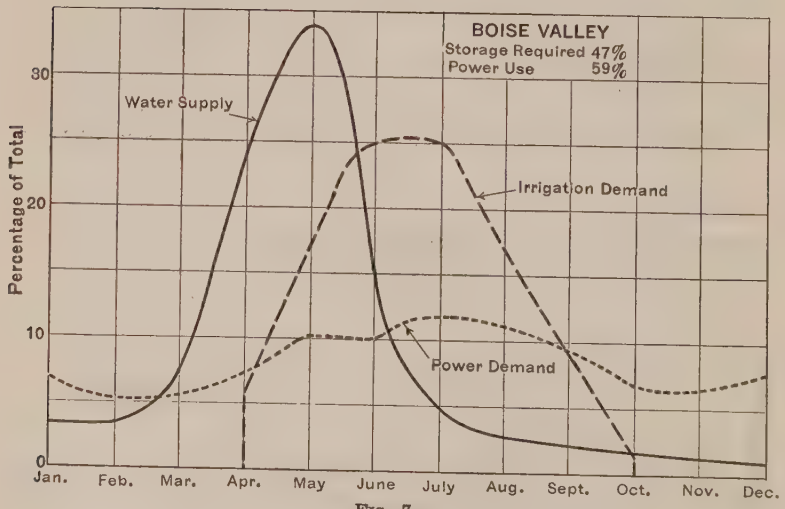
BOISE VALLEY
Storage Required 47%
Power Use 59%
Water Supply
Irrigation Demand
Power Demand
Percentage of Total
30
20
10
0
Jan.
Feb.
Mar.
Apr.
May
June
July
Aug.
Sept.
Oct.
Nov.
Dec.

FIG. 7.

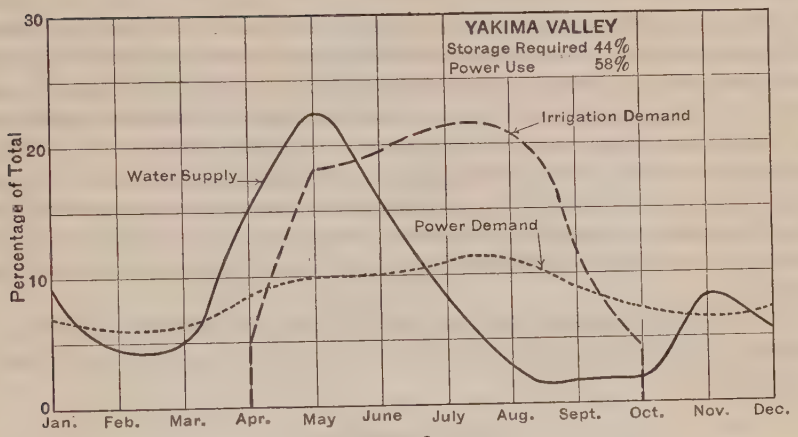
YAKIMA VALLEY
Storage Required 44%
Power Use 58%
Irrigation Demand
Water Supply
Power Demand
Percentage of Total
30
20
10
0
Jan.
Feb.
Mar.
Apr.
May
June
July
Aug.
Sept.
Oct.
Nov.
Dec.

FIG. 8.

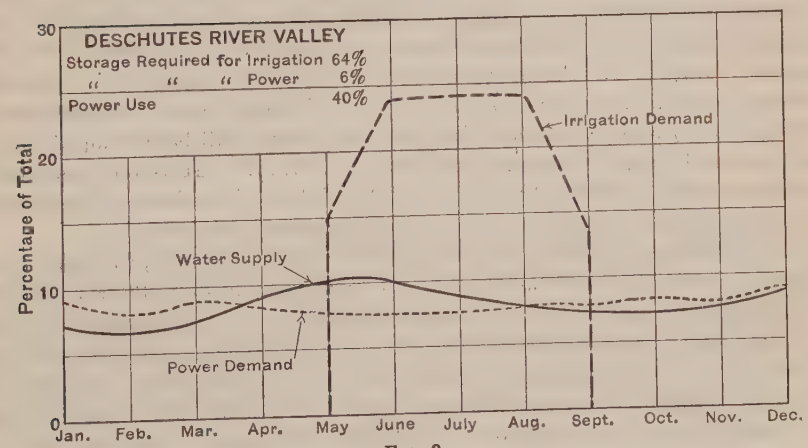
DESCHUTES RIVER VALLEY
Storage Required for Irrigation 64%
" " " Power 6%
Power Use 40%
Irrigation Demand
Water Supply
Power Demand
Percentage of Total
30
20
10
0
Jan.
Feb.
Mar.
Apr.
May
June
July
Aug.
Sept.
Oct.
Nov.
Dec.

FIG. 9.

It should be emphasized that the foregoing data and deductions are in general terms, and are presented to show the maximum characteristic tendencies only, without in any way attempting to outline specific projects or to supplant particular studies that have been made.

In particular the data given concerning the quantity of water available for power after irrigation requirements have been met are subject to a variety of interpretations. If one group of power plants could be placed above irrigation reservoirs and another group below, but above canal diversions, in many cases the entire supply might be almost completely utilized for both power and irrigation without conflict of interest. In some instances, for example, on Deschutes River, power development will be largely confined to the lower river where return water from irrigation becomes a very important factor. On other streams, notably in the Sierras, power plants with their own storage reservoirs, are being extensively developed above irrigation reservoirs and diversions.

It is evident, therefore, that the use of water for irrigation or power, or for both, is strictly localized and general conclusions cannot be applied to specific cases.

Irrigation Power

Where power is developed as a by-product of irrigation just below irrigation reservoirs or at drops in irrigation canals, it is impossible to utilize the full supply for power unless a market having irrigation characteristics is simultaneously developed.

In a very interesting paper entitled "Hydro-Electric Power as a By-Product of Agricultural Storage",* Everett N. Bryan, Assoc. M. Am. Soc. C. E., has completely analyzed this source of power with particular reference to the San Joaquin Valley. Mr. Bryan states that in this Valley, projects completed, under construction, or about to construct, will irrigate nearly 600 000 acres of land by gravity and at the same time sufficient power will be developed to irrigate an equal area by pumping through a 90-ft. lift. It is obvious that if irrigation water is stored 100 ft. above the Valley, the release of that water represents sufficient energy to lift an equal quantity 70 to 80 ft.

The utilization of such irrigation power must depend quite largely on irrigation pumping, or otherwise on its transmission to industrial markets now supplied by steam. By interconnection of large power systems it is possible that this irrigation power may become an important agent in reducing the amount of steam-generated power, but its time distribution is such that it cannot supplant steam auxiliaries, because of the low-water periods prior and subsequent to the irrigation season when this power is not available.

Conflict of Use

There is, and probably always will be, a conflict between power and irrigation in their respective demands for water. It may as well be recognized and admitted that complete harmony of use can rarely be obtained. Devel-

* *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 910.

opment of large water supplies for power that can never be required for irrigation, such as are possible on the Columbia River, and streams draining the western slopes of the Cascade Mountains, coupled with the interconnection of extensive power systems will largely obviate for many years to come the necessity of an attempt to harmonize these conflicting uses.

In particular instances considerable can be done to dovetail the use of water for power with that for irrigation. The Salt Lake Basin, where practically all water used for irrigation is also used for power at some point in its line of travel from mountain to lake, furnishes an excellent example of what may be accomplished in this direction.

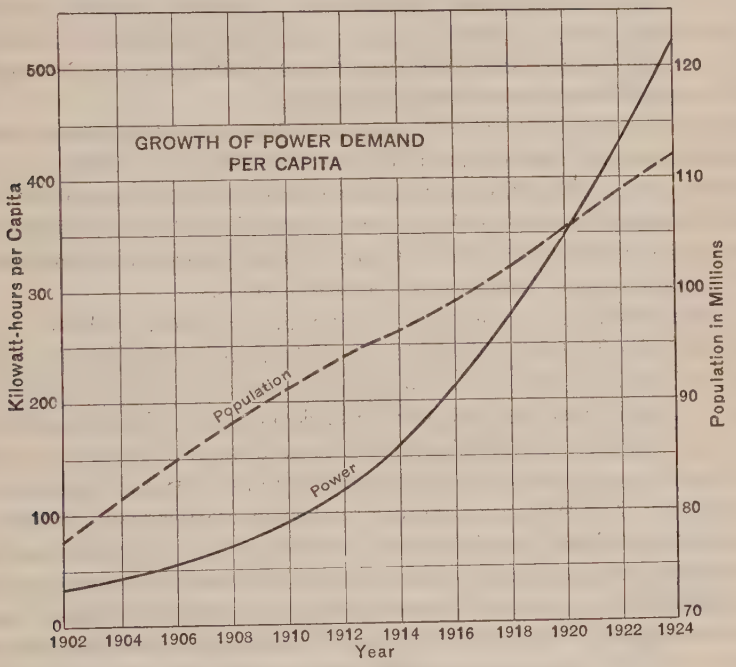

FIG. 10.

The demand for power is increasing much more rapidly than the demand for agricultural products. The demand for foodstuffs increases directly with increase of population, whereas demand for power per capita is compounding at a startling rate. A comparison of the growth of population in the United States with that of generated power in kilowatt-hours per capita is shown in Fig. 10. During the past twenty years population, and with it the demand for agricultural products, has increased 38%, whereas power consumption per capita has increased 1 200 per cent.

The next decade will see an enormous growth in power production, and no one can predict when the saturation point will be reached. On the other hand the growth of agriculture will be comparatively slow, the saturation point having been reached many years ago. In fact, super-saturation has been quite apparent since the World War.

Such conditions will tend to increase the conflict between power and irrigation. Nevertheless, the water supply of the West must be preserved for agriculture regardless of the power demand. Every effort should be made to harmonize the two wherever conditions permit. It is believed that much can be done to accomplish this, but the problem is strictly a local one and no general plan of procedure can be outlined.

Methods of Regulation

Strictly speaking there is only one known method of regulating stream flow, and that is by storage. Storage, however, may be accomplished in more than one way. The storage of water behind dams in suitable reservoir sites is the most direct and efficient method known. It is estimated that in the West suitable reservoir sites exist where water supply sufficient for the irrigation of 15 000 000 additional acres may be developed.

In some localities where suitable reservoir sites are not to be found extensive soil storage may be developed. In Butte Valley, Northern California, water from melting snows that formerly collected into basins and evaporated before it could be applied to the lands, is now pumped back to the margins of the Valley where it sinks into the soil and is later recovered in galleries and applied by gravity and pumping to the valley lands.

It is suggested that a somewhat similar plan is worth serious study for the Los Angeles District. That city is daily pouring 300 to 400 sec-ft. of sewage into the ocean, 99% of which is pure water. If this wasted supply were treated by modern methods and pumped back on to the adjacent valley lands it would replenish the rapidly diminishing ground-water of this region without the slightest danger of contamination or injury to the health of the residents of those sections. Forestation as another method of storing water is being persistently advocated and should not go unchallenged. The writer has been a student of the effect of forests on water supply for many years and has yet to find reliable data to prove that forests have any appreciable value in conserving water supplies. This should not be construed as meaning that he is opposed to the forest policy now in effect or the plans for systematic reforestation that are rapidly taking shape. There is abundant and urgent need for protection and perpetuation of the forests, from the standpoint of timber products and recreation, but why persist in attributing to forests, effects and properties they do not possess?

Propaganda on this subject has been persistent and misleading, as illustrated by the following quotation:* "The connection between forests and rivers is like that between father and son. No forests, no rivers." Exaggeration, of course, but nevertheless an epitome of the intellectual diet on which engineers have been fed.

Great plans have been made for exhaustive research into this question. Comparable areas, forested and treeless, but otherwise similar, have been selected with great care, and elaborate hydrographic and climatic observa-

* "The Fight for Conservation," by Gifford Pinchot, Affiliate, Am. Soc. C. E., Doubleday, Page & Co., N. Y., 1910.

tions have been made on them over a series of years. Long standing records of river flows under varying forest conditions have been scrutinized. Laboratory studies have been made, and volumes have been written, but the only unchallenged truth that has been adduced is that the forest is a great consumer of water, which fact was already known. Forests consume from 15 to 30 in. of precipitation per year, and that is about all there is to it. If other effects exist, such as retarding the melting of snow, absorbing and holding moisture in the forest litter, stimulation of soil infiltration, and retarding of surface run-off, they are too insignificant to be of economic importance.

No one can study the masterly analysis* of this subject by the late H. M. Chittenden, M. Am. Soc. C. E., the reports of the Tenth Navigation Congress held at Milan, Italy, in 1905, the report† by D. W. Mead, M. Am. Soc. C. E., on Wisconsin conditions, the analysis of Merrimac and Connecticut River conditions by Edward Burr,‡ M. Am. Soc. C. E., and Harry Taylor,§ M. Am. Soc. C. E., the writer's paper entitled "Forests and Their Effect on Climate, Water Supply, and Soil,"‖ and other papers and articles that treat this subject strictly on its merits, without being convinced that there is no foundation for the popular belief that reforestation is a practicable means of effecting stream regulation.

In water-sheds where the total precipitation is or will ultimately be required for irrigation and where the entire supply may be regulated by storage reservoirs, the forest on the water-shed is a detriment instead of a benefit, as far as water supply alone is concerned, because if it were not for the forest the total water yield would be increased by the amount the forest now consumes.

On the other hand, in regions of abundant rainfall the dense forests dissipate large quantities of the precipitation that would otherwise run off the area. This dissipation, however, is concurrent with the supply through increased evaporation and, during the growing season, through transpiration. Flood supplies are not held over to augment summer yields as is popularly believed. Forestation, therefore, as a means of water storage and stream regulation is without economic value from the standpoint of practical engineering.

Conclusions

In treating this subject generally no tangible means of effecting harmony between the use of water for power and irrigation has been disclosed. Both uses require artificial stream regulation, but irrigation, having preferential rights, deprives power development of the full utilization of the supply that might otherwise obtain. Water is vitally indispensable for agriculture, but not for power, hence the justice of preferential use.

Regulation for power conserves surplus waters for use throughout the entire year, whereas regulation for agriculture conserves the surplus for use during about one-half the year. Generally speaking, greater storage capacities

* *Transactions*, Am. Soc. C. E., Vol. LXII (1909), p. 245.
† *Bulletin No. 425*, Univ. of Wisconsin.
‡ House Doc. No. 9, 62d Congress.
§ House Doc. No. 1294, 61st Congress.
‖ *Journal*, Assoc. of Eng. Societies, July, 1913, p. 1.

are required for complete utilization for irrigation than for power, and the shorter the growing season the greater the amount of storage required.

Uniformity of stream flow is a valuable characteristic in power development, but loses much of that value when the entire supply is devoted to agriculture.

Complete regulation for both uses will require storage in high altitudes for power alone and foothill re-storage for irrigation. Power obtained from the utilization of released irrigation water will minimize the amount of power storage otherwise required.

Power from irrigation release has its most promising market in irrigation pumping, but the extent of this market is limited, and it can never be hoped to utilize all the power that can thus be developed.

The problem of best utilizing the available water supply for both power and irrigation is strictly a local one and cannot be treated in general terms. Definite water supplies, definite storage opportunities, definite lands to be irrigated, and known power demands are essential prerequisites for its solution.

Primarily, actual regulation may be accomplished through the construction of storage reservoirs, and, secondarily, through soil storage, but not at all by reforestation.

In many cases harmony can be fostered and much can be done to eliminate conflicting interests. Power must concede the superior right to irrigation and, in the long run, will benefit substantially by that concession.

DISCUSSION

EDWARD H. SARGENT,* M. AM. SOC. C. E. (by letter).—The statement has frequently been made that stream regulation for the combined purposes of power development and flood control is impracticable on the ground that for flood-control purposes a reservoir should at all times be empty, which condition would destroy its utility for regulation for power purposes. This thought has probably been due to the fact that frequently it is not feasible to build a storage reservoir of sufficient capacity adequately to control the flow of the water-shed at the dam site. The writer will discuss two projects with which he is familiar where it was feasible, principally because of the effect on power developments, to build reservoirs sufficiently large so that their operation for power purposes to a great extent controlled the flood flow.

In the preliminary studies of the so-called Portage Reservoir, made under the direction of John R. Freeman, Past-President Am. Soc. C. E., it was planned to build large orifices in the spillway section of the dam at such an elevation as would permanently reserve a large amount of storage for flood control—in other words, the top of the reservoir was to be made into a retardation basin, the orifices being of such capacity as to discharge under maximum conditions a flow not exceeding the normal flow of the stream. The remaining capacity of the reservoir was to be used in stream regulation for power purposes. At the time the studies were made (1907-08), only limited stream-flow data were available on the Genesee River, the stream in question. Since that time daily records of the flow of this river have been kept. Extended mass curve studies of these records have convinced the writer that practically all the flood control which would have been obtained under Mr. Freeman's plan would be obtained by using the entire reservoir capacity for power regulation, that reserving the top of the reservoir as a retardation basin would have been equivalent to throwing away that amount of storage which could have been used for power purposes. The water-shed in question was 958 sq. miles in area and the proposed storage totaled 413 000 acre-ft., or 431 acre-ft. per sq. mile of water-shed. Stated in other terms, the storage capacity was approximately 52% of the average yearly run-off at the dam site. The economics of the project made it feasible to develop storage to furnish this high degree of control which was such that, in the period of 1908-24, had the reservoir been operated for power purposes, only once would the normal flow of the river immediately below the dam have been exceeded.

The Sacandaga Reservoir, now being built (1925) under the direction of the writer, is to serve the dual function of regulation of stream flow for power and for flood control. This reservoir has a water-shed at the dam site of 1 044 sq. miles and is the principal tributary of the Hudson River, which at the "center of gravity of power" has a water-shed of 2 780 sq. miles. The water-shed of the Sacandaga River lies in a region of much higher precipitation than that of the Genesee River and the Sacandaga Reservoir is being built to have an available capacity of 689 000 acre-ft., which is equivalent to 658 acre-ft.

* Chf. Engr., Hudson River Regulating Dist., Albany, N. Y.

per sq. mile of water-shed, almost exactly 50% of the total run-off in an average year.

Between the dam and the confluence of the Sacandaga and Hudson Rivers there is a fall of 152 ft., which is undeveloped, and on the Hudson below this confluence there is a fall of 548 ft., most of which is developed. In making the first studies of this reservoir it was felt that, because of the comparatively large amount of fall immediately below the dam, the reservoir would have to be operated to provide the best flow possible at the dam site and that this would seriously impair its usefulness for flood control, inasmuch as, with power plants immediately below the dam, water would have to be released from the reservoir when the Hudson River was in flood. It has since developed that the power which will be generated at the power plants between the storage dam and the Hudson River will be transmitted into the systems of large generating and transmitting companies. These companies either at their own plants on this and other streams, or at those plants from which they purchase energy, have available during the spring high-water months a surplus of electric energy, so that shutting down the plants situated immediately below the storage dam, in order to minimize conditions of high water on the Hudson River, does not seriously impair their usefulness. This condition makes it feasible to operate the reservoir to produce the best flow possible at the "center of gravity of power" on the Hudson River and at the same time makes it unnecessary to release water from the reservoir when the Hudson River is at high flow. This regulation will increase the minimum flow of the Hudson River at the so-called "center of gravity of power" from the present (1925) minimum of 900 sec-ft. to 3 000 sec-ft., increasing the firm or continuous power from 34 000 h.p. to 160 000 h.p., on a load-factor basis of 100 per cent. At the same time, this regulation, which in effect will subtract the flood flow from 1 044 sq. miles, will eliminate most of the damaging floods on the Hudson River immediately below its confluence with the Sacandaga River, and largely reduce those not eliminated. The Hudson River Regulating District is a public corporation created as a result of an Act of the Legislature which is quite similar to the Miami Conservancy Act. The general plan of the District calls for the eventual construction of sixteen storage reservoirs having a total capacity of 1 800 000 acre-ft. It is believed that these reservoirs, although they are to be constructed primarily for the regulation of the flow of the Sacandaga and Hudson Rivers and their tributaries for power purposes, will largely prevent floods on the Hudson River above its confluence with the Mohawk River.

Too little consideration is given in the design of spillways of storage reservoirs to the storage above the spillway crest. Not infrequently plans provide for as great spillway length and capacity as if the proposed dam were merely a power dam without a large flowage area. Engineers are prone to take from one of the well-known curves of flood flows of typical streams a maximum flow for the project in question and design the spillway accordingly. If the project is that of a storage reservoir with a large area, the effect of storage above the spillway crest would largely reduce the necessary spillway capacity.

For example, in the design of the Sacandaga Reservoir, a flood flow on a square-mile basis equal to the Miami flood of 1913 was used (Fig. 11), assuming that such a flood occurred when the reservoir was full. The storage above the spillway crest would have reduced this flood from 120 000 to 60 000 cu. ft. per sec. Inasmuch as the topographic and geologic conditions at the dam site were such as to require a side-hill spillway the reduction in necessary spillway capacity materially reduced the cost of the dam and spillway channel.

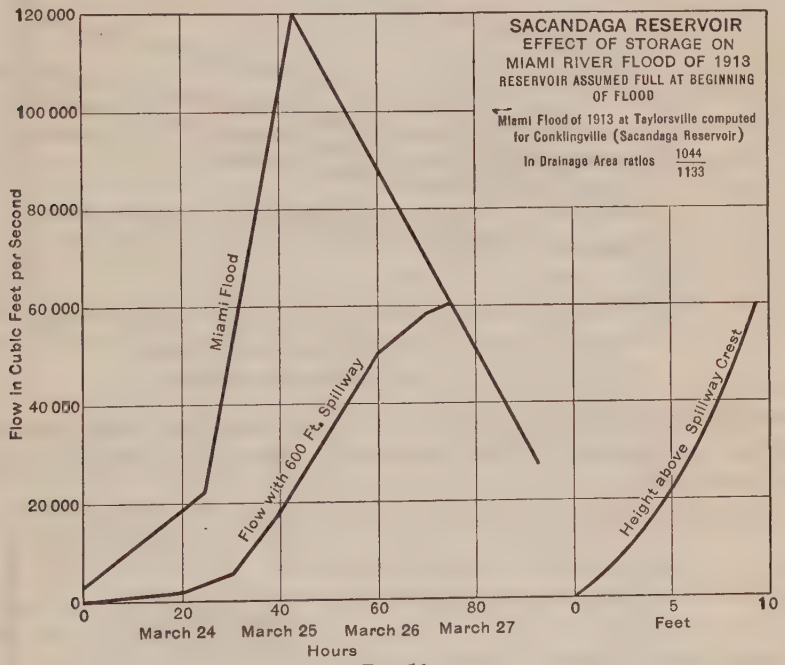

FIG. 11.

Although it is obvious, yet it may be pertinent to note parenthetically that the storage above the spillway crest of the Sacandaga Reservoir would have a great mitigating effect on the floods of the Hudson River even if the reservoir were full at the inception of such floods. For example, the flood of March, 1913, would have filled the reservoir by April 1, on which date the natural flow of the Hudson River was 33 800 sec-ft., of which 11 300 sec-ft. came from the Sacandaga River. The storage above the spillway crest would have reduced this 11 300 sec-ft. to less than 1 000 sec-ft., so that the flood flow in the Hudson River of 33 800 sec-ft. would have been reduced to 23 500 sec-ft.

The writer wishes to take issue with another generally accepted theory that storage reservoir operation which assumes variable rates of discharge for each dry season, to the end that most of the available storage can be used each season, is purely theoretical and cannot be carried out in practice. In connection with his studies of the Sacandaga Reservoir he has made extensive studies of the stream-flow data, which are of unusual accuracy and duration.

As a result of these studies, an operation chart has been prepared, an application of which over the last eighteen years, and without the assumption of

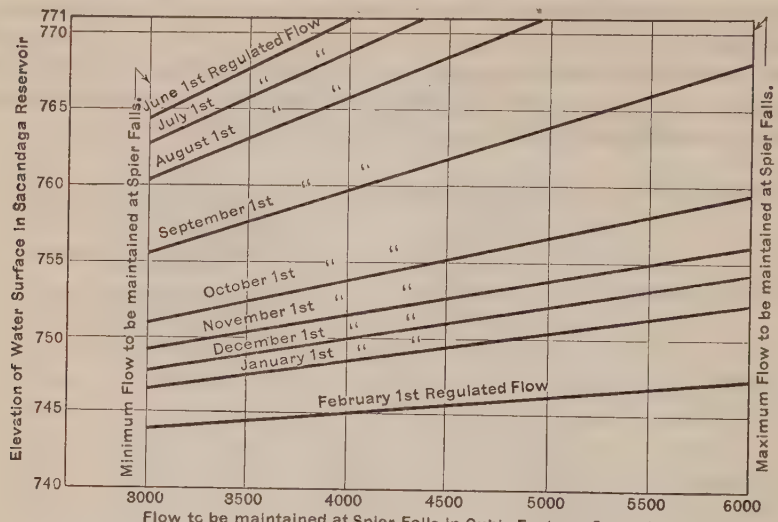

FIG. 12.—OPERATION CHART, SACANDAGA RESERVOIR.

foreknowledge of stream flow, provides what the writer has called "practical regulation," as opposed to "ideal regulation," which assumes foreknowledge of the stream flow.

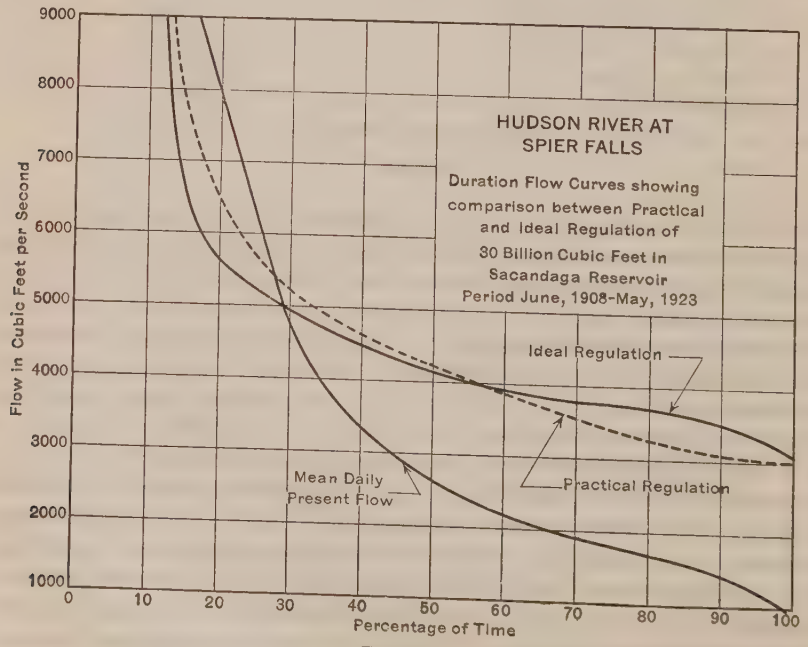

FIG. 13.

The method of obtaining the chart (Fig. 12) was as follows: A mass curve study was made to determine the "ideal regulation," that is, the regulated flow that could have been maintained with foreknowledge of stream flow; next, from the resulting regulated flow mass curve together with the capacity curve of the reservoir the elevation of the reservoir at 10-day intervals was obtained, and these elevations were plotted as ordinates with the corresponding regulated flows as abscissas and lines drawn through the resulting points. This diagram, which has been called the "operation chart," was then used in conjunction with the natural flow mass curve, assuming no foreknowledge of stream flow, and after a number of empirical adjustments were made the diagram as shown was obtained. This chart is not complete and there is in course of preparation a more elaborate chart for the use of the engineer who will operate the reservoir. The chart in its present form, however, has been tested with the stream-flow conditions existing since 1908, assuming no foreknowledge of stream flow. A duration flow curve (Fig. 13) computed from such so-called "practical regulation" has been computed and plotted as has also a duration flow curve from the so-called "ideal regulation." As shown on Fig. 13 the benefits from the "practical regulation" are only slightly less than from the ideal or theoretical regulation.

W. G. Hoyt,* Assoc. M. Am. Soc. C. E. (by letter).—Mr. Stevens has discussed most thoroughly the many important problems that must be considered in the development of storage of water for power and irrigation. It may be of interest to outline the Governmental activity which is being carried on in the best interests of the West, the work of the United States Geological Survey with respect to water resources.

Recently, a report by a prominent Western engineer stated that within twenty-five years there would be a demand for all the water and all the power that could be had and developed from the rivers of the Sierra Nevadas in California. This statement applies not only to the State of California, with its phenomenal growth, but with equal force to practically all the United States west of the Mississippi, with perhaps the exception of a narrow strip along the coast in Northern California, Oregon, and Washington, and other great concentrations of water resources such as are found along the Columbia and Colorado Rivers and some of their principal tributaries. The time limit for this demand may vary to some extent in different parts of this great region, but inasmuch as many areas have already been retarded in development because of an inadequate supply of water, it is hazardous to set the time limit too far ahead.

If it is true, therefore, that the time is not far distant when Western water resources will be in full demand, it is necessary to make future development with a minimum of conflict between the various uses to which water and the lands needed in connection with the utilization of water may be put. Competent agencies functioning singly or in co-operation should be clothed with sufficient power not only to require modification of any project involving the use of water, so as to adapt it to a comprehensive scheme of improvement

* Hydr. Engr., Conservation Branch, U. S. Geological Survey, Washington, D. C.

and utilization, but to require such modification of any other development as may be necessary to prevent material interference with the best use of the water, unless such other development would be the more valuable to the region concerned. The exercise of such power should be based of course on adequate information in order to determine comprehensive plans of development and the relative values of different uses.

Outstanding examples of disastrous conflicting uses are well known. In Oregon, two railroads were constructed, at nearly water grade, along an important power stream, where one at a higher grade could have been constructed at only a slightly greater cost and would apparently take care of all present and probable future transportation requirements. One of these railroads is, therefore, an added burden on power development. In Idaho, there is an irrigation district which must always meet a heavy assessment for pumping water, because prior developments prevented the building of a dam high enough to make the water available by direct diversion at a lower cost. Many other instances of this kind could be cited.

The author has brought out the necessity for storage; but what is to be done if, prior to the date when storage becomes a necessity, a railroad or a National highway has been so constructed across the site that the cost of its relocation precludes the use of storage possibilities, or if the lands have passed to patent without reservation under the Federal Water Power Act.

It is apparent, therefore, that, not only must development take place according to a comprehensive plan, but, further, that before the time for full development is ripe projects of lesser value must not be constructed, that will materially interfere with such development. The whole problem should be studied from the point of view of the highest use of natural resources—a point of view that is already well recognized in the West in connection with the use of water.

Certain steps are being taken to guarantee, in a measure at least, that the resources of the National domain will be put to their highest use. On July 1, 1924, a total of 6 626 474 acres were embraced in withdrawals and classifications relating to water resources. A considerable part of this land is withdrawn from entry because it is covered by applications for permit or license under the Federal Water Power Act. The remainder has been investigated by the U. S. Geological Survey and found to have value in connection with water utilization. It is possible that some engineers in their task of building the West, have been delayed and perhaps been put to a considerable expense and annoyance because of these withdrawals. The principle involved, however, is fundamentally right and the benefit that will eventually accrue to the western country because of these withdrawals will far outweigh any past inconvenience.

In making and administering these withdrawals the results of about 5 000 miles of river surveys have been studied and the records of nearly 1 000 gauging stations have been analyzed. Field studies have been made over a period of 17 years, since the first withdrawal of land for power, which covered 17 080 acres in Utah.

The value of land for the development of water power can not be estimated properly on the basis of the present limits to development, but must be determined by considering the probable limitations of a future day, when by reason of increased demand and increased value of such power a project that is now impracticable because of high cost will become thoroughly feasible. In classifying and administering public land with respect to the possibility of developing water power, it has been necessary to determine whether the land might not be more beneficially devoted to the use of water for domestic supply or irrigation than for power and whether its use for power would be prejudicial to the other uses at the particular site under consideration or at some other site more or less remote.

It has also been necessary to estimate in many cases whether the land is of more value for agriculture, or for a town site, or a manufacturing site than for a power site. Such considerations apply especially to lands that may be suitable for storage reservoirs. Where civic, industrial, or rural improvements have already been made on a reservoir site they largely control the decision on this point; but the sites which lack such improvements are still numerous. Therefore, the field of investigation has extended beyond the facts of the present day, and a forecast of prospective events must be made.

Those who have had to deal with these problems have not forgotten that human activities embrace many things besides the utilization of water—that a community which produces anything must, if its products are of value, have suitable means of transporting them to market. Consideration, therefore, has been given to the relative usefulness of a piece of land as a power site or as a right of way for a railroad. The bottom of a canyon that is suitable for a reservoir may also be an advantageous railroad or highway route. Although an alternate transportation route can usually be located, the cost of utilizing it may be so much greater as to outweigh the advantages that may accrue from the development of power along the stream. In determining such questions it is necessary to appraise the relative value and comparative cost of several possible methods of using the land and to choose the course that is, or is likely to become, of the highest human benefit.

The classification of land for water-power sites, therefore, requires a perspective so broad, and a comprehension of so many variable conditions and probable future changes in public demand, that mistakes are easily made. With respect to the public domain, therefore, an attempt is being made to guide future development according to what are believed to be far-seeing comprehensive plans. However, because of constitutional restrictions many States are still disposing of their remaining water resources on the policy, embodied in the proverb, "the early bird catches the worm." This was undoubtedly the best policy during the first stages of development, but, viewed in the light of present-day conditions, it seems erroneous and should be corrected.

In this discussion there is no intention to imply that present policies have acted to dissipate materially National water resources, but rather to draw attention to the fact that the problems of the future utilization of these resources will demand, not only a careful and complete study of the drainage basins, but also the development of a sound economic policy.

F. F. Henshaw,* M. Am. Soc. C. E. (by letter).—The author has discussed the power and irrigation demands and their relation to regulation by storage for the normal streams of the arid and semi-arid West. He has given as examples six fairly typical streams having a concentrated fall in their head-water portions that is utilized for power development and having irrigable valley land at a low level, with storage capacity above the power plants and, on some of the streams, below the tail-race of the power plant and above the points of diversion for irrigation. A typical stream already developed in this manner is the Tuolumne River, rising in the Sierra Nevada and debouching into the San Joaquin Valley, California. This relative arrangement is normally found in a region of moderately mature topography.

A radically different condition prevails in the Pacific Northwest, where in Tertiary times the great Columbia River lava flow inundated like a flood a large part of Washington, Oregon, Idaho, and Northern California. The Cascade Range dates from an even more recent epoch, and adjustments of drainage have occurred during what may be considered a geologic yesterday. Another series of lavas, the andesites that form the summit of the Cascade Range, is one of the newest and most pervious of all geologic formations of any extent within the United States. As a result of this recent extensive volcanic activity there is in parts of the plateau area west of the Rocky Mountains a topography of extreme youth in which the surface is comparatively little distorted. The major streams have cut steep, narrow canyons in the borders of the plateau, but their head-waters flow near its surface. The later volcanic formations give rise to immense springs—some of the greatest in the United States if not in the world.

The typical stream of the lava area is the Deschutes River in Central Oregon, as mentioned by the author, but the Snake River in Southern Idaho, the Klamath River in Southern Oregon and Northern California, and the Rogue River in Southwestern Oregon, have many characteristics in common with the Deschutes. In the basins of all these streams most of the irrigable lands lie at a relatively high level, on the surface of the plateau. Water must be diverted in the upper portions of the streams before it has reached the deeper canyons. The power sites are below rather than above the sites for irrigation diversion. The water available for developing power on most of the streams is augmented by inflow from springs at points below all possible sites of diversion for irrigation.

The natural flow of the streams in their head-water sections is, in general, that shown by Fig. 5. With no storage regulation, or with partial storage, the flow below the point of irrigation diversion is little affected by the diversion from October to April, but is reduced materially during the summer. When partial storage is provided in the head-water section, the summer flow between the reservoir and the point of diversion of course is augmented, but the only effect on the power is to reduce it uniformly during the non-irrigating season.

The Snake River may be taken as an example. This great stream is fed by immense springs as well as by tributaries that have the usual characteristics

* Dist. Engr., U. S. Geological Survey, Portland, Ore.

of streams rising in high mountains. Its low-water flow from Lewiston, Idaho, to the mouth probably ranged from 20 000 to 30 000 sec-ft. before extensive irrigation was begun in Southern Idaho. The river discharges a fairly steady volume from about August 1 until sometime in the winter. Irrigation development has progressed until below Milner Dam, the lowest point of diversion from the main river to the plateau lands, the river is practically dry each summer. Not far from Milner the river plunges over Twin and Shoshone Falls and enters the series of canyons through which it flows to its junction with the Columbia at Pasco, Wash. Springs discharge large volumes of water into the river in this canyon section, and as a result the flow at the mouth has never been less than 10 000 sec-ft., even in recent years. This quantity is, therefore, available for continuous power generation.

Storage development has been started; Jackson Lake and Arrowrock Reservoirs augment materially the water supply for irrigation in the Upper Snake and Boise Valleys, respectively, but do not appreciably reduce the flow through the lower canyons of the Snake River. The American Falls Reservoir, now (August, 1925) under construction, together with other proposed storage basins, will noticeably diminish the winter and flood discharges below the irrigated areas. The final result will be to produce during late summer and early autumn a fairly uniform flow of large proportions, say, 10 000 to 12 000 sec-ft., at the mouth and to leave available several hundred thousand horse-power, notably in the deep canyons between Huntington, Ore., and Lewiston. The lowest stages will continue no doubt to occur near the end of the irrigation season. The shortage of power during this time and the peak load produced by the requirements of irrigation pumping can be met in a measure by power generated from water released from storage reservoirs, and diverted at lower levels from irrigation. The utilization of such double sources of power may be deferred many years; it will probably await the construction of a superpower network of considerable capacity, but it presents important possibilities.

The Klamath River is almost unique in the lack of relief in the upper part of its basin and the contrasting extreme ruggedness of the lower part. The run-off characteristics harmonize with the topography, being uniform above Klamath Falls and flashy for the section below the gap in the Cascade Range. The land available for irrigation in this particular basin is considerably less than the available water will supply, and the irrigation demand will ultimately consume somewhat less than one-half the average annual run-off. Upper Klamath Lake, with an area at low stages of about 60 000 acres, presents a storage basin of large capacity. It has been developed to provide a controlled range of stage of 7.3 ft., storing somewhat more than 500 000 acre-ft. This range is limited by the cost of protecting by levees the valley lands bordering the lake. Ordinarily, it will be possible to maintain a fairly steady flow of 800 to 1 000 sec-ft. for power development, in addition to supplying all reasonable irrigation requirements. Severe shortages may occasionally occur during dry summers, but these can be provided for by developing the power available from water released from storage in the headwater portion of Deschutes River and elsewhere. Here, again, the utilization

of such a double source of power must await the establishment of a super-power system of considerable capacity.

BARRY DIBBLE,* M. AM. SOC. C. E.—The importance of taking care of available water supplies has been mentioned by both Mr. Stevens and Mr. Hoyt, but the time comes when they are absolutely "extinguished", that is, so far as future development is concerned. In a great many places, particularly in California, it is not realized how far that "extinguishment" has already proceeded. Power development in California has had so much publicity that, in comparing the statistics of that State with those of the United States as a whole, it has been rather surprising to learn that, whereas over a 10-year period the compounded annual increase in power consumption in California is 10.5%, in the United States as a whole it is 11.3 per cent. During the 10-year period California had a large increase in population, so that the per capita use increased at the compound rate of only 6.6% per year as against 11.2% in the whole United States. The power consumption of Southern California dropped even lower with a per capita increase of 4.3 per cent. This decline in the growth of the California power business has undoubtedly been due to difficulty in keeping up with the demand by constructing hydro-electric developments of reasonable cost.

Throughout the world the consumption of electricity is held back because development follows the demand rather than precedes it. Power companies generally have not had to search for customers. They have operated in a "seller's market". There is no good basis for determining to what extent electricity would be used in a community if its sale were energetically and consistently pushed in all lines of application. There is no indication that the saturation point is being approached.

In the territory of the Minidoka Irrigation Project, in Idaho (120 000 acres and 17 000 population), the per capita consumption of power amounts to 2 800 kw-hr. annually, or four times the average in the United States. It has now stopped increasing because temporarily it has reached the limit of the power supply.

Compilations of estimates show that power company managements anticipate that the electrical business will continue to compound at the rate of 11%, which means that it doubles in less than 7 years.

Mr. Woolley's discussion shows that there is often not a great deal of difference between statements that may seem to be contradictory. No one will accuse the speaker of lacking interest in electric power and its application. The fault with Mr. Woolley's statement is that he has left the time factor out of consideration. Mr. Hoyt carefully included that factor. Already there are a number of instances—one at American Falls in Idaho and numerous others—where the economics of the situation have demanded the extinguishment of power rights in favor of irrigation. Apparently, the present plan of classification is to look to the future and prevent installations which may be economic at the moment, but which, in the reasonably near future, may have to be extinguished to permit of better use of the natural possibilities. The

* Cons. Engr., Redlands, Calif.

present demands for power are making such developments feasible in connection with irrigation projects where a few years ago they were not economically possible. Power is becoming of considerable value in meeting summer peaks and in permitting the cutting down of expenditures for fuel. After all, there is no great difference of opinion between engineers on these questions.

RALF R. WOOLLEY, ESQ.*—Those who are interested in reclamation and agriculture do not hesitate to remark that agriculture is the basic industry; those interested in mining feel that mining is the basic industry; and the stock growers are sure that theirs is a basic industry. In each case statistics are not wanting to support the claims of these various advocates.

There is no question but that agriculture, including irrigation, was the foundation on which practically all the permanent communities of the arid West were built. Some of these communities are still solely dependent on this factor for their existence, while others boast of additional forms of industry such as mining, manufacturing, etc. Communities supported by irrigated lands are no doubt more stable than those subject to the ups and downs of prosperity not uncommon to other industries, and, for this basic reason, public opinion in the West has defined certain preferential rights to the use of streams, placing irrigation superior to all other uses except domestic.

To those who have given some study to the question of the greatest economic utilization of Western streams, this fact raises a serious question; namely, whether or not it is advisable always to sacrifice the use of a stream for power in favor of irrigation regardless of the relative economic values of the two uses.

Agriculture in the arid West is based on irrigation. Generally speaking, its future growth is commonly expressed in terms of the total run-off of the streams and undeveloped acres without regard to the economic factor in the problem. The recent investigations of the United States Reclamation projects disclosed the fact that "the reclamation problem is to-day essentially an agricultural and economic problem". Dr. John A. Widtsoe, one of the members of this Fact-Finding Commission, makes the following statement:†

"Irrigation is a complex art, involving the application of the best knowledge of agriculture, especially the relationships among soils, crops, and water; of engineering, especially of hydraulics; of rural economics and sociology; and of the life and response of human beings under the unique conditions always prevailing under the irrigation ditch.

"It is not surprising that so complex an art, at this time, when thoroughgoing changes are reshaping economic and social structures, should present many and serious problems. Such problems pertaining to the development of irrigation in the West do exist, and must be solved, if the great American experiment in irrigation shall become permanently successful. Yet these problems, essential to irrigation development, may be classified very simply into two groups:

"1. To establish on every irrigation project an acceptable economic and social environment.

"2. To extend gradually, as economic needs demand, the irrigated area by completing existing projects and by reclaiming new areas, until the water resources of the West shall be fully utilized."

* Hydr. Engr., Water Resources Branch, U. S. Geological Survey, Salt Lake City, Utah.

† See pp. 682–683.

It seems apparent from this statement that irrigation will become permanently successful only when it is practiced on a sound economic basis, and it should be extended "gradually, as economic needs demand". This is the one point that seems to be entirely overlooked by the irrigationist in his enthusiasm to develop irrigation, the result being that other uses of streams, such as for power, are made subject for all time, to any and all future demands for irrigation developments, regardless of their economic feasibility or the relative merits of the two uses.

Each stream is an individual problem, and should be considered as such in working out its complete utilization. Those communities that are solely dependent on irrigation, will only grow to the capacity of their available irrigable acres, and available water supply; and in many of the communities along the small streams throughout the arid West the limit of such growth already has been reached under present irrigation practice. If any further growth of these towns is to take place, it must either be the result of other industries coming in as, for example, manufacturing and mining, or the limited possibility of increasing the returns from irrigation by improving methods of farming.

Is it not possible, therefore, to conceive the following conditions in the growth of many Western communities that have begun with a small nucleus of irrigated land. As the community grew in population it extended its irrigated area, and continued for many years to supply all the demands of its inhabitants for a livelihood; finally, the limit of economical irrigation growth was reached because the railroad or other transportation agencies brought in supplies and foodstuffs more cheaply than the citizens could produce them. The community was fortunate enough to have a good geographical location for manufacturing industries, raw materials were close at hand, and vast mineral resources were discovered; the community began to grow industrially, and far beyond anything that could possibly have been expected from its irrigation resources. Hydro-electric power was easily developed, and, accordingly, industries of all kinds were encouraged. Is it not conceivable now that the curve of growth of this community based on irrigation is at some point crossed by the curve of growth based on industry, and that henceforth, from this point, the community is not primarily an irrigation or agricultural community, but an industrial one? In all this development power is the one important factor; accordingly is it not the water power resources that will play the largest part in future industry and agriculture in the arid West?

How does power development affect agriculture? Without going into the details of the different combinations of co-ordination of irrigation and power uses of streams and the possible conflicts in use, it is interesting to note the following statements which no doubt are based on considerable study of the problem.

The Committee of Special Advisers on Reclamation to the Secretary of the Interior concluded regarding power that:*

* Federal Reclamation by Irrigation (Presidential Message), Senate Doc. No. 92, 68th Cong., 1st Session, p. 135.

"The place of power in modern agriculture has been well established. The electric current has become the servant of the farmer, by the aid of which one man is enabled to do several times the work that the farmer of old could do. The need for power on an irrigated farm is as legitimate as the need for water. Any future policy relative to the development of Federal reclamation should keep the power problem in mind. The water users, when they take over a project, should be allowed to operate the whole project as a unit, including the power plants that may be constructed; and, further, no reservations should be made regarding possible power sites, except as to the permanent retention of existing power possibilities for the use of the project population."

The California Railroad Commission has stated* that:

"Any halt in the State's hydro-electric development will spell disaster. It is the Commission's firm belief that financially sound utilities, functioning economically and efficiently, will successfully carry the burden of California's development. It is obvious, that industrial and agricultural California cannot reach the peak of development unless there stride, a step in advance, utilities carrying service possibilities 'ready on demand' ".

Similar views are continually being expressed in the technical press. Some striking data have been published,† showing the relation between kilowatts generated and community development: "Each kilowatt of power developed makes possible the building of one home, the irrigation of 4 acres, employment of one person in an industry and means $6 000 to the community."

In conclusion, it is the speaker's opinion that water power really has an important place in the utilization of Western streams. It may be a by-product of irrigation, co-ordinate with it or even superior to it. Each project is an individual problem and should be solved according to the economic factors involved. It is a mistake to impose such restrictions on power projects as will postpone indefinitely or possibly preclude their development without a proper analysis of the whole problem. It would indeed be a short-sighted policy on the part of the person seeking to develop power if he did not foster all possible development of the community through irrigation; likewise, it is a short-sighted policy on the part of the person seeking to develop irrigation to put every obstacle possible in the way of developing power.

Arthur P. Davis,‡ Past-President, Am. Soc. C. E.—Mr. Sargent has mentioned a plan proposed by Mr. Freeman, in which an automatic outlet or spillway was provided below the top of the reservoir in order that a certain amount of that storage would be reserved for flood control at all times. Every project must be stated in accordance with its own characteristics. A spillway that will discharge an amount that is not objectionable as a flood below the dam, and yet will discharge the water so fast that the storage back of it can be utilized for regulation of the next flood, is, in principle, exactly what Mr. Freeman proposed. Evidently, the capacity of his spillway in both cases differs with the head, so that he is not proposing a new principle in either case, but simply to dispose of the flood at a height at which it can be safely discharged. That differs greatly with the characteristics of each different

* *Electrical World*, October 30, 1920, p. 893.

† *Journal of Electricity*, January 1, 1921.

‡ Chf. Engr., East Bay Municipal Utility Dist., Oakland, Calif.

problem. It differs greatly with the manner of flood discharge of the stream in its natural state. It differs also with the requirements of flood control in the stream below.

J. W. SWAREN,* M. AM. SOC. C. E. (by letter).—Commendation is due the author for bringing before the Society in concrete form one of the biggest problems in hydraulic engineering. It involves the economics of agriculture and industry and of finance from the point of view of the investor and the promoter; as well as the purely technical problems of design to meet operating conditions with numerous and changeable variants.

The enthusiastic irrigationist proclaims that all water falling on a watershed draining through, or contiguous to, a sufficient acreage, should be conserved by storage and utilized on the land, leaving only dry stream beds at the entrance to the sea. At the most he would concede only canalized streams to water transportation. The operating power official, on the other hand, would fit the stream flow to his daily and seasonal load curve, with irrigation and navigation surviving as they may.

An engineering undertaking comprises three phases, named in the order of their importance as follows:

(*a*) The accumulation of pertinent data, which is crystallized into a decision or plan of action;
(*b*) The formulation of designs; and
(*c*) The construction of the works.

It is not often that the practicing engineer is equally familiar with the three major interests of power, irrigation, and transportation, so that he can, or does, take an entirely disinterested view. More often such a point of view bears the imprint of a regulatory body, and on this body rests the economic decision as to the maximum beneficial use. These bodies have no, or at the most, a meager, staff for the collection of data. The sole reliance for this important phase must rest entirely on the applicant. Only one who has had experience in the review of such submissions of data can appreciate the wide variance of the material offered. In spite of carefully drawn regulations, specifying the points to be covered so as to conserve the time of both applicant and examining officer, such submissions vary from a mass of heterogeneous irrelevance to avowed briefs of special pleading. It is through these that the examining officer must wade in an endeavor to formulate a sound conclusion of the respective phases of interest.

Regulatory bodies are political (not partisan), and as such must take the political point of view, even if they are headed in many cases by engineers. As a result, the digestion and assimilation of these masses of data must be done by subordinates who formulate technical decisions for consideration by politically minded superiors. These, in turn, look to three parties in interest:

(1) The consuming public—irrigator, power customer, or shipper;
(2) The investing public—security holders and taxpayers; and
(3) The applicant, with its attendants, engineers, promoters, and security vendors.

* Cons. Engr., Washington, D. C.

State appointed bodies have approached the problem almost entirely from the point of view of the first class, having in mind the continued maintenance of rates. Where consideration has been given to earnings as affecting the investing public, the possible absorption of wreckage from failure, as a rate influence, usually has predominated.

A study of decisions handed down by the Federal Power Commission indicates that it takes a much broader view of the problem. It has rarely touched rates. Naturally the immediate interests of the Federal Government have been given first consideration; but where its decisions have departed from the immediate Government interest, its attention has been directed to the wider economic phases of the project under consideration. This is entirely proper, as rates are a local matter but economic utilization is of National import.

The popular conception of the financial phase is that the man who puts up the money is well able to take care of himself. This is true in the first instance, as the securities of large undertakings, the sale of which provides funds for construction, are purchased by interests whose chief concern is re-sale, and who anticipate that a certain percentage will turn "sour". This is not true, however, of the final purchasers. The only way to appreciate how widespread the final distribution of a group of securities extends, is to examine the mailing list of a dividend, or the coupon verification sheet of an interest date.

Needless to say the vendors of securities are not slow to point out that even tacit approval by a commission gives added strength to the offering. Only too frequently this is accepted by the final investor as tantamount to a guaranty, particularly when such approval is by a Federal body; and the vitalness of this matter to engineers is doubly apparent from the fact that it is possible for a security issue to be prepared so that it will pass the accounting analysis of any "blue sky" commission in the United States, when at the same time a brief analysis of its engineering economics will forecast certain failure. Many such instances have come before the writer, and he presumes that other engineers have had similar experiences in the conduct of examinations.

Thus far, securities of hydro-electric companies have a good record. John E. Field, M. Am. Soc. C. E., does not present a happy case for irrigation projects.* Such débacles as he instances are preventable. Their lasting effects are shown in the difficulty of privately financing even an exceptionally meritorious irrigation project.

Any study of a specific stream to determine methods of control for power or irrigation is not complete without a study of the economic phases. It is not safe to assume that any one phase is paramount; only competent analysis can determine the particular phase. Usually the ultimate decision will rest with members of some extraneous regulatory body, whose judgment should be free from purely local influence, having in mind not only the needs of the community, but also the National welfare. A stream study that merely provides data for the design of structures to harmonize an imposed group of physical

* See p. 718.

restrictions is not complete. It must also provide the data from which may be evolved a plan that will result in the maximum economic benefit.

WILLIAM B. LANDRETH,* M. AM. SOC. C. E. (by letter).—The writer has been much interested in Mr. Stevens' statement in regard to forestation as a means of water storage and stream regulation. Study of the flow of Eastern streams leads him to agree that forestation has little if any economic value in water storage or supply.

In discussing the paper, Mr. Sargent describes studies made in forming an operating chart for the proposed Sacandaga Reservoir. This calls to mind an operating diagram made by the late G. Edward Gibson and the writer in 1919, for an existing reservoir at Hinckley, N. Y.† Part of the water supply for the Rome Summit level of the New York State Barge Canal is secured by the use of the Hinckley Reservoir on West Canada Creek. This reservoir has a crest capacity of 77 000 acre-ft. and a usable capacity of 3.85 in. on its watershed. It has been in use since January, 1915, and furnishes water for the Barge Canal during May to November, inclusive.

Several water powers are situated on West Canada Creek both above and below the point of diversion of the canal supply. Any plan of regulation that would distribute water not needed for the Barge Canal would tend to reduce the damage that might result from the diversion of water. It was realized that the plan adopted should be a practical one and rely as little as possible on individual judgment or guesswork. Gaugings had been made of the flow of the creek at various points since 1907 and were continuous from January, 1907, to January, 1918.

A study of the surface elevations of the water in the reservoir showed that during 1915, 1916, 1917, and 1918, the general plan of operation was to keep the reservoir full or nearly so during the canal months and then draw the water down, ready for the next spring flood, which resulted in inefficient operation. The reservoir is not large enough for a complete regulation of the stream. The most beneficial use to which it can be put is to smooth out the flood peaks and to increase the low-water flow.

As the records of stream gaugings were very complete after January, 1907, it was decided to use them in the preparation of an operating diagram. The records of stream flow from 1907 to 1914, inclusive, were used in the study to determine the best plan of operation and those of the four years, 1915 to 1918, inclusive, as a test period. It was believed that the best results would be obtained by dividing each month into three nearly equal periods, beginning, say, on the 1st, 11th, and 21st, of the month. By such a division of the time the necessary equalizing storage would be reduced to a small amount and, in fact, could be disregarded. The rate of inflow on corresponding dates varied from year to year. The rate of draft at any particular time would be dependent on the amount of storage and the rate of inflow at that time. In actual use the quantity of the draft is determined from an operating diagram based on the quantities referred to. The net inflow, the draft, and the storage were expressed in terms of the equivalent depth, in inches,

* Cons. Engr., Schenectady, N. Y.

† Report of New York State Engineer, 1920, p. 7.

on the water-shed. A mass curve was constructed using the mean net inflow for each "one-third-of-a-month" period for the years 1907 to 1914, inclusive. From this mass curve the maximum uniform drafts possible, with varying amounts of storage from an empty to a full reservoir, were determined for the date beginning each "one-third-of-a-month" period. From the data for each date the drafts for various amounts of storage were determined and by plotting those drafts for the various amounts of storage for each "one-third-of-a-month" period in chronological order, an operating diagram was obtained. This diagram showed varying allowable rates of draft for varying amounts of storage through the year.

For storage above the crest of the dam the draft is uniform, the additional storage being disregarded—as was true also for the storage in the lower 3.5 ft. of the reservoir. As a test, the first operating diagram was applied to the 4-year period, 1915 to 1918, inclusive, as those years had not entered into the formation of the diagram. As the reservoir was in use during those years the flow below the reservoir was corrected for the influence of the storage, so as to show the inflow into the reservoir. The operating diagram was then applied to the entire 12-year period, 1907 to 1918, inclusive, and such corrections made in it as seemed warranted and advisable.

During 1919, the diagram was in use and the results of its operation were carefully noted. A final operating diagram was drafted in December, 1919, based on the operating diagram in use. This final diagram has been in use since January, 1920, and differs only from the first one by a slight increase in the amount of draft and a decrease in the reserve storage. The diagram in use calls for a nearly empty reservoir on April 1, in order to obtain the full effect of it during the spring floods that generally come in April or May, and a full reservoir about June 1.

Use is made of the operating diagram as follows: At the beginning of any "one-third-of-a-month" period the gate operator would obtain the surface-water elevation of the reservoir from a gauge. In the operating diagram at the intersection of the vertical date line and the horizontal reservoir-elevation line he would read the draft, in second-feet, for the ensuing "one-third-of-a-month" period and set the reservoir gates to feed that quantity. The operator would check the gate setting by a gauge about 1 mile down stream from the reservoir dam and, if necessary, reset the gates. Observation of the surface elevation in the reservoir and a study of the operating diagram during the "one-third-of-a-month" period would show whether the draft was too great or too small and it could be corrected at the beginning of the next period. For several years the regulation of the draft from the reservoir by the operating diagram has been under the control of the State Engineer.

In Tables 1, 2, and 3 are given the mean monthly drafts from the Hinckley Reservoir since January, 1915, except that for 1919 when the record was badly broken, and the yearly rainfall at or near it. These records are taken from the Annual Reports of the New York State Engineer.

In 1917 the total run-off on the water-shed was 31.68 in.; in 1923, it was 32.49 in. In 1923, when the diagram was used, the draft was much more

uniform than in 1917, when no diagram was used, although the total run-off was nearly the same.

TABLE 1.—Monthly Mean Draft from Hinckley, N. Y., Reservoir When It Was Not Operated by the Aid of a Diagram.

Year.	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.
1915	1119	677	663	1525	731	275	298	871	459	1222	663	723
1916	1505	1423	965	1983	1775	643	475	340	334	370	983	1230
1917	1136	469	460	1693	1086	1485	445	436	304	1034	1487	494
1918	418	472	957	1505	903	434	358	326	451	1133	1333	1359

TABLE 2.—Monthly Mean Draft from Hinckley, N. Y., Reservoir When It Was Operated by the Aid of a Diagram.

Year.	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.
1920	958	688	762	2660	1120	473	434	579	630	695	750	1390
1921	1140	920	2330	1400	870	524	607	695	588	640	1000	1250
1922	730	676	1110	3390	1330	2100	1060	688	624	643	619	523
1923	607	769	728	2690	2060	570	492	382	396	469	647	916
1924	1070	1080	784	1980	2580	636	455	366	498	969	767	646
1925	570	741	1640	2340	1200	666	996	754	684	...	...	....

TABLE 3.—Yearly Rainfall, in Inches, from Stations at or near Hinckley, N. Y., Reservoir.

Year.	Inches.
1915	47.41
1916	44.27
1917	45.73
1918	44.57
1920	43.67
1921	43.56
1922	54.91
1923	47.05
1924	44.83
1925	61.41

The use of an operating diagram in determining the draft from the reservoir has shown that such regulation is a very practical question and can be solved within reasonable limits by a study of the gaugings of the stream. The past performance of a stream, as shown by its gaugings, forms a good measure of what it will do, its value depending on the length of the gauging period.

Regulation for canal supply from May 1 to December 1 in each year does not differ greatly from that for irrigation purposes, during those months.

J. D. Galloway,* M. Am. Soc. C. E. (by letter).—The author has clearly set forth the relations between stream flow, irrigation, and power, but attention should be called to some points in the paper that need explanation. A series of diagrams is presented for different streams and localities that express the general relationship of the three factors in the form of percentages. The author, however, states as one of his conclusions that:

"The problem of best utilizing the available water supply for both power and irrigation is strictly a local one and cannot be treated in general terms. Definite water supplies, definite storage opportunities, definite lands to be irrigated, and known power demands are essential prerequisites for its solution."

The writer is in complete agreement with the conclusion, and for this reason believes that the data set forth in the diagrams are misleading unless properly interpreted. In all instances the elements considered are quantitative, and the percentage method does not serve to express the relation between the three. It is not correct to superimpose on the percentage curve of water supply the additional irrigation and power demand curves, if a true relationship is to be shown. The relationship between the actual quantity of water in a stream and the irrigation and power demand is lost in such a case, and the diagrams are, to this extent, incorrect. For instance, the supply of water in the river is frequently ample for all power and irrigation requirements. In most cases a large quantity of water is wasted. Any diagram, to express the relationship between the three elements, should be quantitative. For this reason, it is believed that the author would have made the subject clearer if his diagrams had expressed the relationship quantitatively and not merely by percentages.

The statement in his conclusion is correct, because each river must be studied by itself in order to obtain a true view of the entire situation. This is expressed more in detail in Fig. 14 which is reproduced from the writer's paper on "Hydro-Electric Developments on the Pacific Coast",† and also from his discussion‡ of the paper by William Kelly, M. Am. Soc. C. E., entitled "Co-Ordination of Irrigation and Power". The diagram is made up of curves similar to those given by the author, except that a hydrograph of the stream is drawn and typical irrigation and power demand curves are superimposed thereon. The author placed considerable stress on the apparent conflict between irrigation and power in the use of the waters of a stream. It is believed that in practically all cases much of this apparent conflict disappears, or can be reconciled by the construction of reservoirs in appropriate situations.

From Fig. 14, it will be noted that stored water, made necessary by low stream flow during the months, June to November, is released for two purposes, which appear in three parts of the diagram. At one time the irrigation demand exceeds the demand of power for stored water, but in September, October, and November, and frequently well into December, stored water

* Cons. Engr., San Francisco, Calif.

† *Transactions*, Am. Soc. C. E., Vol. LXXXVI (1923), p. 814, Fig. 25.

‡ *Loc. cit.*, Vol. 89 (1926), p. 1470, Fig. 17.

must be released for power in excess of that demanded for irrigation. On any ordinary stream where the irrigated lands lie below the power developments, water released from storage for power purposes is available for the use of irrigation, and, to this extent, irrigation benefits by the storage of water for power. The diagram indicates that in May, June, July, and August the irrigation demand for stored water exceeds that for power, and to that extent (indicated by the area in the upper part of the diagram) stored water for irrigation must be supplied independent of the power storage. The correlation between the two, however, is indicated by that section of the diagram beneath the power demand curve, which water, released for power, is available for irrigation. To this extent, power and irrigation work together, and, instead of being in conflict, the two are mutually helpful. In Fig. 14, if there were no power development on the stream, irrigation would have to provide for all the stored water indicated by the area between the hydrograph of the stream and the irrigation demand curve. When, however, power reservoirs are installed, the burden of providing storage for irrigation is automatically lightened.

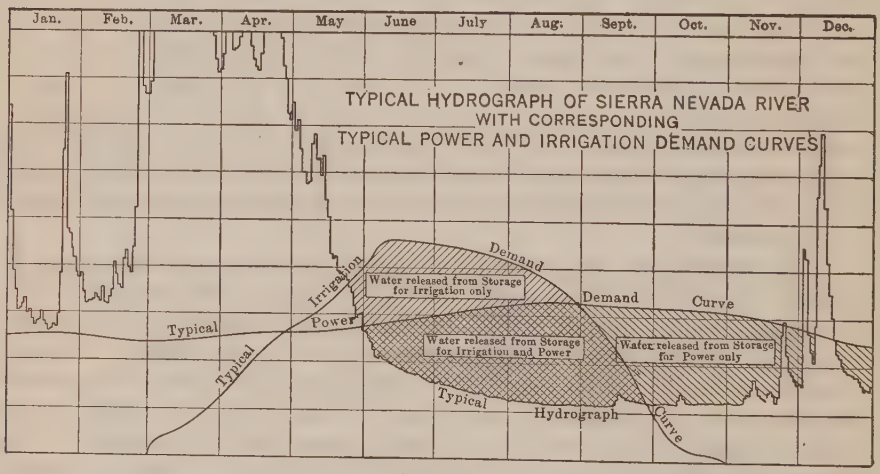

FIG. 14.

However, in addition to the correlation and harmony between the two, there is a further aid to irrigation not often appreciated. On a properly designed system, where the two interests are correlated, the irrigation reservoirs at the end of the irrigating season would be practically empty. Owing to the wide variation in stream flow from year to year, it might happen that, with irrigation reservoirs empty at the end of the season, the next winter's run-off would be so low that the reservoirs would not be filled. Ordinarily, however, where the irrigation reservoir is down stream from the power plants, the water released from the power reservoirs during September, October, and November, and sometimes in December and January, would assist in filling the irrigation reservoirs for the next year's use. In this way the stream is regulated over two years or more, and the possibility of a shortage of water

for irrigation is largely dissipated by the regulation of the stream by power reservoirs.

It is admitted that the physical situation sometimes is exactly reversed, as on the Deschutes River in Oregon and in the Green River Valley in Wyoming, where all the irrigated lands lie above the best power developments. With the Deschutes River, however, although the largest part of the drainage area is tributary to the irrigable lands, the major portion of the stream flow, appearing as springs from the lava formation below the irrigable lands, is available for power without interfering with irrigation.

The purpose in calling attention to these points is to minimize what the writer believes to be the incorrect statements constantly made that the use of the waters of the stream for power and for irrigation are necessarily in conflict. The general case is that of the stream which has its source in the mountains, where power development is possible, and after delivering up its energy flows out on to a plain where the irrigable lands lie. With a proper arrangement of reservoirs—some in the high mountains for power and some in the foothills for irrigation—the regulation of the stream can be accomplished in the manner indicated and in such a way that the two are mutually helpful. The greater profits from power development will stand the higher cost of reservoirs in the mountains, and the cheaper reservoir sites in the foothills are available for irrigation. The effect is to reduce the size and cost of the irrigation reservoirs, which is practically the situation on all the California rivers, where irrigation is practiced most.

The author indicates in another way what seems to be a conflict between power and irrigation. This refers to the fact that only a small percentage of power developed on a stream is used for irrigation pumping, and the remainder (about 95%) is used otherwise. It would seem that this does not entirely explain the situation. In any country where the people depend on irrigation there is a seasonal demand for power which is made up not only of the power used for pumping, but also of that used in all the related industries that go with irrigation. One of the best examples is found in the City of Fresno, Calif., and the surrounding region. The area is largely dependent on irrigation from several rivers coming down from the Sierra Nevada. The business of the city is to supply the wants of an agricultural community. Large packing plants are operated by power during the harvest season. The entire life of the community centers around agriculture and produces for the San Joaquin Light and Power Corporation System a summer demand greater in percentage than any other of the Western systems. In the writer's paper referred to previously, Fig. 23* shows a series of curves similar to those indicated by the author's Fig. 2. The July and August demand of the San Joaquin Light and Power Corporation exceeds the average demand by about 27 per cent. The amount varies from year to year. All this excess demand is caused by the seasonal changes in agriculture which, in the case cited at Fresno, is almost all caused by irrigation. Power is as necessary in such a community as irrigation water.

* *Transactions*, Am. Soc. C. E., Vol. LXXXVI (1923), p. 810.

The writer has dwelt at length on this phase of the relation between power and irrigation because it is believed that statements such as those made by the author have a tendency to emphasize a conflict that does not exist if all the factors are taken into account. Owing to the private ownership of power companies and the public ownership of irrigation supplies, demagogic politicians and short-sighted reformers have constantly endeavored to emphasize a conflict between the two, to the disadvantage of power development. It is of the same type of misinformation as that to which the author refers regarding the relation between forests and stream run-off. It is apparent that water for irrigation is superior in its use to that for power. However, it is seldom the two cannot be correlated to mutual advantage, and, if the total life of the community be taken into account, power is as necessary as irrigation water. It is believed to be incorrect to over-emphasize the apparent conflict between the two, and for this reason the writer has developed the same ideas which he has expressed in several discussions on the subject.

With the author's statement regarding the relation of forests and stream flow, the writer is in complete agreement. For years the public has been fed on incorrect statements of the kind he cites. Engineers cannot too often emphasize the real relationship that exists between the two.

Although some criticism has been made of parts of the author's paper, the writer desires to express his appreciation of the clear manner in which the subject has been set forth.

J. C. Stevens,* M. Am. Soc. C. E. (by letter).—The writer is gratified at the discussion his paper has brought forth.

Mr. Sargent has presented very valuable data on the practical regulation of reservoirs for flood control and water power without forecasts of probable supplies. Doubtless, the same principles can be utilized in many cases in practical regulation for power and irrigation. The storage capacity above spillway level probably can be counted upon and the spillway capacity reduced accordingly in large reservoirs for which complete records of run-off are available. On small reservoirs, however, where stream-flow records are rather "sketchy", it would be safer not to count on it.

The writer wishes every engineer who has to do with the development of natural resources could read Mr. Hoyt's discussion. It is a clear, concise statement of the policy that must govern the practical administration of public land and water resources. As Mr. Woolley points out, conditions may arise that make power development paramount to irrigation. Every water-shed must be studied in the light of the most economic use of its water supply. Behind the conclusions drawn from such an analysis there must exist unprejudiced authority to enforce them in the interest of all concerned, and, obviously, this function is best exercised by the Federal Government.

Mr. Galloway questions the writer's diagrams because they express water supply and demand in percentages of totals. To have given quantitative data would have defeated the purpose of the paper, which, as stated in the Synopsis and elsewhere, was to inquire into the subject in general terms. Quantitative

* Cons. Hydr. Engr. (Stevens & Koon), Portland, Ore.

figures would have made a local problem of each example cited, and the study would have been limited to the present demands. By treating these examples generally, a forecast is gained as to the future ultimate utilization of the entire annual supply.

Fifty or a hundred years hence these streams will be flowing much as they are to-day. In all probability the curves of demand for power and irrigation will not be materially different from those indicated, because they are expressed in percentages of the total rather than in second-feet or kilowatts. The simplicity and value of presentation in the manner adopted by the writer would have been completely lost had he followed Mr. Galloway's suggestion as illustrated by Fig. 14, in which neither percentages nor quantities are given, the curves being merely "typical".

The writer reiterates his statement that water is and always will be vitally necessary for irrigation, but not in the same degree for power. There is stern justice in conceding a preferential use to agriculture, and power interests will benefit in the long run by such a concession. Whether or not there is a conflict of interest is a purely local fact. There is nothing hypothetical about it. The very fact that power companies in California and elsewhere are constructing storage reservoirs and power plants above irrigation reservoirs is proof that a conflict of interest did exist, and that the solution lay in a double system of reservoirs. Many sections, however, are not so topographically fortunate in this respect, and where such solutions are not possible concessions will have to be made on both sides. There is, however, a limit to the concession irrigation can make.

Those discussors who mentioned the subject at all agree with the writer in his remarks about storage by forestation. It is gratifying to know that some engineers at least have done some original thinking on this subject and have penetrated the thick coat of propaganda spread over it.

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Paper No. 1617

DEVELOPMENT OF INDUSTRIAL SITES AND TRACKAGE LAYOUTS AT KANSAS CITY, KANSAS, AND LOS ANGELES, CALIFORNIA, BY THE UNION PACIFIC SYSTEM*

By Walter R. Armstrong,† M. Am. Soc. C. E.

With Discussion by Messrs. A. D. Butler, Lester Bernstein, and Augustus Hunt.

Synopsis

In this paper the writer has given brief descriptions of the Fairfax Industrial District at Kansas City, Kans., one of the eastern terminals of the Union Pacific System, and of the industrial districts at Los Angeles, Calif., one of the western terminals of the System.

Introduction

With the keen competition which enters into the business of the railway transportation systems of the country generally, and, more particularly, with the recent competition of motor vehicles, the railroad companies of the United States have recognized the necessity of providing suitable trackage for various industries, warehouses, and manufactories at their terminals and larger stations, to enable the industries to handle commodities transported in carloads with the least possible delay and at the least expense.

In line with this progressive policy, the Union Pacific System has encouraged the establishment of industrial trackage. At Kansas City and Los

* Presented at the meeting of the City Planning Division, Salt Lake City, Utah, July 9, 1925.

† Gen. Supt., Los Angeles & Salt Lake R. R., Los Angeles, Calif.

Angeles, as here described, it has invested in large areas of land contiguous to its railroad, and has planned thereon a system of trackage to accommodate most conveniently industries seeking locations.

Kansas City, Kans.

With the development of Kansas City it has been apparent for some time that an industrial district should be added to the city, as well as districts for residences, retail business, and manufacturing of certain classes. With this end in view, the Union Pacific System, in line with its policy to aid in the development of worthy institutions within its jurisdiction, in co-operation with other enterprises, has established the Fairfax Industrial District, at Kansas City, Kans. The map (Fig. 1) shows its location north and east of the city, on a tract of land which many years ago was built up by the action of the Missouri River in depositing sands in what was formerly the old channel. Fig. 1 indicates how well the various tracts that have been laid out are to be served by a railroad.

The development has a number of interesting features:

First.—The area of the Fairfax Industrial District is approximately 1 300 acres, including:

(*a*) The area to be used for streets, approximately 200 acres;

(*b*) The area to be used for right of way of spur-tracks, approximately 75 acres; and

(*c*) The area within the district released for dike and storm drainage, 60 acres.

Second.—A beginning has been made in the construction of trackage to the extent of 5 miles.

Third.—The ultimate development of main switching spurs and storage tracks is approximately 40 miles. (This mileage does not include such separate loading spurs as may be constructed for the exclusive use of individual industries, which would be built at the expense of the industry.)

Fourth.—The tract, exclusive of streets and alleys, is made up in lots 100 ft. wide and 600 ft. long, more or less. The lots are to be 100 ft. wide; industries requiring more space can occupy more than one lot.

Fifth.—Arrangements have been made for water supply and drainage. A dike of earth, with rip-rap when necessary, has been thrown up to protect the District from any encroachment by the rivers. This dike was constructed at the expense of the Fairfax Drainage District.

Sixth.—It is proposed to sell industrial sites on time contracts if desired, but more favorable terms can no doubt be made with the owners by cash outlay.

The population of Greater Kansas City, including Kansas City, Kans., was reported in the Federal Census for 1920 as 440 000. There has been a steady, gradual increase during the past five years.

Los Angeles, Calif.

When the Union Pacific System purchased all outside ownership in the Los Angeles and Salt Lake Railroad in 1921 and made Los Angeles one of

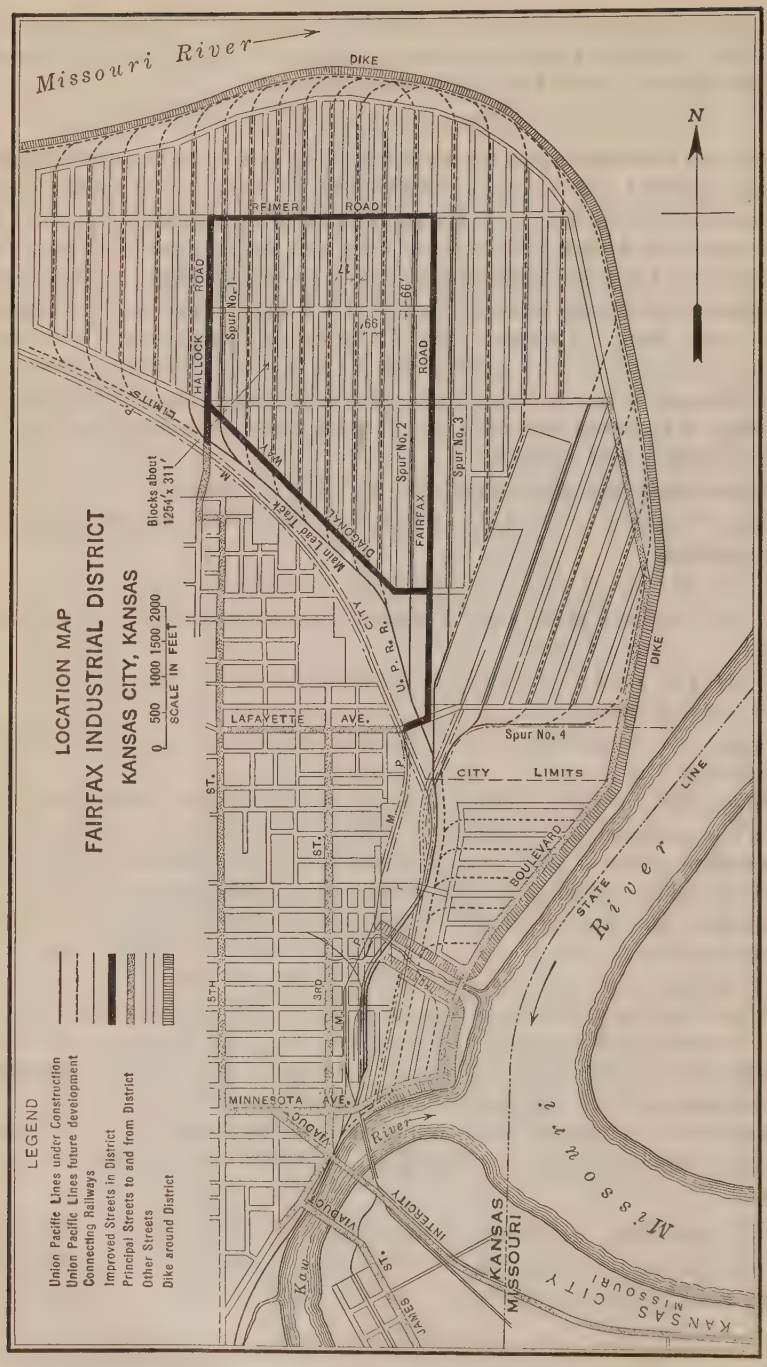
Missouri River
DIKE
N
REIMER ROAD
HALLOCK ROAD
Spur No. 1
Spur No. 2
Spur No. 3
FAIRFAX ROAD
DIAGONAL WAY
Main Lead Track
U. P. R. R.
Blocks about 1254 x 311'
LOCATION MAP
FAIRFAX INDUSTRIAL DISTRICT
KANSAS CITY, KANSAS
0 500 1000 1500 2000
SCALE IN FEET
LAFAYETTE AVE.
Spur No. 4
CITY LIMITS
STATE LINE
BOULEVARD
MINNESOTA AVE.
VIADUCT
INTERCITY
KANSAS
MISSOURI
Missouri River
KANSAS CITY MISSOURI
LEGEND
Union Pacific Lines under Construction
Union Pacific Lines future development
Connecting Railways
Improved Streets in District
Principal Streets to and from District
Other Streets
Dike around District

FIG. 1.

its western terminals, intensive study was made of the city and its environments as a manufacturing and distributing center. In the preceding decade the city had made rapid strides industrially. Construction of every character incident to the rapid increase in population in all Southern California gave impetus to this movement. The improvement of Los Angeles Harbor from a "mud flat" to a harbor of major proportions, and the subsequent development of trans-Pacific and trans-Atlantic as well as intercoastal and coastal shipping gave further impetus.

Study of the existing industrial situation indicated development principally on lands immediately abutting existing rail lines and gradually spreading in all directions from the business center of the city. It was apparent that, in a short time, this character of development would bring about two undesirable results: First, the industrial district would be scattered over large areas; and, second, industrial areas suitable for rail facilities would soon be improved and further industrial development could only be brought about at excessive cost.

The needs of the local situation were resolved into four classes. Ensuing developments by the Union Pacific System may be briefly summarized, as follows:

First.—There was need of close-in property susceptible of improvement by rail facilities and within short truck haul of the central business district for the location of industries the character of whose business, such as warehousing and light manufacturing, will bear the overhead of valuable realty and the type of building construction required by fire protection ordinances for down-town business districts. Such property was to be subdivided to provide for the warehouse type of construction. Lots were to be varied in width from 100 ft. to a maximum of 150 ft., with blocks, in so far as other governing conditions would permit, having a minimum length of 600 ft. and a maximum of about 800 ft.

The matter of industrial spurs was given careful study. The first demand of the average industry is for an individual spur-track. An attempt was made to meet this demand. It was determined, however, that the inconvenience resulting from more than one industry on a service track is more than overcome by the saving in valuable underlying realty and a single service spur to each subdivided block was adopted. To reduce to a minimum the inconvenience to industries caused by moving partly loaded or unloaded cars to make switches for another industry, a third track was designed to serve as a lead track, extending through to the block farthest removed from the holding yards. To insure prompt deliveries and further facilitate switching, the industrial subdivision was provided with its own holding yards of a capacity designed to meet the requirements of the ultimate development of the tract.

Streets were laid out with a minimum width of 60 ft. and paved with concrete, with a center drain. The paving was constructed to a width of 20 ft. on each side of the center line, an additional 10 ft. on each side between the paving and the property line being left for improvement at the option of the industry with a sidewalk or a trucking driveway.

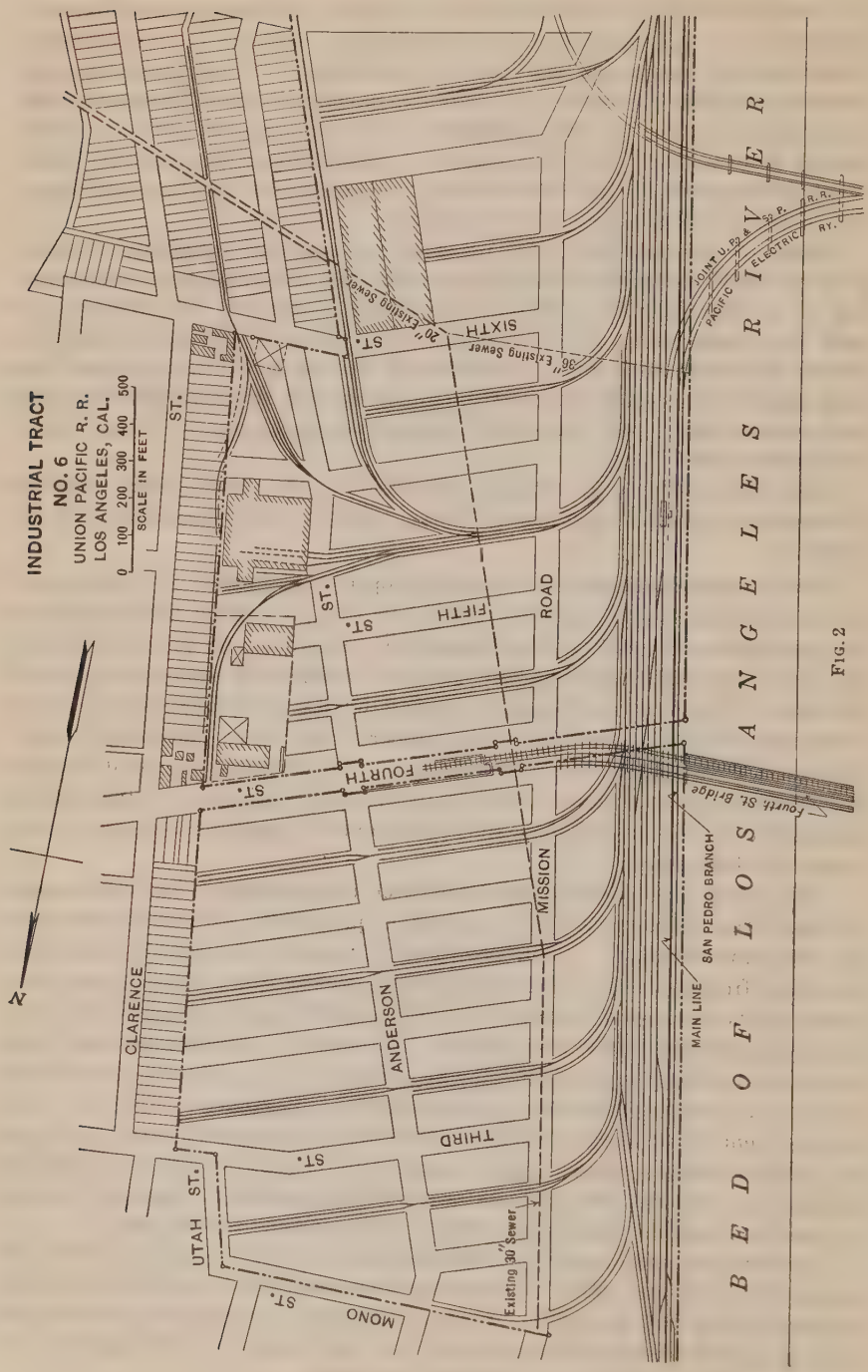
INDUSTRIAL TRACT
NO. 6
UNION PACIFIC R. R.
LOS ANGELES, CAL.
0 100 200 300 400 500
SCALE IN FEET
N
CLARENCE ST.
UTAH ST.
MONO ST.
THIRD ST.
ANDERSON
MISSION ROAD
FOURTH ST.
FIFTH ST.
SIXTH ST.
20" Existing Sewer
36" Existing Sewer
Existing 30" Sewer
Fourth St. Bridge
SAN PEDRO BRANCH
MAIN LINE
JOINT U. P. & S. P. R. R.
PACIFIC ELECTRIC RY.
BED OF LOS ANGELES RIVER

FIG. 2

In accordance with city requirements, sanitary sewers were placed beneath the center lines of streets, house connections being extended to each lot. City water lines were laid in one unpaved 10-ft. strip and gas lines in the other. Telephone and electric light and power lines on pole construction were likewise located on the 10-ft. strips. Storm sewers to meet the needs of the situation were constructed and connected to existing city lines or given their own separate discharge into the adjacent river bed. The property made available for sale to industries excluded the track area, the Railroad Company retaining ownership to the tracks and realty underlying them. A development of this character is shown on Fig. 2, outlining the Union Pacific Metropolitan Warehouse and Industrial District.

Second.—Other property must be reasonably close in but far enough removed from the central business district to reduce the cost and relieve the industry of heavy carrying charges. A lower grade of building construction also would be permissible. This would serve for the location of a warehousing or manufacturing business requiring convenient close contact with the central business district, but unable to support the greater overhead of high land values and expensive building construction, as well as for industries requiring somewhat larger areas and open storage.

In planning the subdivision of this property, an effort was made to vary the width from about 210 ft. maximum to a minimum of 120 ft., with blocks having a minimum length of 600 ft., and subject to requirements of the city in conforming with existing dedicated streets. The layout was so made, as shown on Fig. 3. This subdivision is laid out with 60-ft. streets, paved with concrete as described in the preceding class, and is also furnished with sanitary sewers, storm sewers, high-pressure gas, city water, telephone, electric light, and power service. Service tracks, intermediate lead tracks, and holding yards are designed substantially as described for the Metropolitan Warehouse and Industrial District (Fig. 2), except that for this development the third, or intermediate, track was extended throughout the entire subdivision. This tract covers an area of 70 acres. An additional undeveloped area of about 170 acres, immediately adjacent to the tracks on the east, is held to protect future needs.

Third.—Another class comprised property carrying a still lower land value, well removed from the wholesale and retail business center, suited for the development of a class of industry requiring acreage, or for embryo industries covering small areas but in their early stages unable to sustain the overhead of the more expensive property in the preceding classes.

In selecting these locations, attention was given to direct and adequate highway access to the city and to surrounding towns, but of necessity, in order to secure cheap land, the locations were 5 or 6 miles removed from the heart of the business district. In this character of development, in order to hold the selling price as low as possible, improvements were held down to bare necessities. The properties selected, being located without the city limits, are at this time without sanitary sewers. It is contemplated that cesspools will take care of this feature until the development of projected

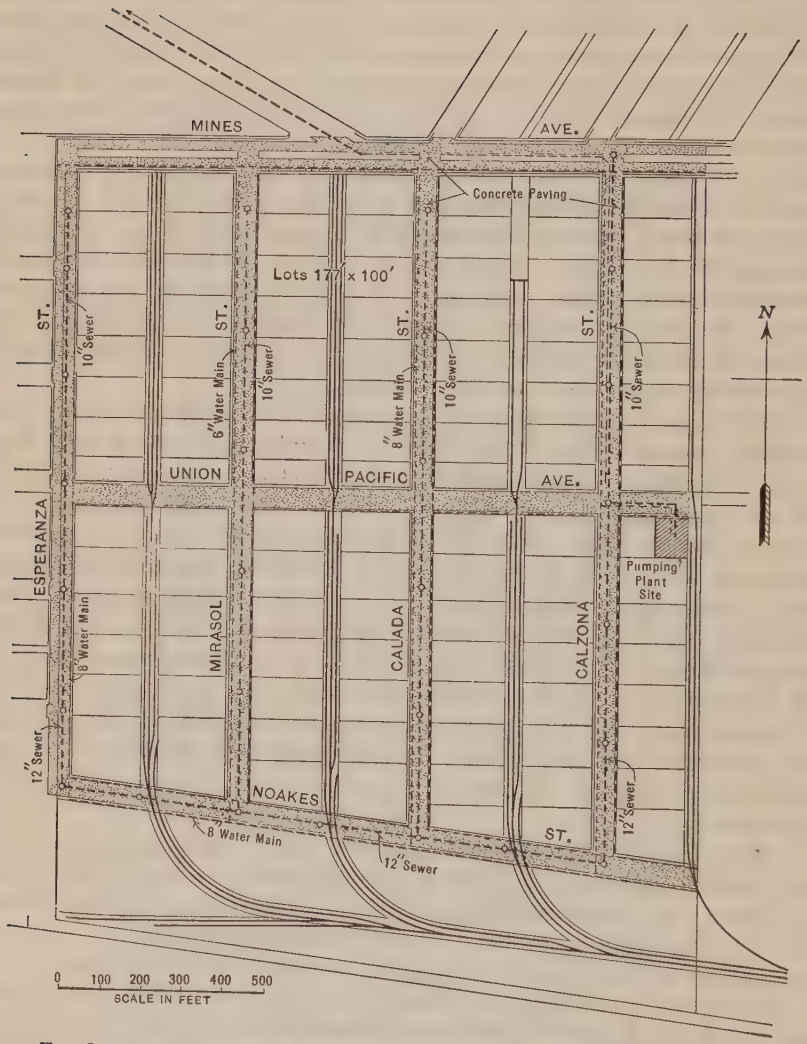

FIG. 3.—IMPROVEMENT OF INDUSTRIAL TRACT NO. 1, UNION PACIFIC SYSTEM, LOS ANGELES, CALIF.

county facilities affords better disposition. Water will be furnished in adequate quantities from a local private system.

The subdivision as designed will provide, on about one-half the area, for lots varying in width from 150 to 300 ft.; the lengths of blocks, in so far as conforming with orderly development, varying from a minimum of 800 ft. to a maximum of 1 200 ft. Streets were designed with a minimum width of 60 ft. Where a central street may be used as the main artery of trucking,

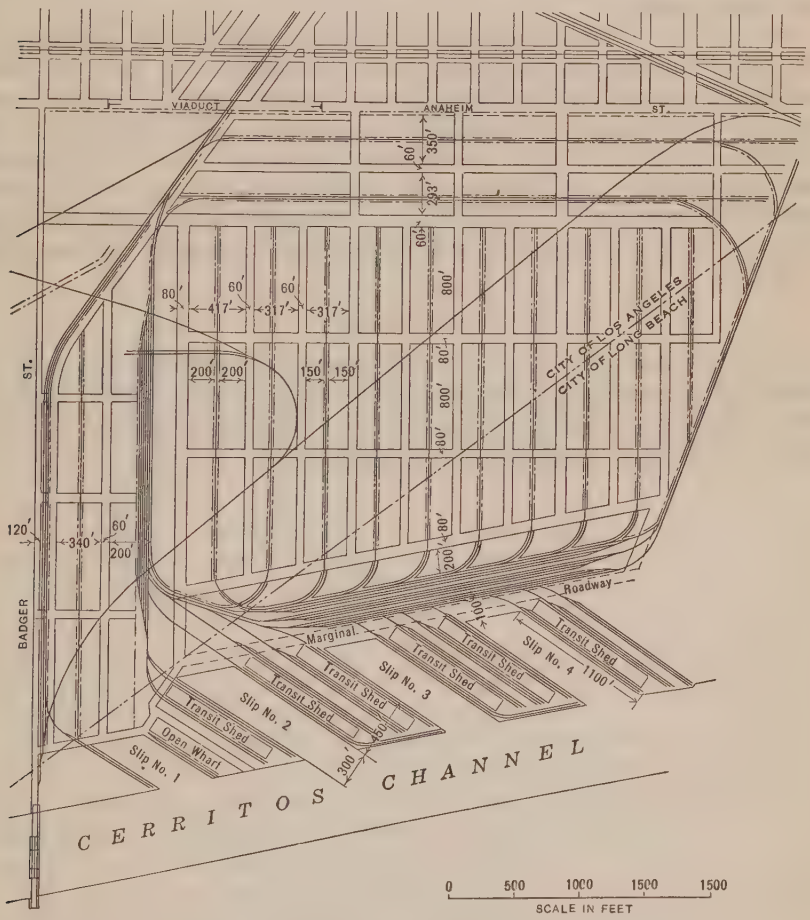

FIG. 4.—UNION PACIFIC WATER-FRONT AND INDUSTRIAL SUB-DIVISION, LOS ANGELES HARBOR.

for entrance and departure, an 80-ft. width is assigned. About one-half of each subdivision is allowed to remain without projected street and trackage development. This will later accommodate the large manufacturing or assembling plant requiring acreage, it being contemplated that development would be built around such requirements.

For trackage, a 17-ft. right of way is withheld between each subdivided street and a lead track constructed thereon. It is expected that owners will

furnish their own spur-tracks to be located on their own right of way. Street paving, as provided, consists of a 24-ft. strip of concrete paving in the center of the street reservation. Industries may, at their option, extend the paving to their property lines. In each instance, gas mains are available within reasonable distance. It may become necessary later to construct service mains on each street. In order to hold the price down, this facility is not now so conveniently accessible to the owner. Electric power and light, and telephone service are provided.

Fourth.—The Union Pacific System acquired with the Los Angeles and Salt Lake Railroad property, a large acreage of land at Los Angeles Harbor, some of which has a direct water frontage, the larger portion, however, being potential industrial land immediately back of this. This property, as yet almost undeveloped, is held for warehousing, light manufacturing, and acreage industrial purposes. An illustration of what may be termed "typical water-front development design", combined with the subdivision of the supporting back area for warehousing and light manufacturing purposes, is represented in Fig. 4.

Conclusion

The establishment of these industrial layouts will undoubtedly enable the Union Pacific System, in a large measure, to keep its transportation facilities abreast of the development of the cities in which they are located, and contribute to a spirit of progressiveness in these cities.

DISCUSSION

A. D. Butler,* M. Am. Soc. C. E.—Cities, like people, have their individuality. It is a strange thing, and one often commented upon, that of all the people in the world no two persons are exactly alike; and this characteristic of individuality is even more pronounced in cities than in persons. The problems that confront each city are those peculiar to its particular needs; and a solution that meets the requirements of one will only apply to another after modification.

When a railroad terminal is to be established in a city built in a flat, level country, it is often a serious problem to select a route that will meet the needs of the road, and yet not seriously invade some residential district, greatly to its detriment.

Nature has been kind to Spokane, Wash., in many ways, and has provided against the problem of a railroad company invading to any considerable extent any of its residential districts. The City of Spokane was started along the banks of the Spokane River, and has developed about equally both north and south of this dividing line; the ground rises on the south to a plateau about 400 ft. above the river and on the north side, more gradually, to an elevation of approximately 100 ft. As is the case with most of the Western cities, the valley through which the Spokane River flows is the only practical place for the railroads to build; so, by nature, all the railroad terminals and industrial sections of the city are in the flat valley on each side of the river and interfere little with any residential section.

The down-town section of the city is badly cut up, however, with three main-line transcontinental railways. This has been very expensive, both to the municipal government and to the railway companies. If, twenty years ago, the railway companies had been able, as they do now, to have seen this problem in the broad sense of transportation and service to the public, rather than in an impassioned desire to get business away from their competitors, such things as this could have been avoided.

The existence of railroads is only justified because they render service to the public; and this fact should be kept uppermost in mind by both railroad and city officials, to the end that the terminals may be designed so that the public will be best served, and not merely that one company may take away some of the business that its competitor receives.

The trend of thought in city planning, however, namely, to seek through experience in the past to provide for the future growth of cities, is certainly an encouraging sign. Although mistakes will be made in future development, yet this method should go a great way toward eliminating some of the unfortunate situations in which some cities now find themselves.

Lester Bernstein,† Assoc. M. Am. Soc. C. E.—As many industries can bear only a limited charge for doing business (including taxes), some of the older cities of the United States will soon have to consider the establishment

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of industrial districts to take proper care of their industries. Taxes have become quite a serious problem with some plants situated in the central parts of towns. In a number of instances taxes, with high real estate values that prohibit plant additions, have compelled long-established industries to move outside the city limits.

Augustus Hunt,* Assoc. M. Am. Soc. C. E.—The arrangements made for the terminal facilities at the Port of Los Angeles are particularly impressive. Of special interest is the kind and amount of aid offered by the City of Los Angeles to further this development. The acquiring of the long strip of territory to be developed into railroad yards, unloading docks, store house, and terminal structures, is an example of what a progressive city can do to assist and procure for its industrial program aids that in the years to come will be of inestimable value to the growth of the city and will provide for more efficient and cheaper freight handling for the railroad. Both the City of Los Angeles and the Union Pacific Railroad System are to be congratulated on their spirit of co-operation.

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Paper No. 1618

FIELD PROCEDURE OF ADJUSTING THE GREAT CIRCLE LINE TO THE RHUMB LINE*

By N. B. Sweitzer,† Assoc. M. Am. Soc. C. E.

With Discussion by Messrs. C. V. Hodgson, D. E. Hughes, R. L. Faris, George L. Hosmer, Joseph James Corti, and N. B. Sweitzer.

Synopsis

Technical literature on the various branches of surveying is rich on the subject of the geodetic problems of establishing various lines on the earth's surface, comprising boundaries, to connect points of known latitude and longitude. The methods of procedure for the actual field operations, however, are not summarized and for the field engineer to compile these results for actual field use covering all phases of this question is a complicated process. To summarize selectively for the needs of the field engineer, to supply shorter and simpler formulas giving results well within the limits of admissible error, and to supply data for field use, is the object of this paper.

Many boundary lines are rhumb lines varying from the parallel of latitude through the quadrant until they merge into the meridian. Ordinarily, such lines were first run by instruments with a solar attachment, or by the needle, but retracements for extreme accuracy are now generally run by the engineer's transit. To convert this great circle or transit line to the rhumb line, or loxodromic curve, a short practical method has been needed. The basis of these calculations is the geodetic proposition involving the lines connecting latitude and longitude positions. These are simplified by the use of the theories governing railroad curves, with all unnecessary refinements eliminated, care being taken that the limit of error in the calculations would be so small that it would not show in the field projections. The formulas given contain all

* Published in May, 1926, *Proceedings*.
† Dist. Cadastral Engr., U. S. Interior Dept., Gen. Land Office, Denver, Colo.

data required in running a rhumb line by the use of the transit or great circle line and also for calculating the latitude and longitude arcs of the earth within the limits of field operation.

National boundaries as well as sub-divisions thereof, and even boundaries as minute as farms and city lots, in the great majority of cases, were originally surveyed and marked on the ground and their bearings platted on their respective maps, giving the courses as of record, and referred to the rhumb line (loxodromic curve), that is, to a line making equal oblique angles with all meridians. The final recorded plats of many railroads, canals, highways, etc., are also thus given. Calculations for closures, areas, etc., are also based on this curve; and the surveyor or engineer, especially in cadastral work, is continually making his calculations on this line as a basis. One of the main reasons for adopting the rhumb line so generally was on account of the instruments in wide use in former years and the consequent formulas and tables adopted for proper reduction and calculations based on this system. The first of these instruments for finding direction or bearing was the magnetic compass which ran an approximate rhumb line. About the middle of the Nineteenth Century, the solar compass came into use; this was a vast improvement in accuracy over the needle compass and ran a true rhumb curve. By the aid of these two instruments practically all the cadastral surveys in the United States were made and recorded on the official or commercial plats as rhumb lines.

On account of inaccuracies frequently caused by the inferiority of the magnetic compass in recording true bearings and also by carelessness in handling the solar compass, a review of early surveys often shows large errors in closure or in areas. This was, originally, not considered particularly detrimental on account of the small monetary value of lands, but as values increased and restorations for lost corner monuments became imperative a new element was introduced into cadastral work—the harmonizing of the error.

These increased land values demanded better and closer field work and better instruments. The legal side in regard to the re-location or re-establishment of these lost corners of boundary lines demanded an exact closure as regards area. The result was a proper legal and scientific treatment of the error.

This naturally brought prominently to the front the improved engineer's transit. In railroad and other engineering work of magnitude this instrument has always been the main reliance for the projection of lines, and its supremacy for cadastral work of prime accuracy is now acknowledged. The engineer's transit, however, runs exclusively a great circle line, and in cadastral restoration work this will have to be referred to the rhumb line as originally run by the engineer or surveyor, who placed monuments that must be restored in their original positions.

The requirement to-day is to run a great circle line with the engineer's transit and to transfer this line on the ground by means of offsets and proper intersection angles to the original survey which was run as a rhumb line on

the ground and recorded as such on the plats. Engineering literature furnishes data and methods for running one kind of a rhumb line only, namely, a parallel of latitude by the tangent or secant method, but there are many variations of a rhumb line from the parallel in practice, and the writer has been unable to find a general method treating all positions or courses of this line, and capable of use in field practice. The general rigid methods given in geodetic surveying for azimuth, with latitude and longitude known, are too involved for a rapidly moving field party.

Leonard S. Smith, M. Am. Soc. C. E., suggests the following:

$$\phi - \phi' = (-d\phi) = B K \cos Z + C K^2 \sin^2 Z + D h^2$$

$$\lambda - \lambda' = (+d\lambda) = A K \sin Z \sec \phi'$$

$$Z + 180^\circ - Z' = -dZ = d\lambda \sin \tfrac{1}{2}(\phi + \phi')$$

$$A' = \frac{1}{N' \text{ arc } 1''}; \quad B = \frac{1}{R' \text{ arc } 1''}; \quad C = \frac{\tan \phi'}{2 R' N' \text{ arc } 1''}$$

$$D = \frac{\frac{3}{2} e^2 \sin \phi' \cos \phi' \sin 1''}{(1 - e^2 \sin^2 \phi')}; \quad m\,\phi = \text{mean latitude} = \frac{\phi + \phi'}{2}$$

in which,

$h = B K \cos Z$;
ϕ = latitude of a known point;
ϕ' = latitude of a second or unknown point $= \phi + d\phi$;
$d\phi = \phi' - \phi$;
λ = longitude of a known point;
λ' = longitude of a second or unknown point $= \lambda + d\lambda$;
$d\lambda = \lambda' - \lambda$;
Z = azimuth of the unknown point from the known, counting from the south point in the direction S. W. N. E.;
Z' = azimuth of the known point from the unknown, or the back azimuth $= Z + dZ + 180^\circ$;
$dZ = Z' - Z + 180^\circ$;
K = length, in meters, of line joining the two points;
e = eccentricity of the earth's meridian section;
N = length of the normal, or radius of curvature of a section perpendicular to the meridian of the middle latitude, in meters;
R = radius of curvature of the meridian, in meters.

This formula is not exactly rigid, but is sufficiently accurate for the most exacting field engineer. The main trouble is that the geodesists have not wholly agreed on the form of the earth and give various values for its flattening or ellipticity, and, apparently, the arcs in the same latitude vary in different longitudes, and conversely. However, the surveyor is not directly interested in this question, as the magnitude of his operations will not be such as to cause his work to be materially affected by these discrepancies.

The tangent method of locating a parallel has its limitations. The main objection against it is that it departs from the parallel rapidly—as the square

of the distance—but for short lines it is good, because of its freedom from complications. The secant method is accurate, but introduces many unnecessary complications. It is never tangent to the parallel, contact being dependent on accurate measurement and alignment; it is first south, then north, then south again of the parallel, all in six miles, and this is likely to cause mistakes in the setting of offsets, thus doubling errors. The variation of lengths of the offsets is difficult to remember and the general problem is more involved than is necessary.

The variation of the tangent method here offered is practically the same as that used on railroad work. The only difference is that instead of using the ordinary functions of the railroad curve, the latitude and longitude radii and the normals of the earth are substituted. Basically, the parallel of latitude is taken. A 6-mile tangent is assumed for a given latitude and the intersection angle calculated by the following equation:

$$\tan \phi \times 5'.2,$$

or,

$$d\lambda \sin (m\phi) = \text{curvature or intersection angle}$$

The offsets, in links, from the tangent to parallel are obtained by the equation:

$$1.01 \times (d\lambda)^2 \times \tan \phi$$

in which, $d\lambda$ is difference of longitude, in miles, $m\phi$ is the mean latitude, and ϕ is the latitude.

Having thus secured the curvature or intersection angle for 6 miles and calculated the offsets for each ½ mile (the first 3 miles of offsets are all that are required as the offsets of the other 3 miles repeat in inverse ratio), obtain by an observation, preferably on Polaris, a true meridian. Then, commencing at A, as in Fig. 1, turn an angle of 90° east and run on a tangent with offsets as follows: ½ mile, no offset; 1 mile, 1 link north to parallel; 1½ miles, 2 links north; 2 miles, 4 links north; 2½ miles, 6 links north; and at 3 miles, 8½ links north to parallel. At this point, turn a deflection angle of 4′.9 to the left and run on a tangent 3 miles, setting the offsets in inverse order always north. The next 3-mile point will be tangent to the parallel and will be due east, the same as at the starting point, and a back-sight of one-half the intersection angle, or 2′.4, will intersect the starting point, 6 miles westward. Repeat this procedure indefinitely if it is required to prolong the parallel, or, as in Fig. 1, 6 miles farther to C. A great many of the old lines to be relocated on the ground, given on the plats as east or west, vary in reality considerably from these cardinals. Manifestly, the intersection angles and offsets of such a line will differ from those given for a parallel.

In Fig. 1, assume the parallel to be revolved about B as an axis, to occupy the position, $A'B'C'$, and assume the course of $A'B'C'$ to be N. 47° E. Its mean latitude will still be 43° N., and, without involving any excessive irregularities, the intersection angles and offsets may be written:

$$\sin \theta \ (\tan \phi \times 5'.2) = \text{intersection angle, 6-mile tangent}$$

or,

$$(d\lambda \sin \phi_m) \sin \theta = \text{intersection angle}$$

$$\sin \theta \ (1.01 \times D^2 \times \tan \phi) = \text{offsets in links from tangent}$$

in which, θ = course of line, ϕ = latitude, D = distance, in miles, $m\ \phi$ = mean latitude, and $d\ \lambda$ = difference in longitude.

Table 1 for Latitudes 33° to 49°, giving offsets in links, deflection angles, tangential angles, and difference of longitude for 6 miles for the given latitudes, will cover the latitude of the United States.

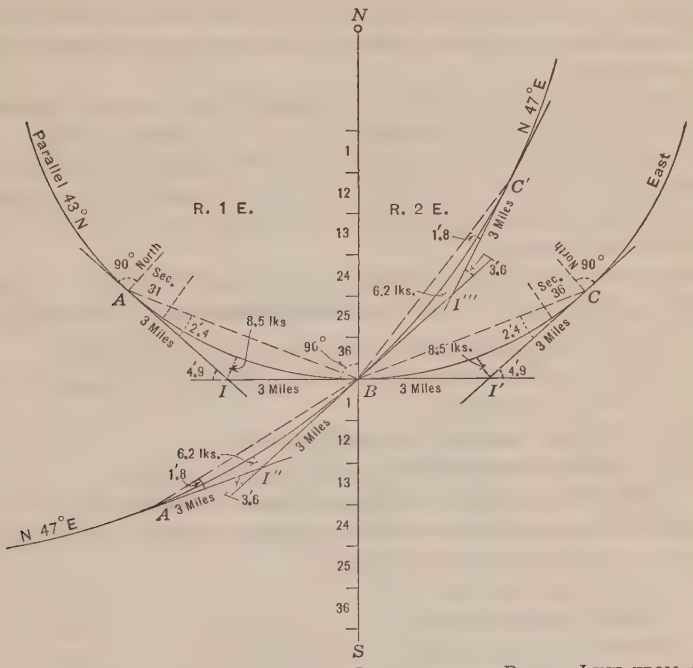

FIG. 1.—PROJECTING A PARALLEL OF LATITUDE OR A RHUMB LINE FROM A TRANSIT OR GREAT CIRCLE LINE.

TABLE 1.—OFFSETS IN LINKS, DEFLECTION ANGLES, ETC., LATITUDE 33° TO 49°.*

Latitude.	Miles.						Deflection angle.	Tangential angle.	6 Miles difference in longitude for:
	½	1	1½	2	2½	3			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
33°	0	1	1	3	4	6	0°3′ 23″	0°1′ 41″	0°6′ 12″
35°	0	1	2	3	4	6	0°3′ 38″	0°1′ 49″	0°6′ 21″
37°	0	1	2	3	5	7	0°3′ 55″	0°1′ 57″	0°6′ 31″
39°	0	1	2	3	5	7	0°4′ 13″	2′ 06″	0°6′ 41″
41°	0	1	2	4	5	8	0°4′ 31″	2′ 15″	0°6′ 53″
43°	0	1	2	4	6	8½	0°4′ 51″	2′ 25″	0°7′ 06″
45°	0	1	2	4	6	9	0°5 12″	2′ 36″	0°7′ 21″
47°	0	1	2	4	7	10	0°5′ 34″	2′ 47″	0°7′ 37″
49°	0	1	3	5	7	10	0°5′ 59″	2′ 59″	0°7′ 55″
	5½	5	4½	4	3½	3			
	Miles.								

* Multiply offsets and deflection angles by sine of course for rhumb line.

Interpolation can easily be made for any of the functions given, as they vary only slightly and are smaller than the least reading of the verniers of the field instruments. These functions, multiplied by the sine of the course of a rhumb line of the same mean latitude, will give the corresponding functions of that line. For a long line the total curvature is equal to the sum of the intersection angles, which will restore at once the desired meridian and will enable the engineer to check accidental or accumulated errors by check observations.

Back-sights or back azimuths are one-half the sum of the intersection angles, or the sum of the tangential angles. A 6-mile tangent is assumed in this example for convenience, as its apex distance is small and is kept near its objective—the parallel or rhumb line—but a tangent of any greater length could be assumed and calculated by the formulas, as well as the new intersection angle and offsets.

Column (10) in Table 1, "Difference of Longitude", can be rigidly calculated by an equation in use by the Naval Observatory. Referring to Fig. 1:

Let, $A\ B\ C$ = parallel of latitude, in 43° N., for 12 miles.
$A'\ B'\ C'$ = rhumb line (N. 47° E.), in mean latitude 43° N., for 12 miles.
$N\ S$ = meridian through Point B.
I, I', I'', I''' = intersection angles, tangents, 3 miles.
C, C' = tangential angles, one-half intersection angles.
Offsets from tangent to parallel, in links $= 1.01 \times D^2 \times \tan \phi$.
D = distance, in miles.
ϕ = latitude.
λ and λ' = longitudes.
Intersection angle, parallel $= \tan \phi \times 5''.2$.

Referring to the rhumb line:

Offsets from tangent to curve $= \sin \theta\ (1.01 \times d^2 \times \tan \phi)$.
θ = course of line.
Intersection angle $= \sin \theta\ (\tan \phi \times 5''.2)$.
Total curvature of line = Σ intersection angles.
Back-sights = Σ tangential angles.
Curvature of meridians $= (\lambda - \lambda') \sin \frac{1}{2} (\phi + \phi')$.
Length of 1° of longitude $= 69.3 \times \cos \phi \pm$ (statute miles).*
1° Lat. (statute miles) $= 69.0569 - 0.3494 \cos 2\phi + 0.0007 \cos 4\phi$.
1° Long. (statute miles) $= 69.2316 \cos \phi - 0.0584 \cos 3\phi + 0.0001 \cos 5\phi$.

Care should be taken in using the longitude formula with the signs of the latitudes. It can be written for the ordinary field practice: Length of a degree in longitude $= 69.30514 \cos \phi =$ statute miles, for Latitude 49° N. and $69.23125 \cos \phi = 1°$ (statute miles) for Latitude 30° N. For intermediate latitudes, by interpolation for a new factor between 69.30514 and 69.23125, a close approximation can be secured, close enough for any ordinary field use and far within

* This is very close for 49° latitude, for 30° latitude use the factor, 69.23.

the least count of the field instrument. It will be wise to check the longer formula by the shorter one, as it is more or less involved. The latitude is not so important as the value do not change rapidly; it can be taken from any good map or obtained by an observation on Polaris at the upper or lower culmination.

In presenting this problem, the main idea has been to simplify complicated mathematical formulas and involved solutions. This is exemplified in the equation for the intersection angle, $\tan \phi \times 5'.2$, which eliminates the necessity for solving the difference of longitude. This study is confined to a tangent of 6 miles, which is naturally the most convenient one to use and is accurate to within 1″ of arc up to 70° latitude, where it varies only 2″ in arc.

The main appeal of this method is to the field engineer who will probably recognize something for which he has been looking. A search through engineering or surveying literature reveals apparently nothing bearing distinctly on this subject, except as previously mentioned, nor, of course, on the reductions for latitude, longitude, azimuth, and back azimuth in geodetic surveying, especially as it applies to primary and secondary traverses and reductions for triangulations. The methods here described are too involved for the ordinary field surveyor's use and for engineering work which requires rapid progress with a minimum of office calculations, at the same time bringing the field data to be placed on the ground well below the least count of the verniers. These calculations will do more than this, however, even if they are simplified as far as possible. It might be well for the field engineer to have a copy of Fig. 1 in his field book, as a glance at it will show him just how far he is departing on any transit line from his objective rhumb-line course, and consequently from his corner position. The course given as N. 47° E. at Latitude 43° N. in Fig. 1 will show (1) just how much the offsets and intersection angles vary from the parallel to the meridian; (2) where they will be at a minimum and finally disappear on the meridian; and (3) where they are at a maximum on the parallel. It should be remembered that the offsets vary nearly as the square of the distance on the tangent and that the intersection angles are proportional to the distance in any given latitude, and, therefore, for the longer tangents that these two functions will materially increase and depart from the objective rhumb line. This is especially important from a cadastral standpoint, as the rhumb line is the legal line on which the corners or monuments are situated.

In regard to the maximum error involved by the use of these short formulas for field purposes in preference to the more extended and laborious methods, it is well to remember that the rigid geodetic calculations are based on a theoretic ellipsoid of revolution which, of course, closely approximates the real form of the earth. The writer's object is to project a rhumb line which will represent, as nearly as field conditions render possible, the actual curve on the earth's surface within the limits of the latitude selected for operations, and to furnish the field parties with amply accurate and simplified formulas. These

limits are from the Equator to 60° latitude. A comparison of the short formulas given herein with those developed by the rigid theoretic ellipsoid of revolution, shows a maximum error well within 1″ of arc for a rhumb line of 24 miles. This is a fairly long line for re-survey work between known original corners, for original work between monuments established by astronomical determination, or, again, for other work before check azimuth determinations are made. At 70° latitude this error is about 2″ of arc. As the engineer's transit seldom reads to less than single minutes of arc and as it is known that few lines of any length projected on the earth's surface actually are kept within the limit of even 1′ of arc on account of physical conditions, the calculations are far more exact than actual field operations can ordinarily achieve.

DISCUSSION

C. V. Hodgson,* M. Am. Soc. C. E. (by letter).—This method for reducing a great circle line to a rhumb line is evidently intended to apply primarily to lines of cadastral surveys for which the distances between control points are ordinarily not greater than six miles. Nevertheless, allusion is made to the application of the method to longer lines such as those found on National and State boundaries. In that connection, it may be of interest to point out some of the other considerations which must be taken into account by an engineer engaged in retracing such a line.

Practically all National and State boundaries, if defined by arcs of the parallel or of the meridian, were established by astronomical methods starting from a known point, such as an astronomical station. A random line was first run by one of the usual methods for a number of miles and another astronomical position was then determined. The random line was then corrected to conform to the position of the new astronomical station. Although there are variations of this procedure possible, these general principles will apply to the great majority of cases.

The accuracy of the determination of the astronomical position at the new point will depend on the instruments and methods used. When the most accurate field instruments available are used with the proper methods, latitude can be determined with an accuracy of about 0.1″, which corresponds roughly to 10 ft. on the earth's surface. Longitude can be determined with an error of about 20 ft. With an engineer's transit, the error will perhaps be ten times as great. There may be, therefore, considerable error from this source to be considered when retracing or establishing a boundary line of great length.

Still another factor to be reckoned with has a much greater effect on the location of the boundary line than instrumental errors or detailed methods of establishing sections of a boundary line between astronomical stations, namely, the deflection of the vertical at the astronomical station. The existence of station error and its effect are known by all engineers, but they often lose sight of the magnitude of the error from this source.

Suppose that at two adjacent astronomical stations at which latitude is observed the deflection of the vertical is in opposite directions, throwing the latitude too far south in one case and too far north in the other. The relative deflection between the two stations affecting the latitude is, of course, the algebraic sum of the deflections in the meridian at the two stations. In country of moderate relief, the relative deflection between two stations, 50 to 100 miles apart, will not usually exceed 1″ to 2″, but in mountainous country or in regions where the rock structure varies greatly in density, the relative deflections may be several times that amount. Between the east and west coasts of the Philippine Islands, for instance, the astronomical distance is in error by more than a mile and a similar error exists between the north and

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south coasts of Porto Rico. In India at Dehra Dun there is a 12″ relative deflection between two points only 5 miles apart, which corresponds to an error of 1 in. to every 100 ft.

The effect of these station errors is strikingly shown by an examination of State boundaries in the western part of the United States where the monuments have been connected to the first order triangulation of the Coast and Geodetic Survey. This triangulation is on the North American Datum (which means that the points have been computed on the spheroid of reference which seems best adapted to this part of the globe) and is connected to the initial datum point in Kansas. The latitude and longitude of this point were so selected as to reduce practically to a minimum the algebraic sum of the deflections of the vertical at astronomical stations symmetrically distributed over the United States. The maps and charts of the nation are based on the triangulation computed from this datum.

An arc of this first order triangulation crosses Kansas near the 98th Meridian. A boundary monument on the north boundary of the State was found to be 888 ft. too far north, while one at the south boundary was 500 ft. too far south, thus making the State about ¼ mile wider at this point than its intended constitutional limits. A boundary monument at the southeast corner of Montana was found to be 650 ft. too far south and the one at the northeast corner of Wyoming was 800 ft. too far south.

An error of that magnitude in the location of a boundary line is bad enough, but a still more disturbing effect from the viewpoint of the surveyor is that the amount and direction of the deflection of the vertical changes from point to point along the boundary line, so that a line established by astronomical methods is a zigzag line. A monument on the eastern boundary of Montana is 1 900 ft. too far east, while another one 150 miles south on the same boundary line and supposedly on the same meridian is 1 400 ft. still farther to the east. On the boundary between Nevada and Utah, two boundary monuments, 2° 10′ apart, are 1 040 and 3 300 ft., respectively, too far east. On the international boundary between the United States and Canada, along the northern boundary of Montana, two astronomical stations 98 miles apart showed a relative error in latitude of more than 1 400 ft., while at another place on the international boundary, there is a relative error of 800 ft. in latitude in 9¼ miles.

The problem, then, of adjusting the random line between astronomical stations is complicated immensely by these station errors and the retracing of a line is correspondingly difficult. At once it is seen why there has been so much litigation over State boundaries. The only chance of making a new boundary line agree approximately with the old is to re-establish the astronomical stations first established, and to interpose no new ones.

The disadvantage of having boundary lines that are so difficult to re-establish cannot now be prevented, for no other method was available at the time they were first located. All political units are concerned, however, with the problem of maintaining those boundaries. This can be done in either of two ways: First, by a strong monumentation of the line and frequent

inspection of the monuments as provided by some States; and, second, by a similar monumentation supplemented by triangulation along the boundary with a sufficient number of stations of the triangulation close to the boundary so that offsets can be measured to the boundary at critical points. Each station and reference point of the triangulation scheme thus becomes a monument from which the boundary can be re-established.

A large scale topographical map with parallels and meridians on the North American Datum, therefore, will show a State astronomical boundary meandering at a considerable distance from the parallel or meridian which it is supposed to follow. Since a boundary is legally fixed after its location is once accepted by proper authority, the only way to bring a boundary into proper relation geographically with the adjacent cadastral and topographic surveys mapped on the North American Datum would be for the States concerned mutually to agree to a re-survey, and that is usually impracticable. Good triangulation and topographical surveys along State and county boundaries, however, would be of immense benefit to the political units concerned, as well as to the land-owners adjacent to the boundaries.

D. E. Hughes,* M. Am. Soc. C. E. (by letter).—The formulas of this interesting and instructive paper, are practically correct for use within the limits contemplated by the author. In Table 1 the angles to single seconds and the offsets to only the nearest link look inharmonious; but, as indicated by the author, ordinary transit readings will only approximate the computed values within 10 to 30″, which would not justify tenths of links; whereas, on the other hand, the odd seconds are useful in other calculations.

Probably no one would lay out a long parallel of latitude from a single great circle tangent, for the offsets determined by the simple formula and the distances of the mile-posts from the starting point would both become appreciably in error, except near the Equator. If he were only setting fence posts the engineer might choose to offset from such lengths of great circle arcs as would make the deflection angles whole minutes; but for setting mile-posts he would use miles of length, and, if on standard parallels, the six miles that range lines are apart would be most convenient. Pointing out, as the author does, the likeness to railroad curves is helpful to many who may have assumed that to run a parallel of latitude is more difficult.

In olden times a student was shown that from any initial point a line of stakes, in Azimuth 90° or 270°, would, if continued, cross the Equator 90° away, and therefore would mark a great circle and not a parallel of latitude, which is the east and west line. Also, he would know that if the near-by stakes are cut off on a straight line tangent at the initial point their stumps will project above level ground varying amounts, each equal to the square of its distance from the initial point divided by the diameter of the earth (this reduces to 8 in. multiplied by the square of the distance in statute miles); and that in the absence of refraction this is the correction for curvature used in leveling.

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Next, he would see that if on level ground other stakes are leaned against the sawed off stumps and pointed each at right angles to the axis of the earth, the leaning stakes will be in a plane parallel to the plane of the Equator, and that their feet on level ground are, therefore, on the parallel of latitude which embraces the initial point. As the angle between the plumb stump and the leaning stake is in each case equal to the latitude of the place, the offset on the level ground from the one to the other will equal the correction for curvature multiplied by the tangent of the latitude.

Later, after learning navigation, the student would see that an oblique rhumb line, if ever brought ashore, could be marked on the ground in the same way from a line of plumb stakes set to the corresponding azimuth; but that in this case the feet of the inclined stakes, although on the rhumb line, would not be at the same distances as the plumb stakes from the instrument, and would need to be moved along the rhumb line to positions square across from the line of plumb stakes, thus making new offsets which would be equal to the old ones multiplied by the sine of the bearing of the line. This then becomes equal to the correction for curvature times the tangent of the latitude, times the sine of the bearing, which, of course, is identical with the formula recorded by the author, except that he uses 1.01 links instead of 8 in.

The navigator, if given a transit instead of a mariner's compass on a wheel-barrow, will know that, to pursue the rhumb line with a succession of short transit lines, he must make deflections at their junctions which shall be proportional to their lengths, and produce an aggregate equal to the convergency of meridians. His calculation of convergency is about the same as the author's formula on page 1008, wherein 6 miles is changed to 5.2 geographic miles, or minutes, and multiplied by both the sine of the course and the tangent of the latitude, which need be neither meridional latitude nor middle latitude when the line is so short.

The rhumb line, so simple to navigators, has been little more than a curiosity to surveyors whose teaching and work have covered plane and geodetic surveying, including the marking of parallels of latitude as required on some State boundaries and on standard parallels or correction lines in the public land survey. Of course, nearly all students were tricked to say that a point going 4 miles per hour on a loxodrome running N. 60° E. would never reach the Pole, in fact, this is still stated in the Standard Dictionary. Those who gave it a second thought, however, realized that every inch along this course, N. 60° E., would make a northing of ½ in., and that the point would therefore reach the Pole as quickly as would another going due north at one-half the speed. This is a pretty illustration of an infinite series decreasing so rapidly that its sum is finite and moderate; also of the fact that although all lines reaching the Pole must come from the south in the geographer's language, yet the navigator or surveyor at the Pole may arbitrarily assume any one of them as reference, and from it turn off any course desired.

It is startling, however, to old Californians to read that farm lines, railroad tangents, etc., are "in the great majority of cases" loxodromic curves, and that such a curve is the "legal line"; for they had tried hard to keep the lines horizontally straight.

Suppose that from an existing corner, say, in Latitude 43° N., a grant line is recorded as having been run 47° E., 6 miles, to a tree now gone. A rhumb line N. 47° E. would miss the stump by a rod; another rhumb line bearing N. 47° 01′ 47″ E. would hit it; but there is no record of such bearing. Furthermore, this new line would have a 4-ft. bow, and the ranchers and the Courts want the fence line straight. Of course, an engineer seeing N. 47° E. written on a line on a plat, can advance a technical interpretation that all parts of the line may be presumed to have that bearing, and therefore that each small part must so bear with respect to its own meridian, and thus make a rhumb line. However, in California, at least, the ranch was defined by plane surveying, with bearings referred to one meridian through the point of beginning, unless there was need to adopt a faulty one to fit an adjoining ranch.

When the magnetic compass was used to initiate a course the effort afterward was, by aid of back-sight or fore-sight, or both, to keep the line straight; and on long lines the varying needle bearings were read so that the last instead of the first could be used if needed at the next set-up, and thus eliminate diurnal variation of declination, local attractions, and convergency of magnetic meridians. When the solar attachment was used, and if along the line new meridians were determined by it, deflections were made to keep the lines straight and the corner angles correct, except when running standard parallels. It should not be lightly assumed that the old surveyor neglected to allow for convergency of meridians. The best mathematicians and most careful thinkers lived long ago. Even were it necessary now to assume that the old surveyor had failed to allow for convergency of meridians, yet it would not be warrantable to assume that he had traced a rhumb line instead of a succession of great circle arcs; and if it is not known at what stations he was guided anew by the sun alone, there is no telling whether his arcs were nearer a rhumb line than a single great circle.

For comparison of methods, assume a triangular tract in Latitude 43° N. with the boundary described as beginning at Station 1, witnessed so and so; thence N. 47° E. 8 915 ft. to Station 2; thence S. 65° E. 14 387 ft. to Station 3; thence W. 19 559 ft. to point of beginning. In that latitude each 1 000 ft. of northing or southing changes the latitude 9.87″; in each 1 000 ft. of easting or westing, the longitude changes 13.53″; and the convergency of meridians is 9.20″. Here, the meridians at Stations 1 and 2 converge 1′ and at Stations 2 and 3, 2′. Hence, the geodetic surveyor would record azimuths as follows:

Stations 1 to 2, 227°; Stations 1 to 3, 270°; Stations 2 to 1, 47° 01′;
Stations 2 to 3, 295° 01′; Stations 3 to 2, 115° 03′; Stations 3 to 1, 90° 03′.

He, however, would agree with the plane surveyor in restoring Corners 2 and 3 and the intermediate line marks, for both surveyors would orient the

same at Station 1, turn the same angles, and run the lines horizontally straight.

But how will the rhumb surveyor restore the corners? If he finds the old surveyor had a solar compass, will he assume he used it incorrectly? And if so, will he use the recorded bearings, which if applied to rhumb lines, will place Station 2 about 1.3 ft. to the northwest and Station 3 about 8.5 ft. north of the former positions? Or will he run his lines on calculated substitute bearings of N. 47° 00′ 30″ E., S. 64° 58′ E., and N. 89° 58′ 30″ W., in order to hit Corners 2 and 3? Even then his corner angles will not be correct, being 111° 58½′, 25° 00½′, and 43° 01′, instead of 112°, 25°, and 43°; his fences will not be straight; and he will have violated the general understanding and the tenor of Court decisions, that lines from point to point are to be straight, horizontally, unless specifically described as following a parallel or other curve.

This discussion is admittedly elementary, not for old men who will not read it, but for some young man who may be prone to apply hastily seemingly new things too widely. It is not a criticism of the author's paper which is clear in giving a simple and correct method of locating a parallel or other rhumb line of moderate length. The author may yet explain more definitely when to use rhumb lines instead of great circles in retracing lost boundaries; and he will be likely to agree that in case of doubt it is safer to use the latter.

R. L. Faris,* M. Am. Soc. C. E. (by letter).—A consideration of the general formulas developed in Helmert's "Geodesy" shows that Mr. Sweitzer's formulas are approximations that are sufficiently accurate for practical use if the distance taken is not much more than that indicated in the paper. When the rhumb line does not coincide with the parallel it will of course not be a circle, and except when it coincides with a meridian or the parallel it will not be a plane section of the ellipsoid. However, for the distance treated in the paper the approximation to a circle is sufficient to answer all practical purposes and the field procedure suggested is justified. The attempt to express the results of general formulas in simpler form for practical use is commendable and no doubt saves engineers much time and trouble in practice.

The writer finds in his field notebook (dated 1892), a discussion of this same problem which appears to be shorter and probably somewhat more approximate, but which gives practically the same results. By way of comparison this old method may be of interest.

Inclination of the Meridian.—In Fig. 2 let $A\,G$ and $B\,G$ be arcs of a quadrant of meridians, 1° apart in longitude; then $A\,B = 1°$ of longitude on the Equator = 69.16 statute miles. Let $D\,E$ be an arc of longitude on any parallel of latitude, and $E\,H$ and $D\,H$ the tangents meeting the earth's axis, produced at H, and corresponding to the parallel, $D\,E$. Then the line, $E\,F = D\,F = \cos\phi = \cos A\,D$, or $B\,E$; $D\,F\,E = 1°$; and the angle, $D\,H\,E$ = inclination of the meridian, which will be called X degrees.

Required to find an expression for X degrees. The triangles, $F\,D\,E$ and $D\,H\,E$, have the same base, $E\,D$, and are isosceles; hence their vertical angles

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vary as their sides (vertical angles, $D\ F\ E$ and $D\ H\ E = X°$). Hence,

$$1° \times E\ F = X° \times E\ H$$

but $E\ F = \cos\ \phi$, $E\ H = \cot\ \phi$, and the angle of $1° = D\ F\ E$, hence,

$$X° \cot\ \phi = 1° \cos\ \phi$$

and,

$$X° = \frac{1° \cos\ \phi}{\cot\ \phi} = 1° \sin\ \phi \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

that is, the inclination of the meridians for any difference of longitude varies as the sine of the latitude.

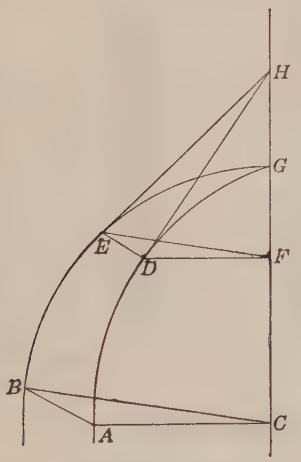

FIG. 2.

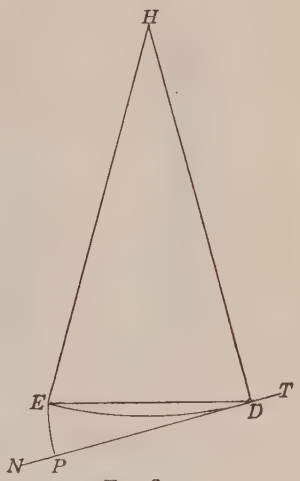

FIG. 3.

To get the inclination in seconds of arc, multiply by 3 600 the number of seconds in 1 degree. The above is the deflection for 1° of longitude. To get the deflection for 1 mile, divide by the number of statute miles in 1° of longitude at the particular latitude under consideration.

Thus, in Latitude 43° (Example 1):

$$\log \sin\ \phi = 9.833783$$
$$\text{“}\ 3600 = 3.556303$$
$$3.390086$$

Divide by 50.66 miles = 1° longitude in Latitude 43°:

$$\log = 1.704682$$
$$3.390086$$
$$= 1.685404 \text{ (in seconds)}$$
$$= 48.''46.$$

which is the inclination for 1 mile of longitude.

Offsets.—In Fig. 3 let $H\ D$ and $H\ E$ be tangent to two meridional arcs at D and E as in Fig. 2.

Let $T\ N$ be a tangent to the arc of parallel at the point, D; then it is an east and west line at D. Then the angle, $E\ D\ N$, is the "angular deflection" in running the distance, $D\ P$, beginning at D on a true east and west course; and $P\ E$, perpendicular to $T\ N$, is the "offset" from the arc of the parallel, at E.

TABLE 2.—Inclination of Meridians.

Latitude, in degrees.	Inclination for:		Deflection of Tangents:	
	1 mile.	6 miles.	1 mile.	6 miles.
30	30.03″	3′ 00″	15.02″	1′ 30.0″
31	31.26″	3′ 07″	15.63″	1′ 33.5″
32	32.49″	3′ 15″	16.24″	1′ 37.5″
33	33.83″	3′ 23″	16.92″	1′ 41.5″
34	35.17″	3′ 31″	17.58″	1′ 45.5″
35	36.50″	3′ 39″	18.25″	1′ 49.5″
36	37.83″	3′ 46″	18.92″	1′ 53.0″
37	39.17″	3′ 55″	19.58″	1′ 57.5″
38	40.67″	4′ 04″	20.34″	2′ 02.0″
39	42.17″	4′ 13″	21.08″	2′ 06.5″
40	43.67″	4′ 22″	21.84″	2′ 11.0″
41	45.17″	4′ 31″	22.58″	2′ 15.5″
42	46.85″	4′ 41″	23.42″	2′ 20.5″
43	48.52″	4′ 51″	24.26″	2′ 25.5″
44	50.19″	5′ 01″	25.10″	2′ 30.5″
45	52.00″	5′ 12″	26.00″	2′ 36.0″
46	53.83″	5′ 23″	26.92″	2′ 41.5″
47	55.67″	5′ 34″	27.84″	2′ 47.0″
48	57.67″	5′ 46″	28.84″	2′ 53.0″
49	59.83″	5′ 59″	29.92″	2′ 59.5″

TABLE 3.—Offsets, in Feet.

Latitude, in degrees.	Distance.					
	1 mile.	2 miles.	3 miles.	4 miles.	5 miles.	6 miles.
30	0.39	1.54	3.47	6.17	9.64	13.88
31	0.40	1.60	3.61	6.42	10.03	14.44
32	0.42	1.67	3.76	6.67	10.42	15.02
33	0.43	1.73	3.90	6.93	10.82	15.60
34	0.45	1.80	4.05	7.20	11.25	16.20
35	0.47	1.87	4.20	7.47	11.68	16.81
36	0.48	1.94	4.36	7.75	12.11	17.41
37	0.50	2.01	4.52	8.04	12.57	18.09
38	0.52	2.08	4.69	8.33	13.02	18.75
39	0.54	2.16	4.86	8.63	13.49	19.43
40	0.56	2.24	5.03	8.95	13.98	20.11
41	0.58	2.32	5.21	9.27	14.48	20.85
42	0.60	2.40	5.40	9.59	14.99	21.59
43	0.62	2.48	5.59	9.93	15.52	22.35
44	0.64	2.57	5.79	10.29	16.07	23.14
45	0.67	2.66	5.99	10.65	16.64	23.96
46	0.69	2.76	6.20	11.02	17.21	24.80
47	0.71	2.85	6.42	11.41	17.83	25.68

The angle at H, or the angle that measures the inclination of the two meridians $= \frac{\cos \phi}{\cot \phi} \times 3\ 600$ for 1° of arc (Equation (1)).

The angle at H = 2 Angle $E\ D\ N$; or the angle of meridian inclination is twice the angle of deflection of an east and west line from an arc of parallel at the point, D. Hence,

$$\text{Angle } E\ D\ N = \frac{1}{2} \text{ Angle } H$$

The "offsets", such as $E\ P$, are computed from the distance (length of course), such as $D\ P$, and the angle of deflection, such as $E\ D\ P$. Thus, from Example 1, the angle of inclination for 1 mile, Latitude 43° = 48.″ 46; hence,

$$\text{Angle of deflection} = \frac{48.''\ 46}{2} = 24.''\ 23 \text{ log tan} = 6.0698637$$

$$1 \text{ mile} = 5280 \text{ ft.; log} = 3.7226339$$

$$\text{log offset} = 9.7924976$$

$$\text{Offset} = 0.62 \text{ ft.}$$

Tables 2 and 3 give the inclination of meridians and offsets for various latitudes and distances.

George L. Hosmer,* M. Am. Soc. C. E. (by letter).—This paper, describing a field method of laying out the loxodromic curve, is a substantial contribution to the literature of this subject, not only because it presents a simple and practical field method, quite within the reach of all engineers, but also because it directs attention to much needed improvements in the methods of dealing with the fixing of boundary lines of all kinds. In the United States the methods of doing this have not received much attention and have not been very fully developed. The use of the loxodrome as the legal line is perfectly sound because it is in accordance with the accepted principle and the most important Court decisions, that in a re-survey the object is to recover the position of the original line, not to correct mistakes which may have been made in laying it down—a principle which has not yet been discovered in some communities.

The author's plan of substituting a circular arc, or a series of circular arcs, for a portion of the loxodrome, is sufficiently accurate and gives practically the same line on the ground. Such curves can be dealt with in a manner familiar to all engineers. The short offsets from the geodetic line to the loxodrome are given by his formulas to the nearest hundreth of a foot in the latitudes of the United States.

The geodetic formulas first mentioned in this paper are, of course, the Puissant formulas as adapted for use on the United States Coast Survey by Hilgard about 1846. The one that applies most directly to this computation is that giving the convergence of the meridians, or change in the direction of the loxodrome, namely,

$$-d\ Z = d\ \lambda \sin \tfrac{1}{2} (\phi + \phi')$$

The author has used this same formula (page 1008), in a slightly modified form.

The formula, $5'.2 \tan \phi$, is based on a 6-mile distance. At the Equator this would subtend an angle of 5′.2. The difference in longitude on any other

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parallel of latitude would be $5'.2 \times \sec \phi$. This is the difference in longitude just as it would be computed by a navigator, the 5'.2 being the departure in nautical miles. This amounts to computing the difference in longitude for a particular distance along the curve, and the process is equivalent to the solution of the second equation in the group referred to. The convergence of the meridians (equal to the change in direction of the loxodrome) is given by:

$$\begin{aligned} d\,Z &= 5'.2 \times \sec \phi \times \sin \phi \\ &= 5'.2 \times \tan \phi \end{aligned}$$

Along a curve having a small course the change in latitude must be allowed for, since it affects the value of $\tan \phi$. The difference in latitude is given by the distance expressed as arc and multiplied by the cosine of the course. This is equivalent to solving the first term of the first equation, which is the only large term of this series. In dealing with any line not lying nearly east and west it is necessary therefore to solve all three of the geodetic equations, in one form or another, although this may not be obvious from the simple equations stated by the author.

These three equations may be re-stated in a simple form, sufficiently accurate for the purpose in hand, as follows: Assuming the earth to be a sphere, a statute mile subtends an angle of 0'.868. (The suppression of the terms due to the ellipticity of the meridian appears to cause an error of less than 0.5%, in the latitudes of the United States.) The three equations become (in the notation already used):

$$\begin{aligned} -d\,\phi &= 0'.868 \times D \times \cos \theta \\ d\,\lambda &= 0'.868 \times D \times \sin \theta \sec \phi_m \\ -d\,Z &= d\,\lambda \sin \phi_m \end{aligned}$$

in which, $d\,\phi$, $d\,\lambda$, and $d\,Z$ are the differences (in minutes) of latitude, longitude, and azimuth; D is the distance, in statute miles; θ is the course, or bearing; and ϕ_m is the mean, or middle, latitude.

It is natural that the use of the loxodrome as the true line should be adopted in regions surveyed by the United States System, since this curve forms the basis of the system. In the Eastern States as a rule the loxodrome has not been adopted as the true line. The original surveys of certain boundaries were run out by transit (some of them as early as 1787), and are therefore properly treated as straight, or "geodetic", lines. Others, although run out by the use of the magnetic compass, have been treated subsequently as if they were straight between fixed points. If the loxodrome were to be used for such boundaries, or portions of them, the preceding formulas might have to be slightly modified, for convenience; but there is no reason for not retaining the fundamental idea, that is, to regard the loxodrome as the true curve, but to substitute the circular curve for a portion of it in the field operations covering comparatively short arcs.

Joseph James Corti,* Assoc. M. Am. Soc. C. E. (by letter).—The problem of locating a loxodrome, or rhumb line, by means of a series of chords, can

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be solved, south of the Equator, through the use of formulas similar to those given by the author, as follows:

$$d\phi = \phi' - \phi = B \cdot K \cdot \cos Z + C \cdot K^2 \cdot \sin^2 Z + D \cdot h^2$$
$$d\lambda = \lambda' - \lambda = A' \cdot K \cdot \sin \cdot Z \cdot \sec \phi'$$
$$dZ = Z' - Z \pm 180^\circ = d\lambda \cdot \sin \phi_m$$

in which, ϕ is measured positively northward and λ, positively westward; and Z is measured counterclockwise from the north.

TABLE 4.—Items for Computing Geodetic Positions.

(Clarke spheroid, 1866, $a = 6\,378\,206$ *m.*; $c = 1 \div 295$.)

ϕ, in degrees.	log $A' + 10$.	log $B + 10$.	log $C + 10$.	log $D + 10$.	ϕ, in degrees.
0	8.50973	8.51268	$-\infty$	$-\infty$	0
2	8.50972	8.51267	9.950-10	1.23	2
4	8.50972	8.51265	0.252	1.54	4
6	8.50971	8.51262	0.428	1.71	6
8	8.50970	8.51259	0.555	1.83	8
10	8.50968	8.51254	0.654	1.93	10
12	8.50966	8.51249	0.734	2.00	12
14	8.50964	8.51242	0.803	2.06	14
16	8.50961	8.51234	0.864	2.12	16
18	8.50959	8.51226	0.918	2.16	18
20	8.50955	8.51216	0.967	2.20	20
22	8.50952	8.51206	1.013	2.23	22
24	8.50948	8.51195	1.055	2.26	24
26	8.50944	8.51183	1.094	2.29	26
28	8.50940	8.51170	1.131	2.31	28
30	8.50936	8.51157	1.167	2.33	30
32	8.50931	8.51144	1.201	2.35	32
34	8.50927	8.51130	1.234	2.36	34
36	8.50922	8.51115	1.266	2.37	36
38	8.50917	8.51100	1.298	2.38	38
40	8.50912	8.51085	1.328	2.39	40
42	8.50907	8.51070	1.359	2.39	42
44	8.50902	8.51054	1.389	2.39	44
46	8.50896	8.51039	1.419	2.39	46
48	8.50891	8.51024	1.449	2.39	48
50	8.50886	8.51008	1.480	1.39	50
52	8.50881	8.50993	1.510	1.38	52
54	8.50876	8.50978	1.542	1.37	54
56	8.50871	8.50964	1.574	1.36	56

The factors, A', B, C, and D, are to be taken from Table 4, A' for Latitude ϕ', and B, C, and D for Latitude ϕ.

Table 4 has been prepared from "Formulae and Tables for the Computation of Geodetic Positions",* by reducing the number of factors and decreasing the number of decimal figures, both to the limits required by the field engineer. It can also be used north of the Equator, with slight changes in the formulas, as shown in the paper.

If from Point M, on Parallel 43° S., a loxodrome of 47° constant azimuth had been located, N and P being its intersections with meridians 5′ and 10′ west of M, the length and azimuth of the chords, $M\,N$ and $N\,P$, would be computed as shown in Table 5. Since the latitude of both points is known

* U. S. Coast and Geodetic Survey, *Special Publication No. 8.*

it makes no difference whether the computation is made first from M to N, and checked afterward from N to M, or in the inverse order. However, if the loxodrome is to be located, the latitude of N being unknown, the computations must be made first from N to M, and checked by computing from M to N with the latitude obtained for N.

The computations from N to M are made as follows: First is computed $\log K \cdot \sin Z = 3.83221\ n$, and $d\ Z = d\ \lambda \cdot \sin \phi_m = 3'\ 24''.6$. It will be sufficient to take ϕ, the latitude of Point M, instead of the unknown latitude, ϕ_m.

The loxodrome is a curve which winds spirally on the surface of the earth, with curvature varying from point to point. For the purpose in view, however, the stretch between the ends of a chord may be taken as an arc of a parabola with the axis perpendicular to the chord at its middle point. This assumption fixes the position of the tangents at M and N, by taking $\frac{1}{2}\ d\ Z = 1'\ 42''.3$ as the angle that either of them makes with the chord, $M\ N$, and to write at the foot of Column (2) in Table 5, $Z' = 47°\ 1'\ 42''.3$ as the azimuth for $M\ N$, and $Z = Z' - 3'\ 24''.6 + 180° = 226°\ 58'\ 17''.7$ as the azimuth for $N\ M$.

Once Z is known, $\log \tan Z = 0.02991$, and by the set of operations,

$$(\log K \cdot \sin Z = 3.83221\ n) - (\log \tan Z = 0.02991)$$
$$= (\log \cdot K \cdot \cos Z = 3.80230\ n)$$
$$(\log \cdot K \cdot \cos Z = 3.80230\ n) + (\log B = 8.51062)$$
$$= (2.31292\ n = \log 205\ 552)$$

is obtained,

$$d\ \phi_1 = -\ 3'\ 25''.552$$

From $\log K \sin Z = 3.832$ and $\log C = 1.374$, m is computed, which in this problem will always have a minus sign; and this leads to $-\ 42°\ 56'\ 34''.557$ as the latitude of Point N.

If the computations were repeated giving to ϕ_m, B, and C new values in accordance with the latitude obtained for N, it would be found that the difference in the results would not justify the additional amount of work involved.

The computation of latitude must be carried to the third decimal place in seconds, in order that the disagreement in the values of azimuths may not exceed a few seconds.

The check from M to N is to be done entirely from the top downward, starting from the latitude obtained when computing from N to M.

Lastly, four values of K will be obtained, two from the computations, M to N, and two from N to M; and the four values must check within a few decimeters.

If it were desirable to locate points of the parabola between the ends of the chord, the offsets would be determined by starting from the middle and longest one, given in meters by $\frac{1}{4}\ K \tan \frac{d\ Z}{2}$.

TABLE 5.—Example of Computations.

Items. (1)		Computations.							
		N to *M.* (2)		*M* to *N.* (3)		*P* to *N.* (4)		*N* to *P.* (5)	
λ	ϕ	5′	−42° 56′ 34.557″	0′	−43° 00′ 00.000″	10′	−42° 53′ 08.925″	5′	−42° 56′ 34.557″
λ'	ϕ'	0′	−43° 00′ 00.000″	5′	−42° 56′ 34.557″	5′	−42° 56′ 34.557″	10′	−42° 53′ 08.925″
$d\lambda$	$d\phi$	−300″	−3′ 25.443″	+300″	+3′ 25.443″	−300″	−3′ 25.632″	+300″	+3′ 25.632″
$\log (K \sin Z)^2$	....	7.664	.....	7.664		7.665		7.666	
$\log C$	....	1.374	.....	1.374		1.373		1.372	
$\log m$	m	9.038	−0.109″	9.038	−0.109″	9.038	0.109	9.038	0.109
....	$d\phi_1$		−205.552″		+205.334″		−205.741″		+205.523″
$\log d\lambda$	$\log d\phi_1$	2.47712*n*	2.31292*n*	2.47712	2.31246	2.47712*n*	2.31332*n*	2.47712	2.31286
" A'	" B	−8.50904	−8.51062	−8.50904	−8.51062	−8.50904	−8.51662	−8.50904	−8.51062
" $\sec \phi'$	....	−0.13587	.....	−0.13547		−0.13547		−0.13507	
$\log K \sin Z$	$\log K \cos Z$	3.83221*n*	3.80230*n*	3.83261	3.80164	3.83261*n*	3.80270*n*	3.83301	3.80224
" $\sin Z$	" $\cos Z$	−9.86393*n*	−9.83401*n*	−9.86433	−9.83356	−9.86393*n*	−9.83401*n*	−9.86433	−9.83355
$\log K$	$\log K$	3.96828	3.96829	3.96828	3.96828	3.96868	3.96869	3.96868	3.96869
K	K	9295.6*m*.	9295.8*m*.	9295.6*m*.	9295.6*m*.	9304.2*m*.	9304.4*m*.	9304.2*m*.	9304.4*m*.
$\log \tan Z$	Z	0.02991	226° 58′ 17.7″	0.03077	47° 01′ 40.8″	0.02991	226° 58′ 17.9″	0.03077	47° 01′ 41.0″
" $d\lambda$	$\pm 180°$	2.4771*n*	−180°	2.4771	+180°	2.4771*n*	−180°	2.4771	+180°
" $\sin \phi_m$	....	9.8338*n*	.....	9.8336*n*		9.8330*n*		9.8331*n*	
" dZ	dZ	2.3109	+3′ 24.6″	2.3107*n*	−3′ 24.5″	2.3101	+3′ 24.2″	2.3102*n*	−3′ 24.3″
....	Z'		47°01′ 42.3″		226° 58′ 16.3″		47° 01′ 42.1″		226° 58′ 16.7″

A comparison of the paper with the results reached by the writer, shows that the author has computed the values in Column (10) of Table 1 by means of the formula, $d\lambda = K \cdot A' \cdot \sec \phi$, with $K = 9\,656$ $m = 1$ mile. From the values thus obtained Column (8) is computed by $d Z = d\lambda \cdot \sin \phi$. With $d Z$ known, he computes Column (7) by $\frac{1}{4} K \cdot \tan \frac{d Z}{2}$; and fills out Columns (2) to (6) with the offsets of a parabola.

The method of the paper, therefore, is shown to be correct; and the author is much to be commended for the preparation and publication of his Table 1, which so greatly simplifies the computations for problems of this type.

As to the location of a parallel, the combination of the tangent and the chord methods, makes it possible to simplify field operations, as will be developed subsequently.

After the direction of the chord and the tangent have been determined and after computing the offsets from each of these two lines to the parallel, along the meridians of the different points to be located, two surveying parties are sent to measure off the abscissas, one along the chord, the other along the tangent. When the two parties reach the foot of the two offsets for each point, the distance between these two must agree with the one computed, and the corresponding point on the parallel can be located with no other instrument than a tape.

For the location of 1° 40′ of the — 36° parallel, part of the southern boundary of the Province of Mendoza, Argentine Republic, the computations were made with reference to two chords of 75 km. each, passing through the middle point of the arc toward the ends, and to the tangent touching the parallel at the middle point. The maximum distance between the foot of the two offsets for a common point was only 321.2 m.

N. B. Sweitzer,* Assoc. M. Am. Soc. C. E. (by letter).—In his discussion Mr. Hodgson brings up the question of control on State and National boundaries and emphasizes the fact that these isolated and unconnected astronomical determinations are not actually on the parallel or meridian intended and as described in the accompanying field notes and calculations. This fact has been well known to many field engineers who have retraced old lines or who have had to tie into these boundaries. What has often appeared as a paradox is the fact that a line astronomically determined with great expense and by undisputed experts in their professions and with the best and most expensive equipment would fall far short of accuracy as regards azimuth and, only too frequently, measurement, when compared with the ordinarily determined azimuths and measurements of the average field engineer, using modern but commercial instruments.

The writer's attention was first called to this fact about 1896 on the boundary between Canada and the United States, north of Roseau Lake in Minnesota. The boundary was run through a cedar swamp, and local surveys were closed on the tangent line, cut as a skyline, and not on the actual parallel, which was not marked except by posts set far apart. This

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gave a ragged traverse and not the actual parallel as described in the field notes. The limit of error of azimuth on most of these major lines, astronomically determined, is given as 2″ or 3″ of arc, but the fact is that the average transit instrument, with 1′ of arc as the least count of the verniers, easily picks up errors of many minutes on the actual lines as fixed on the ground. Mr. Hodgson mentions many specific instances of this condition, scattered over a large area. The writer can add to this list materially but, as his observations have been confined to errors of azimuth and measurement, he has not been in a position to determine the total error.

Isolated determinations of latitude and longitude are beset with many sources of error. Some can be guarded against and eliminated, but not all. Even after one of these isolated positions has been successfully and accurately determined, it is not constant. Latitude and longitude change; they are not permanent institutions. The experts who have been engaged in this work, especially in the past, were thoroughly conscientious, but their enthusiasm for their specialty undoubtedly caused them to overlook certain weaknesses in their methods. Too much emphasis was given to complicated theory and volumes of abstruse calculations with hair-splitting, overlapping niceties which were confusing instead of enlightening and made the authors the sole interpreters of the final results. This condition placed the notes or records in an unassailable position and, when the actual corner positions were legally accepted, there was really nothing to be done about it.

As Mr. Hodgson has pointed out, one of the many errors that beset these isolated astronomical determinations is the deflection of the vertical. This has been pointed out by many authorities on the subject. The late John F. Hayford, M. Am. Soc. C. E., emphasized this in his admirable treatise on "Geodetic Astronomy", but evidently this and other sources of error have been disregarded. The only way to eliminate this deflection is to make adjustments from other accurately determined positions. The answer to all this is that a practical and comprehensive system of triangulation should be an adjunct of every astronomical determination for latitude and longitude; they belong together and are incomplete, unless united.

With the introduction of such modern appliances as radio there is a practical side to astronomical determinations for longitude. This can be taken on almost any condition of terrain. Triangulation is difficult and expensive under some conditions, as in the heavily wooded rolling country west of the Cascade Mountains, in Washington and Oregon, and in, say, the Valley of the Amazon. Lengthy, intermediate points will have to be filled in by closed primary traverses, especially in heavily wooded and difficult country. Where such valuable property as mineral and oil lands are at stake, these traverses should be reduced to the loxodrome and it is here that the writer's method could be profitably used. A triangulation can only touch occasional points of an objective boundary; where instrumental connections to the line are used, this system can be abandoned and other methods adopted, such as the back and fore-sight transit line, direct measurement, etc. All these methods introduce their various sources of error, not found in the triangula-

tion. Manifestly, the calculations for these secondary lines do not have to be carried out to the extreme nicety of those in the triangulation network and, even if they were, it would be useless as, instrumentally, they could not be executed on the ground as calculated. The writer, therefore, thinks that the problem of adjusting the great circle to the rhumb line, as suggested and detailed in this discussion, is amply accurate for any line actually run on the ground and tied into properly adjusted triangulation stations. He heartily agrees with Mr. Hodgson in his discussion.

Mr. Faris suggests the relation of the rhumb line to the circle from the parallel to the meridian and notes that it will not be a plane section of the ellipsoid except at these points. He, however, states that for the distances specified in the paper the approximation of the circle is sufficient to answer all practical purposes, and the field procedure is justified. This element in the problem as to what reception the principle would receive from Government surveying authorities, caused the writer more or less worry. He compared the differences but could find nothing except extremely complicated formulas by which to make comparisons; these were entirely too unwieldy for field purposes and the differences were so small for the usual latitudes that, as far as field work was concerned, they did not exist. Mr. Faris' well-known reputation as a specialist in this particular field confirms the findings. The tendency of such men is extreme conservatism, especially on these subjects, and it is gratifying that he has taken the time to solve this problem. He is to be thanked for his contribution to this subject by the introduction of figures, formulas, and tables. The table of offsets (Table 3) is especially valuable, carried out as such offsets are to smaller decimal places.

Mr. Corti's method of locating a loxodrome by a series of chords is quite interesting, and his mathematical handling of the subject will appeal to many, especially those interested in the problem usually stated in the terms of textbooks on the subject. He has simplified the problem without sacrificing accuracy that could be identified in the field work and has suggested some valuable points in the practical field location; namely, the possible verification of the work by a process of checking the chord against the tangents. This amounts not only to double chaining but checks calculated offsets against each other from tangents and chords. Mr. Corti has used 5′ and 10′ as differences of longitude in his example and this corresponds closely with that used in the example suggested in the paper. This is amply close, however, to compare the different methods and prove the results.

In a letter dated July 16, 1926, Dr. O. S. Adams, Mathematician, U. S. Coast and Geodetic Survey, called attention to and suggested that the primes be omitted from the B, C, and D factors in the formula quoted from Smith's "Surveying", stating that these factors are taken out for the latitudes of the known point and not for the unknown point. As this book is used as a textbook in many colleges, this is important and has undoubtedly caused confusion in presenting this problem.

The writer is greatly indebted to Dr. Adams and wishes to thank him for his trouble in going over this problem. His mathematical ability is so well

known in the profession as to give added value to his opinion that the writer's solution is "sufficiently accurate for all practical purposes".

Mr. Hosmer has brought out some valuable points and, as he also is a recognized authority, his statements will bear weight. Simplicity, lucidity, and accuracy are what the field engineer demands, and the elimination of unnecessary complications.

Stripped of unnecessary detail, the development of the method was as follows: The writer first found what angle at the pole would subtend 1 mile at the Equator, using Clarke's value (1866) of the earth's flattening. This angle was found to be 0′.867403. In terms of a railroad curve, this angle was substituted for the angle at the center; but the angle at the center equals the intersection angle and this, in the present case, will also equal the convergency, which becomes 0′.867403 tan ϕ. The 1-mile unit was found to be impracticable for field use, as back-sights become confused by running this traverse, and deflection angles become too numerous and small—less than the least count of the verniers—and also are apt to come in draws and in places where it was impossible to set up the instrument. Therefore, the 6-mile tangent, was adopted as a unit and the equation became, sin θ (5′.20442 tan ϕ) = intersection angle, 6-mile tangent, which, for field use, can be written in the form, sin θ (5′.2 tan ϕ), and,

$$d\lambda' = \sin\theta \left(5'.2 \times \frac{D}{6} \sec. \phi_m\right) \quad \ldots\ldots (2)$$

$$-d\phi' = \cos\theta \left(5'.2 \times \frac{D}{6}\right) \quad \ldots\ldots (3)$$

$$-dZ' = d\lambda \sin\phi_m \quad \ldots\ldots (4)$$

Equation (2) gives the difference of longitude; Equation (3), the difference of latitude; and Equation (4), the difference of azimuth. Closer results can be secured by using,

$$d\phi' = D \cos\theta\ (0'.8733 - 0.0001 \times \phi_m).$$

If closer results are required for office use, the intersection angle for the 6-mile tangent for the loxodrome, using Clarke's values for the spheroid, can be secured by the following formula:

$$\sin\theta\ (\tan\phi_m \times 5'.20442 \pm d\cos\theta)$$

using the following tabulation for $\pm d$ and interpolating for intermediate values:

Latitude.	$\pm d$.	Latitude.	$\pm d$.
0°	+ 0.0355	54° 45′	0.0000
9°	+ 0.0342	63°	— 0.0067
18°	+ 0.0304	72°	— 0.0125
27°	+ 0.0246	80°	— 0.0160
36°	+ 0.0171	90°	— 0.0176.
45°	+ 0.0089		

For a 1-mile tangent, this will become:

$$\sin\theta \left(\tan\phi_m\ 0'.8674 \pm \frac{d}{6}\cos\theta\right)$$

The results obtained by using the equation for the sphere and applying it to the example under discussion makes the intersection angle for the 6-mile tangent, loxodrome N. 47° E., 3′.5494. The general equation for the spheroid, suggested in this case, that is,

$$\sin \theta \ (\tan \phi_m \times 5'.20442 \pm d \cos \theta)$$

makes this same angle, 3′.5544.

The circular curve on the surface of a sphere and spheroid in terms of the functions affecting a circular curve on a plane surface by methods in use by railroad engineers has been considered herein. It has been only in more recent years that simple curves have given way to more involved spirals to ease off the angles at the point of curve and point of tangent. Different methods were used, namely, lengthening the radius at the point of curve and point of tangent; making a compound curve; or, more generally, by use of the transition curve. These curves, generally speaking, were of a radius of only a few thousand feet and affected the central, or intersection, angle by only a few seconds (a minute or more at most). When these short radii are compared with the radii of the curves under discussion, thousands of miles long, it may be readily seen that the effect of change in the intersection angle by using the equation for a loxodrome instead of the circle will be negligible, especially in the latitudes of the United States and Southern Canada. Farther north, to about Latitude 70° to 75°, by using the 1-mile tangent, they will approach so close as to practically coincide. An illustration of this justifiable substitution of an approximation, which may be said practically to equal a rigid determination, is in the equation for determining the tables in general use for inclined sights in stadia work. These tables are based on the equation,

$$K r \cos^2 V \ (c + f) \cos V$$

in which,

r = vertical reading;
V = observed vertical angle;
K = wire interval, or ratio;
c = distance from center of instrument to object-glass;
f = distance from plane of cross-wire to object-glass.

This is under the assumption that the sight along a "vertical angle" at the inclined rod makes a right angle with the rod held vertical. This is not true, but the angle is so nearly a right angle that it does not affect the practical results and the equation has been adopted as "rigid" by the profession, has become standardized and rightly so, and is correct. The writer is in complete accord with Mr. Hosmer when he suggests that the circular arcs give practically the same line on the ground as the loxodrome.

Mr. Hughes' discussion is interesting, particularly in his description of the evolution of the rhumb line. He brings together the Middle Western farmer with his fence posts and the deep-sea sailor with his mariner's compass on a wheel-barrow, and so illustrates the mathematical principles involved that "he who runs may read." He confirms the correctness and practicability of the formulas used in the paper. The writer is pleased that Mr. Hughes

has joined in the discussion, especially on account of his extensive experience in the practical solutions involved.

In all restoration of old and obliterated lines where such lines are boundaries of deeded lands or accepted political sub-divisions, etc., certain fundamental rules have been laid down by the Courts for the guidance of surveyors and engineers. Justice Cooley, of the Supreme Court of Michigan, has written some valuable opinions and contributed considerably to the subject. Since those opinions were expressed many important cases have been decided by all the Courts, especially the Supreme Court, all along the same lines. Among the general rules are that fixed monuments hold over course and distance and that a properly accepted and confirmed surveyed boundary is unchangeable. There are many fundamental rules of the same tenor. In restoring a lost boundary engineers should follow in the footsteps of the original surveyor. To do this they have his field notes and plat and, possibly, if the survey is not too old, living witnesses as to what he did and where he placed his monuments. This is all along the line of evidence and competent testimony. When this is lacking or incompetent and contradictory, then the skill of the engineer or surveyor has to be exercised, but, following proper legal procedure and accepted mathematical and technical practice. There are always errors to harmonize, and in harmonizing these errors the skill of the engineer is shown. They are not to be crudely guessed at, but to be scientifically treated, both mathematically and legally. In regard to the original California surveys mentioned by Mr. Hughes, apparently these are mostly in two classes, Spanish grants and cadastral surveys made by the Government. Now, as the case under discussion relates to transit or great circle lines and the loxodrome or solar and needle lines, it is better not to take up extrinsic matters.

The Government survey was based on the loxodromic curve, not only mathematically in fact and by execution, but by legal enactment and judicial decision. First, the major lines—standard parallels—follow the parallel of latitude, and guide meridians follow the proper meridian; then come the ordinary correction lines and check of township, town, and range lines, all following this same system. In the sub-division of the unit township, the section lines correspond to this general system. In the effort to make each section 1 mile square, the loxodromic condition is recognized by running adjacent north and south section lines with a bearing corresponding to the latitude of the township. To confirm all this, the field notes, maps, and returns of all lines, major and minor, give the courses and bearings as a loxodromic curve. These surveys are throughout loxodromic, *de facto* and *de jure*.

Now, as regards the larger tracts known as grants, it will be found that both the great circle and the rhumb line are used. This will be shown first by referring to the record or wording of the grant and then by the surveyor's notes or the method used in running out these boundaries. The wording of some boundaries will call for a line from some peak to another, a great circle, along some river or water-shed, being metes and bounds; and then it will

revert to the loxodrome, with course and distance. In retracing or restoring these grant lines, those conditions will have to be taken into consideration. The method used for restoration must correspond to the wording of the original record of the grant, the treaty, proclamation, or instrument creating it, and also to the methods used by the engineer or surveyor whose final survey has been properly accepted.

In Mr. Hughes' examples, which are hypothetical, so many unknown conditions may exist respecting transfers, deeds, methods of the original survey, and the question of the authority by which the survey was made, that it is difficult to give an answer. The example as to the method of running a line in Latitude 43° N. a course of N. 47° E., 6 miles, would depend upon what was intended in the wording of the deed or grant and what the original surveyor did. A description of the line would help materially. Was it an air-line from peak to peak of a mountain, or through dense timber and brush? One condition would predispose a great circle; the other that a rhumb line was used, assuming that the description in the deed was ambiguous.

In regard to Mr. Hughes' query about a straight line and the preference of ranchers for the great circle line, they must remember they are living on a spheroid. This was very well and good before the days of Columbus. When the good padres came to California land was cheap and abundant, but land values have appreciably increased, especially in California, and recent investors are more critical about having their land boundaries restored and re-established.

The second problem suggested by Mr. Hughes resolves itself into two parts, namely, (1) determining the class of lines required, whether great circles or rhumb lines; and (2) determining and adjusting the error of closure. The first has been discussed herein in the first problem suggested, and the second part, namely, the error adjustment, has many angles. The writer is rather partial to the form of adjustment suggested in the discussion of his paper on "Retracement-Resurveys, Court Decisions and Field Procedure", by Leonard S. Smith, M. Am. Soc. C. E.,* for adjustment of traverses. This consists of the adjustment of both azimuth errors and proper proportional chaining. Fig. 4 shows a plane graphic field adjustment, unclosed. Course and distance are taken to only the nearest degrees, minutes, and links. The method, however, is rigid.† While the adjustments for error in azimuth and proportional chaining are sound legally and mathematically, local conditions should be studied and the history of the case thoroughly considered.

In these examples of Mr. Hughes, he may have in mind the adjusting of a triangle. If the original work was a primary triangulation executed by scientific and competent authorities the angles, of course, should be adjusted by the method of least squares and the side adjustments applied. However, as the triangle illustrated in the example is small and apparently not connected with a triangulation network, the original work would probably have been a secondary triangulation, or even of less importance; in this case, the angles would have been reduced by taking their mean values. All this is

* *Transactions*, Am. Soc. C. E., Vol. LXXV (1912), p. 429.

† "Retracement Resurveys," by N. B. Sweitzer, 1918.

discussed in textbooks on higher surveying. As to the examples suggested by Mr. Hughes the main point to bear in mind is that a reproduction on the ground of the actual lines run by the original surveyor or engineer and the re-establishment of all monuments in their original positions are the prime objects. A great many of these cases suggested have to be passed on by the Courts; and the Courts see through legal eyes and precedents and not so much by mathematical reasoning.

The method given in the paper is suggested primarily for the convenience and aid of the field engineer. The theoretical problems concerning the shape of the earth and the elaborate formulas evolved for securing terrestrial measurements in various latitudes and longitudes have been thoroughly and accurately calculated by the most eminent specialists in this field. These formulas and problems are mostly reduced to the geodetic position and are referred to Clarke's spheroid of revolution giving results to 0.001″ of arc.

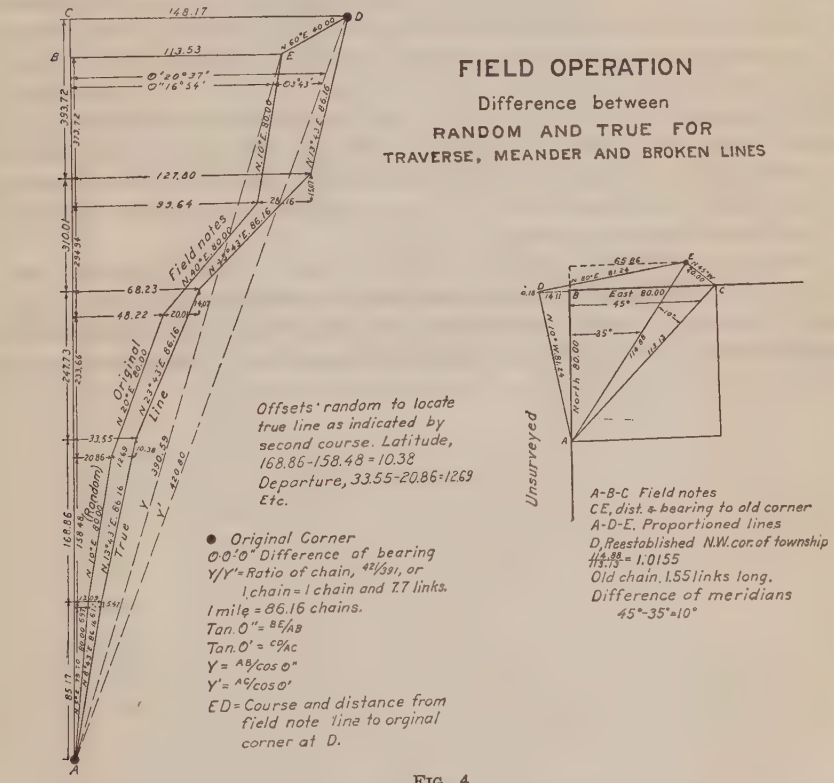
FIELD OPERATION
Difference between
RANDOM AND TRUE FOR
TRAVERSE, MEANDER AND BROKEN LINES
Offsets random to locate true line as indicated by second course. Latitude, 168.86-158.48 = 10.38
Departure, 33.55-20.86=1269
Etc.
Original Corner
0°0'0" Difference of bearing
Y/Y'=Ratio of chain, 42/391, or 1 chain = 1 chain and 7.7 links.
1 mile = 86.16 chains.
Tan. 0" = BE/AB
Tan. 0' = CD/AC
Y = AB/cos 0"
Y' = AC/cos 0'
ED = Course and distance from field note line to orginal corner at D.
Unsurveyed
A-B-C Field notes
C-E, dist. & bearing to old corner
A-D-E. Proportioned lines
D, Reestablished N.W. cor. of township
4/4.88/113.75 = 1.0155
Old chain. 1.55 links long.
Difference of meridians
45°-35°=10°

FIG. 4.

Nowadays, engineers are engaged mostly in retracements and relocations of lost or obliterated monuments. As original surveys are fast nearing completion it is therefore necessary to restore such monuments, as nearly as possible, in their original positions. This requires taking into consideration the methods used in making the original survey. Practically all these original

surveys were made with a needle compass, solar instrument, or simple transit. On major lines (such as State boundaries), sextants, zenith telescopes, and other more refined instruments were used for the latitude and longitude determinations. Practically all these instruments secured their zenith or horizon by the action of gravity, or the plumb line. This determination, however, is astronomic, not geodetic, and is referred to the actual irregular figure of the earth at that particular instrumental station. This, of course, would include the station error with other errors which, in some instances, would amount to as much as 10″ of arc or more, if reduced to the geodetic position.

It is evident therefore that the more elaborate formulas, including reduction to the geodetic center, would not produce results any closer to the proper final original positions sought than those suggested by the method of the paper. Practically all instrumental field operations incidental to the restoration of lost monuments or original surveys are limited in azimuth readings to the least count of the verniers. Modern practical field experience has generally shown this to be 1′ or 30″ of arc. A more refined instrument than this is too clumsy for rapid and economic transportation and handling. Therefore, why furnish the field engineer with equations and data refined beyond his field execution? In highly refined original work more involved equations are appropriate, especially in obtaining extremely exact results for the final reports and notes. The writer's simpler equations could be used for checks on the final results. If these results varied more than 1″ of arc it would be advisable to check the rigid and more involved methods.

The writer is grateful for the interest taken in this discussion and, especially for the expansion and development of the ideas presented in this method. The unanimity of those taking part, especially from so many widely separated places, and representing different specialties in the profession, is gratifying.

AMERICAN SOCIETY OF CIVIL ENGINEERS
INSTITUTED 1852

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Paper No. 1619

INTERSTATE WATER PROBLEMS AND THEIR SOLUTION*

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With Discussion by Messrs. Frank C. Emerson, D. C. Henny, and M. C. Hinderlider and R. I. Meeker.

Synopsis

The growth and future prosperity of the arid States rest primarily upon the efficient use of the waters of the Western rivers which are a common resource, interstate in character. The use and re-use of these waters from their sources down, is imperative to self-preservation of the States and the welfare of the Nation. Water consumption in the Western United States has attained that stage of development wherein such uses in one or more States frequently give rise to fear of encroachment on the use in adjoining States. Under such conditions wherein the sovereignty of two or more States or nations comes in conflict, it is readily seen that there may be fruitful ground for trouble. The problem is one between States equal in powers and with equal rights of self-preservation, and not one between mere private users whose rights are derived from their respective States.

The rapid development of rivers for municipal, irrigation, and hydro-electric power purposes has brought many interesting engineering and legal problems to the fore during the past fifteen years. None has been more intricate or difficult of solution and settlement than that pertaining to interstate water rights.

Friction between States and nations over the use of interstate or international rivers may easily arise from the fact that political divisions generally do not conform to river-basin boundaries, but cut across the latter in every

* Presented at the meeting of the Irrigation Division, Salt Lake City, Utah, July 9, 1925.
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conceivable direction and overlap on adjacent river basins. Due to physical conditions present in practically all major drainage basins, demands are frequently made on the water supply of one State for use in another State or in another river basin, and hence all phases of water utilization are affected by such conditions. Not the least of the difficulties arising over the administration of interstate streams is that due to differences in State water laws, no two of which are the same. Each State may use the waters of its streams as it may see fit save only for extra territorial burdens imposed: First, by internal treaties; second, by National control of navigation; third, by interstate treaties or compacts; and, fourth, by decisions of the United States Supreme Court.

This paper will be confined to interstate river problems of the Western United States, although reference will be made to like problems of the Eastern United States.

General Statement

In the arid and semi-arid States in the Western United States, where irrigation is essential to sucessful agriculture, interstate river problems are becoming more and more apparent, due in part to politics and in part to rapid growth and consequent uses of water.

Table 1 gives a summary of the more important rivers on which irrigation is paramount, and shows the interstate character of these streams. A brief study of the table discloses in a measure the far-reaching effect which questions relating to stream control may have where the interests of sovereign nations are involved. Many tributaries of the rivers noted in Table 1 and other independent streams are also interstate in character.

TABLE 1.—Interstate Character of Rivers in the Western United States.

River.	States and Nations affected.
Arkansas	Colorado and Kansas.
Colorado	Wyoming, Utah, Colorado, New Mexico, Arizona, Nevada, California and Mexico.
Columbia	Wyoming, Montana, Idaho, Washington, Oregon, and Canada.
Missouri	Montana, Wyoming, North Dakota, South Dakota, and Canada.
North Platte	Colorado, Wyoming, and Nebraska.
Rio Grande	Colorado, New Mexico, Texas, and Mexico.

Until the passage of the Reclamation Act in 1902 irrigation development was chiefly intrastate in character. Then came the era of development under Corporate, Carey, Irrigation District, and Federal Reclamation Acts, each imposing, more or less silently, its demands on Western rivers. As the colonization and development of irrigation projects is a gradual process, there occurs a lagging effect in the utilization of water appropriated for such reclamation, the cumulative effect of which results in reduced water supply which is sometimes not apparent for from ten to fifteen years. It generally happens in Western streams that this condition is only temporary and is largely corrected through return flow to the stream.

Interstate river problems were foreshadowed in the dry cycle of 1902 which, in some river basins, brought this subject of water shortage prematurely to the attention of irrigators, engineers, attorneys, and legislators. A preponderance of wetter years immediately following, together with reservoir development and the healing effect of return flow from irrigated areas, deferred general action until recently. Since the dry cycle of 1902, recurring dry years, in some instances, reflected water shortage in some of the smaller river basins, and pointed the finger of caution to potential interstate conflicts, emphasizing the need of early interstate action. Although in the main, careful studies have proved or will demonstrate the supposed injuries to be chimerical, yet these are causes which will lead to conflict and must be adjusted sooner or later.

Reference to the U. S. Census reports shows the rapid expansion of irrigation during the past twenty years. The figures in Table 2 have been extracted therefrom and are offered for their comparative value.

TABLE 2.—Comparative Data. Irrigation in the United States.*

Year.	Acres irrigated.	Capital invested.	Reservoir capacity, in acre-feet.
1889	3 631 000	$29 534 000	890 000
1899	7 518 000	67 000 000	2 561 000
1902	8 874 000	82 582 000	4 525 000
1909	13 738 000	308 000 000	12 580 000
1919	19 192 000	697 657 000	21 246 000

* See Tables 7, 8, and 15, U. S. Census Report on Irrigation, 1920.

From these data it is quite apparent that reclamation of arid lands reached a culmination in the 1909-19 decade. The effect of such development, together with that of the past five years, on water consumption and return flow is now commencing to be reflected in river flow, especially with the recurrence of dry years, which emphasizes interstate water problems.

Table 3 indicates the growth by Western river basins of irrigated areas for the seventeen-year period of 1902 to 1919, and shows that such areas were increased from 41 to 211 per cent.

TABLE 3.—Comparative Data. Irrigation in the United States.*

River basin.	Acreage Irrigated.		Percentage of increase, 1902 to 1919.
	1902	1919	
Great Basin streams	1 640 000	2 313 000	41
Missouri River and tributaries	2 533 000	4 147 000	64
Arkansas River and tributaries	393 000	851 000	116
San Joaquin River and tributaries	933 000	2 104 000	126
Colorado River and tributaries	927 000	2 312 000	149
Rio Grande River and tributaries	497 000	1 294 000	161
Columbia River and tributaries	1 297 000	3 873 000	198
Sacramento River and tributaries	203 000	641 000	211

* From U. S. Census reports.

Reference to similar data on the tributaries of the rivers mentioned in Table 3 shows that in many instances the increase for the seventeen-year period mentioned was frequently as great as 400 and 500%, and even more for areas of considerable size.

Reference to Table 2 shows that the total reservoir capacity in arid and semi-arid States trebled between 1902 and 1909, and more than doubled during the period of 1909 to 1919. Reservoir data by river basins for 1902 are not available for comparative purposes. Table 4 is offered to show roughly the distribution of reservoirs by river basins.

In recent years, prospective and potential irrigation, power, and municipal projects of Western river basins have been outlined more or less by engineering surveys and studies, and from these studies it is quite apparent that future water development will have to be in large units, will be chiefly interstate in character, and will encounter legal and financial difficulties unless interstate agreements are perfected whereby State titles to river flow are settled. No arid State is immune because State lines do not conform to river basins. Interstate problems concerning municipal water supplies, sanitation, flood control, power development, and related matters are not confined to the West alone; they also occur on rivers of the Eastern United States, thus emphasizing the importance of this subject.

TABLE 4.—Developed Reservoir Capacity, Western United States: River Basins for Year 1919.*

River basin.	Developed reservoir capacity, 1919, in acre-feet.
San Joaquin River and tributaries	330 000
Sacramento River and tributaries	348 000
Arkansas River and tributaries	792 000
Colorado River and tributaries	1 676 000
Independent rivers	1 899 000
Great Basin drainage	2 395 000
Rio Grande and tributaries	3 233 000
Missouri River and tributaries	4 861 000
Columbia River and tributaries	5 712 000
Total	21 246 000

* From Table 17, U. S. Census Report on Irrigation, 1920.

Tremendous property values depending on the utilization of interstate streams are at stake, and future water development should be freed from the clouds of possible conflict which act as a bar to the successful financing of such enterprises. Prior to the development of the Compact Method, the only means available for the settlement of interstate controversies was through an appeal to the Supreme Court of the United States.

The experience of Colorado in interstate water litigation has not been an enviable one, although she has never been the aggressor in any such suit. During the period from 1903 to 1925, inclusive, State appropriations for the investigation and protection of Colorado interests in interstate rivers amounted

to about $400 000, and reservoir and ditch companies spent a like amount in joint litigation over such matters. Roughly, a total of $1 000 000 has been expended by Colorado alone in necessary defense. The interstate problem of the Arkansas River has been in litigation for more than twenty years with still no final settlement, despite one decision by the U. S. Supreme Court, and three suits in the U. S. District Court, two of which are still active (1925). Eleven years elapsed between the date of the inception of, and decision in, the Laramie River case. In addition to the expenditures mentioned, irrigation development in Colorado has been retarded in certain areas or indefinitely postponed, because of interstate litigation. The decisions of the Courts have been unsatisfactory to both sides of each litigated case.

Colorado has been a pioneer on interstate river problems because of her physical location astride the Continental Divide with four major river systems radiating to the four points of the compass. As previously stated, interstate water litigation has not furnished a satisfactory solution of Colorado's interstate river problems. The compact method for settlement of such problems is the outgrowth of a long period of years of interstate water conflicts.

To the present time interstate litigation over Colorado streams has concerned comparatively small water supplies which are relatively over-developed and yet four States were involved, the streams in question being:

The Laramie River, with an average annual flow at the Colorado-Wyoming State line of.....	200 000	acre-ft.
The Republican River, with an average annual flow at the Colorado-Nebraska State line of.....	5 000	" "
The Arkansas River, with an average annual flow at the Colorado-Kansas State line of about.....	200 000	" "

The heavy expense and long delays incident to securing decisions in these litigated cases, coupled with the general dissatisfaction with the decisions rendered, have pointed to the need of constructive treatment of interstate water problems on the larger rivers where the yearly flow approximates millions of acre-feet, where present and prospective land and other property values run into hundreds of millions of dollars, and where from two to seven States claim joint rights to the natural resource.

In many of the Western river basins where the use of water has not attained a mature stage, comprehensive plans for the greatest use can be made, and conditions are still elastic for interstate adjustments. In some of the smaller interstate river basins the consumption of water has approximated total utilization, and interstate adjustments, whether by litigation or mutual agreements, are increasingly difficult.

Origin of Compact Idea

The compact method of treatment of interstate river rights was the direct outgrowth of the Laramie River interstate water suit between Wyoming and Colorado. The first compact proposal was made in conjunction with engineering and legal studies of the South Platte River problem in 1916-17; later, in

connection with the La Plata River investigations during 1919 and 1920, followed by the Colorado River problems which appeared on the horizon of interstate strife in 1920.

It should be recalled that as far back as 1903 both Elwood Mead, M. Am. Soc. C. E., and Mr. R. P. Teele, of the U. S. Department of Agriculture, directed attention to interstate river problems and discussed the ultimate need of agreements between the States concerning the utilization of interstate waters. Recently, both Herbert Hoover, Hon. M. Am. Soc. C. E., and Mr. Mead have placed the stamp of approval on the interstate water compact principle as a desirable means for the settlement of interstate river problems. It is to Delph E. Carpenter, Interstate River Commissioner of Colorado, however, that the principal credit is due for the working out of the compact idea and the application of it to the settlement of controversies over Western interstate streams.

Object of River Compacts

The purpose of interstate water compacts is to settle the title to river flow between or among the States claiming a river as a common resource. In making compact adjustments, the underlying principles have been to ascertain and define the relative needs and rights as between the States in interest and to safeguard future development against unnecessary delays and the unsettled status of title or wasteful and protracted litigation.

Legal Basis for Interstate River Compacts: Treaty Power of States

Each State of the United States is a sovereign power and may legislate as it pleases concerning its internal problems, except as such powers were relinquished under the Constitution. With reference to external problems, Article I, Section 10, Paragraph 3, of the Constitution of the United States recognizes compacts or agreements between States, as follows: "No State shall, without consent of Congress * * * enter into any agreement or compact with another State * * *."

In fact, heretofore, interstate controversies and differences respecting boundaries, fisheries, and other matters, have been frequently settled by interstate compacts through the use of the reserved treaty powers of the States. Among the many boundary disputes thus settled may be mentioned those of Virginia and Pennsylvania, 1780 (11 Pet., 20); Virginia and Pennsylvania, 1784 (3 Dall., 425); Kentucky and Tennessee, 1820 (11 Pet., 207); Virginia and Tennessee, 1802, 1856, (148 U. S., 503, 511, 516); and Virginia and Maryland, 1785 (153 U. S., 155, 162). Compact adjustments of fishery disputes on the Columbia River have been made between Washington and Oregon, and also between Maryland and Virginia in like matters in respect to the Potomac River (153 U. S., 155). For a full discussion respecting the rights of the States to enter into treaties or compacts, with the consent of Congress, see Rhode Island *vs.* Massachusetts (12 Pet., 657, 725-731); Virginia *vs.* Tennessee, (148 U. S., 503); and Wharton *vs.* Wise (153 U. S., 155).

Summarizing, it is found that the States have the same power (with the consent of Congress) to enter into compacts with each other on all matters not delegated to the Federal Government, as independent nations have to make joint treaties.

International River Problems

A review of international river controversies discloses that such matters are usually settled by treaty (Heffter Droit Ind. Appendix VIII; Hall International Law, Sec. 39, 21 Opinion, U. S. Atty. General, 274, 282). The Rio Grande controversy between Mexico and the United States concerning river depletion and water utilization was settled by treaty in 1906. There is also the Canadian treaty of 1909 concerning the present and future use of the waters of the Milk and St. Marys Rivers, the former being a large tributary of the Missouri River. About 1915 an agreement was made in Australia over the waters of the Murray River. The signatories to that agreement were the States of New South Wales, Victoria, South Australia, and the Commonwealth Government.

An effort has been under way for several years to settle the controversy between Egypt and The Sudan over the waters of the Nile. Likewise, discussions looking to the adjustment of the problems arising over the use of the waters of the Great Lakes and the St. Lawrence River are in active progress between Canada and the United States; and between Mexico and the United States over the Rio Grande below Fort Quitman, Texas.

Interstate River Decisions: U. S. Supreme Court

The two important Supreme Court decisions concerning the use of Western interstate rivers are those of Kansas *versus* Colorado (May 13, 1907), and Wyoming *versus* Colorado (June 5, 1922).

Kansas versus Colorado.—This interstate river suit arising over the use of the waters of the Arkansas River, was filed by the State of Kansas in May, 1901. The Kansas suit was predicated on the Doctrine of Riparian Rights. In Colorado, the Doctrine of Prior Appropriation prevails. The State of Colorado contended that constitutional provisions entitled it to ownership of all waters within its borders.

The United States intervened in this suit in the interest of its National reclamation policy, and although contending for the Doctrine of Prior Appropriations, raised the question of Federal *versus* State control of water supplies. In substance, the outstanding points of the Supreme Court decision were:

1.—It affirmed State ownership and control of waters of non-navigable streams. This was the primary question involved.
2.—It defined interstate rights to be an equitable division or apportionment of the benefits arising through the use of waters in an interstate stream.
3.—It dismissed the Kansas petition without prejudice on the grounds of insufficient proof of injury.

At the time of the Kansas-Colorado suit (1902), the irrigated area from the Arkansas River and tributaries, according to U. S. Census reports, approxi-

mated: In Colorado, 300 000 acres; and in Kansas, 22 000 acres. This interstate river suit occurred during a dry cycle of stream flow and prior to any material development of reservoirs in the Arkansas Basin.

Wyoming versus Colorado.—The Laramie River interstate suit was filed by the State of Wyoming in 1911 over a threatened depletion of the Laramie River by a trans-mountain canal system and tunnel. Many conjectures have been made and several fallacies have obtained concerning the Doctrine of Priority and its application to interstate river problems. Perhaps the chief fallacy concerning this doctrine and its application to interstate streams has been the assumption that administration of river flow should be basin-wide in character regardless of State lines, and that such administration should be by the Federal Government.

The far-famed Laramie River Supreme Court decision of June, 1922, has been repeatedly cited as sustaining this contention. The Laramie River decision was based on the fundamental principles underlying the Doctrine of Priority, but the respective private rights in each State were grouped and the allocation made was based on the grouped demands and not on a basin-wide administration of priorities of both States. The Court gave to the most junior project (Colorado Tunnel) a preferred right of diversion as against senior canals in Wyoming, which were compelled to build reservoirs to supply the deficiency. No Federal bailiff was appointed and no super-administration placed in the hands of the Federal Government. The internal administration of priorities of the Laramie River in Colorado or in Wyoming remains undisturbed. No interposition of Colorado priorities was plastered upon the Wyoming priorities, or *vice versa*. In substance, 43 000 acre-ft. per year of the Laramie River water supply was allocated to Colorado, wet and dry years alike, if obtainable. The remainder of the river flow was allocated to Wyoming, and the burden of reservoir regulation placed on Wyoming users.

In substance, the Wyoming-Colorado interstate river decision was as follows:

1.—It affirmed the fundamental principle of priority as the proper basis for allocating river flow to established uses in each State when both States recognize the same doctrine, but re-affirmed the Kansas-Colorado decision where local laws differ.
2.—It held untenable the Colorado contention that a State may use all the water of an interstate stream which originates within its boundaries.
3.—It held untenable the Wyoming contention that appropriations of Laramie River water should be limited to use in the Laramie River water-shed.
4.—It declined to pass on the Government contention that all unappropriated water of Western streams belonged to the United States and are wholly removed from State control.
5.—It held that the burden of reservoir development is on the lower State.

The Doctrine of Priority is an intrastate doctrine, is primarily a rule of local administration, and its application in a sweeping way to a large river basin affecting several States where various water rights date back as much as

fifty years, would disturb water rights in the several States and attendant large property values. Furthermore, the transition from semi-arid to humid conditions in a lower State whereby a different rule would apply, would make basin-wide administration impracticable.

Physical conditions and the element of time in river flow are also opposed to operating vast river basins (such as those of the Colorado or North Platte River), hundreds of miles in length, with hundreds or, perhaps, thousands of ditches diverting water, according to priority regardless of State lines. Such vast river basins would necessarily have to be sectionized and water administration localized, which State lines do automatically. Another objection to the administration of river basins as a whole, according to the Doctrine of Priority, is that changing conditions and the pressure of need may modify the present method and character of water administration within a State. Any means of interstate settlement should avoid disturbance of the internal relation of State priorities.

The application of the Doctrine of Priority as a principle in determining interstate apportionment is, however, sound. Such an apportionment was made in the Laramie River case between the States of Wyoming and Colorado where State priorities were lumped in determining each State's quota. Definite quantities per annum were allocated to the upper State, the remainder going to Wyoming, the lower State. State administration was left intact. No Federal administration was imposed on the Laramie River, and none is necessary on that or any other interstate river. There has been a growing tendency to demand too much Federal assistance on interstate relations to the detriment of State autonomy. The theory of Federal administration of interstate rivers is an apt illustration of such tendencies. In May, 1925, President Coolidge pointed out the fallacy of too much Federal intervention in State affairs and the need of more State responsibility in interstate matters.

With reference to State or Federal control of interstate streams, in 1903,* Mr. Mead commented as follows:

"It would be unscientific and in the highest degree unfortunate if the principle of local self-government, on which this nation is founded, and the opportunity to exercise self-reliance and self-control, which has done so much for its manhood in the past, should be taken away from the irrigators of the West by the transfer of the local regulation of streams to some centralized bureau."

Under the compact method of settlement, State autonomy in water matters is preserved and intrastate priorities undisturbed as to position. In compact adjustments of river supplies where development and water utilization are far advanced, definite State allotments are necessary. In river basins where water utilization is in its infancy, State allocations can be made more or less elastic to fit changing conditions and the needs of the future.

Under the litigation method of settlement of interstate water disputes, all efforts are centered on winning the suit. A vast amount of information is submitted in an unco-ordinated way, and inadequate attention given to potential water needs.

* "Irrigation Institutions," p. 378.

Under the compact or treaty method of adjustment, opportunity is had to study a river basin as a whole, and from such study to arrive at some amicable settlement based on established facts as to total water supply, irrigated and irrigable areas, municipal necessities, reservoir opportunities, power possibilities, and future development.

Such studies must necessarily include questions of the relative necessities of each State as determined by climatic conditions, environment, natural resources, and facilities for utilizing them in the most efficient manner, ever bearing in mind that for all time to come Nature has placed a limit on a common resource far below the requirements, thereby making it incumbent on all those who would avail themselves of its use to conserve and re-use this resource in the most efficient way. A lower State which permits water, the very life-blood of an upper State, to flow unused through the lower State should not be heard to complain of any unfair apportionment of the benefits arising out of the use of that stream by the upper State. To hold otherwise would be abhorrent to all the theories of justice, equity, and economics, as well as the fundamental principles on which American irrigation institutions have been founded. Owing to climatic differences and accidents of settlement in a river basin development has frequently occurred first in the lower States. Development on the head-water States is more gradual, and if interstate rights are determined entirely by priority rule, the upper State might be largely deprived of a natural resource having its origin principally in its own territory. It would be impossible to remedy this evil by the exercise of eminent domain because such exercise would be limited by the boundaries of the upper State and could not affect property in the lower States. The upper State is, therefore, left one of two alternatives: First, to prevent development in the lower State; or, second, to secure a recognition of its rights for future development by interstate agreement or compact.

Recent Interstate Legislation

Recent legislation by the States of Wyoming and Utah provides for cooperation with adjacent States in the matter of acquiring, determining, supervising, and regulating water and water rights of interstate streams. This is a constructive step destined to have a considerable field of application in the future. Oregon has older similar reciprocal legislation concerning the State line canals, but it is less broad than other features of the Utah Act of 1921.

Several States and Federal agencies have entered into joint engineering studies of all phases of water utilization of several intrastate and interstate rivers. Comprehensive reports with recommendations have been made, and those on interstate streams will probably be the basis of future water agreements between the States affected and the Federal Government.

Reference is made to the study and report on the Columbia River Basin published by the Federal Power Commission in 1923. The States of Washington, Idaho, and Montana, and several departments of the Federal Government participated in this study.

During the winter of 1924-25, North Dakota passed legislative acts providing for two interstate water commissions—the Missouri River Commission to be composed of representatives of Montana, South Dakota, and North Dakota, and the Red River Flood Commission which provides for participation by Minnesota, South Dakota, and North Dakota.

Historical Review of Interstate River Compacts

The object of interstate water compacts is to determine the respective rights of the various interested States to the use of a common river supply. The compact idea of interstate river apportionment was formulated in 1916 and 1917 in connection with legal and engineering studies of the South Platte River in a controversy then pending between the States of Nebraska and Colorado. The first application of the compact plan of settlement, however, occurred on two other interstate rivers, namely, the Colorado River and the La Plata River. The necessary State and National legislation providing for interstate river compacts was passed in 1921, and the first compacts were drafted in November, 1922, for the Colorado River and the La Plata River, at Santa Fé, N. Mex.

A marked contrast occurs on these two interstate river problems. The Colorado River water supply involves seven States, 20 000 000 acre-ft. of water per year, water rights to 3 000 000 acres of irrigated land, and large potential irrigation and power possibilities. Water utilization is still far from having attained a complete stage of development in the Colorado River Basin, and about 13 000 000 acre-ft. per year passes unused to the Pacific.

The La Plata River involves two States, 23 000 acres of irrigated land, and about 60 000 acre-ft. of water per year, with the direct-flow rights heavily overdecreed, and with storage as the final step in water utilization.

Six years after the commencement of engineering studies on the La Plata River, and four years after the initial legislative act, an interstate compact is in operation. The Colorado River Compact is still an open problem, although five of the seven States have ratified unconditionally and one conditionally. Considerable progress has been made when it is realized that engineering studies usually require two or more years, legislative ratification can only occur at two-year intervals, and Congressional ratification can seldom be accomplished in less than one year.

In a comparative way the Colorado River problem may be classed as a major one for accomplishment, and the La Plata River a minor interstate problem when volume of water is considered, although the La Plata River problem required a large amount of study and effort to reach a settlement. The South Platte Compact is now awaiting Congressional ratification.

Interstate compact accomplishments may be briefly summarized as follows (status as of May, 1925):

River compacts concluded, under negotiation, or provided for by legislative act	10
River compacts concluded	5*
River compacts ratified by interested States, with or without reservations	5
River compacts in operation	1

Table 5 gives the salient features of rivers and States affected by compacts concluded, under negotiation, or provided for by legislative act.

Prospective Interstate Water Problems

A review of water utilization and prospective development on other interstate rivers discloses many nascent and potential interstate water problems. Among these may be mentioned the following (Table 6), although there are undoubtedly many others.

Résumé

A résumé of the foregoing discussion, it is believed, will reveal the following pertinent and outstanding facts:

First.—The vast importance of the subject herein discussed as affecting relationship between sovereign States, the protection of existing rights in each State, and the systematic, necessary, and logical development of the latent resources of not only the Western United States, but all parts of the Nation where such resources are of an interstate character.

Second.—That with respect to the status of interstate streams, the Supreme Court of the United States has made a number of well-defined rulings which have quite definitely established certain limitations to the exercise of State and Federal control of a resource common to two or more States, and which point the way for future action in interstate matters. These may be enumerated as follows:

(*a*) State ownership and control of waters of non-navigable streams.
(*b*) Equitable apportionment between States of the benefits arising through the use of the waters of an interstate stream.
(*c*) Recognition of the fundamental principle of priority as a proper basis for allocating river flow to users claiming water from an interstate stream when the interested States recognize the common doctrine of priority of use.
(*d*) Holding as untenable the theory that a State may rightfully claim the exclusive use of all the waters originating within the boundaries of such State to the serious detriment of a lower State where such water supply is a common resource, and where each State recognizes the doctrine of appropriation.
(*e*) Affirms the principle that the locus of application of the waters of a stream common to two or more States is not limited to the drainage basin of such stream.
(*f*) Among other things the Supreme Court has refused to pass upon the contention of the U. S. Government that all the unappropriated waters of the Western streams belong to the United States and are wholly removed from State control, but a previous ruling does recognize the States' control of such waters.

* Includes the Delaware River Compact between New York, New Jersey, and Pennsylvania.

TABLE 5.—Status of Interstate River Compacts, May, 1925.

River.	States affected.	Status.
Western United States.		
Arkansas	Colorado and Kansas	Negotiations in progress.
Colorado	Arizona, California, Colorado, Nevada, New Mexico, Utah, and Wyoming	Compact concluded. Ratified intact by all States except Arizona and California, which attached reservations.
La Plata	Colorado and New Mexico	Compact concluded. Ratified by both States: ratified by Congress; signed by President, January 29, 1925. In operation season of 1925.
North Platte	Colorado, Nebraska, and Wyoming	Engineering investigations completed; hearings held in 1924 in three States; active negotiations under way.
Pecos	New Mexico and Texas	Compact concluded December 19, 1924, amended February 10, 1925.
Rio Grande	Colorado, New Mexico, and Texas	Engineering investigations completed; hearings scheduled for 1925.
South Platte	Colorado and Nebraska	Compact concluded; ratified by both States awaits Congressional ratification.
Eastern United States.		
Delaware	New York, New Jersey, and Pennsylvania	Compact concluded. New Jersey: Referred to a Board of Seven for consideration to report to 1926 Legislature. Pennsylvania: Passed Senate; amended in Assembly with reservations proposed by New Jersey. Note.—Ratification by Pennsylvania impossible before 1927. Compact as amended passed Assembly, but failed in Senate. New York: Compact with reservations and amendments proposed by New Jersey passed without opposition, and signed by Governor Smith on March 18, 1925.

TABLE 6.—Interstate Water Problems.

Stream.	States affected.	Nature of problem.
Surface Streams.		
James River	North and South Dakota	Flood control and drainage.
Big Sioux River	South Dakota and Iowa	Flood control and drainage.
Columbia River	Montana, Idaho, Washington, and Oregon	Irrigation and power chiefly. Columbia Basin Project involved.
Snake River	Colorado and Wyoming	Irrigation uses.
South Canadian River	Arkansas, Kansas, New Mexico, and Oklahoma	Flood control and conservation.
Arkansas River	Arkansas, Kansas, and Oklahoma	Flood control and conservation.
Grand River	Kansas, Missouri, and Oklahoma	Flood control and conservation.
Caney River	Arkansas, Kansas, and Oklahoma	Flood control and conservation.
Underground Water.		
Artesian Basin	North and South Dakota	Regulation of present waste from about 10 000 artesian wells.

As a result of such decisions it is apparent that no State is supreme in matters pertaining to the utilization of the waters of a stream common to two or more States; that there must be recognition of a common right of all States concerned to share therein. It naturally follows that some means must exist or be found for the determination and allocation of the benefits arising under such right. As between two nations, such means are limited to arbitration or war. As between the States forming the nation, such means are limited to arbitration or to appeals to the U. S. Supreme Court, resort to force being prohibited under the Constitution. The treaty or compact method would seem to offer great possibilities and a rational basis for the settlement of interstate disputes, as such method provides ample opportunity for a dispassionate analysis of all conflicting claims and equities, in an atmosphere of fairness and in the light of all established facts.

It is the opinion of the writers that the opportunities for the adjustment of such disputes, offered by such means, should be resorted to first, and that only after every effort has been completely exhausted should either State appeal to the Court of last resort. It is believed that such procedure is scientifically and economically sound and is certainly conducive to that sense of interstate comity which should prevail between sister States as one of the requisites of Government stability.

Any decision based on an incomplete presentation of the facts, or a perversion of the same, which is likely to occur in any Court proceeding, and which leaves the losing party honestly convinced of the unfairness of the decision, is fundamentally wrong, and, in a true sense, cannot be regarded as a real settlement of the dispute. Court decisions follow Constitutional provisions and statutes. The former of necessity are fundamental in their application and may permit of considerable variation in interpretation. The latter, although supposedly based on the elements of equity, justice, and common sense, often fall far short of the objective sought, and the Court interpretation of such is frequently susceptible of many shadings.

As long as conditions which permit of litigation between States are allowed to exist through failure to provide a more rational method for the elimination of such disputes, expensive litigation resulting in an unsettled state of affairs will continue as a clog on future development. Suits between individuals or corporations of different States are expensive, both in time and money, and even when decisive, the results are limited to the parties in action and to the particular question before the Court. Suits between States which must be brought in the U. S. Supreme Court usually result in more or less definite determinations, but such decisions require long periods of time and the expenditure of large sums of money. Some of the decisions indicate that the Court was more interested in enunciating some principle applicable to the case in question than in a definite and constructive settlement of the differences at issue.

On the other hand, under the treaty method of adjusting interstate controversies, full play is given for the application of constructive methods looking to a realization of the legitimate aspirations of each party. Furthermore,

when a common ground has been found, the consummation of a compact between States eliminates all future litigation of the matters treated, not only between the States, but between the residents thereof as well, thereby promoting orderly development through the determination of the relative rights to the use of a common resource under conditions to be determined by the owners thereof.

Furthermore, a treaty between sovereign powers is not susceptible of arbitrary change or revocation or modification without the consent of the contracting parties, which tends for greater stability as against Court decisions which are susceptible of reversal or such modifications as may seem meet and proper to the Court. On the other hand, in the drafting of an interstate compact, provision may be made by the contracting parties for desirable modification which might appear necessary as time passes, which would provide a flexible and practical method of administration, compatible with the necessities of an advancing age. Such conditions would not ordinarily obtain under the inflexible rules and limitations which usually hedge about the administration of a Court decree.

Not the least of the advantages to be secured to each State through the use of the compact method of settling interstate disputes is the elimination of a real menace to State institutions and constitutional guaranties, that of Governmental interference in the allocation and administration of public water supplies. The writers know of no theory advanced by proponents of Federal control of the resources of Western States that would be so inimical to the welfare of irrigation institutions as that of Governmental supervision of the administration or control of public water supplies. If for no other reason than this alone it is highly important that the Western States adopt some method for the adjustment of controversies over interstate streams which would eliminate the necessity for an appeal to a Federal tribunal with the consequent opportunity for imposing Federal supervision.

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DISCUSSION

FRANK C. EMERSON,* M. AM. SOC. C. E.—The State of Wyoming shares with Colorado the position of sitting at the high point in the United States and watching the waters of interstate streams flow across the borders of the State to the four points of the compass. To the north the Big Horn River rushes into Montana to join the Yellowstone and eventually the Missouri; to the east the North Platte flows into Nebraska also to join the Missouri; to the west, the Snake River departs into Idaho as a tributary of the Columbia; and to the south the Green and Little Snake Rivers flow into Utah and Colorado as tributaries of the mighty Colorado. This situation provides Wyoming with many interstate water problems of importance. No State has made greater progress in meeting these problems than Wyoming and the speaker feels assured that the facts given in this discussion will bear out the assertion.

The interstate water problems of the arid West were just beginning to present themselves with force when the speaker assumed the office of State Engineer of Wyoming in 1919. This was due no doubt largely to the extremely dry season of 1919 combined with the increasing use of water throughout the inter-mountain country. During the speaker's six years' tenure of office the problems have been coming with ever greater force, and it has been necessary to take an active part in attempting to meet them to the best advantage of the State. It is hoped that this discussion may convey some information and ideas that may be helpful in the solution of the problems under consideration, which mean so much to the development of this great Western country.

A few thoughts will be given in support of the conclusions of Messrs. Hinderlider and Meeker that the best solutions of interstate water problems will be found in amicable agreements largely expressed through interstate compacts. Wyoming has certainly had convincing experience that litigation should be avoided whenever possible in favor of effort to solve questions with other States through intelligent conferences and mutual agreements. It was stated that the case of Kansas *vs.* Colorado has been in the Courts for a period of twenty years and is still undecided, and that the Wyoming *vs.* Colorado case respecting the use of water from the Laramie River was before the U. S. Supreme Court eleven years before decision was made. With the latter case, despite the fact that much time, energy, and money were spent by both litigants while this case was running the gauntlet of the Courts, the results could not be considered to be of material advantage to either Wyoming or Colorado when the decision finally came. It is true that the earlier priorities on the Laramie River in Wyoming were sustained in a general way according to the wording of the Court opinion, but in practical effect the verdict has resulted in little advantage to Wyoming.

As State Engineer and President of the State Board of Control, the regulation and distribution of water from the streams of the State come within the speaker's jurisdiction. During the season of 1922, when the flow of the Laramie River was considerably below normal, it was found that water rights

* Governor of Wyoming, Cheyenne, Wyo. At the time this discussion was presented, July 9, 1925, Mr. Emerson had served as State Engineer of Wyoming since 1919.

in Wyoming of 1883 priority were forced to go without water although, under the decree of the Court, the diversion of water into the Laramie-Poudre Tunnel in Colorado was allowed, notwithstanding the fact that the Court recognized only the late priority of 1909 for the right. The Colorado right was being served while water users in Wyoming having rights of twenty-six years seniority on paper saw their ditches "bone" dry. Why this situation? Simply because the Court of last resort in its wisdom granted to Colorado rights a certain volume of water for use each year, regardless of the seasonal flow of the river. As a matter of fact, under the conditions that prevail, the situation is such that the 1909 right for the Colorado tunnel diversion could never exceed the quota of water allowed this right by the Court, although Wyoming rights of more than a quarter of a century seniority will at times be without water unless they undertake a large program of reservoir construction. What was heralded as a victory for Wyoming was indeed an empty one.

How futile the time, effort, and expense devoted to this litigation, and, curious though it may be, Wyoming fought for the establishment of a principle in this case (which in itself only concerned the water supply of a comparatively small stream), that would work against the State in relation to practically every large stream in the State. What was to be gained by Wyoming as a State from the logical decision of the Court that no State could use the waters of an interstate stream to the material prejudice of rights established in other States. The small Laramie River happens to rise in the mountains of Colorado and flow into Wyoming; practically every large stream of the State flows from Wyoming to States beyond its borders. Again, the eminent members of the legal fraternity who presented the case for Wyoming, contended that water could not be legally diverted from one water-shed to another. The Court wisely decided that the inter-water-shed diversions were permissible as long as damage was not inflicted on established rights. An adverse decision would have been disastrous to a considerable development in Wyoming in sections where it has been found that conducting water from one drainage area to another has resulted in the most valuable use of the water supply available.

The foregoing comment is made on the Laramie River case by reason of the fact that it is probably the most important irrigation case decided by the U. S. Supreme Court. In this controversy Wyoming engaged in legal combat with its neighbor, Colorado, and after years of litigation the decision came without bringing material gain to either State. The speaker ventures to assert that representatives of the two States in intelligent conference could have reached within two years an agreement that would have been reasonably satisfactory to both Commonwealths. It is not intended to infer that the decision of the Court was unfair or poorly founded. To the contrary, the findings of the Court are generally logical throughout and have decided clearly and definitely several mooted points concerning water-right jurisprudence. It was well in a way to have them decided but, at the same time, the particular matters concerned by this case certainly did not warrant the struggle in the Courts.

Other cases concerning interstate water problems of interest to Wyoming could be cited. Suffice it to say, however, that the usual experience in cases

wherein appeal has been made to the Courts for solution of water-right questions, has proved that litigation is a poor method indeed. In contrast to this unfavorable experience when resort has been made to the Courts, is the record of accomplishment obtained through friendly conference and mutual agreements. Favorable progress toward the solution of interstate water problems has been made by Wyoming along what may be considered as three different lines of effort, namely:

1.—By what may be considered as a "gentleman's agreement", as exemplified by the North Platte River Co-Operative Investigations;

2.—By reciprocal statutory enactments and agreements between States, as is evidenced by the negotiations and agreements concerning interstate streams between Utah and Wyoming; and

3.—By interstate compact such as is now under negotiation in respect to both the Green and North Platte Rivers of Wyoming. Comment will be made on these three plans in order.

North Platte River Co-Operative Investigations.—In 1904, the State of Wyoming granted to the United States a permit for the construction of the Pathfinder Reservoir and the storage of more than 1 000 000 acre-ft. of water per annum of the North Platte River. The U. S. Reclamation Service forthwith proceeded to construct a monument of good engineering and by the Pathfinder Dam provided for the conservation of the waters of the North Platte. Although this great conservator of water filled a definite need, it proved for many years to be a curse rather than a blessing to the State of Wyoming which granted the right. With its large investment in the dam the United States became much concerned about the adequacy of the North Platte to supply more than the areas of land lying under the canals constructed by the Reclamation Service, which were looked to for the return of the cost of the reservoir. On the supposition, or the fear, that these lands, lying principally in Nebraska, needed all the water of the river for their proper irrigation, an affective embargo was placed upon any additional developments in Wyoming.

Development on the North Platte in Wyoming was effectively prevented for years in any large way by the Pathfinder situation. Carey Act segregations were refused, desert land entries were denied, rights of way across Government lands were not granted, and, in other ways, developments which proposed a use of water in Wyoming from the North Platte River and its tributaries were blocked by the Federal Government. The situation became intolerable until in 1918 a co-operative investigation of the whole matter was agreed upon between the State of Wyoming and the United States. The field investigations were thorough and resulted in what is known as the Conkling-Meeker report, Harold Conkling, M. Am. Soc. C. E., representing the United States, and R. I. Meeker, M. Am. Soc. C. E., representing the State of Wyoming. On this preliminary report the Board of Review on North Platte Co-operative Investigations founded its report carrying the final recommendations and conclusions concerning the use and disposition of the waters of the North Platte River. The Board of Review was composed of E. A. Moritz, M. Am. Soc. C. E., for the

United States, the speaker for the State of Wyoming, and Mr. L. G. Carpenter, of Denver, Colo., as the third party.

The primary finding of the Board was to the effect that there was sufficient water in the North Platte River, when properly conserved, for the reclamation of all lands practical of irrigation in Colorado and Wyoming and for additional extensive areas in the State of Nebraska. This led to the further conclusion that artificial restriction of developments on the upper river were entirely unnecessary.

The report of the Board of Review received the approval of the Secretary of the Interior, the Director of the U. S. Reclamation Service, and the Governor of Wyoming. With its adoption the embargo against further Wyoming developments from the water supply of the North Platte River was immediately lifted and the unfair situation relieved. The report of the Board of Review has been referred to as a "gentleman's agreement" and such it is in practical effect. Although not a real adjudication, the findings of the Board have been respected, and the strain has been removed from the condition that existed. By intelligent study and friendly conference this "gentleman's agreement" recorded real accomplishment. Negotiations are now under way to form a compact between the States of Colorado, Wyoming, and Nebraska, concerning the use of the waters of the North Platte River and it is proposed that the report of the Board of Review be the basis for the compact.

Reciprocal Statutory Enactments and Agreements Between States Concerning Interstate Streams.—The example afforded by effort along this line by the States of Utah and Wyoming is noteworthy, and what may be accomplished through such a plan, is well illustrated by the present agreements concerning the use and administration of the waters of interstate streams of interest between these two States. In 1921, the Legislature of Wyoming passed a bill proposing reciprocity with Utah in water-right matters. After this bill was enacted into law by the Wyoming Legislature, the speaker made a trip to Salt Lake City to work with the State Engineer of Utah in the matter and a bill similar to that passed by Wyoming was prepared and introduced in the Utah Legislature. Favorable action was taken by the Legislature and, therefore, similar statutes are now in force in the two States, providing for conference and mutual agreements concerning the waters of the Bear River and other interstate streams. For the information of the membership the Wyoming statute is, as follows:

"Co-Operation with Utah in Water Right Matters

"Sec. 1. That the state engineer is hereby authorized and empowered to receive and grant applications to appropriate water from streams in Wyoming where such water is to be conveyed and used for beneficial purposes within the state of Utah, and the board of control is hereby authorized and empowered to issue certificates of appropriation under such permits as may be granted by the state engineer of Wyoming upon certification from the state engineer of Utah that the waters appropriated have been put to the beneficial uses set forth in the permits; the state engineer and the board of control of Wyoming are also authorized and empowered to co-operate with the state engineer of Utah in the determination, supervision, regulation and control of all water and water rights on all Utah-Wyoming interstate streams; and

to these ends the state engineer and board of control of Wyoming, by and with the consent of the governor, may enter into the necessary agreements with the state engineer of Utah to carry out the purposes of this act, provided that such agreements are not in conflict with the provisions of the irrigation laws now in force in this state; provided further that such authority shall not be exercised by the state engineer or board of control until after the state of Utah has passed a law granting its state engineer like authority to that granted to the state engineer and board of control of Wyoming by this act. (L. 1921, ch. 47.)"

Advantage has been taken of these statutes and certain plans have been put into operation under the provisions thereof. In May, 1925, the speaker took a comprehensive trip by automobile with George M. Bacon, M. Am. Soc. C. E., State Engineer of Utah, over the several interstate streams between Utah and Wyoming, for the purpose of obtaining information concerning the situation on the ground in order to work out forms of agreements between the States that would be fair to both. Certain definite arrangements were consummated whereby intelligent administration of the waters of certain streams is made possible where before no satisfactory system was in force. For example, take the situation on the Bear River, an important stream from which a large use of water is made both in the State of Utah and the State of Wyoming.

The Bear River rises in the mountains of Utah and flows across the line into Wyoming and traverses Wyoming territory for a distance of 30 miles in the region of Evanston; then passes back into Utah to supply water to a large acreage of land in that State; then back into Wyoming to supply water for the irrigation of the territory in the vicinity of Cokeville; then into Idaho and finally again back into the State of Utah. Many ditches have their points of diversion of water in one State with the irrigable lands situated in whole or in part in another State. It will be readily seen that interstate problems of importance come into such a situation. Through and under the provisions of the reciprocal statutory measures between Utah and Wyoming an intelligent and effective administration of the waters of the Bear River between these two States is being obtained.

To illustrate what has been accomplished, consider the authority that has been given to Mr. David Dean, Water Commissioner of Wyoming, in charge of the regulation and distribution of water from the Bear River in the Evanston District. In addition to authority over the head-gates located in Wyoming, Mr. Dean has been given specific authority by the State Engineer of Utah over three certain head-gates which are in Utah, but which irrigate lands situated in Wyoming. These three ditches cover a large area of Wyoming lands and have comparatively recent priorities. Heretofore, in times of shortage of water in the river, the Wyoming Water Commissioner has been powerless to act, and has been forced to see large diversions of water for the benefit of these recent rights, while rights of many years seniority in both Wyoming and Utah have been forced to go without. As a Water Commissioner of Wyoming, Mr. Dean had no authority whatever over the diversions situated in Utah. Having been given the power to regulate these diversions, he was immediately placed in a position to take such action as might be

fair in the premises and serve the proper interests of users of water in both States.

On another section of the Bear River an opposite situation exists to the one described. In this instance three certain canals divert water in Wyoming for the irrigation of lands largely situated in Utah. Mr. Dean has been instructed, as Wyoming Commissioner, to allow such quantity of water to pass into the head-gates of these ditches as the lands irrigated thereunder would be entitled to under their priorities, the quantity of water to be governed by the acreage on which beneficial use is now being made. On other streams of interstate nature between Utah and Wyoming agreement has been had whereby water commissioners in the two States co-operate with each other in the administration of streams in accordance with priorities established. This co-ordidination allows fair and intelligent administration of these streams where formerly no competent system of regulation was operative between the two States.

Thus, in these and other ways, the States of Utah and Wyoming are working together in the solution of vexing interstate problems. Intelligent consideration is given to them, the physical facts on the ground are taken into consideration, and otherwise the matter is approached in a manner that, with little exception, provides for decisions and plans that are to the equitable and best interests of parties involved. It is believed that much has been and is being accomplished through these mutual agreements made possible through reciprocal statutory enactments by Utah and Wyoming.

Compact or Treaty Between States.—To this plan the paper of Messrs. Hinderlider and Meeker has been largely devoted and an excellent presentation of the subject has been given. The writer wishes to present certain comment in regard to the now famous proposed Colorado River Compact as an outstanding example along the line of solution by interstate compact of interstate water problems.

Through the Colorado River Compact, agreement was sought between the seven States of the Colorado River Basin concerning the division and use of the waters of the Colorado River and tributaries. No other stream in the arid West affects so many States as the Colorado River. Extensive additional uses of water, beyond the important uses now established, are proposed. Another element of material importance entering into the problem is that of the necessity of flood-control measures in order to relieve the menace of floods to the valleys on the lower river. Conflict of interests between States in relation to these and other phases of the Colorado River problem has led to the adoption of the treaty plan of solution. Without an agreement between States there is little question but that the river system would be involved in litigation for years in event any large additional developments were attempted on any section of the river. By treaty between States, such as would become effective on the ratification by all the States of the proposed Colorado River Compact, or a modified form of such compact that might be agreed upon, the way would be clear for all practical developments that may be proposed.

The Colorado River question is one that has been discussed at length at various times and at different places. It may not be amiss, however, to call

attention to the general physical situation which exists in the water-shed of this great river in view of the necessity of compact or agreement between States if the problems of the interstate use and division of the waters of the Colorado are to be properly solved to the best interests of all parties concerned. The speaker, therefore, takes the liberty of the following recital of physical facts which apply.

The Colorado River Basin in the United States naturally divides itself into two great basins separated by hundreds of miles of deep barren canyon cutting through high and rough plateaus. The Upper Basin embraces areas in the four States of Colorado, New Mexico, Utah, and Wyoming, and these States furnish about 85% of the flow of the river. Millions of acres of land are irrigable in the Upper Basin, and possibilities exist for large developments of hydro-electric power. Throughout the canyon region separating the two basins large power possibilities also exist, although it is impractical to divert water in quantities for irrigation. The Lower Basin comprises areas chiefly in the States of Arizona, California, and Nevada, supplying only about 15% of the water to the river, but which have extensive possibilities for the use of water for domestic, agricultural, and power purposes. The agricultural and economic conditions in the Upper and Lower Basins are entirely different. Development of the water supply for the benefit of the Lower Basin will be much more rapid than for the Upper Basin. Conflict of interest therefore arises between the two basins, and the logical plan of solution is an apportionment of water between them.

The Green River rising in the mountains of Wyoming is one of the most important tributaries of the Colorado River. The Little Snake is also a Wyoming tributary. These two streams drain about one-fifth the area of Wyoming, and furnish about 14% of the total flow of the Colorado River System. Present irrigation from these streams in Wyoming covers areas of about 435 000 acres. Additional irrigation possibilities to the extent of 765 000 acres are estimated. There is, therefore, a possible total development of 1 200 000 acres on Wyoming tributaries of the Colorado. The developments of the future will not be rapid, but will be none the less desirable as projects become feasible. Wyoming, therefore, has looked with much concern on the proposed large developments on the Lower Colorado River that would establish priorities to the use of water from the river that might well cause an embargo against future developments in this State.

Having its inception as an undertaking at the meeting of the League of the Southwest held in Denver, Colo., during August, 1920, the matter has gone far. Through the refusal of the State of Arizona to ratify the form of compact agreed to by its representatives on the Colorado River Commission, the Compact has thus far failed to become operative, although ratified in its original form by the other six interested States. California has apparently taken a backward step in the matter recently by attaching a reservation to its ratification of the Compact under the six-States plan, when the proposal was considered by the Legislature to make the Compact binding on the

States of California, Colorado, Nevada, New Mexico, Utah, and Wyoming, regardless of the failure of the State of Arizona to approve it.

However, the fact that necessary enabling legislation to allow consideration of the matter of the Compact was first passed by all seven States in the Colorado River Basin and by Congress; that the representatives of the seven States and of the United States who formed the membership of the Colorado River Commission could unanimously agree on a definite form of compact; and that this form of compact should be ratified and approved by the Legislatures of six of the seven interested States, is in itself a marked accomplishment, whether or not the Compact ever becomes operative in fact. Although falling short of the final mark the speaker believes that the terms of the Compact will be found in the future largely to govern the use and division of the waters of the Colorado River and, therefore, that much indeed has been accomplished by this effort to solve the interstate situation on this great river.

Having represented Wyoming on the Colorado River Commission, the speaker naturally is convinced of the merits and fairness of the present proposed form of compact. It is possible that certain modifications or amendments might be made to meet the objections of the present opponents of the Compact. However this may be, it is his firm conviction that some such agreement must be had between States before the way will be properly clear for meritorious development of the great water supply involved. It, therefore, behooves all parties to wipe out as far as possible those natural prejudices which exist between conflicting interests on the river and seek to learn as much as possible of the viewpoint of the other fellow and of his aims and possibilities. For the purpose of obtaining first-hand information, it was the speaker's pleasure to make a rather comprehensive trip over the Lower Colorado River when the question of the Compact first came up for consideration. The serious flood menace to the Imperial Valley and other sections on the lower river was made plain; conditions were noted which witnessed the need for conservation of water for irrigation, power, industrial, and other uses; and opportunity was given the speaker to inform himself generally concerning the situation in the Lower Basin warranting certain developments at an early date. He wonders whether many of the representatives of the States of the Lower Basin understand those conditions in the Upper States that warrant the insistent attitude for preservation of their natural resources that prompts those States to demand an equitable agreement in regard to the use of water from the Colorado River before their support can be given to lower-river developments. The speaker thinks not and, speaking for Wyoming, he wishes to make brief reference to facts concerning the State.

The average traveler passing through Southern Wyoming over the Union Pacific Railroad, on his journey across country, would obtain no real conception of the resources and possibilities of this great State. To give some idea of what may be expected from Wyoming Table 7, compiled from data from the U. S. Census reports of 1880, 1900, and 1920, is presented to show the remarkable increase in the agricultural and livestock industries of the State, in twenty-year periods, beginning with 1880.

The increase shown by Table 7 of from less than $6 000 000 in valuations in 1880 to more than $67 000 000 in 1900, and then to the comparatively large total of more than $334 000 000 in 1920, demonstrates that Wyoming is a rapidly growing State in relation to its agricultural development. This development is largely dependent on the great natural resource of water supply. It should not be handicapped by artificial restrictions that might come from early development on the Lower Colorado River. For many years, as has already been stated, Wyoming was seriously handicapped by the embargo placed on the developments from the North Platte River under the supposition that the water supply of that river was not adequate to take care of further extensions of irrigation in Wyoming. It was finally determined that the water supply of the North Platte was sufficient and that the embargo had indeed been suffered unnecessarily. No such repetition in relation to the water supply of the Green and Little Snake Rivers is desired as actually did obtain on the North Platte. Wyoming would welcome being placed in a position whereby support and aid could be given to all legitimate development for the benefit of the States in the Lower Basin of the Colorado River. In the present situation, however, the citizens of Wyoming cannot look with favor on large developments on the lower river and in just consideration of their own interests, their position must so remain as long as no agreement is in force to protect properly the future use of water in the Upper Basin. Litigation to solve the questions that are bound to arise in the future would certainly result in a waste of much time, energy, and money by all parties concerned. It is, therefore, to be sincerely hoped that amicable agreement between the States of the Colorado River Basin may finally be reached, whether through the proposed Colorado River Compact or through a modification of it.

TABLE 7.—Valuations of Farms and Farm Property.

Year.	Land, fences, and buildings.	Machinery and implements.	Livestock.
1880	$835 895	$95 482	$5 007 107
1900	26 965 530	1 366 000	39 145 871
1920	234 748 125	11 777 949	87 884 516

In the matter of interstate compact, engineers should feel called on to take a hearty interest. The political question of interstate relations seems to have received little attention. Why should engineers devote themselves so industriously to considerations of a technical character to the practical exclusion of the larger political questions on interstate streams that would handicap, if not entirely preclude, any large development of the water resources until the political phases of the matter receive intelligent attention and solution. It should be as much the duty of engineers to give their attention and assistance to the solution of the political questions involved by interstate problems as to the engineering phases. Until they take such attitude they will fall far short of the mark of the service that they are in position to render.

In conclusion, the speaker wishes again to voice both his hope and firm conviction that the best solutions of interstate water problems will come through intelligent conference and amicable agreement. Litigation means that decision will need to be made from a mass of testimony, generally conflicting in the extreme and well flavored with prejudice throughout. Solution through interstate agreements, worked out in harmony through the careful consideration of all facts and equities involved, means that justice will be done as far as humanly practicable to all parties in interest, and with the greatest saving possible in time and expense. Engineers as a body should give their attention to the political phases of interstate questions without the solution of which all technical devotions may be largely in vain. Let the profession take its place as a leader in the solution of these admittedly complicated and vexing problems, and it will certainly be the winner thereby.

D. C. Henny,* M. Am. Soc. C. E. (by letter).—The argument made by the authors in favor of amicable settlement of water disputes between States on interstate streams as against Court proceedings carries a strong appeal. Engineers familiar with trials of water cases in the Courts realize fully the difficulty inherent in this method of reaching practicable and just conclusions and will be the first to admit the desirability of substituting for Court proceedings some less cumbersome, rigid, time-consuming, and expensive method of dealing with water questions.

The progress made in recent years with interstate compacts, as recorded by the authors, adds force to the argument, whereas on the other hand the obstacles which the compact method has encountered expose certain weaknesses which deserve careful consideration.

States at the head of an interstate stream system hold a strong strategical position. As one of such States, Colorado granted and still grants to its citizens the right of diversion and storage limited only by prior rights within the State. Prior rights in lower States were not only disregarded in practice but were alleged by reason of their position to be legally subsequent.

The effect of increased diversions in Colorado on lower States was not injurious until, through irrigation expansion, shortage resulted and thus the controversy started and is still waging. The State of Colorado made full use of its strategical position and of the doubt as to interstate water rights and at least in one case has for a long time worked serious damage on one of the lower States.

Two of the lower States suffering or anticipating injury resorted to the Courts for redress, which gave rise to the famous Kansas-Colorado and the Wyoming-Colorado cases. Technically, of course, Kansas and Wyoming were plaintiffs in these suits, but it can hardly be pleaded from this that Colorado was not the real aggressor, as was done by the authors.

The money spent by Colorado on these cases, reported by the authors as totaling about $1 000 000, was intended to make good its sweeping claims to all water arising within its borders, and similar expenditures in lower States were incurred in defense of their prior claims.

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The earlier Kansas-Colorado suit resulted in a negative decision by the U. S. Supreme Court, as the Court found no injury had been suffered up to that time by Kansas and failed to settle respective State rights. It was not until June 5, 1922, that the highest Court gave its verdict on this main point of controversy and decided in favor of the lower State, Wyoming.

This decision brought about a radical change in the attitude of Colorado. When it found that lower States could find effective redress in the Courts, it began to point out the objections to Court proceedings and to advocate amicable negotiations and settlement by treaty, otherwise called the compact idea.

A compact, to be satisfactory and lasting, should be based not on the somewhat haphazard result of a clash between opposing extreme claims but on the primary willing adoption by all sides of the views expressed by the U. S. Supreme Court, which virtually wipes out State lines in the consideration of respective water rights, at least as regards States with substantially similar water laws and water practice. It should be approached in a mutual desire for fairness and in a truly friendly and co-operative spirit. Pending compact negotiations, no effort should be made by either side, directly or indirectly, to change radically the *status quo* as any such step would tend to destroy mutual confidence, without which no real progress in reaching a compact agreement can be expected.

The idea of central bureau administration is as obnoxious to most Westerners, including the writer, as it is to the authors. The growing tendency to demand too much Federal assistance is distinctly injurious to the much lauded pioneer spirit of the Western farmers. It is believed, therefore, to be for the best interests of the West that local control shall prevail as much as possible.

It should not be inferred from this, however, that decisions of the U. S. Supreme Court do more than declare what the law is. They do not as such interfere with local self-government. Resort to the Courts cannot be logically objected to as setting up some central bureaucratic authority.

In regard to river control, no matter whether rights are settled by the Court or adjusted by negotiation, it is well to bear in mind that a mere verdict, by whomsoever rendered, cannot execute itself, and of necessity will call for some regional authority to carry it into effect, especially as against the States which have the geographical advantage of being at the head of a stream.

Actual conditions of river flow as affected by flow from the head, side inflow, and diversion, present an ever-changing problem for division of waters. This problem is of sufficient complexity when confined within the limits of a single State, but if extended to an interstate stream system its treatment requires the highest administrative talent.

Additional diversions will affect the stream in a manner which cannot be definitely foretold. The upper State may claim that benefit through return flow late in the season will offset injury during the earlier part, and this claim may be denied by lower States and can only be definitely judged after the

event, dependent as it is on the extent to which drainage is provided and on other factors only partly known.

Actual administrative control, therefore, must be provided in any case, and such control will probably be local in character, possibly by boards on which interested States are represented through an administrative officer. Whatever solution may be found of this phase of the question, it is independent of whether the fundamental scheme of division of waters is the result of a verdict by the Court or by a compact commission.

A possible weakness in compact agreements is that their fairness depends on the wisdom of a number of men, generally appointed by the respective Governors of the States involved. In spite of the fact that appointments made in the past may have been of a high order, it is nevertheless conceivable that the exigencies of local politics may influence future appointments. It is, therefore, quite possible, if not probable, that representatives of States will be less trained to render impartial decisions than the judges of the Federal Courts, than whom as to previous training and impartiality no better men can be found to deal with difficult problems judiciously. Some lawyers also argue that the rigidity and circumscription of Court proceedings to which general objection is made and which are to be avoided in compact commission hearings, are nevertheless the result of vast experience and their irksomeness is offset by material advantages.

It may be inferred from the authors' arguments that the compact method eliminates Court proceedings. It is believed to be probable that such proceedings will be lessened. However, the States and even Congress have no power by legislative or other process to interfere with vested rights so long as the Fifth Amendment of the Constitution stands. Claimants in one State may have good reason to believe that their interests have not been properly safeguarded in the final compact, even if they have been given full opportunity to be heard before the Commission. They may be tempted, therefore, even against great odds, to obtain redress through the Courts.

The compact idea offers the greatest chance of success where only a few States are involved. It becomes more cumbersome as the number of interested States increases because any one State may fail to give the compact its legislative sanction and may make a strong effort also to prevent Congressional approval. This is understood to be the present status of the Colorado River Compact, Arizona being in this instance the objecting State.

There are cases in which conflict is not merely as between States, but in which the United States itself is an interested party. In most Western States there are Indian tribes living on reservations for whom the United States acts as guardian, whose prosperity is in part dependent on water rights often secured by treaty. By the very fact of their legal incompetency they need to be represented by their guardian and not by State representatives. Similarly, there are in every arid State reclamation projects, claiming water through filings made on their behalf by the United States, on which, therefore, should rest in large part the obligations of defense against illegal encroachments such as may unwittingly be committed by compact commissions. In addition, there are in almost every State great possibilities for further reclamation of public lands in which the entire nation has an interest.

The compact idea, therefore, would gain in attractiveness and be less subject to criticism and active opposition if it included in its program a composite commission on which the Department of the Interior, by which most of these Federal interests are administered, is directly represented by a commissioner.

The authors devote a few paragraphs to international river problems and refer to various treaties, among others to the treaty of 1906 between the United States and Mexico, dealing with the division between the two nations of the water in the Rio Grande above Fort Quitman, Texas.

This treaty provides for the delivery to Mexico of 60 000 acre-ft. per annum from the Elephant Butte Reservoir in New Mexico, built by the Reclamation Bureau in connection with its Rio Grande Project in New Mexico and Texas. The supply to the reservoir is seriously threatened by contemplated storage in Colorado and although the quantity assigned by the treaty to Mexico is small in relation to the full capacity of the reservoir, a series of dry years, such as has occurred in the past, may so deplete the reservoir as to render delivery of the stipulated quantity impossible. If this results merely from climatic conditions, the treaty itself holds the United States harmless, but if this condition can be proved to have been definitely aggravated by new diversions above, it is not unlikely that Mexico may demur and come forward with a large claim of damages as it did prior to the date of the treaty.

Thus, it would seem that in cases of this kind the United States is interested not merely through its Department of the Interior, but also through the Department of State, and that the latter also should be represented by a commissioner. Where this is not provided the compact idea will find opponents in land owners whose individual interests will be seriously jeopardized when at some time in the future the United States, through the Department of State, should be called on to take hold of the situation to make good its international obligations.

The objections to the compact idea as developed to date, carried out by State Commissioners under the chairmanship, at times, of some United States representative, are thus brought forward, not for the purpose of belittling its great possibilities, but to indicate its weak points and to explain why it has met with some well-founded opposition.

In order to be reasonably successful the compact idea should be conceived in a spirit of true fairness without any effort to take undue advantage; it should be carried through without permitting any change in the *status quo* as to main questions in dispute, after initiation of a proposed compact, so as not to destroy the feeling of mutual confidence which is essential to success; it should be handled by a commission on which all interests are fairly represented and the resultant decisions should be sufficiently elastic to permit the authority entrusted with their execution to deal with the ever-changing river conditions as they arise. It is believed that as to some of these points the foundation for arriving at compact agreements has not at all times been satisfactory.

M. C. Hinderlider* and R. I. Meeker,† Members, Am. Soc. C. E. (by letter).—In the brief period that has elapsed since this paper was presented, many events have emphasized the growing importance and the advantages of the treaty or compact method as a means of solution of interstate river problems. The Colorado River problem is still an open question, both interstate and international. Flood control problems of interstate rivers of New Mexico, Oklahoma, and Texas have been under discussion, and an agreement has been drafted to be submitted to the Legislatures of the three States. The South Platte River Compact between Nebraska and Colorado has been ratified by Congress and is in operation. The Lower Rio Grande situation between Mexico and the United States is pressing for solution, and efforts are being made to entangle this river problem with the international phase of the Colorado River. The proposed Colorado River Aqueduct for Los Angeles, Calif., and associated municipalities, has taken definite form. An interstate commission is studying the Snake River for a compact or treaty among the States of Wyoming, Idaho, Washington, and Oregon. The consummation of a North Platte River Compact between Colorado, Nebraska, and Wyoming is looked for shortly, and interstate litigation is threatened on the Rio Grande.

The discussion by Mr. Emerson is a substantial contribution to the subject and touches on one additional phase of interstate river conflicts not covered by the writers.

General concurrence in the treaty method as a sane means for solving interstate river controversies is expressed by Mr. Emerson. Wyoming first tried the litigation method of settlement, and, like Colorado, found that method less satisfactory than agreement through friendly negotiations; this, too, despite the Laramie River decision of the U. S. Supreme Court, which is improperly referred to by many as completely wiping out State lines and as a victory decision for the lower State. Unfortunately, this erroneous interpretation of the decision persists. In other words, some attorneys and engineers believe that the Laramie River decision can be applied in blanket form to interstate streams whereby all waters of a river basin may be administered as a unit, thereby resulting in a common administration of all priorities from the upper end of such river basin to the lower end, irrespective of State authority. Until this viewpoint is dispelled, many false assumptions will be injected into interstate river problems. Those who hold such views and look on the Laramie River decision as a panacea for interstate river problems should re-read and study the decision with care, and should read the decree as well as the opinion.

In the Wyoming case the Supreme Court did not declare that appropriations of individuals supersede State lines. When the opinion and decree are read together and are applied to the facts in that case, it is found that the Court decided that where each of two adjoining States recognizes the justice of the fundamental principles of prior appropriation in the distribution of water of its streams among its citizens, and a controversy arises

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between the two States respecting the use of the waters of a river common to both, the Court, as between the States, will apply the same general principle of justice in making an equitable distribution of the waters of the stream; and after determining the average dependable supply of the stream, will so allocate or apportion such average dependable supply between the States (not between individual appropriators in the different States) in such a manner as to furnish a reasonably dependable supply of water for the irrigation of the land already irrigated in each State, and will apportion the surplus between the States for proposed new projects. The Court, however, qualified even this general rule by certain conditions unknown to the law of appropriation, namely, that the burden is upon each State to use every reasonable means to conserve the common water supply and, in a case such as the Laramie River, that the entire burden of providing storage is placed upon the lower State; that the water will be allocated to the States in bulk for interstate distribution by State officials; and that the State of origin (where storage facilities are limited) shall be entitled to a preferential right of annual diversion, irrespective of water supply and irrespective of the fact that such diversion is by the most junior appropriator of the entire stream system, and that such preferential diversions may interfere with the supply of the most senior appropriators, located in the lower State.

One of the writers (Mr. Meeker) was the Hydraulic Engineer for the State of Wyoming in that case, and was surprised to find that by the much heralded decision of "priority regardless of State lines", the Court had, in fact, decreed to the Laramie-Poudre Tunnel enterprise an absolute right to divert a given quantity of water each year, regardless of river conditions and in the face of the fact that, by the opinion, the tunnel enterprise was found to be the junior appropriator in both States.

When it is considered that this decision places the burden of river regulation and water storage entirely upon the lower State for the benefit of junior appropriators in an upper State, and when the other phases of the decision are considered in the light of the facts, there is little difference between the principles applied in the Laramie River case and in the Kansas-Colorado decision. In both cases the Supreme Court equitably apportioned the waters of the river between the two States in a manner conforming to the fundamental principles of justice recognized within each State.

The Laramie River conflict was a concrete interstate water problem which did not contain all the seeds of conflict embraced in interstate issues of larger and longer rivers like the Colorado, North Platte, Rio Grande, and other major interstate river controversies. The decision, therefore, will have only a limited application to the larger interstate river problems, which, if litigated for solution, must rest on some additional features of interstate river uses.

Considerable space is devoted by Mr. Emerson to the North Platte River embargo imposed by the Federal Government, which resulted in the intolerable Pathfinder situation created in the North Platte Basin in Wyoming whereby "Carey Act segregations were refused, desert land entries were denied,

rights of way across Government land were not granted, and, in other ways, developments which proposed a use of water in Wyoming from the North Platte River and its tributaries were blocked by the Federal Government" due to the supposition or fear of an inadequate water supply.

The North Platte embargo included also that portion of the North Platte Basin in Colorado known as "North Park" where the river has its source. Irrigation development in that area was also effectively withheld for a period of twenty years on the erroneous theory of insufficient water supply for Pathfinder Reservoir (capacity, 1 070 000 acre-ft.). Applications for rights of way for canals and reservoirs (of more than 1 000 acre-ft annual use) in Colorado over Government land were likewise refused. Construction of canals and reservoirs in North Park was stopped and, after protracted delays, this resulted in the destruction of the enterprises and total loss to the claimants. On one Carey Act project that was involved about twenty miles of canal actually had been constructed. Engineering studies of the water supply of the North Platte Basin, including studies of water requirements of irrigated and irrigable lands, consumptive use, return flow, and drainage recovery, show that the North Platte embargo was imposed without adequate knowledge of the water supply. The pathetic feature of the whole embargo program, however, is that the action of the Government was illegal and that private development was prevented or destroyed without legal or moral rights.

A parallel case occurred on the Rio Grande in Southern Colorado and Northern New Mexico, where a similar interstate river embargo, declared in 1907, was laid against the construction of reservoirs and canals involving the use of water in excess of 1 000 acre-ft. per year, on the erroneous theory that larger projects might interfere with the water supply of the Elephant Butte Reservoir (capacity, 2 638 000 acre-ft.), and with the treaty obligation of a mere 60 000 acre-ft. per year to Mexico. The Rio Grande embargo has been just as intolerable to Colorado as the North Platte embargo was to Wyoming, for needed reservoir development was prevented and projects were ultimately abandoned.

In May, 1925, the Rio Grande embargo was declared to be, and admitted to have always been, illegal and was cancelled by the Secretary of the Interior. Comprehensive engineering studies of water requirements and water consumption of the Rio Grande Basin in Colorado, New Mexico, and Texas (above Fort Quitman, 80 miles below El Paso), have also shown that the Rio Grande embargo was unwarranted. Surely projects in Colorado cannot be prejudiced in their relations with other lower river enterprises by the effects of an illegal embargo which constituted a cause of delay beyond their control.

The North Platte and Rio Grande embargoes have withheld development in four States for a period of twenty years. Although fathered at Washington and imposed by a Federal Bureau, they have been in fact measures favoring Lower Basin States, since they have effectively neutralized the strategic head-water position of Colorado and Wyoming on the North Platte, and Colorado and New Mexico on the Rio Grande.

In his excellent discussion Mr. Henny points out a few of the many problems that have confronted those who have pioneered the field of settlement of interstate water problems. His suggestions are timely and valuable in developing the subject-matter on a broad basis.

It so happened that of the many Western States which will ultimately face the same problems, Colorado, by reason of her more rapid development, was the first to meet assertions of adverse water claims on behalf of neighboring States. Colorado was the first to be attacked and the first to be forced to defend. Many have entertained an erroneous view of the issues in both the Kansas-Colorado and the Wyoming-Colorado cases, and particularly the views of those who formulated the written pleadings in those cases. It is a rule well established that in all litigation each side asserts its most extreme doctrines and brings forward its maximum claims, and those who are prone to criticize the attitude of Colorado, as expressed in the pleadings in that case, should withhold such criticism until they have acquainted themselves with the extreme assertions set forth in the bills of complaint. That is particularly true in fairness to one of the States which brought suit against Colorado and now finds itself confronted with a situation similar to that occupied by Colorado, and so presents the same arguments advanced by Colorado in both the cases mentioned.

Unconsciously, Mr. Henny does injustice to Colorado and her people in his statement that the decision in the Wyoming case caused "a radical change in the attitude of Colorado", and that not until after that decision did Colorado begin to "point out the objections to Court proceedings and to advocate amicable negotiations and settlement by treaty, otherwise called the compact idea." There has been no change in Colorado's attitude.

The decision in the Wyoming case was announced June 5, 1922. The interstate treaty method of settlement of interstate water controversies was initiated in 1916; it was advocated and had been in process of development since 1911, by one of the attorneys who represented Colorado in the Wyoming case. The proceedings which resulted in the compact between Colorado and Nebraska concluded April 27, 1923, were commenced in 1916, and were authorized by the Legislature of Colorado in 1921, more than a year prior to the decision in the Wyoming case. The resolution adopted by representatives of the Colorado River States at Denver, Colo., on August 27, 1920, calling for the formation of the Colorado River Commission to negotiate the Colorado River Compact, was suggested by Colorado and drawn by the Colorado representative. The Colorado Legislature, with the Legislatures of the six other Colorado River States, authorized the formation of the Commission during the early part of 1921, and the Commissioner for each State had been appointed prior to May 10, 1921, when the Governors of the seven States met at Denver and formulated resolutions addressed to the President and the Congress requesting action by Congress for participation by the United States in the conclusion of the interstate compact. The Act of Congress was approved August 19, 1921, and Mr. Hoover was appointed as a Federal representative shortly thereafter. The Colorado River Commission had organ-

ized, had concluded public hearings, and was prepared to reconvene in executive and final session prior to the decision in the Wyoming case. The Colorado and New Mexico Legislatures authorized the La Plata River Commission in 1921, and the Commission was appointed and its deliberations were largely concluded during that year. The 1921 Session of the Colorado Legislature also authorized the appointment of a commissioner for Colorado to join with a similar commissioner for Kansas in the settlement of the pending interstate litigation on the Arkansas River, and authorized a similar commission to conclude an interstate compact between Colorado and Wyoming with respect to the waters of the Laramie River. The settlement of interstate river controversies by interstate compact was considered and discussed in the brief filed by the Colorado attorneys in the Wyoming case, and the methods of procedure in such matters had been under discussion since 1912.

Suggestion of the use of the treaty-making powers of the States, by consent of Congress, in the settlement of controversies respecting interstate rivers, was prompted by the early recognition of the futility of settling such matters by litigation, save as a last resort, and was earlier suggested by Colorado by reason of the fact that Colorado happened to be the first of the Western States with which the interstate water issue was drawn. Most of the Western States are in more or less the same position as that occupied by Colorado; even by 1920, they were not only urging the same arguments of State sovereignty theretofore advanced by Colorado, but were recognizing the necessity of prompt and effective measures in the settlement of title to interstate rivers as a preliminary to further major construction on which might be predicated monopolistic claims of "vested rights". The injustice of subjecting rights of States to the judgment of masters appointed by Federal Courts, wholly or largely uninformed of the subject matter litigated, recommended the treaty method of settlement and brought about the prompt action resulting in the conclusion of the Colorado River, La Plata River, and South Platte River Compacts.

Apparently from his discussion Mr. Henny entertains the view that so-called "vested rights" of individuals exist in rivers, which rights are so protected by the Fifth Amendment to the Constitution that both Congress and the State Legislatures are powerless to provide for the equitable distribution of the use of the waters of interstate streams between the interested States—in other words, that the rights of a user of water in one State are greater than the rights of his State or of a neighboring State. The writers fail to comprehend how the rights of an appropriator in any State can be any greater than the rights of his State, the source of his title. The rights of his State may be limited by the rights of neighboring States, as determined either by interstate compact or by decision of the U. S. Supreme Court in a controversy between the two interested States, and certainly his rights would be defined by the limitations placed on the rights of his State. In other words, the rights of the State's grantee—the water user— at all times are subject to whatever limitations may be defined with respect to the relative rights of his State either by compact or Supreme Court decision. The grantee

(the appropriator) can have no greater right than his State has power to convey. To urge that a water user has a right greater than that of his State, and derives his title from a source of power foreign to, independent of, and superior to, the jurisdiction of his State, is to challenge the autonomy of the State and to declare that each of the Western States was not admitted on an equality with the other States. It is true, there are advocates of the doctrine that all water rights are derived, not from the State, but from the United States, and that the disposition of the waters of the Western streams, and, perforce, of all the streams of all States admitted to the Union since the adoption of the Constitution, is in the keeping of Congress and wholly removed from State control save as permitted by Congress. Only by this doctrine could an appropriator pretend to claim rights superior to those of his State, or immune from the terms of any compact to which his State is a party and which shall have received the sanction of Congress. The writers hold Mr. Henny's opinions in too great respect to believe that his suggestions are founded on a doctrine so abhorrent to, and destructive of, the fundamental principles of the union of equal States of equal power which compose the Nation.

Suggestion is made of the need of a Federal representative on a compact commission to protect treaty rights of the Indians; also concerning the desirability of a representative from the Department of the Interior to protect inchoate water rights of Federal Reclamation projects.

Mr. Henny seems to overlook the fact that the negotiations involve the rights of sovereignties and are similar to international proceedings. The protection and safeguarding of such water rights are properly focused through the Federal representative who may have as many assistants and advisers as he desires. In negotiations of the Colorado River Compact the Federal Commissioner was not a representative of any Federal Bureau, but was a Presidential appointee selected for that specific task and was assisted by special counsel and engineers for the United States representing the Government projects, Indian obligations, international relations, navigation, and any other interest the Government may claim under the Constitution.

In any interstate treaty respecting the use of the waters in interstate rivers, the duties imposed on the river by international treaties, rights of control of navigation, and other rights in the stream predicated on the surrender of powers by the States to the Federal Government, must be respected and, preferably, by representation on the Commission as well as by the action of Congress in approving the interstate compact. However, these rights are represented by different Federal executive branches. International rights are represented by the Department of State; rights of navigation by the Department of War; and Indian rights by the Department of the Interior. National reclamation, which by the terms of the Reclamation Act, as well as by the debates of Congress prior to the adoption of that Act, is at best but Federal aid of State development, and in which under the Act all proceedings must be in accordance with the laws of the States where the projects are constructed, is, for the time being, in the control of the Department of

the Interior. Any of these several branches of Federal interest may be transferred from one Department to another by mere Act of Congress, and may be changed about from time to time and as frequently as Congress may determine.

It would appear that while it is advisable that a Federal representative participate in the work of any interstate river commission, such representative should not be from any one Department, or sub-branch, but should be one specially advised on the problems involved, representing the United States by Presidential appointment, and not merely some executive branch of Federal activities. This is well illustrated by the fact that while irrigation is at present uppermost in the minds of the American public, it probably will be overshadowed shortly by the development of electrical energy which is in charge of a separate Executive Department under the control of the Secretaries of War, Agriculture, and the Interior, while the Forestry Department, controlling the greater part of the National public lands, is a branch of the Department of Agriculture. It would appear that the promotion of interstate peace and comity is of sufficient importance and dignity to justify the appointment of a special representative by the United States to participate in the deliberations of each interstate river commission where Federal interests are in any manner involved. The negotiations are between sovereignties, each independent in its own sphere. It is hardly sufficient that a representative of some Bureau, or the substitute of some Executive Department, be "handy man" in such matters.

It has been suggested that some regional authority is required on an interstate river to execute either a Court decision or a compact. This may be desirable for some of the larger unsolved interstate river problems, but is not always necessary. No regional authority was designated in the Laramie River decision by the U. S. Supreme Court and none is required. The State Engineers of Colorado and Wyoming designated as the administrative authorities in control of State waters, naturally have charge of the execution of the Court decisions pertaining to water. In the La Plata and South Platte Compacts provisions are made for the State Engineers jointly to supervise and administer the compact obligations. The Colorado River Compact likewise provides for some administration supervision by the respective State Engineers, the Director of the U. S. Reclamation Service, and the Director of the U. S. Geological Survey.

Concerning the statement that the compact method will not eliminate Court proceedings but may only lessen litigation—while isolated cases may occur which have to follow the legal route of solution, the writers believe that compact or treaty solutions will generally and practically eliminate Court proceedings.

A criticism has been offered to the effect that the compact method is cumbersome where six or seven States are involved on an interstate river. This is more true of litigation, for litigation compounds its delays and costs directly in proportion to the number of States involved. If eleven years are required for a Court decision in a two-State dispute, how many years would

very likely be required to litigate a six or seven-State river dispute with Federal Reclamation projects and a foreign nation involved?

Mr. Henny refers to the Rio Grande international situation concerning the treaty with Mexico and the yearly obligation of 60 000 acre-ft., and comments as follows:

"The supply of the reservoir [Elephant Butte] is seriously threatened by contemplated storage in Colorado, and although the quantity assigned by the treaty with Mexico is small in relation to the full capacity of the reservoir, a series of dry years, such as has occurred in the past, may so deplete the reservoir as to render delivery of the stipulated quantity impossible."

The writers believe the latter view scarcely probable and impossible of substantiation. The international obligation is less than 3% of the average annual water production of the Rio Grande Basin above El Paso, and less than 8% of the average yearly flow of the Rio Grande originating in New Mexico above San Marcial at the head of the Elephant Butte Reservoir. The Elephant Butte Reservoir, with a capacity of 2 600 000 acre-ft., is the largest reservoir in the United States, and the capacity is two and one-half times the average yearly flow at San Marcial. However, it is a well-known fact that the mere seepage and return waters from irrigation of lands between Elephant Butte and El Paso are far more than adequate to insure delivery of water under the International Treaty with Mexico, irrespective of drought on the head-waters of the Rio Grande. It appears that, in the effort to make the water supply for the Elephant Butte doubly secure, at the expense of needed reclamation in the upper States, the insignificant quantity guaranteed Mexico under the International Treaty has been overworked until the situation is much like the proverbial tail wagging the dog. Going a step farther, the water supply of the Elephant Butte Reservoir is not seriously threatened by the proposed projects on the Middle Rio Grande Valley at Albuquerque, on the Puerco, on the Santa Cruz near Espanola (all in New Mexico), or by the proposed reservoir construction on the head-waters in Colorado. Practically all these proposed developments are to aid lands now irrigated, or to make possible the drainage of lands now water-logged and wasting large quantities of water through evaporation. Such lands were irrigated before the Elephant Butte Reservoir was contemplated. Drainage recovery of water now wasted by evaporation will be equivalent to an increased water supply in the river.

The upper river structures in both New Mexico and Colorado will assure drainage of the upper irrigated areas, will regulate river flows, will furnish clear water to the Middle Basin, and will assist in decreasing the silt menace with which the Elephant Butte Reservoir is confronted, namely, about 20 000 acre-ft. of silt deposition per year.

When considering interstate river problems, engineers have been prone to overlook certain fundamental jurisdictional phases. Among these are: That the question arises between States, possessing all the sovereignty of independent nations, except for those powers and that limited jurisdiction surrendered by each State to the United States; that principles of international law are involved; that claims of foreign servitude for the benefit of one State

are made over the territory of another State, without its consent; that such claims are abhorrent to principles of international law; that assertions of right are made which, if sustained, would destroy the equality of the States and the autonomy of one State for the exclusive benefit of another State (or its citizens) without the consent of the former and without compensation for the property taken; that water is necessary to the life and preservation of each of the arid States, and the right to its use must be protected in justifiable self-defense; that the injection of principles of Federal control or interference with State streams under assumption of powers not granted by the Constitution or predicated on theories calling for conclusions contrary to the intent of the Constitution, whether prompted by motives of temporary expediency or desire to over-ride State jurisdiction, only tend to destroy interstate comity and to bring the States to a position of resistance; that all doctrines of water law are, after all, but rules of local administration by which a sovereign—the State—regulates the use of its natural resources among its nationals, and are not applicable to interstate problems because of lack of jurisdiction to create machinery to enforce such rules of administration; and that to attempt to effect such administration would be wholly to supersede all State authority over a resource which is essential to the preservation of the State and which must ever remain subject to local eminent domain in order that the State may re-adjust the use of its life-giving fluid to the ever-changing necessities of its people.

When these and many other factors are considered in their true light, and with a desire to protect the autonomy of the States for the necessary preservation of the Nation, which is but a Union of otherwise independent States, the whole subject is taken out of the plane of ordinary local rules and thought, and must be approached from a much broader standpoint than the mere protection of selfish individual property rights within a State. This the interstate compact method seeks to accomplish by bringing about contractual relations between sovereignties with full realization of the sanctity of treaties and the necessity of their fulfillment without the interposition of external annoyances and intermeddling, at all times having due regard to the rights granted the United States by the Constitution and the protection of these rights by the contracting States.

AMERICAN SOCIETY OF CIVIL ENGINEERS
INSTITUTED 1852

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Paper No. 1620

DETERMINATION OF THE DUTY OF WATER IN WATER-RIGHT ADJUDICATIONS*

REPORT OF THE DUTY OF WATER COMMITTEE OF THE IRRIGATION DIVISION

WITH DISCUSSION BY MESSRS. E. A. PORTER AND LLOYD GARRISON

SYNOPSIS

This report discusses features of water adjudications of interest to engineers. The principles on which the duty of water is defined as indicated by Court decisions are analyzed. The character of evidence likely to be most useful in assisting the adjudicating agencies in such determinations is discussed. Illustrations of results and practices in existing decrees are included. Suggestions are made as to desirable practices in such determinations.

DETERMINATION OF THE DUTY OF WATER IN WATER-RIGHT ADJUDICATIONS

The adjudication of a water right for irrigation acquired by appropriation includes the determination of the priority of the right, the place of use, and the quantity or extent of the right. Priority and place of use are mainly questions of fact and although important are outside the immediate interest of this discussion. The determination of the quantity of water represented by the right, the rate of diversion which may be found to represent beneficial use, or the duty of water as it is frequently called, is a matter in which engineers participating in such adjudications are directly concerned.

Under the various administrative and judicial procedures now in use in the different Western States, adjudications of water rights may be made by State

* This report represents a part of the results of the work undertaken by the Duty of Water Committee of the Irrigation Division. It was presented, in somewhat condensed form, by the Chairman of the Committee, S. T. Harding, M. Am. Soc. C. E., at the meeting of the Irrigation Division at Salt Lake City, Utah, July 9, 1925.

administrative officers, such as the State Engineer; by the Courts, either State or Federal; or by a combination of State officers and Courts. Engineers come into contact with such determinations both from the point of view of those engaged in the conduct of the proceedings in those States where the determinations may be wholly or partly in the hands of the engineering officers of the State, or as representatives of individual claimants in all forms of determinations. All engineers having to do with irrigation development or operation are concerned in the basis of the procedure which may be followed and the effectiveness of the results which may be obtained, as these results determine the ability of all canals concerned to secure the water supply necessary for their successful operation with a minimum of controversy and uncertainty. Engineers are also interested directly in the procedure for the determination of the extent of the rights, as it is on this question that they may most often be asked to present evidence.

The quantity of water which may represent a reasonable standard of beneficial use is a question of opinion rather than of law. The character of evidence which may influence such determinations, the standards of practice to be followed, and the terms and form of the resulting decree are all matters of engineering interest.

The following discussion attempts to analyze the present practices in such duty-of-water determinations. It is based on the results of an inquiry in which data were secured on the decrees now in effect on about thirty-five streams and also on a study of Court decisions in an effort to secure information on the principles followed by the Courts. Opinions were sought from those responsible for the administration of present decrees on the experience secured from their operation. The Committee is under grateful obligation to the large number of those who have assisted it in the collection of these data.

Standards of Practice Used in Determinations of Beneficial Use

There are different standards that may be adopted to define the extent of use that is considered beneficial. Among these are:

(1) Claims in original notice or in applications;
(2) Capacity of constructed works; and
(3) Requirements of lands served.

(*1*).—*Claims in Original Notice or in Applications.*—Appropriation rights were initiated in the earlier periods of development of the Western States by posting a notice of the appropriators' intention at the point of diversion and recording the notice in some county office. This has been changed in most States so that a formal application must be made to some State officer who has a varying amount of jurisdiction over the application during the period of completion. Wyoming adopted a centralized system of application in 1890 and all the remaining States in which irrigation is of importance have since adopted similar systems except Montana, which still follows the former method. The claims to water rights of a great many systems now in use were initiated by the posting of such a notice of intention.

Such notices were required to include a statement of the quantity of water claimed. As there were no limitations on the extent of such claims, it was

natural that the appropriators should state a quantity amply liberal to cover any probable needs. This practice is so well recognized that little weight would ordinarily be given to the quantity claimed in the notice. The notices are useful as evidence of the date of initiation, of the general character of the works contemplated, and of the location of the point of diversion. (Hufford *v.* Dye, 121 Pac. 400 (California, 1912).)

(*2*).—*Capacity of Constructed Works.*—The capacity of the constructed works is a more definite guide to the quantity required than the claim in the notice as it represents the extent to which the appropriator was willing to incur costs in providing for his needs. However, as many early diversions were constructed with less definite understanding of the carrying capacities of canals or the water requirements of lands than is available at present, the carrying capacity will not usually be accepted as controlling against evidence of the actual requirements. If other standards result in a larger use than the capacity of the works, the capacity will usually define the limit of the right. (Conrow *v.* Huffine, 138 Pac. 1094 (Montana, 1914); Nichols *v.* Hufford, 133 Pac. 1084 (Wyoming, 1913); Felsenthal *v.* Warring, 180 Pac. 67 (California, 1919); Northern California Power Co. *v.* Flood, 199 Pac. 315 (California, 1921); Hough *v.* Porter, 98 Pac. 1085 (Oregon, 1909).)

(*3*).—*Requirements of Lands Served.*—The requirements of the lands served represent the usual present standard by which an appropriation right for irrigation is defined. There is room for much difference of opinion in regard to the character of practice that should be used in defining such requirements and also in expressing any standard in numerical terms for any specific diversion. Among the standards which may be used are the following: (1) Rates of use specified by State statutes; (2) earlier local or individual practice; and (3) present practice. The terms used are necessarily general.

Statutory Limitations.—Several States have statutes specifying the maximum rate of use which will be considered beneficial. Wyoming was the first State to adopt such a provision, diversions being limited to not to exceed 1 sec-ft. for each 70 acres irrigated. Other States have similar statutes with the specified rates varying from 50 to 100 acres served per second-foot diverted. Some States, in addition, specify the maximum number of acre-feet per acre per season. In some States in which there are no statutes on this point, a similar result is secured by the regulations of the State office having supervision of appropriation.

These provisions are an expression of legislative opinion regarding the rates of use which, in general, would be beneficial. They do not prevent a claimant from receiving larger quantities if he can present proof of a need for such larger use, and there are a number of cases in which quantities in excess of such statutory limitations have been allowed. In fixing a rate of use to apply to a State as a whole, the Legislature would naturally select one sufficiently liberal to cover all usual conditions. This rate would exceed the quantity needed under more favorable conditions and a smaller rate of use would be justified in such cases. (Joyce *v.* Rubin, 130 Pac. 793 (Idaho, 1913); Conrow *v.* Huffine, 138 Pac. 1094 (Montana, 1914); Nichols *v.* Hufford, 133 Pac. 1084 (Wyoming, 1913); Hedges *v.* Riddle, 146 Pac. 99 (Oregon, 1915).)

Earlier Local or Individual Practice.—To use past practice as the basis of the extent of a right is practically equivalent, for developed systems, to accepting the constructed capacity as measuring the requirements. It is well recognized that in the earlier periods of development the use of water is less economical than in later periods. Although such excessive use may have been permitted in such earlier periods, the Courts do not consider themselves bound to define the rights on the basis of past use where wastefulness can be shown. This principle appears to be generally accepted by the Courts although there may be much difference of opinion in individual cases as to what constitutes a sufficient showing of wasteful use to justify a decree less than former diversions. The lay and expert views on waste frequently differ.

It appears to be equally accepted that each claimant is entitled to have his right defined on the basis of the reasonable needs for the character of practice under which the right was acquired, provided such practice is within the limits of beneficial use. An appropriator irrigating wild hay may be limited to a proper use for such a crop, but will not be limited to the quantities required for other crops of less requirements because other claimants are irrigating such crops.

Cases supporting these conclusions are, as follows: Hufford *v.* Dye, 121 Pac. 400 (California, 1912); Calif. Pastoral & Agricultural Co. *v.* Madera Canal & Irrigation Co., 138 Pac. 718 (California, 1914); Union Colonization Co. *v.* Madera Canal & Irrigation Co., 178 Pac. 957 (California, 1919); Felsenthal *v.* Warring, 180 Pac. 67 (California, 1919); Stinson Canal & Irrigation Co. *v.* Lemoore Canal & Irrigation Co., 188 Pac. 77 (California, 1919); Northern California Power Co. *v.* Flood, 199 Pac. 315 (California, 1921); Oliver *v.* Robnett, 210 Pac. 408 (California, 1922); Farmers Co-operative Ditch Co. *v.* Riverside Irrigation District, 102 Pac. 481 (Idaho 1909); Washington State Sugar Co. *v.* Goodrich, 147 Pac. 1073 (Idaho, 1915); Conrow *v.* Huffine, 138 Pac. 1094 (Montana, 1914); Rodgers *v.* Pitt, 89 Fed. 420 (Nevada, 1904); Hough *v.* Porter, 98 Pac. 1085 (Oregon, 1909); Little Walla Irrigation Union *v.* Finis Irrigation Co., 124 Pac. 666 (Oregon, 1912); *in re* Willow Creek, 144 Pac. 505 (Oregon, 1914); Salt Lake City *v.* Gardner, 114 Pac. 147 (Utah, 1911); U. S. *v.* Bennett, 207 Fed. 524 (Washington, 1913); and Nichols *v.* Hufford, 133 Pac. 1084 (Wyoming, 1913).

Present Practice.—Many decisions contain such phrases as "an amount actually necessary for his use"; "a sufficient amount of water to irrigate the land in a proper manner"; "the highest and greatest possible duty from the waters of the State"; "the largest duty and the greatest use"; "the amount actually necessary"; "the inquiry was therefore not what he had used, but how much was actually necessary"; "we can require them only to use the water economically and reduce the quantity to a minimum by reasonable and cheap methods according to their situation and condition"; and, "reference should always be had to lands that have been prepared and reduced to a reasonably good condition for irrigation."

It is difficult to express in definite terms the standard of practice which is represented by a decree, and the following conclusions are necessarily general.

What may be termed present local use or practice appears to have been the factor of greatest influence. This does not mean that each canal will be decreed a quantity based entirely on its present practice, but that considering all canals concerned the adjudicating bodies are unwilling to fix the decrees on any basis that will require material changes in present methods. Where variations in use by different ditches serving similar areas are shown, those using larger quantities may be limited to the quantities used by others.

Actual practice on each canal may be guiding unless evidence can be presented which demonstrates that better results are actually being secured voluntarily on some canals within the area. The standards of practice used in decrees are usually more liberal in the quantities allowed than would be required in order to make the canal systems feasible. A new project in the same area may be feasible with a water supply smaller than that to which the decrees may limit the older canals. Users under new canals may be willing to practice methods of irrigation in order to use successfully much smaller supplies than the Courts are willing to require of those who have developed their systems under conditions of more liberal supply and use. Of the decrees on which data have been obtained no instance was found in which such decrees are based on a higher standard than would be regarded as reasonable for present local practice. The decrees appear to follow rather than to lead in improvements in irrigation practice and the quantities decreed appear to have been reduced from those previously used only where strong proof could be submitted to show that many irrigators have adopted such improvements voluntarily and that the resulting practice has come to represent a reasonable standard for all users. This position on the part of the determining bodies is a natural one as the burden of proof of the reasonableness of any reduction in present practice should rest on those who would benefit by such reduction. The effort to secure a higher standard of practice is usually made by the later priorities who would benefit by such a standard.

Among decisions including such principles are the following: State of Colorado *v.* State of Wyoming, 42 Sup. Ct. 552; Union Colonization Co. *v.* Madera Canal & Irrigation Co., 178 Pac. 957 (California, 1919); Farmers Co-operative Co. *v.* Riverside Irrigation District, 102 Pac. 481 (Idaho, 1909); Washington State Sugar Co. *v.* Goodrich, 147 Pac. 1073 (Idaho, 1915); Beasley *v.* Engstrom, 168 Pac. 1145 (Idaho, 1917); Vineyard Land & Stock Co. *v.* Twin Falls Salmon River Land & Water Co., 245 Fed. 22 (Idaho, 1917); Muir *v.* Allison, 191 Pac. 206 (Idaho, 1920); Conrow *v.* Huffine, 138 Pac. 1094 (Montana, 1914); Rodgers *v.* Pitt, 129 Fed. 932 (Nevada, 1904); Hough *v.* Porter, 98 Pac. 1085 (Oregon, 1909); Little Walla Irrigation Union *v.* Finis Irrigation Co., 124 Pac. 666 (Oregon, 1912); *in re* Willow Creek, 144 Pac. 505 (Oregon, 1914); Salt Lake City *v.* Gardner, 114 Pac. 147 (Utah, 1911); and U. S. *v.* Bennett, 207 Fed. 524 (Washington, 1913).

Factors Affecting the Duty of Water

The more important factors affecting the duty of water are the soil, character of crops, climate, preparation of land, cost of water, and the skill and attention used by those actually applying the water. Of these, climate and

character of crops, although varying over large areas, are usually fairly constant within the areas covered by any adjudication. The preparation of land and the attention given to the application of water vary with the individual farm. The soil may vary widely within the area covered by an adjudication. It is not subject to material change and is not within the control of the individual user. In consequence, the soil is the variable factor which is most often considered in fixing differences in rates of use within any area.

Of the adjudications on which data were secured, about one-half were based on some differences in the duty of water within the area covered and one-half made a uniform allowance. In some cases variations as great as 100% were made where conditions differed materially; the variations used were based mainly on soil conditions rather than on crops.

Soils.—The texture of the soil will materially affect irrigation practice both as to the frequency of application, the quantity applied at each irrigation, and the resulting total use. If it is conceded that the use of water does vary with the soil texture and that account of such variations should be taken in fixing the duty of water, the rates of use for different areas included in a determination will need to be variable if such soil variations occur. The soils in nearly all irrigated areas are sufficiently variable to warrant taking account of such differences.

The uncertainty regarding the practicability of making a soil classification of sufficiently definite character to justify varying the rate of use based on such a classification appears to have prevented a more complete consideration of soil texture. Although soil texture has been considered in some cases, the tendency appears to have been to allow an additional quantity of use to soils of coarse texture or to allow enough use to all soils to cover the needs of the more porous types. To reduce the rate of use on soils of more favorable texture is just as logical as to increase the use on those of less favorable character. A practical limit must be set in any effort to base rates of use on a soil classification, but even in areas of considerable mixture of soils a practical classification can be made which will be more nearly representative of actual needs than might be obtained on the basis of generalities only.

Cases in which differences in rates of use due to differences in soils have been upheld are, as follows: Washington State Sugar Co. *v.* Goodrich, 147 Pac. 1073 (Idaho, 1915); Joyce *v.* Rubin, 130 Pac. 793 (Idaho, 1913); Rodgers *v.* Pitt, 129 Fed. 932 (Nevada, 1904); *in re* Waters of Umatilla River, 168 Pac. 922 (Oregon, 1917).

The determinations of the Division of Water Rights of California have varied from a diversion of 1 sec-ft. for 20 acres on Hat Creek for soils considered particularly difficult to handle to 1 sec-ft. for 90 acres on heavy soils on Oak Creek. For coarse soils on the upper areas of Oak Creek, 1 sec-ft. to 25 acres was decreed, the rights of all users being accepted by stipulation. On the largest stream adjudicated—Stanislaus River—the decrees for the smaller ditches serving mountain areas generally gave 1 sec-ft. to 40 acres, whereas those of the Oakdale and the South San Joaquin Irrigation Districts in the San Joaquin Valley gave 1 sec-ft. to 80 acres.

The Salt River decree in Arizona, in which the rights of about 4 800 owners were separately defined in terms of the irrigable area of each owner, was based on a uniform delivery equivalent to 1 sec-ft. for 133 acres.

In Idaho, 1 miner's inch per acre was used for bench-lands and 1.1 miner's inches per acre for bottom-lands on the Boise River, and 1 to 5 miner's inches per acre for different areas on Big Wood River. In Idaho, 50 miner's inches are equal to 1 sec-ft.

In Montana, in the West Gallatin decree, the rates varied from 0.75 miner's inch per acre on heavy black loam to 1.25 miner's inches per acre on gravelly land. On the smaller streams of the State different rates of use have not been general, many such decrees being based on 1 miner's inch per acre. In Montana, 40 miner's inches are equal to 1 sec-ft.

The decision of the Special Master of the Federal Court in the adjudication of Truckee River in Nevada (still pending) decrees different rates of use to different areas based at least partly on difference in soil conditions.

In Oregon, rates of use of 2.0 to 3.0 acre-ft. per acre were decreed for different areas on the Deschutes River and 1 sec-ft. for 40 to 80 acres, or 3 to 6 acre-ft. per acre, for different soil conditions on the Umatilla River.

The quantities given apply to the quantities diverted unless otherwise stated.

Crops.—The crops within any area are subject to variation both over longer cycles of years and temporarily due to changing economic conditions. The more complete development of an area usually results in greater diversity of crops which, in turn, tends to reduce the average quantity of water used, as the crops first grown are generally ones of larger water requirement, such as forage. This is not the case, however, in areas where grain is largely grown in the earlier years. General crop changes occur gradually; temporary changes may be rapid, such as the growth in single seasons of large areas of some crop for which the prospective price is attractive. Where rights have been acquired based on the irrigation of one class of crops and it is proposed to change the crops so as to result in a material change in the time of use or an increase in the quantity used, such changes may be opposed by other users who have made appropriations based on the previously existing conditions. If the earlier practice has been well established such opposition would be expected to be successful.

Court decisions indicate that a claimant is entitled to a quantity of water sufficient to meet the needs of his lands for the type of crop which he has been accustomed to growing or which he may grow under reasonable crop rotations. Decrees for individual users or under small canals may be based on the requirement for the crop of largest use, usually forage. For larger areas, diversity of crops is to be expected and the needs can be based on the requirements for the proportion of different crops which has obtained during recent years. Although there may have been a definite tendency toward crops of smaller needs and although it may reasonably be anticipated that such tendency may continue, it is hardly to be expected that those making the decision will reduce the right below the quantity needed for the crops being grown at the time of the determination.

Courts are guided by local practice as to the crops the irrigation of which may be considered beneficial. The irrigation of native grasses is a recognized beneficial use, but it must be practiced under conditions representing proper local standards in the handling of the water. The fact that some claimants may desire to irrigate crops of smaller water requirement will not justify limiting all rights to the needs of such crops where other rights have been used for crops of larger need.

In the decree on Salmon Falls Creek, in Idaho, the Federal Court based its findings on a use of 2.5 acre-ft. per acre for forage crops and 1.5 acre-ft. per acre for other crops, one-half of the area being used for each type of crop. These quantities are to be measured at the farm head-gate.

The recent determination on Humboldt River made by the State Engineer of Nevada is based on a use of 3 acre-ft. per acre for harvest crops, 1.5 acre-ft. per acre for meadow pasture, and 0.75 acre-ft. per acre for diversified pasture. (Rodgers *v.* Pitt, 129 Fed. 932 (Nevada, 1904); U. S. *v.* Bennett, 207 Fed. 524 (Washington, 1913); *in re* North Powder River, 144 Pac. 485 (Oregon, 1914); Muir *v.* Allison, 191 Pac. 206 (Idaho, 1920).)

Climate.—Climate enters into water-right determinations due to its influence on the length of the irrigation season, the crops grown, and the need for irrigation, as water used in irrigation is supplemental to that obtained from natural precipitation. There is less difference in the rate at which water will be used during the period of maximum demand than there is in the length of the irrigation season in different localities. The amount and occurrence of rainfall will influence the amount of irrigation required. Such differences in rainfall more usually affect the time of the beginning of the irrigation season. In localities where the rainfall continues into the growing season, as in the Plains Area east of the Rocky Mountains, the irrigation season may be delayed until later in the crop season than is the case where the rainfall occurs almost wholly in the winter months as in the Pacific Coast States.

Differences in use based on differences in climate were adopted on the Beaverhead River in Montana where different rates of use were decreed for the upper and lower valleys. These differences may be due to the combination of partial separate decrees rather than to a distinct difference based on climate. A decree in Apache County, Arizona, allowed 1 sec-ft. to 90, 110, and 180 acres, respectively, for areas at different altitudes.

Preparation of Land.—The character of preparation of the land may make a great difference in the quantity of water used. Where the surface is uneven, greater average depths must be applied in order to cover the higher areas. The flooding of areas of too large size results in excess percolation loss.

It is usual to find a definite tendency toward improvement in the preparation of land as an area has been longer under irrigation. This is due both to the increased resources of the owners which enable them to incur greater costs in preparing land and to the benefit of longer experience under local conditions and the development of methods best suited to such conditions. The best support for a contention that older users should have their rights reduced to the quantity required under better methods of land preparation would

appear to be a showing that such improved methods were in voluntary use in the area concerned by a sufficient proportion of the users to have demonstrated their advantages and economy. Efforts to secure decrees which would require general changes in the methods of preparing land where such support was not available have not been successful. (Foster *v.* Foster, 213 Pac. 895 (Oregon, 1923); Farmers Co-operative Ditch Co. *v.* Riverside Irrigation District, 102 Pac. 481 (Idaho, 1909); Beasley *v.* Engstrom, 168 Pac. 1145 (Idaho, 1917).)

Cost of Water.—The cost of water is usually an indirect factor. It may affect the duty of water due to its influence on the methods of preparing land and handling water. The cost which it may be feasible to incur is limited by the value of the crop returns. The rates charged for water may influence the use of water in actual practice, but do not appear to have entered directly into the determination of water rights.

Skill and Attention Used in Applying Water.—The skill or attention of the labor used in applying water is closely related to the character of land preparation. Claims for a quantity of water which will permit the land to be irrigated with only occasional attention to the water are frequently made in areas irrigating pasture or forage. Such practices are judged by their reasonableness in comparison with the general practice of the area. In many areas the cost of labor in the application of water still exceeds the cost of the water itself, so that the extent to which improvements which would increase the labor costs can be expected, may be limited.

Summary of Factors Considered in Fixing the Duty of Water

Although there are a number of decrees in which a uniform rate of use was awarded to all claimants, these illustrations appear to justify the conclusion that determining agencies, whether an administrative officer, such as the State Engineer, or a Court, either State or Federal, will decree varying rates of use to different claimants whenever they conclude that differences in soils, crops, or other conditions exist which warrant differences in the rate of use. Although soil conditions may vary within relatively small areas, the tendency appears to have been to distinguish mainly differences in the conditions for general classes or divisions of the area rather than for individual owners or ditches. Presumably, however, if evidence could be presented supporting such differences the determining body would feel free, as far as principles are concerned, to vary the use for smaller areas. With the increasing attention that is being given to the factors which affect the duty of water it is reasonable to expect that the adjudicating agencies will tend to give closer attention to the details of the water requirements of the lands served, with the result that variations in the rate of use allowed may be more frequent in the future than in the past.

Terms Used to Express the Quantities Which Any Right May Be Entitled to Divert and the Season of Diversion

The older decrees were expressed almost entirely in terms of continuous flow. As storage has become of importance it has been necessary to express

some decrees in terms of the total quantity per season and to limit the period of diversion for direct use. In the older decrees, shortages in supply generally occurred sufficiently late in the season to be within the period of maximum rate of use so that it was not necessary to define the beginning of the irrigation season. The periods of diversion were left to the discretion of the users, the right permitted diversion up to its maximum rate at any time flow was available and a beneficial use could be made of the water. A reduction in diversion could be made when waste occurred.

The Salt River decree in Arizona permits diversion throughout the year as climatic conditions result in continuous use. On Hat Creek, in California, the decrees are effective from May 1 to October 27, no limitations being placed on use during the remainder of the year. On the West Fork of Carson River, in California, Big Lost River, in Idaho, Silvies and Umatilla Rivers, in Oregon, Logan River, in Utah, and West Okanogan, in Washington, diversion is limited to periods between specified dates.

The Bear River decree of the Federal Court for the Idaho District specifies a diversion season from April 20 to September 30, with the rate of diversion from April 20 to April 30 and from September 15 to September 30 limited to 40% of that allowed during the remainder of the period.

On the Boise River, the decree now in present use is not final on the duty of water, the tentative allowances being adjusted during the season as the stream flow diminishes. The following order illustrates the practice used:

"The various rights as adjudicated in the so-called Stewart Decree shall receive 100% until the natural flow of the water of Boise River shall decrease or until all the rights in said decree cannot receive 100%, at which time the various rights as adjudicated in the so-called Stewart Decree shall first be cut to 75% of the amount of water decreed by the Stewart Decree as the natural flow of Boise River decreases, beginning with the latest rights and proceeding to the earliest rights in the order fixed in said Stewart Decree, and after all rights shall have been reduced to 75% of the amount fixed in the Stewart Decree, should the natural flow of the waters in the Boise River decrease below the amount necessary to supply said 75% of the water rights as decreed in said Stewart Decree, then the various rights, beginning with the latest and proceeding to the earliest as aforesaid, shall be reduced to 60% of the amount specified in said Stewart Decree and 60% of the amount specified in said Stewart Decree is hereby fixed and determined as the highest duty of water for the year 1919."

This is equivalent to using a higher standard of practice during periods of small supply in order to reduce the hardship of such shortages on rights of later priority. This method has been in use for several years, apparently with generally satisfactory results.

On the Weber River, in Utah, the proposed determination by the State Engineer is based on the use during high-water stages of 1 sec-ft. to 60, 65, and 70 acres for the upper, middle, and lower river areas, with all rights reduced to 1 sec-ft. to 80 acres before any rights are cut off.

On the Big Lost River, in Idaho, a decree accepted by stipulation by all parties provides for diversion from April 20 to October 31, diversion during April and October to be at the discretion of the Water Commissioner.

On Logan River, in Utah, all the older rights were decreed a priority of May 1, 1860, and a schedule provided for stream flows of 400 sec-ft. or less by which different rights receive somewhat different percentages of reduction as the flow diminishes. A similar general method is followed in the temporary schedule now in use on Kings River, in California, in which different rights are entitled to divert different quantities at different stages of stream flow, the rights being listed for each variation of 100 sec-ft. in stream flow up to a total discharge of 10 000 sec-ft.

For the Humboldt River, in Nevada, the rates of use in acre-feet per acre for different crops, previously mentioned, are proportioned to the length of the irrigation season, harvest crops being irrigated over a 180-day season. The diversion rights are expressed in the decree in terms of second-feet.

The definition of rights in terms of acre-feet per acre per season has been more usual in the administrative determinations in Oregon. Total acre-feet per season for each right, with a limiting rate in second-feet, were used on the Deschutes, Grande Ronde, Hood, Umatilla, and Wallowa Rivers. On the Silvies River, 1.5 acre-ft. per acre per month prior to June 1 and 1 acre-ft. per month after June 1, to be taken at a rate of not to exceed 1.40 sec-ft. per acre was used.

The following quotation from the decision in the case of the State of Wyoming *v.* State of Colorado (42 Sup. Ct. 568) represents a recent expression of the U. S. Supreme Court:

"A much larger amount is claimed, but our finding restricts the amount to what the evidence shows is reasonably required, which is 1 acre-ft. per acre for the larger part of the lands, 2 acre-ft. per acre for a part, and 2½ acre-ft. per acre for the remainder."

It appears to be well established that the determining bodies may define the rights in such terms as they consider to represent beneficial use; that the season may be restricted to those periods during which use is beneficial and that specific dates may be defined limiting the season of use; and that in addition to the more usual former practice of defining merely the maximum rate of diversion, the total quantity of use for the season may be defined. The limiting of the total quantity that may be diverted during the season, leaving the user to adjust the time of its diversions to his needs subject to some limitation on the maximum rate at which he may divert, seems to represent a desirable basis on those streams where storage as well as direct rights exist. Such a basis protects the prior right in an adequate supply under reasonably flexible conditions of use. It also defines the total use so that the supply available for storage can be determined. Its terms are sufficiently direct and specific to permit practical operation of the stream.

Decisions involving maximum rate of diversion with total acre-feet per acre limitation, are: Foster *v.* Foster, 213 Pac. 895 (Oregon, 1923); *in re* Willow Creek, 144 Pac. 505 (Oregon, 1914); *in re* North Powder River, 144 Pac. 485 (Oregon, 1914); Laurence *v.* Brown, 185 Pac. 761 (Oregon, 1919); and Vineyard Land & Stock Co. *v.* Twin Falls Salmon River Land & Water Co., 245 Fed. 22 (Idaho, 1917).

Decisions involving different rates in low water period are: *In re* Willow Creek, 144 Pac. 505 (Oregon, 1914); *in re* North Powder River, 144 Pac. 485 (Oregon, 1914); and Laurence *v.* Brown, 185 Pac. 761 (Oregon, 1919).

Decisions involving season limited by fixed dates include Foster *v.* Foster, 213 Pac. 895 (Oregon, 1923).

Conveyance Losses

In general, all decrees define the extent of the right at the point of diversion. This is essential usually for purposes of practical administration. The decree includes any losses in conveyance to the point of delivery to the land owner. In some determinations no distinction is made separately for conveyance losses. In others, the requirement at the land is determined and the diversion right based on increasing this by some specified quantity for conveyance losses. In the Salt River decree, 1% per mile of canal was used. Actual measurements of loss for each ditch have been used in some cases, such as the West Gallatin in Montana and the Deschutes and Grande Ronde in Oregon. The following provision is contained in the decree for Umatilla River, in Oregon:

"In all cases the water master, where the works are in good condition, may at his discretion allow an increased diversion for such seepage and evaporation, which increased diversion shall be determined by the water master according to the actual seepage and evaporation in the diversion works, but in no case shall such increased diversion exceed 20% of the amount allowed by this finding."

The diversion requirement can be determined best by estimating the delivery requirement and the conveyance loss separately and combining the result. The delivery requirement and the conveyance loss have no consistent relationship. The lengths of canals serving similar soils and having similar delivery requirements may vary widely so that a decree based on average diversion requirements may be unfair to canals of greater loss.

The construction and maintenance of canals must represent similar standards of reasonably good practice to that expected in the application of water to the land. Where conveyance losses are excessive, the right may be limited to a reasonable loss.

Cases covering conveyance losses include Town of Sterling *v.* Pawnee Extension Ditch Co., 94 Pac. 339 (Colorado, 1908); Wheat *v.* Cameron, 210 Pac. 761 (Montana, 1923); Basinger *v.* Taylor, 211 Pac. 1085 (Idaho, 1923); Clark *v.* Hansen, 206 Pac. 808 (Idaho, 1922); Santa Cruz Reservoir Co. *v.* Ramirez, 141 Pac. 120 (Arizona, 1914).

Rotation in Delivery

Nearly all decrees include some diversions serving relatively small areas. If the right for such an area is expressed in terms of its average need the resulting diversion may be too small for effective use. To meet this condition, rotation in delivery is frequently practiced. Under larger canals such rotation is practiced among the different users without affecting the diversion from the

stream. Under small canals, the users may desire to divert at twice the average rate for one-half the time or make such other adjustments as may meet their needs. To avoid confusion in the decrees it is usual to define each right in terms equivalent to its continuous diversion, leaving such adjustments to be handled by the one in charge of administering the stream under general clauses in the decree.

Where such conditions exist, it is usual to permit rotation subject to provisions under which later priorities may be protected in securing the supply to which they are entitled. Such provisions usually require the one seeking to divert a surplus to secure the approval of the water commissioner and to arrange an exchange with other owners desiring to rotate.

There appears to be some uncertainty as to the right of a Court to require rotation or to define each right so that rotation must be practiced in order to make irrigation feasible. The general tendency has been toward a recognition of rotation as a part of usual practice and toward holding that beneficial use may be defined on a basis which may result in requiring rotation for successful irrigation.

Rotation is permitted in the Salt River decree, in all decisions of the California Division of Water Rights, in the West Gallatin in Montana, the Big Wood River and Salmon Falls Creek in Idaho, permitted or required in various determinations of the State Engineer in Oregon, and permitted in the Bear River decree in Utah. No cases were found in which it was prohibited; the remaining streams appear to represent omission of any such provisions rather than objection to it unless the case of Muir *v.* Allison is considered as an exception to this statement. (*In re* Willow Creek, 144 Pac. 505 (Oregon, 1914); *in re* North Powder River, 144 Pac. 485 (Oregon, 1914); Muir *v.* Allison, 191 Pac. 206 (Idaho, 1920); and Reno *v.* Richards, 178 Pac. 81 (Idaho, 1919).)

Character of Evidence on the Duty of Water Which Appeared to Be of Most Influence with the Adjudicating Body

In the older cases little evidence of an expert character was presented. Technical evidence, such as measurements of canals, was often used, but this was generally for the purpose of basing the claims on records of past diversion rather than to determine the proper duty of water. More recent irrigation developments usually include large units which plan to utilize all or nearly all the remaining unappropriated flow of streams. The water supply available is the difference between the total run-off and the existing rights. The more closely the existing rights can be limited in their quantity, the larger is the remaining unappropriated water. Such limitations are in the public interest in permitting a larger total utilization of a given stream provided the limitations placed on the earlier rights are not unreasonable. As previously noted, the adjudicating bodies have not used such high standards in defining existing rights as to impose unreasonable limitations.

The advantage to be gained by such later rights from a restriction of the earlier users and the extent of their interest due to the size of many of these larger projects have resulted in the employment in many cases of expert

witnesses to testify on the proper use of water. Such witnesses have more usually testified in favor of higher standards of practice than those advocated by the users themselves or those directly engaged in the operation of the existing systems. This has led to the general classification of such expert witnesses as high duty advocates, as contrasted with the usual farmer or direct experience advocates of larger quantities of water. Many attempts have been made to rebut such expert testimony on the ground that it was not based on sufficient familiarity with the areas involved, that it represented theoretical results rather than practical, that the costs necessary to accomplish the results recommended would be beyond the means of those now irrigating, and that prior users were legally entitled to such quantities of use as might be required under the general methods existing in the area. Rebuttal to the evidence of users in the area is usually based on the claim that the ordinary irrigator although experienced in the application of water is not qualified in water measurement so that his opinions on the quantities he had used are not competent evidence. As the numerical results presented by the two classes of witnesses are frequently relatively far apart the resulting decision may offer opportunity for at least a general judgment as to the evidence which appears to have been given weight in the decisions.

On the basis of the information assembled by the Committee, the decrees made by Courts appear to have been based largely on the testimony of those directly engaged in the use of water in the area under adjudication. Where expert witnesses have been able to qualify on the basis of both general experience and of experience in the locality, or where their opinions are supported by records of use by the better irrigators of the area or by experimental determinations under local field conditions, their conclusions appear to have been of more assistance to the case. Where the conclusions have not been supported by local data or have been based on experiments under conditions not representative of field conditions, little weight appears to have been given to the evidence.

For the West Gallatin and Beaverhead Rivers in Montana expert irrigators were engaged by the Court to make determinations in the field and advise the Court in defining the water requirements. Irrigators having local experience were selected. In the adjudication of Hood River, in Oregon, local trials of the use of water were made.

The water requirements depend on the number of irrigations and the quantity used at each irrigation. The one irrigating under a given set of conditions is usually fully competent to testify regarding the frequency of irrigation required although he may not be able to measure properly the quantity of water used. A logical determination would be secured by utilizing direct experience on the number of irrigations required, together with competent expert evidence on the quantity required for each irrigation, based on actual measurement under proper field conditions representing the standards of practice that the Court desired to follow in its decision. Such a basis for the delivery requirement when increased to include canal conveyance losses based on actual observations would appear to utilize both direct and expert evidence to the best advantage.

These comments apply mainly to adjudications by Courts. In the adjudications by administrative officers of the State, direct observational data are collected by the engineer conducting the adjudication. The entire procedure is in the hands of those experienced in such work under conditions where the one making the determination is not limited to evidence brought before him, but may and usually does conduct his own investigations. For such determinations controversies between expert and lay witnesses are less usual. There have been relatively few cases in which the conclusions of such procedures have been modified on appeal to the Courts.

Among the decisions discussing the character of evidence on the duty of water are: Stinson Canal & Irrigation Co. *v.* Lemoore Canal & Irrigation Co., 188 Pac. 77 (California, 1919); Pabst *v.* Finmand, 211 Pac. 11 (California, 1922); Farmers Co-operative Ditch Co. *v.* Riverside Irrigation Dist., 102 Pac. 481 (Idaho, 1909); Idaho Irrigation Co. *v.* Gooding, 285 Fed. 453 (Idaho, 1922); Rodgers *v.* Pitt, 129 Fed. 932 (Nevada, 1904); Anderson *v.* Bassman, 140 Fed. 28 (Nevada, 1905); Little Walla Irrigation Union *v.* Finis Irrigation Co., 124 Pac. 666 (Oregon, 1912); Foster *v.* Foster, 213 Pac. 895 (Oregon, 1923); Sharp *v.* Whitmore, 168 Pac. 273 (Utah, 1917); Pasco Fruit Lands Co. *v.* Timmermann, 152 Pac. 675 (Washington, 1915); and Nichols *v.* Hufford, 133 Pac. 1084 (Wyoming, 1913).

Should Jurisdiction Be Retained to Permit Revision of the Duty of Water?

Standards of irrigation practice change similarly to the standards of other features of crop production. Increasing land values call for increasing average yields in order to support such values. Such increases in yields require and support better methods of preparing land and more attention to the application of water. Public interest in the best utilization of the limited water resources supports a continually higher standard of practice as the basis of defining beneficial use. All these factors tend toward a reduction in the quantity of irrigation water used for each acre of land.

As a recognition of such changing conditions, and in order to permit later adjustments in decrees, it has frequently been urged that jurisdiction should be retained in water-right adjudications so that the terms of the decrees may be later modified to meet such changes. The Courts retain partial jurisdiction from year to year in order to provide for the appointment of water commissioners for the administration of the decree. Provisions for retaining jurisdiction to permit re-opening the case for further evidence on the duty of water and possible modifications in its terms have been made in the Salt River decree in Arizona, the Boise River and Salmon Falls Creek decrees in Idaho, and the Silvies River decree in Oregon. The Powder River decree in Oregon is left open for further evidence on seepage and evaporation. The determination of the State Engineer on the Humboldt River in Nevada is to be interlocutory for three years for further consideration of the duty of water. The findings of the Special Master of the Federal Court on the Truckee River in the same State propose that jurisdiction shall be retained for purposes of possible revision of the duty of water allowances.

The State Engineer of Utah has provided in the recent adjudications made by his office that the decrees may be re-opened after five years to review the

duty of water findings. The Utah statutes provide for proceedings by which a re-determination of rights may be made where it can be shown that wasteful use occurs.

Although provisions for re-opening of decrees as stated were found, no case was reported in which such re-opening had occurred unless the general determination on the Sevier River in Utah by the State Engineer is regarded as such a re-opening of the different old Court decrees on this stream. As none of these Court decrees included all users on Sevier River, the present determination is hardly a re-opening of procedure in which all rights have participated. In all cases those supplying information regarding these streams expressed the opinion that such a re-opening would at present be inadvisable, the reason more usually given being that the probabilities of securing a more favorable decree on the duty of water would be more than offset by the disadvantages of controversy, cost, and uncertainty in rights that would result.

The conclusion appears warranted that provisions for retaining jurisdiction so that such cases may be re-opened may be desirable on many streams, but that the probabilities of such re-opening are rather limited at least until conditions and standards have changed materially from those existing at the time of the original decree.

Decisions discussing the right to retain jurisdiction include: Mays *v.* District Court, 200 Pac. 115 (Idaho, 1921); and Big Cottonwood Tanner Ditch Co. *v.* Shurtleff, 189 Pac. 587 (Utah, 1920).

Has the Operation of the Decree Been Satisfactory and in What Ways, if Any, Could It Be Improved?

Although the comment is frequently made that methods of adjudicating water rights, particularly by Court procedure, are unsatisfactory, the replies to a questionnaire in nearly all cases were to the effect that the decrees were operating fairly satisfactorily. As the opinions secured were generally from people directly connected with the adjudication or its operation, although usually not representing any particular interest on the stream, the results may be regarded as representative of the general opinion. A noticeable feature of many determinations made by State administrative officers is the proportion of such cases in which the findings by such officers are accepted by stipulation before the decision is made. In some cases the opinion was expressed that a higher standard in defining the duty of water, particularly in the earlier decrees, would have been in the interest of the public. Greater definiteness in the terms and conditions were also mentioned as possible improvements. In general, however, it appeared to be the opinion of those replying that any such advantages that might be gained by revision would be offset by the disadvantages of delay, cost, and controversy.

Conclusions

The following general conclusions are the result of the Committee's study of principles and practice in the present procedure for defining the duty of water in water-right determinations:

1.—Determining agencies are guided mainly by present standards of reasonable practice and will not accept a standard of practice which is not well supported by rather extensive actual use, adopted voluntarily by existing local canals.

2.—Those endeavoring to secure the adoption of higher standards of practice in such determinations may well be guided by the previous statement and will usually find their efforts more productive of practical results if it is observed. An endeavor to secure decrees representing a standard of practice not so supported may result in the determination being based on a less desirable standard that might have been followed if a more practical presentation of good practice had been made.

3.—The use of the direct experience of local irrigators to determine the number of irrigations required, with evidence on the proper depth per irrigation based on measurement under actual local field conditions representing the standard of practice it is desired to follow, represents the most logical method of using both direct experience and expert opinion.

4.—A form of decree which will meet the requirements for direct-flow rights on streams where storage is to be used will be one defining the maximum rate of diversion with an additional limitation on the total quantity that may be diverted during the season. This form of decree has been used sufficiently to establish its legality. The fixing of the irrigation season by specifying the dates for beginning and ending the diversion may be desirable or not depending on local conditions; where a limit is placed on the total seasonal diversion in quantitative units such dates for diversion are less essential. Rotation should be permitted and encouraged and the decree if based on a reasonable standard of use will generally result in practically requiring rotation among smaller diversions.

5.—Improvements in the standards of practice used as a basis in defining water rights are dependent mainly on the adoption in general irrigation practice of more of the present knowledge regarding the advantages of a more economical use of water. Legal definitions of water rights can be expected to follow such improvements in general practice, but cannot be expected to force such improvements by limiting rights on the basis of a better practice than may be considered reasonable at present. It is desirable to have provisions in decrees for later adjustments in the quantities decreed whenever conditions may warrant such re-determinations. Such provisions will probably only be used at infrequent intervals so as to avoid general disturbance and expense in maintaining water rights.

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DISCUSSION

E. A. Porter,* M. Am. Soc. C. E.—This report covers one of the most complicated and vexing problems in irrigation engineering and operation. It is a problem for which a reasonable solution has been attempted, from time to time, since the beginning of irrigation in the arid regions. It is a subject which ever seems to meet with changeable factors and conditions and which arouses the wrath and suspicion of the old water user more quickly than any other in irrigation. For this reason the report is not only timely but should be of great value to those engineers who are interested in irrigation and kindred problems.

It is well known that any one entering into a discussion in an attempt to solve the proper duty of water for a given area, is likely to be severely criticized. The speaker has been connected with problems involving the reasonable duty of water for several years. Such experiences have taught him that the problem must be met in a spirit of tolerance and with a practical mind, and that the factors of water supply, soil conditions, and methods of use in combination with crop production, although of great importance, may all be changed by the important, but variable, factor of climate. The 1925 season in Utah furnishes an example of this condition. The water duty for that season may be quite out of place for another year. Those in charge of water-right adjudications should be able to appreciate the fact that the user's interest, although paramount, should be analyzed in connection with the interests of the Commonwealth in which he lives. It will only be a matter of time until a reasonable water right will have assumed as definite a value as any other property right. Therefore, the water supply of the arid regions must ultimately be made to perform its maximum duty. Realizing the ultimate insufficiency of water for all the irrigable lands within the arid regions it does not seem consistent that final decrees should be permitted in water-right determinations. Neither does it seem consistent that States should permit further appropriations on most of the streams until accurate data are available to show a surplus.

The results secured by the Committee have been ably presented. However, it appears evident after a review of the report that there still remains a vast opportunity for improvement in the collection of data to determine the proper duty of water under practical methods of use. All who have had experience in irrigation management or operation realize that in most instances throughout the arid West, some District Court will pass on the final recommendations for a determination suit. The Committee is right when it explains that the testimony of the actual user of water will carry more weight than that of the expert. This is probably a bold statement, but, nevertheless, it is true, for the reason that the expert is usually employed to help a client win his point. It is well known that some so-called experts often introduce as facts testimony which they know to be untrue.

Under the revised Water Rights Law of the State of Utah (1919), the State Engineer is empowered to re-determine the water rights along any river

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system in the State after the water users under such stream have taken the necessary legal preliminary steps. At present (1925), this official is engaged with the problems of a proposed determination for the entire Sevier River Basin in Southern Utah which data will later be submitted to the water users and District Court for confirmation. This river system is typical of most of the rivers in the inter-mountain region, and for various reasons presents an ideal example for use in connection with other systems not involving priority rights for power development. There are no important power developments within the Sevier Basin. It is solely an irrigation stream for the reason that the early settlers filed on all the normal water supply for surface irrigation purposes. Power developments will probably come in later years, but such installations will require equalizing storage reservoirs.

At present, the upper valleys of the Sevier Basin are irrigated almost solely by natural flow, diverted either from the main channel or from tributary streams. The area served in these valleys is about 25% of the total area irrigated within the Basin. The remainder of the irrigated, or irrigable, area lies within the storage zone where the water supply can be accurately controlled by the aid of several storage reservoirs. The largest of these reservoirs has been created during recent years, prior to which the natural flow rights, now within the storage zone, had been filed upon, or used, for a long time. The Courts had adjudicated these waters to the rightful owners. Later, the storage reservoirs were created to conserve all the surplus water supplies. Prior to 1914 the Federal Government and the State had established three or four water-measuring stations within the Basin, but in 1914 these two agencies, in co-operation with the water users, saw fit to determine accurately the water resources of the Basin at all critical points.

To-day, the engineer, water user, or public have available before them a mass of accurate data showing the seasonal head-gate diversion of every important canal in the Basin, the daily, monthly, or annual flow of the river at a score of points and the approximate area of irrigated and irrigable lands under each canal system. Without doubt the stream-flow data on the Sevier System are as complete as those on any system in the country. With the aid of these data the factors of canal capacities, gross diversions, and return flow from important areas can be ascertained. There is, however, one weak link in the chain of data for this river system, and this is the duty of water, which is of such importance that it may delay the adjudication suit for years.

However, the importance of the reasonable water requirement for the irrigated lands had been appreciated by a few. The steps taken to secure data on this subject have been along practical lines. The first step was to persuade the older canal companies, operating under the former natural flow rights, to appreciate the value of a stored water supply as compared with their variable and uncertain decreed rights. In 1920 one of the older canal companies agreed to exchange, for trial only, its decreed right for a definite stored water supply for use as desired during the irrigation season. The result was so surprising that all suspicion against the plan was immediately dispelled, and this particular canal has now pooled its former natural flow right with the stored water interests. Several other canal companies have followed the plan, after this experience.

This method of procedure has brought together many conflicting interests which stood in the way of the most efficient adjudication. It has demonstrated, in a practical manner and without any additional cost, that a stored water supply is superior to a fluctuating natural right. It has taught the older water users that successful crops, common to the intermountain region, can be matured with 2 to 3 acre-ft. of water instead of 4 to 7 acre-ft. as formerly used on the land. These results have been secured almost solely by the Water Commissioner who has had field charge of water distribution and utilization. The Commissioner has operated under the supervision of the Court or State Engineer, and his policy has been to distribute the water in co-operation with the water users and storage reservoirs. In years of stress and seasons of plenty this Commissioner, who is a technical engineer as well as a diplomat, has been able to protect the interests of all satisfactorily.

It would appear that all water supplies within a storage zone should be made common and that the interests should be pooled on a priority basis. Data in connection with the determination of the duty of water should be secured in co-operation with the actual farmer or irrigator. Water-right decrees should not be made final as a general rule. Water commissioners should be selected carefully and after a selection is made, the commissioner should be given full charge of all field duties with the necessary authority and backing of a legal body. The length of the irrigation season should be definitely set prior to the start of each season.

Lloyd Garrison,* Esq.—One or two notes based on the speaker's experience in Utah may be of interest. In 1919, the Legislature of Utah passed a law which provided for the adjudication of water rights, and under this law all the streams of the State will eventually be adjudicated. It has been the practice of the State Engineer to make a determination of all rights as provided by the law and to submit his findings to the Court with the recommendation that jurisdiction of the case be retained by the Court for a period of five years. Duty of water in the determination is fixed by general experience and such local information as is available. During the five-year period it is contemplated that tests of duty will be conducted on representative tracts of land. These tests will include records of water used, crops raised, soil determinations, and such other data as will give as complete information as possible concerning the duty required. At the end of the five years it is hoped that sufficient data will be collected to permit the Court to render a final decree.

The speaker hardly agrees with Mr. Porter that the suit should be held open indefinitely. That leaves the settlement for too long a period. Five years should be sufficient for an accurate determination of duty. The law in the State of Utah is good and if given a chance will be very helpful to the farmers in effecting a permanent settlement of their difficulties.

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AMERICAN SOCIETY OF CIVIL ENGINEERS
INSTITUTED 1852

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Paper No. 1621

ANALYSIS OF CONTINUOUS CONCRETE ARCH SYSTEMS*

By Charles S. Whitney,† M. Am. Soc. C. E.

With Discussion by Messrs. Hardy Cross and Charles S. Whitney.

Synopsis

A number of methods varying greatly in the labor involved and in degree of accuracy have been proposed for the analysis of concrete arches that form a series supported by elastic piers. In general, the more accurate methods are quite laborious and the approximate methods are quite easily applied. Little information is available as to the limits within which suitably accurate results may be obtained by the approximate methods, and it is hoped that this paper with its discussion will throw more light on this subject.

The writer presents herein a simple method, which he believes to be new, for analyzing a system composed of two arch spans with an elastic pier. This is an application of a general method for analyzing symmetrical statically indeterminate structures called the "Verfahren der Belastungsumordnung", several interesting examples of which are given by W. L. Andrée.‡ It consists in replacing a single eccentric load by two pairs of symmetrically placed loads which have the single load for their resultant. The analysis of the structure under this new loading is considerably simpler because of the symmetry. Advantage is also taken of the treatment of single arch ribs given in the writer's paper, "Design of Symmetrical Concrete Arches",§ with which the reader is assumed to be familiar, and tables of influence-line ordinates are given which make the solution easy. By this method it is possible to study readily the effect on the adjacent spans of the elasticity of one pier and to determine the accuracy of approximate methods for different cases.

* Published in May, 1926, *Proceedings*.
† Cons. Engr., Milwaukee, Wis.
‡ "Das B-U. Verfahren," by W. L. Andrée, R. Oldenbourg, Berlin, Germany.
§ *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 931.

In addition, an easily applied approximate method is given for the analysis of a system of any number of spans, which the writer believes to be accurate enough for the majority of cases occurring in practice.

Analysis of Two-Span System

The following analysis shows the effect of the elasticity of a pier and the yielding of its foundation on the stresses in the adjacent arch ribs. The arch ribs are assumed to be similar and symmetrical. In order to simplify the equations, the outer ends of the arch ribs are assumed to be fixed but, if desired, the effects of elastic supports can readily be considered. The inner ends of the arch ribs are rigidly attached to the top of an elastic pier, the base of which is fixed against horizontal and vertical displacement, but permitted to rotate slightly by the elasticity of the supporting foundation. The effect of shortening of the pier due to vertical loading is not considered, as it is negligible in all cases. A bending moment in an arch rib is considered to be positive when it causes compression at the top and, in the pier, when it compresses the right-hand side. A horizontal thrust is considered to be positive when it tends to shorten an arch rib or to cause a displacement of the base of the pier toward the left.

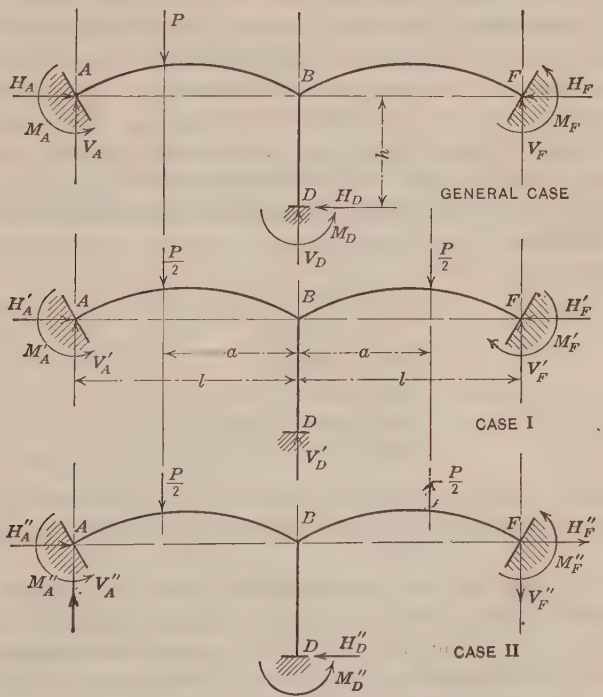


Fig. 1.

A single concentrated vertical load placed on one of the ribs of a two-span system, as shown in Fig. 1, will produce a horizontal and vertical thrust

and a moment at each of the two abutments and at the base of the pier, or a total of nine reactions the size of which cannot be determined by the three statical equations. The analysis can be greatly simplified by substituting for the single load two pairs of symmetrically placed loads which are equal to the single load. This substitute loading is shown in Fig. 1. If the loading of Case II be superimposed on that of Case I, the loads on the right-hand span are neutralized, and the sum of those on the left is equal to P. The determination of the reactions for the two cases taken separately is quite simple, and, if combined, they will give the reactions for the single load, P.

In Case I, the loads, $\frac{P}{2}$, both act downward and are equi-distant from the center line of the pier. From symmetry, the reactions at A and F are the same. If the shortening of the pier due to direct compression is neglected, there is no displacement, horizontal, vertical, or rotational, of the head of the pier. The symmetrically placed loads produce at the springing line of the two arch ribs next to the pier equal moments and thrusts, which exactly balance, the resultant being a vertical thrust down the axis of the pier. The reactions at the end of each rib are then the same as if the ribs were supported separately by rigid abutments. The reaction for Case I can be found, therefore, by considering the rib, A-B, to have fixed ends, and the following equations can be written.

Case I.—

$$\left.\begin{aligned} H_A' &= H_F' \\ V_A' &= V_F' \\ M_A' &= M_F' \\ H_D' &= 0 \\ V_D' &= 1 - 2\,V_A' \\ M_D' &= 0 \end{aligned}\right\} \qquad \text{.....(1)}$$

The solution of Case I can be effected by the methods given in the writer's paper, "Design of Symmetrical Concrete Arches", referred to previously.

For Case II, the loads are symmetrically placed but oppositely directed. The reactions at A and F are then equal in size but opposite in sign. The vertical reaction at the pier end of one rib is equal and opposite to that of the other rib so that they balance each other. The horizontal thrusts and the moments, however, are exerted in the same direction from both ribs and they produce a horizontal and angular displacement of the head of the pier. From these considerations, the following formulas are derived.

Case II.—

$$\left.\begin{aligned} H_A'' &= -\,H_F'' \\ V_A'' &= -\,V_F'' \\ M_A'' &= -\,M_F'' \\ H_D'' &= 2\,H_A'' \\ V_D'' &= 0 \\ M_D'' &= 2\,M_A'' - 2\,H_A''\,h + 2\,V_A'' - P\,a \end{aligned}\right\} \qquad \text{.....(2)}$$

By adding together the reactions for Cases I and II, the reactions for the single load of the general case are obtained:

$$H_A = H_A' + H_A'' \qquad H_F = H_A' - H_A''$$
$$V_A = V_A' + V_A'' \qquad V_F = V_A' - V_A''$$
$$M_A = M_A' + M_A'' \qquad M_F = M_A' - M_A''$$

In the rib at the pier:

$$H_B = H_A$$
$$V_B = V_A - P$$
$$M_B = M_A + V_A\, l - P\, a$$

At the pier base: .....(3)

$$H_D = H_D''$$
$$V_D = V_D'$$
$$M_D = M_D''$$

At the pier top:

$$H_E = H_D''$$
$$V_E = V_D'$$
$$M_E = M_D'' - H_D''\, h$$

Method of Analysis for Case II

Case II will be analyzed by considering first the pier base to be cut away from the foundation, leaving the two arch ribs acting as a curved beam fixed between the end abutments, A and F, with the pier as a cantilever projecting down from the center of the beam. Aside from the reactions at A and F, the forces acting on this beam are the couple formed by the two loads, $\frac{P}{2}$, and the two reactions at the base of the pier, M_D'', and H_D'', which will be applied as loads. The effects of these forces will be considered separately and the influence lines for M_D'' and H_D'' obtained.

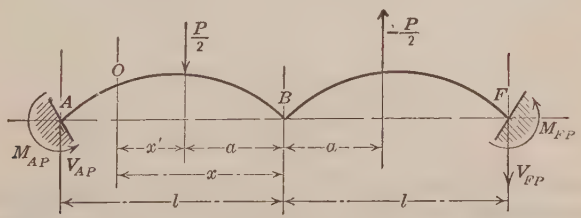


Fig. 2.

Effect of Loads Applied to Arch Ribs.—The reaction at A and F and the shear at B produced by the loads, $\frac{P}{2}$, will first be considered. The two spans are acting as a single fixed beam, as shown in Fig. 2. The pier is not shown, as it need not be considered at this time.

As the two applied forces are equal, and are symmetrically placed about the point, B, it will be seen that both tend to produce horizontal and angular displacements of the beam at B that are equal in size and direction. Also,

the load on the left side would produce a vertical displacement of B downward, and the load on the right side would produce an equal vertical displacement upward. The resultant vertical displacement of B with both loads acting is, therefore, zero. Because of the symmetry, it will be sufficient to take only one rib from A to B and consider that the end, A, is entirely fixed while the end, B, is free to move horizontally and to rotate, but is fixed vertically as shown in Fig. 3.

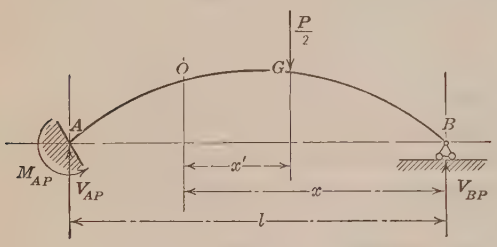


FIG. 3.

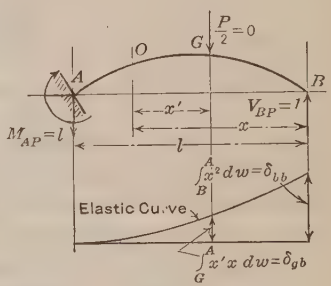


FIG. 4.

As the vertical displacement at B is zero,

$$\int_B^A M_O \, x \, \frac{d\,s}{E\,I} = \int_B^A M_O \, x \, d\,w = 0$$

in which, $d\,w = \dfrac{d\,s}{E\,I}$ and M_O is the moment at any point, O, or,

$$M_O = V_{BP}\,x - \frac{P\,x'}{2}$$

Then,

$$\int_B^A M_O \, x \, d\,w = V_{BP} \int_B^A x^2 \, d\,w - \frac{P}{2} \int_G^A x' \, x \, d\,w = 0$$

from which,

$$V_{BP} = \frac{P}{2} \frac{\int_G^A x' \, x \, d\,w}{\int_B^A x^2 \, d\,w} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

The influence line for the value of V_{BP} is the same as the elastic line for the rib, $A\,B$, when acted on only by a unit value of V_{BP} as shown in Fig. 4.

This is shown as follows: In Fig. 4, the vertical displacement δ_{gb} at the load point, G, due to a unit vertical force at B is, $\int_G^A M_O \, x' \, d\,w$, and as M_O in this case is equal to $V_{BP}\,x = x$,

$$\delta_{gb} = \int_G^A M_O \, x' \, d\,w = \int_G^A x' \, x \, d\,w$$

which is the numerator in Equation (4). Also, the displacement, δ_{bb}, at B is,

$$\delta_{bb} = \int_B^A M_O \, x \, d\,w = \int_B^A x^2 \, d\,w$$

which is the denominator in Equation (4). Therefore,

$$V_{BP} = \frac{P}{2} \frac{\delta_{gb}}{\delta_{bb}} \qquad (4a)$$

This can also be demonstrated by Maxwell's theorem. If δ_{bg} is the vertical displacement of the point, B, produced by a unit vertical load at G, the total displacement at B, due to both $\frac{P}{2}$ and V_{BP} will be,

$$\frac{P}{2} \delta_{bg} - V_{BP} \delta_{bb} = 0$$

or,

$$V_{BP} = \frac{P}{2} \frac{\delta_{bg}}{\delta_{bb}}$$

As the displacement, δ_{bg}, at B produced by a unit load at G is equal to the displacement, δ_{gb}, produced by a unit load at B,

$$V_{BP} = \frac{P}{2} \frac{\delta_{gb}}{\delta_{bb}}$$

The values of δ_{gb} and δ_{bb} can be determined by graphical or numerical integration or, using the ribs as established in the writer's paper previously mentioned, they may be derived by direct integration. Equation (54),* given therein, is,

$$d\,w = \frac{l_1}{E\,I_c} [1 - (1 - m)\,z]\,d\,z$$

After integrating,

$$\int_B^A x^2\,d\,w = \delta_{bb} = \frac{l^3\,(7 + 9\,m)}{48\,E\,I_c}$$

in which,

$$l_1 = \frac{l}{2}.$$

$$z = 1 - \frac{2\,x}{l}.$$

$$m = \frac{I_c}{I_s \cos \phi_s}.$$

I_c = moment of inertia of rib at crown.

I_s = moment of inertia of rib at springing line.

$\cos \phi_s$ = cosine of angle between the horizontal chord and the tangent to the rib axis at the springing line.

This value of δ_{bb} can be taken from Table 1. The equation for the elastic curve of Fig. 4 has also been obtained by direct integration and, from this, Table 2 has been computed, giving the value of δ_{gb}:

$$\delta_{gb} = 2\,l\,\delta_s$$

* *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 983.

Then, for any particular rib,

$$V_{BP} = \frac{P}{2}\,\frac{\delta_{gb}}{\delta_{bb}} = \frac{P\,l\,\delta_3}{\int x^2\,d\,w} = P\,\frac{C_2}{C_1} \qquad \text{.....(5)}$$

The values of C_2 and C_1, are given directly by Tables 1 and 2.

The reactions at A (Fig. 2) produced by the symmetrically placed loads are then as follows, when $P = 1$,

$$\left.\begin{aligned} M_{AP} &= V_{BP}\,l - \frac{P\,(l-a)}{2} = V_{BP}\,l - \frac{l-a}{2} \\ H_{AP} &= 0 \\ V_{AP} &= \frac{P}{2} - V_{BP} = \frac{1}{2} - V_{BP} \end{aligned}\right\} \qquad \text{.....(6)}$$

Effect of Reactions at Base of Pier.—The effect of a moment, M_D, applied at the base of the pier after it has been cut free from its foundation, will be considered.

As the two arch ribs are equally stiff, this moment will be divided equally between the two ribs. The two ribs may be considered as if cut apart with the moment, $\frac{M_D}{2}$, applied to the end of each rib at B. The right end of the left-hand rib will be displaced horizontally to the left and, from symmetry, the left end of the right-hand rib will be displaced horizontally to the left an equal amount. When the ends are joined, there is, therefore, no horizontal restraint at B and hence no horizontal thrust is produced by the moment, M_D. There is, however, a tendency for the end of the left-hand rib to move down and that of the right-hand rib to move up an equal amount. As they are attached and are symmetrical about the pier axis, the ribs will restrain each other so that no vertical displacement of B occurs and a vertical shear, V_{BM}, will be produced.

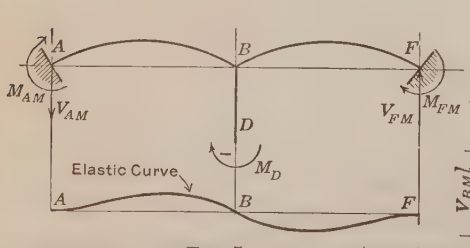


FIG. 5.

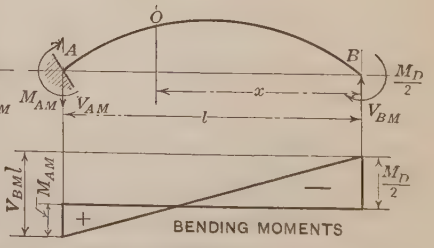


FIG. 6.

The rib, $A\ B$, will be considered separately (see Fig. 6) and the reactions, M_{AM} and V_{AM}, at A produced by M_D will be determined. At A the rib is entirely fixed and at B restrained vertically, although free to move horizontally and to rotate. Therefore, the vertical displacement at B is,

$$\int_B^A M_O\,x\,d\,w = 0$$

but,

$$M_O = \frac{M_D}{2} + V_{BM}\, x$$

so,

$$\int_B^A M_O\, x\, d\, w = \frac{M_D}{2}\int_B^A x\, d\, w + V_{BM}\int_B^A x^2\, d\, w$$

Then,

$$V_{BM} = -\frac{M_D}{2}\,\frac{\int_B^A x\, d\, w}{\int_B^A x^2\, d\, w} = -\frac{M_D}{2}\,\beta$$

in which,

$$\frac{\int_B^A x\, d\, w}{\int_B^A x^2\, d\, w} = \beta$$

Then, when $M_D = -1$,

$$\left.\begin{aligned} V_{AM} &= -V_{BM} = -\frac{\beta}{2} \\ M_{AM} &= \frac{M_D}{2} + V_{BM}\, l = \frac{\beta\, l - 1}{2} \\ M_{OM} &= \frac{\beta\, x - 1}{2} \\ M_{BM} &= \frac{M_D}{2} = -\frac{1}{2} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

Considering the elasticity of the ribs and the pier, the angular displacement at D due to $M_D = -1$ will be,

$$\phi_{DM} = \int_D^A M\, d\, w = \frac{1}{2}\int_B^A (\beta\, x - 1)\, d\, w - \int_D^B d\, w$$

In order to take into consideration also the elasticity of the foundation, the reactions at D will be thought of as acting, not directly on the end of the pier, but on a rigid member attached to the end of the pier by a hinge that has the same elastic properties as those assumed for the foundation. It will be assumed further that a unit value of M_D causes this hinge to rotate through a small angle which will be called ε. The angle due to the yielding of the foundations will be $M_D\, \varepsilon$ and the expression for the total angular displacement produced by $M_D = -1$ at its point of application then becomes,

$$\phi_{DM} = \frac{1}{2}\int_B^A (\beta\, x - 1)\, d\, w - \int_D^B d\, w - \varepsilon \quad \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Effect of Horizontal Thrust at D.—In so far as the effect on the arch ribs is concerned, the horizontal thrust acting at D (Fig. 7(*a*)) may be re-

placed by a thrust, H_D, acting through the elastic centers of gravity of the two ribs together with a moment at D equal to $-H_D\,(h+b)$. The resultant of this thrust and moment is the thrust, H_D, acting at D.

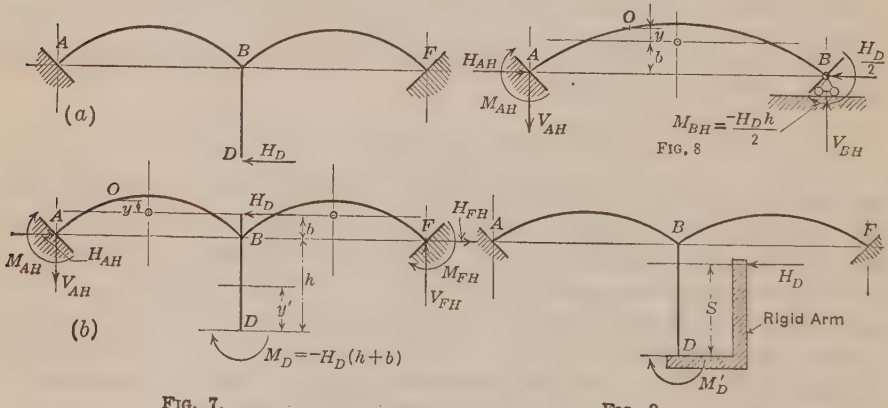
The thrust acting through the elastic centers of gravity of the two ribs produces only a horizontal displacement at B with no vertical or angular displacement. Because of symmetry, this thrust is equally divided between the two ribs, producing a tension in the right-hand rib and a compression in the left-hand rib.

Then,

$$H_{AH} = \frac{H_D}{2} \text{ and } M_{OH} = \frac{H_D\,y}{2}$$

This thrust produces moments at A and B equal to $\frac{H_D\,b}{2}$. The effect of a moment at the pier base, as just determined, is given by Equation (7). From this, the combined effect of thrust and moment when $H_D = 1$ will be as follows,

$$\left.\begin{aligned} V_{AH} &= -V_{BH} = -\frac{\beta}{2}(h+b) \\ M_{AH} &= \frac{b}{2} + (h+b)\frac{\beta l - 1}{2} = \frac{(h+b)\,\beta l}{2} - \frac{h}{2} \\ M_{BH} &= \frac{b}{2} - \frac{h+b}{2} = -\frac{h}{2} \\ M_{OH} &= \frac{y}{2} + (h+b)\frac{\beta x - 1}{2} = \frac{(h+b)\,\beta x}{2} - \frac{h+b+y}{2} \\ H_{AH} &= \frac{1}{2} \end{aligned}\right\} \quad \ldots\ldots(9)$$



These equations can also be derived by considering the rib, $A\ B$, as fixed at A and restrained vertically at B as in Fig. 8. Placing the vertical displacement equal to zero,

$$\int_B^A M_O\,x\,d\,w = 0$$

in which,

$$M_O = V_{BH}\,x - \frac{H_D}{2}(b + y) - \frac{H_D\,h}{2} = V_{BH}\,x - \frac{H_D}{2}(b + y + h)$$

Then,

$$\int_B^A M_O\,x\,d\,w = V_{BH}\int_B^A x^2\,d\,w - \frac{H_D}{2}(h + b)\int_B^A x\,d\,w$$
$$-\frac{H_D}{2}\int_B^A x\,y\,d\,w = 0$$

As the axis of ordinates, y, was chosen through the elastic center of gravity of the rib, $\int x\,y\,d\,w$ and $\int y\,d\,w$ are zero. Hence,

$$V_{BH}\int_B^A x^2\,d\,w - \frac{H_D}{2}(h + b)\int_B^A x\,d\,w = 0$$

and,

$$V_{BH} = \frac{H_D}{2}(h + b)\frac{\int_B^A x\,d\,w}{\int_B^A x^2\,d\,w} = \frac{H_D\,\beta}{2}(h + b)$$

from which Equations (9) follow.

The angular displacement, ϕ_{DH}, at the foot of the pier, D, produced by a unit value of H will be,

$$\phi_{DH} = \int_D^A M_O\,d\,w = \frac{1}{2}\int_B^A [(h + b)\,\beta\,x - (h + b + y)] - \int_D^B y'\,d\,w$$

or,

$$\phi_{DH} = \frac{h + b}{2}\int_B^A (\beta\,x - 1)\,d\,w - \int_D^B y'\,d\,w \ldots\ldots\ldots\ldots (10)$$

The angular displacement at D produced by unit values of thrust and moment at D have now been determined, and an equation can be written for the total angular displacement from the two combined:

$$\phi_D = H_D\,\phi_{DH} + M_D\,\phi_{DM}$$

from which,

$$M_D = \frac{\phi_D - H_D\,\phi_{DH}}{\phi_{DM}}$$

If this angular displacement is equal to zero,

$$M_D = -H_D\frac{\phi_{DH}}{\phi_{DM}}$$

When $H_D = 1$,

$$M_D = -\frac{\phi_{DH}}{\phi_{DM}}$$

This means that, if H_D acts on a rigid member connected to the base of the pier and is placed so that it produces at the base of the pier a moment equal to $-H_D\frac{\phi_{DH}}{\phi_{DM}}$, it will cause no rotation of this member. This condition will be satisfied when, in Fig. 9,

$$s = \frac{\phi_{DH}}{\phi_{DM}}$$

The total moment M_D at D will then be,

$$M_D = M'_D - H\,s\,.$$

As H_D thus placed produces no angular displacement at D, by Maxwell's theorem of reciprocal displacements, M'_D will produce no horizontal displacement of the point of application of H_D. These two forces, therefore, are independent of each other and can be considered separately.

Substituting Equations (8) and (10) in the expression for s:

$$s = \frac{\phi_{DH}}{\phi_{DM}} = \frac{\frac{h+b}{2}\int_B^A (\beta x - 1)\,d w - \int_D^B y'\,d w}{\frac{1}{2}\int_B^A (\beta x - 1)\,d w - \int_D^B d w - \varepsilon}$$

Placing

$$\frac{1}{2}\int_B^A (\beta x - 1)\,d w = \psi \qquad \text{.....(11)}$$

$$s = \frac{(h+b)\,\psi - \int_D^B y'\,d w}{\psi - \int_D^B d w - \varepsilon} \qquad \text{.....(12)}$$

Then Equations (9) become:

$$\left.\begin{aligned} V_{AH} &= -V_{BH} = -\frac{\beta}{2}(h+b-s) \\ M_{AH} &= \frac{(h+b-s)\,\beta\,l}{2} - \frac{h-s}{2} \\ M_{BH} &= -\frac{h-s}{2} \\ M_{OH} &= \frac{(h+b-s)\,\beta\,x}{2} - \frac{h+b-s+y}{2} \\ H_{AH} &= \frac{1}{2} \end{aligned}\right\} \qquad \text{.....(13)}$$

The values of β and ψ and the integrals for the pier can be obtained from the accompanying tables, as will be explained subsequently. The value of the angle, ϵ, can be computed if the size of the pier base and the elasticity of the supporting foundation are known. The latter value must be determined experimentally. The relative importance of the yielding of the foundation is indicated by the effect of ϵ on the value of s. If ϵ can be omitted with no appreciable effect on s, the yielding of the foundations can be neglected. It should be noted that the formulas herein derived can be used for the special case of a frictionless hinge at the base of the pier. In that case ϵ is infinitely great and s becomes zero.

Influence Line for M'_D.—The influence line for M'_D is the elastic curve of the rib, which gives the vertical displacement when $M'_D = -1$.

In Fig. 10, ϕ_{DM} is the angular displacement of the rigid member attached to the base of the pier produced by a unit value of M'_D. Let δ_m be the vertical

displacement of the points where the loads are applied to the rib when $M'_D = -1$, which displacement, by Maxwell's theorem, is equal to the angular displacement at D produced by the two loads.

The total angular displacement at D is, therefore,

$$\phi_D = M'_D \phi_{DM} - \left[\frac{P}{2}\delta_m + \frac{-P}{2}(-\delta_m)\right] = M'_D \phi_{DM} - P\delta_m$$

Since $\phi_D = 0$,

$$M'_D = P\frac{\delta_m}{\phi_{DM}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

From Equation (8),

$$\phi_{DM} = \frac{1}{2}\int_B^A (\beta x - 1)\, dw - \int_D^B dw - \varepsilon$$

or,

$$\phi_{DM} = \psi - \int_D^B dw - \varepsilon \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

The values of ψ and $\int_D^B dw$ are given in Tables 1 and 6, respectively.

The elastic curve for the vertical displacements of the rib is obtained from two elementary elastic curves. The bending moments in the rib are as shown in Fig. 11. (See, also, Equation (7).)

δ_1 is the vertical displacement of the rib when $M_A = M_B = -\frac{1}{2}$ and δ_2 is the displacement when $M_A = \frac{1}{2}$ and $M_B = 0$. The value of δ_m is then,

$$\delta_m = \delta_1 - \beta l \delta_2$$

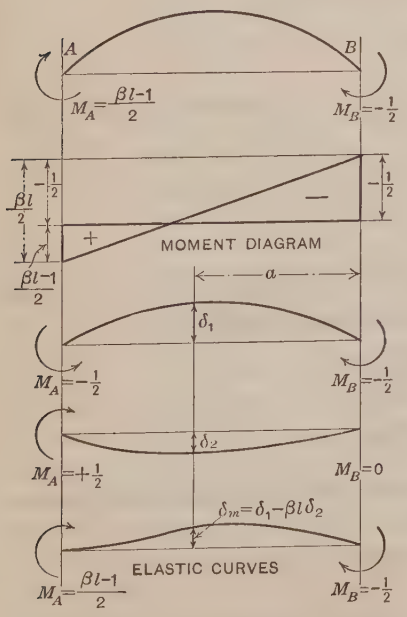


Fig. 11.

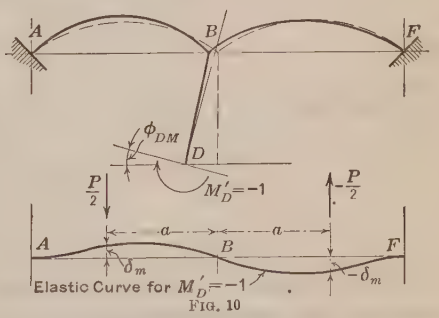


Fig. 10

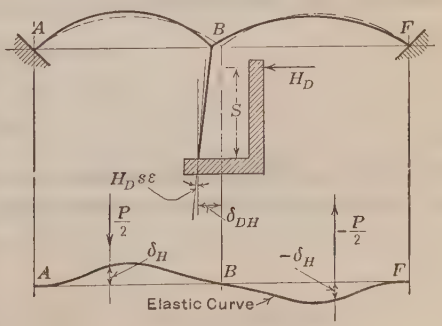


Fig. 12.

The values of δ_1 and δ_2 can be obtained from Tables 4 and 5 for any particular value of m, but δ_m is also given directly in Table 3. These values were obtained by integrating the differential equation for the elastic curves as explained for δ_8.

Influence Lines for H_D.—The influence line for H_D is the elastic curve of vertical displacements produced by $H_D = 1$. Let δ_{DH} be the horizontal displacement of D produced by $H_D = 1$, and let δ_H be the vertical displacement at the load point of the rib produced by $H_D = 1$, which, by Maxwell's theorem, is also the horizontal displacement at D produced by a unit vertical load on the rib. Then, the total horizontal displacement at the point of application of H_D is,

$$\Delta_D = H_D\,\delta_{DH} + \left[\frac{P}{2}\,\delta_H + \frac{-P}{2}(-\delta_H)\right] = H_D\,\delta_{DH} + P\,\delta_H$$

or, as $\Delta_D = 0$,

$$H_D = -P\,\frac{\delta_H}{\delta_{DH}} \qquad (17)$$

The value of δ_{DH} is negative and can be obtained from the equation,

$$\delta_{DH} = \int_D^A M_O\,y''\,d\,w$$

in which, y'' is the ordinate measured from the line of action of H_D.

For the arch rib, from A to B,

$$y'' = (h + b - s + y)$$

From Equation (13),

$$M_O = \frac{\beta\,x - 1}{2}(h + b - s) - \frac{y}{2}$$

then,

$$\int_B^A M_O\,y''\,d\,w = \frac{1}{2}\int_B^A\left[\left(h+b-s+y\right)\left\{\left(\beta x-1\right)\left(h+b-s\right)-y\right\}\right]d\,w$$

$$= \frac{1}{2}(h + b - s)^2\int_B^A(\beta\,x - 1)\,d\,w - \frac{1}{2}\int_B^A y^2\,d\,w$$

$$= (h + b - s)^2\,\psi - \frac{1}{2}\int_B^A y^2\,d\,w$$

If it is desired to consider the effect of shortening of the rib by direct stress, the last term should be multiplied by $(1 + u)$, in which,

$$u = \frac{\int\frac{\cos\phi\,d\,x}{E\,A}}{\int y^2\,d\,w}*$$

The horizontal displacement at the point of application of H_D, due to bending and direct stress in the rib, is then equal to:

$$(h + b - s)^2\,\psi - \frac{1+u}{2}\int_B^A y^2\,d\,w$$

* *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 945, Equation (20*a*).

For the pier,

$$M_O = s - y' \text{ and } y'' = -(s - y')$$

hence,

$$\int_D^B M_O\, y''\, d\,w = -\int_D^B (s - y')^2\, d\,w$$

$$= -s^2 \int_D^B d\,w + 2\,s \int_D^B y'\, d\,w - \int_D^B (y')^2\, d\,w$$

For the elastic foundation,

$$\int M_O\, y''\, d\,w = -H_D\, s^2\, \varepsilon = -s^2\, \varepsilon$$

then the complete expression is,

$$\delta_{DH} = (h + b - s)^2\, \psi - \frac{1+u}{2} \int_B^A y^2\, d\,w - s^2 \left(\varepsilon + \int_D^B d\,w\right)$$

$$- \int_D^B (y')^2\, d\,w + 2\,s \int_D^B y'\, d\,w \qquad (18)$$

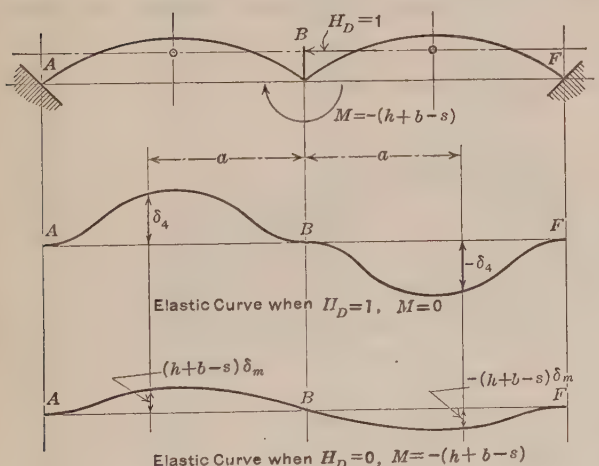


FIG. 13.

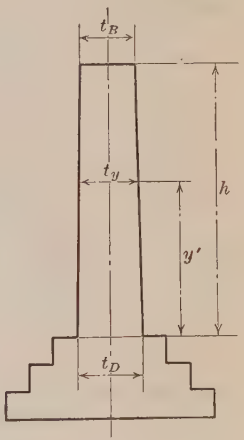


FIG. 14.

The value of ψ is given in Table 1 and the integrals for the pier in Table 6. The value of $\int_B^A y^2\, d\,w$ can be found in Table 18*, of the writer's paper entitled "Design of Symmetrical Concrete Arches."

By combining elastic curves that have already been determined, the value of δ_H can be obtained. As shown in Fig. 13, the thrust, H_D, can be resolved into a thrust, H_D, acting along the horizontal gravity axis of the arch rib, and a moment equal to $(h + b - s)\, H_D$. The elastic curves for the thrust and moment are obtained separately and their ordinates combined.

From Fig. 13 it is seen that this combined ordinate for the displacement produced by H_D is,

$$\delta_H = \delta_4 + (h + b - s)\, \delta_m \qquad (19)$$

* *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 991.

The value of δ_m has already been obtained (Table 3) and δ_1 can be computed by means of Figs. 28 to 32* and Table 18 in the writer's paper previously mentioned, from the formula,

$$\delta_4 = \frac{C\, C_H\, r\, l^2}{2\, E\, I_c} \qquad (20)$$

in which, C is found from Table 18 and C_H from Figs. 28 to 32.

Influence Lines for Case II.—When the reactions at the base of the pier have been obtained for different positions of the load, the moments and thrusts at the desired sections, due to the loads and to M_D' and H_D, are to be computed from Equations (5), (6), (7), and (13). These values are to be added together in order to obtain the total moments or thrusts, as, for instance:

$$M_A'' = M_{AP} + M_{AM} + M_{AH}$$

Value of Integrals for Arch Ribs and Piers.—In order to facilitate the solution of the foregoing equations the values of the integrals for the ribs and the piers have been determined and are given in Tables 1 to 6. Using the arch rib as presented in the previous paper, the following formulas have been derived (Fig. 15):

$$d\,w = \frac{l}{2\, E\, I_c}\,[1 - (1 - m)\, z]\, d\, z\dagger$$

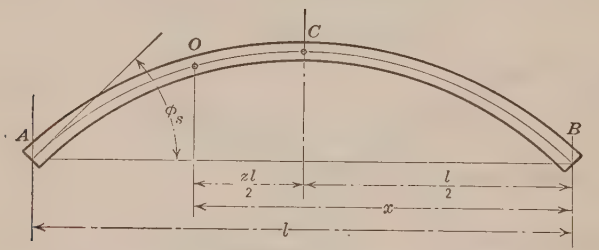


FIG. 15.

By integration,

$$\int_B^A d\,w = \frac{l}{E\, I_c}\,\frac{1+m}{2} \qquad (21)$$

$$\int_B^A x\, d\,w = \frac{l^2}{E\, I_c}\,\frac{1+m}{4} \qquad (22)$$

$$\int_B^A x^2\, d\,w = \frac{l^3}{E\, I_c}\,\frac{7+9\,m}{48} \qquad (23)$$

$$\beta = \frac{\int x\, d\,w}{\int x^2\, d\,w} = \frac{12}{l}\,\frac{1+m}{7+9\,m} \qquad (24)$$

$$\beta \int x\, d\,w = \frac{l}{E\, I_c}\,\frac{3\,(1+m)^2}{7+9\,m} \qquad (25)$$

$$\psi = \frac{\beta}{2}\int_B^A x\, d\,w - \frac{1}{2}\int_B^A d\,w = -\frac{l}{E\, I_c}\,\frac{1+4\,m+3\,m^2}{28+36\,m} \qquad (26)$$

* *Transactions*, Am. Soc. C. E., Vol. 88 (1925), pp. 1004–1008.

† From Equation (54), *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 983.

Tables 2 to 5 give the elastic curves for the conditions indicated which were obtained by integrating the differential equation,

$$\frac{d^2\,\delta}{d\,z^2} = -\frac{M\,l^2}{4\,E\,I_c}\,[1-(1-m)\,z]^*$$

It was not thought necessary to give the resulting equations here.

TABLE 1.—Arch Rib Constants.

Value of m.	$\frac{E\,I_c}{l}\int d\,w$ (Equation (21)).	$\frac{E\,I_c}{l^2}\int x\,d\,w$ (Equation (22)).	$\frac{E\,I_c}{l^3}\int x^2\,d\,w$ (Equation (23)).	$\beta\,l$ (Equation (24)).	$\frac{E\,I_c}{l}\psi$ (Equation (26))
0.15	0.575	0.2875	0.1740	1.652	—0.04991
0.20	0.600	0.3000	0.1833	1.637	—0.05455
0.21	0.605	0.3025	0.1852	1.633	—0.05546
0.22	0.610	0.3050	0.1871	1.630	—0.05638
0.23	0.615	0.3075	0.1890	1.627	—0·05730
0.24	0.620	0.3100	0.1908	1.625	—0.05821
0.25	0.625	0.3125	0.1927	1.622	—0.05912
0.26	0.630	0.3150	0.1946	1.619	—0.06003
0.27	0.635	0.3175	0.1965	1.616	—0.06094
0.28	0.640	0.3200	0.1983	1.614	—0.06185
0.29	0.645	0.3225	0.2002	1.611	—0.06275
0.30	0.650	0.3250	0.2021	1.608	—0.06366
0.31	0.655	0.3275	0.2040	1.605	—0.06456
0.32	0.660	0.3300	0.2058	1.603	—0.06547
0.33	0.665	0.3325	0.2077	1.601	—0.06636
0.34	0.670	0.3350	0.2096	1.598	—0.06727
0.35	0.675	0.3375	0.2115	1.596	—0.06817
0.36	0.680	0.3400	0.2133	1.594	—0.06906
0.37	0.685	0.3425	0.2152	1.592	—0.06996
0.38	0.690	0.3450	0.2171	1.589	—0.07085
0.39	0.695	0.3475	0.2190	1.587	—0.07175
0.40	0.700	0.3500	0.2208	1·585	—0.07264
0.45	0.725	0.3625	0.2302	1.575	—0.07709
0.50	0.750	0.3750	0.2396	1.567	—0.08152

In order to derive formulas for the integrals for the pier, it has been assumed that the length of the pier is uniform from top to bottom and that the sides have a straight batter (Fig. 14). Then, placing $\frac{t_D}{t_B} = k$, it follows that,

$$\frac{t_y}{t_B} = 1 + (k-1)\,\frac{h-y'}{h} = \frac{1}{h}\,(k\,h - k\,y' + y')$$

Then assuming that,

$$\frac{I_y}{I_B} = \frac{t_y^3}{t_B^3} = \frac{(k\,h - k\,y' + y')^3}{h^3}$$

$$I_y = \frac{I_B}{h^3}\,(k\,h - k\,y' + y')^3$$

in which,

I_B = moment of inertia of the section at the top.

I_y = moment of inertia of any section.

Then,

$$\int_D^B d\,w = \int_0^h \frac{d\,y'}{E\,I_y} = \frac{h^3}{E\,I_B}\int_0^h \frac{d\,y'}{[k\,h + (1-k)\,y']^3} = \frac{h}{E\,I_B}\,\frac{1+k}{2\,k^2} \quad \ldots(27)$$

Similarly,

$$\int_D^B y'\,d\,w = \frac{h^3}{E\,I_B}\int_0^h \frac{y'\,d\,y'}{[k\,h + (1-k)\,y']^3} = \frac{h^2}{E\,I_B}\cdot\frac{1}{2\,k} \quad \ldots(28)$$

* Equation (55), *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 983.

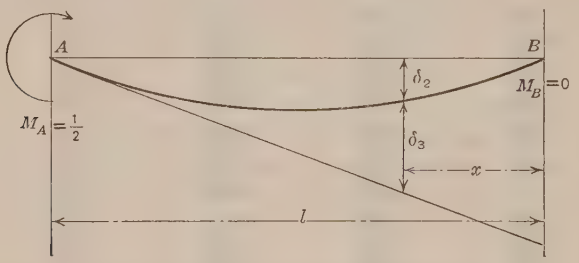
Also,

$$\int_D^B (y')^2 = \frac{h^3}{E\,I_B}\int_0^h \frac{(y')^2\,d\,y'}{[k\,h + (1-k)\,y']^3}$$

$$= \frac{h^3}{E\,I_B}\,\frac{k^2 - 4\,k + 3 + 2\log k}{2\,(k-1)^3} \qquad (29)$$

By adjusting the value of k, these formulas can be made to apply quite closely when there is a batter at the ends of the pier as well as the sides.

TABLE 2.—VALUE OF $\delta_3 \frac{E\,I_c}{l^2} = C_2$.

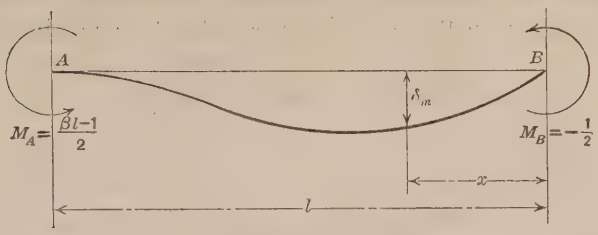


$\frac{x}{l}$	VALUES OF m.					
	0.15	0.20	0.25	0.30	0.40	0.50
1.00	0·00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.95	0.00011	0.00014	0.00017	0.00019	0.00026	0.00032
0.90	0.00050	0.00061	0.00073	0.00083	0.00106	0.00129
0.85	0.00125	0.00148	0.00173	0.00198	0.00245	0.00293
0.80	0.00242	0.00282	0.00324	0.00363	0.00455	0.00527
0.75	0.00408	0.00469	0.00529	0.00589	0.00709	0.00830
0.70	0.00628	0.00711	0.00793	0.00875	0.01039	0.01204
0.65	0.00907	0.01013	0.01119	0.01224	0.01435	0.01648
0.60	0.01245	0.01376	0.01507	0.01637	0.01898	0.02160
0.55	0.01646	0.01802	0.01960	0.02114	0.02427	0.02740
0.50	0.02109	0.02291	0.02474	0.02656	0.03021	0.03385
0.45	0.02634	0.03841	0.03050	0.03258	0.03675	0.04092
0.40	0.03210	0.03444	0.03680	0.03913	0.04388	0.04852
0.35	0.03626	0.04078	0.04351	0.04610	0.05135	0.05657
0.30	0.04477	0.04765	0.05056	0.05343	0.05922	0.06500
0.25	0.05151	0.06469	0.05787	0.06103	0.06738	0.07373
0.20	0.05844	0.07191	0.06538	0.06883	0.07576	0.08269
0.15	0.06549	0.07926	0.07302	0.07678	0.08430	0.09183
0.10	0.07262	0.08669	0.08075	0.08482	0.09295	0.10108
0.05	0.07979	0.08417	0.08855	0.09292	0.10166	0.11042
0.00	0.08698	0.09167	0.09635	0.10104	0.11042	0.11979

NUMERICAL EXAMPLES

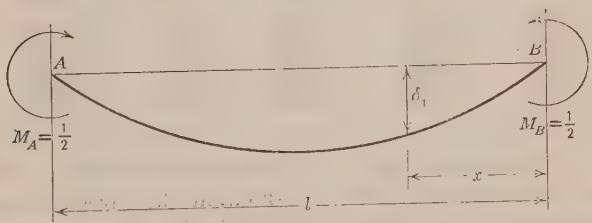
In order to show the application of the foregoing method of analysis and the effect of varying the proportions of the pier, two examples will be solved and the resulting influence lines compared with the influence lines for fixed arches. The proportions of the pier in the first example are more slender than would be used in bridge construction. For the second example, the ribs have been kept the same, but the pier has been doubled in thickness for the purpose of bringing the proportions within the limits for practical design. The second

TABLE 3.—VALUE OF $\delta_m \frac{E I_c}{l^2}$.



$\frac{x}{l}$	Values of m.					
	0.15	0.20	0.25	0.30	0.40	0.50
1.00	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.95	0.00007	0.00008	0.00010	0.00011	0.00014	0.00016
0.90	0.00031	0.00036	0.00042	0.00048	0.00058	0.00066
0.85	0.00074	0.00085	0.00097	0.00109	0.00129	0.00147
0.80	0.00138	0.00155	0.00174	0.00192	0.00226	0.00254
0.75	0.00221	0.00245	0.00271	0.00297	0.00343	0.00384
0.70	0.00321	0.00352	0.00385	0.00418	0.00477	0.00530
0.65	0.00434	0.00472	0.00511	0.00551	0.00621	0.00685
0.60	0.00553	0.00597	0.00642	0.00687	0.00769	0.00843
0.55	0.00671	0.00720	0.00771	0.00820	0.00911	0.00994
0.50	0.00779	0.00831	0.00886	0.00939	0.01037	0.01128
0.45	0.00866	0.00921	0.00977	0.01034	0.01138	0.01236
0.40	0.00924	0.00985	0.01040	0.01098	0.01206	0.01308
0.35	0.00946	0.01007	0.01064	0.01122	0.01235	0.01338
0.30	0.00930	0.00980	0.01046	0.01105	0.01216	0.01321
0.25	0.00871	0.00928	0.00985	0.01040	0.01148	0.01252
0.20	0.00771	0.00824	0.00876	0.00927	0.01028	0.01124
0.15	0.00630	0.00675	0.00721	0.00766	0.00852	0.00938
0.10	0.00451	0.00486	0.00519	0.00556	0.00622	0.00687
0.05	0.00238	0.00258	0.00279	0.00300	0.00337	0.00375
0.00	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

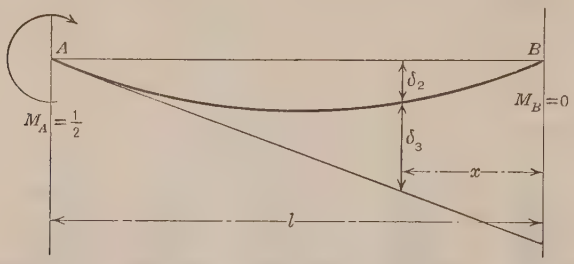
TABLE 4.—VALUE OF $\delta_1 \frac{E I_c}{l^2}$.



$\frac{x}{l}$	Values of m.						$\frac{x}{l}$
	0.15	0.20	0.25	0.30	0.40	0.50	
1.00	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.00
0.95	0.007076	0.007358	0.007641	0.007923	0.008487	0.009052	0.05
0.90	0.013858	0.014367	0.014875	0.015383	0.016400	0.017417	0.10
0.85	0.020241	0.020925	0.02161	0.022294	0.023663	0.025031	0.15
0.80	0.026117	0.026933	0.027750	0.028567	0.030200	0.031833	0.20
0.75	0.031380	0.032292	0.033203	0.034115	0.035937	0.037760	0.25
0.70	0.035925	0.036900	0.037875	0.038850	0.040800	0.042750	0.30
0.65	0.039645	0.040658	0.041672	0.042685	0.044712	0.046740	0.35
0.60	0.042433	0.043467	0.044500	0.045533	0.047600	0.049667	0.40
0.55	0.044183	0.045225	0.046266	0.047306	0.049387	0.051469	0.45
0.50	0.044792	0.045833	0.046875	0.047917	0.050000	0.052083	0.50

system has also been analyzed by an approximate method and the influence lines compared with the results of the more exact method in order to determine the importance of the errors introduced by the approximations.

TABLE 5.—VALUE OF $\delta_2 \frac{E I_c}{l^2}$.



$\frac{x}{l}$	Values of m.					
	0.15	0.20	0.25	0.30	0.40	0.50
1.00	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.95	0.00424	0.00444	0.00465	0.00486	0.00526	0.00567
0.90	0.00820	0.00856	0.00891	0.00927	0.00998	0.01069
0.85	0.01180	0.01227	0.01273	0.01319	0.01411	0.01504
0.80	0.01498	0.01551	0.01604	0.01657	0.01763	0.01869
0.75	0.01766	0.01823	0.01880	0.01937	0.02051	0.02165
0.70	0.01981	0.02039	0.02098	0.02156	0.02273	0.02390
0.65	0.02137	0.02196	0.02254	0.02312	0.02429	0.02545
0.60	0.02234	0.02291	0.02348	0.02404	0.02518	0.02632
0.55	0.02268	0.02323	0.02377	0.02432	0.02541	0.02651
0.50	0.02240	0.02292	0.02344	0.02396	0.02500	0.02604
0.45	0.02150	0.02200	0.02250	0.02299	0.02398	0.02496
0.40	0.02009	0.02056	0.02102	0.02149	0.02242	0.02335
0.35	0.01828	0.01870	0.01913	0.01957	0.02042	0.02129
0.30	0.01612	0.01651	0.01690	0.01729	0.01807	0.01885
0.25	0.01372	0.01406	0.01440	0.01475	0.01543	0.01611
0.20	0.01114	0.01142	0.01171	0.01200	0.01257	0.01314
0.15	0.00844	0.00866	0.00888	0.00910	0.00955	0.00999
0.10	0.00566	0.00581	0.00597	0.00611	0.00642	0.00673
0.05	0.00284	0.00292	0.00299	0.00306	0.00323	0.00338
0.00	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

Example I.—The system used in the first example is shown in Fig. 16. The ribs and pier are assumed to be 1 ft. thick in the direction normal to the plane of the rib axis. The base of the pier is fixed rigidly, as are the outer ends of the ribs at the abutments. The ribs are the same as were used in the examples of the writer's paper on "Design of Symmetrical Concrete Arches", and detailed dimensions are given in Fig. 19* therein. The effect of rib-shortening has been neglected in these examples.

The properties of this rib are, as follows:

$l = 100$ ft.	$y_c = 3.15$ ft.	$I_c = 6\,399$ in.$^4 = 0.309$ ft.^{4}
$r = 15$ ft.	$d_c = 18$ in.	$I_s = 24\,560$ in.$^4 = 1.185$ ft.^{4}
$g = 4.70$	$d_s = 27$ in.	$b = 11.85$ ft.
$m = 0.339$	$\cos \phi_s = 0.769$	$h + b = 71.85$ ft.

* *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 961.

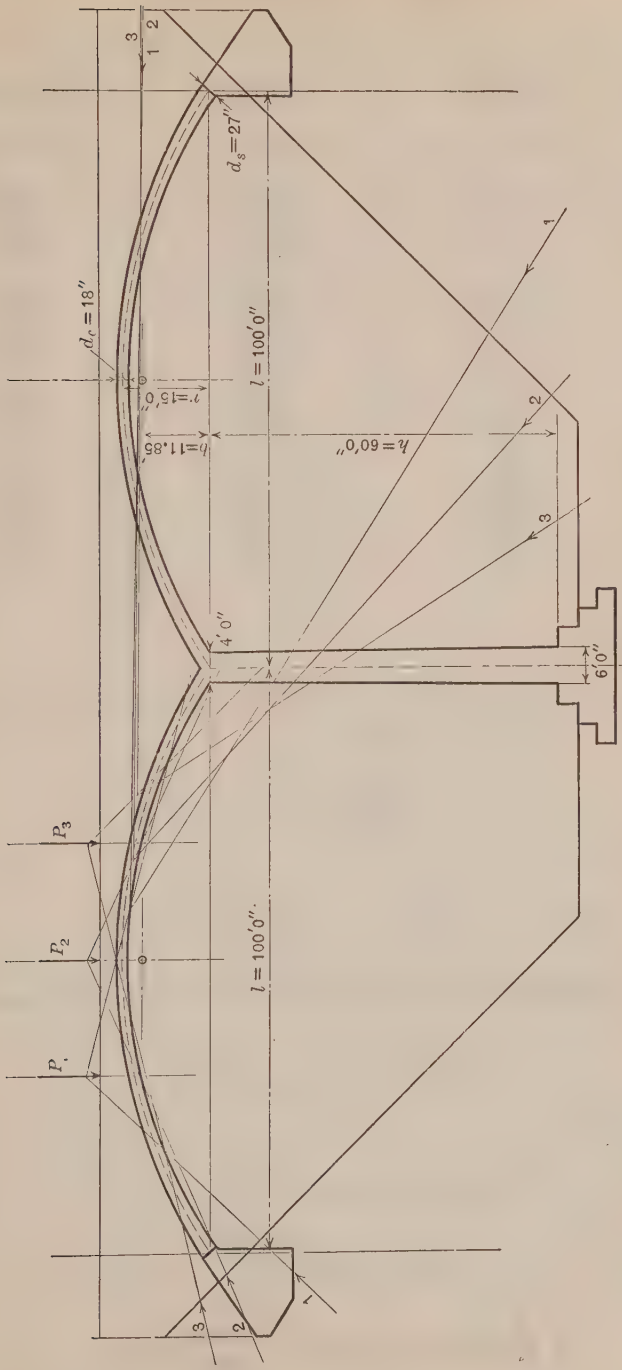

3
1 2
$d_s = 27''$
$d_c = 18''$
$l = 100'0''$
$r = 15'0''$
$h = 11.85$
$h = 60'0''$
1
2
3
4'0''
6'0''
P_3
P_2
P_1
$l = 100'0''$
3
2
1

FIG. 16.

TABLE 6.—Pier Constants.

Value of k.	$\frac{E I_B}{h}\int d\,w$. (Equation (27)).	$\frac{E I_B}{h^2}\int y'\,d\,w$. (Equation (28)).	$\frac{E I_B}{h^3}\int (y')^2\,d\,w$. (Equation (29)).
1.0	1.0000	0.5000	0.3333
1.1	0.8677	0.4545	0.3100
1.2	0.7632	0.4167	0.2902
1.3	0.6815	0.3846	0.2727
1.4	0.6122	0.3571	0.2576
1.5	0.5555	0.3333	0.2437
1.6	0.5078	0.3125	0.2315
1.7	0.4674	0.2941	0.2205
1.8	0.4321	0.2739	0.2105
1.9	0.4017	0.2632	0.2014
2.0	0.3750	0.2500	0.1931
2.1	0.3514	0.2381	0.1855
2.2	0.3306	0.2273	0.1785
2.3	0.3119	0.2174	0.1720
2.4	0.2951	0.2083	0.1659
2.5	0.2800	0.2000	0.1604
2.6	0.2663	0.1923	0.1552
2.7	0.2538	0.1852	0.1503
2.8	0.2423	0.1785	0.1457
2.9	0.2319	0.1724	0.1414
3.0	0.2222	0.1667	0.1373

Then, from Table 1,

$$\beta\, l = 1.60\,; \quad \beta = 0.016$$

$$\frac{E I_c}{l}\,\psi = -\,0.06718$$

$$E\,\psi = \frac{-\,0.06718 \times 100}{0.309} = -\,21.73$$

For the pier,

$$h = 60 \text{ ft.}; \quad t_B = 4.0 \text{ ft.}; \quad t_D = 6.0 \text{ ft.}; \quad k = \frac{6.0}{4.0} = 1.5$$

Increasing the moment of inertia of the concrete 10% to allow for reinforcement,

$$I_B = 1.1 \times \frac{4^3}{12} = 5.87 \text{ ft.}^4$$

From Table 6,

$$E\int d\,w = 0.5555 \times \frac{60}{5.87} = 5.67$$

$$E\int y'\,d\,w = 0.3333 \times \frac{3\,600}{5.87} = 204.4$$

$$E\int (y')^2\,d\,w = 0.2437 \times \frac{216\,000}{5.87} = 8\,970$$

Then,

$$s = \frac{(h + b)\,\psi - \int_D^B y'\,dw}{\psi - \int_D^B dw - \varepsilon} = \frac{-(71.85 \times 21.73) - 204.4}{-21.73 - 5.67 - 0} = 64.4 \text{ ft.}$$

$$h + b - s = 71.85 - 64.4 = 7.45 \text{ ft.}$$

The reactions due to the loads can now be determined by Equations (5) and (6). Table 7 gives the computations. From Table 1, C_1 is 0.2094.

TABLE 7.—Effect of Loads.

Load point, $\frac{x}{l}$.	C_2 (From Table 2).	V_{BP} $\frac{C_2}{C_1}$.	M_{AP} $\left(V_{BP} l - \frac{l-a}{2}\right)$.	V_{AP} $\left(\frac{1}{2} - V_{BP}\right)$.
10	0.00000	0.0000	— 0.00	0.5000
9	0.00092	0.0044	— 4.56	0.4956
8	0.00399	0.0191	— 8.09	0.4809
7	0.00939	0.0448	—10.52	0.4552
6	0.01739	0.0831	—11.69	0.4169
5	0.02798	0.1337	—11.64	0.3664
4	0.04096	0.1956	—10.44	0.3044
3	0.05569	0.2660	— 8.40	0.2340
2	0.07153	0.3420	— 5.84	0.1580
1	0.08799	0.4200	— 3.00	0.0800
0	0.10470	0.5000	— 0.00	0.0000

The influence line for H_D will next be determined by Equations (17), (18) and (19):

$$H_D = \frac{-\delta_H}{\delta_{DH}}$$

$$\delta_{DH} = (h + b - s)^2\,\psi - \frac{1}{2}\int_B^A y^2\,dw - s^2\left(\varepsilon + \int_D^B dw\right) - \int_D^B (y')^2\,dw + 2s\int_D^B y'\,dw$$

$$(h + b - s)^2\,\psi = \frac{-7.45 \times 7.45 \times 21.73}{E} = -\ 1\,205 \div E$$

$$-\frac{1}{2}\int_B^A y^2\,dw = -\ 0.0408^* \frac{100 \times 225}{E \times 2 \times 0.309} = -\ 1\,487 \div E$$

$$-s^2\left(\varepsilon + \int_D^B dw\right) = \frac{-64.4 \times 64.4\,(0 + 5.67)}{E} = -\ 23\,530 \div E$$

$$-\int_D^B (y')^2\,dw = -\ 8\,970 \div E$$

$$\text{Sum} = -\ 35\,192 \div E$$

$$+2s\int_D^B y'\,dw = \frac{204.4 \times 2 \times 64.4}{E} = +\ 26\,350 \div E$$

$$\delta_{DH} = -\ 8\,842 \div E$$

$$\delta_H = \delta_4 + (h + b - s)\,\delta_m$$

* Value of C.

in which,

$$\delta_4 = \frac{C\, C_H\, r\, l^2}{2\, E\, I_c}$$

The value of C from Table 18* of the writer's paper previously mentioned, is 0.0408 and the values of C_H are taken from Figs. 28 to 32† of the same paper. δ_m is given by Table 3. The computation of H_D is given in Table 8.

TABLE 8.—COMPUTATION OF H_D.

Load point, $\frac{x}{l}$	C_H.	$E\,\delta_4 = \frac{C\, C_H\, r\, l^2}{2\, I_c}$.	$\delta_m \frac{E\, I_c}{l_2}$ (Table 3).	$(h + b - s)\, \delta_m\, E$.	$E\,\delta_H$.	$H_D = \frac{-\delta_H}{\delta_{DH}}$.
10	0.0000	0	0.00000	0.	0.	0.0000
9	0.0287	284	0.00052	125.	409.	0.0463
8	0.0972	965	0.00205	494.	1 459.	0.1651
7	0.1762	1 748	0.00441	1 062.	2 810.	0.3180
6	0.2385	2 365	0.00719	1 738.	4 098.	0.4635
5	0.2630	2 604	0.00977	2 358.	4 962.	0.5615
4	0.2385	2 365	0.01140	2 747.	5 112.	0.5780
3	0.1762	1 748	0.01148	2 765.	4 513.	0.5110
2	0.0972	965	0.00966	2 330.	3 295.	0.3725
1	0.0287	284	0.00582	1 404.	1 688.	0.1910
0	0.0000	0	0.00000	0.	0.	0.0000

The moments and thrusts produced by H_D are found from Equations (13), which were derived for unit values of H_D. These are given in Table 9.

TABLE 9.—EFFECT OF H_D.

Load point, $\frac{x}{l}$	$V_{AH} = -V_{BH} = \frac{-\beta}{2}(h+b-s)\, H_D$.	$H_{AH} = \frac{H_D}{2}$.	$M_{BH} = -\frac{h-s}{2} H_D$	$M_{AH} = H_D \left[\frac{(h+b-s)\,\beta\, l}{2} - \frac{h-s}{2}\right]$.	$M_{DH} = H_D\, s$.
10	— 0.0000	0.0000	0.000	0.000	0.00
9	— 0.0028	0.0232	0.102	0.378	2 98
8	— 0.0098	0.0826	0.363	1.345	10.64
7	— 0.0182	0.1590	0.700	2.590	20.50
6	— 0.0276	0.2318	1.020	3.775	29.85
5	— 0.0334	0.2808	1.235	4.580	36.15
4	— 0.0344	0.2890	1.270	4.715	37.25
3	— 0.0304	0.2555	1.122	4.170	32.90
2	— 0.0222	0.1863	0.820	3.036	24.00
1	— 0.0114	0.0955	0.420	1.556	12.30
0	— 0.0000	0.0000	0.000	0.000	0.00

The influence-line ordinates for M_D' are next computed from Equation (14):

$$M_D' = P\, \frac{\delta_m}{\phi_{DM}}$$

$$\phi_{DM} = \psi - \int_D^B d\, w - \varepsilon = -\frac{21.73}{E} - \frac{5.67}{E} - 0 = -\frac{27.4}{E}$$

The computation of M_D' is given in Table 10; also, the effect of M_D' from Equation (7).

* *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 991.

† *Loc. cit.*, pp. 1004–1008.

TABLE 10.—COMPUTATION OF M_D'.

Load point, $\frac{x}{l}$.	$\delta_m \frac{E I_c}{l^2}$ (Table 3).	$M'_D = \frac{\delta_m}{\phi_{DM}}$.	$V_{AM} = -V_{BM} = M_D' \frac{\beta}{2}$.	$M_{AM} = -M_D' \left(\frac{\beta l - 1}{2}\right)$.	$M_{BM} = \frac{M_D'}{2}$.
10	0.00000	— 0.00	—0.0000	0.00	—0.00
9	0.00052	— 0.62	—0.0049	0.18	—0.31
8	0.00205	— 2.43	—0.0194	0.73	—1.22
7	0.00441	— 5.21	—0.0417	1.56	—2.61
6	0.00719	— 8.52	—0.0680	2.55	—4.26
5	0.00977	—11.55	—0.0923	3.46	—5.78
4	0.01140	—13.50	—0.1080	4.05	—6.75
3	0.01148	—13.60	—0.1087	4.08	—6.80
2	0.00966	—11.42	—0.0913	3.43	—5.71
1	0.00582	— 6.88	—0.0550	2.06	—3.44
0	0.00000	— 0.00	—0.0000	0.00	—0.00

The final value of moments and thrusts for Case II loading are obtained by adding together algebraically the partial values for loading, H_D and M_D', as follows:

$$M_A'' = M_{AP} + M_{AM} + M_{AH}$$
$$M_B'' = M_{BP} + M_{BM} + M_{BH}$$
$$M_D'' = M_D' + M_{DH}$$
$$H_D'' = H_D$$
$$H_A'' = \frac{H_D}{2}$$
$$V_A'' = V_{AP} + V_{AM} + V_{AH}$$
$$V_B'' = V_{BP} + V_{BM} + V_{BH} = \frac{1}{2} - V_A''$$
$$V_D'' = 0$$

The moment at the crown section has also been computed from the formula,

$$M_c'' = \frac{V_A'' \, l}{2} - H_A'' \, r + M_A'' - \frac{P \, x'}{2}$$

All these final values are given in Table 11. For the right-hand span all values are the same as corresponding values for the left-hand span with the sign changed. The values given in Table 11 for Case I were obtained by dividing by 2 the corresponding values found for the fixed arch rib in the writer's previous paper.*

The sums of the values for Case I and Case II are the values for the general case as presented in Table 11. For comparison these values have been plotted in Figs. 18, 19, and 20, together with the corresponding influence lines for the two solutions of Example II and for the single-span fixed arch. Fig. 16 shows the reaction lines for a load at three different points on the left-hand arch rib.

Example II.—A complete solution has been made for a two-span system similar to that treated in Example I, except that the pier has been made twice

* *Transactions*, Am. Soc. C. E., Vol. 88 (1925), Table 19, p. 1001.

TABLE 11.—EXAMPLE I. COMPUTATION OF STRESSES.

(Comparison of Influence Lines for Examples I, II, and II(a), Is Given in Figs. 18, 19, and 20.)

$\frac{x}{l}$.	H_A'.	H_A''.	H_A.	V_A'.	V_A''.	V_A.	V_B'.	V_B''.	V_B.	M_D.
1.0	0.0000	0.0000	0.000	0.500	0.5000	1.000	0.000	0.000	0.000	0.000
0.9	0.096	0.023	0.112	0.489	0.488	0.977	0.011	0.012	0.023	2.36
0.8	0.324	0.088	0.412	0.457	0.452	0.909	0.043	0.048	0.091	8.21
0.7	0.587	0.159	0.746	0.403	0.395	0.798	0.097	0.105	0.202	15.29
0.6	0.795	0.232	1.027	0.332	0.321	0.653	0.168	0.179	0.347	21.35
0.5	0.876	0.281	1.157	0.250	0.241	0.491	0.250	0.259	0.509	24.60
0.4	0.795	0.289	1.084	0.168	0.162	0.330	0.332	0.338	0.670	23.75
0.3	0.587	0.256	0.843	0.097	0.095	0.192	0.403	0.405	0.808	19.30
0.2	0.324	0.186	0.510	0.043	0.044	0.087	0.457	0.456	0.913	12.58
0.1	0.096	0.096	0.192	0.011	0.014	0.025	0.489	0.486	0.975	5.42
0.0	0.000	0.000	0.000	0.000	0.000	0.000	0.500	0.500	1.000	0.00
0.0′	0.000	0.000	0.000	0.000	0.000	0.000	0.500	−0.500	0.000	0.00
0.1′	0.096	−0.096	0.000	0.011	−0.014	−0.003	0.489	−0.486	+0.003	− 5.42
0.2′	0.324	−0.186	0.138	0.043	−0.044	−0.001	0.457	− 0.456	+0.001	− 12.58
0.3′	0.587	−0.256	0.331	0.097	−0.095	+0.002	0.403	−0.405	−0.002	−19.30
0.4′	0.795	−0.289	0.506	0.168	−0.162	+0.006	0.332	−0.338	− 0.006	−23.75
0.5′	0.876	−0.281	0.595	0.250	−0.241	+0.009	0.250	−0.259	−0.009	−24.60
0.6′	0.795	−0.232	0.563	0.332	−0.321	+0.011	0.168	−0.179	−0.011	−21.35
0.7′	0.587	−0.159	0.428	0.403	−0.395	+0.008	0.097	−0.105	−0.008	−15.29
0.8′	0.324	−0.088	0.236	0.457	−0.452	+0.005	0.043	−0.048	−0.005	− 8.21
0.9′	0.096	−0.023	0.073	0.489	−0.488	+0.001	0.011	−0.012	−0.001	− 2.36
1.0′	0.000	0.000	0.000	0.500	−0.500	0.000	0.000	0.000	−0.000	0.00

TABLE 11.—(*Continued*).

$\frac{x}{l}$.	H_D.	V_D.	M_A.	M_A''.	M_A.	M_B'.	M_B''.	M_B.	M_C'.	M_C''.	M_C.
1.0	0.046	0.000	0.00	0.00	0.00	0.000	0.00	0.00	0.000	0.000	0.00
0.9	0.165	0.022	−3.19	−4.00	−7.19	+0.76	−0.21	+0.55	−0.15	+0.04	−0.11
0.8	0.318	0.087	−3.36	−6.02	−9.38	+2.31	−0.84	+1.47	−0.39	+0.31	−0.08
0.7	0.463	0.195	−1.59	−6.37	−7.96	+3.69	−1.88	+1.81	−0.27	+0.98	+0.71
0.6	0.561	0.337	+0.94	−5.36	−4.42	+4.07	−3.20	+0.87	+0.58	+2.23	+2.81
0.5	0.578	0.500	+3.10	−3.60	−0.50	+3.10	−4.54	−1.44	+2.46	+4.22	+6.68
0.4	0.511	0.663	+4.07	−1.67	+2.40	+0.94	−5.46	−4.52	+0.58	+2.09	+2.67
0.3	0.373	0.805	+3.69	−0.15	+3.54	−1.59	−5.66	−7.25	−0.27	+0.77	+0.50
0.2	0.191	0.913	+2.31	+0.67	+2.98	−3.36	−4.88	−8.24	−0.39	+0.11	−0.28
0.1	0.000	0.978	+0.76	+0.62	+1.38	−3.19	−3.02	−6.21	−0.15	−0.13	−0.28
0.0	0.000	1.000	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.0′	0.000	1.000	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.1′	−0.000	0.978	+0.76	−0.62	0.14	−3.19	3.02	−0.17	−0.15	+0.13	−0.02
0.2′	−0.191	0.913	+2.31	−0.67	1.64	−3.36	4.88	+1.52	−0.39	−0.11	−0.50
0.3′	−0.373	0.805	+3.69	+0.15	3.84	−1.59	5.66	+4.07	−0.27	−0.77	−1.04
0.4′	−0.511	0.663	+4.07	+1.67	5.74	+0.94	5.46	+6.40	+0.58	−2.09	−1.51
0.5′	−0.578	0.500	+3.10	+3.60	6.70	+3.10	4.54	+7.64	+2.46	−4.22	−1.76
0.6′	−0.561	0.337	+0.94	+5.36	6.30	+4.07	3.20	+7.27	+0.58	−2.23	−1.65
0.7′	−0.463	0.195	−1.59	+6.37	4.78	+3.69	1.88	+5.57	−0.27	−0.98	−1.25
0.8′	−0.318	0.087	−3.36	+6.02	2.66	+2.31	0.84	+3.15	−0.39	−0.31	−0.70
0.9′	−0.165	0.022	−3.19	+4.00	0.81	+0.76	0.21	+0.97	−0.15	−0.04	−0.19
1.0′	−0.046	0.000	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

as thick. The properties of the ribs remain the same and those of the pier are as follows:

$$h = 60 \text{ ft.}; \; t_B = 8.0 \text{ ft.}; \; t_D = 12.0 \text{ ft.}$$

$$k = 1.5; \; I_B = 1.1 \times \frac{8^3}{12} = 46.85$$

$$E \int d\,w = 0.5555 \times \frac{60}{46.85} = 0.710$$

$$E \int y'\, d\,w = 0.3333 \times \frac{3\,600}{46.85} = 25.6$$

$$E \int (y')^2\, d\,w = 0.2437 \times \frac{216\,000}{46.85} = 1\,123$$

$$s = \frac{-(71.85 \times 21.73) - 25.6}{-21.73 - 0.71} = 70.75$$

$$h + b - s = 71.85 - 70.75 = 1.10 \text{ ft.}$$

This value is very much smaller than that in the previous example. The size of $(h + b - s)$ may be used as a criterion of the importance of the effect of the elasticity of the pier. It should be noted that with a perfectly rigid pier,

$$\int_D^B d\,w = 0$$

$$\int_D^B y'\, d\,w = 0$$

and,

$$s = \frac{(h + b)\,\psi + 0}{\psi + 0} = h + b$$

Then,

$$h + b - s = 0$$

The final results of this analysis are given in Table 12. The Case I values are the same as those for the first example given in Table 11. Influence lines are plotted in Figs. 18, 19, and 20.

Example II.—Approximate Methods.—The arch system of Example II will be analyzed by a simple approximate method, which appears to give results that are reasonably close and err on the side of safety.

The effect on the arches of the elasticity of the pier is to permit the end of the arch ribs at the pier to move horizontally without vertical movement. This decreases the horizontal thrust in the loaded span by an amount, ΔH, and changes the bending moment by an amount, ΔHy. If H' and M_O' are the horizontal thrust and moment, respectively, at any point in the arch rib when its ends are fixed, the corresponding values with elastic supports will be, approximately,

$$\left.\begin{aligned} H &= H' + \Delta H \\ M_O &= M_O' - \Delta H\, y \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (30)$$

This assumption is made for the unloaded as well as for the loaded span. The effect of the elasticity of the supports on the vertical reactions is neglected. In the case of the symmetrical two-span system, the decrease in horizontal

thrust in the loaded span will be equal to the increase in the unloaded span, and the thrust resisted by the pier will be,

$$H_D = H' + 2\,\Delta H \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (31)$$

This is true because the increase in length of the loaded span is the same as the decrease in the unloaded span, and, as their resistances are equal, the values of ΔH must be equal. This decrease in the length of the unloaded span is,

$$\delta_B = -\Delta H\,(1 + u) \int_B^C y^2\, d\,w$$

which is also equal to the movement of the top of the pier. The forces acting on the pier will be assumed as shown in Fig. 17, and the displacement will be computed at the springing line, B.

The reason for these assumptions should be noted. Actually, the resultant thrust from the loaded span does not follow the horizontal gravity axis of the ribs but intersects the pier axis at some point below, depending on the position of the load. For the position at the crown at which the load produces the greatest thrust, the resultant cuts the pier axis not far from the springing line. The average position of H, therefore, could be assumed to be at the springing line without great error.

Also, the displacement that effects the value of the horizontal thrust in the rib is not the displacement at B but that at B' which lies on the horizontal gravity axis of the rib. On account of the bending of the pier, the displacement at B' is greater than at B. It would be proper, therefore, to assume the thrust, H', to act at B, the thrusts, ΔH, to act at B' and to compute the displacement at B'. However, the displacement at B' produced by H' acting at B is the same as the displacement at B produced by H' acting at B'. It would, therefore, give the same results to use the displacement at B produced by H' at B' and $2\,\Delta H$ at B. For the sake of simplicity the thrust, $2\,\Delta H$, is also assumed to act at B', but this does not introduce a great error as ΔH is comparatively small. This error tends to compensate that introduced by assuming the thrust, H', to act at the springing line.

The thrust on the pier from the loaded span will be $(H' + \Delta H)$ and will be assumed to act along the horizontal gravity axis of the arch rib. The thrust from the unloaded span is $-\Delta H$, which likewise acts along the horizontal gravity axis of the corresponding rib. For the purpose of determining the deflection of the pier, the moment at any section of the pier will be,

$$M = (H' + 2\,\Delta H)\,(h + b - y')$$

The horizontal displacement at the top of the pier, B, due to bending of the pier will then be,

$$\delta_B = \int_B^D M\,(h - y')\,d\,w = (H' + 2\,\Delta H) \int_B^D (h + b - y')\,(h - y')\,d\,w$$

The horizontal displacement due to the elasticity of the foundation will be,

$$M_D\,h\,\varepsilon = (H' + 2\,\Delta H)\,(h + b)\,h\,\varepsilon$$

TABLE 12.—EXAMPLE II. COMPUTATION OF STRESSES.

(Comparison of Influence Lines for Examples I, II, and II(a), Is Given in Figs. 18, 19, and 20.)

$\frac{x}{l}$.	H_A'.	H_A''.	H_A.	V_A'.	V_A''.	V_A.	V_B'.	V_B''.	V_B.	M_D.	H_D.
1.0	0.000	0.000	0.000	0.500	0.5000	1.000	0.000	0.000	0.000	0.000	0.000
0.9	0.096	0.056	0.152	0.489	0.489	0.978	0.011	0.011	0.022	7.17	0.112
0.8	0.324	0.202	0.526	0.457	0.454	0.911	0.043	0.046	0.089	25.69	0.405
0.7	0.587	0.372	0.959	0.403	0.398	0.801	0.097	0.102	0.199	46.1	0.743
0.6	0.795	0.510	1.305	0.332	0.325	0.657	0.168	0.175	0.343	61.8	1.020
0.5	0.876	0.558	1.434	0.250	0.243	0.493	0.250	0.257	0.507	67.2	1.150
0.4	0.795	0.540	1.335	0.168	0.163	0.331	0.332	0.337	0.669	59.8	1.079
0.3	0.587	0.419	1.006	0.097	0.094	0.191	0.403	0.406	0.809	42.6	0.838
0.2	0.324	0.255	0.579	0.043	0.042	0.085	0.457	0.458	0.915	22.2	0.510
0.1	0.096	0.096	0.192	0.011	0.011	0.022	0.489	0.489	0.978	5.1	0.191
0.0	0.000	0.000	0.000	0.000	0.000	0.000	0.500	0.500	1.000	0.0	0.000
0.0′	0.000	0.000	0.000	0.000	0.000	0.000	0.500	−0.500	0.000	0.0	0.000
0.1′	0.096	−0.096	0.000	0.011	−0.011	0.000	0.489	−0.489	0.000	− 5.1	− 0.191
0.2′	0.324	−0.255	0.069	0.043	−0.042	0.001	0.457	−0.458	−0.001	−22.2	− 0.510
0.3′	0.587	−0.419	0.168	0.097	−0.094	0.003	0.403	−0.406	−0.003	−42.6	− 0.838
0.4′	0.795	−0.540	0.255	0.168	−0.163	0.005	0.332	−0.337	−0.005	−59.8	− 1.079
0.5′	0.876	−0.558	0.318	0.250	−0.243	0.007	0.250	−0.257	−0.007	−67.2	− 1.150
0.6′	0.795	−0.510	0.285	0.332	−0.325	0.007	0.168	−0.175	−0.007	−61.8	− 1.020
0.7′	0.587	−0.372	0.215	0.403	−0.398	0.005	0.097	−0.102	−0.005	−46.1	− 0.743
0.8′	0.324	−0.202	0.122	0.457	−0.454	0.003	0.043	−0.046	−0.003	−25.69	− 0.405
0.9′	0.096	−0.056	0.040	0.489	−0.489	0.000	0.011	−0.011	0.000	− 7.17	− 0.112
1.0′	0.000	0.000	0.000	0.500	0.500	0.000	0.000	0.000	0.000	0.000	0.000

TABLE 12.—(*Continued*).

$\frac{x}{l}$.	M_A'.	M_A''.	M_A.	M_B'.	M_B''.	M_B.	M_C'.	M_C''.	M_C.
1.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.9	−3.19	−3.63	−6.82	+0.76	+0.23	+0.99	−0.15	−0.04	−0.19
0.8	−3.36	−4.67	−8.03	+2.31	+0.70	+3.01	−0.39	−0.03	−0.42
0.7	−1.59	−3.97	−5.56	+3.69	+0.71	+4.40	−0.27	+0.35	+0.08
0.6	+0.94	−2.21	−1.27	+4.07	+0.30	+4.37	+0.58	+1.39	+1.97
0.5	+3.10	−0.23	+2.87	+3.10	−0.90	+2.10	+2.46	+3.30	+5.76
0.4	+4.07	+1.24	+5.31	+0.94	−2.50	−1.56	+0.58	+1.30	+1.88
0.3	+3.69	+1.80	+5.49	−1.59	−3.80	−5.39	−0.27	+0.23	−0.04
0.2	+2.31	+1.57	+3.88	−3.36	−4.20	−7.56	−0.39	−0.16	−0.55
0.1	+0.76	+0.71	+1.47	−3.19	−3.20	−6.39	−0.15	−0.17	−0.32
0.0	+0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.0′	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.1′	+0.76	−0.71	0.05	−3.19	+3.20	0.01	−0.15	+0.17	+0.02
0.2′	+2.31	−1.57	0.74	−3.36	+4.20	0.84	−0.39	+0.16	−0.23
0.3′	+3.69	−1.80	1.89	−1.59	+3.80	2.21	−0.27	−0.23	−0.50
0.4′	+4.07	−1.24	2.83	+0.94	+2.50	3.44	+0.58	−1.30	−0.72
0.5′	+3.10	+0.23	3.33	+3.10	+0.90	4.00	+2.46	−3.30	−0.84
0.6′	+0.94	+2.21	3.15	+4.07	−0.30	3.77	+0.58	−1.39	−0.81
0.7′	−1.59	+3.97	2.38	+3.69	−0.71	2.98	−0.27	−0.35	−0.62
0.8′	−3.36	+4.67	1.31	+2.31	−0.70	1.53	−0.39	+0.03	−0.36
0.9′	−3.19	+3.63	0.44	+0.76	−0.23	0.53	−0.15	+0.04	−0.11
1.0′	0.00	0.00	0.00	+0.00	0.00	0.00	0.00	0.00	0.00

Adding this to the previous expression for displacement, the total due to elasticity of pier and foundation will be,

$$\delta_B = (H' + 2\,\Delta\,H)\left[\int_B^D (h + b - y')\,(h - y')\,d\,w + (h + b)\,h\,\varepsilon\right]$$

Placing,

$$\left[\int_B^D (h + b - y')\,(h - y')\,d\,w + (h + b)\,h\,\varepsilon\right] = K$$

$$\delta_B = H'\,K + 2\,\Delta\,H\,K$$

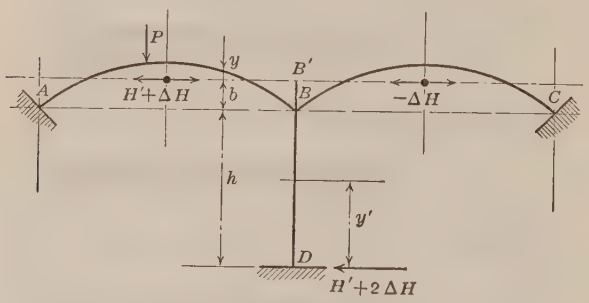


Fig. 17.

From consideration of the arch rib it was found that,

$$\delta_B = -\,\Delta\,H\,(1 + u)\int_B^C y^2\,d\,w = -\,\Delta\,H\,(1 + u)\int_A^B y^2\,d\,w$$

Then, as the end of the rib is attached to the end of the pier, these displacements are the same, and,

$$-\,\Delta\,H\,(1 + u)\int_A^B y^2\,d\,w = H'\,K + 2\,\Delta\,H\,K$$

from which,

$$\Delta\,H = -\,H'\,\frac{K}{(1 + u)\int_A^B y^2\,d\,w + 2\,K} \qquad \ldots\ldots\ldots\ldots (32)$$

The value of K reduces to,

$$K = h\,(h + b)\int_D^B d\,w - (2\,h + b)\int_D^B y'\,d\,w + \int_D^B (y')^2\,d\,w + (h + b)\,h\,\varepsilon \qquad \ldots\ldots\ldots\ldots (33)$$

The value of the integrals can be taken from Table 6.

The moment at the top of the pier will be approximately,

$$M_E = M_B' + 2\,\Delta\,H\,b \qquad \ldots\ldots\ldots\ldots (34)$$

and at the foot of the pier,

$$M_D = M_B' + 2\,\Delta\,H\,b + (H' + 2\,\Delta\,h)\,h$$
$$= M_B' + H'\,H + 2\,\Delta\,H\,(h + b) \qquad \ldots\ldots\ldots\ldots (35)$$

M_B' and H' are the values for the fixed-end arch.

Numerical Example of Approximate Method, Two Spans.—This approximate solution will be applied to the system of Example II so that the results may be compared with those given by the more exact method. It was found that,

$$\frac{1}{2}\int_A^B y^2\,dw = \frac{1\,487}{E}, \text{ or } \int_A^B y^2\,dw = \frac{2\,974}{E}$$

Also,

$$E\int_D^B dw = 0.710$$

$$E\int_D^B y'\,dw = 25.6$$

$$E\int_D^B (y')^2\,dw = 1\,123$$

Then,

$$\begin{aligned} K = +\,60 \times 71.85 \times 0.71 \div E &= +\,3\,060 \div E \\ -\,131.85 \times 25.6 \quad \div E &= -\,3\,375 \div E \\ &\quad +\,1\,123 \div E \\ \text{Sum} &= +\quad 808 \div E \end{aligned}$$

and,

$$\Delta H = -H'\frac{808}{2\,974 + 1\,616} = -0.176\,H'$$

$$H = 0.824\,H'$$

For a load at the crown,

$$H = 0.824 \times 1.752 = 1.443$$

The value obtained by the accurate method was 1.434. The calculations of influence line ordinates for the moments and horizontal thrust are given in Table 13. The vertical reactions are assumed to be the same as for the fixed arch.

The influence lines found by this approximate method are plotted in Figs. 18, 19, and 20, and show a fairly close agreement with those obtained by the other method. It should be noted that the proportions of the system analyzed are unusually slender. The pier is unusually high and the arch ribs have a low rise; also the width of the pier has been taken the same as that of the arch ribs. If separate ribs are used the pier will be wider than the ribs and the relative stiffness of the pier will be increased. As the relative stiffness of the pier increases and its effect on the stresses in the rib decreases, the accuracy of the foregoing approximate method increases. It is suggested that the value of $\frac{h + b - s}{b}$ might be used to indicate the magnitude of the effect of the elasticity of the pier. In Example I,

$$\frac{h + b - s}{b} = \frac{7.45}{11.85} = 0.628$$

TABLE 13.—EXAMPLE II, APPROXIMATE METHOD. COMPUTATION OF STRESSES.

(Comparison of Influence Lines for Examples I, II, and II (*a*), Is Given in Figs. 18, 19, and 20.)

$\frac{x}{l}$.	H_A'.	ΔH.	H_A.	M_A'.	ΔM_A.	M_A.	M_B'.	ΔM_B.	M_B.	M_C'.	ΔM_C.	M_C.	$H_A' h$.	$2\Delta H(h+b)$.	M_D.*	H_D.
1.0	0.000	0.000	0.000	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	00.0	0.0	00.0	0.000
0.9	0.191	−0.034	0.157	−6.38	−0.40	−6.78	+1.52	−0.40	+1.12	−0.31	0.11	−0.20	11.5	4.9	08.1	0.123
0.8	0.648	−0.114	0.534	−6.72	−1.35	−8.07	+4.61	−1.35	+3.26	−0.77	0.36	−0.41	38.9	16.4	27.1	0.420
0.7	1.174	−0.207	0.967	−3.18	−2.45	−5.63	+7.38	−2.45	+4.93	−0.53	0.65	+0.08	70.5	29.8	48.1	0.760
0.6	1.590	−0.280	1.310	+1.87	−3.32	−1.45	+8.14	−3.32	+4.82	+1.16	0.88	+2.04	95.4	40.3	63.2	1.030
0.5	1.751	−0.308	1.443	+6.19	−3.65	+2.54	+6.19	−3.65	+2.54	+4.92	0.97	+5.89	105.6	44.3	67.5	1.135
0.4	1.590	−0.280	1.310	+8.14	−3.32	+4.82	+1.87	−3.32	−1.45	+1.16	0.88	+2.04	95.4	40.3	57.0	1.030
0.3	1.174	−0.207	0.967	+7.38	−2.45	+4.93	−3.18	−2.45	−5.63	−0.53	0.65	+0.08	70.5	29.8	37.5	0.760
0.2	0.648	−0.114	0.534	+4.61	−1.35	+3.26	−6.72	−1.35	−8.07	−0.77	0.36	−0.41	38.9	16.4	15.8	0.420
0.1	0.191	−0.034	0.157	+1.52	−0.40	+1.12	−6.38	−0.40	−6.78	−0.31	0.11	−0.20	11.5	4.9	0.2	0.123
0.0	0.000	0.000	0.000	0.00	0.00	0.00	0.0	0.00	0.00	0.00	0.00	0.00	00.0	0.0	0.0	0.000
0.0	.....	0.000	0.000	.....	0.00	0.00	.....	0.00	0.00	.....	0.00	0.00	00.0	0.0	00.0	0.000
0.1	.....	0.034	0.034	.....	0.40	0.40	.....	0.40	+0.40	.....	−0.11	−0.11	− 11.5	− 4.9	− 0.2	−0.123
0.2	.....	0.114	0.114	.....	1.35	1.35	.....	1.35	+1.35	.....	−0.36	−0.36	− 38.9	−16.4	−15.8	−0.420
0.3	.....	0.207	0.207	.....	2.45	2.45	.....	2.45	+2.45	.....	−0.65	−0.65	− 70.5	−29.8	−37.5	−0.760
0.4	.....	0.280	0.280	.....	3.32	3.32	.....	3.32	+3.32	.....	−0.88	−0.88	− 95.4	−40.3	−57.0	−1.030
0.5	.....	0.308	0.308	.....	3.65	3.65	.....	3.65	+3.65	.....	−0.97	−0.97	−105.6	−44.3	−67.5	−1.135
0.6	.....	0.280	0.280	....	3.32	3.32	.....	3.32	+3.32	.....	−0.88	−0.88	− 95.4	−40.3	−63.2	−1.030
0.7	.....	0.207	0.207	.....	2.45	2.45	.....	2.45	+2.45	.....	−0.65	−0.65	− 70.5	−29.8	−48.1	−0.760
0.8	.....	0.114	0.114	.....	1.35	1.35	.....	1.35	+1.35	.....	−0.36	−0.36	− 38.9	−16.4	−27.1	−0.420
0.9	.....	0.034	0.034	.....	0.40	0.40	.....	0.40	+0.40	.....	−0.11	−0.11	− 11.5	− 4.9	−08.1	−0.123
1.0	.....	0.000	0.000	.....	0.00	0.00	.....	0.00	0.00	.....	0.00	0.00	00.0	0.0	00.0	0.000

* $M_D = H_A' h + M_B' - 2\Delta H(h+b)$.

and in Example II,

$$\frac{h + b - s}{b} = \frac{1.10}{11.85} = 0.093$$

The computation of this value will give some idea as to whether or not the approximate solution may be used.

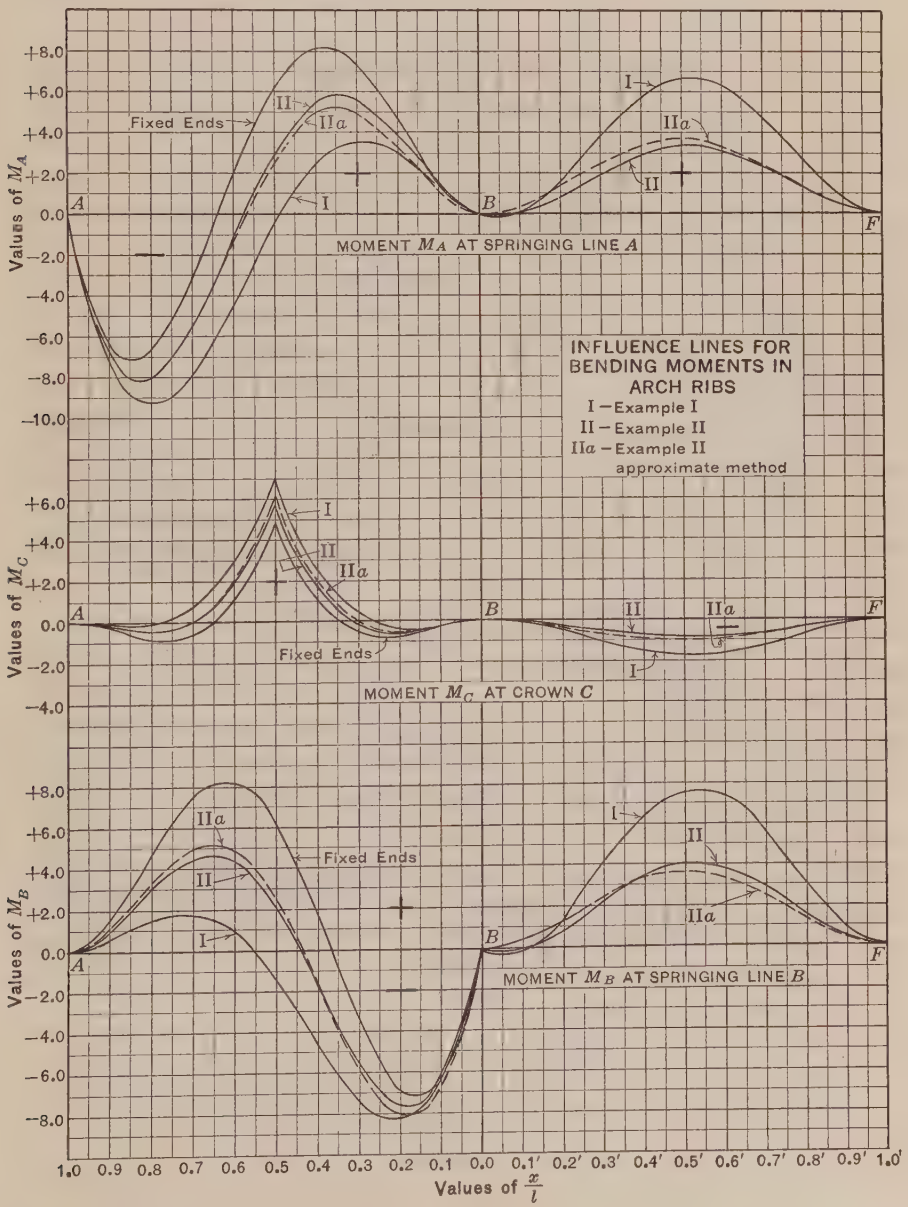


FIG. 18.

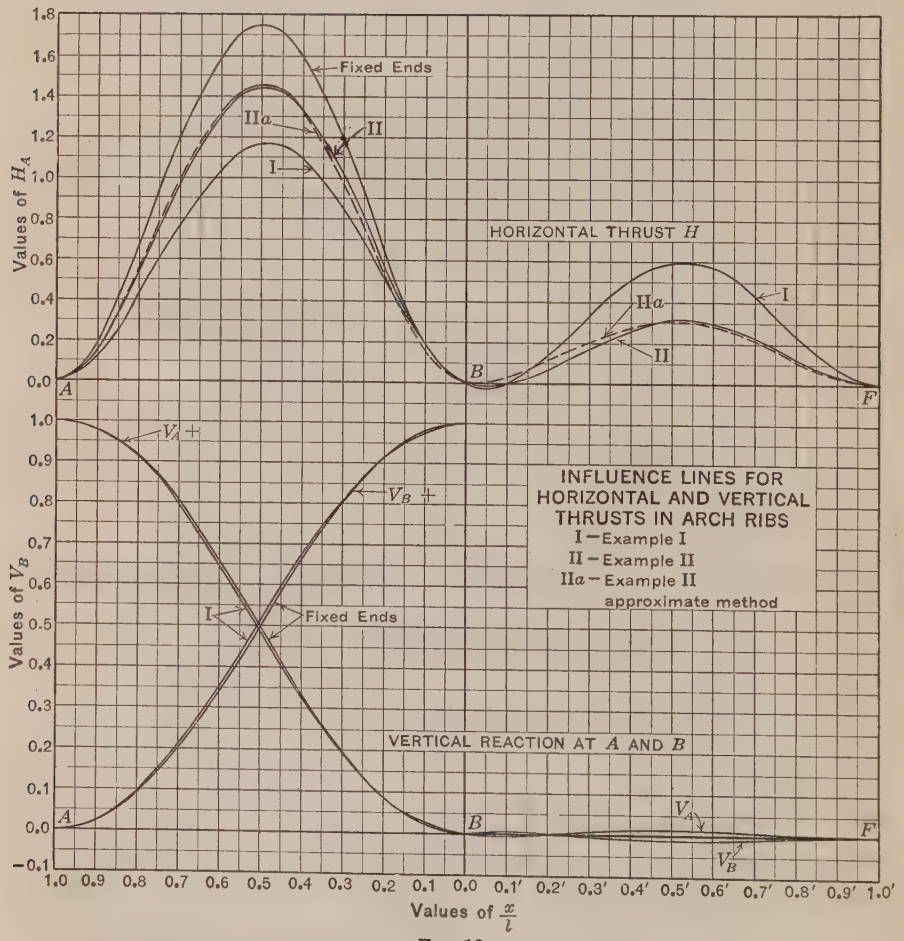

Values of H_A
Fixed Ends
IIa
II
I
HORIZONTAL THRUST H
A
B
F
Values of V_B
V_A +
V_B +
INFLUENCE LINES FOR HORIZONTAL AND VERTICAL THRUSTS IN ARCH RIBS
I — Example I
II — Example II
IIa — Example II approximate method
Fixed Ends
VERTICAL REACTION AT A AND B
V_A
V_B
Values of $\frac{x}{l}$

FIG. 19.

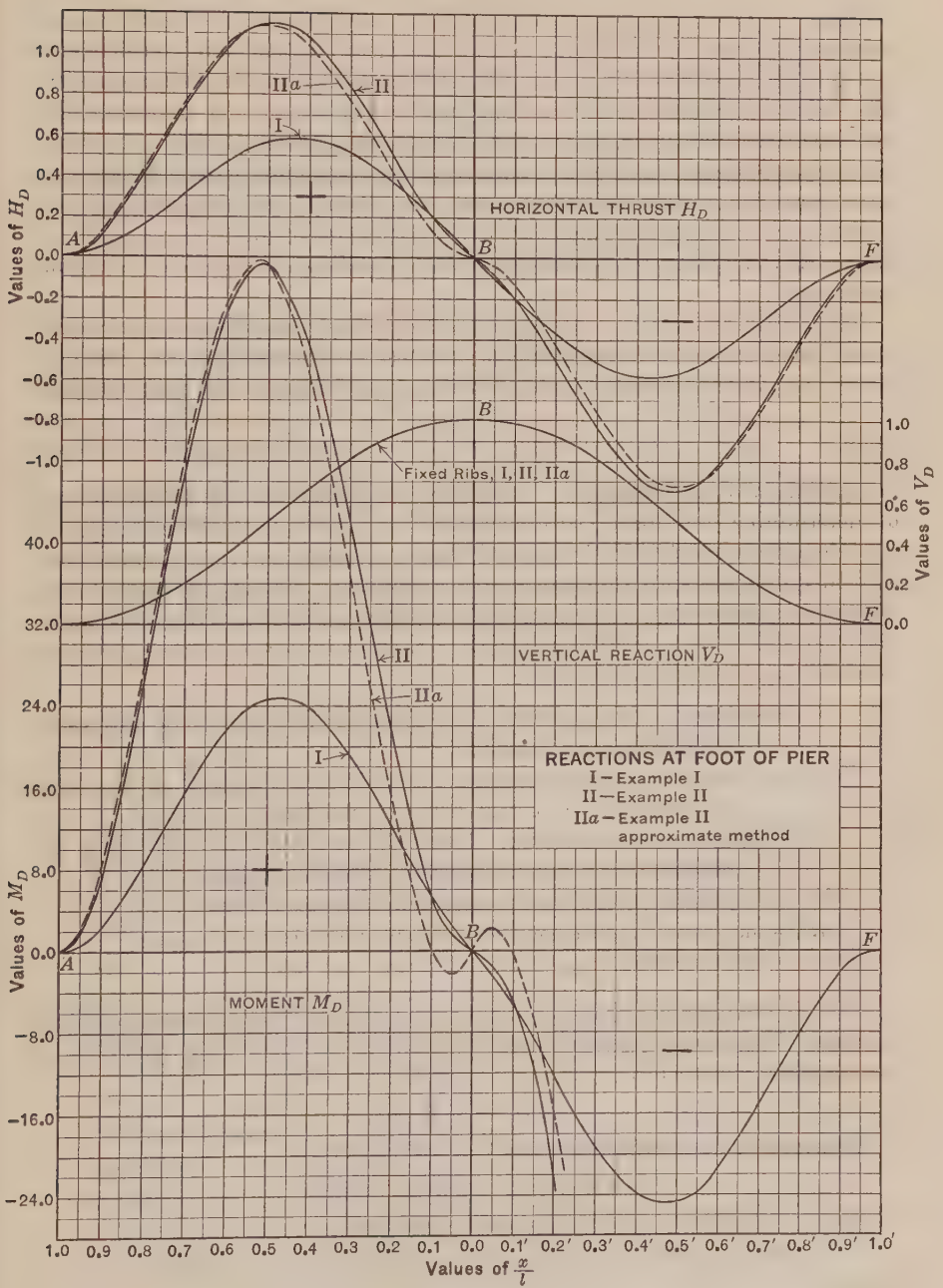

HORIZONTAL THRUST H_D
Values of H_D
VERTICAL REACTION V_D
Values of V_D
MOMENT M_D
Values of M_D
Fixed Ribs, I, II, IIa
REACTIONS AT FOOT OF PIER
I—Example I
II—Example II
IIa—Example II approximate method
Values of $\frac{x}{l}$

FIG. 20.

Approximate Method for Any Number of Spans

Thus far, only the two-span system has been considered. If an arch system consisted of a number of spans, and all the piers but one were considered to be rigid, the methods previously presented could be used to determine the effect of the flexibility of that one pier on the adjoining spans, assuming that the outer ends of the spans are fixed. If the approximate method is accurate enough to determine the effect of this one pier, the assumptions on which it is based must be suitable for determining the effect of the flexibility of any other pier or the simultaneous effect of the flexibility of all piers, provided, of course, that the spans are similar and the proportions of the piers are about the same.

The writer believes that an approximate method for systems with several spans, based on the same assumptions as those herein used for the two-span system, will have the same degree of accuracy as the latter and that the same value of $\frac{h + b - s}{b}$ that was computed for the two-span system can be used as an indication of its suitability. If this is true, it provides a simple method of determining the limits within which the approximate method may be used to obtain results of the desired degree of accuracy. An approximate method for the analysis of systems of several spans will now be derived from the same assumptions as were used for the two-span system. It is not necessary that all the spans shall be the same. In order to derive general formulas, the following notation will be used:

δ_h = change in length of the span of the rib produced by a unit value of the horizontal thrust.
δ_r = horizontal displacement of the right-hand support of the arch rib produced by a unit value of the horizontal thrust.
δ_l = horizontal displacement of the left-hand support of the arch rib produced by a unit value of the horizontal thrust.
H' = horizontal thrust for the arch rib with fixed ends.
H = horizontal thrust for the arch rib with elastic supports.
$H = H' + \Delta H$

ΔH is then the change in horizontal thrust, due to the elasticity of the supports. The change in span length of the arch rib, due to the flexibility of the supports, is seen to be,

$$\delta = -\Delta H \, \delta_h$$

The sum of the displacements of the two supports is,

$$\delta = (H' + \Delta H)(\delta_r + \delta_l)$$

These two quantities must be the same, therefore,

$$-\Delta H \, \delta_h = (H' + \Delta H)(\delta_r + \delta_l) = H'(\delta_r + \delta_l) + \Delta H (\delta_r + \delta_l)$$

Then,

$$-\Delta H (\delta_h + \delta_r + \delta_l) = H'(\delta_r + \delta_l)$$

and,

$$\Delta H = - H' \frac{\delta_r + \delta_l}{\delta_h + \delta_r + \delta_l} \ldots\ldots\ldots\ldots\ldots\ldots (36)$$

The value of δ_h for the arch rib is given by the expression,

$$\delta_h = \int y^2 \, d \, w \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (37)$$

If it is desired to take into account the effect of rib-shortening due to thrust, this expression becomes,

$$\delta_h = (1 + u) \int y^2 \, d \, w \, * \ldots\ldots\ldots\ldots\ldots\ldots (38)$$

in which,

$$u = \frac{\int \frac{\cos \phi \, d \, x}{E \, A}}{\int y^2 \, d \, w}$$

Referring to Fig. 21, the value of δ_l, the horizontal displacement of the left support, L, of the loaded span when $H = 1$, will now be determined.

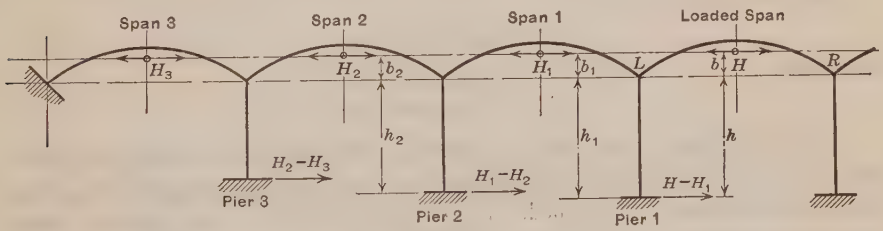


FIG. 21.

The displacement of the right end of the rib of Span 1 will be $H \, \delta_{l1}$, due to the yielding of its left-hand support, L_1, plus $H_1 \, \delta_{h1}$ due to the shortening of the rib itself, or

$$\delta_l = H_1 (\delta_{h1} + \delta_{l1}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (39)$$

The terms, δ_{h1} and δ_{l1}, have the same meaning in regard to Span 1 as δ_h and δ_l have for the loaded span.

If δ_{P1} is the horizontal displacement of the top of Pier 1 produced by a unit value of $(H - H_1)$, then the yielding at L will also be equal to,

$$\delta_l = (H - H_1) \, \delta_{P1}$$

As the displacement of the pier top is the same as that of the end of the arch rib at L,

$$H_1 (\delta_{h1} + \delta_{l1}) = (H - H_1) \, \delta_{P1}$$

$$H_1 (\delta_{h1} + \delta_{l1} + \delta_{P1}) = H \, \delta_{P1}$$

$$H_1 = H \frac{\delta_{P1}}{\delta_{h1} + \delta_{l1} + \delta_{P1}} \ldots\ldots\ldots\ldots\ldots\ldots (40)$$

* *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 945, Equation (20*a*).

Similarly,

$$H_2 = H_1 \frac{\delta_{P2}}{\delta_{h2} + \delta_{l2} + \delta_{P2}}$$

Placing,

$$\frac{\delta_{P1}}{\delta_{h1} + \delta_{l1} + \delta_{P1}} = v_1$$

and,

$$\frac{\delta_{P2}}{\delta_{h2} + \delta_{l2} + \delta_{P2}} = v_2, \text{ etc.}$$

$$\left.\begin{aligned} H_1 &= H\, v_1 \\ H_2 &= H_1\, v_2 = H\, v_1\, v_2 \\ H_3 &= H_2\, v_3 = H\, v_1\, v_2\, v_3 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (41)$$

If all spans and piers are identical, v_1, v_2, etc., are all the same, and,

$$H_n = H\, v^n \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (41a)$$

Combining Equations (41) and (39),

$$\delta_l = H_1 (\delta_{h1} + \delta_{l1}) = H (\delta_{h1} + \delta_{l1})\, v_1$$

Since H is unity,

$$\delta_l = (\delta_{h1} + \delta_{l1})\, v_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (42)$$

Similarly,

$$\delta_{l1} = (\delta_{h2} + \delta_{l2})\, v_2$$

These formulas can be used to analyze a system of any number of spans. Starting at the left abutment, the values of δ_{ln}, $\delta_{l(n-1)}$, etc., are computed until the loaded span is reached. The same procedure is to be followed from the right abutment to obtain the value of δ_r. The value of H is then determined and the thrusts in the unloaded spans are computed successively back to the abutments.

The value of δ_{P_1} is derived as follows: The forces acting on the pier are shown in Fig. 22. When $(H - H_1) = 1$, the moment at any section of the pier is,

$$M_O = (h + b - y')$$

The displacement of the pier-head will be (considering also the elasticity of foundation),

$$\int_B^D M_O\, (h - y')\, d\, w + M_D\, h\, \varepsilon = \int_B^D (h + b - y')\, (h - y')\, d\, w + (h + b)\, h\, \varepsilon$$

which was placed equal to K. (See Equation (33).)

Then, $\delta_{P_1} = K_1$ and $\delta_{P_2} = K_2$, etc., and the equations for v_1, etc., become,

$$\left.\begin{aligned} v_1 &= \frac{K_1}{\delta_{h1} + \delta_{l1} + K_1} \\ v_2 &= \frac{K_2}{\delta_{h2} + \delta_{l2} + K_2}, \text{ etc.} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (43)$$

After the horizontal thrusts have been obtained, the bending moments are found in the same manner as that used for the solution of the two-span system:

$$H = H' + \Delta H$$
$$M = M' - \Delta Hy.$$

Vertical reactions are assumed to be the same as for fixed ribs.

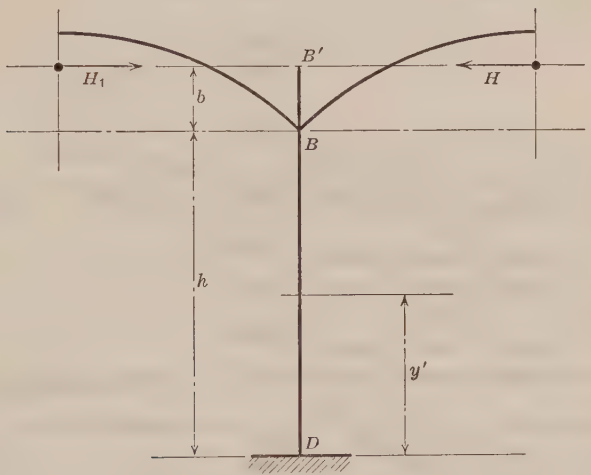


FIG. 22.

Application to Two-Span System.—Applying the general formulas to the two-span example with a load on the right-hand span,

$$\delta_{l_1} = 0$$

and,

$$v_1 = \frac{K_1}{\int y^2\, d\,w + 0 + K_1}$$

$$\delta_l = v_1 (\delta_{h1} + \delta_{l1}) = \frac{K_1 \int y^2\, d\,w}{\int y^2\, d\,w + K_1}$$

As the right support of the loaded span is fixed,

$$\delta_r = 0$$

Then,

$$\Delta H = -H' \frac{\delta_l}{\int y^2\, d\,w + \delta_l} = -H' \frac{K_1}{\int y^2\, d\,w + 2\,K_1}$$

which is the same as Equation (32).

Numerical Example.—The general method will now be applied to a system of seven spans having ribs and piers the same as those of Example II. All

values of $\int y^2\, d\, w$ and K are equal. The thrust produced by a load on the center span will be determined.

It was found that,

$$\int y^2\, d\, w = \frac{2\,974}{E}$$

and,

$$K = \frac{808}{E}$$

Then,

$$v_3 = \frac{K}{\delta_{h3} + \delta_{l3} + K} = \frac{808}{2\,974 + 0 + 808} = 0.2135$$

$$\delta_{l2} = v_3\,(\delta_{h3} + \delta_{l3}) = 0.2135\,(2\,974 + 0) = 636$$

$$v_2 = \frac{808}{2\,974 + 636 + 808} = 0.1828$$

$$\delta_{l1} = 0.1828\,(2\,974 + 636) = 660$$

$$v_1 = \frac{808}{2\,974 + 660 + 808} = 0.1820$$

$$\delta_l = 0.1820\,(2\,974 + 660) = 661 = \delta_r \text{ (from symmetry)}$$

$$\Delta H = -H' \frac{661 + 661}{2\,974 + 661 + 661} = -0.308\,H'$$

$$H = H' - 0.308\,H' = 0.692\,H'$$

$$H_1 = v_1 H = 0.1820 \times 0.629\,H' = 0.126\,H'$$

$$H_2 = v_2 H_1 = 0.1828 \times 0.126\,H' = 0.023\,H'$$

$$H_3 = v_3 H_2 = 0.2135 \times 0.023\,H' = 0.0049\,H'$$

From this it is seen that the effect on the end span is negligible. If there were five spans in the system, the thrust in the center span when loaded would be:

$$H = H' - H' \frac{660 + 660}{2\,974 + 660 + 660} = 0.6925\,H'$$

With three spans, it would be

$$H = H' - H' \frac{636 + 636}{2\,974 + 636 + 636} = 0.7005\,H'$$

This shows that, for a system with piers of the relative proportions assumed in the example, the horizontal thrust is little affected by the consideration of more than one span on each side of the loaded span. The value of ΔH is $-0.2995\,H'$ for the three-span system as against $-0.176\,H'$ for the two-span system, so that one might expect the errors introduced by neglecting the effect of the rotation of the pier-heads to be greater. It should be noted, however, that the thrust resisted by the pier in the former case is smaller than in the latter. There would, therefore, be less bending of the pier in the former case and the error would not be proportional to the value of ΔH. With somewhat stiffer piers, the value of ΔH would be smaller and the effect of the second span from the loaded span would also be reduced.

The conclusion drawn by the writer from this is that for any system to which this approximate method is applicable, it is necessary to consider the effect of only one span on each side of the loaded span, except when the end span is analyzed, in which case the effect of the two spans at one side should be considered.

For the end span loaded, considering only one additional span, ΔH was found to be $-0.176\ H'$. Considering two additional spans, it would be,

$$\Delta H = -H' \frac{660 + 0}{2\,974 + 660 + 0} = -0.1815\ H'$$

Considering three spans to the side,

$$\Delta H = -H' \frac{661 + 0}{2\,974 + 661 + 0} = -0.1817\ H'$$

For the three-span system analyzed by Pichl,* regarding which he states that the effect of rotation of pier-heads could be neglected without serious error, the value of $\frac{h + b - s}{b}$ was 0.198. In the case of the more flexible system which he analyzed, $\frac{h + b - s}{b}$ was 1.79. In the writer's Example I, the value was $\frac{7.45}{11.85} = 0.628$. In neither of these latter cases would the approximate method given herein be satisfactory, and in order to obtain results close enough for practical design it would be necessary to use a more accurate method. In both cases, the pier bases were considered fixed. The rotation of the pier (head produced by rotation) about the base does not introduce as great an error as the bending of the pier itself, because the center of rotation is farther from the end of the rib and the horizontal displacement is proportionately larger. Therefore, if the effect of rotation of the pier base is considerable, a large value of $\frac{h + b - s}{b}$ is not necessarily an indication that the approximate method here presented is not suitable. This method might give very accurate results for a system with tall stiff piers on frictionless hinges at the base, although in that case the value of s would be equal to $(h + b)$.

For such a case the value of s computed with ϵ placed equal to zero would be some indication of the size of error introduced. The approximate method fails to take into consideration the bending moments produced in the arch ribs by the rotation of the pier-head; when $(h + b - s)$ is small, this rotation is slight.

Conclusion

The writer believes that the data in Tables 1 to 6 make the application of the accurate method presented herein for the analysis of two-span arch systems so simple that the use of an approximate method is unnecessary, except perhaps for the preparation of a preliminary design. This accurate method also provides an easy way to study the effect of the elasticity of a single pier of varying proportions so that the designer may form an idea of its im-

* "Der durchgehende gelenklose Bogen auf elastischen Stützen," by Ernst Pichl.

portance and may judge the accuracy of the approximate methods when applied to systems of several spans. The influence which the elasticity of the piers will have on the design of the arch ribs will depend on the relative importance of live and dead loads, as well as on the proportions of the piers and ribs, so that the preliminary design will be largely a matter of judgment. A fixed-end rib design may be used as a basis and the thickness modified to suit the particular condition.

As all arch-rib design is a matter of trial, the writer feels that the methods of this paper and of the one previously published* are of great value, because they permit a complete analysis to be made without designing the rib in each case. It is merely necessary to establish the span, rise, and loading and select suitable values of m, N, I_c, and I_s, which then fix all the characteristics of the rib. The design is completed after its suitability is determined. This is particularly advantageous in the case of multiple-arch systems in which the characteristics of a number of spans are involved.

The approximate method presented will probably be sufficiently accurate for most practical cases, but the designer should determine in each instance whether or not it is suitable. The test suggested by the writer, consisting of the computation of the value of $\frac{h + b - s}{b}$, should be helpful in this respect if properly applied. The large number of variables involved in bridge design makes it inadvisable to draw too definite conclusions, but the writer hopes that a discussion by engineers who have had experience in the design of multiple-span arch bridges will help to establish the limits within which the approximate method may be used.

For the analysis of systems of more than two spans when the piers are very flexible, or when there is doubt as to the sufficiency of the approximate method, the writer would suggest the use of the method given by Hardy Cross, M. Am. Soc. C. E.,† in connection with one similar to that presented by J. Melan.‡ Professor Melan gives a method of analysis of single-span ribs with elastic supports and Professor Cross shows how to determine the elastic properties of the supports of any rib in a series of any number of spans. Using the elastic characteristics of arch ribs as presented by the writer, these methods can be applied to obtain an accurate solution without an unreasonable amount of labor.

For the analysis of three-span systems the writer has developed a means of design in the same way as for two spans by the accurate method. This requires considerably more labor than the two-span solution, but is probably as simple as any other method by which full influence lines for three spans can be obtained.

The writer is indebted to Mr. W. J. Fuller, of the Extension Division of the University of Wisconsin, for checking over the manuscript.

* *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 931.

† *Loc. cit.*, p. 1196.

‡ "Plain and Reinforced Concrete Arches," by J. Melan, translated by D. B. Steinman, M. Am. Soc. C. E., p. 71.

DISCUSSION

HARDY CROSS,* M. AM. SOC. C. E. (by letter).—This paper has real design value. The importance of studies in continuous arches is beginning to be realized, but there is still a wide gap between methods of analysis for general studies of such structures and design methods that will lead to better proportioning. In estimating the effect of continuity the changes in the horizontal reaction are the most important consideration, the rotation of the pier tops being usually of minor value. If the piers are relatively stiff, studies of three spans only are sufficient.

General Form of Influence Lines for Continuous Arches.—The outstanding questions are those that usually occur in indeterminate structures, in the following order:

1.—When and where should such structures be built?
2.—What are the proper proportions of piers, span, and rise?
3.—What points are critical in the design of such structures?
4.—How are these problems most readily analyzed?

As usual, the sequence of study is necessarily in reverse order to the questions.

Influence Lines.—Qualitative Studies.—As for the general action of arches on slender piers, the influence line for the crown moment, say, may be thought of as made up of three parts:

(*a*) The influence line for the fixed arch;
(*b*) The influence line for the effect of deflection of the pier tops; and,
(*c*) The influence line for the effect of rotation of the pier tops.

The effect of pier rotation is less pronounced than that of pier deflection. A combination of Parts (*a*) and (*b*) gives an influence line for the crown moment as shown in Fig. 23 (*a*). It is evident that the piers are pulled inward in all spans on either side of the span considered, causing the arch ring to rise (plotted down for graphic reasons) in the span considered and to drop in all spans on either side (negative influence ordinate). The curve of rise or fall, if it is assumed there is no rotation of the pier tops, has the shape of an influence line for crown thrust, but the ordinates become smaller outward from the pier considered.

Similarly, Fig. 23 (*b*) shows the two important constituent parts of the influence line for moment at the springing line, the influence line for fixed-ended conditions, and that for the effect of the reduction in crown thrust. This, again, shows the tendency to produce an influence line with all ordinates positive outside the span considered.

It is not difficult to see that the effect of pier rotation will be to modify Fig. 23 (*a*) and Fig. 23 (*b*) somewhat as shown in Fig. 23 (*c*) and Fig. 23 (*d*).

The Approximate Method of the Author.—The approximate method given by Mr. Whitney for determining the change in crown thrusts seems to the writer the most valuable part of the paper. This method consists of assuming

* Prof. of Structural Eng., Univ. of Illinois, Urbana, Ill.

the change in span in the arches to be due entirely to a horizontal thrust along the horizontal neutral line of the arch and that in the pier, to a thrust along this same line; and then of equating the displacements at the pier top. This assumes that the springing line of the arch does not rotate, but that the top of the pier is free to rotate. This is inconsistent, but seems to give good results.

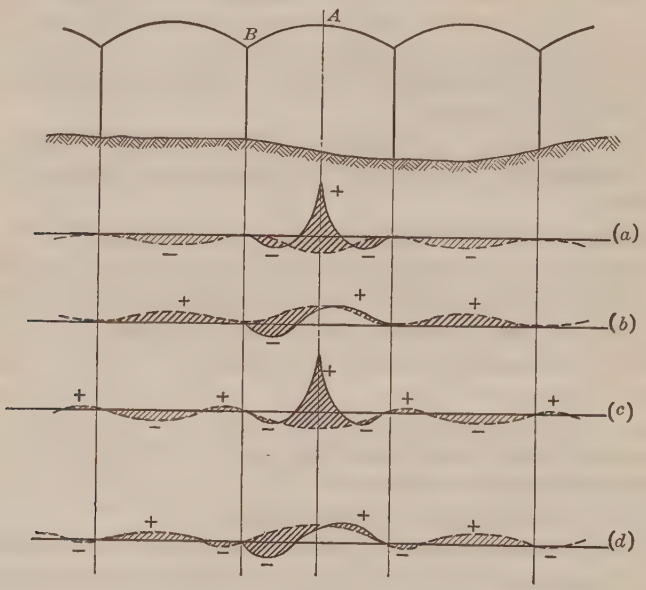


FIG. 23.

As the author points out, a tremendous amount of work is necessary to determine the limits within which this approximation is satisfactory and also the limits within which the consideration of only three spans is satisfactory. He suggests that the criterion as to the value of the first assumption is to be found in the value, $(h + b - s)$. The writer has no good substitute to offer, and yet he ventures the opinion that the definition of the terms used in this expression might be made simpler. The values of h (height of pier) and of b (distance from springing line of arch to horizontal neutral line) are clear enough, but s is defined by a long expression for $\frac{\phi_{DH}}{\phi_{DM}}$.

The writer has roughly applied this approximation to continuous arch systems on which he has collected data, and had hoped to compare systematically the exact with the approximate results in these cases. A comparison of influence ordinates for crown moment in one case, however, is given in Fig. 24.

Apparently, in practical cases, the approximation indicated may well be used for preliminary design, the final design being checked by more exact methods. While the effect of live load is more pronounced in continuous arches than in fixed arches, the live load stress is less than one-half the total stress and the effect of errors is diminished accordingly.

Exact Method of Analysis.—The exact analysis indicated by the writer* and referred to by the author, is based on the idea that the action of any elastic body under the influence of terminal forces can be completely described in terms of (*a*) the co-ordinates of the center of rotation of this terminus for moments applied to it; (*b*) the parameter of the value of this rotation for a unit terminal moment; (*c*) the horizontal displacements of this center for a unit horizontal force and for a unit vertical force acting through this center; and (*d*) the corresponding vertical displacements of the center for unit vertical and horizontal forces through the center.

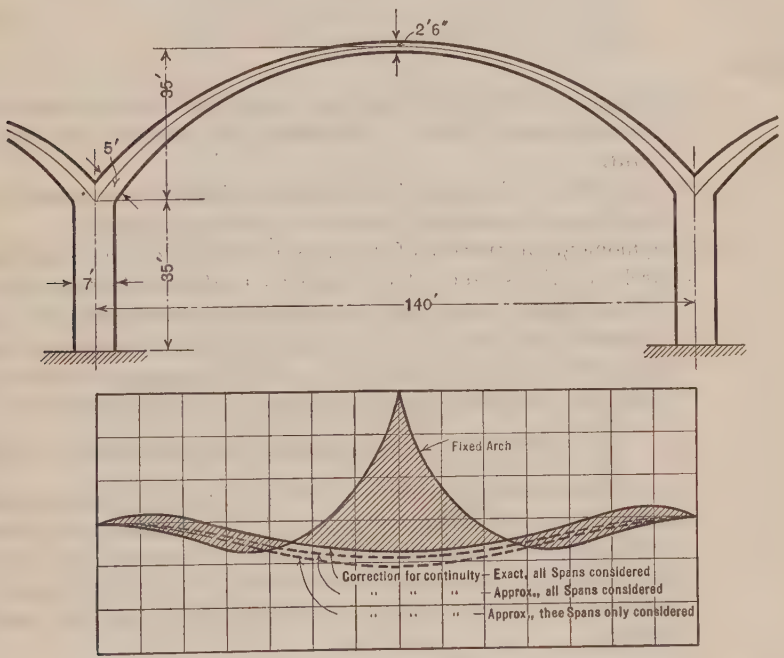


FIG. 24.—CONTINUOUS ARCH SERIES. INDEFINITE NUMBER OF SPANS. INFLUENCE LINE FOR CROWN MOMENT.

Determine the elastic properties of the end span, combine this with the second arch, and proceed thus until the arch under consideration is reached. Proceed similarly from the other abutment to the span in question. This gives the elastic properties of the arch under consideration and of two equivalent elastic piers on which it is regarded as resting. The solution for stresses in the span then may be made by the usual methods.

This gives an exact and definite procedure; the expressions used are not very formidable and may be simplified without appreciable loss of accuracy to give convenient solutions. The properties of each arch in the series are systematically determined and from these the stresses in the various arches.

* *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 1196.

The fundamental equations for the reduction process in the exact analysis are:

$$\frac{1}{J_s} = \frac{1}{J_p} + \frac{1}{J_a'}$$

or,

$$J_s = \frac{J_a' J_p}{J_a' + J_p} \quad \ldots\ldots (44)$$

$$y_p = \frac{J_s}{J_a'} y' \quad \ldots\ldots (45)$$

$$\frac{1}{W_s} = \frac{1}{W_a} + \frac{1}{W_p} + \frac{x^2}{I_a} + \frac{(y')^2}{J_a' + J_p} \quad \ldots\ldots (46)$$

in which

J_s = the moment of inertia of the elastic weight of the equivalent pier for the system about a horizontal axis through its neutral point.

J_p = the moment of inertia of the elastic weight of the actual pier about a horizontal axis through its elastic centroid.

J_a' = the moment of inertia of the elastic weight of the equivalent arch for the system about the axis conjugate to the vertical axis through the neutral point of the combined arch and pier (about H_a'-H_a' in Fig. 25). Note that the arch becomes unsymmetrical when the pier on one side is combined with it or when the equivalent piers on either side are not alike; or

$J_a' = J_a - \frac{Z_a^2}{I_a}$, in which J_a = the moment of inertia of the axis weight of the equivalent arch about a horizontal axis through its neutral point; Z_a = the product of inertia of the equivalent arch about simultaneous horizontal and vertical axes through its neutral point; and, I_a = the moment of inertia of the elastic weight of the equivalent arch about a vertical axis through its neutral point.

y_p = the vertical distance from the neutral point of the actual pier to the neutral point of the equivalent pier.

y' = the vertical distance from the neutral point of the actual pier to the conjugate axis, H_a'-H_a' (see Fig. 25), or, $y' = y_{np} + x_{np} \frac{Z_a}{I_a}$, in which, y_{np} = the vertical distance from the neutral point of the actual pier to the neutral point of the equivalent arch; x_{np} = the horizontal distance from the neutral point of the equivalent arch to the neutral point of the actual pier; and Z_a' and I_a are the same as previously defined.

W_s = the total elastic weight of the equivalent pier.

W_a = the total elastic weight, $\int \frac{d\,s}{E\,I}$, of the equivalent arch.

W_p = the total elastic weight of the actual pier.

Arch on Equivalent Elastic Piers.—The properties of the arch on equivalent piers may be expressed in terms of I_0, J_0, Z_0, and W_0, in which I_0 and J_0 are the moments of inertia, respectively, about the vertical and horizontal axes through the neutral point of the equivalent arch, of all the elastic weights including the equivalent piers on each side of the span in question; Z_0 is the product of inertia of all the elastic weights about simultaneous horizontal and vertical axes through the neutral point; and W_0 is the total elastic weight of the arch and equivalent piers.

The value of each of these functions may be stated in the following general equations (see Fig. 25):

$$I_0 = I_a + \Sigma (W_s x^2) - 2 W_0 E_x^2 \quad \ldots\ldots (47)$$

$$W_0 = W_a + \Sigma W_s \quad \ldots\ldots (48)$$

$$J_0 = J_a + \Sigma J_s + \Sigma (W_s y^2) - W_0 E_y^2 \quad \ldots\ldots (49)$$

$$Z_0 = \Sigma W_s x y - W_0 E_x E_y \quad \ldots\ldots (50)$$

$$\tan \beta = \frac{Z_0}{I_0} \quad \ldots\ldots (51)$$

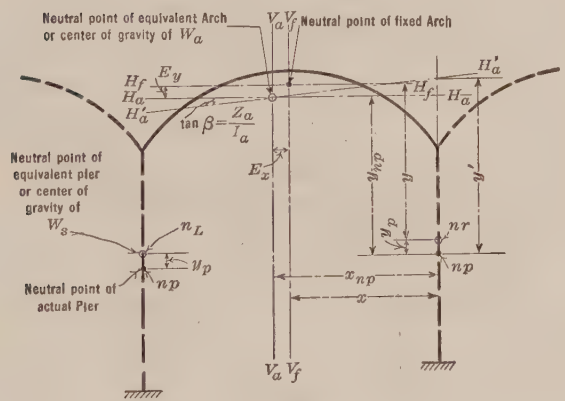


FIG. 25.

As noted in Fig. 25,

E_x and E_y = the co-ordinates of the neutral point of the combined system with respect to the neutral point of the fixed arch as the center of co-ordinates;

β = the angle which the axis conjugate to the vertical axis through the neutral point of the system makes with the horizontal axis through the same neutral point; and,

x and y = the horizontal and vertical distances, respectively, from the neutral points of the equivalent piers to the neutral point of the fixed arch.

The other terms were defined previously.

When the critical arch is the center span of a symmetrical series, the equivalent elastic piers on either side have the same properties, $E_x = 0$, and Equations (47), (49), (50), and (51) simplify to the following:

$$I_0 = I_a + 2\,W_s\,x^2 \quad \ldots\ldots(52)$$

$$J_0 = J_a + 2\,J_s + 2\,W_s\,y^2 - W_0\,E_y{}^2 \quad \ldots\ldots(53)$$

$$Z_0 = 0 \quad \ldots\ldots(54)$$

$$\tan\beta = 0 \quad \ldots\ldots(55)$$

The influence ordinates for moment, M_0, the horizontal thrust, H_0, and the vertical shear, V_0, at the neutral point of the arch on elastic piers are then determined, as follows, for a symmetrical case (see Fig. 26):

$$M_0 = \frac{W_s\left(\frac{L}{2} - X\right) + \Sigma\, w\, x'}{W_0} \quad \ldots\ldots(56)$$

$$V_0 = \frac{W_s\,\frac{L}{2}\left(\frac{L}{2} - X\right) + \Sigma\, w\, x'\, x}{I_0} \quad \ldots\ldots(57)$$

$$H_0 = \frac{W_s\left(\frac{L}{2} - X\right)y' + \Sigma\, w\, x'\, y}{J_0} \quad \ldots\ldots(58)$$

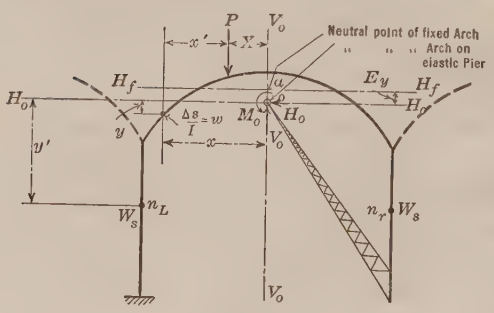


FIG. 26.

These are the typical equations for the neutral point method. The notation used is indicated in Fig. 26:

x = horizontal distance from the neutral point to the elastic weight under consideration.

x' = horizontal distance from the load point to the elastic weight under consideration.

X = horizontal distance from load to the neutral point.

y = vertical distance from the neutral point of the combined system to the elastic weight under consideration (= y' for the pier).

Approximate Method of Analysis.—An approximate method of reduction has been developed which considerably simplifies and shortens the process of computation indicated by Equations (44), (45), and (46) and yet is exact enough for all practical purposes. In Equation (44), $J_a' = J_a$ for the approxi-

mate solution. This eliminates the use of the correction factor, $\frac{Z_a^2}{I_a}$, although J_a changes for each equivalent arch. J_a is then computed as the moment of inertia of the elastic weight of the equivalent arch about a horizontal axis through the neutral point of the fixed arch.

In Equation (45), y' is made equal to y. This eliminates the correction factor, $x_{np}\frac{Z_a}{I_a}$. Theoretically, y is the distance from the neutral point of the equivalent arch to the neutral point of the actual pier. However, in the approximate method, y is taken as the distance from the neutral point of the fixed arch to the neutral point of the actual pier.

The equations for the approximate analysis can then be re-stated as follows, referring to Fig. 27:

$$\frac{1}{J_s} = \frac{1}{J_p} + \frac{1}{J_a} \quad \ldots\ldots(59)$$

$$y_p = \frac{J_s}{J_a} y \quad \ldots\ldots(60)$$

$$\frac{1}{W_s} = \frac{1}{W_a} + \frac{1}{W_p} + \frac{x^2}{I_a} + \frac{y^2}{J_a + J_p} \quad \ldots\ldots(61)$$

Extended studies have shown that no considerable error results from use of the more convenient formulas.

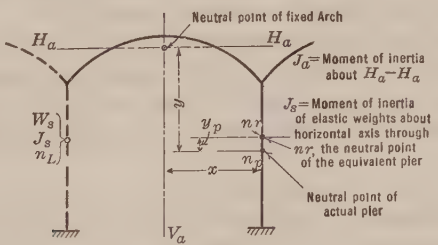


FIG. 27.

Other Considerations in Continuous Arch Studies.—Thus far this discussion has dealt entirely with the solution of a problem in mechanics. The engineering problems involved in the design are much more complicated. Precision in the solution of the problem indicated in mechanics is not very valuable in any particular case unless some estimate can be made of (*a*) the effect of distortions of the pier foundation; (*b*) the factor of safety to be provided for highly improbable conditions of live load distribution; and (*c*) the effect of the floor system.

Foundation Distortions.—The effect of foundation distortions is not difficult to include if one has any idea of their value; but the prediction of these distortions is one of the most difficult of the unsolved problems of civil engineering. The unlimited variety of foundation conditions for different strata, for strata of different thicknesses, and for the same strata under different con-

ditions of wetness, makes the collection of adequate data almost prohibitive because of the time and money required. Furthermore, the absence of any workable theory as to the laws governing these distortions makes it difficult to use the data for any one stratum. It is not to be supposed that if a pressure of 4 tons per sq. ft. on 100 sq. ft. causes ¼ in. of settlement, a pressure of 2 tons per sq. ft. on 100 sq. ft. will cause ⅛ in. of settlement, and it almost certainly will not be true that a pressure of 4 tons per sq. ft. on 50 sq. ft. will also cause ¼ in. of settlement. Existing ideas are so vague as to have practically no quantitative value whatever; but in a continuous series of arches the matter is very important. What might seem a slight rotation at the base of a pier may produce at the pier top a displacement several times as great as the elastic distortion of the pier. Preliminary reports on the work of W. M. Wilson, M. Am. Soc. C. E., at the University of Illinois, and of Clyde T. Morris, M. Am. Soc. C. E., at Ohio State University, indicate that this does occur, but their measurements were on the effect of loads of considerable duration—dead load and temperature. The effect of transient live load would not be so great.

In a continuous series of arches, economy based on the topographic conditions and æsthetic considerations usually indicates that the span lengths shall not be equal, but shall decrease toward the abutments. In such cases if the dead load thrusts are balanced at the pier heads, the temperature thrusts are unbalanced and *vice versa,* and if the thrusts are balanced, the moments at the pier heads are unbalanced. The unbalancing of the moments is not very important, but unbalanced thrusts are very important.

Probably a safe and sane treatment of the problem is to equalize any unbalanced thrusts from dead load and temperature throughout the series, thus neglecting any resistance from the piers, and then to neglect the effect of foundation distortions on live load stresses because of the transient effect of live load.

Live Load Criteria.—In addition to giving much time to the study of existing data on arches and especially on multiple arches, the writer has had the pleasure of directing three very able investigations by graduate students in this field. In 1923-24, Mr. Lorenz Straub made an extensive study along the lines indicated previously for exact analysis, determining the influence lines and maximum stresses for different pier heights. In 1924-25, Mr. H. E. Wessman extended the work previously done by Mr. Straub and made studies of the probability of the indicated live load conditions and the effect of these conditions on the design. This investigation resulted in a set of specifications for computing live load maxima, together with a simplified procedure of computation of which the approximate method indicated is a part. Perhaps the most interesting part of the work was in showing the heavy burden which could be put on the design by a too literal interpretation of influence lines. Thus, under certain conditions, the pier would be required to carry as a cantilever beam unsupported by adjacent spans nearly all the full live load thrust of one span. It is easy to see that this is a tremendous burden, but it can be shown that the probability of the required congested loading is small.

There is, however, an unfortunate tendency in the literature of indeterminate structures to assume that the construction of influence lines gives a complete solution of design problems, whereas interpretation of these graphs is a much more difficult problem, and an influence line not properly interpreted may become an expensive luxury to a client.

Economic Proportions.—In 1925-26, Mr. E. A. MacLean (now Associate Professor of Civil Engineering at the Agricultural and Mechanical College of Texas) took up the study of the economic proportions of continuous arch series, investigating twelve different combinations of pier proportion and rise ratio. This problem, the ultimate aim of structural studies, is one of great complexity. Yet, Professor MacLean was able to lay down certain tentative principles to guide the designer. That the rise ratio should increase with the pier slenderness was, of course, indicated from the beginning.

Influence of the Floor.—The writer has in hand extensive studies the first object of which is to determine the desirability of considering the effect of the integral action of the floor and arch ring in fixed arches and especially the effect of expansion joints in the floor. Later, it is intended to study the influence of the floor in continuous arches. In the case of fixed arches, the question of considering the floor is apparently one of economy, because (except in some cases for the effect of improperly located expansion joints on temperature stresses) no danger is to be expected from neglecting it. In a continuous arch series, on the other hand, where the spandrel posts are stiff, neglect of the floor invalidates the whole computation. The controlling element in the analysis of the continuous arch problem is the distribution of unbalanced live load thrusts at the pier-head between the pier and the two connecting arches in proportion to their resistance to horizontal displacement. It is evident that the resistance of an arch to horizontal displacement (change of span) is decidedly affected by the interaction of arch ring and floor system.

Difficulty of Correlation of Data.—A continuous arch series offers possibilities of economy and graceful beauty which are unusually attractive to bridge engineers. The problems involved, however, are complicated, their study requires both time and patience, and a synthesis of the results is not to be made hastily. Many existing series are neither economical nor beautiful, and past practice cannot here be considered to be a good guide to future performance. It is, therefore, of the greatest importance that the principles be laid down after full and careful discussion by those who have studied the problem intensely and not by hasty dicta or arbitrary dogmas.

Charles S. Whitney,* M. Am. Soc. C. E. (by letter).—The discussion by Professor Cross is sincerely appreciated because of the amount of labor involved in a careful discussion of such a paper. On account of the general tables (Tables 1 to 6) the methods presented can be applied with much more ease than would be indicated by the formidable appearance of the paper.

The value of s in the expression, $\frac{h + b - s}{b}$, is very readily found from Equation (12) and Tables 1 and 6. This term may be described as the

* Cons. Engr., Milwaukee, Wis.

vertical distance from the base of the pier to the neutral point of the two-rib and pier system. The expression, $\frac{h + b - s}{b}$, is, therefore, the quotient of the vertical distance from the neutral point of the single arch rib to the neutral point of the two-rib and pier system divided by the vertical distance from the neutral point of the single arch to its springing line. This value obviously approaches zero as the stiffness of the pier increases, and the smaller it is the less the effect of the bending of the piers.

Professor Cross has mentioned enough of the unsolved problems encountered in arch design to indicate that the single arch and the continuous arch series will provide a fertile field for investigation for some time to come. It is, of course, well to keep in mind that advanced mathematical theories are only sharp tools which must be used intelligently in the light of experience.

MEMOIRS OF DECEASED MEMBERS

GUILLIAEM AERTSEN, M. Am. Soc. C. E.*

DIED APRIL 20, 1926.

Guilliaem Aertsen, the son of the late James Musgrave and Harriet (Romyn) Aertsen, was born in Philadelphia, Pa., on March 2, 1855. He was educated at the Germantown Academy, an old and well-known preparatory school, and, later, at Racine College, in Wisconsin.

After leaving college Mr. Aertsen entered the employ of the Midvale Steel Works (afterward the Midvale Steel Company) in 1876 as Assistant Superintendent of the Forge. Two years later he and a young fellow-townsman and school classmate started for Oregon to raise sheep. After a short time at this work, he left the West and returned to the Midvale Works, remaining until 1887, first as Assistant Superintendent and, later, as Superintendent of the Works. It was while at Midvale that he laid the foundation for his extensive fund of knowledge of steel manufacture, his understanding of men, and his executive ability.

In 1887, he was employed by the Homestead Steel Company as Superintendent under Mr. Julian Kennedy. He remained in this position until 1888, following Mr. Kennedy who then left the Company. Shortly after this, in 1888, Mr. Aertsen was engaged as Manager of the Latrobe Steel Company, at Latrobe, Pa., a new company, just organized under the Presidency of Mr. Marriot C. Smith. When the Latrobe Company was purchased in 1906 by the Railway Steel Spring Company, Mr. Aertsen continued as General Manager of the Latrobe Plant until 1911.

It was while at Latrobe that Mr. Aertsen become widely acquainted in the railroad world and when he re-entered the employ of the Midvale Steel Company in 1911 it was as Consulting Engineer on Railroad Products and their sale. When this Company was purchased by the Midvale Steel and Ordnance Company in 1915, Mr. Aertsen remained with it in the capacity of Engineer, and, later, of Auditor of the Company. The Midvale Steel and Ordnance Company was taken over by the Bethlehem Steel Company in 1923, and Mr. Aertsen remained with that Company until his last illness.

He died at his home in Germantown, Philadelphia, on April 20, 1926, and is survived by his widow, formerly Edith Price, of Cleveland, Ohio, and by two children, Mrs. Murray Watts and Guilliaem Aertsen, Jr.

Mr. Aertsen's profound knowledge of metallurgy, in particular that of the manufacture of steel products for railroads, made him a much sought-after man by the various technical societies of his calling. He was connected with the following organizations: The Engineers' Club of Philadelphia, of which he was President in 1920-1921; the American Society for Testing Materials, of which he was Vice-President in 1921-1923, and President in 1923-1924; the

* Memoir prepared by Joseph B. King, Esq., Philadelphia, Pa.

American Institute of Mining and Metallurgical Engineers; American Iron and Steel Institute; American Electric Railway Engineers Association; the Geographic Society of Philadelphia; the National Geographic Society; the Chicago Engineers' Club; the Engineers' Club of New York; and the Union League, of Philadelphia.

Mr. Aertsen had great influence over his men, due, first, to a keen insight into human nature; and, second, to his unfailing justice and fairness in his dealings with others. His judgment was sound and was fortified by a large fund of excellent common sense. He was loved by friends and respected by antagonists—a gentleman.

Mr. Aertsen was elected a Member of the American Society of Civil Engineers on October 1, 1890.

GEORGE ROCKWELL BASCOM, M. Am. Soc. C. E.*

DIED JUNE 26, 1925.

George Rockwell Bascom, the son of William Russel and Abby (Hovey) Bascom, was born at Bloomington, Ill., on June 21, 1882. He was educated in the public schools of Bloomington and Chicago, Ill., and in the University of Illinois from which he was graduated with the degree of Bachelor of Science in Municipal and Sanitary Engineering in 1905.

In the summer of 1905, Mr. Bascom was employed by the Chicago Engineering and Contracting Company, and had charge of the engineering work on the lock and dam on the Washita River, at Monroe, La. From the autumn of 1905 to the autumn of 1908, he was with John A. and Edward S. Cole, Members, Am. Soc. C. E., Consulting Engineers, with offices in New York, N. Y., and Chicago, on various phases of municipal and hydraulic engineering. During the following winter, he served as Special Engineer for the City of Muskogee, Okla., in charge of the design of the water-works and sewer extension.

Thence, until March, 1913, Mr. Bascom was engaged in the private practice of engineering and contracting, acting as Engineer for the Heman Construction Company and for Fuller and Fuller, Contractors. He also served as Secretary and Manager for the Muskogee Crushed Stone Company and the Hyde Park Sand and Gravel Company. In the autumn of 1912, he went to San Antonio, Tex., to take charge of special hydraulic concrete work for the firm of Fuller and Fuller.

From March, 1913, to January, 1918, he was Assistant Professor of Municipal and Sanitary Engineering, in charge of a Department of the Extension Division, at the University of Wisconsin, Madison, Wis. In 1917, he also served as Consulting Engineer for water-works and sewerage for Alpha, Mich.

In January, 1918, Mr. Bascom entered the Army as a Major in the Sanitary Corps, and during the World War acted as Camp Sanitary Engineer at Camp Pike, Arkansas. After the Armistice he was sent to Washington, D. C., to assist in writing that part of the medical history of the war, represented by the activities in the Sanitary Engineering Corps, which included a history of malaria control for fifty-four camps, posts, and stations. He resigned from the Service in March, 1920, and on May 14, 1920, was commissioned a Lieutenant-Colonel in the Quartermaster Corps of the Officers' Reserve Section.

In March, 1920, Mr. Bascom became associated with the Barber-Greene Company, Aurora, Ill., as Branch Manager of the Chicago District, and continued in that capacity to the time of his death on June 26, 1925.

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* Memoir prepared by W. B. Greene, Vice-Pres. and Treas., Barber–Greene Co., Aurora, Ill.

He is survived by his widow, who was Litta Bauschbach, of Princeton, Ill., and two children, William Russel and Elizabeth Anne Bascom.

He was a member of the American Water Works Association and the Wisconsin Engineering Society and also belonged to the Phi Gamma Delta Fraternity.

Mr. Bascom was elected an Associate Member of the American Society of Civil Engineers on September 11, 1917, and a Member on June 2, 1920.

HARRY AUSTIN BELDEN, M. Am. Soc. C. E.*

DIED OCTOBER 30, 1925.

Harry Austin Belden, the son of James Belden, was born on March 28, 1865, at Wellington, Ohio. He was graduated from the High School in his native town and spent one year at the Case School of Applied Science, Cleveland, Ohio.

In 1887, he served as Rodman and Draftsman on cable railway construction for the Grand Avenue Railroad Company, Kansas City, Mo. In 1888, he was employed as Engineer at the Kansas City Stock Yards and, in 1889, he again accepted a position with the Grand Avenue Railroad Company as Draftsman on cable railway construction. Later, in the same year, he was similarly engaged by the Washington and Georgetown Railroad Company, at Washington, D. C.

Returning to Kansas City in 1890, Mr. Belden served as Assistant Chief Draftsman in the City Engineer's Office, but, in 1891, he returned to Washington to serve again as Draftsman on cable railway construction for the Washington and Georgetown Railroad Company. In 1892 he entered the service of the Baltimore City Passenger Railway Company, at Baltimore, Md., as Draftsman on cable railway construction and, in 1893, he was appointed Principal Assistant Engineer in charge of the Drafting Office and the construction of trolley lines of the City and Suburban Railway Company, at Baltimore.

In 1894, he became Assistant Engineer in the Frog, Switch, and Signal Department of the Pennsylvania Steel Company and, in 1895, was appointed Principal Assistant Engineer in charge of the Drafting Office and of the construction of conduit electric railway lines for the Metropolitan Railway Company of Washington, D. C. During 1896 and 1897, he served as Principal Assistant Engineer in charge of the extension of the cable line and of the erection of the Union Passenger Station for the Capitol Traction Company of Washington.

Mr. Belden was connected with the National Contracting Company of New York, N. Y., during 1898 and 1899, as Engineer in charge of construction of conduit electric railway lines for the Metropolitan Street Railway Company on the Eighth Avenue Line above 59th Street and on the Fifth Avenue Line below 50th Street.

From 1900 until 1902, he held the position of Engineer for Shewan, Tomes, and Company, an English firm of Hong Kong, China, and was in charge of the erection of electric plants and tramways in Canton and Hong Kong. He made many friends among the Chinese who called him the "Heavenly Light Man". After spending three years in China, Mr. Belden resigned his position to accept another in the Philippine Islands, as Superintendent of Construction for the J. G. White and Company, of New York, N. Y., where his work consisted of the construction of a street railway system, including 40 miles of

* Memoir compiled from information on file at the Headquarters of the Society.

track, an electric lighting system for the City of Manila, and harbor improvement contracts at Cebu and Iloilo.

During the World War Mr. Belden served as Chief Engineer on the construction of Camp Hancock, Augusta, Ga. After the completion of this work his health became impaired and on the advice of his physician he moved to Margaretville, N. Y. The change proved so beneficial that he was able in a short time to take charge of work in Porto Rico. For the greater part of three years prior to his death, Mr. Belden was engaged as Special Expert in the valuation of the properties of the Colorado Electric Light and Power Company, and spent most of his time in Denver, Colo.

In 1904 he was married to Susie C. McCay, of Covington, Ga., who, with a daughter, Jane, survives him.

He was a member of the English Club, of Hong Kong, the English and University Clubs, of Manila, and of the Engineers' Club, of New York.

Mr. Belden had the gift for making true and lasting friendships, and, with a mind enriched by the study of peoples and customs of the various countries visited during his extensive travels, was a most delightful companion. He was passionately devoted to the Orient, took a keen interest in Oriental art, and had made an interesting collection of Chinese paintings and Japanese color prints. He was an engineer whose professional integrity and devotion to any undertaking in his line of work gave him a high standing in the estimation of his business associates, and many of his most enthusiastic friends and admirers were among the members of his profession.

Mr. Belden was elected a Member of the American Society of Civil Engineers on October 4, 1905.

AARON MOULTON BURT, M. Am. Soc. C. E.*

DIED APRIL 20, 1925.

Aaron Moulton Burt, the second of three sons of Dr. John Otis and Helen (Moulton) Burt, was born at Syracuse, N. Y., on May 1, 1866. His father was a descendant of Henry Burt who came from England to the Massachusetts Colony in 1639, settling in what is now Springfield, Mass., and who married Eulalia Marche, a Huguenot of Artois, France, whose ancestors had been driven out of France by the religious persecutions of the Duke of Alva. The descendants of Henry Burt bore a considerable part in the development of Massachusetts and the neighboring Colonies. During the troublous years of the French and Indian Wars and, later, throughout the controversy with England, ending in the Revolutionary War, Burts were found serving in town councils, Colonial Assemblies, and Colonial armies.

Aaron Burt, great-grandfather of Aaron Moulton Burt, was a private in Captain Nathum Ward's Company, in Colonel David Well's regiment of militia belonging to the County of Hampshire, and participated in the capture of General Burgoyne.

Aaron Burt, grandfather of Aaron Moulton Burt, at the age of 18 left his parental home in Marlboro, Vt., to seek his fortune in the West. He intended to go to Ohio, but stopped in the spring of 1812 at Dover, N. Y. In 1813, he visited New York, N. Y., traveling there on horseback. The next year he resumed his journey to Ohio, but, stopping at Oneida, N. Y., to visit a sister, gave up the idea of continuing his journey and settled, first, in Manlius, N. Y., where, in 1815, he was married to Lucy Burke. After her death he married Eleanor Ross Otis. He was an industrious man and developed much skill in building mills which were greatly needed in that new country. With the means thus accumulated he purchased a farm near Fayetteville where he resided for fourteen years. During this period he represented his district for three sessions in the State Legislature at Albany, becoming intimate with Martin Van Buren and Henry Seymour among other distinguished men of that time.

Early in the history of the Erie Canal he was appointed one of its Superintendents and held that position on the Middle Sections for many years. Having acquired important interests at Syracuse, N. Y., he moved his family there and became identified with its early history and growth. He secured a large tract of land in and around the city, much of which he subdivided into building lots and later sold. His own home occupied a commanding hill site among extensive lawns and trees. As Commissioner he took an active part in the construction of the railway between Syracuse and Utica, N. Y., now a part of the New York Central System, between which points the entire line was constructed under his immediate supervision. He died in 1848, having earned the respect and highest esteem of his fellow citizens.

* Memoir prepared by the following Committee of the Northwestern Section: Messrs. Louis Yager, M. Am. Soc. C. E., *Chairman*, J. L. Campbell, W. L. Darling, B. O. Johnson, H. R. Safford and H. E. Stevens, Members, Am. Soc. C. E., and H. M. Stout, Record Engr., Northern Pacific Ry. Co., St. Paul, Minn.

Dr. John Otis Burt, father of Aaron Moulton Burt, became a prominent physician of Syracuse. He was graduated from Harvard University in the Class of 1858, traveled in Europe, and on his return began the study of medicine at the Jefferson Wyman Medical School, at Cambridge, Mass., and the Harvard Medical School. In 1860 he entered the College of Physicians and Surgeons in New York, but, in the following year, aroused by the attack on Fort Sumter, he enlisted in the 9th New York Regiment "for the war". Restive of delays in fitting out this regiment he sought and obtained entrance to the Medical Corps of the United States Navy, passed the required medical examination, and received his commission from President Lincoln as an Assistant Surgeon. He was immediately ordered to Admiral Farragut's Squadron and was with the fleet when the forts below New Orleans, La., were taken and through its advance up the Mississippi River and, later, through the siege of Vicksburg, Miss. In the autumn of 1863, after the fall of Vicksburg, he was invalided home, and having been discharged from the Navy, returned to his interrupted medical studies. He was married to Helen Narcissa Moulton, an orphaned daughter of Dr. Franklin Moulton, of Marcy, N. Y., whose father, Solomon Moulton, a soldier of the Colonial Army in the Revolutionary War, had been mentioned for conspicuous bravery at the Battle of Long Island. They sailed for Europe where for two years Dr. Burt studied surgery in the celebrated hospitals of Paris and Vienna and on their return he was graduated in New York. He then began the practice of medicine in Syracuse. He was Visiting Physician at St. Joseph's Hospital for a number of years, physician to the Board of Health, and for seven years was connected with the Medical Department of Syracuse University as Assistant Professor of Chemistry, and, later, as Professor of Materia Medica. His wife died in 1873, after a few days' illness of typhoid pneumonia, leaving three small boys, nine, seven, and four years old.

Each person is, and must be, largely the creator of his own character and fortune; still, there is back of his existence a power and an influence which gives direction to his thoughts and actions. Great intellectual attainment and high moral purposes are as well founded in inherited ability from a family line as from acquirement. "As ye sow, so shall ye reap", is as true to-day as it was when it had utterance 2 000 years ago. Sound ancestry leaves a rich inheritance; the parent will ever live in his children, and, in some degree direct the thought and action of the generations that are to follow where he has led. In the application of such law the ancestry of Aaron Moulton Burt sent him forth in this world with a background in which stood out in bold relief, industry, energy, public spiritedness, capacity, and leadership.

The modest education with which Mr. Burt was equipped was obtained in the public and private schools of Syracuse and at the Allen School in West Newton, Mass. Ill-health and financial reverses on the part of his father terminated his education, and at the age of 15 he returned to Syracuse and began work in a drug store. There he stayed for three years; but becoming dissatisfied because of long hours and low wages, in company with a boyhood chum he started West to seek his fortune, ending his journey in a wild ride

over the mountains in a stage coach with a drunken driver, at Aspen, Colo., just over the Saguache Mountains of the Continental Divide. During the next year he had a varied, rough, frontier experience as a ranch hand, a grocery clerk, and a drug store clerk, sleeping in the drug store at night.

In March, 1885, in his nineteenth year, Mr. Burt secured a position with a surveying party on the Colorado Midland Railroad, then just starting its program of locating and building its line into Buena Vista and down the Colorado River into Leadville, thence to Glenwood Springs and New Castle, with a branch from Glenwood Springs to Aspen. He began as Assistant Transitman; on him devolved the work of carrying the transit. The first survey on which he was engaged was from Leadville to Glenwood Springs. Until that time he had had no experience whatever with surveying instruments or in engineering work but by exercise of pluck and application he soon mastered the transit which he liked to describe as "an old 'Stackpole' that weighed 38 pounds".

Mr. Burt remained with the Colorado Midland Railroad until the end of 1888, locating main lines and investigating branch lines to coalfields. He next served on construction of the line and, finally, on maintenance on a division out of Leadville, having progressed from Assistant Transitman to Rodman, to Assistant Engineer on Construction, and Division Engineer on Maintenance. Despite his limited schooling he was described by one of his party on the Colorado Midland Railroad as the youngest but still, mentally, the most alert of the group, and even then he was showing evidences of those characteristics which later were so strongly marked in him. During the time of his Colorado employment he became intimately associated with John Cotton Dana, now Librarian at the Public Library in Newark, N. J. Between them there was formed an attachment which had no small influence, as he has himself said, on his subsequent life, and which grew even stronger to the end.

In April, 1889, on the completion of the work for the Colorado Midland Railroad, Mr. Burt accepted a position with the Big Horn Southern Railway Company as Assistant Engineer in charge of the location of a line from Sheridan, Wyo., to Custer, Mont., now a part of the Chicago, Burlington and Quincy System. In September, 1889, he left this work and joined the Engineer Corps of the Northern Pacific Railroad Company as a Transitman on the location of the Cœur d'Alene Line from Missoula, Mont., westward, up the St. Regis River over the Cœur d'Alene Mountains, through the St. Regis Pass to Cœur d'Alene City, on Cœur d'Alene Lake, at the foot of the western slope of the Continental Divide. In May, 1890, he was made a Resident Engineer on the construction of this line under L. R. Lothrop, Assistant Engineer in Charge. His residency lay along the St. Regis River, a mountain stream, adjoining that of W. L. Darling, M. Am. Soc. C. E., later Chief Engineer of the Northern Pacific Railway, under whom Mr. Burt served for a number of years. After the completion of construction of this line, Mr. Burt continued with the Northern Pacific Railway Company as Assistant Engineer on surveys for various line changes and grade revisions until February, 1892. In March of that year he went to the Adirondack and St. Lawrence Railway Company

(now a part of the New York Central System) as Resident Engineer in charge of 10 miles of construction in the vicinity of Old Forge, N. Y. This work was finished in September, 1892, when he accepted a position with the Wisconsin Central Railway Company in Chicago, Ill., as Division Engineer, remaining until August, 1894.

From August, 1894, to 1896, Mr. Burt was Assistant Engineer with the Chicago and Northern Pacific Railroad Company (now a part of the Baltimore and Ohio Terminal Railroad). During the rainroad strike of 1894 he worked as a Signalman in the signal towers in Chicago for this Company. From March to December, 1896, he was with the Wisconsin Northern Railway, a branch of the Chicago and Northwestern Railway, as Resident Engineer on the location and construction of 15 miles of road between Leona and Newald, Wis. This work having been completed and with no further construction work in prospect on that road, he left it at the end of 1896 and, on January 1, 1897, re-entered the service of the Northern Pacific Railway, with which Company, except for nine months with the United States Railroad Administration, he continued an uninterrupted service until his death.

His initial employment in this period was as Supervisor of Bridges and Buildings on the Minnesota Division from January 1, 1897, to March 1, 1902, with headquarters at Staples and Minneapolis, Minn. From March 1, 1902, to October 10, 1903, he was Assistant Division Superintendent at Grand Forks, N. Dak. On October 10, 1903, he was appointed Division Superintendent of the Dakota Division, with headquarters at Jamestown, N. Dak., in charge of the maintenance and operation of that Division, and continued in that position until his transfer to the Fargo Division on January 1, 1911. He was Superintendent of the Fargo Division, with headquarters at Fargo, N. Dak., from January 1 to August 14, 1911, when he was transferred to Missoula, Mont., as Superintendent of the Rocky Mountain Division. On April 25, 1912, he was transferred to the Idaho Division as Superintendent at Spokane, Wash., where he remained until January 1, 1914, when he was appointed Chief Engineer of Maintenance of Way of the entire System with headquarters in St. Paul, Minn., in which capacity he served until April 1, 1918.

On the latter date Mr. Burt was promoted to be Acting General Manager on Lines East of Paradise, Mont., serving in that capacity until August 1, 1918, when he was made Assistant General Manager. He held this office until June 1, 1919, when he was chosen Assistant Director, Division of Operation, United States Railroad Administration, with headquarters at Washington, D. C. On March 1, 1920, he returned to the Northern Pacific Railway Company as Assistant to the Vice-President in charge of Operation. He was promoted to the position of Assistant Vice-President on May 1, 1923, holding that position until January 26, 1925, when, on the death of Mr. J. M. Rapelje, he was appointed Vice-President in charge of maintenance and operation, serving in that capacity until his death at Jamestown, N. Dak., on April 20, 1925, of pneumonia contracted while on an inspection trip.

Mr. Burt's patriotism prompted him to make considerable effort to take an active part in the World War and soon after the United States entered

the conflict he made application to secure service abroad in some capacity. His request, however, was denied with the encomium that his expert knowledge and operating skill were essential for transportation problems at home.

Mr. Burt's career with the Northern Pacific Railway was almost coincidental with that Company's history since its re-organization. He was among the last of those who played a conspicuous part in this chapter of its history during the past generation. He knew intimately the story of its development from a single-track line with 66-lb. steel rails, narrow gauge, light engines, and cars carrying a traffic of less than 4 000 000 tons, a line mileage of 4 527, and a total track mileage of 5 325, to a system brought to a high state of operating efficiency by grade and line changes, much double-track, increase in weight of rail to 100 and 130 lb., and an increase in trackage to 6 701 miles of line, with 10 417 miles of track, carrying 24 000 000 tons of traffic per year.

Mr. Burt's experience in railroad service was broad and complete. From a subordinate in a surveying party he rose to the highest operating position on one of the great American railway systems. Although 16 years of his business life were given to engineering, 24 years were devoted to transportation operations, and in this branch he made his greatest contributions. His outstanding donation to American railway operation was through his possession of a vision beyond that possessed by most American operating men. No operating problem to him constituted a problem by itself, for he always considered it in its broadest aspects, its relation to other elements of operation, and its effect on future operation. Out of this breadth of vision came certain terminal developments on the Northern Pacific Railway so constructed to serve the present day and yet designed so as to require but a simple enlargement to care for future needs. He was one of the first American railway engineers to see clearly the economy in the standardization both of facilities and practices. The high degree of such standardization and the broad lines of intelligent, comprehensive development of the Northern Pacific Railway will stand as his individual contribution to American transportation.

During the period in which he served the United States Railroad Administration as Assistant Director, Mr. Burt was also Chairman of the Automatic Train Control Committee, and planned and organized the work of that body in a most judicious manner, as is evidenced by its report to the Director-General in 1919. After he left the Railroad Administration he was appointed the representative of the Engineering Division of the American Railway Association on the Joint Committee on Automatic Train Control which acted in co-operation with the Interstate Commerce Commission in working out details for a practicable plan for equipping the railroads with automatic train-control systems.

In physique, Mr. Burt was much above the average height, being 6 ft. 3 in. tall. Straight, upstanding, and of moderate weight, his bearing was dignified, yet not stiff, impressing his assurance and capacity at the first meeting. His blue eyes, quickly kindling in friendly humor or steeling in penetration of knotty problems, showed his sociability and power. He was possessed of a melodious, resonant voice which lent agreeable support to personal bearing

in public speech or friendly talk, and of that saving grace of dry American humor which could smooth a tense situation and which always made him a delightful companion. As a young engineer on construction, he gave evidence of that tact and diplomacy which in later years served him so well in handling many perplexing human problems which come to men of heavy responsibilities.

He was possessed of an analytical mind, a conservative manner, and a temperate expression as a result of which his opinions carried great weight and were readily accepted. He was tolerant with the differing views of others and was a diligent searcher for truth in the fertile places of an open mind. Mr. Burt was a man of exemplary habits and moderation and had a pleasing personality in all business and social contacts. He never permitted his mind to be exclusively engaged with his professional duties, but always found time to participate in the play of social forces around him.

His dealings with subordinates were marked by exceptional fairness and unusual consideration. He was a keen student of human nature and his knowledge of and interest in the human element in railroad operation contributed largely to his success in this field. He always took a deep interest in young professional men and devoted much of his time to providing opportunities for deserving individuals. His entire life was an inspiration to the young engineer. Having been thrown into the world early in life with only a modest education he, nevertheless, by perseverance and constant use of the powers with which he was endowed, neglected no opportunity for learning and enlarging his experience and thus pushed courageously forward to a place of leadership. His career should be a wholesome stimulus to youths of this day who are forced to start life's battle with meager equipment.

He loved intensely the rugged Western mountains in which so much of his life had been spent and explored many of the promising trout streams in his short periods of summer recreation. Mr. Burt possessed a peculiarly gifted ability to use carpenter's tools which inclination constituted with him an avocation and his chief recreation from his daily work. After a day's work in summer and on week ends he could divest himself of his cares and find extreme pleasure in the actual building of things at his cottage by a pleasant Minnesota lake. At other times he utilized this love of tools to the making of many novel, quaint, and useful things.

In 1898, Mr. Burt was married to Margaret James, a sculptress and artist of ability, the daughter of Mr. H. C. James, of St. Paul, Minn. Mrs. Burt, with two adopted children, Lynda Olive Burt, and Harold Glenn Burt, survives him.

He was a member of the American Railway Engineering Association from March 28, 1911. For eight years, from 1916 to 1924, inclusive, he was a member of the Committee on Signals and Interlockers, and a member of its Board of Direction from March, 1924, until his death. He was also a member of the Engineers Society of St. Paul from 1916, the Minnesota Club, the St. Paul Athletic Club, White Bear Yacht Club, and the Informal Club, a local literary society.

Mr. Burt always showed a keen, lively, and productive interest in all worthy movements and enterprises seeking the betterment of his community. He was a Charter Member, Vice-President, and Member of the Executive Committee of the Metropolitan District Planning Commission which has for its object a definite plan for the development of the area comprising the Twin Cities of St. Paul and Minneapolis, Ramsey and Hennepin Counties, in which these two cities are located, and contiguous territory. He was among the most active of Minnesota engineers in the promotion of this large undertaking and his loss has been keenly felt by that organization. Although he held no office, elective or appointive, in the municipal organization, he was always interested in public enterprises and public welfare. A call for meetings, conferences, or some other form of assistance relating to the public welfare, was rarely issued but that he would find it possible to be present and to encourage the movement by public speech and private counsel.

He was a member of Unity Church, of St. Paul, a member of its Board of Trustees, and active in its welfare and financial support. He was also much interested in art and music and lent them his earnest encouragement and support although he was not gifted in either.

In 1913, by invitation, Mr. Burt lectured before the Graduate School of Business Administration of Harvard University on "Railroad Problems". This address dealt mainly with phases of railroad organization and administration from the viewpoint of a Division Superintendent. The lecture was greatly appreciated as exceedingly interesting and stimulating. At various times thereafter he was urged to address the same school on transportation subjects but natural modesty and press of other duties prevented. We may pause and reflect that through his own achievement and self-acquired accomplishments he was sought as a transportation authority to lecture before one of the schools of the great university from which his father was graduated and subsequently obtained part of his professional education.

With simple and satisfying devotion, Mr. Burt, throughout his life, adhered to his creed that "the greatest thing a man can do for the world is to do his own job well". To this end he gave in his work all his loyalty and energy.

He was elected a Member of the American Society of Civil Engineers on May 31, 1916, and at the time of his death was President of the Northwestern Section of the Society. He had represented the Section on the Metropolitan Planning Committee from 1924 until January, 1925, and had served as Vice-President of the Committee during this period.

ROBERT WALTER CREUZBAUR, M. Am. Soc. C. E.*

DIED JUNE 2, 1926.

Robert Walter Creuzbaur was born in Austin, Tex., on August 6, 1860. He was the only son of Robert and Lydia Rachel (Kirk) Creuzbaur. His father was a German-Austrian mechanical engineer of great ability, who emigrated as a young man to Texas and served during the Civil War in the Confederate Army. On the restoration of peace he moved with his family to New York, N. Y., where his son received his primary and High School education. Financial reverses prevented a university career.

Mr. Creuzbaur began his engineering training with his father and was subsequently employed as Draftsman in the office of the Central Railroad of New Jersey.

In 1877 and 1878, he was engaged on the construction of the Sixth Avenue Elevated Railroad of New York. On the completion of this work, he went to Austin, where he spent about a year on surveys and on railroad and dam construction. In 1880, he was employed by Mr. E. W. Bowditch, of Boston, Mass., on landscape and sanitary engineering work.

From 1881 to 1884, Mr. Creuzbaur was engaged in the Drafting Office of the New York, West Shore and Buffalo Railroad Company, where he designed highway, transfer, and ferry bridges, coaling pockets, docks, masonry, etc.

From 1881 to 1891, he held a similar position with the Aqueduct Commissioners of New York under whose charge the New Croton Aqueduct was being built. For this work Mr. Creuzbaur designed, under the direction of the Chief Engineer, the late Alphonse Fteley, Past-President, Am. Soc. C. E., steel roofs, gate-valves, and sluice-gates, some of which were to be subjected to unusually great water pressure. He was next retained for a short time by the New York Underground Railroad Company for which he made designs and estimates of cost. This work being interrupted by adverse legislation, Mr. Creuzbaur opened an office as Consulting Engineer in Southern California. Leaving a partner in charge of the work, he soon returned to New York to complete the designs of the mechanical work of the New Croton Aqueduct.

Mr. Creuzbaur's next engagement, from 1891 to 1893, was to prepare, as Assistant Engineer of the New York Central Railroad, under the direction of the late J. D. Fouquet, M. Am. Soc. C. E., the contract and working plans for the "Depew Shops". On the completion of this work he was transferred by the Chief Engineer, the late Walter Katté, M. Am. Soc. C. E., to "The Park Avenue Improvement"; he designed the masonry substructure for a four-track bridge across the Harlem River.

In May, 1893, he entered the service of the City of New York, as Assistant Engineer in the Finance Department. In 1896, he resigned this position to go into private practice in Guatemala, Central America, where he was engaged in a number of private undertakings, including the hydro-electric develop-

* Memoir prepared by Edward Wegmann, M. Am. Soc. C. E.

ment of the Rio Nigel for lighting the City of Antigua. On his return to the United States to provide equipment for one of these projects, he was obliged to give up his work in Central America, on account of the sudden death of his partner. During an interesting journey from "coast to coast" he examined the work in progress in the Pacific States, in the mining region of Montana, and on the Chicago Drainage Canal.

After a short service in 1897 as Assistant Engineer in the Department of Docks of the City of New York, in charge of sea-wall construction, Mr. Creuzbaur was transferred to the Department of Highways and placed in charge of repaving on Manhattan Island for which about $3 000 000 was available in 1897. On June 1, 1898, he was transferred as Assistant Engineer to the Finance Department of the City of New York and, on October 1, 1898, was promoted to the position of Principal Assistant Engineer of this Department, serving throughout the terms of Comptrollers Coler and Grout in supervision of all municipal work in the Boroughs of Brooklyn and Queens.

In 1902, while on a six months' leave of absence, Mr. Creuzbaur took charge of the construction of the Oak Lane Reservoir at Philadelphia, Pa., and on the expiration of this leave of absence he acted as Consulting Engineer to the New York Continental Jewell Filtration Company and until the reservoirs were completed.

On April 26, 1906, he was transferred to the office of the President of the Borough of Brooklyn, who appointed him as his Consulting Engineer. While in this position, which he held for five years, Mr. Creuzbaur re-organized the Engineering Staff of the Public Works of Brooklyn and introduced many reforms in the engineering methods.

In May, 1911, he resigned his position in the City's service to enter private practice as Consulting Engineer, having opened an office for this purpose with the late Charles L. Parmelee, M. Am. Soc. C. E.

In 1914, Mr. Creuzbaur was appointed Consulting Engineer for the City of Elizabeth, N. J., to supervise that City's part in the elimination of grade crossings and improvement work of the Pennsylvania Railroad and the Central Railroad of New Jersey. This work entailed an expenditure of about $4 000 000.

In 1917, he was appointed Consulting Engineer by the Emergency Fleet Corporation in connection with the construction of three well-known plants at Hog Island, Bristol, Pa., and at Newark, N. J. On the completion of this work in September, 1919, he resumed his private practice in New York as Consulting Engineer.

In summing up Mr. Creuzbaur's engineering activities, which extended over more than fifty years, one is impressed with the wide range of subjects covered. As regards his character, he was a man of few words, a hard, indefatigable worker, faithful, honest, and loyal in the performance of his duties, and of excellent judgment in dealing with important engineering questions. His services were of great value to the City of New York. In this connection, former Comptroller Bird S. Coler states:

"Mr. Creuzbaur was very efficient in rapid transit work, and was my main adviser in what was accomplished when, as Comptroller, I was a member of

the first Rapid Transit Commission. He prepared the arguments and facts which were used in persuading that Commission to make its first extension to Brooklyn. I thought so highly of Mr. Creuzbaur that when I became Borough President of Brooklyn I asked him to become Consulting Engineer to the Borough, and in that capacity he rendered to me the same conscientious and able work as he did in the Comptroller's office."

He died at his country home at East Hampton, N. Y., on June 2, 1926, and is survived by his widow, Mrs. Jessie B. Creuzbaur.

Mr. Creuzbaur was elected a Junior of the American Society of Civil Engineers on April 2, 1890, an Associate Member on April 4, 1894, and a Member on April 4, 1900. He was also a Charter Member of the Municipal Engineers of the City of New York.

GEORGE EDWIN GIFFORD, M. Am. Soc. C. E.*

DIED APRIL 14, 1926.

George Edwin Gifford, the son of Edwin Sands and Harriett (Searles) Gifford, was born at Long Ridge (Stamford), Conn., on March 20, 1864. The family is a Colonial one, his great-grandfather, a native of Rhode Island, having served in the Revolutionary Army. His grandfather was a Baptist minister who held several charges in Eastern New York and Connecticut. His father when a young man taught school, but later engaged in carriage manufacturing and continued in this business until his death.

Mr. Gifford received his early education at Long Ridge, having attended both public and private schools there. He also attended a boarding-school in Noroton, Conn., after which he entered the Stamford High School. For a year or two after graduating from High School, he taught in the country schools near Stamford. In the fall of 1883, he entered Rensselaer Polytechnic Institute, at Troy, N. Y., from which he was graduated in 1887 with the degree of Civil Engineer.

Immediately after his graduation Mr. Gifford was employed by the Louisville Bridge and Iron Company, of Louisville, Ky., as a Draftsman and Shop Inspector. In 1888, he accepted a position with the King Bridge Company, of Cleveland, Ohio, and except for a short time was with that Company until 1905. He was located in Cleveland until 1894, as Principal Assistant Engineer in the Designing and Contracting Department, and, later, was in charge of designing and contracting. Among other work, he designed a 312-ft. swing bridge for the United States Government at Oakland, Calif.; a cantilever bridge over the Willamette River, at Albany, Ore.; a preliminary design for the New York Central and Hudson River Railroad Company's four-track swing bridge over the Harlem River, at New York, N. Y., and a swing bridge and viaduct for Tacoma, Wash. He also designed extensions to the plant of the Bridge Company at Cleveland.

In 1894, Mr. Gifford was placed in charge of the New York Office of the Company, and took up his residence at Stamford. The New York office handled all matters pertaining to the sale and erection of bridges and structural work in the Eastern territory. During this period, he was also in charge, for the Contractor, of the erection of the New York Central and Hudson River Railroad Company's four-track swing bridge over the Harlem River, and the Harlem Ship Canal Bridge at Kingsbridge, N. Y.

In 1898, Mr. Gifford was, for a short time, a member of the R. H. Hood Company, engaged in contracting in and near New York. At this time, he moved from Stamford to Maplewood, N. J. His connection with the H. R. Hood Company proving unsatisfactory, he returned to his position with the King Bridge Company, and was placed in charge of the New York Office. He remained in this position until 1905, when he became connected with the firm of Milliken Brothers, of New York, as Contracting Engineer.

* Memoir prepared by Thomas Earle, M. Am. Soc. C. E.

When this Company went into the hands of Receivers, the latter appointed him Chief Engineer.

In 1909, Mr. Gifford assisted in the organization and became Vice-President of the Bowles-Gifford Company, Contracting Engineers, of New York. In 1911, he found it advisable to interest others in this Company, and his efforts resulted in the formation of its successor, The Jobson-Gifford Company, of which he became Vice-President. His active connection with the management of this Company was of short duration, although he retained his interest in it until his death.

In 1911, Mr. Gifford became Secretary of the Bridge Builders' Society and of the Structural Steel Society, both of which had headquarters in New York. Shortly afterward, when these two associations were merged, becoming the Bridge Builders' and Structural Society, he was appointed as Secretary and retained that position until the dissolution of the Society in 1922. Thereafter, he was not active in his profession, but devoted his time to personal and local municipal affairs.

When he became a resident of Maplewood, Mr. Gifford identified himself actively with all its local affairs. He was a Director of both the Maplewood Bank and the Maplewood Building and Loan Association from their organization. He was one of the early Presidents of the Maplewood Country Club; was active in church and Masonic affairs, having been a Past-Master in his lodge. He served as Tax Assessor of the town for a number of years.

While of a rather retiring disposition, Mr. Gifford was held in high esteem by all who were closely associated with him, and they will miss his quiet cheerful sympathy and consideration.

He was married in Cleveland, Ohio, in 1890, and is survived by his widow, Emma (LeVake) Gifford, two sons, a married daughter, and also by his mother.

Mr. Gifford was elected an Associate Member of the American Society of Civil Engineers on October 7, 1891, and a Member on January 1, 1896.

EDLOW WINGATE HARRISON, M. Am. Soc. C. E.*

DIED NOVEMBER 27, 1925.

Edlow Wingate Harrison was born in New York, N. Y., on May 9, 1851, the son of Samuel E. and Sarah Edlow (Williams) Harrison. He received his early education in the public schools of his native city and, later, at the New York Free Academy. In 1866 he entered the Engineering Classes of Cooper Union from which he was graduated in 1872. During the same period he was tutored in Mechanical Engineering.

Between 1872 and 1884 he was in the office of the firm of Bacot, Post, and Camp, at Jersey City, N. J., laying the foundation of that broad detailed experience and technique that he used so successfully in his later career as Administrative and Consulting Engineer.

From 1885 to 1893 Mr. Harrison served as Engineer of the State Board of Assessors of New Jersey and as such originated the methods of physical valuation of all railroad property that are still used in taxation. He also served as Expert for the State in all the litigation establishing the legality of the tax legislation.

In 1888 he held the position of Expert Engineer for the State of New Jersey and the Pennsylvania Railroad Company in the litigation in opposition to the Baltimore and Ohio Railroad Crossing of the Arthur Kill without the consent of the State.

From 1888 to 1890 he projected, and with his associates incorporated and built, the Raritan River Railroad of which he served as Chief Engineer and Director until 1907 and as Director and Vice-President until his death.

Mr. Harrison was keenly interested in public affairs both in Jersey City and in the State of New Jersey. He was one of the first to project the Hudson County Boulevard, running from Guttenberg to Bergen Point, Bayonne, of which he was Chief Engineer from 1892 to 1898. This highway which was planned with great skill and foresight, is considered one of the greatest assets of Hudson County.

He was largely instrumental in securing an additional gravity supply of pure water for Jersey City which for many years had been supplied with the badly polluted water from the Lower Passaic River near Belleville. About 1900 a contract was made by the Jersey City Water Supply Company, of which Mr. Harrison was Chief Engineer, to supply 70 000 000 gal. daily from the Rockaway tributary of the Passaic River. This work involved an expenditure of about $7 500 000 and was completed in 1903.

Jersey City and the adjoining territory were the termini of numerous trunk-line railroads, and Mr. Harrison was called in consultation in many controversies which arose as to the proper valuation for taxation to be placed on the railroads and also on questions of valuation regarding docks and ship-

* Memoir prepared from information supplied by Frederic Molitor and J. Waldo Smith, Members, Am. Soc. C. E.

ping. He was frequently in Court as an expert witness on valuation in connection with the diversion of water, power, and many other matters.

From 1897 to 1911 he held the position of Expert Engineer for the Central Railroad of New Jersey, the Lehigh Valley, the Erie, the Pennsylvania, the Delaware, Lackawanna and Western, and the New York Central (West Shore) Railroad Companies, as well as the North German Lloyd Steamship Company, in the examination and determination of the physical value of the terminal yards, piers, and structures on the New Jersey shore front of New York Harbor and the settlement of State and municipal taxes.

He also acted as Consulting Engineer in the examination and report as to the true present value of the Hudson River Tunnels in the settlement of taxation by the State, and made an examination and report for the City of Hoboken, N. J., on the value of lands and improvements along its water-front.

Mr. Harrison was consulted in the planning of the trunk sewer in the Passaic River Valley from Paterson, N. J., to the point of discharge in New York Harbor.

In addition to his other work, he carried on a general engineering practice, which included roads, bridges, and other important construction. He had studied the sewerage systems and road works of Great Britain, France, Germany, and Switzerland, and had used some of the foreign methods of design and construction in his work in this country.

Mr. Harrison had an attractive personality and was a man of unusual mentality, being possessed of a remarkable memory for detail, which served him well during his engagements on important cases in Court. Sanitary matters interested him, and he was influential in promoting better conditions of living. He was an engineer of remarkable qualifications with deep insight and a seeker and finder of truth.

He was interested in military matters and served in the Fourth Regiment, National Guard of New Jersey, for fifteen years. He was a member of the American Institute of Consulting Engineers, the Railroad Club of New York, the New Jersey State Sanitary Association, the New Jersey State Chamber of Commerce, the Hudson County Historical Society, the Carteret Club of Jersey City, and the Yountakah Country Club. Mr. Harrison was also a member of St. John's Protestant Episcopal Church of Jersey City, until he moved to Montclair, where he became affiliated with St. James Protestant Episcopal Church.

In 1885, he was married to Martha A. Bumsted who died the following year. In 1889, he was married to Harriet Taylor McLaughlin who died in 1910.

Mr. Harrison was elected a Member of the American Society of Civil Engineers on June 3, 1885.

KENNETH PHIPSON HAWKSLEY, M. Am. Soc. C. E.*

DIED MAY 2, 1924.

Kenneth Phipson Hawksley was born on September 15, 1869, in London, England. He received his education at Wellington College, Sandhurst, and, subsequently, at Trinity College, Cambridge.

He was first employed under the direction of his father, the late Charles Hawksley, a Past-President of the Institution of Civil Engineers, who was a partner in the firm of T. and C. Hawksley. He also served as Apprentice in the workshops of the Glenfield Company, at Kilmarnock, Scotland, from 1889 to 1892, and in 1893 studied at University College, London.

From 1893 to 1896, Mr. Hawksley was engaged on various Parliamentary surveys and in superintending the construction of engineering work for the firm of T. and C. Hawksley, including water-works and pumping stations, of which the most important were those at Sunderland, Bristol, Coventry, and Banbury, as well as impounding reservoirs at Newcastle-on-Tyne, Bristol, Huddersfield, and other places. He was also engaged in the construction of gas-works at Darlington, Sunderland, Lowestoft, and Folkestone, and of sewerage works at Birmingham and Yeovil, as well as numerous smaller works. He continued in similar work until 1899 and took part in the responsibility of the design, as Engineering Assistant for the firm.

On January 1, 1900, Mr. Hawksley entered into partnership with his father and with him carried on a large practice as Consulting and Constructional Engineers, principally with reference to water, gas, and sewerage works. He became well known as one of the foremost experts in water supply engineering in England and was in constant request in giving evidence before Parliamentary Committees on behalf of water corporations and companies.

During the World War he was appointed Consulting Engineer for Water Supplies in connection with the Ministry of Munitions and designed and carried out the construction of large works for the Gretna, Avonmouth, Queen's Ferry, Henbury, and other large explosive plants.

At the time of his death Mr. Hawksley was a Member of the Council of the Institution of Civil Engineers, having been elected in 1921, and was the third generation of his family to serve in this capacity. This constituted a record, as there was no other instance of the kind in the history of the Institution. He was also a member of several other scientific societies.

Mr. Hawksley was elected a Member of the American Society of Civil Engineers on May 6, 1903.

* Memoir compiled from data on file at Society Headquarters and from information supplied by H. W. Davey, Esq., London, England.

WILLIAM HENRY HEUER, M. Am. Soc. C. E.*

DIED APRIL 28, 1925.

William Henry Heuer was born in St. Louis, Mo., on March 2, 1843, of German parentage. He was a graduate of the United States Military Academy at West Point, N. Y., and was commissioned a First Lieutenant of Engineers, U. S. Army, on June 23, 1865.

His first duty appears to have been in California, where he was placed as an Assistant Engineer on the river and harbor work of the Pacific Coast, remaining there from 1865 until 1870. During that period he superintended the removal of Blossom Rock (a menace in the Harbor of San Francisco), to a depth of 24 ft. below low-water. This work was done under contract by the eminent pioneer civil engineer of California, the late Col. Alexis W. von Schmidt. Lieut. Heuer also made a reconnaissance for a military roadway from Fort Churchill, Nev., to Boise City, Idaho.

In 1870 he was promoted to the rank of Captain, and during the two following years was on duty at Willets Point, N. Y., as Adjutant to the U. S. Battalion of Engineers stationed there.

From 1872 to 1873, Captain Heuer was placed in local charge of the removal of Halletts Point, known as Hell Gate, in New York Harbor, and prior to its removal he had charge of the sinking of a shaft on Flood Rock, an important piece of preliminary work.

In 1874, he joined a Commission of Military Engineers as a Junior Officer to examine and report on the possible interoceanic canal routes at Darien, Nicaragua, and Panama, in Central America. During a part of the same year he was also in charge of certain measurements to determine, with great accuracy, the length of the Central Pacific Railway from Sacramento, Calif., to Battle Mountain, Nev., a distance of 387 miles.

From 1876 to 1880, Captain Heuer was on duty at Key West, Fla., as Lighthouse Engineer, and also in charge of the fortifications of this station and the Dry Tortugas Islands. During that period he built two first-class lighthouses, one on Fowey Rocks and the other on the American Shoal of the Florida Reef.

From 1880 to 1884, he was on duty at New Orleans, La., in charge of the river and harbor works, and fortifications; he also served as Lighthouse Engineer. He commenced the construction of the jetties at Sabine Pass, Texas, in 1882, and continued this work for two years, building in this connection the lighthouses at South Pass and Sabine Pass, Texas, in Mobile Bay, Alabama, and also one at Fort Point, Galveston, Tex.

He became a Major in 1884, and was on duty at Philadelphia, Pa., during 1885. While in charge of the improvement of rivers and harbors, fortifications, and lighthouses, he designed a project for a permanent improvement of the Delaware River, and under his direction the lighthouse on the 14-ft. bank was built in Delaware Bay.

* Memoir prepared by Otto von Geldern, M. Am. Soc. C. E.

From 1885 to 1887 Major Heuer was again at New Orleans continuing the work he had left in 1884. He returned to the Pacific Coast in 1887, and with the exception of a year's absence between 1896 and 1897, during which he was on duty at Cincinnati, Ohio, in charge of the Ohio River improvements and as a member of the Missouri River Commission, he remained in California until his death.

He was in charge of the river and harbor work in San Francisco and became a member of the California Débris Commission appointed by the State to regulate hydraulic mining. He also began the construction of the Humboldt Bay jetties during this period.

In 1900, he was made a Lieutenant-Colonel and continued on the work of the improvement of rivers and harbors, together with the erection of lighthouses in San Francisco Harbor and in other localities on the Pacific Coast. The destruction of several well-known obstructions in San Francisco Harbor—Arch Rock and the two Shag Rocks—was successfully accomplished under his immediate direction.

During the Spanish-American War Colonel Heuer was in charge of the submarine defense of San Francisco Bay.

He was promoted to the rank of Colonel in 1904 and was placed on the retired list in March, 1907. After his retirement he opened an office in San Francisco, as a Consulting Engineer, and was connected with a number of very important works on the Pacific Coast for a period of ten years.

In April, 1917, Colonel Heuer was again called to active duty and took charge of the office of the Division Engineer for the Districts of San Francisco, Los Angeles, and Honolulu, Hawaiian Islands, which charge he held during the period of the World War until 1919.

Colonel Heuer was not married. During his later years, his niece, Miss Elizabeth P. Heuer, came from St. Louis, Mo., to care for him. Until his eightieth year he retained his good health and faculties and was well able to take care of himself; afterward old age developed within him the symptoms of nephritis and without very much suffering he passed away, after a comparatively short illness, on April 28, 1925.

His funeral was held in the Masonic Temple, in San Francisco, on April 30, 1925, with Masonic rites and with simple military honors, and was attended by many of his old friends, colleagues, and military comrades.

Like so many of the early visitors to the Golden State, Colonel Heuer became infatuated with it, and whether living in California or far away from it, he considered it his home, always returning to San Francisco when the opportunity offered. He became thoroughly identified with the people, and later in life was widely and favorably known throughout the Pacific Coast. He was a very popular and highly respected public character.

The characteristics of Colonel Heuer were those of a real man. He had the courage of his convictions and was never afraid to express an opinion and to defend vigorously whatever he thought to be just and right. It was his great principle of life to be just and fearless, and accordingly he feared nothing, neither physical dangers nor the opinions of others. He invariably

asked himself the question: "Is it right?" and when he came to his conclusions nothing could swerve him, neither pleadings nor threats.

Generous to a fault, he became known in all his official dealings as the most scrupulously honest custodian of a trust. With him it was not only a matter of dollars but one of cents, and all those who engaged in business with his office knew they must deal with a strict accountant.

Those who had his confidence he trusted and defended, but those who failed him were sure to be called to a severe accounting, He was quick to act and did not mince his words. He was a critical reader and a man of sound logic. Reports sent to him had to stand the "fire test", for he was sure to find the weak points in them if they had any.

Colonel Heuer was a true American of the old type, one of those who under a somewhat rough exterior have the tenderest feelings, who unostentatiously, reservedly, and almost shyly, do great acts of humanity as if they were afraid that some one might notice.

He had a pronounced dislike for all pomposity and outward show and he was, indeed, one of the finest representatives of the United States Military Academy, one of those to whom the noble traditions of West Point had become sacred articles of faith.

Colonel Heuer was elected a Member of the American Society of Civil Engineers on March 3, 1880.

HOWARD VERNON HINCKLEY, M. Am. Soc. C. E.*

DIED MARCH 24, 1926.

Howard Vernon Hinckley was born in Marstons Mills, Mass., on November 22, 1855. He was the son of Nathaniel and Ann (Judson) Hinckley.

His technical education was received at Worcester Polytechnic Institute, Worcester, Mass., from which he was graduated in 1876.

From 1877 to 1893, Mr. Hinckley was employed in the Engineering Department of the Atchison, Topeka and Santa Fé System. As Assistant Chief Engineer, which position he filled for twelve years—1881 to 1893—his duties included the designing of bridges, buildings, shop yards, track appliances, water stations, freight and passenger terminals; hydraulic problems on the Mississippi, Missouri, Kansas and Colorado Rivers; interlocking signals; inspection of pavements, sewers, and other public improvements in Chicago, Ill., Kansas City, Mo., Topeka and Wichita, Kans., and Pueblo, Colo.; and passenger and freight terminals in Chicago, Kansas City, Pueblo, Topeka, Denver, Colo., and El Paso, Tex.

From 1894 to 1899, Mr. Hinckley, as a Consulting Engineer, was engaged on various irrigation, railroad, bridge, water-works, sewer, light plant, and paving improvements. In 1897, he built the Melan Arch Bridge over the Kansas River at Topeka at a cost of $150 000, and also designed four municipal water-works systems. He acted as Sewer Expert in a suit against the City of Topeka in 1898. As Consulting Engineer, in 1899, he designed reinforced concrete bridges for Des Moines, Iowa, Columbus, Ohio, and several other cities; chief among the improvements handled at that time, were the valuation of the Topeka Water-Works, and the surveying of Muskogee Townsite. From 1900 to 1903 Mr. Hinckley was United States Supervising Engineer for the Indian Territory.

Opening an office in 1904, he practiced as a Consulting Engineer for five years, in charge of various municipal improvements in Oklahoma. Two bridges were built in Platt National Park, at Sulphur, Okla., following Mr. Hinckley's designs. These designs were selected in Washington, as being better than any of those submitted by several bridge companies. Sewage and garbage disposal problems took up his time from 1910 to 1913. Since 1913, his work had been largely confined to bridge and highway construction, and for the last few years he was connected with the State Highway Bridge Department of Oklahoma.

Always interested in Society affairs, Mr. Hinckley was a leader in all matters of interest to Oklahoma engineers, and for years his untiring work as Secretary of the Oklahoma Section of the Society, kept Oklahoma engineers together. He also served as President of the Oklahoma Section in 1919.

* Memoir prepared by V. V. Long, M. Am. Soc. C. E.

Mr. Hinckley was a man of strong opinions and principles and could always be depended upon to fight for what he thought was right. He was respected and honored by all engineers who knew him and his death leaves a vacancy in the Oklahoma Section that will be difficult to fill.

On November 17, 1880, he was married to Lettie S. Cooldaugh, of Towanda, Pa., who, with their two children, Mrs. Hila H. Wunderlich, of Oklahoma City, and Charles M. Hinckley, of Ponca City, Okla., survives him.

Mr. Hinckley was elected a Member of the American Society of Civil Engineers on December 5, 1883.

JOSEPH WARREN HOOVER, M. Am. Soc. C. E.*

DIED JUNE 18, 1925.

Joseph Warren Hoover was born on November 19, 1850, on a farm about five miles north of Canton, Ohio. His parents were Daniel and Mary (Kryder) Hoover. With his two brothers, William H. and Frank K., his boyhood was spent among surroundings typical of what might be called the second stage of pioneer development, newly cleared land, rail fences, neighborhood flour mills, brick kilns, tanneries, etc.

His education began with the country school at the nearest crossroads and was carried on at Mount Union College, Mount Union, Ohio.

A combination of circumstances determined Mr. Hoover's life work, the first incident occurring on a sweltering day when his breath was knocked out of him by the handle of a plow which he was guiding. The result was a definite rejection of farming as a life occupation. The fact that his father, Daniel Hoover, through self-training, had become a competent Surveyor and frequently surveyed the farms of his neighbors for drainage purposes, etc., probably aroused his interest in surveying. At this time a new railroad called the Valley Railroad was being run south from Akron, Ohio, passing about three miles to the west of the old farm. This may have suggested to Mr. Hoover employment other than farm work. In any event, he secured work during the summers on this new line which gave him experience in railroad surveying.

This experience doubtless led to his entering the University of Michigan in the fall of 1872. He was graduated from the Engineering Department three years later, in the spring of 1875. In this period he covered four years of work in Civil Engineering and of the three working years nearly one-half of each was spent, during the construction season, in field work for the Valley Railway Company. Of his work and character at that time, P. H. Dudley, Chief Engineer of the Valley Railway Company, wrote on September 21, 1874, as follows, "His ability, fidelity, and strict integrity are such that they commend him to any one".

After his graduation, Mr. Hoover's first professional employment was as Assistant in an Observatory in Cincinnati, Ohio. This work was very interesting, but not sufficiently lucrative, and in the fall of 1875 he secured employment as Chief Engineer of the Indianapolis Bridge Company. He invested in the Company all the money he had saved from his summer employment in previous years. Within a year, however, the Bridge Company encountered financial difficulties, and Mr. Hoover lost the investment he had made and his employment as well. The effect of this experience was to implant in him a deep-seated and permanent conservatism which resulted in a life of hard work, careful planning, and avoidance, as far as possible, of large hazards.

He returned to Canton and for a time was engaged in designing and building highway bridges. In 1878, he was employed as Engineer by the Wrought

* Memoir prepared by H. P. Treadway, M. Am. Soc. C. E.

Iron Bridge Company, of Canton. He found this work interesting, sufficiently remunerative, and well appreciated, but much too confining, and, in 1884, having made a contract with the Wrought Iron Bridge Company to represent it as General Western Agent, Mr. Hoover moved with his family to Kansas City, Mo.

His contract was such that he was justified in devoting himself to building up a creditable business, which he did through the succeeding years. The facilities of the Company were adapted only to the fabrication of highway bridges and steel viaducts and his field of operations, therefore, followed that line.

In the West at that time there were few engineers engaged in the design of such work. Most of the county engineers were only surveyors and not trained for economical bridge or structural design, and Mr. Hoover's services were often sought by engineers for cities, counties, street railway companies, etc. In the face of severe competition, he built up this business largely through his ability as a designing engineer. He was particularly in demand in the case of structures which were out of the ordinary and which could be properly analyzed and designed only by a man of thorough training and experience.

As time passed his business became more and more that of a builder and less that of an engineer, and his later success was based on two characteristics, tireless attention even to detail and absolute faithfulness to the interests of his customers. These deep-seated characteristics made possible his success in a field marked with the wrecks of many failures.

Mr. Hoover operated continuously under his contract with the Wrought Iron Bridge Company for sixteen years until 1900, when that Company, with about twenty-five other shops, was merged into and became part of the American Bridge Company.

In the next two years, 1901 and 1902, the American Bridge Company passed through a period of re-organization of its various plants and Mr. Hoover continued as its General Agent in a limited territory. That temporary association, however, was not to the best advantage of either party and, in 1902, he became the General Western Agent for the Canton Bridge Company, of Canton.

At some time in 1904 or 1905 he sustained a physical injury which resulted about 1906 in the loss of his sight. After continuing for some time with the hope of recovery, he retired in 1909 as General Agent of the Canton Bridge Company. For a few years more he gave active attention to certain enterprises in which he was interested. As soon as possible, however, he put his affairs in order. During the remainder of his life he attended to his investments and with the aid of his family and others devoted much time to the reading of history and current events, and to music.

He tolerated motor cars when necessary, but preferred his one-horse carriage until it became the single survivor of its kind in Kansas City. He was a Mason, York and Scottish Rite, and a Shriner, a member of the University Club, and of the Unitarian Church, of Kansas City.

Mr. Hoover's strongest characteristic, perhaps, was accuracy. He was intolerant of careless error, of negligence, and of duplicity, but patient with failure where there had been genuine effort. He was faithful to all trusts placed in him. His word was as good as any bond, and he held the complete confidence of all who really knew him. Although naturally cautious, or perhaps because he was cautious and conservative in his general plan of life, he was a good loser in the case of any unexpected, unforeseen disaster. Such occurrences, after analysis and diagnosis, were simply "spilled milk".

His life was one of faithful service to others. It was successful in the matter of financial reward, but the loss of his sight deprived him of the real fruits of his many years of effort.

Mr. Hoover was married on December 30, 1875, to Mary Catherine Ruthrauff, of Stark County, Ohio, who, with a daughter, Helen Hoover Secrest, and a son, Frederick R. Hoover, Assoc. M. Am. Soc. C. E., survives him.

Mr. Hoover was elected a Member of the American Society of Civil Engineers on June 6, 1888.

PHELPS JOHNSON, M. Am. Soc. C. E.*

Died February 20, 1926.

Phelps Johnson was born at Warwick, N. Y., on October 23, 1849. His school and academy training was received at Springfield, Mass., and when seventeen years of age he joined the R. F. Hawkins Iron Works as Draftsman, later becoming Assistant Engineer. In January, 1879, he accepted a position with the Wrought Iron Bridge Company as Assistant Engineer and remained with that Company until he went to Canada in the beginning of 1882 to join the Toronto Bridge Company as Assistant Engineer.

The Dominion Bridge Company, with its principal works at Lachine, Que., Canada, was formed in 1883 by the interests owning the Toronto Bridge Company, and Mr. Johnson then became the Manager and Engineer of the Toronto Works, a position he held until June, 1888, when these works were closed and he moved to the main office as Chief Engineer of the Company. While Manager of the Toronto Works he was also in charge of the erection of the cantilever bridge over the Reversible Falls at St. John, N. B., Canada, and of the St. Lawrence River Bridge at Lachine, both important structures in their time.

Mr. Johnson continued as Chief Engineer of the Dominion Bridge Company until October, 1891, when he was appointed General Manager. In February, 1910, desiring to be relieved from some of the details of management, he was appointed Managing Director, and, in October, 1913, President, a position which he held until January, 1919, when he retired from active work. He was elected a Director of the Company in 1903, and remained a valued Director and Member of the Executive Committee until his death.

It will be seen from his record that Mr. Johnson occupied a leading position in the Dominion Bridge Company from the time of its inception until he retired from active business in 1919. This was the period of Canada's growth in the construction of railways and highways, during which notable bridges were constructed, many of them under his direction. These structures, the success of the Company, and the originality displayed by Mr. Johnson in solving difficult steel construction problems and in devising new erection methods, many of which are now common practice, are witness to the genius of the man.

For many years after the formation of the Dominion Bridge Company there were few consulting engineers in Canada, and Mr. Johnson was called upon frequently for advice in engineering matters both without and within the field of bridge superstructures. He occupied a position of peculiar authority and trust among those who came into professional or business contact with him, and his judgment was highly valued.

The erection of the 1 800-ft. span at Quebec placed the unprecedented problem before engineers of finding a practicable design that could be safely constructed with reasonable economy of plant and preparation. That engineer-

* Memoir prepared by G. H. Duggan, M. Am. Soc. C. E.

ing monument, the Forth Bridge, in Scotland, of practically the same span, had been built with consummate skill, plate by plate at the site, but climatic and industrial conditions in Canada prevented this method from being followed and it was necessary to construct the Quebec Bridge of members completely finished in the shops so that they could be quickly assembled in the field.

Modern traffic conditions also required the Quebec Bridge to be more than twice as heavy as the Forth Bridge, so that members of very large size were needed which would have been exceedingly difficult to handle if the bridge were constructed with any of the ordinary arrangements of web-bracing. Secondary stresses and distortions due to cantilever erection also became formidable considerations.

It was natural that Mr. Johnson should have been consulted and in close touch with the several projects for bridging the St. Lawrence River at Quebec and, although he was not successful in obtaining for his Company the first contract for the bridge, it may be said fairly that his suggestions enabled the Company to obtain the order eventually and successfully construct the bridge.

During the preparation of the competitive designs for tendering on the final construction of the bridge, Mr. Johnson suggested that many of the most serious obstacles would disappear by adopting a new form of web-bracing, making what is now known as the K-truss, but designated by him at the time as "half shear bracing" because of its function of dividing the shears between two members in each panel. It took little investigation to confirm his judgment that this offered the most desirable solution of the problem with decided advantages in economy of material and plant, uniformity in deflections, and safety and ease of erection.

To carry out this great work the St. Lawrence Bridge Company was formed and Mr. Johnson became its President and General Manager, at all times taking a deep interest in the engineering side of the work, particularly in its erection. It was an application on a large scale of the qualities which distinguished his work throughout his career, his practical turn of mind and a wonderful insight into difficult problems of handling material, supplemented by the skill to devise methods for overcoming the difficulties ahead.

Although called to executive positions with his Company he never lost his interest in the engineering side of the work and kept closely in touch with it even after his retirement from the actual direction of affairs.

Mr. Johnson was a bachelor, and led a very simple life, without any home ties. He had few recreations and did not mingle extensively in the social activities of his associates; there was thus nothing to interfere with his dominant interests, that of his profession and of his Company. He was, however, a public-spirited citizen, generously supporting the interests of others. No man in his community has been more revered among his associates and those who came in contact with him in business.

A little more than forty years ago, the writer became associated as a Junior with Mr. Johnson, and the respect which he early formed for his character and ability was never shaken but only deepened in that time. He feels that he owes much to his example and counsel.

Mr. Johnson became a Member of the Canadian Society of Civil Engineers (now the Engineering Institute of Canada) in 1883. He served on its Council for six years, was Vice-President for one year, and President in 1913.

McGill University, Montreal, Que., Canada, conferred on him the honorary degree of Doctor of Laws in 1921, in recognition of his eminence in the Engineering Profession and particularly for the part he played in the construction of the Quebec Bridge.

Mr. Johnson was elected a Member of the American Society of Civil Engineers on July 1, 1891.

WILLIAM PIERSON JUDSON, M. Am. Soc. C. E.*

DIED FEBRUARY 12, 1925.

William Pierson Judson, the son of Col. John Work and Emily (Pierson) Judson, was born at Oswego, N. Y., on May 20, 1849. Both his father and his mother were of American Colonial descent, his mother having been a direct descendant of Abraham Pierson, the first President of Yale College.

He was educated in the public schools of Oswego, having been graduated from the High School in 1866. His education was continued by study and active practice under his father and Capt. Jared A. Smith, Corps of Engineers, U. S. Army, both of whom were graduates of the United States Military Academy, West Point, N. Y.

From 1866 to 1898, Mr. Judson was in active service as a Civilian Engineer with the United States Corps of Engineers on fort, river, and harbor surveys with headquarters at the U. S. Engineer Office at Oswego, as follows:

From 1866 to 1868, he served as Draftsman at Fort Ontario, Oswego, under Captain Smith and Maj. Charles E. Blunt, Corps of Engineers, U. S. A. For the next four years he was engaged as Draftsman in connection with fort and harbor construction on Lake Ontario, and from 1873 to 1874, he served as Surveyor and Draftsman for forts, rivers, and harbors. In 1874, Mr. Judson was appointed U. S. Assistant Engineer in executive charge of the surveys, design, and construction of river and harbor works, lighthouses, and forts on Lake Ontario and the St. Lawrence River, including Lake Champlain; the harbors of Buffalo, N. Y., and Erie, Pa.; the rebuilding and repair of Fort Wayne, Detroit, Mich.; Fort Montgomery, Rouse's Point, N. Y.; Fort Niagara, New York; and many surveys of projected works, including 10 miles of the Narrows of Lake Champlain.

He also made canal surveys and presented reports, as follows: In 1890 he reported on the Niagara Ship Canal, advising a barge canal from Oswego to the Hudson River rather than a ship canal.† In 1896, he made a survey and report for the United States Deep Waterway Commission on "The Oswego-Oneida-Mohawk Valley Route from Oswego to Troy", advising the canalizing of the channels of these rivers.‡ In 1897, he was detailed as Assistant Engineer to Maj. Thomas W. Symons, Corps of Engineers, U. S. A., at Buffalo, and there wrote the body of the "Report on Ship Canal from the Great Lakes to the Hudson River", the construction advised being for barges rather than for ships.§ The report was the basis for the plans for the Barge Canal as it was built by the State of New York (1905-20).

In 1898, Mr. Judson entered private practice and was engaged as Engineer for the Contractor on the construction of three miles of the Erie Canal, at Buffalo.

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* Memoir compiled from information on file at the Headquarters of the Society.

† H. R. No. 283, 52d Cong., 1st Session, 1892.

‡ Report of the Commission, H. R. No. 149, 56th Cong., 1st Session, 1900.

§ H. R. No. 86, 55th Cong., 1st Session, 1897.

From 1899 to 1905, he served as Deputy State Engineer of New York for three terms, having been in executive charge of the organization and direction of the New York State Highway System during its first six years. He also edited the publications of the Department, directed the surveying and monumenting of county lines through the Adirondack Mountains, and co-operated with the U. S. Geological Survey in the survey of the State and the gauging of its streams.

From 1905 until his death, Mr. Judson was engaged in practice as a Consulting Civil and Electrical Engineer. During this time he was also President of the Broadalbin, N. Y., Electric Light and Power Company and the Broadalbin Supply and Construction Company, Incorporated, as well as a Director of the Broadalbin Knitting Company, Limited, and the Broadalbin Bank. He had also served on the Commission on the Varick Water Power Canal, at Oswego, since 1876.

He was married on October 9, 1888, to Mrs. Anna Littlejohn Thompson McWhorter, of Oswego, who survives him.

Mr. Judson was a man upon whom one could rely through all the vicissitudes and experiences of life, true to the end. Whether among those whom he knew well or others, he was recognized as one who possessed the highest conception of integrity and honor. He was devoted to his profession and maintained a deep interest in it as long as he lived. His death was a great loss to those who were privileged to enjoy his friendship.

He was the author of the following books: "From the West and Northwest to the Sea by Way of the Niagara Ship Canal" (1890); "An Enlarged Waterway from the Great Lakes to the Atlantic" (1893); "Lake Ontario to the Hudson River Through the Oswego-Oneida-Mohawk Valley" (1896); "History of Projects from Great Lakes to Tidewater, 1768-1901"; "City Roads and Pavements", editions of which were issued in 1894, 1902, 1906, and 1909; and "Road Preservation and Dust Prevention" (1908). He also wrote many engineering reports and papers.

Mr. Judson was a member of the Institution of Civil Engineers of Great Britain; American Society of Municipal Improvements; Massachusetts Highway Association; International Navigation Congress; American Association for the Advancement of Science; Genealogical and Biographical Society of New York; New York State Historical Association; American Defense Society; Sons of the American Revolution; and the Society of the War of 1812. He was also a member of the following clubs: Fortnightly, City, Country, and Yacht Clubs of Oswego; Antlers Club, of Amsterdam, N. Y.; Fort Orange, and Institute of History and Art, of Albany, N. Y; Lake Placid Club, Lake Placid, N. Y.; and the Authors' Club, of London, England. He was also a Mason, and a member of the Protestant Episcopal Church. In politics he was a Republican.

Mr. Judson was elected a Member of the American Society of Civil Engineers on September 7, 1881.

WILLARD DATUS LOCKWOOD, M. Am. Soc. C. E.*

DIED FEBRUARY 22, 1926.

Willard Datus Lockwood was born at Mt. Kisco, N. Y., on November 29, 1870. He was a son of the late Ezekiel Sturgis and Caroline M. (Ensign) Lockwood. The Lockwood family were early settlers in Connecticut, being mentioned in the history of the State as far back as 1652.

His early life was spent in Rochester, N. Y., where he was educated. He was graduated from the Rochester High School and spent four years in the Rochester Atheneum Mechanics Institute. While in the Institute he acted as Instructor and Lecturer in descriptive geometry and took special courses in structural design, hydraulics, and strength of materials.

From 1888 to 1892, Mr. Lockwood was with the Rochester Bridge and Iron Works as Draftsman and Designer; the following year he became Assistant to the City Engineer of Rochester on the trunk sewer, a rock tunnel 10 miles long, and in 1893 and 1895 served as Engineer in connection with the Rochester Water Supply on the location and construction of 26 miles of steel water pipe.

In 1895 and 1896, he held the position of Assistant Engineer for the State of New York in charge of a division of surveys of State lands, and the following year was Engineer for contracting firms in charge of tunnel work and difficult foundations. During 1898 and 1899, he served as Principal Assistant Engineer with the United States Board of Engineers in the survey and investigation of water supply and power and the location of dams.

Following this work in 1899 and 1900, Mr. Lockwood was with the Isthmian Canal Commission in charge of surveys for the location of the Boca San Carlos Dam and the survey for Graytown Harbor. He was also in charge of estimates of cost for the proposed canal and the design of the harbor terminals.

For a year, in 1901, he was in Ecuador as Engineer on 50 miles of mountain railroad for the Guayaquil and Quito Railway. He again entered the Government service with the United States Corps of Engineers, in charge of fortifications and river and harbor work during 1901 and 1902.

Going to England in 1903, Mr. Lockwood was Engineer and Superintendent for the construction of a large blast furnace at Frodsham, and built a complete American plant for a British Steel Company.

Returning to Rochester in 1904 he became Assistant Engineer and Engineer in Charge of the construction of the Cobbs Hill Reservoir.

Leaving the professional engineering field, he was connected with several large contracting firms in the construction of dams, tunnels, dry docks, and submarine pipe lines in various parts of the United States and also in Santo Domingo, where he was in charge of the construction of concrete piers for the Dominican Government.

In 1913, Mr. Lockwood was made Manager for the Frederick Snare Corporation, Engineers and Contractors, at Philadelphia, Pa., and until 1925 had

* Memoir prepared by E. S. Skillin, Assoc. M. Am. Soc. C. E

charge of all construction in and around Philadelphia for that Company. His work included the construction of several large steamship piers, both substructures and superstructures, and the supervision of a large part of the track installation on the elevated railway structure in Philadelphia. During the World War the construction of the first shipways at the Hog Island Yards and the large Quartermaster's Terminal at Philadelphia was carried out, as well as other important works.

Due to ill health Mr. Lockwood resigned his position with the Frederick Snare Corporation and in 1925, on the advice of his physician, moved to Miami, Fla. During the few months of his residence there he established himself in business, also taking great interest in the various engineering problems in the large developments of Florida, and was elected Vice-President of the Engineers' Club of Miami.

His active life was spent in the various fields of engineering and construction in many places, in the United States, South America, the West Indies, and England. He had a very large acquaintance and many friends among engineers and those connected with engineering matters. He was an untiring worker and never saved himself when his time or energies were needed by his friends or business associates. He was always intensely active and interested in any problems to be solved and had a keen sense of ways and means to overcome unusual and difficult conditions.

By nature extremely sociable, Mr. Lockwood was interested and took an active part in the several societies and organizations of which he was a member, such as the Engineers' Club of Philadelphia, The Philadelphia Board of Trade, and the Manufacturers' Club of Philadelphia. He is survived by his widow and two daughters.

Mr. Lockwood was elected a Junior of the American Society of Civil Engineers on April 3, 1894, an Associate Member on October 7, 1896, and a Member on October 2, 1906.

ROBERT HUGH McCORMICK, M. Am. Soc. C. E.*

DIED FEBRUARY 23, 1925.

Robert Hugh McCormick, a son of David and Ann (McNaughton) McCormick, was born at Detroit, Mich., on May 1, 1853.

Mr. McCormick was educated in the public schools of Detroit and at the Bryant and Stratton Business College. In 1869 he found employment in his father's crockery and glass store. Upon the death of his father in 1872 and the closing of the business, young McCormick entered the Pingree and Smith Shoe Factory as a machine operator.

In the fall of 1878 Mr. McCormick became a Recorder on a Government survey of the Mississippi River. During the summer and fall of 1879 he was in charge of a soundings cutter on the St. Mary's River in connection with a survey for a new Hay Lake channel, made under the direction of the late Alfred Noble, Past-President, Am. Soc. C. E. From the fall of 1879 to the fall of 1880 Mr. McCormick was engaged as Rodman, Leveler, and Assistant Engineer on railroad location and construction in Eastern Michigan, and during the winter of 1880 he located and built twenty-five miles of narrow-gauge railroad for the late Gen. Russell A. Alger. From January, 1882, to July, 1883, Mr. McCormick was a Division Engineer for the Grand Trunk Railway Company on construction between Pontiac and Jackson, Mich.

In July, 1883, Mr. McCormick entered the office of the City Engineer at Detroit, as Assistant Engineer in charge of the Street Department. He became City Engineer in 1898 and continued in that position, except for a period of about one year, until January 1, 1917, when he resigned and retired from active work. The increase in population of Detroit during the time of Mr. McCormick's contact with the office of the City Engineer was from about 150 000 in 1883 to about 800 000 in 1917, or more than 400 per cent.

He was a painstaking public official, possessed of a high sense of his responsibility to the public and to his profession. He was by nature careful and conservative and although quiet of disposition was keenly appreciative of the humorous. Mr. McCormick never married.

Mr. McCormick was elected a Member of the American Society of Civil Engineers on September 7, 1898.

* Memoir prepared by John W. Reid, M. Am. Soc. C. E.

CHARLES LEONARD ROWLAND, M. Am. Soc. C. E.*

DIED APRIL 18, 1926.

Charles Leonard Rowland was born in Brooklyn, N. Y., on November 28, 1852, the son of James and Mary Eliza (Smith) Rowland. While he was still quite young, his father who was a native of Fairfield, Conn., removed his family to that place, and the boy was educated at the public schools there and at Fairfield Academy. He also took a special course in Mechanical Engineering.

For a number of years Mr. Rowland was employed by the Continental Iron Works, in Brooklyn, N. Y., as Mechanical Engineer and in other capacities. His service with this Corporation terminated in 1887 and in the following year he established a similar business for himself—the Morton Iron Works, in Brooklyn—which he continued until 1906.

In 1906 he went to Carbondale, Pa., where in association with the late Alfred P. Trautwein and Mr. Frank A. Dempsey, he founded, and became President of, the American Welding Company, with a plant at Simpson, Pa. This Company specializes in the manufacture of steel products, such as tanks and containers, that require highly developed forge welding processes, and supplies many of the tanks used by the E. I. DuPont Company, the Standard Oil Company, and the Texas Oil Company. Forged welded cylinders, high-pressure tanks, furnaces, flues of large diameter, boiler shells, and other custom machine work are also manufactured and shipped to all parts of the United States. When it was first opened, the plant had only 20 employees, but its growth has been steady and substantial, and when it was sold to the Shippers' Car Line, a subsidiary of the American Car and Foundry Company, on March 6, 1926, it covered 4 acres, with 15 buildings and 225 employees. Mr. Rowland, who made his home at Carbondale, died at the Post-Graduate Hospital in New York, N. Y., on April 18, 1926.

During the World War, he invented and patented a container for chlorine gas, of 1-ton capacity, which was adopted and used by the United States and the Allied Governments. More than 8 000 of these tanks were manufactured during this period, and proved to be a factor in helping to win the war.

On October 24, 1883, Mr. Rowland was married to Alice Lucille Ackerly, of Brooklyn, and on July 20, 1917, to Kathryn Ethel Schweizer, of New York, who survives him.

He was President of the Parkway Improvement Company and the Great Neck Realty Company and a Director of the Manufacturers' Association of New York. He was also a member of the American Society for Testing Materials and the Engineers' Club of New York. He belonged to the Presbyterian Church and was a Republican in politics.

Mr. Rowland was elected a Member of the American Society of Civil Engineers on September 1, 1886.

* Memoir prepared by Frank Orchard, Esq., Works Mgr., American Welding Co., Carbondale, Pa.

OBERLIN SMITH, M. Am. Soc. C. E.*

DIED JULY 18, 1926.

Oberlin Smith, the eldest son of George R. and Salome Kemp Smith, was born in Cincinnati, Ohio, on March 22, 1840. His father was of English parentage, the ancestral home having been at Stoke, Dorsetshire, England, where Mr. Smith's grandfather operated a flour mill and dealt largely in wheat and other grains. The Napoleonic Wars brought heavy losses to the English people and were doubtless responsible for the migration of the family to America between 1830 and 1840.

Oberlin Smith's parents owned and lived on two or three Ohio farms during his early years. The boy, however, had a bent for mechanics and when only fifteen years of age, he built a steam engine designed entirely from descriptions given him by his father. He also constructed toy water-wheels, pumps, and other apparatus, finally attempting a cylinder electrical machine made from bottles of various sizes with holes cut in the bottom with a sharp file. He made frequent visits to neighboring saw-mills and on his return built engines which produced successful results.

About 1857 his father died and his mother, with her three children, removed to Bridgeton, N. J., where relatives were living. Notwithstanding his youth, Oberlin Smith took his father's place, assisting his mother in superintending the education of his brother and sister. He attended the West Jersey Academy and studied engineering at the Polytechnic Institute, Philadelphia, Pa.

While quite young, he entered the machine shop of the Cumberland Nail and Iron Company of Bridgeton and his exceptional mechanical ability was soon recognized. While working in the pipe-mill attached to the plant, he noticed that when the ends of pipe were cut off, it took the time of an additional man to remove the burr on the inside edge caused by the cutting process. He thereupon invented a simple attachment which enabled the operator to cut and ream the pipe at the same operation. The value of this labor-saving device was so apparent, that the Superintendent gave him $50. Mr. Smith frequently declared that no reward subsequently received for his inventions was so gratifying as this unexpected "bonus".

In 1863, he went into business for himself, building a small shop and conducting a general trade in gas and steam-fitting, plumbing, and architectural iron work, such as iron fences, verandas, etc. He also engaged in a general jobbing and repair business, followed by the making of a few sizes of foot-presses for canning factories, a beginning which subsequently led to the specialty of presses and dies as an almost exclusive product.

On January 1, 1864, Mr. J. Burkitt Webb (afterward a Professor at Cornell University and Stevens Institute of Technology) became a partner, under the firm name of Smith and Webb. Mr. Webb was also noted for his originality and with Mr. Smith frequently startled their fellow townsmen. For instance, they built the first automobile ever seen in Southern New Jersey.

* Memoir prepared from information on file at the Headquarters of the Society.

The machine consisted of an old buggy carrying a small boiler and engine that drove the rear axle by means of sprockets. The contrivance was crude, the steam valve, for instance, being operated by a loose socket wrench instead of a hand-wheel. The car traversed the streets of Bridgeton to the delight of the inhabitants, but came to grief when descending a hill, a jolt causing the socket wrench to fall into the street, and there being no brake, it was impossible to stop. The result was disastrous to the car, but fortunately the occupants were not seriously hurt.

The firm of Smith and Webb was dissolved after several years and Mr. Frederick F. Smith, a younger brother, became a partner under the name of Oberlin Smith and Brother. In 1873, the shop and a part of the tools were sold and new buildings were erected in East Bridgeton, N. J., and on January 1, 1877, the Ferracute Machine Company was organized, with Mr. Smith as President, the regular products of presses and dies being supplemented by a number of special automatic machines for the working of metals. These machines were largely abandoned later, in order to concentrate on the development and improvement of presses with their dies and other legitimate attachments.

On September 27, 1903, the buildings of the Company were completely destroyed by fire, together with most of their contents. Plans for new and improved shops were immediately prepared and erection was begun. The new buildings are fireproof and fitted with modern devices, many of them designed by Mr. Smith, all the construction having been subject to his oversight and superintendence which was performed with the energy of a much younger man.

Mr. Smith made several European tours of engineering observation, and, in addition to his wide acquaintance among American engineers, he knew the leading mechanical engineers of Germany and England. He was a frequent contributor to the technical press and, in 1896, published a book entitled "Press-Working of Metals", which had a wide sale. This book covers a field that had been but little exploited to that time.

He had taken out about seventy-five patents for inventions a number of which are connected with the manufacture of presses and other machines for cutting and forming metals. His inventive genius, however, covered a wide field as testified by patents on improvements in automatic egg boilers, pill machines, milk-shakers, a system of gearing for turbines of ocean steamers, etc.

In common with other self-made men, Mr. Smith possessed an unusual amount of perseverance and determination. Nothing gave him so much pleasure as to attack hard problems, especially if his competitors had failed in their solution. In his younger days he started his daily task at an early hour and frequently worked long into the night. In recent years he had become more careful of his health and allowed nothing to interfere with his daily automobile rides. He always took an interest in the mechanical features of the better makes of automobiles of which he had several. Among his inventions is an electrical device by which the pressure of a button at a considerable distance from his garage caused the doors to automatically open and be ready for the car when it arrived.

He was one of the most approachable of men and seemed to enjoy being interrupted, having had the rare faculty of immediately resuming his previous occupation at the point where he left it on the arrival of the visitor.

Mr. Smith served the State of New Jersey as Commissioner to the Pan-American Exposition in 1901 and as a member of the New Jersey Department of Conservation and Development. He was also a Director of the Cumberland National Bank; a member of the Bridgeton Commercial League; a Past-President of the American Society of Mechanical Engineers, and a member of the American Institute of Mining and Metallurgical Engineers; American Institute of Electrical Engineers; American Iron and Steel Institute; American Academy of Political and Social Science; Society for Encouragement of Arts, Manufactures, and Commerce; Franklin Institute, Art Club, and Engineers Club, of Philadelphia, Pa.; Lotus Club and Engineers Club, of New York, N. Y.; Cohanzick Country Club, Bridgeton, and other organizations.

On December 25, 1876, he was married to Charlotte Hill of Bernardston, Mass., who died in 1918. He is survived by his son, Percival N. Smith, a daughter, Madame Velko Roditchevitch, a brother, Fred F. Smith, and a sister, Emily Smith.

Mr. Smith was elected a Member of the American Society of Civil Engineers on September 3, 1884.

ROBERT SOMERVILLE, M. Am. Soc. C. E.*

DIED OCTOBER 19, 1925.

Robert Somerville was born on May 24, 1851, in Richmond, Va., and was reared there, living on Franklin Street, near where the Jefferson Hotel now stands. His early education was received largely at the University School of Richmond, a private school, of which the Assistant Principal was Thomas R. Price, Jr., who was called from this school to be Professor of Mathematics at the University of Virginia and, later, at Columbia University in New York, N. Y. It was the thorough instruction received by Mr. Somerville from this eminent mathematician which lent so much to his success as an engineer.

After the Civil War Mr. Somerville attended the University of Virginia, where he became a member of the Phi Kappa Psi Fraternity, to which Chapter, a few years later, President Wilson was elected. Mr. Somerville did not take any degree at the University of Virginia, but completed the resident work necessary for a Civil Engineer and later would have been entitled to the degree had he applied for it.

On leaving the University of Virginia he was employed by the Chesapeake and Ohio Railroad Company and worked under Maj. Peyton Randolph for several years in the vicinity of Huntington, W. Va. The lack of home life and social advantages in that country caused Mr. Somerville to abandon his profession for a time and he returned to Richmond and formed a partnership with his father in the grain and commission business, under the firm name of R. B. Somerville and Son. He did the bookkeeping and kept all the records for the firm, and his father did all the buying and selling.

While in Richmond and in this business he was married to Mary Lee, a close relative of Gen. Robert E. Lee. They had long been sweethearts and the marriage had been delayed by her ill-health. When she died a year later, Mr. Somerville left Richmond and again entered the engineering field, this time to stay for the remainder of his life.

He was again employed by the Chesapeake and Ohio Railroad Company on the reconnaissance and location surveys for the extension from Huntington to Cincinnati, Ohio. It was while doing this work in Kentucky that he was offered a position with the Board of Mississippi Levee Commissioners. He accepted this offer and went to Greenville, Miss., where he lived until his death on October 19, 1925. With the exception of a short employment with the Georgia Pacific Railway Company (now the Columbus and Greenville Railway Company) on the location of its lines through the Mississippi Delta, Mr. Somerville was steadily employed by the Levee Board for the remainder of his life, and it was in this position that he did his real life work. He was employed as an Assistant Engineer for many years and, finally, the position of Assistant Chief Engineer was created, to which he was promoted. His

* Memoir prepared by C. H. West and W. E. Elam, Members, Am. Soc. C. E.

principal work which will live after him was the keeping of records that show all the work of the Levee Board.

Mr. Somerville was always careful, conscientious, and conservative in all matters coming under his care; and his passion for record keeping gave the Mississippi Levee District the most complete records of any Levee District as far as the writers know. In passing, it might be added that a few years ago one of the writers (Mr. Elam) tried to collect data from all the Levee Districts along the Mississippi River, similar to the information contained in the records of the Mississippi Levee District, and had to abandon it because so many districts could only supply the total expenditures and could not separate the cost of construction from the other expenses, such as overhead, interest, etc. The desired information was what had been spent on actual construction, and in the other districts it could only be estimated. Yet Mr. Somerville, at a time when cost accounting was practically unknown, devised a system of record keeping which was complete enough to supply reliable data when legislation was being urged in Congress for the improvement of the Mississippi River.

Mr. Somerville had a mild and most gentlemanly manner, coupled with firmness when necessary. His devotion to duty will ever be a landmark in the community.

He was prominent in church and fraternal circles. For at least forty years he was an active member of the Board of Stewards of the First Methodist Episcopal Church of Greenville, and he served the church in practically every capacity possible for a layman. In each and every place he was ready, competent, and willing to serve at all times. He was also quite active in Masonic circles, having been a member of the Blue Lodge, Chapter, Council, Commandery, Shrine, and Eastern Star. He served as the highest official of each of these locally, and held many other local offices in the bodies; he also served the Grand Chapter, Royal Arch Masons, as Grand High Priest for the year 1913.

On September 16, 1885, Mr. Somerville was married to Nellie Nugent, daughter of Col. William L. Nugent, formerly of Greenville, but then residing in Jackson, Miss. To this union there were born four children, Robert N., Abe D., Eleanor (Mrs. A. W. Shands), and Lucy R., all of whom survive him. He will be missed by his devoted family and those who came in contact with him in a social and professional way.

Mr. Somerville was elected a Member of the American Society of Civil Engineers on June 1, 1887.

MILTON FREDERICK STEIN, M. Am. Soc. C. E.*

DIED JANUARY 2, 1926.

Milton Frederick Stein, the son of Max and Elvira Stein, was born in Chicago, Ill., on December 17, 1886. He received his preliminary education in the Englewood Schools of Chicago and was graduated from the University of Illinois in 1909 with the degree of Bachelor of Science in Municipal and Sanitary Engineering. In 1916, Mr. Stein received the degree of Civil Engineer. His outstanding scholarship and unusual thesis merited special graduating honors and election to Sigma Xi.

After his graduation, Mr. Stein entered the employ of Chester and Fleming, Consulting Engineers (now The J. N. Chester Engineers), of Pittsburgh, Pa. Under Mr. Chester, he designed a number of reservoirs, pumping stations, water filtration plants, and other structures, at various places, in some instances acting also as Supervisor of construction and operation. Among these municipalities may be mentioned Charleroi, New Kensington, Phœnixville, and Erie, Pa.

In 1913 Mr. Stein was employed by the Water Department of the City of Cleveland, Ohio, where he supervised the preliminary tests for filtration of the water supply of that city. He was in immediate charge of the design of certain parts of the concrete work in the Division Avenue Filtration Plant, and assumed a major responsibility for the design of the filter details, chemical treatment equipment, laboratories, and the clear water reservoir. He also made some special investigations on the recovery of water sludge and its utilization.

In 1917, Mr. Stein left the employ of the City of Cleveland and joined the Engineering Staff of Hering and Gregory, Consulting Engineers, of New York, N. Y., where he remained for about a year, working particularly on the design of the sewerage works at Albany, N. Y. He then returned to Cleveland and was employed by the Division of Sewage Disposal on special studies relating to the Westerly Sewage Treatment Works. In 1917, in order to be at home, Mr. Stein returned to Chicago and became connected with the work of the Sanitary District of Chicago, especially in the design of the Des Plaines activated sludge sewerage works, one of the first large plants of this type. For a short time afterward, he was employed by Pearse, Greeley and Hansen, Hydraulic and Sanitary Engineers, of Chicago, and subsequently, became associated with the International Filter Company as Research Chemist and Engineer. It is understood that in this position he perfected a number of water-works improvements, particularly in rate controllers.

About 1921, owing to failing health, Mr. Stein gave up regular employment and spent his time in special investigation and study. In this connection, he developed and patented with Mr. C. A. Jennings, of Chicago, a fine screen for the removal of suspended solids from sewage. From 1923 until

* Memoir prepared by George B. Gascoigne, M. Am. Soc. C. E.

his death, most of his professional work was done as an associate of the writer as a special designing engineer, particularly on water-works problems.

Mr. Stein was a man of marked ability, very painstaking, and a great student. His contributions to the technical press were frequent and of high caliber. Aside from the magazine articles that appeared from time to time, his best known work is his book entitled "Water Purification Plants and Their Operation", which was first published in 1915, the second edition coming out in 1919, while a third edition, completed just before his death, has since been published. This book, which is of very high order, is considered a standard textbook in its field. Mr. Stein not only had a brilliant and analytical mind, but was extremely versatile. Not only was he at home in the field of sanitary engineering but he liked to delve into unusual and abstruse problems, requiring the most intricate mathematical solutions. Furthermore, he was an artist of no mean ability and an accomplished musician. By nature he was quiet and reserved and there were few who knew him intimately. These recognized in him a true friend as well as a capable engineer. His passing removes from the field of water purification engineering one who, perhaps more than any other, understood and was thoroughly conversant with the design, construction, and operation of such works.

Mr. Stein was elected an Associate Member of the American Society of Civil Engineers on December 3, 1912, and a Member on September 9, 1919.

JOSEPH STRACHAN, M. Am. Soc. C. E.*

DIED MARCH 20, 1926.

Joseph Strachan was born at Newburgh, N. Y., on December 31, 1859. He was a son of James Strachan and Susan (McCullough) Strachan, both of Ulster-Scotch stock.

His earlier education was obtained in the public schools of Brooklyn, N. Y., to which city his family moved when he was a child. In 1879, he entered New York University and was graduated in 1883 with Phi Beta Kappa honors and with the degrees of Bachelor of Science and Civil Engineer; in 1886, the degree of Master of Science was conferred on him by his Alma Mater.

After graduation Mr. Strachan began his professional career by accepting a position on construction work with the South Pennsylvania Railroad Company, where he passed from the grade of Rodman to Chief of a construction party. On this work he spent about two years and then returned to his home in Brooklyn and began work with the Kings County Elevated Railroad Company in December, 1885, on its new elevated structure. For nearly four years he was engaged on elevated and surface railroad construction in Brooklyn, serving as Draftsman, Inspector, and Construction Superintendent. His work during this period embraced the design and construction of trestles, track layouts, coal pockets, conveyors, etc., and included work on the old New York and Bath Beach Railroad, the New York and Sea Beach Railroad, and the Fulton Elevated Railway.

In July, 1889, Mr. Strachan entered the employ of the then City of Brooklyn, N. Y., as an Assistant Engineer in the Department of City Works, Bureau of Water-Works Extension. In this capacity he prepared detail plans of masonry for the extension of the water mains east of Rockville Center, Long Island, designed changes of structures on the conduit line, made surveys, detail plans, and estimates for the relief of Baiseley's Pond from Jamaica sewage, and surveys and estimates for proposed changes in the grades of streets in Brooklyn.

In 1894 and 1895, he conducted the work of strengthening a part of the brick conduit near Liberty Avenue, Brooklyn, designed the connections of a proposed 66-in. steel pipe conduit with an existing conduit and the Milburn Pumping Station, and had charge of land purchases along the line of the conduit. From 1896 to 1900, he was engaged on the design of various pieces of work along the aqueduct, including the location survey for a proposed 66-in. steel pipe conduit from Milburn to Spring Creek; he superintended the layout and construction of the Oconee and Shetucket deep-well stations and of the change of the Springfield Pond Pumping Station to a deep-well station.

In 1901 and 1902, he had charge of the construction of a double pipe line of 48-in. cast-iron pipe from the Milburn Engine House to the Milburn Efflux Chambers.

* Memoir prepared by Willis R. Tenney, M. Am. Soc. C. E.

From November, 1902, until his retirement from municipal service on January 31, 1926, Mr. Strachan was a Division Engineer in the Bureau of Highways, Borough of Brooklyn, New York, N. Y. For a considerable part of this time he was in charge of construction work, supervising the grading, paving, and repaving of many miles of streets. During his connection with street work he saw modern pavements of asphalt and granite on concrete foundation increase from about 95 miles to more than 870 miles in the Borough of Brooklyn, and the disappearance of the old cobble-stone and Belgian block pavements to the extent of 260 miles. In this work he had a large part, not only in the supervision of construction but in the preparation of the plans and specifications, in the study of local traffic conditions, and in the collection of data relating to improved methods in highway construction in various other cities.

Mr. Strachan was a Charter Member and Past-President of the Brooklyn Engineers' Club and had served as its Secretary for more than twenty-three years, from January, 1903, until his death. During his incumbency of this office the Club grew rapidly, increasing in membership from about 200 to nearly 500, a result due in a very large measure to his efforts and his active interest in everything pertaining to it. He was also a Charter Member of the Brooklyn University Club, a member of the Zeta Psi Fraternity and of the Zeta Psi Club of New York.

As Secretary of the Brooklyn Engineers' Club he arranged, in 1914, the first Industrial Exposition held in the Borough of Brooklyn. This has now become a regular feature of the life of the city.

In 1888, Mr. Strachan was married to Mary Elizabeth Griswold, of Brooklyn, who died in 1921. A daughter, Louise, and two sons, Kenneth and Malcolm, survive him.

In August, 1925, he was compelled to give up the active practice of his profession, owing to heart trouble. From that date he gradually became weaker until his death on March 20, 1926.

Mr. Strachan was a man of exemplary habits, of a patient and kindly disposition, and one who had the respect of his associates and subordinates. His courteous manner, his willingness to assist others, and his consideration for the feelings of those about him made of him a companion whose loss will be a personal one to hosts of those who were fortunate in having had the opportunity of associating with him.

Mr. Strachan was elected an Associate Member of the American Society of Civil Engineers on May 4, 1892, and a Member on May 4, 1898.

WILLIAM ANDREW THOMPSON, M. Am. Soc. C. E.*

DIED DECEMBER 15, 1925.

William Andrew Thompson was born at Greenwich, N. Y., on December 16, 1853. He was the third son of Andrew A. and Eliza (Stevens) Thompson. His great-grandparents emigrated to this country from the north of Ireland in early Colonial days and both parents had several ancestors who served in the War of the Revolution.

After pursuing his studies in the local schools, he entered the University of Vermont and was graduated as Civil Engineer in the Class of 1877.

Immediately after his graduation Mr. Thompson sought employment with the United States Engineer Corps then being organized for improving the Upper Mississippi River from St. Paul, Minn., to the mouth of the Missouri River. From July, 1878, to March, 1879, he was in charge of surveys, and from March to December, 1879, served as Inspector in charge of contract work on the improvement of the river.

From August, 1880, to 1925, he was employed continuously on various engineering works for improving the Upper Mississippi, among which were the following: In 1887 he was in responsible charge of the construction of a breakwater in Lake Pepin, near Lake City, Minn., and in 1888 rebuilt the breakwater in Lake Pepin near Stockholm, Wis. From May, 1896, to 1925, he was in charge, as Assistant Engineer, of the river improvements from Winona, Minn., to the mouth of the Wisconsin River, a distance of 96 miles. In this period he built another breakwater at Kings Coulee in Lake Pepin and planned and executed the river improvement works between Winona and the Wisconsin River, the first section to attain the full projected depth of 6 ft. at extreme low water. For this work he was greatly complimented by his superior officers.

Mr. Thompson was the first to advocate and use large quantities of brush in constructing the training works used to confine the channel. This method resulted in great economies in construction and in cost to the Government.

His Headquarters' Office at La Crosse, Wis., established in 1895, was relinquished on December 1, 1925, as the improvement work in that section was completed, and the office force was transferred to St. Paul, Minn. Mr. Thompson died at his home in La Crosse, on December 15, 1925.

It is a curious coincidence that the date of the completion of his immediate work, of his retirement from service under the Civil Service Retirement Act, and of his death, all occurred on December 15, and that the anniversary of his birth was December 16. He was a man who spent a long life under one employer—the United States Government—and in one class of work, who attained the highest position to which he could aspire in that service and who was contented with the modest emolument allowed his position, but who gave his whole heart to his work and died respected and loved by his friends and associates.

* Memoir prepared by M. Meigs, M. Am. Soc. C. E.

Mr. Thompson was of a highly sociable nature and enjoyed the company of his many friends. He took a great interest in the civic affairs of his home town. He was a member of the La Crosse Club and the La Crosse Country Club.

His age prevented him from participating actively in the World War, but he gave a son to the service, Capt. Alexander M. Thompson, who went overseas and is now head of the La Crosse Dredging Company, at Minneapolis, Minn. Mr. Thompson took an intense and patriotic interest in the war and was active in promoting the sale of Liberty Bonds, serving on committees for that purpose.

On March 10, 1885, he was married to Myra Arntzen, of Quincy, Ill., the daughter of the Hon. Bernard Arntzen. His widow and son survive him.

Mr. Thompson was elected a Member of the American Society of Civil Engineers on October 7, 1896.

REUBEN JOSEPH WOOD, M. Am. Soc. C. E.*

DIED OCTOBER 15, 1925.

Reuben Joseph Wood was born at Kansas City, Mo., on June 19, 1884. He was the only child of William and Josephine Wood. He received his early education in the public schools of San Francisco, Calif., which city had been his home since 1885. In 1903, he entered the University of California and was graduated with the degree of Bachelor of Science in Civil Engineering in 1907.

After his graduation Mr. Wood was employed for about two years with the Southern Pacific Railroad Company, designing steel and reinforced concrete structures. Following this engagement he entered the contracting business, specializing in building work.

In 1911, he joined the Staff of the City Engineer of San Francisco as an Assistant Engineer. From this position he advanced to that of Consulting Structural Engineer, in which capacity he served until his death.

During his employment in the City Engineer's Office, Mr. Wood was intimately connected with the design of the structural elements of many important public improvements, such as the high-pressure fire system, which included some large reinforced concrete tanks and a great number of cisterns; the Municipal Railway with its reinforced concrete car barns and trolley poles; the Stockton Street and Twin Peaks Tunnels and Stations; the Hetch Hetchy Project, including the Moccasin Power House and the steel towers on the transmission line, and the O'Shaughnessy, Priest, Lake Eleanor, and Early Intake Dams.

Although never of robust health, Mr. Wood possessed to a high degree those qualities of energy, perseverance, resourcefulness, and loyalty which won recognition for him in the engineering service of the City. Intensely interested in civil engineering, he found time to acquire a large store of accurate information in the broader field of life. The quality of his work as an engineer, the ability he displayed, and his steadfast devotion to his profession gave promise of a useful and honorable career had his life been prolonged.

He was unmarried and made his home with his mother, by whom he is survived.

Mr. Wood was elected a Member of the American Society of Civil Engineers on August 31, 1925.

* Memoir prepared by Nelson A. Eckart and L. H. Nishkian, Members, Am. Soc. C. E.

CARL DONALD BOSSERT, Assoc. M. Am. Soc. C. E.*

DIED SEPTEMBER 12, 1925.

Carl Donald Bossert was born in Washingtonville, Ohio, on May 3, 1884. He received his early education in the local grade and high schools and the Leetonia High School. He then entered Ohio State University and was graduated with the Class of 1908 with the degree of Civil Engineer.

During the summer vacations and immediately after his graduation from college Mr. Bossert was with the Pennsylvania Railroad Lines West of Pittsburgh, as Instrumentman and Draftsman on location, working in Eastern Ohio. In 1909, he resigned to accept a position with the United States Government, laying out Government lands in Minnesota. He was also stationed at San Francisco, Calif., and from there went to the Navy Yard at Charleston, S. C., as Instrumentman.

Upon the completion of his work at the Charleston Navy Yard in 1910, he again engaged in railway work as Instrumentman on the location of the Sandy Valley and Elkhorn Railroad, in Eastern Kentucky.

In May, 1911, Mr. Bossert returned to Eastern Ohio to become Deputy County Surveyor of Columbiana County, where he had charge of the location and construction of several important roads, with bridges and drainage. He held this position until April, 1915, when he transferred to Wayne and Holmes Counties, Ohio, as Deputy Surveyor, which position he held until the World War. While in this position, he worked on the design of the 8 000-acre drainage project for these counties.

In October, 1918, he was commissioned First Lieutenant of Engineers, U. S. Army, and was about to go over seas when the Armistice was signed. He was discharged from the Army in January, 1919.

From March, 1919, to January, 1923, Mr. Bossert served as Bridge Engineer in Franklin County, Ohio, working on preliminary surveys and plans for the Greenlawn Avenue Bridge and the changes in design of the Mound Street Bridge, at Columbus. He had complete charge during the construction of the latter bridge.

In January, 1923, he entered the employ of Carl B. Harrop, Engineers and Constructors, as Construction Engineer and, later, became Superintendent of Construction. This position took him into the ceramic field in different parts of the United States in which as in all former enterprises he was highly successful.

Mr. Bossert ranked high as a student. possessed a keen mind, and had the faculty of combining theoretical and practical engineering knowledge to a high degree. He was well liked among his fellow workers and, although a hard worker, always had time to help a younger engineer. He was an engineer in every sense of the word and was always very active in technical

* Memoir prepared by C. R. Oberst, Esq., Columbus, Ohio.

societies. Courteous, unassuming, and self-retiring, he held the esteem of all who knew him.

He was a member of the American Association of Engineers, an Associate Member of the American Ceramic Society, and a member of the Engineers' Club of Columbus. He was a member of the Masonic Fraternity and had taken all the degrees possible, having been a Knight Templar, Scottish Rite Mason, and Shriner. He was also a member of Acacia, National Collegiate Fraternity.

Mr. Bossert was elected a Junior of the American Society of Civil Engineers on October 31, 1911, and an Associate Member on December 6, 1915.

THOMAS PATRICK O'SHAUGHNESSY, Assoc. M. Am. Soc. C. E.*

DIED JULY 11, 1924.

Thomas Patrick O'Shaughnessy was born in San Francisco, Calif., on July 21, 1893, the son of Michael and Anna E. O'Shaughnessy. He was the eldest of four children and was educated in San Francisco and Oakland, Calif., receiving the degree of Bachelor of Science from St. Mary's College, in Oakland, in 1915.

Following his graduation Mr. O'Shaughnessy worked as Assistant Engineer for the California Highway Commission until November, 1917, when he entered the U. S. Army as a Cadet Pilot in the Air Service of the Signal Corps. He was in this service for more than a year, after which he was engaged with the Bureau of Public Roads of the U. S. Department of Agriculture until July, 1919. He then re-entered the service of the California Highway Commission, as Assistant Engineer of Construction.

In July, 1921, Mr. O'Shaughnessy became Concrete Inspector on the Hetch Hetchy Dam for the City of San Francisco, in charge of the pouring of precast work and, later, in charge of the night pouring at the dam. After almost a year in this position he was made Resident Engineer for the Tule and Baxter Creek Irrigation Districts in Lassen County.

This memoir must of necessity be brief, for the life of "Tom" O'Shaughnessy was a short one, his death occurring a few days before his thirty-first birthday. During his few years of active work he made countless friends, with whom he has left ineradicable memories of a dauntless spirit of youth, coupled with a keen sense of responsibility and judgment, which made it possible to employ him in positions of trust. His was a valiant, high-hearted nature, and it is hard to think that he is no more a part of the actual life of his associates, that henceforward he will be only a happy memory. His friends are happy to have loved him, happy that he has been happy with them, happy to have been "a port where he hath fitted himself for what sea he saileth to."

One of his fellow engineers on the Hetch-Hetchy work writes of him:

"To those who knew him best, Tom's dominant characteristic was a warm heart coupled with a fearless and independent mental attitude which was an inspiration to his associates. He was a fine fellow and I loved him."

He had a "way with him" which endeared him to all, old and young, and, because his work was for the most part in mountainous country where Nature is at its strongest and most beautiful, his associates will look for him in all that is most beautiful and strong:

"In the sunlight of a morning in June, in the soft green grass that carpets the uplands in May, in the sturdy oak whose head is unbowed by the winds of a hundred winters, in the mellowed radiance of the harvest moon, in all that we love in Nature we will look for our departed friend."

Mr. O'Shaughnessy was elected an Associate Member of the American Society of Civil Engineers on May 28, 1923.

* Memoir prepared by a Committee of the San Francisco Section, Am. Soc. C. E.

CLARENCE OWEN SHIVELY, Assoc. M. Am. Soc. C. E.*

DIED MARCH 25, 1926.

Clarence Owen Shively was born at Dayton, Ohio, on August 14, 1890. He was the son of John D. and Elizabeth (Brubaker) Shively.

His early education was obtained in the public schools of Dayton. In September, 1911, he entered Purdue University, Lafayette, Ind., and in June, 1915, received the degree of Bachelor of Science in Civil Engineering. During his University course he spent two summers on topographic and sanitary engineering and one on transmission line work in Dayton.

Mr. Shively was a good student and was actively interested in student affairs. He was a member of Delta Tau Delta Fraternity and of the Student Council, and was also Fraternity Editor of *The Débris,* the Year Book published by the Seniors at Purdue University.

Immediately after graduation he entered the employ of the Miami Conservancy District. For three years he was engaged as Draftsman, and, later, as Chief Draftsman, in the preparation of topographic maps and other general drawings for the District in connection with the development of the plans for the Miami Valley Flood Protection System. In October, 1918, Mr. Shively was assigned to the construction work of the Germantown Dam as Assistant Division Engineer, and continued on this unit of the project until its completion in the summer of 1920. He was then transferred to a similar position at the Taylorsville Dam, where he remained until November, 1921. Throughout his connection with the work of the Miami Conservancy District, Mr. Shively won and retained the friendship and good will of all his associates, and the work under his supervision was carried through in a most commendable manner.

In November, 1921, he was appointed Instructor in Civil Engineering at Purdue University and, in June, 1923, he was promoted to be Assistant Professor, the position which he held at the time of his death. He was advancing rapidly toward maturity of judgment in his professional field and had spent the summer of 1925 in making studies and designs of storm and sanitary sewers for the City of Dayton. He died on March 25, 1926, of double pneumonia after a short illness.

At Purdue, Professor Shively's work was mainly in Hydraulic and Sanitary Engineering. On account of his sympathetic contact with students, his ability to organize and carry out instruction, he was placed in charge of the administration of the course in Engineering Problems for Freshman Civil Engineers. He always maintained the most cordial relations with his students and at the same time commanded their respect and maintained high standards in his class work. His unfailing courtesy and genuine spirit of helpfulness will long be remembered by Faculty and Alumni.

* Memoir prepared by W. K. Hatt and R. B. Wiley, Members, Am. Soc. C. E., and C. S. Bennett, Assoc. M. Am. Soc. C. E.

Professor Shively was actively interested in the affairs of the Society, having given generously of his time to the Purdue University Student Chapter. Only two weeks before his death he delivered an excellent illustrated address before the Chapter on the "Construction of Dams".

In 1916, he was married to Katherine E. Faber who, with two sons, Richard Faber and Harold Owen, survives him.

Professor Shively was elected an Associate Member of the American Society of Civil Engineers on April 19, 1920.

CLARK LUZERNE WILCOX, Assoc. M. Am. Soc. C. E.*

DIED MARCH 11, 1926.

Clark Luzerne Wilcox was born in Mexico, Oswego County, N. Y., on March 4, 1878. He was the son of Luzerne Wilcox and Mary Wilcox, deceased. He received his education in the public schools of his native town and at Cornell University, from which he was graduated in Civil Engineering with the Class of 1901.

Immediately after his graduation Mr. Wilcox was employed as Draftsman by the American Bridge Company for several months, following which he was engaged as Engineer Office Assistant for nine months at the Bureau of Filtration of the City of Philadelphia, Pa., on experimental work at the Schuylkill Testing Station. For two years, beginning in March, 1902, he was with the firm of Waring, Chapman, and Farquhar, New York, N. Y., on sanitary engineering projects at Pensacola, Fla., Athol, Mass., and Larchmont, N. Y., with which firm—the members of which were pioneer engineers in sanitary design—were engaged.

In May, 1904, Mr. Wilcox removed to Pittsburgh, Pa., where he resided continuously until his death, almost 22 years. His first engagement was with the Bureau of Filtration of the City of Pittsburgh, as Designing and Constructing Engineer on various parts of the filtration plant at Aspinwall, Pa.

In 1906 he became associated with The Pitt Construction Company, Inc., as one of the organizers, and acted as Treasurer of the Company until his death. Aside from his duties as Treasurer, he supervised the construction of many important projects, among which were the Conneaut Viaduct, a reinforced concrete, 7-span, arch-ribbed structure, 90 ft. high and 1 500 ft. long; the mechanical equipment of the Filtration Plant at Buffalo, N. Y., consisting of forty 4 000 000-gal. filters; and the Longbridge Viaduct on the Lincoln Highway near Ligonier, Pa.

In 1911, Mr. Wilcox was married to Corbly Wood, of Pittsburgh, who survives him. He is also survived by his father and one brother, Perley S. Wilcox, of Kingsport, Tenn.

He was active in civic and church affairs, and was a Mason, a member of the Mexico, N. Y., Lodge, and the Duquesne Commandery and Syria Shrine of Pittsburgh. Among other affiliations were the American Water Works Association, Engineers Society of Western Pennsylvania, Associated General Contractors of America, and the University Club of Pittsburgh.

Mr. Wilcox was a man of the highest character. Being of a quiet and modest nature, a full appreciation of his many exceptional qualities was enjoyed only by his close friends and associates.

* Memoir prepared by E. D. Harshbarger, Assoc. M. Am. Soc. C. E.

In addition to his many admirable personal characteristics, Mr. Wilcox was noted for his thoroughness in every detail of his undertakings. His influence and example was strongly impressed on his associates and his death leaves a gap which will be hard to fill.

Mr. Wilcox was elected a Junior of the American Society of Civil Engineers on December 3, 1901, and an Associate Member on March 1, 1910.

EDWARD JAMES CONNOR, Affiliate, Am. Soc. C. E.*

DIED DECEMBER 9, 1925.

Edward James Connor was born in New York, N. Y., on October 26, 1877. He received his early education at the Hoboken, N. J., High School, from which he was graduated with honors. He received a scholarship at Stevens Institute, Hoboken, but was unable to accept it on account of his father's death.

In 1896, he entered the office of Mr. William H. Jenks, a dock building contractor in New York, as Draftsman and, later, as Superintendent of Construction, which position he retained until 1909, when the business was dissolved after the death of the owner. During the period from 1896 to 1901, Mr. Connor took a night course in Civil Engineering at Cooper Union. He continued his studies in later years, until he became a proficient engineer, capable in surveying, designing, and supervising construction.

Between 1909 and 1925 he was associated with the Deuville Artificial Stone Company and John Monks and Sons, both in New York, and also with Barth S. Cronin, Incorporated, of Brooklyn, N. Y. For five years previous to and at the time of his death he was in the employ of Allen N. Spooner and Son, Incorporated.

Mr. Connor was of genial disposition with keen ability, remarkable for accurate details, especially competent in the design of piers, wharves, and sheds, besides his ability in supervising construction. A long and varied experience in harbor engineering gave him knowledge, that was of value to a constructor —such knowledge as can be obtained only after many years of hard study and work. He therefore has left a vacancy not easy to fill. He was beloved by his associates in business, by his wife and family, and commanded a high respect of his subordinates. He leaves a widow and two sisters.

Mr. Connor was elected an Affiliate of the American Society of Civil Engineers on January 31, 1911.

* Memoir prepared by Allen N. Spooner, M. Am. Soc. C. E.

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OF THE

AMERICAN SOCIETY OF CIVIL ENGINEERS

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